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G. I. Ghandour and R. G. Moorhouse

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DISCLAIMER

FIXED-t DISPERSION RELATIONS AND POLARIZATION IN PION NUCLEON CHARGE EXCHANGE SCATTERING*

G. I. Ghandour†

Department of Physics and Lawrence Berkeley Laboratory
University of California, Berkeley, California 94720

and

R. G. Moorhouse

Glasgow University, Glasgow, Scotland

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ABSTRACT

We evaluate the real parts of the pion nucleon charge exchange amplitudes using fixed-t dispersion relations with the imaginary parts of the amplitudes found at low energy from partial wave analysis and at high energy from a simple s-channel absorptive model. We find that the calculated polarization, though quantitatively somewhat small, has all the correct qualitative features.

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In a dual absorptive model for dips in inelastic hadron processes, Harari [1] has hypothesized that while the imaginary amplitudes of such processes are peripheral the real parts are not necessarily so, but should rather be derived from the imaginary part through analyticity. In particular the model uses the assumption of Bessel function-like t behavior of the imaginary part together with the assumption of low energy onset of the asymptotic (Regge) phase condition relating the real to the imaginary part, to explain dip phenomena. However, in the explanation of the dips, the real part of the s-channel helicity nonflip amplitudes, $\mathrm{Re}\, f_{\Delta\lambda=0}(s,t)$, are not critical, and so their features are left largely undetermined. In this context Ringland and Roy [2] have recently used the polarization data to evaluate $\mathrm{Re}\, f_{\Delta\lambda=0}(s,t)$ for the reaction $\pi^- p \to \pi^0 n$ at 8 GeV/c, finding a markedly nonperipheral nature for this amplitude.

In this dual absorption model the consistent explanation of the dips assumes that the $\rho - A_2 - f^0 - \omega$ Regge phase holds for the spin flip, $f_{\Delta\lambda=1}$, amplitude. In $\pi^- p \to \pi^0 n$ we know from the nonzero polarization that this cannot also be true for the spin nonflip, $f_{\Delta\lambda=0}$, amplitude. In this model we extend the application of analyticity by evaluating $\mathrm{Re}\, f_{\Delta\lambda=0}(s,t)$ for $\pi^- p \to \pi^0 n$ using fixed-t dispersion relations; our evaluation gives a phase for $f_{\Delta\lambda=0}(s,t)$ unequal to the Regge phase, and a qualitative agreement with the polarization and the Ringland and Roy (experimental) value of $\mathrm{Re}\, f_{\Delta\lambda=0}(s,t)$.

We use the unsubtracted fixed-t dispersion relations for the invariant amplitudes, $A'^{(-)}(s,t)$ and $B^{(-)}(s,t)$, of charge exchange pion-nucleon scattering [3], to determine $\mathrm{Re}\, f_{\Delta\lambda=0}$ and $\mathrm{Re}\, f_{\Delta\lambda=1}$ from the input imaginary parts, $\mathrm{Im}\, f_{\Delta\lambda=0}$ and $\mathrm{Im}\, f_{\Delta\lambda=1}$. At pion laboratory momenta below 2.8 GeV/c we determine the imaginary amplitudes from pion-nucleon elastic partial wave analysis [4] and above that momenta we take the simple form

$$\mathrm{Im}\, f_{\Delta\lambda=0}(\nu,t) = \beta_0(t)\, \nu^{\alpha(t)} \tag{1a}$$

$$\mathrm{Im}\, f_{\Delta\lambda=1}(\nu,t) = \beta_1(t)\, \nu^{\alpha(t)} \tag{1b}$$

where $\alpha(t)$ is the ρ-meson Regge trajectory, taken as

$$\alpha(t) = 0.55 + t\,. \tag{2}$$

In the dual absorptive model of Harari [1], $\beta_0(t) \sim J_0[R(-t)^{\frac{1}{2}}]$ and $\beta_1(t) \sim J_1[R(-t)^{\frac{1}{2}}]$; also the energy dependence $\nu^{\alpha(t)}$ is implicit in that model for the dominant spin flip amplitude, (1b), since the asymptotic phase relation with phase angle $\frac{1}{2}\pi\,\alpha(t)$ is crucial to the explanation of the dip phenomena. Here we suggest that the simple energy dependence $\nu^{\alpha(t)}$ is also appropriate for the imaginary part of the spin nonflip amplitude, though, at the energies we evaluate, the asymptotic phase relation is not. This is consistent with duality enforcing energy dependence appropriate to exchange degenerate trajectories on the imaginary parts of amplitudes.

We follow Ringland and Roy [2] in using the following explicit forms for the residue functions of equations (1):

$$\beta_0(t) = B_0 \exp[B_0'\, t]\, J_0[R(-t)^{\frac{1}{2}}] \tag{3a}$$

$$\beta_1(t) = B_1 \exp[B_1'(t + 0.21)]\, J_1[R(-t)^{\frac{1}{2}}] \tag{3b}$$

with their phenomenologically determined values of

$$R = 5.2\ \mathrm{GeV}^{-1} \approx 1\ \mathrm{fm}$$

$$B_0 = -34.5(\mu b)^{\frac{1}{2}} \qquad B_0' = 1.3\ \mathrm{GeV}^{-2}$$

$$B_1 = 58.7(\mu b)^{\frac{1}{2}} \qquad B_1' = 1.6\ \mathrm{GeV}^{-2}\,. \tag{4}$$

The value of B_0' comes from without the πN system, being from an estimate by Davier and Harari [5] from $K^{\pm}p$ cross sectional differences. (For a variation of B_0' of ±20% our results for the polarization at its maximum vary by ~ ±13%.) In fig. 1 we show, for $t = -0.4$ as a typical example, Im A'(s,t) as calculated from (3) and (4), which is used in the dispersion relation for $p_{lab} > 2.8$ GeV/c, and Im A'(s,t) as calculated from the partial wave analysis for $p_{lab} < 2.8$ GeV/c. (We have varied the value of p_{lab} below which we use the phase shifts to lower values in the range 2 to 2.8 GeV/c and we find no qualitative difference in our results.) We show some results from the dispersion relations using (3) and (4) as input in figs. 2 and 3. Figure 2a shows our dispersion relation calculated $\mathrm{Re}\, f_{\Delta\lambda=0}$ at 8 GeV/c compared with the real part found using the asymptotic phase relation: $\mathrm{Re}\, f_{\Delta\lambda=0} = \tan\frac{1}{2}\pi\alpha(t)\ \mathrm{Im}\, f_{\Delta\lambda=0}$. Also shown in fig. 2a are the values of $\mathrm{Re}\, f_{\Delta\lambda=0}$ found by Ringland and Roy [2] from the measured polarization [6] using the form (3) for $\mathrm{Im}\, f_{\Delta\lambda}$ <u>and</u> the asymptotic phase relation for the spin flip amplitude: $\mathrm{Re}\, f_{\Delta\lambda=1} = \tan\frac{1}{2}\pi\alpha(t)\ \mathrm{Im}\, f_{\Delta\lambda=1}$. We see that our calculated spin nonflip real part disagrees with that found from the asymptotic phase relation for the spin nonflip amplitude, but agrees rather well with the values found by Ringland and Roy. There is a contrast with fig. 2b which shows the real part of the spin flip amplitude as calculated from the dispersion relation to be in much better agreement with the real part as found from the asymptotic phase relation.

In figs. 3a and 3b we show our predicted polarization compared with experiment [6] at 8 GeV/c and 5 GeV/c respectively. We see that there is qualitative agreement between the experimental features and the prediction of the dispersion relation; we already expect such a qualitative agreement from the qualitative agreement with $\text{Re}\, f_{\Delta\lambda=0}$ calculated from the polarization experiments. The maxima in the calculated polarization at 5, 8, 11, and 18 GeV/c are respectively 0.43, 0.26, 0.20, and 0.10; the decrease with energy of the polarization is expected in our calculation since analyticity ensures that both amplitudes have the Regge phase for large enough energies. The polarization appears to be too small and this is probably associated with the curve of fig. 2a for $\text{Re}\, f_{\Delta\lambda=0}$ being still somewhat too high near $t = -0.3$. More extensive and more accurate pion nucleon elastic partial wave information and a modification of the energy dependence or other parameters of the high energy form (3a) of $\text{Im}\, f_{\Delta\lambda=0}$ in the energy region up to ~ 6 GeV/c might correct this situation. In this connection we emphasize the qualitative nature of our remarks and the fact that no parameters are adjusted to obtain a better fit.

In fig. 3c we show our predicted differential cross section compared with experiment [7] at 3.67, 5.74, and 13.3 GeV/c respectively. The agreement is rather satisfactory up to $|t|$ corresponding to the dip.

We have also evaluated $\text{Im}\, f_{\Delta\lambda=0}$ and $\text{Im}\, f_{\Delta\lambda=1}$ using the lowest moment FESR [8] and the πN partial waves and have used this result, instead of (3), in the dispersion relation to calculate

Re f. The resulting polarization is nearly zero. This result may be expected, since imposing the lowest moment FESR without a high enough cutoff amounts to imposing* the same (Regge) phase upon the amplitudes $f_{\Delta\lambda=0}$ and $f_{\Delta\lambda=1}$. A similar approach was used for the $K^- n \to \pi^- \Lambda$ and $\pi^+ n \to K^+ \Lambda$ reactions by R. D. Field [9], but his purpose was not the same as ours.

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* Re A as calculated by fixed-t dispersion relations differs from $\text{Re}\, A_c$ as given by the Regge pole ρ by

$$-\frac{2}{\pi\nu}\int_0^N d\nu'\, \text{Im}[A(\nu') - A_\rho(\nu')] + O\left(\frac{1}{\nu^3}\right);$$ and this quantity is set equal to zero by the lowest moment FESR calculation of the residue in A_ρ.

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FIGURE CAPTIONS

Fig. 1. The imaginary part of the charge exchange invariant amplitude, $A'^{(-)}(s,t)$, for $t = -0.4$: (i) ——— from πN partial wave analysis; (ii) – – – the high energy form calculated from eqns. (1) - (4) of the text.

Fig. 2. (a) Re $f_{\Delta\lambda=0}$ plotted as a function of t at pion laboratory momentum 8 GeV/c: (i) ——— calculated from the fixed-t dispersion relation; (ii) - - - calculated using the asymptotic phase relation $\mathrm{Re}\, f = \tan \frac{1}{2}\pi\alpha(t)\, \mathrm{Im}\, f$; (iii) ⫯ as found by Ringland and Roy [2] from the observed polarization.

(b) Similar plot to fig. 2a for Re $f_{\Delta\lambda=1}$.

Fig. 3. (a) Polarization as a function of t at 8 GeV/c calculated using fixed-t dispersion relations is shown by the curve; the points are experimental [6] values.

(b) Calculated and experimental polarization as a function of t at 5 GeV/c shown as in (3a).

(c) Differential cross sections as a function of t at 3.67, 5.74, and 13.3 GeV/c, calculated using fixed-t dispersion relation are shown by the curve, the points are experimental [7] values.

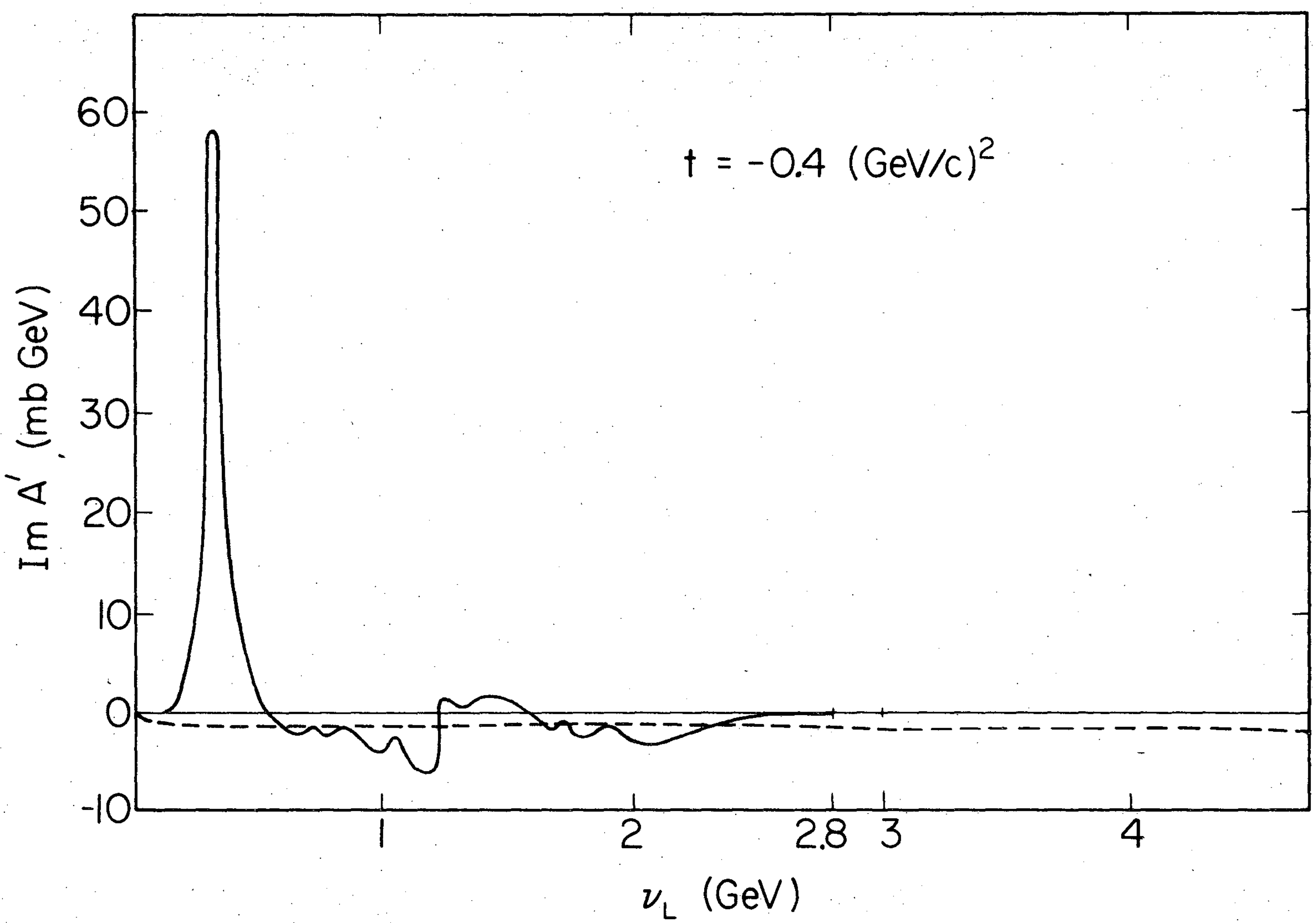

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Fig. 1

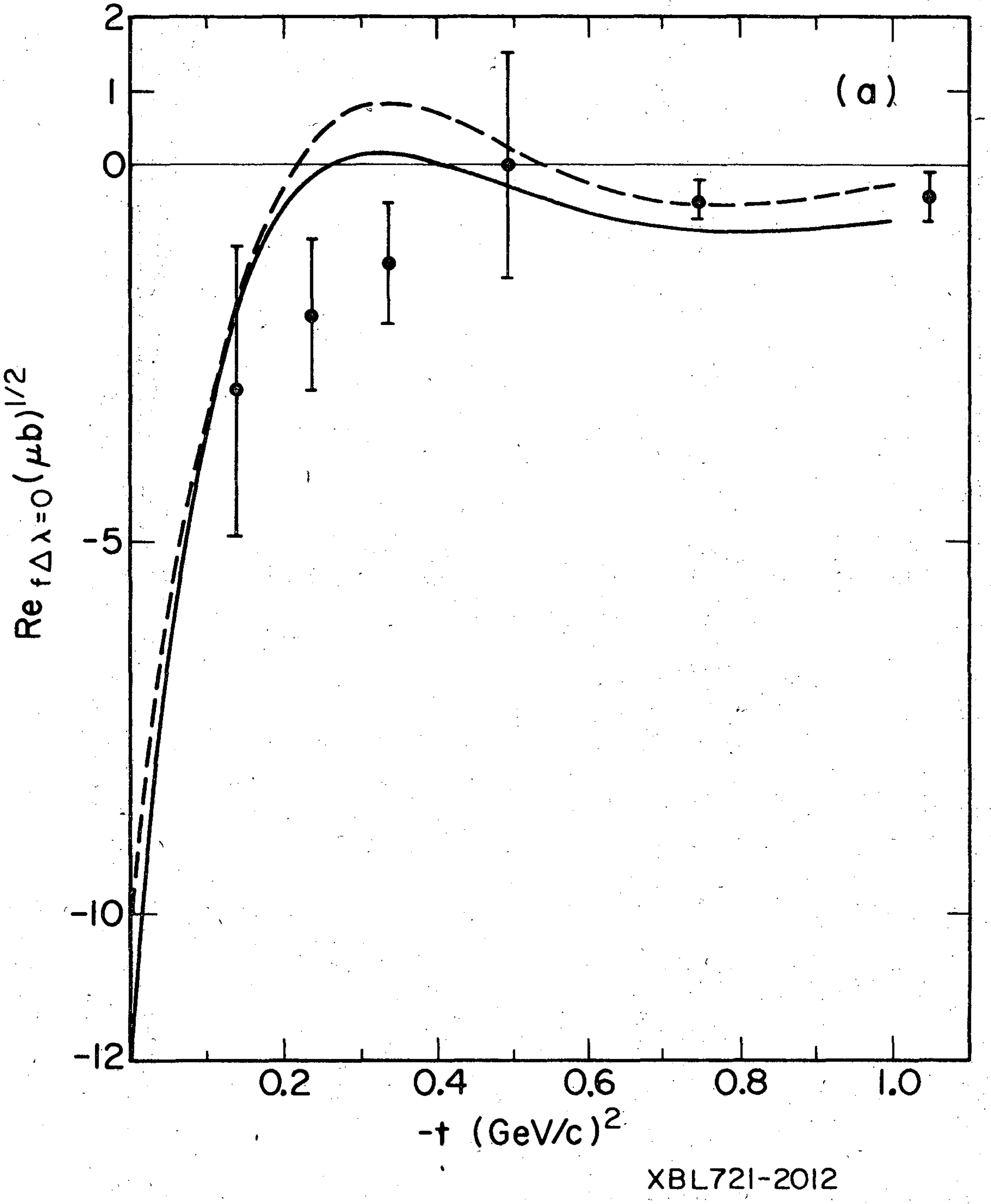


Fig. 2a

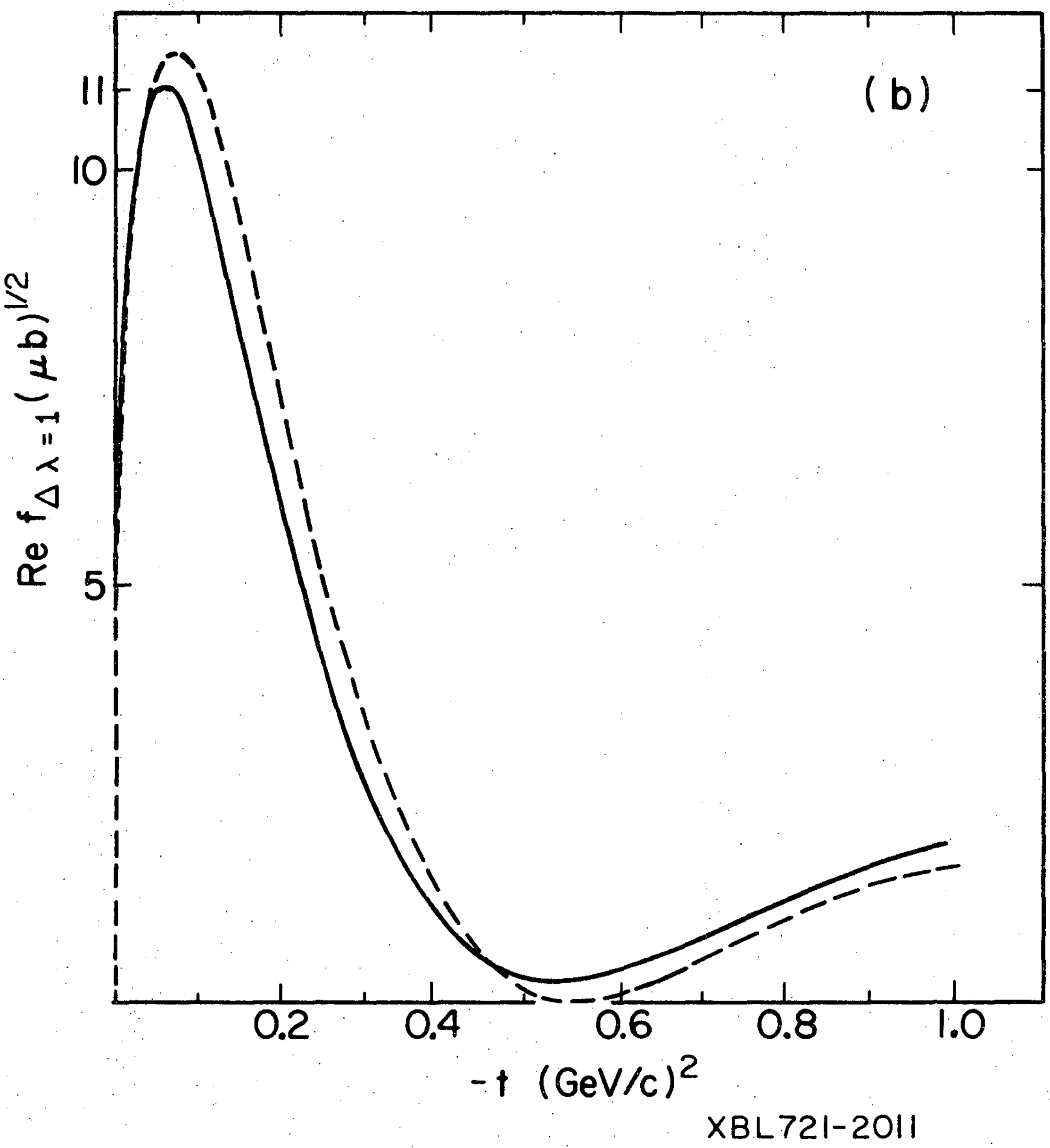

(b)
11
10
5
Re $f_{\Delta\lambda=1}$ $(\mu b)^{1/2}$
0.2
0.4
0.6
0.8
1.0
$-t$ $(GeV/c)^2$
XBL721-2011

Fig. 2b

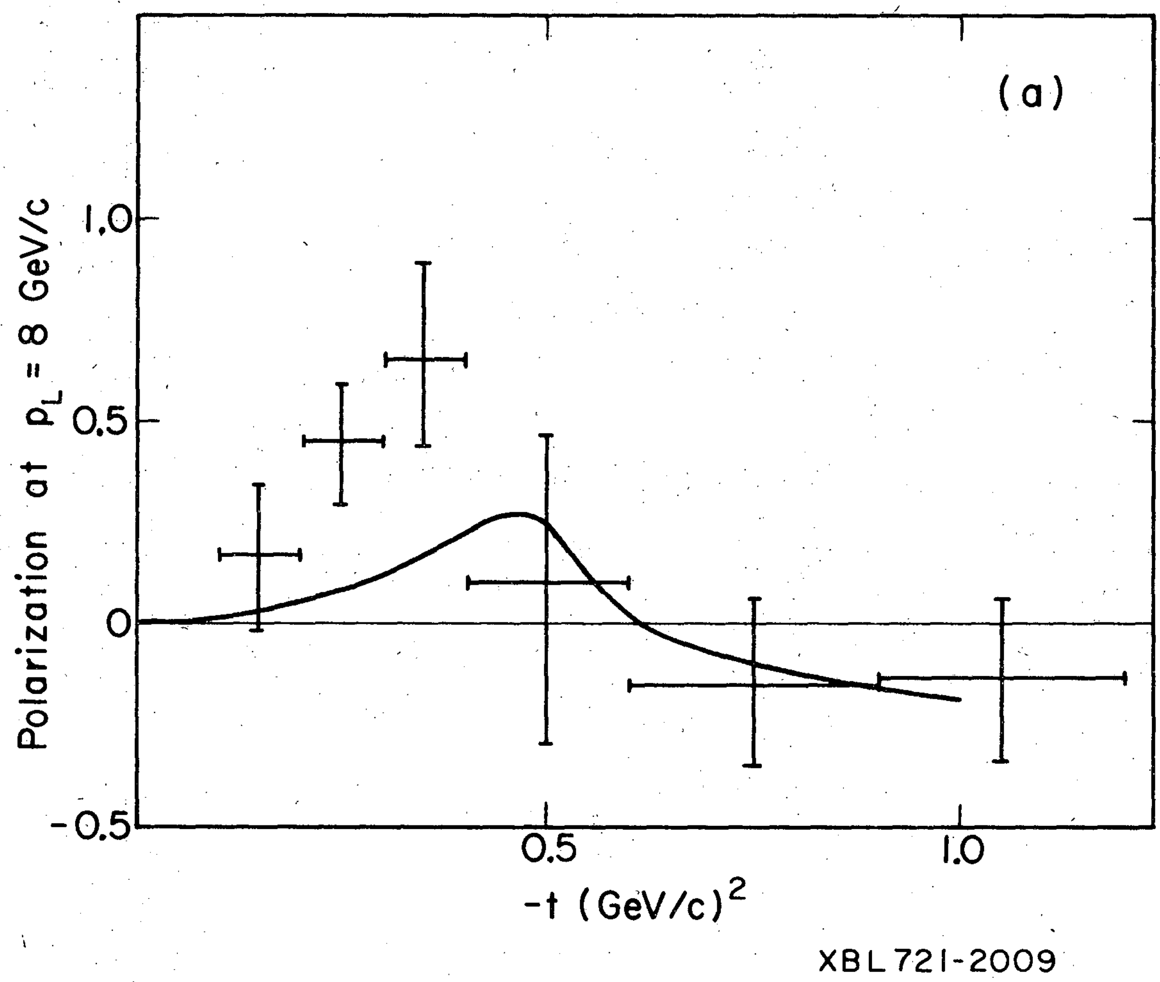


Fig. 3a

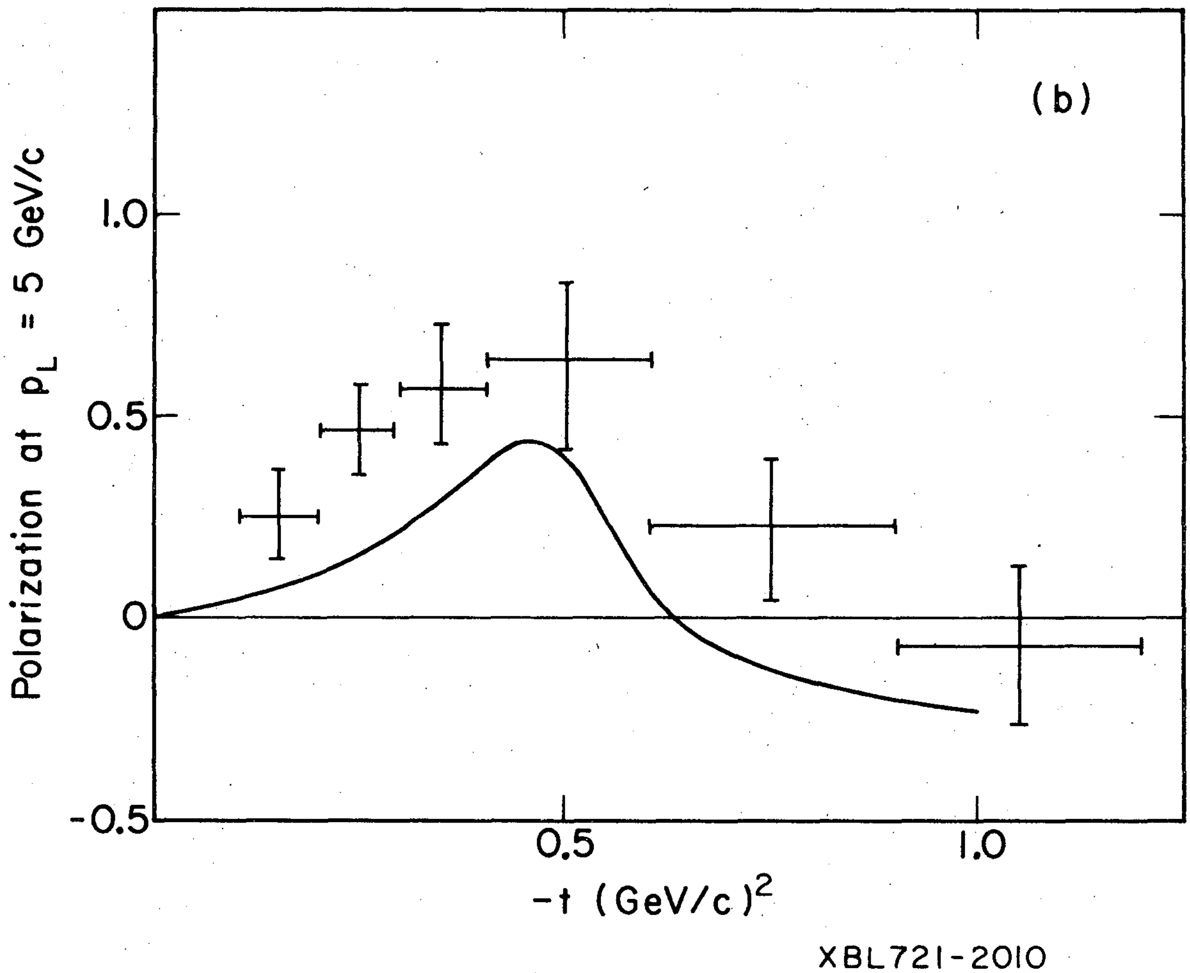
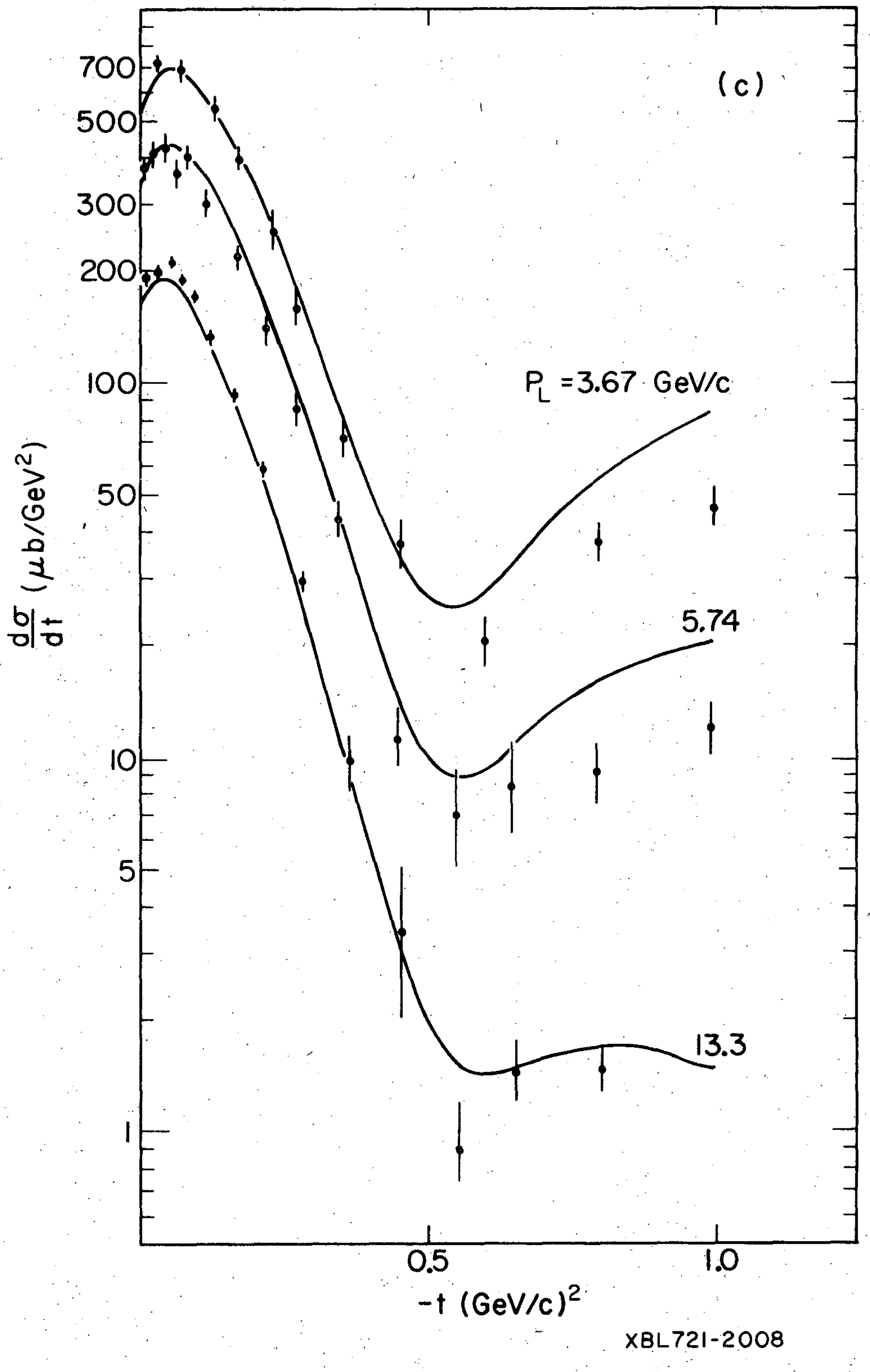


Fig. 3b



700
500
300
200
100
50
10
5
1
(c)
P_L =3.67 GeV/c
5.74
13.3
$\frac{d\sigma}{dt}$ (μb/GeV^2)
0.5
1.0
-t (GeV/c)^2
XBL721-2008

Fig. 3c