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**Author**
Hall, L.J.

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# Towards A Theory of Quark and Lepton Masses

L.J. Hall

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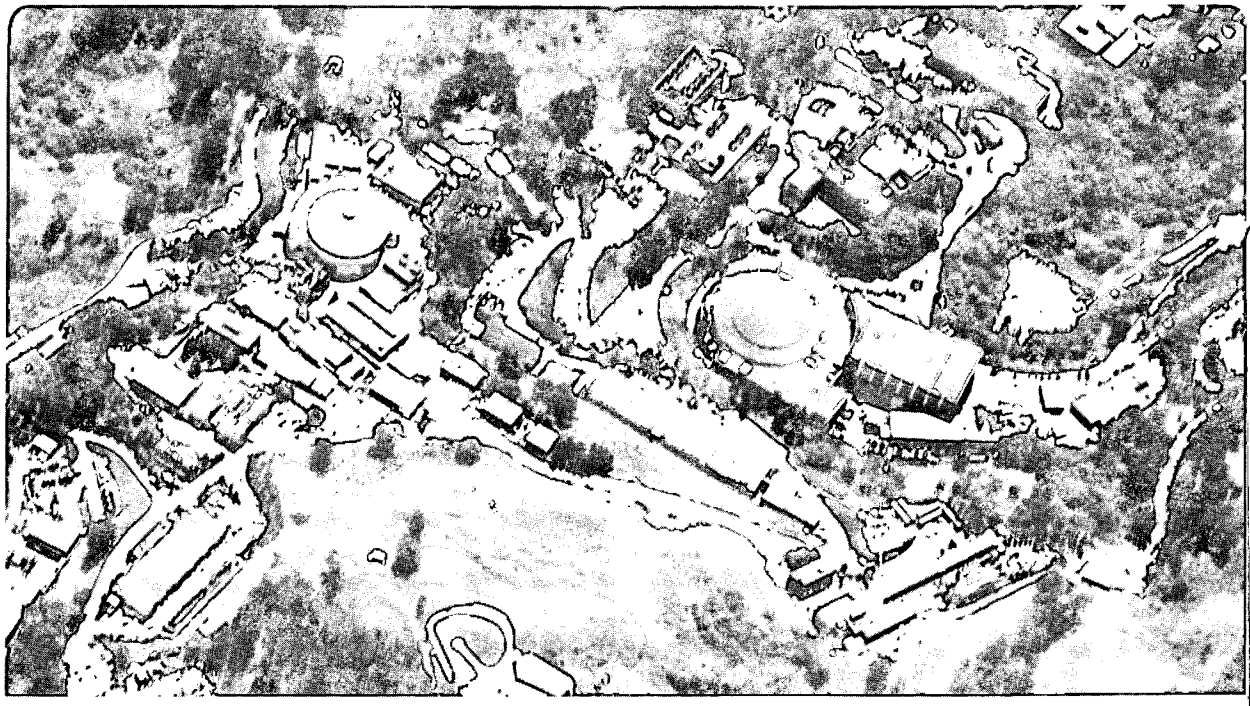



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# Towards A Theory Of Quark and Lepton Masses. * †

L. J. Hall

Theoretical Physics Group
Lawrence Berkeley Laboratory
University of California
Berkeley, CA 94720

## Abstract

Has any progress been made on understanding and predicting the 13 parameters which describe the observed masses and mixing angles of the quarks and leptons? Arguments are given in favor of pursuing schemes in which grand unified and family symmetries provide many relations among these 13 parameters. A sequence of simple assumptions leads to a supersymmetric SO(10) theory with 8 predictions: $\tan\beta, m_t, V_{cb}, m_s, m_s/m_d$, $m_u/m_d, V_{ub}$ and the amount of CP violation $J$. These predictions are presented, together with experiments which will test them.

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## Disclaimer

A major component of experimental high energy physics is the measurement of the masses of the quarks and leptons and their couplings to the $W$ boson. There is no mystery about why this is so: we are interested in learning the fundamental parameters of the standard model, and 13 out of 18 of these correspond to quark and lepton masses and mixing. I am not trying to minimize the importance of the 5 parameters of the gauge and Higgs sectors, which can be taken as $\alpha, \alpha_s, M_Z, G_F$ and $M_H$; but it is a simple fact that the majority of the fundamental parameters belong to the flavor sector. These 13 parameters consist of 9 masses: for the up-type quarks $m_u, m_c$ and $m_t$, the down-type quarks $m_d, m_s$ and $m_b$, and the charged leptons $m_e, m_\mu$ and $m_\tau$; and the 4 independent parameters of the Kobayashi-Maskawa (KM) mixing matrix, which I take as the Cabbibo angle $\sin\theta_c = |V_{us}|$, $|V_{cb}|$, $|V_{ub}|$ and the parameter $J$ which describes the amount of CP violation in the KM matrix.

Each of the 18 fundamental parameters is represented in the standard model by a coupling constant. I do not know how to construct a fundamental theory and perform a first principles calculation of these coupling constants. Does this mean I have no hope of making predictions? **No.** It is always possible to obtain predictions by *reducing the number of free parameters.* The Balmer formula provides an illustration of this. A large number of observables (the hydrogenic spectral wavelengths) are described by a single free parameter (the Rydberg constant). Twenty eight years after this incredibly successful formula was written down, it was understood by Bohr; indeed, his atomic model gave a theoretical prediction for the Rydberg constant, $R = 2\pi^2 m Z^2 e^4/h^3$. This crowning achievement was the birth of the quantum theory of atomic structure. It may well be that a predictive scheme for fermion masses, depending on far fewer than the 13 flavor couplings of the standard model, is a prerequisite for the development of a fundamental theory of fermion masses.

It is interesting to recall in a little more detail how the development of atomic theory and quantum mechanics grew out of studies of spectral wavelengths, and to compare this evolution with the spectroscopy of today: that



of quark and lepton masses. I would argue that the development proceeds from the experimental measurements of the phenomena, first to a recognition of regularities amongst the measurements, then to the physical insight which gives some understanding of these regularities and finally to a fundamental theory, which allows the totality of the phenomena to be understood from a few general principles. The spectral lines of hydrogen were first measured accurately, to 1 part in $10^4$ for four of the lines of the Balmer series, by Angstrom in 1860. A simple regularity in this series of wavelengths was noticed by Balmer in 1885, which he described by the formula $\lambda = C\frac{n^2}{n^2+4}$ where n is an integer and C a constant. Few would disagree that the great leap of physical insight came from Bohr, with his atomic model of 1913. This provided a picture of what was going on in terms of discrete energy levels, together with a derivation of Balmer's formula in the now more familiar form $\nu = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ and a theoretical prediction for the one parameter $R$ in terms of $\alpha$ and $m_e$. Over the next fifteen years this led to the development of quantum mechanics, a radical new foundation underlying all physics.

At what stage of the development process do we stand today with regard to quark and lepton spectroscopy? I would guess that if you asked this to a cross-section of particle physicists, most would say that we are somewhere between Angstrom (1860) and Balmer (1885). We have some reasonable data, but essentially no understanding of the regularities or of the underlying theory. This may indeed be the situation. Attempts to predict the quark and lepton masses in gauge theories began in 1972 [1], immediately after these theories were shown to be renormalizable (ie predictive), and there have been a variety of approaches, each with an interesting history. Some schemes have been very ambitious, suggesting an origin for fermion masses very different than the description provided by the standard model. Two such examples are extended technicolor [2] and string theory [3]; however, despite considerable effort, it is not known whether these ideas are consistent with the observed masses, and they are certainly very far from providing predictive relations that can be tested. The same criticism cannot quite be leveled at

the scheme known as top condensates [4] as this does predict the top quark and Higgs boson masses. Nevertheless, while it would be interesting to find $m_t$ near 230 GeV and $m_H$ near 260 GeV, it would hardly be persuasive, because these results correspond to infrared fixed points and are therefore quite insensitive to the underlying physics at high mass scales. There are other ideas, such as gauged generation symmetries and radiative hierarchies, which are very well motivated, but which again have not led to concrete accurate predictions which can be experimentally tested. One can therefore argue quite persuasively that, not only are we far from having a theory of fermion masses, but many avenues are open precisely because the regularities of the quark and lepton masses have yet to be found.

In this talk I would like to argue the case for an alternative viewpoint: that we can already see and understand some of the regularities. This viewpoint may be completely mistaken, but it should be taken seriously because it is the only direction which has provided a sufficient number of accurate predictions to qualify as being "testable". This direction is the one of *parameter reduction obtained by imposing symmetries.* If this viewpoint is correct, our stage of development is somewhere between Balmer (1885) and Bohr (1913), perhaps even close to Bohr.

The regularities are not embodied in a simple formula of the Balmer type, but in the framework of todays tools of theoretical physics: symmetries. In fact, such successful predictions have only been obtained by the combination of three very different types of symmetry. The first, grand unified gauge symmetry is both very elegant and very powerful. It allows relations between the up, down and lepton masses. The second, family symmetry, is also very powerful leading to a substantial parameter reduction, however at the moment it is very ad hoc and is the weak link in the chain. The final symmetry, supersymmetry, actually leads to an increase in the number of parameters, but is apparently required by data since otherwise many of the predictions are not correct. Although there is a simple group theoretic understanding of the Yukawa coupling structure, these simple regularities are not immediately



manifest in the observed masses because the Yukawa couplings are modified by calculable dynamical effects.

There is an experimental hint that the above viewpoint of parameter reduction in the flavor sector is worth pursuing: progress has been made in reducing the number of parameters in the gauge sector. In grand unified theories (GUTs) the three independent gauge couplings become related [5]. This implies predictions for the weak scale gauge couplings $g_i(M_W)$, i = 1,2,3 of the form [6]:

$$g_i(M_W) = C_i\, g_G\, \eta_i \qquad (1)$$

where $g_G$ is the GUT gauge coupling, $C_i$ are numerical group theory constants and the $\eta_i$, which are radiative corrections computed with the renormalization group, depend on mass ratios such as $M_W/M_G$, where $M_G$ is the GUT scale. How many predictions occur in the gauge sector of GUTs? While the $C_i$ are purely numerical group theory constants, the $\eta_i$ depend on ratios of various mass scales. If there are two or more mass ratios on which the $\eta_i$ depend, then there are no predictions: together with $g_G$ there are three or more free parameters for the three standard model parameters $g_i$. The only hope is for the maximally predictive possibility that the $\eta_i$ depend only on the single mass ratio $M_W/M_G$, in which case there will be one prediction, usually chosen to be the weak mixing angle $\sin^2\theta$.

There are many possible GUTs which have no new scale other than $M_G$. How many different predictions for $\sin^2\theta$ can they give? The answer is basically just two: .211 without supersymmetry and .233 with weak-scale supersymmetry [7]. What is the accuracy of these predictions? There are threshold corrections from GUT [8], Planck [9], and weak scales [10] which are typically around .002. Since the standard model is consistent with any value of $\sin^2\theta$ from 0 to 1, I think that it is very significant that the minimal supersymmetric scheme predicts precisely the experimental value of .233 $\pm$ .001. Many people shrug this off, pointing out that it is just one number. However *it is the only significant prediction* of any of the 18 parameters of

the standard model, and hence I take it as a valuable indication from experiment that these theories are worth pursuing further. In particular it lends support for two of the three symmetries which we will use to obtain flavor predictions: the grand unified gauge symmetry and supersymmetry.

The successful prediction of $\sin^2\theta$ resulted from requiring a larger symmetry than required by experiment. It is well known that this same enlargement of the gauge symmetry can also yield predictions in the flavor sector. Flavor observables at the weak scale, $F_a(M_W)$, can be given by relations of the form

$$F_a(M_W) = C_a\, F_Q\, \eta_a \qquad (2)$$

where $C_a$ are again purely numerical group theory constants, while the dynamical factors $\eta_a$ depend on several parameters, including $\alpha_s$ and mass ratios such as $M_W/M_Q$. $F_Q$ represents the set of independent flavor parameters of the GUT. Clearly a predictive theory must have fewer such parameters than the 13 flavor parameters of the standard model. The first such prediction in GUTs was for $m_b/m_\tau$[11]. However, we now know that in this case $\eta_a$ depends on $m_t$ and $\alpha_s$, leading to uncertainties of 30% and 10% respectively. Hence this successful prediction is much less significant than $\sin^2\theta$, especially as one successful prediction out of so many flavor parameters is not convincing. The first successful prediction of type (2) following from family symmetries was $\sin\theta_c = \sqrt{m_d/m_s}$ [12]. While successful, this is again a single relation at the 10% accuracy level.

What level of significance can be expected in general from these type of flavor predictions? This is determined by the experimental uncertainties, both of the predicted quantities and of the inputs used to determine the free parameters of the theory. For example if the muon mass could be predicted with only the electron mass needed as input, then the significance would be extremely high. However, nobody is even close to being able to do this. In fact the best we are able to do is to use the six most accurately measured flavor parameters as inputs: $m_e, m_\mu$ and $m_\tau$ are known at the 1 in $10^3$ level or better, the Cabibbo angle to 1%, and $m_c$ and $m_b$ to 5 - 10%. In addition, to



calculate the dynamical effects one needs to know the strong gauge coupling, which is known at the 10% level. Hence in this case the level of significance of the predictions is dominated by how well the predicted quantity is known, and this varies from around 15% to 60%. The crucial lesson is that no single prediction of this sort can possibly be very significant. *The only hope that this approach will lead to significant successes is if there are a large number of predictions.* Imposing grand unified, family and super-symmetries still allows a vast number of possible theories. How are we to decide which such theories to study? My answer is that we will simply study those which offer the hope of obtaining the largest number of predictions within a simple set of assumptions. We are hoping that nature is kind to us and that the flavor sector of the GUT depends on only a very few parameters. While this 'principle of maximal predictivity' could be criticized as arbitrary, I would argue that either nature is kind or the approach is not worth pursuing.

The power of combining family and GUT symmetries was realized by Georgi and Jarlskog [13] who wrote down a simple pattern for the Yukawa coupling matrices at the GUT scale. This led to relations of the form of equation (2) allowing successful predictions for all down type quark masses: $m_d, m_s$ and $m_b$. Harvey, Ramond and Reiss [14] showed how to obtain this same pattern in the context of an SO(10) GUT. They found that such a scheme violated CP, led to a prediction for $m_t$ from $V_{cb}$ and allowed predictions to be made in the neutrino sector as well. More recently Dimopoulos, Hall and Raby [15] showed that this Georgi-Jarlskog pattern, when used in a supersymmetric theory, was consistent with everything we know about the flavor parameters. In addition to $m_d, m_s, m_b$ and $m_t$, we found that the form of the KM matrix allows $|V_{ub}/V_{cb}|$ and $J$ to be computed. We were astonished to find that these two predictions and the top quark mass prediction were all successful. While it is a relatively simple matter to construct a theory which gets any one or two of these relations it is very non-trivial to get all six simultaneously. Despite the success of this framework I will not discuss the predictions further. In the rest of this talk I describe a very different class of

SO(10) theories which have just 6 flavor parameters and are therefore even more predictive [16].

The following assumptions are used to define this class of theories:

- The gauge group is SO(10). This is the smallest gauge group that allows an entire family to be described by a single irreducible representation. Thus the three families are written as $16_i$ with i=1,2,3 and $16_3$ being the heaviest family. Perhaps the most elegant feature of SO(10) is the way in which all the measured gauge charges of the fermions can be simply understood in terms of this 16 dimensional spinor.

- The GUT is supersymmetric. Below the grand unification scale we take the theory to be the minimal supersymmetric standard model, as this is the unique minimal possibility for obtaining the successful $\sin^2\theta$ prediction.

- The two low energy Higgs doublets of this theory lie in a single 10 dimensional representation of SO(10). This is the unique minimal possibility.

- The masses of the heavy generation ($m_t, m_b$ and $m_\tau$) come from a single renormalizable operator

$$A\ 16_3\ 10\ 16_3 \tag{3}$$

where 10 is the multiplet containing the light doublets. This elegant picture of the unification of the Yukawa couplings $\lambda_t$, $\lambda_b$ and $\lambda_\tau$ is reminiscent of the unification of the three gauge couplings $g_1, g_2$ and $g_3$ and is due to Ananthanarayan, Lazarides and Shafi [17, 18].

- All the masses of the quarks and leptons of the lightest two generations, and the mixing angles of the KM matrix, are entirely due to non-renormalizable operators which give masses suppressed compared to those from (3) by powers of $M_G/M_P$ where $M_P$ is the Planck scale. Thus the mass hierarchy between generations and the smallness of the KM angles is to be understood in terms of powers of $M_G/M_P$. We study those models with the fewest such operators required for consistency with the known masses and mixings.



- These non-renormalizable operators have the form

$$O_{ij} \equiv 16_i \frac{45_1}{M_1} \cdots \frac{45_k}{M_k} 10 \frac{45_{k+1}}{M_{k+1}} \cdots \frac{45_\ell}{M_\ell} 16_j \tag{4}$$

The mass terms result when the various 45 dimensional adjoint representations acquire vacuum expectation values (vevs) of order $M_G$.

- Simple relations amongst the masses in the up, down and electron sectors follow because each 45 vev lies in a definite direction in the SO(10) group space: in the hypercharge, $B-L$, $T_{3R}$ or X direction, where X preserves an SU(5) subgroup. To see this, recall that when a 45 vev acts on a fermion in a 16, it gives a numerical "Clebsch", which is the charge of the fermion under the particular group generator corresponding to the direction of this vev. It is the SO(10) group theory "Clebschs" which allow an understanding of the regularities of the fermion mass matrix [19].

- SO(10) may be broken to SU(5) by a vev of $45_X$ at a larger scale than SU(5) is broken. This means that the objects appearing in the denominators in (4) can be $< 45_X >$ as well masses of order $M_P$.

At least two operators of the form (4) are needed in order to give all quark and leptons a mass. Two such operators, together with (3), allow the $3 \times 3$ Yukawa matrices to have non-zero determinants. However the coefficients of these three operators can all be made real by rotating the phases of the three $16_i$ fields. Hence this case is excluded because the CP violation in the KM matrix, $J$, vanishes.

The most predictive theories of this sort therefore have three operators of type (4) in addition to the operator (3). Now only three of the four operator

coefficients can be made real, so that there are five independent GUT flavor parameters. In addition, the quark and lepton masses depend on $\tan\beta$, the ratio of the two Higgs doublet vevs, so there are a total of six independent flavor parameters. We choose to determine these from the six best measured flavor parameters: $m_e, m_\mu, m_\tau, \theta_c, m_c$ and $m_b$. Hence the theory predicts $\tan\beta$ and the seven standard model flavor parameters $m_t, V_{cb}, m_s, m_d, m_u, V_{ub}$ and $J$. We are currently performing a numerical search for all successful theories of this type, and while the search has not yet been completed, we already know that there is a unique favored class of models. In this talk I will discuss only this favored class, which is selected by the additional requirement that

- there must be a natural understanding of why $m_c/m_t \approx V_{cb}^2 \ll m_s/m_b \approx m_\mu/m_\tau$ for quantities renormalized at the GUT scale.

A lengthy but straightforward argument shows that the set of assumptions marked above by bullets leads to Yukawa coupling matrices renormalized at the GUT scale of the form:

$$\mathbf{U} = \begin{pmatrix} 0 & \frac{1}{27}C & 0 \\ \frac{1}{27}C & 0 & x_u B \\ 0 & x'_u B & A \end{pmatrix}$$

$$\mathbf{D} = \begin{pmatrix} 0 & C & 0 \\ C & Ee^{i\phi'} & x_d B \\ 0 & x'_d B & A \end{pmatrix}$$

$$\mathbf{E} = \begin{pmatrix} 0 & C & 0 \\ C & 3Ee^{i\phi'} & x_e B \\ 0 & x'_e B & A \end{pmatrix} \qquad (5)$$

where A occurs in (3), and $B, C$ and $Ee^{i\phi'}$ are proportional to the coefficients of the three non-renormalizable operators of type (4), which must be chosen to contribute to the 23, 12, and 22 entries of the matrices respectively. Notice that while operator (3) yields $U_{33} = D_{33} = E_{33}$, a similar equality is not found



for the non-renormalizable contributions. For these the 45 vevs introduce simple numerical Clebsch factors: $U_{22} : D_{22} : E_{22} = 0 : 1 : 3$ and $U_{12} : D_{12} : E_{12} = 1 : 27 : 27$. The 22 entry is infact the one similarity of this scheme with the Georgi-Jarlskog pattern. While we have proved that there is a unique successful Clebsch ratio for the 12, 21, and 22 entries, the 23 and 32 cases are quite different. Several Clebsch ratios are possible and we have parameterized these discrete possibilities by $x_i$ and $x'_i$ in eq. (5). Infact the low energy predictions depend on only two combinations of these Clebsch parameters.

To demonstrate the power of these theories I will write down the analytic formulas for the eight predictions. The predictions follow from relations of the form of eq. (2). A technical problem is that the dynamical renormalization group factors, $\eta_a$, depend not only on $\alpha_s$, but also on the third generation Yukawa parameter $A$. Hence the determination of $A$, and of the $\eta_a$, is a non-linear problem, which has no analytic solution. Of course A and $\eta_a$ can be numerically computed with good accuracy. Hence I will give the predictions in terms of the 6 input parameters, $A$ and $\eta_a$ and one should simply remember that $A$ and $\eta_a$ are understood to be computed numerically from the inputs.

Since the predictions are obtained from relations of the type of eq. (2) with the GUT parameters $F_G$ determined from the inputs, the predictions take the form

$$\begin{pmatrix}\text{predicted}\\ \text{quantity}\end{pmatrix} = \begin{pmatrix}\text{group theory}\\ \text{GUT Clebsch}\end{pmatrix}\begin{pmatrix}\text{input}\\ \text{parameter}\end{pmatrix}\begin{pmatrix}\text{dynamical}\\ \text{RG factor}\end{pmatrix} \qquad (6)$$

The eight predictions are as follows. The ratio of electroweak vevs $\tan\beta$ is obtained from

$$\cos\beta = \frac{\sqrt{2}m_\tau}{v}\frac{\eta_1}{A} \qquad (P1)$$

where $v = 247$ GeV. The top quark mass parameter is

$$m_t = \tan\beta\, m_\tau\, \eta_2 \qquad (P2)$$

with $\beta$ determined from eq. (P1). These two predictions follow from just operator (3) for the heaviest generation and in these cases the GUT Clebsch

factor is unity. The mixing between the two heaviest generations is given by

$$V_{cb} = \chi\sqrt{\frac{m_c}{m_t}}\,\eta_3 \qquad (P3)$$

Thus the GUT relation $V_{cb} = \sqrt{\frac{m_s}{m_b}}$ [14, 15, 20] is modified by a Clebsch factor $\chi$, which we discuss below. Whenever $m_t$ appears on the right-hand side of a prediction, it is understood that the value given by (P2) is to be used. The strange mass is given by

$$m_s = \frac{1}{3}(1+\delta)\frac{m_b}{m_\tau}m_\mu\,\eta_4 \qquad (P4)$$

which is one of the Georgi-Jarlskog relations, except for a small correction $1+\delta$ which we discuss below. From the determinant of **D** and **E** one finds

$$\frac{m_s}{m_d} = \frac{1}{9}(1+2\delta)\frac{m_\mu}{m_e} \qquad (P5)$$

which is a small modification of the second Georgi-Jarlskog relation. In this prediction the renormalization factors cancel. The prediction for $m_u/m_d$ follows from the determinants of the Yukawa matrices, with $m_s$ substituted from (P4):

$$\frac{m_u}{m_d} = \frac{1}{3^7}(1+\delta)\frac{m_\mu}{m_\tau}\frac{m_t^2}{m_c m_b}\eta_8 \qquad (P6)$$

The last two predictions are for parameters of the KM matrix. Diagonalization of **U** and **D** yield a KM matrix of the form

$$V = \begin{pmatrix} c_1c_2 - s_1s_2e^{-i\phi} & s_1 + c_1s_2e^{-i\phi} & s_2s_3 \\ -c_1s_2 - s_1e^{-i\phi} & c_1c_2c_2e^{-i\phi} - s_1s_2 & c_2s_3 \\ s_1s_3 & -c_1s_3 & c_3e^{i\phi} \end{pmatrix} \qquad (7)$$

where $s_1$, $s_2$ and $\phi$ are renormalization group invariants, and $s_3 = V_{cb}$ has a simple scaling behaviour. The CP violating phase $\phi$ is derived from, but not identical to, the phase $\phi'$ of eq. (5). The angles $s_1$ and $s_2$ are given by

$$s_1 = \sqrt{\frac{m_d}{m_s}} \qquad (8)$$



$$s_2 = \sqrt{\frac{m_u}{m_c}} \qquad (9)$$

and $\phi$ is determined from the Cabibbo angle via

$$\sin\theta_c = |V_{us}| = |s_1 + c_1s_2e^{-i\phi}| \qquad (10)$$

The two quantities of **V** which are predicted are

$$\left|\frac{V_{ub}}{V_{cb}}\right| = s_2 \qquad (11)$$

and the amount of CP violation

$$J = s_1s_2s_3^2s_\phi \qquad (12)$$

It is very interesting to note that equations (7) - (12) also hold in the Georgi-Jarlskog scheme [15]. Indeed it has recently been shown that the successful predictions (11) and (12) follow from very simple assumptions about the form of **U** and **D** [21]. However, the class of theories under study here is much more predictive and makes specific predictions for $\sqrt{\frac{m_d}{m_s}}$ and $\sqrt{\frac{m_u}{m_c}}$ which can then be substituted in eq. (8) and (9). The prediction for $\sqrt{\frac{m_d}{m_s}}$ is obtained from (P5) while the prediction for $\sqrt{\frac{m_u}{m_c}}$ gives

$$\left|\frac{V_{ub}}{V_{cb}}\right| = s_2 = \frac{1}{27}\frac{m_e^{1/2}m_\mu^{1/2}}{m_\tau}\frac{m_t}{m_c}\eta_7 \qquad (P7)$$

The final prediction is for the amount of CP violation in the KM matrix obtained by using the above expressions for $s_1, s_2$ and $s_3 = V_{cb}$ in eq. (12)

$$J = \frac{\chi^2}{9}(1-\delta)\frac{m_e}{m_\tau}s_\phi\,\eta_8 \qquad (P8)$$

where $\phi$ is obtained from (10).

The class of models under discussion does not have a unique operator contributing to the 23 and 32 entries of the Yukawa matrices. This is reflected in (5) by the appearance of the Clebschs $x_i$ and $x_i'$ which can assume a set

of discrete values. Nevertheless all models of this class lead to the above 8 predictions and the only dependence on these Clebschs is through the two parameters

$$\chi = \frac{x_u - x_d}{\sqrt{x_u x'_u}} \tag{13}$$

which only enters the relation for $V_{cb}$ and

$$\delta = \frac{x_e x'_e - 3x_d x'_d}{x_u x'_u} \frac{m_\tau m_c}{m_\mu m_t} \eta_\delta \tag{14}$$

The prediction (P3) impies that the only theoretically allowed values of $\chi$ which are experimentally acceptable are: $\chi = 2/3, 5/6$ and $8/9$. The case $\chi = 1$ [14, 15, 20] is disfavored in the present theories which contain operator (3) because the resulting values for $V_{cb}$ are uncomfortably large. For all models of interest $\delta \ll 1$ and hence the $\delta$ dependence of $m_s$ in (P4) and of $J$ in (P8) is much less than the experimental uncertainties on these quantities, and can be dropped. The more interesting effects of $\delta$ are to be found in (P5) and in (P6), where they give small modifications to the ratios $m_u/m_d$ and $m_s/m_d$.

In summary our eight predictions for $\beta, m_t, V_{cb}, m_s, m_s/m_d, m_u/m_d, V_{ub}/V_{cb}$ and $J$ are given in (P1) - (P8). They all agree with present experimental values, and the predictions are sufficiently accurate that future experiments will provide critical tests of these theories. The most important advances which can test our scheme via predictions (P1) - (P8) are

- A measurement of $m_t$.
- A high statistics study of semi-leptonic B meson decay to measure $V_{cb}$. In addition, better theoretical understanding of this matrix element is required, which looks likely in view of recent developments in heavy quark effective field theory.
- A measurement of the CP violating decays of neutral B mesons, which will test our predictions for the KM matrix.



- A better theoretical understanding of the values for $m_u/m_d$ and $m_s/m_d$ implied by experiment.

There are two obvious objections to the above scheme:

1) While there are only six independent continuous flavor parameters, there are millions of operators of the form of (4), and therefore there are extra discrete variables: the Clebschs. If Clebschs can be found to fit any values of the standard model parameters, then there is no significance to our results.

Infact we find the set of possible Clebschs to be very coarse-grained. As Clebschs are varied from one set to the next set, the value of a predicted quantity is found to jump by amounts typically much larger than its experimental error bar, hence successful predictions are significant. A case where this is not true is the prediction (P3) for $V_{cb}$. In this case the experimental error bar is of order the interval generated by successive possible values of the Clebsch $\chi$. A modest decrease in the experimental error bar will simply serve to choose one of the three presently allowed values of $\chi$.

2) Our scheme is based on a large number (9) of assumptions, suggesting that it is unlikely to be the one chosen by nature.

My response to this is mixed. It may well be that the "zeroth order" assumption is wrong and that this whole approach to fermion masses is incorrect. However, I have argued that this is the only known approach which yields predictions of any significance which can be compared to experiment. Given that this approach is worth pursuing, I would argue that the set of 9 assumptions which we have made is *the simplest* that leads to models of such *high predictivity*. There are undoubtedly more complicated sets of assumptions, and obviously there are less predictive theories, but *without major additions to the basic tools it is unlikely that there is a simpler, more predictive model.* The success of the predictions gives me optimism that nature may have chosen the very simplest direction.

SO(10) grand unified theories offer the hope that neutrino masses can

be predicted once a sufficiently simple flavor sector has been written down. This is because both left and right-handed neutrinos lie in the same 16 dimensional spinor representation as the quarks and charged leptons. Actually, it is the neutrino mass *ratios* and the leptonic mixing angles which can be accurately determined. The overall neutrino mass scale involves knowing the scale of lepton number violation responsible for the right-handed neutrino Majorana masses, and this cannot be determined from charged lepton or quark masses. The implementation of the Georgi-Jarlskog ansatz in SO(10) can yield specific forms for the neutrino mass matrices [14]. Two very specific forms predict all the mass ratios and mixing angles as shown in the Table [22]:

| | I | II |
|---|---|---|
| $\theta_{e\mu}$ | $(6.5 \pm .3)10^{-2}$ | $.15 \pm .04$ |
| $\theta_{\mu\tau}$ | $.081 \pm .008$ | $-.027 \pm .003$ |
| $\theta_{e\tau}$ | $(5.7 \pm .6)10^{-4}$ | $(1.9 \pm 0.2)10^{-4}$ |
| $m_{\nu_\tau}/m_{\nu_\mu}$ | $208 \pm 42$ | $1870 \pm 370$ |
| $m_{\nu_\mu}/m_{\nu_e}$ | $(3.1 \pm 1.0)10^3$ | $38 \pm 12$ |
| $m_{\nu_\tau mass}$ | 2.5 eV | 710 eV |

**Table 1**

While extra assumptions, beyond those of Georgi and Jarlskog, are required to obtain these numbers, the very fact that such precise predictions can be made is an important result in itself. The best hope for testing these neutrino masses is offered by searches for $\nu_\mu\nu_\tau$ oscillations by the CHORUS and NOMAD experiments at CERN and by P803 at Fermilab [23]. The SO(10) scheme which leads to predictions (P1) - (P8) cannot be directly used to predict quantities of the neutrino sector. Substantial modifications are required, and these are presently being studied.



Extraordinary effort is involved in measuring the 18 parameters of the standard model. Why bother? Two answers are frequently given:

- because they are there and they are fundamental.
- By measuring them more accurately, via a variety of methods, one could uncover inconsistencies in the standard model which would indicate new physics.

While both of these arguments have considerable merit, a third reason is also important:

- the accurate determination of the 18 parameters of the standard model may lead us to a deeper understanding of particle physics: we may be led to a predictive theory behind the standard model in the same way that atomic spectra were crucial in pointing the way to the Bohr model and to quantum mechanics.

I have argued that we have all the symmetry tools we need to construct predictive theories of fermion masses. Should this direction be correct, does this mean we have no need of a revolution in the underlying theory? Quite the reverse: parameter reduction gets us far along the road, but it cannot be the whole story. Eventually we do need a new framework to address such questions as why the symmetries are what they are and why the free parameters (there are always some) take the observed values. However, it may be possible to go very far down the road guided by experiment, and unexpected features of the underlying theory may only then become apparent.

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LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
TECHNICAL INFORMATION DEPARTMENT
BERKELEY, CALIFORNIA 94720