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A minimally fine-tuned supersymmetric standard model

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Abstract

We construct supersymmetric theories in which the correct scale for electroweak symmetry breaking is obtained without significant fine-tuning. We calculate the fine-tuning parameter for these theories to be at the 20% level, which is significantly better than in conventional supersymmetry breaking scenarios. Supersymmetry breaking occurs at a low scale of order 100 TeV, and is transmitted to the supersymmetric standard-model sector through standard-model gauge interactions. The Higgs sector contains two Higgs doublets and a singlet field, with a superpotential that takes the most general form allowed by gauge invariance. An explicit model is constructed in 5D warped space with supersymmetry broken on the infrared brane. We perform a detailed analysis of electroweak symmetry breaking for this model, and demonstrate that the fine-tuning is in fact reduced. A new candidate for dark matter is also proposed, which arises from the extended Higgs sector of the model. Finally, we discuss a purely 4D theory which may also significantly reduce fine-tuning.

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1. Introduction

Low-energy supersymmetry is an attractive framework for physics beyond the standard model. It not only stabilizes the electroweak scale against potentially large radiative corrections, but also leads to a successful prediction for the low-energy gauge couplings through gauge coupling unification [1]. However, non-discovery of both superparticles and a light Higgs boson at LEP II puts strong constraints on theories with low-energy supersymmetry. To avoid the constraints from LEP II, especially those on the physical Higgs-boson mass, masses of superparticles must generically be pushed up to larger values. This then leads to a large negative Higgs-boson mass-squared parameter at radiative level, requiring fine-tuning among parameters to reproduce the correct scale for electroweak symmetry breaking.

In this paper we construct a supersymmetric theory that does not suffer from significant fine-tuning. We do this without spoiling attractive features of supersymmetry such as the successful prediction associated with gauge coupling unification. We consider the case in which all the states in the minimal supersymmetric standard model (MSSM) are elementary up to high energies of order the unification scale $\simeq 10^{16}$ GeV. We find that, to evade severe fine-tuning, an additional contribution to the physical Higgs-boson mass is needed, beyond those in the MSSM. We provide it by coupling the two Higgs doublets to a singlet superfield S , as in the case of the next-to-minimal supersymmetric standard model (NMSSM) [2]. The form of the superpotential in the Higgs sector of our model, however, is not identical to that of the NMSSM, allowing parameter regions that are not available in the NMSSM Higgs potential. The mass of the physical Higgs boson can also be heavier than in the simplest NMSSM due to additional contributions to the evolution of couplings arising from the sector that dynamically breaks supersymmetry. Raising the Higgs-boson mass, however, is not sufficient to give a significant reduction of fine-tuning. We find that reducing the fine-tuning also requires a low mediation scale for supersymmetry breaking, and a superparticle spectrum that does not respect simple unified mass relations. Our theory naturally accommodates all these features.

The superpotential of the Higgs sector in our theory is effectively given by

$$W_{\text{Higgs}} = \lambda S H_u H_d + L_S^2 S + \frac{M_S}{2} S^2 + \frac{\kappa}{3} S^3, \quad (1)$$

where H_u and H_d are the two Higgs doublets of the MSSM, λ and κ are dimensionless coupling constants, and L_S and M_S are dimensionful parameters of order the electroweak scale. This superpotential can be obtained by integrating out a set of singlet fields, collectively called P and X , that couple both with the S field and with the dynamical supersymmetry breaking (DSB) sector. Due to interactions with the DSB sector, these singlet fields feel the scale of dynamical supersymmetry breaking, which is of order 100 TeV in our theory, and generate mass parameters of order the weak scale, such as L_S and M_S , in the Higgs sector. We find that some of the singlet fields (X fields) naturally develop vacuum expectation values (VEVs) of order the weak scale, while others (P fields) do not. Because the masses of the singlet fields are naturally of order the weak scale, they may affect phenomenology at the electroweak scale. Moreover, our theory possesses an unbroken Z_2 symmetry, under which the P fields, which we call pedestrian fields, are odd and all other

fields are even. This makes the lightest particle in the P multiplets a good candidate for the dark matter of the universe.

To explicitly realize a theory incorporating all the features described above, we employ a warped space construction, in which dynamical supersymmetry breaking is described as supersymmetry breaking on the infrared brane in 5D warped space truncated by two branes. In particular, our model employs the basic structure of the model constructed in Ref. [3]—the gauge group in the bulk of the truncated 5D AdS space is $SU(5)$ but it is broken to the $SU(3)_C \times SU(2)_L \times U(1)_Y$ (321) subgroup both at the ultraviolet and infrared branes. One of the virtues of constructing models in warped space is that they provide calculable theories of dynamical supersymmetry breaking [4]. In the present context, we can explicitly quantify the degree of fine-tuning of our theory by studying the fractional change of the weak scale in response to fractional changes of fundamental parameters a_i of the theory, $\Delta^{-1} = \min_i |(a_i/M_Z^2)(\partial M_Z^2/\partial a_i)|$ [5]. We find that the required “fine-tuning” in our theory is quite mild,

$$\Delta^{-1} \simeq 20\%. \quad (2)$$

This is a significant improvement over conventional supersymmetry breaking scenarios such as supergravity mediation, which typically require a fine-tuning of $\Delta^{-1} \simeq 2\text{--}3\%$ or even worse.

The organization of the paper is as follows. In the next section we discuss sources of fine-tuning in supersymmetric theories, and motivate the particular construction we adopt in later sections. In Section 3 we discuss the basic structure of our theory, and provide details of its Higgs sector. In Section 4 we present an explicit model constructed in 5D warped space, including a discussion of the singlet sector. Electroweak symmetry breaking is studied in Section 5, where we perform a renormalization group analysis, minimize the Higgs potential, and find that the fine-tuning can be significantly reduced in our model. We also calculate the spectrum of superparticles for a few sample points in the parameter space. Some phenomenological issues, including pedestrian dark matter, are discussed in Section 6. In Section 7 we discuss an alternative class of theories, constructed purely in 4D, which also may significantly reduce fine-tuning. Finally, our conclusions are given in Section 8.

For recent, alternative ideas to address the supersymmetric fine-tuning problem, see, for example, Refs. [6–14].

2. Sources of the supersymmetric fine-tuning problem

In this section we discuss possible sources of fine-tuning in generic classes of supersymmetric theories. The leading contribution to the negative Higgs-boson mass-squared parameter in the supersymmetric standard model (SSM) comes from loops of top quark and squarks. This contribution is given approximately by

$$m_h^2 \simeq -\frac{3y_t^2}{4\pi^2} m_{\tilde{t}}^2 \ln\left(\frac{M_{\text{mess}}}{m_{\tilde{t}}}\right), \quad (3)$$

where y_t is the top Yukawa coupling, $m_{\tilde{t}}$ represents the masses of two top squarks, which we have taken to be equal for simplicity, and M_{mess} is the scale at which superparticle masses are generated (or at which supersymmetry breaking is mediated to the SSM sector). If supersymmetry breaking is mediated at a high scale, this gives a large contribution to m_h^2 . For example, in the minimal supergravity scenario, $M_{\text{mess}} \simeq M_{\text{Pl}}$, so that Eq. (3) gives $-m_h^2$ as large as $(500 \text{ GeV})^2$ even for $m_{\tilde{t}}^2 \simeq (300 \text{ GeV})^2$. In fact, the value of $m_{\tilde{t}}$ should typically be larger to obtain sufficiently large Higgs-boson and superparticle masses, as discussed later. This leads to fine-tuning for electroweak symmetry breaking, as the electroweak scale is determined at tree level by the equation

$$\frac{M_Z^2}{2} \simeq -m_h^2 - |\mu|^2, \quad (4)$$

where μ is the supersymmetric mass for the two Higgs doublets. The cancellation required between the two independent parameters m_h^2 and μ is, therefore, at the level of a few percent or worse. To perform a precise analysis, however, we must use renormalization group (RG) equations because the logarithm appearing in Eq. (3) is large. Such an analysis can be found, for example, in Ref. [15] for the case of the minimal supergravity scenario [16], giving fine-tuning at the 2–3% level or worse. Note that Eqs. (3) and (4) are valid for moderately large values for $\tan \beta \equiv \langle h_u \rangle / \langle h_d \rangle$, e.g., $2 \lesssim \tan \beta \lesssim 40$, where h_u and h_d are the two Higgs doublets giving masses for the up-type and down-type quarks, respectively.

While it is possible that the fine-tuning described above may just be an artifact arising from some special relation among parameters derived in some fundamental theory, the simplest possibility for reducing fine-tuning is to make the logarithm in Eq. (3) smaller, i.e., to lower the mediation scale of supersymmetry breaking. The extent to which this helps depends on the top-squark mass $m_{\tilde{t}}$, and the mediation scale M_{mess} . As we are about to see, just making the logarithm small is not enough to eliminate fine-tuning in the simplest supersymmetric theories.

In the minimal supersymmetric standard model (MSSM), the non-discovery of the Higgs boson at LEP II requires the mass of the lightest CP -even Higgs boson, M_{Higgs} , to be larger than 114 GeV in most of the parameter space [17]. Since the value of M_{Higgs} at tree level is bounded from above by M_Z , this requires a significant radiative contribution to M_{Higgs} . Such a contribution arises from top quark and squark loops, whose dominant piece is proportional to $\ln(m_{\tilde{t}}/m_t)$ [18]. The experimental bound $M_{\text{Higgs}} \gtrsim 114 \text{ GeV}$ then implies that the top-squark masses, $m_{\tilde{t}}$, should be larger than about 800–1000 GeV for relatively small top-squark mixing and about 500–600 GeV even if M_{Higgs} is maximized by allowing a large top-squark mixing [19]. We thus find that the constraint from the Higgs-boson search alone is enough to make the fine-tuning (much) worse than 10%, especially for the case of a small top-squark mixing, even if the logarithm in Eq. (3) is just a factor of two or so (corresponding to M_{mess} of order TeV).

Another constraint on $m_{\tilde{t}}$ comes from the non-discovery of superparticles at LEP II. This constraint, obviously, depends on the model we consider, because it depends on the spectrum of the superparticles. What types of models should we consider? Since generic spectra for the superparticles lead to the supersymmetric flavor problem, the mediation of supersymmetry must be flavor universal. For small values of M_{mess} , and preserving the supersymmetric desert, the most natural mechanism that gives flavor-universal superparticle

masses is to mediate supersymmetry breaking through standard-model gauge interactions (this includes, for example, gauge mediation models [20,21] and models in warped space with supersymmetry breaking mediated by gauge interactions [3,22–27]). Imagine, then, that supersymmetry is dynamically broken in a sector respecting a global $SU(5)$ symmetry ($\supset SU(3)_C \times SU(2)_L \times U(1)_Y$) and that the breaking is mediated to the SSM sector by standard-model gauge interactions. This is a natural assumption because the supersymmetry breaking sector should not disturb the success of the gauge coupling unification prediction, and the simplest possibility is that this sector respects $SU(5)$. In this case the ratio of the top-squark mass to the right-handed selectron mass will be about

$$\frac{m_{\tilde{t}}^2}{m_{\tilde{e}}^2} \simeq \frac{(4/3)g_3^4}{(3/5)g_1^4} \simeq (7-9)^2, \quad (5)$$

where g_3 is the $SU(3)_C$ gauge coupling and g_1 is the $U(1)_Y$ gauge coupling in the $SU(5)$ normalization, both renormalized at the scale of order M_{mess} .¹ The non-discovery of the right-handed selectron at LEP II pushes up its mass to be above $\simeq 100$ GeV. This forces $m_{\tilde{t}}$ to be at least 700 GeV, which in turn leads to $-m_{\tilde{t}}^2$ larger than about $(300 \text{ GeV})^2$ even for $\ln(M_{\text{mess}}/m_{\tilde{t}})$ as small as a factor of a few (see Eq. (3)). This therefore requires a fine-tuning worse than 10%. Although this constraint may appear somewhat model-dependent, it in fact applies to rather large classes of low-scale supersymmetry breaking theories. For example, it applies to minimal gauge mediation models [21], in which the messenger sector (referred to as the supersymmetry-breaking sector here) respects an approximate global $SU(5)$ symmetry, i.e., messenger fields fill out complete representations of $SU(5)$, and the leading supersymmetry-breaking effects are approximately $SU(5)$ symmetric.

It is clear, then, that just making the logarithm smaller does not entirely eliminate the fine-tuning. How much can it help? To answer this question, we make a rough estimate of the minimum value of M_{mess} in the following way. Since the supersymmetry-breaking sector is charged under the standard-model gauge group, it contributes not only to the superparticle masses but also to the evolution of the standard-model gauge couplings above M_{mess} . Suppose now that this sector carries the Dynkin index of $\hat{b}$ under $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ (they are universal if this sector respects $SU(5)$). The requirement that the standard-model gauge couplings do not hit the Landau pole below the unification scale then gives a constraint on the value of $\hat{b}$. For $M_{\text{mess}} = O(1-100 \text{ TeV})$, it is $\hat{b} \lesssim 5$ (for gauge mediation models $\hat{b}$ corresponds to the number of messenger fields in $\mathbf{5} + \mathbf{5}^*$ of $SU(5)$). The masses of the gauginos $\tilde{M}$ and the sfermions $\tilde{m}$ will then be bounded as $\tilde{M} \lesssim (g^2/16\pi^2)\hat{b}M_{\text{mess}}$ and $\tilde{m}^2 \lesssim (g^2/16\pi^2)^2 C \hat{b} M_{\text{mess}}^2$, where g and C represent the standard-model gauge coupling and a Casimir factor, because these masses are generated as threshold effects at M_{mess} at order g^2 and g^4 , respectively. This gives a bound on the mediation scale $M_{\text{mess}} \gtrsim 20 \text{ TeV}$.

¹ This ratio could even be larger because the running contributions to $m_{\tilde{t}}^2$ and $m_{\tilde{e}}^2$ below M_{mess} , $\delta m_{\tilde{t}}^2$ and $\delta m_{\tilde{e}}^2$, typically have a larger ratio $\delta m_{\tilde{t}}^2/\delta m_{\tilde{e}}^2 \simeq (4/3)g_3^2 M_3^2/(3/5)g_1^2 M_1^2 \simeq (4/3)g_3^6/(3/5)g_1^6 \simeq (16-24)^2$, where M_3 and M_1 are the gluino and bino masses, respectively, and the quantities appearing in the equation are evaluated at the scale M_{mess} . This effect, however, is not very large for small values of $M_{\text{mess}} = O(1-100 \text{ TeV})$.

With large mixing for the top squarks, and making M_{Higgs} just as large as 114 GeV, one can have $m_{\tilde{t}}$ as small as about 700 GeV (or 500 GeV if one somehow breaks the unified relation of Eq. (5)). The degree of fine-tuning is given by the degree of cancellation between $-m_h^2$ and $|\mu|^2$ needed to satisfy Eq. (4). Since Eq. (4) is derived by minimizing the tree-level Higgs potential, and the tree-level Higgs quartic coupling of $M_Z^2/4v^2$ is raised to $M_{\text{Higgs}}^2/4v^2$ by radiative corrections, where $v \equiv (\langle h_u \rangle^2 + \langle h_d \rangle^2)^{1/2}$, the real degree of cancellation is better measured by Eq. (4) with M_Z^2 replaced by M_{Higgs}^2 , i.e., by the fine-tuning parameter

$$\hat{\Delta}^{-1} \equiv \frac{M_{\text{Higgs}}^2/2}{-m_h^2}. \quad (6)$$

We then find that the fine-tuning can be ameliorated to $\hat{\Delta}^{-1} \simeq 5\%$ ($\hat{\Delta}^{-1} \simeq 9\%$ in the absence of Eq. (5)) in this scenario. Note, however, that in a theory where supersymmetry breaking is mediated by standard-model gauge interactions, the fundamental parameter of the theory is not m_h^2 but the mass scale of the supersymmetry breaking sector, which is proportional to the square root of m_h^2 . Therefore, if we define the fine-tuning parameter as the sensitivity of the weak scale to the fundamental parameters of the theory [5], it is instead given by

$$(\Delta')^{-1} = \frac{1}{2} \frac{M_{\text{Higgs}}^2/2}{-m_h^2} \lesssim 3\% \quad (7)$$

($\hat{\Delta}^{-1} \lesssim 5\%$ in the absence of Eq. (5)).² Although this level of fine-tuning may not be a real disaster for the theory, it is certainly uncomfortable. Moreover, the origin of the large top-squark mixing, which requires a large scalar trilinear coupling (A term) of order a few TeV, is not entirely clear.³ The origin of the structure of the Higgs sector, e.g., the origin of the supersymmetric Higgs mass term (μ term), is also a mystery, and only more so for small values of M_{mess} . In the next section we present a scenario which significantly reduces the fine-tuning and avoids these difficulties.

To summarize, we have seen that conventional theories of supersymmetry breaking, considered widely in the literature, require fine-tuning worse than about 3% to reproduce the correct scale of electroweak symmetry breaking. Here we emphasize that the fine-tuning discussed in this section becomes even severer as the experimental constraints on

² In fact, the fine-tuning parameter defined in Eq. (7) overestimates the required amount of the tuning because it also takes into account generic sensitivity of the weak scale to the fundamental parameters [28]. Nevertheless, we will use this parameter to compare the relative amount of tunings required in different theories, since it is computed relatively easily and unambiguously. We will discuss this issue in somewhat more detail later.

³ A possible way to realize this scenario is to consider theories in supersymmetric warped space with the Higgs fields propagating in the bulk. Supersymmetry is broken on the TeV (infrared) brane while matter fields are localized on the Planck (ultraviolet) brane, on which the Yukawa couplings are located. Then, through the TeV-brane coupling between the up-type Higgs doublet H_u and the supersymmetry breaking field Z ($\langle Z \rangle = \theta^2 F_Z \neq 0$) of the form $\delta(y - \pi R) \int d^4\theta (Z + Z^\dagger) H_u^\dagger H_u$, the required large top-squark mixing (the large trilinear scalar interaction, A term) is generated. In this theory, however, the operators leading to tree-level Higgs soft masses, such as $\delta(y - \pi R) \int d^4\theta Z^\dagger Z H_u^\dagger H_u$, must be suppressed somehow.

the Higgs-boson and superparticle masses become tighter. In particular, an increase of the Higgs boson mass of a few GeV will push up the value of $m_{\tilde{t}}$ by a large amount (because the sensitivity of M_{Higgs} to $m_{\tilde{t}}$ is logarithmic) and make the degree of fine-tuning significantly worse. The theory we present in the subsequent sections is stable against this change, i.e., the theory does not suffer from a severe fine-tuning even if the value of M_{Higgs} is increased, as long as M_{Higgs} is not much larger than 130 GeV.

3. Basic structure of the theory

In this section we present a set of general ingredients for constructing a theory without severe fine-tuning. The ideas introduced here will be used in constructing an explicit, complete theory in the next section.

3.1. Basic elements

The discussion from the previous section leads us to demand the following properties:

- (1) We want the scale M_{mess} to be low: we here imagine M_{mess} to be of $O(10\text{--}100 \text{ TeV})$.
- (2) We need an additional source of the physical Higgs-boson mass other than those from the $SU(2)_L$ and $U(1)_Y$ D -terms and loops of top quark and squarks.
- (3) We want the superparticle spectrum to be different than that arising from an “ $SU(5)$ symmetric” supersymmetry-breaking sector. In particular, we do not want a large mass hierarchy between colored and non-colored superparticles as is the case in Eq. (5).

To construct a fully realistic and attractive theory with low-energy supersymmetry, we want to satisfy these requirements without introducing phenomenological problems, and without destroying the successes of the SSM. Specifically,

- (a) We want to preserve the successful MSSM prediction associated with gauge coupling unification.
- (b) We do not want the supersymmetric flavor problem to be reintroduced: here we require flavor universality for the squark and slepton masses.
- (c) We want to have a dark matter candidate in the theory. In particular, we want the candidate to have generic weak-scale cross sections so that it naturally provides the observed dark-matter energy density as a thermal relic left from the early universe.

In addition, we also require the absence of dangerous dimension four or five proton decay, and a successful implementation of the see-saw mechanism for generating small neutrino masses.

Satisfying all of requirements (1)–(3) and (a)–(c) is clearly not an easy task. For example, condition (3) apparently requires that the supersymmetry breaking sector does not respect $SU(5)$. Such a sector generically affects the gauge coupling prediction of the MSSM, leading to contradiction with (a). Condition (1) makes the lightest supersymmetric particle the gravitino, whose mass is expected to be of order $M_{\text{mess}}^2/M_{\text{Pl}} \simeq 0.1\text{--}10 \text{ eV}$, and

then we lose the lightest neutralino as a dark matter candidate. Below we will consider a theory that reconciles these seemingly contradictory requirements.

The requirements above lead us to the following set of ingredients for our theory:

- (i) We assume that the fundamental scale of supersymmetry breaking is low and close to TeV: $M_{\text{mess}} = O(10\text{--}100 \text{ TeV})$. This implies that we have a sector that induces dynamical supersymmetry breaking (DSB) at a scale $\Lambda \sim M_{\text{mess}}$. The DSB sector is charged under the standard-model $SU(3)_C \times SU(2)_L \times U(1)_Y$ (321) gauge group and leads to 321 gaugino masses of order $(g^2 \hat{b}/16\pi^2) M_{\text{mess}}$. The squark and slepton masses are generated through 321 gauge interactions and thus are flavor universal.
- (ii) The DSB sector possesses an approximate global $SU(5)$ symmetry, whose $SU(3) \times SU(2) \times U(1)$ subgroup is weakly gauged and identified as the standard model gauge group, 321. This $SU(5)$ symmetry is then broken to the 321 subgroup at the scale Λ . Since the global $SU(5)$ is broken at $\Lambda \sim M_{\text{mess}}$, the three gaugino masses are completely independent and the scalar masses do not obey “unified relations” such as Eq. (5). Nevertheless, this broken $SU(5)$ symmetry is enough to ensure that the contribution of the DSB sector to the 321 gauge coupling evolution is universal at energies larger than $\Lambda = O(10\text{--}100 \text{ TeV})$, so that the MSSM prediction for gauge coupling unification is preserved. This class of theories was first constructed in [3].
- (iii) We assume that an additional contribution to the physical Higgs-boson mass arises from the superpotential coupling $\lambda S H_u H_d$, where S is a singlet chiral superfield and H_u and H_d are the two Higgs doublets of the MSSM. While it is possible that some of these fields are composite states and their interactions become non-perturbative at low energies, in this paper we mainly concentrate on the case where all these fields are elementary up to a scale close to the 4D Planck scale.
- (iv) Supersymmetry breaking is mediated to the Higgs sector through singlet chiral superfields that directly interact both with the DSB and the Higgs sectors. There are many possible variations for the singlet sector. In this paper we mainly consider two classes of singlet fields, which we collectively call X and P . The X field couples to S through a superpotential term of the form $S^2 X$, while the P field couples through $S P^2$. Through interactions with the DSB sector, X receives a VEV and P receives supersymmetry breaking masses. These in turn generate supersymmetric and supersymmetry-breaking terms of order TeV in the Higgs sector, generating VEVs for S , H_u and H_d of the right size.
- (v) In the explicit model we consider later, we also introduce a singlet field P' (or a set of singlets) with exactly the same property as P except that it does not directly interact with the DSB sector. The general couplings of the P and P' fields to S then take the form $S P^2 + S P P' + S P'^2$. The theory thus has a Z_2 discrete symmetry under which the P and P' fields are odd while the other fields are even. If this symmetry is unbroken, the lightest member of the P and P' multiplets is a stable dark matter candidate. We call the fields P and P' *pedestrian fields*, and the Z_2 parity acting on these fields *pedestrian parity*.

A schematic depiction of this framework is given in Fig. 1.

Fig. 1. The basic structure of our framework.

At first sight, it may seem that constructing a theory possessing all these ingredients must require some extremely complicated model building. However, in Section 4 these features will be incorporated in an explicit model in a relatively simple way. Before constructing this theory, we first study the Higgs sector in more detail.

3.2. Higgs sector

The Higgs sector of our theory contains, in general, a singlet chiral superfield S , the two Higgs doublets of the MSSM, H_u and H_d , and a set of singlet fields, X , P and P' . To illustrate the basic dynamics of the Higgs sector, here we mainly consider only a single pedestrian field P that directly interacts with the DSB sector. The superpotential of the Higgs sector then contains the terms

$$W_H = \lambda S H_u H_d + \frac{\eta}{2} S P^2, \quad (8)$$

where λ and η are coupling constants. Since the pedestrian field P directly interacts with the DSB sector, it feels the effects of supersymmetry breaking through operators of the form

$$\mathcal{L} \sim \int d^4\theta \left((\hat{Z}^\dagger P^2 + \hat{Z} P^{\dagger 2}) + \hat{Z}^\dagger \hat{Z} P^\dagger P + \hat{Z}^\dagger \hat{Z} (P^2 + P^{\dagger 2}) + (\hat{Z} + \hat{Z}^\dagger) P^\dagger P \right), \quad (9)$$

where $\hat{Z}$ is a supersymmetry-breaking spurion field, $\langle \hat{Z} \rangle = \theta^2 \hat{F}_Z$ with $\hat{F}_Z = O(\Lambda)$, and we have omitted the coefficients of the operators. This generates an effective supersymmetric mass term for the P field

$$W_{\text{eff}, P} = \frac{M_P}{2} P^2, \quad (10)$$

as well as soft supersymmetry breaking masses

$$\mathcal{L}_{\text{soft}, P} = -m_P^2 |P|^2 - \left(\frac{M_P B_P}{2} P^2 + \frac{\eta A_\eta}{2} S P^2 + \text{h.c.} \right), \quad (11)$$

where we have used the same symbol for a chiral superfield and its scalar component. The parameters M_P , B_P and A_η are of order $c\Lambda$, and m_P^2 is of order $(c\Lambda)^2$, where c is a coefficient of order $(\hat{b}/16\pi^2)$, so that we naturally expect $|M_P|, |B_P|, |A_\eta|, |m_P^2|^{1/2} = O(1\text{--}10 \text{ TeV})$. The Lagrangian given by Eqs. (8), (10), (11) defines our minimal Higgs sector at tree level.

Fig. 2. The diagrams inducing (a) F_S , (b) S , (c) $F_S^\dagger S$, and (d) $|S|^2$ terms in the Lagrangian. The crosses on internal lines represent insertions of supersymmetry-breaking masses.

At one loop, the soft supersymmetry breaking terms in Eq. (11) induce linear terms in F_S and S in the Lagrangian, through the diagrams of Fig. 2(a) and (b), where F_S and S represent the highest and lowest components of the chiral superfield S , respectively. Terms of the form $(F_S^\dagger S + \text{h.c.})$ and $|S|^2$ are also generated through the diagrams of Fig. 2(c) and (d), respectively. The term linear in F_S is effectively represented by a superpotential term

$$W_{\text{eff},S} = L_S^2 S, \quad (12)$$

which gives terms of the form $(\lambda L_S^{*2} H_u H_d + (\eta/2) L_S^{*2} P^2 + \text{h.c.})$ in the potential. The other terms in the Lagrangian, $(S + \text{h.c.})$, $(F_S^\dagger S + \text{h.c.})$ and $|S|^2$, give the soft supersymmetry breaking Lagrangian

$$\mathcal{L}_{\text{soft},S} = -(L_S^2 C_S S + \lambda A_\lambda S H_u H_d + \text{h.c.}) - m_S^2 |S|^2, \quad (13)$$

after eliminating the auxiliary field F_S . Here, L_S^2 , $L_S^2 C_S$, A_λ and m_S^2 are coefficients.⁴ Although the diagrams in Fig. 2 are logarithmically divergent, the divergences are cut off at the scale Λ , so that the coefficients L_S^2 , $L_S^2 C_S$, A_λ and m_S^2 are approximately given by

$$L_S^2 \approx -\frac{\eta}{16\pi^2} M_P^* B_P^* \ln\left(\frac{\Lambda}{|M_P|}\right), \quad (14)$$

$$L_S^2 C_S \approx -\frac{\eta}{16\pi^2} A_\eta M_P^* B_P^* \ln\left(\frac{\Lambda}{|M_P|}\right), \quad (15)$$

$$A_\lambda \approx -\frac{|\eta|^2}{16\pi^2} A_\eta \ln\left(\frac{\Lambda}{|M_P|}\right), \quad (16)$$

$$m_S^2 \approx -\frac{|\eta|^2}{8\pi^2} m_P^2 \ln\left(\frac{\Lambda}{|M_P|}\right), \quad (17)$$

where the logarithm $\ln(\Lambda/|M_P|)$ is expected to be of $O(1)$. Altogether, our minimal Higgs sector is effectively given by the superpotentials of Eqs. (8), (10), (12) and the supersym-

⁴ In the spurion language, the term in Eq. (12) arises from the operator of the form $\int d^4\theta \{ \mathcal{D}^2 (\hat{Z}^\dagger \hat{Z}) S + \text{h.c.} \}$, where $\mathcal{D}^2 \equiv \mathcal{D}^\alpha \mathcal{D}_\alpha$ represents a supercovariant derivative. The three terms in Eq. (13), $(S + \text{h.c.})$, $(F_S^\dagger S + \text{h.c.})$ and $|S|^2$, arise from $\int d^4\theta \{ \hat{Z} \mathcal{D}^2 (\hat{Z}^\dagger \hat{Z}) S + \text{h.c.} \}$, $\int d^4\theta \{ \hat{Z} S^\dagger S + \text{h.c.} \}$ and $\int d^4\theta \{ \hat{Z}^\dagger \hat{Z} S^\dagger S \}$, respectively.

metry breaking Lagrangian of Eqs. (11), (13)⁵:

$$W = W_H + W_{\text{eff},P} + W_{\text{eff},S}, \quad (18)$$

$$\mathcal{L}_{\text{soft}} = \mathcal{L}_{\text{soft},P} + \mathcal{L}_{\text{soft},S} + \mathcal{L}_{\text{soft},H}. \quad (19)$$

Here, we have added in Eq. (19) soft supersymmetry-breaking mass terms for H_u and H_d :

$$\mathcal{L}_{\text{soft},H} = -m_{H_u}^2 |H_u|^2 - m_{H_d}^2 |H_d|^2, \quad (20)$$

which arise from 321 gauge loops, with H_u also receiving a contribution from the top-squark loop given by Eq. (3). Note that it is possible to modify the minimal Higgs sector by introducing additional terms in the tree-level superpotential W_H given in Eq. (8). We will discuss such a modification later in this section.

We now study the vacuum of our Higgs sector. The VEVs for the fields, H_u , H_d , S and P , are given by minimizing the potential derived from Eqs. (18), (19). We want to study whether our Higgs sector has a phenomenologically acceptable vacuum with $\langle S \rangle \neq 0$, $\langle H_u \rangle \neq 0$ and $\langle H_d \rangle \neq 0$, and with all physical excitations heavy enough to evade experimental constraints. We are interested in the following parameter regions:

- The coupling λ in W_H should be relatively large so that it can give a sizable contribution to the physical Higgs-boson mass, M_{Higgs} . Specifically, we consider the region where $\lambda \simeq 0.6$ – 0.8 . These values for λ are consistent with the requirement that the couplings in the Higgs sector do not hit the Landau pole below the unification scale $\simeq 10^{16}$ GeV. Note that because the DSB sector is charged under the 321 gauge group, high energy values for the 321 gauge couplings in our theories are larger than those of the MSSM, which allows relatively larger values for λ at low energies when evolved down from a high scale. (A detailed analysis of the evolution of the couplings will be given in Section 5.1.)
- We consider the region where $\tan \beta \equiv \langle H_u \rangle / \langle H_d \rangle$ is not large, say $\tan \beta \lesssim 3$. This is because the contribution of λ to the square of the physical Higgs-boson mass is proportional to $\lambda^2 v^2 \sin^2 2\beta$, which is sizable only when $\tan \beta$ is not so large. Here, $v \equiv (\langle H_u \rangle^2 + \langle H_d \rangle^2)^{1/2}$.
- We assume that the vacuum preserves pedestrian parity (or P parity), under which the P field is odd and the other fields are even. In the context of the present analysis, this is equivalent to $\langle P \rangle = 0$.

In a given vacuum, which could be a local minimum of the potential, the degree of fine-tuning can be parameterized by the quantity

$$\Delta^{-1} \equiv \min_i \left| \frac{a_i}{M_Z^2} \frac{\partial M_Z^2}{\partial a_i} \right|^{-1}, \quad (21)$$

which measures the sensitivity of the weak scale to a change of the fundamental parameters a_i of the theory [5]. This quantity reduces to $\hat{\Delta}^{-1}$ in Eq. (6) for the case of the MSSM

⁵ There are also terms in the Lagrangian induced by finite corrections, such as $S^\dagger H_u H_d$, $S^\dagger S^2$, $S^\dagger P^2$ and $S^\dagger P^\dagger P$, but they are generically suppressed and do not affect our analysis significantly.

with $a_i = m_h^2$, and to $(\Delta')^{-1}$ of Eq. (7) for $a_i \approx \Lambda \propto |m_h^2|^{1/2}$. The parameter Δ^{-1} , however, could overestimate the required amount of fine-tuning in certain cases. For example, suppose that M_Z itself is the fundamental parameter of the theory, $a_i = M_Z$; then Eq. (21) gives $\Delta^{-1} = 50\%$ despite the fact that the theory is not tuned at all. This is because Δ^{-1} also takes into account generic sensitivities of M_Z^2 to a_i , in addition to the actual amount of fine-tuning [28]. To correct this, we consider a slightly modified parameter

$$\tilde{\Delta}^{-1} \equiv \min_i \left| \eta_i \frac{a_i}{M_Z^2} \frac{\partial M_Z^2}{\partial a_i} \right|^{-1}, \quad (22)$$

where η_i are parameters introduced to eliminate generic sensitivities of M_Z^2 to a_i : if $M_Z \propto a_i^n$ in some generic parameter region we take $\eta_i = 1/n$. A difficulty associated with this parameter is that it is not easy to estimate η_i reliably. We thus consider both Δ^{-1} and $\tilde{\Delta}^{-1}$ when we perform a detailed analysis of electroweak symmetry breaking later.

Since we are looking for a theory which is not severely fine-tuned, we demand that the theory has a parameter region that gives $\Delta^{-1} \gtrsim O(0.1)$ at the minimum of the potential. We consider the minimal Higgs sector defined by Eqs. (8), (10), (11) as well as straightforward modifications obtained by adding terms to W_H in Eq. (8).

We first observe that the properties of our Higgs potential significantly depend on the sign of m_S^2 . Let us first consider the case with $m_S^2 < 0$, which corresponds to $m_P^2 > 0$. In this case the Higgs potential has an unstable direction—for $H_u = H_d = P = 0$, there is a direction in the complex S plane in which the potential is not bounded from below, i.e., $V \rightarrow -\infty$ for $|S| \rightarrow +\infty$. This gives a large VEV of S , at least of order Λ , and is phenomenologically unacceptable. We thus have to introduce a stabilizing term in this case, and the simplest possibility is to add a term $(\kappa/3)S^3$ to W_H , where κ is a dimensionless parameter (this also requires the addition of $-(\kappa A_\kappa S^3/3 + \text{h.c.})$ to $\mathcal{L}_{\text{soft},S}$ in Eq. (13) with $A_\kappa = 3A_\lambda$). The theory, then, is essentially the next-to-minimal supersymmetric standard model (NMSSM) [2]. An important point is that to have a relatively large value for λ at low energies, κ must be small, $\kappa \lesssim 0.3$ (0.4) for $\lambda \gtrsim 0.7$ (0.6). This is because the RG equation for λ contains a term proportional to $\lambda\kappa^2$, $d\lambda/d\ln\mu = \lambda\kappa^2/8\pi^2 + \dots$, which gives an asymptotically non-free contribution to the evolution of λ . With this hierarchy between λ and κ (with λ^2/κ^2 larger than a factor of a few), it is difficult to find a parameter region that gives a phenomenologically acceptable vacuum with only mild fine-tuning. One typically finds either that the VEV of S is hierarchically smaller than the electroweak scale or that there is no electroweak symmetry breaking, i.e., either $\langle S \rangle \ll v$ or $v = 0$. (For earlier analyses of the NMSSM Higgs sector, though not necessarily for $\lambda^2/\kappa^2 \gg 1$, see, e.g., [29].) This leads to phenomenologically unacceptable consequences. Therefore, here we do not pursue this possibility further and focus on the other case, $m_S^2 > 0$.

The case with $m_S^2 > 0$ arises when the supersymmetry-breaking mass squared for P is negative, $m_P^2 < 0$. This does not necessarily contradict the requirement of unbroken P parity, as long as the supersymmetric mass for P , $|M_P|$, is sufficiently larger than $|m_P^2|^{1/2}$. In this case, the potential does not have an unstable direction. Therefore, we do not necessarily have to add any additional term to stabilize the potential—our Higgs sector could be simply given by Eqs. (8), (9) at tree level. Taking loop effects into account, the Higgs

sector is then effectively described by the superpotential

$$W = \lambda S H_u H_d + L_S^2 S, \quad (23)$$

and the soft supersymmetry-breaking Lagrangian

$$\mathcal{L}_{\text{soft}} = -m_{H_u}^2 |H_u|^2 - m_{H_d}^2 |H_d|^2 - m_S^2 |S|^2 - (\lambda A_\lambda S H_u H_d + L_S^2 C_S S + \text{h.c.}), \quad (24)$$

where we have set $P = 0$. The parameters L_S^2 , m_S^2 , A_λ and $L_S^2 C_S$ are related to the fundamental parameters of the theory through Eqs. (14)–(17). (For earlier studies of the Higgs potential of the form Eqs. (23), (24), see, e.g., [30].)

It is possible that there are additional terms in the Higgs-sector superpotential of Eq. (23). An obvious example is

$$\delta W = \frac{\kappa}{3} S^3. \quad (25)$$

It is also possible that the Higgs-sector superpotential has the term

$$\delta W = \frac{M_S}{2} S^2, \quad (26)$$

where M_S is a parameter of order the weak scale. A term of this form can arise effectively if there is a singlet field X that couples to S as $W = S^2 X$ and receives a VEV of order the weak scale through direct interactions to the DSB sector (an explicit realization of this will be considered in the next section). The presence of the terms in Eqs. (25), (26) can affect the phenomenology of the theory, especially electroweak symmetry breaking and the neutralino spectrum. In the rest of this subsection, however, we focus on the simplest superpotential of Eq. (23), and discuss its consequences on electroweak symmetry breaking. The terms Eqs. (25), (26) will be considered in later sections.

Denoting the neutral components of H_u and H_d , i.e., the components that will get VEVs, as H_u^0 and H_d^0 , and setting the charged components to be zero, the scalar potential derived from Eqs. (23), (24) is

$$V = V_F + V_D + V_{\text{soft}}, \quad (27)$$

where V_F , V_D and V_{soft} are given by

$$V_F = |\lambda H_u^0 H_d^0 - L_S^2|^2 + |\lambda|^2 |S|^2 (|H_u^0|^2 + |H_d^0|^2), \quad (28)$$

$$V_D = \frac{g^2 + g'^2}{8} (|H_u^0|^2 - |H_d^0|^2)^2, \quad (29)$$

$$V_{\text{soft}} = m_{H_u}^2 |H_u^0|^2 + m_{H_d}^2 |H_d^0|^2 + m_S^2 |S|^2 + (-\lambda A_\lambda S H_u^0 H_d^0 + L_S^2 C_S S + \text{h.c.}). \quad (30)$$

Here, V_D arises from the $SU(2)_L$ and $U(1)_Y$ D -terms, and g and g' represent the $SU(2)_L$ and $U(1)_Y$ gauge couplings.

An important feature of the Higgs potential of Eq. (27) is that, unlike the case where the VEV of S is stabilized by a small coupling κ , the VEVs of S and the Higgs doublets can be determined essentially by independent conditions. To see this, let us look at the derivative of the potential in terms of the fields. Assuming that the parameters in the potential are all

real, for simplicity, we find

$$\begin{aligned} \frac{\partial V}{\partial H_u^{0\dagger}} &= \lambda H_d^{0\dagger} (\lambda H_u^0 H_d^0 - L_S^2) + \lambda^2 H_u^0 |S|^2 \\ &\quad + \frac{g^2 + g'^2}{4} H_u^0 (|H_u^0|^2 - |H_d^0|^2) + m_{H_u}^2 H_u^0 - \lambda A_\lambda S^\dagger H_d^{0\dagger}, \end{aligned} \quad (31)$$

$$\begin{aligned} \frac{\partial V}{\partial H_d^{0\dagger}} &= \lambda H_u^{0\dagger} (\lambda H_u^0 H_d^0 - L_S^2) + \lambda^2 H_d^0 |S|^2 \\ &\quad - \frac{g^2 + g'^2}{4} H_d^0 (|H_u^0|^2 - |H_d^0|^2) + m_{H_d}^2 H_d^0 - \lambda A_\lambda S^\dagger H_u^{0\dagger}, \end{aligned} \quad (32)$$

$$\frac{\partial V}{\partial S^\dagger} = \lambda^2 S (|H_u^0|^2 + |H_d^0|^2) + m_S^2 S - \lambda A_\lambda H_u^{0\dagger} H_d^{0\dagger} + L_S^2 C_S. \quad (33)$$

We want our vacuum to be at $v^2 \equiv |\langle H_u^0 \rangle|^2 + |\langle H_d^0 \rangle|^2 \simeq (174 \text{ GeV})^2$, $\tan \beta \equiv |\langle H_u^0 \rangle| / |\langle H_d^0 \rangle| \lesssim 3$ and $|\mu_{\text{eff}}| \equiv \lambda |\langle S \rangle| \gtrsim 100 \text{ GeV}$, and these values must be given as a solution of $\partial V / \partial H_u^{0\dagger} = \partial V / \partial H_d^{0\dagger} = \partial V / \partial S^\dagger = 0$.

Suppose now that the supersymmetry-breaking masses associated with S , specifically $|L_S^2 C_S|^{1/3}$ and $(m_S^2)^{1/2}$, are somewhat larger than v , with sizes $\approx 400\text{--}800 \text{ GeV}$. In this case, $\partial V / \partial S^\dagger$ is dominated by the second and the last terms in the right-hand side of Eq. (33). This implies that the VEV of S is essentially determined by the balance between these two terms in $\partial V / \partial S^\dagger = 0$, i.e., the balance between the linear and quadratic terms in S in the potential, irrespective of the dynamics determining the VEVs of H_u^0 and H_d^0 :

$$\langle S \rangle \simeq -\frac{L_S^2 C_S}{m_S^2}. \quad (34)$$

Since the mass squared of S is given by $m_S^2 \gg v^2$, the VEV of S can be regarded as essentially fixed when one considers the minimization with respect to the VEVs of H_u^0 and H_d^0 using $\partial V / \partial H_u^{0\dagger} = \partial V / \partial H_d^{0\dagger} = 0$. Assuming real VEVs, we obtain from Eqs. (31), (32)

$$\frac{g^2 + g'^2}{4} v^2 = \frac{m_{H_d}^2 - \tan^2 \beta m_{H_u}^2}{\tan^2 \beta - 1} - \lambda^2 S^2, \quad (35)$$

$$(m_{H_u}^2 + m_{H_d}^2 + 2\lambda^2 S^2) \frac{\sin 2\beta}{2} = \lambda (L_S^2 + A_\lambda S - \lambda \sin \beta \cos \beta v^2). \quad (36)$$

These equations are identical to the MSSM minimization conditions, with the μ and μB parameters of the MSSM identified with the effective μ and μB parameters defined by $\mu_{\text{eff}} \equiv \lambda \langle S \rangle$ and $(\mu B)_{\text{eff}} \equiv \lambda (L_S^2 + A_\lambda \langle S \rangle - \lambda \sin \beta \cos \beta v^2)$. Therefore, we find that for $\mu_{\text{eff}} \approx (\mu B)_{\text{eff}}^{1/2} \approx 100\text{--}200 \text{ GeV}$, our Higgs sector produces realistic electroweak symmetry breaking without severe fine-tuning. For smaller $|L_S^2 C_S|^{1/3}$ and $(m_S^2)^{1/2}$, the situation is somewhat more complicated as the dynamics determining $\langle S \rangle$ and v are coupled with each other.

The desired parameter region, $\mu_{\text{eff}} \approx (\mu B)_{\text{eff}}^{1/2} \lesssim v$, corresponds to the region where the combination of parameters $|A_\eta M_P B_P / m_P^2|$ is smaller than its “natural value” of order $c \Lambda$

by one or two orders of magnitude, depending on the size of η (for $\eta \gtrsim 1$, $|M_P B_P|$ must also be somewhat suppressed compared with $(c\Lambda)^2$). This may be achieved, for example, by simply taking the value of $M_P B_P$ smaller than its “natural value” $\approx (c\Lambda)^2$. It is important to notice that this does not necessarily lead to a fine-tuning in electroweak symmetry breaking, since we have simply chosen some parameters to be small and have not required any precise cancellation between independent quantities. In fact, any fractional change of parameters in our Higgs potential leads to a fractional change of the weak scale, M_Z , of roughly the “same” amount, so that the fine-tuning parameter Δ^{-1} defined in Eq. (21) is not much smaller than one: typically $\Delta^{-1} = O(20\text{--}30\%)$.⁶

We finally emphasize some of the virtues of our Higgs potential, effectively described by Eqs. (23), (24), over the NMSSM Higgs potential, which is more commonly considered in the literature. First of all, our Higgs potential leads to realistic electroweak symmetry breaking much more easily, as we have seen in this subsection. This is particularly true when the coupling λ is taken to be large to push the physical Higgs-boson mass larger. The sensitivity of M_Z to the fundamental parameters is much weaker, allowing for reduced fine-tuning. Moreover, our Higgs potential does not have any approximate continuous symmetry that leads to an unwanted light state in the spectrum when it is spontaneously broken by the non-zero VEVs of S , H_u and H_d . The discrete Z_3 symmetry of the NMSSM potential, which leads to the cosmological domain wall problem, is also absent in our potential, as is evident from the form of Eqs. (23), (24). Finally, it is also interesting to note that we do not need a large S^3 coupling in the superpotential to stabilize the S VEV. This helps us obtain larger values of λ at the weak scale, and thus larger values of the physical Higgs-boson mass through the contribution from the $SH_u H_d$ term in the superpotential.

4. Models

In this section we explicitly construct a theory accommodating all the features described in the previous section. We first explain how the picture described in Section 3.1 leads us to consider a certain class of theories—theories in 5D warped spacetime with supersymmetry broken at the infrared (IR) brane and the bulk unified symmetry broken both on the ultraviolet (UV) brane and on the IR brane. We then describe explicit models accommodating not only the basic structure of Section 3.1 but also the structure of the Higgs sector discussed in Section 3.2. Electroweak symmetry breaking in these models will be studied in the next section.

⁶ In the context of an explicit model, one must check that the desired values $\mu_{\text{eff}} \approx (\mu B)_{\text{eff}}^{1/2} \lesssim v$ are obtained without any hidden fine-tuning. For example, $M_P B_P \ll (c\Lambda)^2$ may not be realized naturally if the DSB sector is truly strongly coupled and all the parameters obey naive dimensional analysis. In our explicit model given in the next section, we obtain the desired values naturally by introducing an additional pedestrian field P' that interacts with S but does not interact with the DSB sector.

4.1. Supersymmetric unification in warped space

As discussed in Section 3.1, the basic ingredients of our theory are as depicted in Fig. 1. In particular, there must be a sector, the DSB sector, that dynamically breaks supersymmetry at the scale near TeV. We denote the gauge group of this sector as G , and its gauge coupling and the size (the number of “colors”) as $\tilde{g}$ and $\tilde{N}$, respectively. The DSB sector is charged under the standard-model 321 gauge interaction. Once supersymmetry is broken, the breaking is directly transmitted to the 321 gauginos, giving them masses of order TeV. The squarks and sleptons feel supersymmetry breaking only through the 321 gauge loops, so that they receive flavor universal masses at loop level. At first sight, it may seem that this type of scenario does not allow for a calculable theory because it necessarily involves strong dynamics near the TeV scale, $\Lambda \approx 10\text{--}100$ TeV. However, if the DSB sector satisfies certain special properties we can formulate a calculable theory using a “dual” higher-dimensional description of the theory.

Suppose that the gauge group G , responsible for dynamical supersymmetry breaking, has a large 't Hooft coupling and a large number of colors, i.e., $\tilde{k} \equiv \tilde{g}^2 \tilde{N} / 16\pi^2 \gg 1$ and $\tilde{N} \gg 1$. We also assume that the coupling $\tilde{k}$ is almost constant above the TeV scale. In this case the AdS/CFT correspondence [31,32] suggests that we can formulate this theory in 5D anti-de Sitter (AdS) spacetime truncated by two branes. The resulting 5D theory appears as a supersymmetric theory on warped space ($0 \leq y \leq \pi R$) with the metric given by

$$ds^2 = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \quad (37)$$

where y is the coordinate for the extra dimension and k denotes the inverse curvature radius of the AdS space [33]. The two branes are located at $y = 0$ and $y = \pi R$, which are called the UV brane (or the Planck brane) and the IR brane (or the TeV brane), respectively. The scales are chosen such that the scales on the UV and IR branes are roughly the 4D Planck scale and the scale Λ , respectively: $k \sim M_5 \sim M_* \sim M_{\text{Pl}}$ and $kR \sim 10$. Here, M_5 is the 5D Planck scale, and M_* the 5D cutoff scale, which is taken to be somewhat (typically a factor of a few) larger than k . The 4D Planck scale is given by $M_{\text{Pl}}^2 \simeq M_5^3/k$ and the scale on the IR brane is defined as $k' \equiv k e^{-\pi k R} \sim \Lambda$.⁷

The standard model gauge fields propagate in the 5D bulk and quarks and leptons are localized to the Planck brane. Supersymmetry breaking caused by the IR dynamics of G then corresponds to supersymmetry breaking localized on the TeV brane. The masses for the 321 gauginos are generated at tree level though their interactions on the TeV brane, which in turn generate squark and slepton masses at one loop. This, therefore, corresponds to the class of theories considered in [3,22–26]. As shown in [23], this class of theories leaves many of the most attractive features of conventional unification intact. In particular, the successful MSSM prediction for the low-energy gauge couplings is preserved, provided that the 5D bulk possesses an $SU(5)$ gauge symmetry which is broken at the Planck brane and that matter and two Higgs doublets are localized towards the Planck

⁷ The description of the various scales here is quite rough, and does not discriminate two scales that differ by one or two orders of magnitude. A more precise choice of scales will be made later, where we will identify k as the unification scale $\simeq 10^{16}$ GeV and choose $k' \simeq 10\text{--}100$ TeV.

brane or have conformally-flat wavefunctions (for earlier work see [34]). Any physics that uses high scales can be accommodated without any obstacle; for example, small neutrino masses can be generated naturally through the see-saw mechanism. Note, however, that for certain purposes the theories reveal their higher-dimensional nature at a scale not far from a TeV, through the appearance of Kaluza–Klein (KK) towers and an $N = 2$ supermultiplet structure. This cuts off divergences in supersymmetry-breaking quantities at the KK scale k' and allows small values of $M_{\text{mess}} \sim k' = O(10\text{--}100 \text{ TeV})$ [25]. This class of theories, therefore, naturally incorporates ingredient (i) in Section 3.1.

Here we emphasize that we should not take the view that we have solved the hierarchy problem twice by introducing both a warped extra dimension and supersymmetry. Rather, our 5D theory is obtained by requiring certain properties on the DSB sector, which is necessarily present in any low-energy supersymmetric theory. For example, we require that the parameters $\tilde{\kappa}$ and $\tilde{N}$ in the DSB sector are large, the evolution of $\tilde{\kappa}$ is very slow over a wide energy interval between k and k' , and the IR dynamics of G produces certain gaps among the anomalous dimensions of various different G -invariant operators. These requirements then naturally lead to a supersymmetric warped extra dimension in the “dual” higher-dimensional (5D) description.⁸ This viewpoint was particularly emphasized in Ref. [4], which we follow here.

Let us now look at the group theoretical structure of our theory more carefully. To preserve the successful MSSM prediction for the low-energy gauge couplings, the 5D bulk must respect, at least, $SU(5)$, which is broken to the 321 subgroup at the Planck brane. In the 4D picture this corresponds to the DSB sector G possessing a global $SU(5)$ symmetry, of which only the 321 subgroup is gauged (at least at energies below $\sim k$). This is crucial for the G sector not to destroy the successful prediction of the MSSM. In our framework the global $SU(5)$ symmetry in the DSB sector should further be broken down to 321 at the scale Λ , in order for the superparticle spectrum not to have $SU(5)$ -symmetric features (this is ingredient (ii) from Section 3.1). In the 5D picture this implies that the TeV brane respects only the 321 subgroup of $SU(5)$, as can be attained by breaking the bulk $SU(5)$ symmetry to 321 by boundary conditions at the TeV brane. We therefore arrive at the following picture for the structure of our 5D theory. The bulk gauge group is $SU(5)$ and it is broken to the 321 subgroup both at the Planck and TeV branes. The MSSM quark and lepton fields are localized on the Planck brane, together with the two Higgs doublets H_u and H_d and a singlet field S . The schematic picture of this 5D theory is depicted in Fig. 3.

Supersymmetry breaking effects are mediated to the sector localized to the Planck brane in essentially two different ways—through 321 gauge interactions and through bulk singlet fields. We introduce a singlet field P in the bulk so that it has non-negligible interactions both to the Planck and TeV branes. This is one of the pedestrian fields discussed earlier, which couples to S on the Planck brane as $W = SP^2$. We also introduce one (or more) pedestrian field P' on the Planck brane, which also couples to S , as was discussed in ingredient (v) in Section 3.1. Finally, depending on the specific model, we may also introduce

⁸ Strictly speaking, it is not necessarily guaranteed that the 4D theory corresponding to our 5D theory (or the consistent embedding of the 5D theory into string theory) exists. Here we do not address the issue of constructing a fully UV completed theory, and treat our 5D theory in the context of an effective higher-dimensional field theory.

Fig. 3. The 5D picture of our theory.

singlet fields, which we collectively call X , that transmit the scale of the TeV brane to the Planck brane and generate supersymmetric masses of S and/or P' fields of order the weak scale. Our explicit models will be described in more detail in the next two subsections.

4.2. Structure of models

We have identified the gauge symmetry structure of our 5D theory—the bulk possesses an $SU(5)$ symmetry, which is broken to the 321 subgroup both on the Planck and the TeV branes. Warped unified models with the bulk $SU(5)$ gauge symmetry broken to 321 both at the Planck and TeV branes were constructed in Ref. [3]. Here we construct our model along the lines of Section 3, following the basic construction of [3].

The theory is formulated in a 5D warped spacetime with the extra dimension y compactified on S^1/Z_2 ($0 \leq y \leq \pi R$). The metric is given by Eq. (37), and parameters are chosen as $k \sim M_5 \sim M_* \sim M_{\text{Pl}}$ and $kR \sim 10$. We consider a supersymmetric $SU(5)$ gauge theory on this gravitational background, with the bulk $SU(5)$ symmetry broken by boundary conditions both at $y = 0$ and πR .⁹ Specifically, the 5D gauge multiplet can be decomposed into a 4D $N = 1$ vector superfield $V(A_\mu, \lambda)$ and a 4D $N = 1$ chiral superfield $\Sigma(\sigma + iA_5, \lambda')$, where both V and Σ are in the adjoint representation of $SU(5)$. The boundary conditions for these fields are given by

$$\begin{aligned} \begin{pmatrix} V \\ \Sigma \end{pmatrix} (x^\mu, -y) &= \begin{pmatrix} \hat{P} V \hat{P}^{-1} \\ -\hat{P} \Sigma \hat{P}^{-1} \end{pmatrix} (x^\mu, y), \\ \begin{pmatrix} V \\ \Sigma \end{pmatrix} (x^\mu, -y') &= \begin{pmatrix} \hat{P} V \hat{P}^{-1} \\ -\hat{P} \Sigma \hat{P}^{-1} \end{pmatrix} (x^\mu, y'), \end{aligned} \quad (38)$$

⁹ Alternatively, the $SU(5)$ breaking could be caused by an $SU(5)$ -breaking Higgs field(s) localized on a brane(s). For the TeV-brane breaking, the VEV of the brane Higgs field must be close to the (local) cutoff scale so that the resulting superparticle spectrum does not have a characteristic feature arising from the $SU(5)$ symmetry. On the other hand, the Planck-brane breaking can be caused by an $SU(5)$ -breaking VEV not necessarily close to the cutoff scale, in which case the unified scale is identified as the smaller of the VEV and the AdS curvature scale k .

where $y' = y - \pi R$, and $\hat{P}$ is a 5×5 matrix acting on gauge space: $\hat{P} = \text{diag}(+, +, +, -, -)$. This reduces the gauge symmetry to 321 both at the Planck and TeV branes. The gauge symmetry at low energies is 321. The zero-mode sector contains not only the 321 component of V , which is the gauge multiplet of the SSM, but also the $SU(5)/(SU(3)_C \times SU(2)_L \times U(1)_Y)$ (XY) component of Σ . The typical mass scale for the KK towers is $\pi k' = O(100 \text{ TeV})$, so that the lowest KK excitations of the standard model gauge fields and the lightest XY gauge bosons both have masses of order 100 TeV.

Matter and Higgs fields are introduced on the Planck brane. We introduce a standard set of matter chiral superfields $Q(\mathbf{3}, \mathbf{2})_{1/6}$, $U(\mathbf{3}^*, \mathbf{1})_{-2/3}$, $D(\mathbf{3}^*, \mathbf{1})_{1/3}$, $L(\mathbf{1}, \mathbf{2})_{-1/2}$ and $E(\mathbf{1}, \mathbf{1})_1$ for each generation, where the numbers represent the transformation properties under 321 with the $U(1)_Y$ charges normalized in the conventional way.¹⁰ For the Higgs sector, we introduce three chiral superfields $S(\mathbf{1}, \mathbf{1})_0$, $H_u(\mathbf{1}, \mathbf{2})_{1/2}$ and $H_d(\mathbf{1}, \mathbf{2})_{-1/2}$ on the Planck brane.¹¹ The Yukawa couplings are then written on the Planck brane:

$$S_{\text{Yukawa}} = \int d^4x \int_0^{\pi R} dy 2\delta(y) \left[\int d^2\theta (y_u Q U H_u + y_d Q D H_d + y_e L E H_d) + \text{h.c.} \right], \quad (39)$$

where we have suppressed generation indices.

As discussed before, we introduce a pedestrian field P in the bulk and one (or more) pedestrian field P' on the Planck brane. The field P transmits supersymmetry breaking from the TeV brane to the Higgs sector on the Planck brane. Using notation where a bulk hypermultiplet is represented by two 4D $N = 1$ chiral superfields $\Phi(\phi, \psi)$ and $\Phi^c(\phi^c, \psi^c)$ with opposite quantum numbers, the bulk pedestrian field P can be written as $\{P, P^c\}$. Without loss of generality, we choose the boundary conditions for this field as

$$\begin{aligned} \begin{pmatrix} P \\ P^c \end{pmatrix} (x^\mu, -y) &= \begin{pmatrix} P \\ -P^c \end{pmatrix} (x^\mu, y), \\ \begin{pmatrix} P \\ P^c \end{pmatrix} (x^\mu, -y') &= \begin{pmatrix} P \\ -P^c \end{pmatrix} (x^\mu, y'). \end{aligned} \quad (40)$$

A bulk hypermultiplet $\{\Phi, \Phi^c\}$ can generically have a mass term in the bulk, which is written as

$$S_\Phi = \int d^4x \int_0^{\pi R} dy \left[e^{-3k|y|} \int d^2\theta c_\Phi k \Phi \Phi^c + \text{h.c.} \right], \quad (41)$$

¹⁰ In the case of boundary condition breaking, there is a priori no reason why the hypercharges for matter fields are quantized with $SU(5)$ normalization. The quantization could be understood either in the case of Higgs $SU(5)$ breaking or in a more fundamental theory such as one with a larger gauge group or in higher dimensions. Alternatively, we can obtain a partial understanding of the matter quantum numbers by slightly delocalizing matter fields in the bulk. Such a delocalization, in fact, may be needed to avoid a dangerous thermal relic of the lightest XY state of Σ , A_5^{XY} , in the universe [35].

¹¹ These fields could alternatively be introduced in the bulk as hypermultiplets obeying appropriate boundary conditions. This reproduces essentially the same physics as the brane-field case if we localize the zero-mode wavefunctions towards the Planck brane by large bulk hypermultiplet masses, $c_S, c_{H_u}, c_{H_d} \gg 1/2$ (for the definition of c parameters see Eq. (41)).

in the basis where the kinetic term is given by [36]

$$S_{\text{kin}} = \int d^4x \int dy \left[e^{-2k|y|} \int d^4\theta (\Phi^\dagger \Phi + \Phi^c \Phi^{c\dagger}) + \left\{ e^{-3k|y|} \int d^2\theta (\Phi^c \partial_y \Phi - \Phi \partial_y \Phi^c) / 2 + \text{h.c.} \right\} \right].$$

The parameter c_Φ controls the wavefunction profile of the zero mode. For $c_\Phi > 1/2$ ($< 1/2$) the wavefunction of a zero mode arising from Φ is localized to the Planck (TeV) brane; for $c_\Phi = 1/2$ it is conformally flat. We choose $c_P \simeq 1/2$ so that the zero mode arising from P has a nearly conformally flat wavefunction and that a large exponential suppression does not arise when transmitting supersymmetry breaking from the TeV brane to the Planck brane.

The couplings of the pedestrian fields to the Higgs sector are given by

$$S_{\text{Higgs}} = \int d^4x \int_0^{\pi R} dy 2\delta(y) \times \left[\int d^2\theta \left(\lambda S H_u H_d + \frac{\eta}{2} S P^2 + \eta' S P P' + \frac{h}{2} S P'^2 \right) + \text{h.c.} \right]. \quad (42)$$

Note that the superpotential of Eqs. (39) and (42), as well as the bulk Lagrangian, is invariant under a $U(1)_R$ symmetry under which various fields transform as $V(0)$, $\Sigma(0)$, $Q(1)$, $U(1)$, $D(1)$, $L(1)$, $E(1)$, $H_u(0)$, $H_d(0)$, $S(2)$, $P(0)$, $P^c(2)$ and $P'(0)$ in the normalization where the superpotential has a charge of $+2$. Imposing this symmetry, potentially dangerous operators on the Planck brane, such as the ones leading to rapid proton decay and a large mass for the Higgs doublets, are forbidden. In particular, all dimension four and five proton decay operators are forbidden by the $U(1)_R$ symmetry. We also introduce a discrete Z_2 symmetry, under which P , P^c and P' are odd and all the other fields are even. This is the pedestrian parity (or P parity) discussed in Section 3. After supersymmetry is broken on the TeV brane, $U(1)_R$ is broken to the $Z_{2,R}$ subgroup, which is exactly the R parity of the MSSM. We assume that the P parity remains unbroken even after supersymmetry breaking. The unbroken P parity ensures the stability of the lightest component of the pedestrian fields, making it a candidate for the dark matter of the universe (this issue will be discussed in Section 6.2).

With the above configuration of fields, the successes of conventional supersymmetric unification are preserved [3]. In particular, assuming that tree-level brane-localized kinetic terms are small as suggested by naive dimensional analysis (which corresponds in the 4D picture to the assumption that the 321 gauge couplings become strong at the scale $\approx k$),¹²

¹² This assumption is not needed for the Planck-brane localized kinetic terms if the $SU(5)$ breaking on the Planck brane is caused by the $SU(5)$ -breaking Higgs field with the VEV hierarchically (one or two orders of magnitude) smaller than the cutoff scale. In this case, the $SU(5)$ -breaking VEV appears in Eq. (43), instead of k , and is identified as the unification scale.

the low-energy 321 gauge couplings g_a ($a = 1, 2, 3$) are given by

$$\begin{pmatrix} 1/g_1^2(k') \\ 1/g_2^2(k') \\ 1/g_3^2(k') \end{pmatrix} \simeq (SU(5) \text{ symmetric}) + \frac{1}{8\pi^2} \begin{pmatrix} 33/5 \\ 1 \\ -3 \end{pmatrix} \ln\left(\frac{k}{k'}\right). \quad (43)$$

This exactly reproduces the MSSM gauge coupling prediction at leading-logarithmic level, with the AdS curvature scale k identified as the conventional unification scale of $\simeq 10^{16}$ GeV (for more details see [3]). There is no proton decay problem—dimension four and five proton decay is suppressed due to the $U(1)_R$ symmetry and its unbroken $Z_{2,R}$ subgroup, and dimension six proton decay is suppressed because the wavefunctions of the XY gauge fields are strongly localized towards the TeV brane and have negligible overlaps with matter fields localized on the Planck brane. Small neutrino masses are also naturally obtained through the see-saw mechanism by introducing right-handed neutrino superfields N on the Planck brane, together with their Majorana masses and Yukawa couplings to the lepton doublets:

$$S_v = \int d^4x \int_0^{\pi R} dy \, 2\delta(y) \left[\int d^2\theta \left(\frac{M_N}{2} N N + y_v L N H_u \right) + \text{h.c.} \right]. \quad (44)$$

Here, N fields carry a $U(1)_R$ charge of $+1$ and are even under the P parity.

In the supersymmetric limit, the spectrum of the theory contains exotic massless states. Specifically, the KK spectrum of the gauge tower, m_n , is approximately given by

$$\begin{cases} V^{321}: & m_0 = 0, \\ \{V^{321}, \Sigma^{321}\}: & m_n \simeq (n - \frac{1}{4})\pi k', \end{cases} \quad \begin{cases} \Sigma^{XY}: & m_0 = 0, \\ \{V^{XY}, \Sigma^{XY}\}: & m_n \simeq (n + \frac{1}{4})\pi k', \end{cases} \quad (45)$$

where $n = 1, 2, \dots$, so that the zero modes consist of not only the 321 component of V , V^{321} , but also the XY component of Σ , Σ^{XY} , which transforms as $(\mathbf{3}, \mathbf{2})_{-5/6} + (\mathbf{3}^*, \mathbf{2})_{5/6}$ under 321 (these exotic states, however, do not affect the gauge coupling prediction nor lead to rapid proton decay as we have seen before). Once supersymmetry is broken on the TeV brane, these exotic states obtain masses [3]. The fermion component λ'^{XY} and the real-scalar component σ^{XY} in Σ^{XY} obtain masses of $O(10\text{--}100 \text{ TeV})$ through the TeV-brane operators of the form

$$\begin{aligned} & -\frac{\rho e^{-2\pi k R}}{4M_*^2} \int d^4\theta \, Z^\dagger Z \text{Tr}[\mathcal{P}[\mathcal{A}]\mathcal{P}[\mathcal{A}]] \\ & + \left\{ \frac{\xi e^{-2\pi k R}}{2M_*} \int d^4\theta \, Z^\dagger \text{Tr}[\mathcal{P}[\mathcal{A}]\mathcal{P}[\mathcal{A}]] + \text{h.c.} \right\}, \end{aligned}$$

where Z represents a chiral superfield responsible for supersymmetry breaking, $\langle F_Z \rangle \neq 0$, and $\mathcal{A}$ is defined by

$$\mathcal{A} \equiv e^{-V} (\partial_y e^V) + (\partial_y e^V) e^{-V} - \sqrt{2} e^V \Sigma e^{-V} - \sqrt{2} e^{-V} \Sigma^\dagger e^V.$$

(The trace is over the $SU(5)$ space and $\mathcal{P}[\mathcal{X}]$ is a projection operator: with $\mathcal{X}$ an adjoint of $SU(5)$, $\mathcal{P}[\mathcal{X}]$ extracts the $(\mathbf{3}, \mathbf{2})_{-5/6} + (\mathbf{3}^*, \mathbf{2})_{5/6}$ component of $\mathcal{X}$ under the decomposition to 321.) The mass of A_5^{XY} is generated at one loop through 321 gauge interactions.

Supersymmetry breaking on the TeV brane also generates masses for the 321 gauginos at tree level through the operators

$$S_{\text{gaugino}} = \int d^4x \int_0^{\pi R} dy 2\delta(y - \pi R) \sum_{a=1,2,3} \left[- \int d^2\theta \frac{\zeta_a}{2M_*} Z \text{Tr}[\mathcal{W}_a^\alpha \mathcal{W}_{a\alpha}] + \text{h.c.} \right], \quad (46)$$

where $\mathcal{W}_{a\alpha} \equiv -(1/8)\bar{\mathcal{D}}^2(e^{-2V}\mathcal{D}_\alpha e^{2V})$ represent field-strength superfields, and $a = 1, 2, 3$ denotes $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$, respectively. An important point is that the coefficients ζ_a for the operators in Eq. (46) do not respect the $SU(5)$ symmetry, as $SU(5)$ is broken to 321 on the TeV brane by boundary conditions. These operators, therefore, generate non-universal gaugino masses at the TeV scale [3]. This is the 5D realization of the condition (ii) in Section 3. The non-universality in the gaugino masses is transmitted to the squark and slepton masses, which are generated at one loop through the 321 gauge interactions. This allows us to break unwanted unified relations for the scalar masses, such as the one in Eq. (5).

The Higgs sector of our theory consists essentially of the four Planck-brane fields, S , H_u , H_d and P' , and a bulk pedestrian field, $\{P, P^c\}$, which are coupled through the superpotential interactions of Eq. (42). After supersymmetry is broken, the bulk pedestrian field obtains supersymmetry breaking masses through the TeV-brane operators

$$S_P = \int d^4x \int_0^{\pi R} dy 2\delta(y - \pi R) e^{-2\pi k R} \\ \times \int d^4\theta \left[\frac{1}{2M_*^2} (\zeta_P Z^\dagger P^2 + \zeta_P^* Z P^{\dagger 2}) - \frac{\xi_P}{M_*^3} Z^\dagger Z P^\dagger P \right. \\ \left. - \frac{1}{2M_*^3} (\rho_P Z^\dagger Z P^2 + \rho_P^* Z^\dagger Z P^{\dagger 2}) - \frac{1}{2M_*^3} (\eta_P Z P^\dagger P + \eta_P^* Z^\dagger P^\dagger P) \right], \quad (47)$$

where ζ_P , ξ_P , ρ_P and η_P are dimensionless parameters. These operators generate soft supersymmetry breaking masses of the form given in Eq. (11), as well as the effective supersymmetric mass term for P , as given in Eq. (10). Imposing CP invariance that is explicitly broken only on the Planck brane (e.g., by the Yukawa couplings), we can eliminate the supersymmetric CP problem because then we can take the basis in which all supersymmetry breaking masses as well as λ , η and η' are real, which is sufficient to suppress unwanted supersymmetric contributions to electric dipole moments. In the rest of the paper we assume the existence of such a basis.

4.3. Singlet sector

The singlet sector of our model has a number of possible variations. We have a singlet S that couples to the two Higgs doublets and the pedestrian fields $\{P, P^c\}$ and P' . In addition,

we can add a set of singlet fields, which we collectively call X , that transmit the scale of the TeV brane to the Planck brane, giving supersymmetric masses of order the weak scale to S and P' .

The effects of variations of the singlet sector appear essentially only in the Higgs sector. Specifically, the Planck-brane superpotential of Eq. (42) can have additional terms if we extend the singlet sector of our theory. We first note that the superpotential of Eq. (42) can have an additional term

$$\delta S_{\text{Higgs}} = \int d^4x \int_0^{\pi R} dy \, 2\delta(y) \left[\int d^2\theta \frac{\kappa}{3} S^3 + \text{h.c.} \right]. \quad (48)$$

This term explicitly breaks the $U(1)_R$ symmetry to the $Z_{4,R}$ subgroup, but this $Z_{4,R}$ is still sufficient to forbid dangerous operators such as the ones leading to a large Higgs mass and dimension four and five proton decay. The coupling κ cannot be very large so that it does not give too large of an asymptotically non-free contribution to the evolution of λ . This constrains the size of κ as $\kappa \lesssim 0.3$ (0.4) for $\lambda \gtrsim 0.7$ (0.6). While we mostly concentrate on the case with $\kappa = 0$, the case with $\kappa \neq 0$ will also be considered when we discuss electroweak symmetry breaking later.

The Higgs-sector superpotential can also contain a supersymmetric mass term for S :

$$\delta S_{\text{Higgs}} = \int d^4x \int_0^{\pi R} dy \, 2\delta(y) \left[\int d^2\theta \frac{M_S}{2} S^2 + \text{h.c.} \right], \quad (49)$$

where M_S is a mass parameter of order the weak scale. This term can naturally arise if there is a singlet field $\{X, X^c\}$ in the bulk that couples both to the TeV brane and the S field on the Planck brane. Suppose that $\{X, X^c\}$ has a bulk mass $c_X \simeq 1/2$ and couples to S on the Planck brane as

$$\delta S = \int d^4x \int_0^{\pi R} dy \, 2\delta(y) \left[\int d^2\theta \frac{\lambda_S}{2} X S^2 + \text{h.c.} \right]. \quad (50)$$

Then, if the TeV-brane physics gives the VEV of the X field, say through the superpotential term $\delta(y - \pi R) \int d^2\theta \, Y(X^2 - M^2)$, where M is some mass parameter and Y is a Lagrange multiplier, the generated mass for S on the Planck brane, $M_S = \lambda_S \langle X \rangle$ is naturally of order the weak scale (this has been used in [23] to generate a weak-scale μ term for the Higgs doublets on the Planck brane). Here we assume that the VEV for F_X , which can be generated through supersymmetry breaking effects, is parametrically suppressed. Similarly, the $\{X, X^c\}$ field could also generate a supersymmetric mass term for P' on the Planck brane:

$$\delta S_{\text{Higgs}} = \int d^4x \int_0^{\pi R} dy \, 2\delta(y) \left[\int d^2\theta \frac{M_{P'}}{2} P'^2 + \text{h.c.} \right], \quad (51)$$

where $M_{P'}$ is naturally of order the weak scale.¹³

The terms in Eqs. (48), (49) affect the phenomenology of the Higgs sector, including electroweak symmetry breaking and the chargino/neutralino spectrum. The presence of the term in Eq. (51) could be important for keeping P parity unbroken. In the analysis in later sections, we will treat κ , M_S and $M_{P'}$ as free parameters with M_S and $M_{P'}$ of order the weak scale. It should be remembered, however, that these terms can naturally arise without introducing any hierarchically small parameters.

4.4. Supersymmetry-breaking parameters

Here we present simple, approximate formulae for the gaugino and scalar masses, using the holographic 4D description of our theory. These formulae were derived in [4] and will be used in the analysis in later sections.

Let us suppose that supersymmetry breaking on the TeV brane is not very strong (i.e., the parameters $\zeta_a F_Z$ are not large compared with the appropriately rescaled curvature scale), which is the case we concentrate on in this paper. In this case, the gaugino and scalar masses are generated in the 4D picture by 321 gauge interactions that link them to the DSB sector. Using a scaling argument based on the large- N expansion [37], the masses for the gauginos, $M_a \equiv m_{\lambda_a^{321}}$ ($a = 1, 2, 3$), are estimated as

$$M_a \simeq g_a^2 \frac{\tilde{N}}{16\pi^2} \hat{\zeta}_a m_\rho, \quad (52)$$

where g_a are the 4D 321 gauge couplings, $\hat{\zeta}_a$ are dimensionless parameters of $O(1)$ that depend on the gauge group, $\tilde{N}$ is the size of the DSB gauge group G , and m_ρ is the typical mass scale of resonances in the DSB sector (i.e., the bound states arising from the IR dynamics of G).¹⁴ Similarly, the squared masses for the scalars, $m_{\tilde{f}}^2$, are estimated as

$$m_{\tilde{f}}^2 \simeq \sum_{a=1,2,3} \frac{g_a^4 C_a^{\tilde{f}}}{16\pi^2} \frac{\tilde{N}}{16\pi^2} \hat{\zeta}_a^2 m_\rho^2, \quad (53)$$

where $\tilde{f} = \tilde{q}, \tilde{u}, \tilde{d}, \tilde{l}, \tilde{e}$ represents the MSSM squarks and sleptons, and $C_a^{\tilde{f}}$ are the group theoretical factors given by $(C_1^{\tilde{f}}, C_2^{\tilde{f}}, C_3^{\tilde{f}}) = (1/60, 3/4, 4/3), (4/15, 0, 4/3), (1/15, 0, 4/3), (3/20, 3/4, 0)$ and $(3/5, 0, 0)$ for $\tilde{f} = \tilde{q}, \tilde{u}, \tilde{d}, \tilde{l}$ and $\tilde{e}$, respectively. Since these masses are generated through gauge interactions, they are flavor universal and the supersymmetric flavor problem is absent. Note that, because of the presence of $\hat{\zeta}_a^2$ in Eq. (53), which is required to correctly pick up the effect of supersymmetry breaking, squark and slepton masses do not obey relations arising from the $SU(5)$ symmetry.

¹³ To preserve $Z_{4,R}$ on the Planck brane (up to the weak-scale effects), $Z_{4,R}$ charges of $\{X, X^c\}$'s generating Eqs. (49) and (51) need to be different.

¹⁴ In a theory where G is almost conformal above the dynamical scale Λ , the parameter $\tilde{N}$ may actually represent the square of the number of “colors” of G , and not the number of “colors” itself. Discussions on this and related issues in the AdS/CFT correspondence can be found, for example, in Ref. [38].

The gaugino and scalar masses in Eqs. (52), (53) can be expressed in terms of 5D quantities in the following way. Let us first identify the relevant parameters in 5D. In our 5D theory, the tree-level brane-localized gauge kinetic terms are assumed to be small (this assumption does not necessarily have to be made for the Planck-brane localized terms in the case that $SU(5)$ is broken by a Higgs VEV on the Planck brane). Now, the relevant parameters for the superparticle masses are the coefficients of the bulk and brane gauge kinetic terms renormalized at the scale k' , measured in terms of the 4D metric $\eta_{\mu\nu}$. This implies that, while the coefficients for the TeV-brane gauge kinetic terms can still be regarded as small, the (renormalized) coefficients for the Planck-brane gauge kinetic terms are not, because they are enhanced by a large logarithm, $\ln(k/k')$, through their RG evolution from the scale k down to the scale k' . We can thus write the gauge kinetic part of the Lagrangian, renormalized at the 4D scale k' , as

$$\mathcal{L}_{\text{ren. 5D}} \approx -\frac{1}{4g_B^2} F_{\mu\nu} F^{\mu\nu} - 2\delta(y) \frac{1}{4\tilde{g}_{0,a}^2} F_{\mu\nu}^a F^{a\mu\nu}, \quad (54)$$

where g_B is the $SU(5)$ -invariant 5D gauge coupling and $1/\tilde{g}_{0,a}^2$ the renormalized coefficients for the Planck-brane gauge kinetic terms. The 4D gauge couplings g_a are then given by

$$\frac{1}{g_a^2} = \frac{\pi R}{g_B^2} + \frac{1}{\tilde{g}_{0,a}^2}, \quad (55)$$

at the scale k' . Identifying the contribution to $1/g_a^2$ from the bulk, $\pi R/g_B^2$, as the RG contribution to the 321 gauge couplings from the DSB sector, $(\tilde{N}/16\pi^2) \ln(k/k')$, we obtain the correspondence relation

$$\frac{\tilde{N}}{16\pi^2} \approx \frac{1}{g_B^2 k}. \quad (56)$$

The scale for the resonance masses, m_ρ , corresponds in the 5D picture to the scale for the KK masses, $\pi k'$, so

$$m_\rho \approx \pi k'. \quad (57)$$

The parameter $\hat{\zeta}_a$ can then be read off by matching the gaugino mass expression of Eq. (52) to the approximate tree-level gaugino mass expression in 5D, $g_a^2(\zeta_a F_Z/M_*)(k'/k)$, as

$$\hat{\zeta}_a \approx \frac{\zeta_a g_B^2 F_Z}{\pi M_*}, \quad (58)$$

where the parameters ζ_a and M_* appear in Eq. (46) and F_Z is the VEV of the highest component of the chiral superfield Z .¹⁵ If we assume that the sizes of various parameters

¹⁵ The definition of F_Z in this paper is given as follows. In the normalization where the kinetic term of Z is canonically normalized in 4D, F_Z is defined by $F_Z = e^{\pi k R} \partial Z / \partial \theta^2|_{\theta=\bar{\theta}=0}$. The natural size for F_Z is then of order $k^2 \sim M_*^2 \sim M_{\text{Pl}}^2$ (no exponential suppression factor).

are given by naive dimensional analysis [39], we obtain $g_B^2 \simeq 16\pi^3/M_*$, $F_Z \simeq M_*^2/4\pi$ and $\zeta_a \simeq 1/4\pi$, and we find that $\hat{\zeta}_a$ are in fact of $O(1)$.

Using the correspondence relations Eqs. (56), (57), (58), we finally obtain the following simple formulae for the gaugino and scalar masses:

$$M_a = g_a^2 \frac{\zeta_a F_Z}{M_*} \frac{k'}{k}, \quad (59)$$

and

$$m_{\tilde{f}}^2 = \gamma \sum_{a=1,2,3} \frac{g_a^4 C_a^{\tilde{f}}}{16\pi^2} (g_B^2 k) \left(\frac{\zeta_a F_Z}{M_*} \frac{k'}{k} \right)^2, \quad (60)$$

where g_a are the 4D gauge couplings given by Eq. (55) and γ is a numerical coefficient of $O(1)$. The quantity $M_{\text{SUSY},a} \equiv (\zeta_a F_Z/M_*)(k'/k)$, which sets the overall mass scale in Eqs. (59), (60), is of $O(M_* e^{-\pi k R}/16\pi^2)$, and so is naturally of $O((100 \text{ GeV})-(1 \text{ TeV}))$ for $k' \simeq 10-100 \text{ TeV}$. These expressions can be checked (numerically) by 5D calculations, as was done in Ref. [25] (for $\zeta_1 = \zeta_2 = \zeta_3$, Eqs. (59), (60) reproduce the mass spectrum given in [25]). The numerical coefficient γ takes values $\gamma \simeq 5-6$, and is not very sensitive to the parameters of the model.

5. Electroweak symmetry breaking

In this section we study electroweak symmetry breaking in our theory. We show that the correct value for the electroweak scale is obtained without severe fine-tuning. We also work out the superparticle spectrum of the theory and discuss its generic features. Some phenomenological analyses, especially those for the neutralino and pedestrian sectors, will be deferred to the next section.

5.1. Parameters in the Higgs sector

The Higgs sector of our theory consists of the Planck-brane fields S , H_u , H_d and P' , and the bulk pedestrian field $\{P, P^c\}$, together with the interactions of Eqs. (42), (47). There can also be additional terms Eqs. (48), (49), (51). After dimensional reduction, the Higgs sector consists of S , H_u , H_d , P' and the zero mode of P , which have the interactions of the form of Eqs. (8), (9) but with Eq. (8) having the additional piece $\delta W_H = \eta' S P P' + (h/2) S P'^2$ (and the pieces coming from Eqs. (48), (49), (51)). After integrating out the P field, which is expected to be somewhat heavier because of the supersymmetric mass term at tree level, we obtain the effective Higgs sector, given by Eqs. (23), (24) but supplemented by additional terms involving the P' fields:

$$W = \lambda S H_u H_d + L_S^2 S + \frac{h}{2} S P'^2 + \frac{M_{P'}}{2} P'^2, \quad (61)$$

$$\begin{aligned} \mathcal{L}_{\text{soft}} = & -m_{H_u}^2 |H_u|^2 - m_{H_d}^2 |H_d|^2 - m_S^2 |S|^2 - m_{P'}^2 |P'|^2 \\ & - \left(\lambda A_\lambda S H_u H_d + \frac{h}{2} A_\lambda S P'^2 + L_S^2 C_S S + \text{h.c.} \right). \end{aligned} \quad (62)$$

Here, we have included a term coming from Eq. (51), which could potentially be present. The Higgs potential we study is then given, for $\langle P' \rangle = 0$, by Eq. (27). In the case that the superpotential terms in Eqs. (48), (49) are added, W and $\mathcal{L}_{\text{soft}}$ in Eqs. (61), (62) are supplemented by the terms

$$\delta W = \frac{M_S}{2} S^2 + \frac{\kappa}{3} S^3, \quad (63)$$

$$\delta \mathcal{L}_{\text{soft}} = -(M_S A_\lambda S^2 + \kappa A_\lambda S^3 + \text{h.c.}), \quad (64)$$

where the terms in $\delta \mathcal{L}_{\text{soft}}$ arise after integrating out the F_S field, through the term $(F_S^\dagger S + \text{h.c.})$ radiatively generated via the diagram in Fig. 2(c).

To discuss electroweak symmetry breaking quantitatively, we need the sizes of parameters appearing in the Higgs potential. In particular, we need to know the sizes of the coupling λ (and κ) and the dimensionful parameters L_S^2 , $m_{H_u}^2$, $m_{H_d}^2$, m_S^2 , A_λ and $L_S^2 C_S$ (and M_S). We first consider the coupling λ . The upper bound on the size of λ at the weak scale is given by the condition that it does not blow up below the unification scale of $\simeq 10^{16}$ GeV. To derive the bound, therefore, we have to evolve parameters from a high scale down to the weak scale. For this purpose it is useful to consider the theory in the holographic 4D picture. The 4D theory is defined at the UV cutoff scale of order $k \sim M_{\text{pl}}$ and contains a sector (DSB sector) that has a gauge interaction with the group G , whose coupling $\tilde{g}$ evolves very slowly over a wide energy interval below k . Denoting the size of the group G to be $\tilde{N}$, the correspondence is given by $\tilde{g}^2 \tilde{N} / 16\pi^2 \approx M_*/\pi k$ and $\tilde{N} \approx 16\pi^2/g_B^2 k$ (so $\tilde{g} \simeq 4\pi$ and $\tilde{N} \gtrsim 1$ here, see also Eq. (56)). The bulk gauge symmetry and the Planck-brane boundary conditions in the 5D theory imply that the G gauge sector possesses a global $SU(5)$ symmetry whose $SU(3)_C \times SU(2)_L \times U(1)_Y$ subgroup is explicitly gauged. Since the DSB sector is charged under 321, it contributes to the running of the 321 gauge couplings, g_a . The RG equations for the 321 gauge couplings in the 4D picture are thus given by

$$\frac{d}{d \ln \mu} \left(\frac{1}{g_a^2} \right) = -\frac{1}{8\pi^2} (b_a^{\text{MSSM}} + b^{\text{DSB}}), \quad (65)$$

where $(b_1^{\text{MSSM}}, b_2^{\text{MSSM}}, b_3^{\text{MSSM}}) = (33/5, 1, -3)$ are the MSSM beta-function coefficients, and b^{DSB} represents the contribution from the DSB sector, which is given by $b^{\text{DSB}} = 8\pi^2/g_B^2 k$ in terms of the 5D quantities. Because of the global $SU(5)$ symmetry of the DSB sector, b^{DSB} is universal, i.e., b^{DSB} does not depend on a . In the case that tree-level Planck-brane kinetic terms are small, as must be the case for the boundary-condition $SU(5)$ breaking on the Planck brane, the 321 gauge couplings in the 4D picture, g_a , approach a Landau pole at the scale $\sim k$, and the DSB contribution is determined as $b^{\text{CFT}} \simeq 4.8$. For the Higgs $SU(5)$ -breaking case, g_a at the scale k can be smaller, so that $b^{\text{CFT}} \lesssim 4.8$. A schematic description for the evolution of the gauge couplings in the 4D picture is given in Fig. 4.

The modes localized to the Planck brane correspond to the fields singlet under G (elementary fields). In particular, the MSSM quarks, leptons and the S , H_u , H_d and P' fields are elementary states. The 4D theory also contains the P field as an elementary field, which

Fig. 4. Schematic description for the evolution of the gauge couplings in our theory.

interacts with the DSB sector through the interaction of the form

$$W = \lambda_P P \cdot \mathcal{O}_P, \quad (66)$$

where $\mathcal{O}_P$ is a G -singlet operator of the DSB sector, whose dimension is close to 2: $[\mathcal{O}_P] \simeq 2$, and λ_P is a coupling which is almost marginal. The KK states for the $\{P, P^c\}$ field in the 5D picture correspond to mixtures of the elementary P state and the composite states interpolating the operator $\mathcal{O}_P$. The interactions among the elementary fields in the Higgs sector are given by

$$W = \lambda S H_u H_d + \frac{\eta}{2} S P^2 + \eta' S P P' + \frac{h}{2} S P'^2, \quad (67)$$

where we have set $M_S = \kappa = 0$, for simplicity. According to the supersymmetric non-renormalization theorem, the couplings λ , η , η' and h run only through anomalous dimensions of the S , H_u , H_d , P and P' fields, which receive contributions from the gauge and Yukawa interactions as well as the interactions in Eqs. (66), (67). Let us now write down the RG equations for η and η' , incorporating the contribution from the DSB sector, Eq. (66), along the lines discussed in [13]. Denoting the 5D couplings of the superpotential terms $S P^2$ and $S P P'$ (η and η' appearing in Eq. (42)) as $\hat{\eta}$ and $\hat{\eta}'$, respectively, the couplings η and η' in the 4D picture (η and η' appearing in Eq. (67)) at the RG scale μ are given by

$$\eta(\mu) = \frac{\hat{\eta} M}{\sqrt{Z_S(\mu) Z_P(\mu)^2}}, \quad \eta'(\mu) = \frac{\hat{\eta}' \sqrt{M}}{\sqrt{Z_S(\mu) Z_P(\mu) Z_{P'}(\mu)}}, \quad (68)$$

where $Z_S(\mu)$, $Z_P(\mu)$ and $Z_{P'}(\mu)$ are wavefunction renormalization factors for the S , P and P' fields, which obey the RG equations

$$\frac{d \ln Z_S}{d \ln \mu} = -\frac{1}{8\pi^2} \left(2\lambda^2 + \frac{1}{2} \frac{\hat{\eta}^2 M^2}{Z_S Z_P^2} + \frac{\hat{\eta}'^2 M}{Z_S Z_P Z_{P'}} + \frac{1}{2} h^2 \right), \quad (69)$$

$$\frac{d \ln Z_P}{d \ln \mu} = -\frac{1}{8\pi^2} \left(\frac{\hat{\eta}^2 M^2}{Z_S Z_P^2} + \frac{\hat{\eta}'^2 M}{Z_S Z_P Z_{P'}} \right) - \frac{M}{k Z_P}, \quad (70)$$

$$\frac{d \ln Z_{P'}}{d \ln \mu} = -\frac{1}{8\pi^2} \left(\frac{\hat{\eta}'^2 M}{Z_S Z_P Z_{P'}} + h^2 \right) \quad (71)$$

(the 5D couplings $\hat{\eta}$ and $\hat{\eta}'$ do not run: $d\hat{\eta}/d\ln\mu = d\hat{\eta}'/d\ln\mu = 0$). Here, M is a spurious parameter relating the 5D and 4D P fields: $P_{5D} = \sqrt{M} P_{4D}$; the physics should not depend on it. The boundary conditions for Z_S , Z_P and $Z_{P'}$ are given by

$$Z_S(k) = Z_{0,S}, \quad Z_P(k) = M Z_{0,P}, \quad Z_{P'}(k) = Z_{0,P'}, \quad (72)$$

where $Z_{0,S}$, $Z_{0,P}$ and $Z_{0,P'}$ represent tree-level kinetic terms, $2\delta(y) \int d^4\theta (Z_{0,S} S^\dagger S + Z_{0,P} P^\dagger P + Z_{0,P'} P'^\dagger P')$, localized on the Planck brane. For the case of strong coupling at the fundamental scale, the UV parameters are estimated as $\hat{\eta} \approx 4\pi^2/M_*$, $\hat{\eta}' \approx 4\pi\sqrt{\pi/M_*}$, $Z_{0,S} \approx Z_{0,P'} \approx 1$ and $Z_{0,P} \approx \pi/M_*$ (using naive dimensional analysis), so that $\eta(k) \approx \eta'(k) \approx 4\pi$. We can also easily see that $Z_S(\mu)$, $Z_{P'}(\mu) \propto M^0$ and $Z_P(\mu) \propto M$ at an arbitrary scale μ , so that $\eta(\mu)$ and $\eta'(\mu)$ in fact do not depend on the spurious parameter M .

Solving the RG equations for η and η' given by Eqs. (68)–(71), we find that the values of η and η' are suppressed at low energies due to the contribution from the DSB sector, represented as the second term in the right-hand side of Eq. (70). This is simply the 4D realization of the fact that the couplings η and η' receive volume suppressions in the 5D theory because the P field propagates in the bulk. This is advantageous for the evolution of λ , since the RG equation for λ is given by

$$\frac{d\lambda}{d \ln \mu} = \frac{\lambda}{16\pi^2} \left(4\lambda^2 + \frac{1}{2}\eta^2 + \eta'^2 + \frac{1}{2}h^2 + 3y_t^2 - \frac{3}{5}g_1^2 - 3g_2^2 \right), \quad (73)$$

so that smaller η and η' help to obtain larger values for λ at low energies. Here, y_t is the top Yukawa coupling, which obeys the RG equation

$$\frac{dy_t}{d \ln \mu} = \frac{y_t}{16\pi^2} \left(6y_t^2 + \lambda^2 - \frac{13}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2 \right), \quad (74)$$

and we have neglected the bottom and tau Yukawa couplings (since we are interested in a small $\tan\beta$ region) as well as the Yukawa couplings for the first two generations.

Our theory has the following three features that allow larger values of λ at the weak scale compared with the conventional NMSSM. (i) The theory has larger gauge couplings at high energies than the conventional MSSM/NMSSM (see Eq. (65)) so that λ is less asymptotically non-free (due to the last two terms in Eq. (73)). (ii) Larger gauge couplings at high energies give smaller values for the top Yukawa coupling at high energies with a fixed value of the top quark mass (due especially to the last term in Eq. (74)), which reduces the asymptotically non-free contribution to the λ running from the top Yukawa coupling given by the third term of Eq. (73). (iii) The superpotential Higgs cubic coupling κ does not have to be large (κ can even be zero), as the stabilization of the VEV of S does not require this term in our theory. This eliminates a potentially large asymptotically non-free contribution coming from non-zero κ , which would add a term $\lambda\kappa^2/8\pi^2$ to the right-hand side of Eq. (73). The first two features (i) and (ii) were considered earlier in [40]. Note that, unlike κ in the NMSSM, the coupling h need not be sizable so that its effect on the evolution of λ can be quite small. The point (ii) is especially significant, which gives values of the top Yukawa coupling at the scale k as small as $y_t(k) = O(10^{-2})$. Together with the

fact that the couplings η and η' are strongly asymptotically non-free, we find that we can easily obtain values of λ as large as

$$\lambda \simeq 0.8 \quad (75)$$

at the weak scale, and we will mainly use this value in our analysis below. The couplings η and η' at the weak scale can be as large as $\eta \simeq 0.1$ and $\eta' \simeq 0.3$, without affecting the result for the maximum value of λ in Eq. (75). A value for the coupling h smaller than about $\simeq 0.4$ does not affect the value in Eq. (75), either. For $\kappa \neq 0$, RG equations are modified such that h^2 in Eqs. (69), (73) is replaced by $h^2 + 4\kappa^2$, and the RG equation for κ is given by

$$\frac{d\kappa}{d\ln\mu} = \frac{3\kappa}{16\pi^2} \left(2\kappa^2 - 8\pi^2 \frac{d\ln Z_S}{d\ln\mu} \right).$$

The condition for Eq. (75) to hold is then given by $h^2 + 4\kappa^2 \lesssim (0.4)^2$ at the weak scale.

The value of y_t should not be very large so as not to give a large asymptotically non-free contribution to the evolution of λ . This gives a lower bound on $\tan\beta$, roughly given by $\tan\beta \gtrsim 1.7$ for $\lambda \simeq 0.8$ ($\tan\beta \gtrsim 1.4$ for $\lambda \simeq 0.7$). Note that this bound does not come from the requirement of the perturbativity of y_t up to the unification scale $\approx k$, but instead comes from the requirement of having large enough λ in the IR and thus a large enough physical Higgs-boson mass. In fact, for these values of $\tan\beta$, the top Yukawa coupling is strongly asymptotically free and thus perturbative up to the scale k , due to large contributions from the gauge couplings to the evolution of y_t .

Next we consider the sizes of the dimensionful parameters L_S^2 , $L_S^2 C_S$, A_λ , m_S^2 and $m_{P'}^2$, appearing in Eqs. (61), (62). These parameters are generated through the diagrams of Fig. 2 and similar diagrams, from supersymmetry breaking masses for P 's given by Eq. (47). Their values are given by the expressions like Eqs. (14)–(17), but now with the cutoff Λ identified as the KK mass scale, $\Lambda = O(k')$, due to locality in the 5D theory. For $\eta' \gtrsim \eta$, they are given by

$$L_S^2 \approx -\frac{\eta}{16\pi^2} M_P^* B_P^* \ln\left(\frac{k'}{|M_P|}\right), \quad (76)$$

$$L_S^2 C_S \approx -\frac{\eta}{16\pi^2} A_\eta M_P^* B_P^* \ln\left(\frac{k'}{|M_P|}\right), \quad (77)$$

$$A_\lambda \approx -\frac{|\eta'|^2}{8\pi^2} A_\eta \ln\left(\frac{k'}{|M_P|}\right), \quad (78)$$

$$m_S^2 \approx m_{P'}^2 \approx -\frac{|\eta'|^2}{8\pi^2} m_P^2 \ln\left(\frac{k'}{|M_P|}\right). \quad (79)$$

We will take the values of L_S^2 , $L_S^2 C_S$, A_λ and m_S^2 essentially as free parameters in our analysis of the Higgs potential, for the following reasons. First of all, the four parameters L_S^2 , $L_S^2 C_S$, A_λ and m_S^2 depend on the quantities η , η' , $M_P B_P$, A_η and m_P^2 , which are free parameters of the theory. Therefore, in general there is no particular relation among the parameters L_S^2 , $L_S^2 C_S$, A_λ and m_S^2 . One may still worry that there may be upper bounds on the sizes of these parameters for a given value of k' , especially because low-energy values

of η and η' are bounded as $\eta \lesssim 0.1$ and $\eta' \lesssim 0.3$. In fact, for the lowest KK gauge-boson mass of order 200 TeV, which allows $|m_p^2|$ as large as $\approx (20 \text{ TeV})^2$, the value of m_S^2 is bounded as $m_S^2 \lesssim (800 \text{ GeV})^2$. Similar bounds also apply to the other parameters, giving $L_S^2 \lesssim (600 \text{ GeV})^2$, $L_S^2 C_S \lesssim (2 \text{ TeV})^3$ and $A_\lambda \lesssim 20 \text{ GeV}$ for the same value of k' (or the KK mass). Since we are interested in the region where the lowest KK gauge-boson mass is smaller than a few hundred TeV, i.e., $k' \lesssim 100 \text{ TeV}$, we expect that these parameters should not take values far in excess of the above bounds. In fact, we will see in the next subsection that correct electroweak symmetry breaking is obtained for the values of A_λ and m_S^2 almost saturating these bounds (see Table 1). The values of L_S^2 and $L_S^2 C_S$ must be somewhat suppressed compared with the bounds, but this can easily be attained by taking the coupling η small, $\eta \approx O(10^{-3} - 10^{-2})$ (see Eqs. (76)–(79)). Note that small values of η are natural because η is a superpotential coupling located on the UV brane.¹⁶

Finally, M_S in Eqs. (63) can take essentially any value of order the weak scale. We thus treat it as a free parameter in our analysis below.

5.2. Minimization of the Higgs potential

We now present examples of parameters for the model of Section 4 that lead to realistic electroweak symmetry breaking and acceptable phenomenology. In Table 1, we list three points (A, B and C) in the parameter space and the corresponding values of the fine-tuning parameters defined in Eqs. (21), (22). The square bracket in the table is defined as $[X]^n \equiv \text{sgn}(X) \cdot |X|^n$, and the sign convention is such that $\tan \beta \equiv \langle H_u \rangle / \langle H_d \rangle > 0$. We also list the physical Higgs-boson mass, some parameters in the Higgs sector, and the soft supersymmetry breaking masses for each point.

The procedure to obtain the numbers in the table is as follows. (i) We first choose the rescaled AdS curvature scale, k' , and choose λ , M_S , κ , L_S^2 , $L_S^2 C_S$, A_λ and m_S^2 as free parameters, which are roughly within the bounds discussed above. (ii) The values of $m_{H_u}^2$ and $m_{H_d}^2$ are also chosen arbitrarily. This gives $\langle S \rangle$, $\langle H_u \rangle$ and $\langle H_d \rangle$ through the minimization of the Higgs potential of Eq. (27) supplemented by the terms in Eqs. (63), (64). The values of λ , M_S , κ , L_S^2 , $L_S^2 C_S$, A_λ , m_S^2 , $m_{H_u}^2$ and $m_{H_d}^2$ should satisfy one constraint $v \equiv (\langle H_u \rangle^2 + \langle H_d \rangle^2)^{1/2} \simeq 174 \text{ GeV}$. However, the correct values for these parameters are easily obtained by starting from arbitrary values, and then rescaling all the parameters according to their dimensions such that they give $v \simeq 174 \text{ GeV}$. (iii) At this point, we obtain three numbers in the table: $\tan \beta = \langle H_u \rangle / \langle H_d \rangle$, $\mu_{\text{eff}} \equiv \lambda \langle S \rangle$ and $(\mu B)_{\text{eff}} \equiv \lambda(L_S^2 + A_\lambda \langle S \rangle - \lambda \sin \beta \cos \beta v^2)$. We also obtain the tree-level Higgs-boson mass $M_{H, \text{tree}}$ by diagonalizing the 3×3 scalar mass-squared matrix in the space of $\{\text{Re } S, \text{Re } H_u^0, \text{Re } H_d^0\}$ and finding the lightest eigenvalue. The corresponding eigenvector in this space is parameterized as $\{\sin \theta_H, \cos \theta_H \cos \varphi_H, \cos \theta_H \sin \varphi_H\}$, and we also list the value of θ_H , i.e., the amount of a singlet component in the lightest Higgs boson mass. (iv) We then choose the right-handed selectron mass m_e^2 as an input parameter, which satisfies the experimental

¹⁶ It is non-trivial, in fact, that we obtain correct electroweak symmetry breaking in the parameter region in which only η is small and all the unprotected IR-brane parameters take the values determined by naive dimensional analysis. This implies that our theory does not have any hidden fine-tuning and thus is technically natural.

Table 1

Values for the parameters of the model for three sample points, A, B and C. The resulting soft supersymmetry breaking parameters as well as the quantities in the Higgs sector are also listed. Here, $[X]^n \equiv \text{sgn}(X) \cdot |X|^n$, and all masses are given in units of GeV

	A	B	C
λ	0.8	0.8	0.8
M_S	317	0	0
κ	0	0.2	0
$[L_S^2]^{1/2}$	−85	203	243
$[L_S^2 C_S]^{1/3}$	−523	−464	−535
$[A_\lambda]$	−21	21	25
$[m_S^2]^{1/2}$	808	683	787
$[m_{Hu}^2]^{1/2}$	−106	−102	−124
$[m_{Hd}^2]^{1/2}$	192	192	193
k'	8×10^4	7×10^4	7×10^4
$\tan \beta$	1.8	1.8	1.7
μ_{eff}	156	159	192
$[(\mu B)_{\text{eff}}]^{1/2}$	−132	167	208
$M_{H,\text{tree}}$	123	116	111
θ_H	0.013	0.066	0.081
M_{Higgs}	140	134	130
M_1	346	259	173
M_2	253	274	289
M_3	305	307	328
$(m_q^2)^{1/2}$	312	319	339
$(m_u^2)^{1/2}$	295	283	290
$(m_d^2)^{1/2}$	271	269	285
$(m_l^2)^{1/2}$	192	192	193
$(m_e^2)^{1/2}$	200	150	100
A_t	−198	−195	−204
$M_{\tilde{t}_1}$	221	214	212
$M_{\tilde{t}_2}$	385	387	404
Δ^{-1}	19%	18%	12%
$\tilde{\Delta}^{-1}$	26%	24%	22%

bound of $m_e^2 \gtrsim (100 \text{ GeV})^2$. This determines the $U(1)_Y$ component of the supersymmetry breaking parameters $M_{\text{SUSY},1}$, defined by $M_{\text{SUSY},a} \equiv (\zeta_a F_Z / M_*)(k'/k)$ ($a = 1, 2, 3$), through Eq. (60). Similarly, $m_{H_d}^2$ determines the $SU(2)_L$ component, $M_{\text{SUSY},2}$. Namely,

$$\begin{cases} m_e^2 = m_{\tilde{f}}^2 (C_1^{\tilde{f}} = \frac{3}{5}, C_2^{\tilde{f}} = 0, C_3^{\tilde{f}} = 0) \\ m_{H_d}^2 = m_{\tilde{f}}^2 (C_1^{\tilde{f}} = \frac{3}{20}, C_2^{\tilde{f}} = \frac{3}{4}, C_3^{\tilde{f}} = 0) \end{cases} \longrightarrow M_{\text{SUSY},1}, M_{\text{SUSY},2}. \quad (80)$$

Here, $m_{\tilde{f}}^2$ is given in Eq. (60), and for the theory with boundary condition $SU(5)$ breaking the value of $(g_B^2 k)$ is determined by the condition that the 321 gauge couplings become strong at the scale k in the 4D picture: $(g_B^2 k) = 8\pi^2/b^{\text{DSB}} \simeq 16$. (v) In our theory the difference between $m_{H_u}^2$ and $m_{H_d}^2$ must come essentially from the top Yukawa contribution, so approximately

$$m_{H_u}^2 = m_{H_d}^2 - \frac{3y_t^2}{8\pi^2} \left\{ m_{\tilde{q}}^2 \ln\left(\frac{k'}{m_{\tilde{q}}}\right) + m_{\tilde{u}}^2 \ln\left(\frac{k'}{m_{\tilde{u}}}\right) \right\}. \quad (81)$$

Here, we have simply cut off the UV-divergent logarithm arising in the 4D one-loop calculation by the rescaled AdS scale k' , which approximates the full 5D finite computation reasonably well.¹⁷ The top Yukawa coupling is given by $m_t/v \sin \beta$. This equation fixes $M_{\text{SUSY},3}$, the only parameter in Eq. (81) still undetermined:

$$\begin{cases} m_{\tilde{q}}^2 = m_{\tilde{f}}^2 (C_1^{\tilde{f}} = \frac{1}{60}, C_2^{\tilde{f}} = \frac{3}{4}, C_3^{\tilde{f}} = \frac{4}{3}) \\ m_{\tilde{u}}^2 = m_{\tilde{f}}^2 (C_1^{\tilde{f}} = \frac{4}{15}, C_2^{\tilde{f}} = 0, C_3^{\tilde{f}} = \frac{4}{3}) \end{cases} \longrightarrow M_{\text{SUSY},3}. \quad (82)$$

Therefore, from Eqs. (59), (60), We obtain the soft supersymmetry breaking masses for the gauginos M_1 , M_2 and M_3 , and for the scalars $m_{\tilde{q}}^2$, $m_{\tilde{u}}^2$, $m_{\tilde{d}}^2$, $m_{\tilde{l}}^2$ and $m_{\tilde{e}}^2$. (vi) The physical top-squark masses, $M_{\tilde{t}_1}$ and $M_{\tilde{t}_2}$ ($M_{\tilde{t}_2} > M_{\tilde{t}_1}$), are given by diagonalizing the mass-squared matrix

$$M_{\tilde{t}}^2 = \begin{pmatrix} m_{\tilde{q}_3}^2 + m_t^2 + \left(\frac{1}{2} - \frac{2}{3} \sin^2 \theta_w\right) \cos 2\beta M_Z^2 & y_t v (A_t \sin \beta - \mu_{\text{eff}} \cos \beta) \\ y_t v (A_t \sin \beta - \mu_{\text{eff}} \cos \beta) & m_{\tilde{u}_3}^2 + m_t^2 + \frac{2}{3} \sin^2 \theta_w \cos 2\beta M_Z^2 \end{pmatrix} \quad (83)$$

where θ_w is the Weinberg angle. Here, the soft supersymmetry breaking masses for the third-generation squarks, $m_{\tilde{q}_3}^2$ and $m_{\tilde{u}_3}^2$, are given by

$$m_{\tilde{q}_3}^2 \simeq m_{\tilde{q}}^2 - \frac{1}{3}(m_{H_d}^2 - m_{H_u}^2), \quad (84)$$

$$m_{\tilde{u}_3}^2 \simeq m_{\tilde{u}}^2 - \frac{2}{3}(m_{H_d}^2 - m_{H_u}^2), \quad (85)$$

while the scalar trilinear coupling, A_t , is given by

$$A_t \simeq -\frac{1}{16\pi^2} \left\{ \frac{32}{3} g_3^2 M_3 \ln\left(\frac{k'}{M_3}\right) + 6g_2^2 M_2 \ln\left(\frac{k'}{M_2}\right) + \frac{26}{15} g_1^2 M_1 \ln\left(\frac{k'}{M_1}\right) \right\}. \quad (86)$$

(vii) With the numbers obtained so far, we can calculate the radiative correction to the Higgs potential arising from the top Yukawa coupling, which is the dominant source of radiative corrections. It gives a correction to the Higgs potential

$$\delta V = \frac{\lambda_{H,\text{top}}}{2} |H_u|^2, \quad (87)$$

¹⁷ In many 5D calculations, the effective cutoff scales of 4D divergent integrals are smaller than the KK gauge-boson mass of $m_{\text{KK}} \simeq (3\pi/4)k' \simeq 2.4k'$.

where $\lambda_{H,\text{top}}$ is given, at one loop, by [41]

$$\lambda_{H,\text{top}} \simeq \frac{3y_t^4}{16\pi^2} \left[\ln\left(\frac{M_{\tilde{t}_2}^2 M_{\tilde{t}_1}^2}{m_t^4}\right) + \left(\frac{M_{\tilde{t}_2}^2 - M_{\tilde{t}_1}^2}{4m_t^2} \sin^2 2\theta_{\tilde{t}}\right)^2 f(M_{\tilde{t}_2}^2, M_{\tilde{t}_1}^2) \right. \\ \left. + \frac{M_{\tilde{t}_2}^2 - M_{\tilde{t}_1}^2}{2m_t^2} \sin^2 2\theta_{\tilde{t}} \ln\left(\frac{M_{\tilde{t}_2}^2}{M_{\tilde{t}_1}^2}\right) \right], \quad (88)$$

where $f(x, y) \equiv 2 - ((x + y)/(x - y)) \ln(x/y)$, and $\theta_{\tilde{t}}$ is the mixing angle for the top squarks needed to go from the basis of Eq. (83) to the mass eigenbasis. With this correction added to the Higgs potential, we can now iterate the procedure from (i) to (vii) until it converges. All the values listed in the table are then corrected by this iteration procedure, except $M_{H,\text{tree}}$ which is by definition a tree-level quantity. We find that the convergence is rather quick, and the corrections are not so large. (viii) Finally, the fine-tuning parameter Δ^{-1} is obtained by slightly varying the fundamental parameters of the theory and measuring the response of M_Z under that variation. This parameter is defined in Eq. (21), and the fundamental parameters a_i are taken as $k', \lambda, M_S, \kappa, L_S^2, L_S^2 C_S, A_\lambda, m_S^2, M_{\text{SUSY},1}, M_{\text{SUSY},2}$ and $M_{\text{SUSY},3}$. In most of the parameter space, the dominant contribution comes from $a_i = M_{\text{SUSY},3}, L_S^2 C_S$ or m_S^2 . In calculating Δ^{-1} we do not include the gauge and Yukawa couplings in a_i , because M_Z^2 has large generic sensitivities to these parameters. These parameters, however, are included in a_i when we calculate $\tilde{\Delta}^{-1}$ defined in Eq. (22). The parameters η_i are estimated naively from the dependence of M_Z^2 on each parameter in “generic” parameter regions. For the relevant parameters this gives: $\eta_i = 1/2$ for $\{\lambda, L_S^2 C_S, m_S^2, M_{\text{SUSY},i}, y_t\}$, $\eta_i \lesssim 1/3$ for $\{M_S, L_S^2\}$ and $\eta_i = 1/4$ for $\{g_1, g_2, g_3\}$. The dominant contribution to $\tilde{\Delta}^{-1}$ comes from $a_i = y_t$. The parameters Δ^{-1} and $\tilde{\Delta}^{-1}$ provide rough measures for fine-tuning required in our theory.

The numbers in Table 1 are subject to errors at the 10% level for the soft superparticle masses. The error could be somewhat larger, at the 20% level for M_3 , due to the strong sensitivity of g_3 to the renormalization scale. The error for the Higgs-boson mass is expected to be at the level of a few GeV. Note that, in contrast to the MSSM case, the two-loop radiative correction to the Higgs potential is not very large in our theory. This is because the top squarks are rather light, ≈ 300 GeV, so that the logarithm appearing in the radiative correction, $\ln(M_{\tilde{t}_2}^2 M_{\tilde{t}_1}^2 / m_t^4)$, is not so large. Comparing with the full two-loop calculation of the radiative correction [42], we estimate that the two-loop contribution to the physical Higgs boson mass (overestimate of M_{Higgs} in Table 1) is about 4 GeV.

The superparticle masses in the parameter points A, B and C, are listed in Table 2, in which we present the mass eigenvalues for the 2 charginos, $\chi_{1,2}^\pm$, 5 neutralinos, $\chi_{1,2,3,4,5}^0$, 3 neutral scalar Higgs bosons, $H_{1,2,3}^0$, 2 neutral pseudo-scalar Higgs bosons, $P_{1,2}^0$, and charged Higgs bosons, $H^\pm$. We also list the masses for the scalars, $\tilde{u}_L, \tilde{u}_R, \tilde{d}_L, \tilde{d}_R, \tilde{e}_L, \tilde{e}_R$ and $\tilde{\nu}_L$, which include the D -term contributions. The masses for the 2 top squarks are listed separately, as they split from the other superparticles by non-negligible amounts. All three points evade phenomenological constraints such as direct collider searches for the superparticles (the issue of evading the constraints on neutralinos for points B and C will be discussed in Section 6.1). As expected, we find that the superparticle masses

Table 2

The masses for the superparticles and the Higgs bosons for three sample points A, B and C given in Table 1. All masses are given in units of GeV

	A	B	C
$\tilde{g}$	305	307	328
$\chi_{1\pm}^{\pm}$	115	121	150
$\chi_2^{\pm}$	297	314	332
χ_1^0	103	88	56
χ_2^0	193	162	132
χ_3^0	288	221	200
χ_4^0	353	262	263
χ_5^0	365	321	336
H_1^0	140	134	130
H_2^0	298	304	332
H_3^0	872	718	802
P_1^0	305	315	343
P_2^0	888	687	799
$H^{\pm}$	288	293	323
$\tilde{u}_L$	309	317	337
$\tilde{u}_R$	294	281	289
$\tilde{d}_L$	315	322	341
$\tilde{d}_R$	272	270	285
$\tilde{e}_L$	195	195	196
$\tilde{e}_R$	203	153	105
$\tilde{\nu}_L$	186	187	188
$\tilde{t}_1$	221	214	212
$\tilde{t}_2$	385	387	404

are rather light and close to experimental bounds. We also see from the table that the superparticle spectrum is quite different from one characteristic of conventional unified theories. In particular, the hierarchy among the three gaugino masses is typically much smaller than the one arising from the unified gaugino mass relation $M_1 : M_2 : M_3 \simeq g_1^2 : g_2^2 : g_3^2$.

Because of the rather small masses for the charged Higgs boson, the $b \rightarrow s\gamma$ process could potentially give strong constraints on our theory. The branching ratio for this process is measured fairly accurately: $\text{Br}(b \rightarrow s\gamma) = (3.3 \pm 0.4) \times 10^{-4}$ at the 1σ level [43], which agrees well with the standard model prediction. The contribution from the charged Higgs boson always interferes constructively with the standard-model contribution. For a charged Higgs boson mass in the range $\approx 250\text{--}350$ GeV, which covers the values obtained in the points presented in Table 2, the next-to-leading order QCD calculation gives the sum of the contributions from the standard model and the charged Higgs boson at the level $\text{Br}(b \rightarrow s\gamma) \simeq (4\text{--}5) \times 10^{-4}$ [44], which is somewhat larger than the observed value. There is, however, also a contribution from chargino loops. In our theory, this contribution

interferes destructively (constructively) with the standard-model one if the sign of μ_{eff} is positive (negative). This, therefore, prefers the positive sign for μ_{eff} (and thus certain signs for the fundamental parameters). With $\mu_{\text{eff}} > 0$, we find that the prediction for $\text{Br}(b \rightarrow s\gamma)$ in our theory can naturally be consistent with experimental data. The contributions from neutralino and gluino loops are negligible.

Finally, we emphasize that it is significant that our theory reduces the fine-tuning down to the level $\Delta^{-1} = O(10\text{--}20\%)$ ($\tilde{\Delta}^{-1} = O(20\text{--}30\%)$) as presented in the table. As we have seen in Section 2, most existing supersymmetry breaking scenarios leads to a fine-tuning at the 3% level or even worse. We have also seen that even with rather general superparticle masses, the fine-tuning is still worse than about 5% in the MSSM. Our theory does not need such an accurate cancellation among different parameters. In fact, we expect that the level of tuning (given by the sizes of Δ^{-1} and $\tilde{\Delta}^{-1}$) obtained in Table 1 is close to the best we can attain in theories that accommodate the MSSM sector in a perturbative way.

6. Phenomenological issues

In this section we discuss some of the phenomenological issues in our theory, focusing on neutralino phenomenology and pedestrian dark matter in particular.

6.1. Neutralino phenomenology

As we saw in the previous subsection, our theory generically predicts light superparticles, and the neutralinos can be particularly light. We here consider the phenomenology of the neutralino sector.

Let us start with the point A in Tables 1 and 2. For this point, the mass of the lightest neutralino is larger than half of the LEP II center-of-mass energy $\sqrt{s} \simeq 200$ GeV, so there is no constraint from direct searches. On the other hand, for the sample points B and C, the mass of the lightest neutralino is around 90 GeV and 60 GeV, respectively (see Table 2). Such light neutralinos could be dangerous, as they contain non-negligible Higgsino components and are easily produced at e^+e^- colliders through s -channel Z exchanges. Whether these light neutralinos evade experimental constraints or not, then, depends on their compositions and decay channels.

The masses and compositions of the lightest neutralino, χ_1^0 , for three sample points A, B and C are given in Table 3. Here, the coefficients $c_{1,\tilde{B}}, c_{1,\tilde{W}_3}, c_{1,\tilde{H}_d}, c_{1,\tilde{H}_u}$ and $c_{1,\tilde{S}}$ are defined through the relations between the mass and gauge eigenstates:

$$\chi_i = c_{i,\tilde{B}}\tilde{B} + c_{i,\tilde{W}_3}\tilde{W}_3 + c_{i,\tilde{H}_d}\tilde{H}_d + c_{i,\tilde{H}_u}\tilde{H}_u + c_{i,\tilde{S}}\tilde{S}, \quad (89)$$

Table 3

The masses and compositions of the lightest neutralino χ_1^0 for the three sample points A, B and C given in Table 1

	$m_{\chi_1^0}$	$\{c_{1,\tilde{B}}, c_{1,\tilde{W}_3}, c_{1,\tilde{H}_d}, c_{1,\tilde{H}_u}, c_{1,\tilde{S}}\}$
A	103 GeV	$\{-0.15, 0.45, -0.60, 0.63, -0.14\}$
B	88 GeV	$\{0.20, -0.33, 0.36, -0.69, 0.50\}$
C	56 GeV	$\{0.16, -0.15, -0.060, -0.54, 0.81\}$

where $i = 1, \dots, 5$, and c 's are normalized as $c_{i,\tilde{B}}^2 + c_{i,\tilde{W}_3}^2 + c_{i,\tilde{H}_d}^2 + c_{i,\tilde{H}_u}^2 + c_{i,\tilde{S}}^2 = 1$. From the table, one sees that the Higgsino components in χ_1^0 are in fact non-negligible. This is because the sizes of μ_{eff} and M_1 are comparable for these parameter points (see Table 1). One also finds that χ_1^0 generically contains non-negligible amounts of the fermionic component of S .

As we already discussed, the point A evades direct search constraints regardless of the decay of χ_1^0 , but this is not true for the points B and C. What are the decay modes of χ_1^0 ? Since χ_1^0 is R -parity odd, its decay products must include an R -parity odd particle with mass smaller than $m_{\chi_1^0}$. An obvious candidate for such particle is the gravitino, which is generically very light, with mass $m_{3/2}$ given by

$$m_{3/2} \simeq \frac{F_Z'^2}{M_{\text{Pl}}} \simeq 0.1\text{--}10 \text{ eV}, \quad (90)$$

where $F_Z' \equiv F_Z e^{-2\pi k R} \approx \{10\text{--}100 \text{ TeV}\}^2$ is the rescaled supersymmetry breaking scale (see, e.g., Eq. (59)). If this were the dominant decay channel, these points would be excluded, because then a significant fraction of the decay of χ_1^0 would go to the gravitino and some visible particles, such as the photon, and such a signal would have already been observed at LEP II. In our theory, however, χ_1^0 can also decay into a pair of the pedestrian fields, depending on the masses of these fields. Therefore, it is not obvious that these points are excluded by the present experimental data.

To illustrate this point, we introduce a Planck-brane pedestrian field P'' which has sufficiently small mixing with P , i.e., a sufficiently small coefficient for the superpotential term of the form $W = S P P''$. The superpotential for this field is then given by

$$W \simeq \frac{h''}{2} S P'^2 + \frac{M_{P''}}{2} P''^2. \quad (91)$$

In order for P'' not to have a VEV, we need $h'' \langle F_S \rangle \lesssim |M_{P''}|^2$. The lightest neutralino can then decay into fermionic and scalar components of P'' , if $|M_{P''}| \lesssim m_{\chi_1^0}/2$. Since $\langle F_S \rangle \simeq (200 \text{ GeV})^2$ in generic parameter regions, this implies that $h'' \lesssim 0.05$ and 0.02 for the points B and C, respectively. Such small couplings do not affect the RG analysis of Section 5.1.

With the coupling of S to P'' in Eq. (91), the constraints from direct searches are evaded. The constraints from the Z -pole data at LEP I are also easily evaded due to phase-space suppression and the fact that the dominant decay of χ_1^0 is invisible. At LEP II with $\sqrt{s} \simeq 200 \text{ GeV}$, two on-shell χ_1^0 's can be produced. However, since the branching ratio of χ_1^0 decay into visible particles is smaller than about 10^{-4} , this process evades detection and thus the corresponding parameter region is not excluded.

For parameter point C, there is a potential danger coming from $\chi_1^0\text{--}\chi_2^0$ associated production. While the production rate receives a suppression of about 0.03 (see the compositions of χ_2^0 in Table 4), this still requires the branching ratio of visible χ_2^0 decays to be smaller than about 3%, if the production occurs with full strength. The precision of our calculation, however, allows errors of order $O(10\%)$ for the masses, so we cannot conclude that the production actually occurs for this parameter point. In any event, since χ_2^0

Table 4

The masses and compositions of the next-to-lightest neutralino χ_2^0 for three sample points A, B and C given in Table 1

	$m_{\chi_2^0}$	$\{c_2, \tilde{B}, c_2, \tilde{W}_3, c_2, \tilde{H}_d, c_2, \tilde{H}_u, c_2, \tilde{S}\}$
A	193 GeV	{0.021, -0.046, -0.69, -0.67, -0.25}
B	162 GeV	{-0.16, 0.26, -0.57, 0.086, 0.76}
C	132 GeV	{-0.61, 0.29, -0.57, 0.31, 0.34}

dominantly decays into χ_1^0 and visible particles, this process constrains the sum of the masses of χ_1^0 and χ_2^0 to be larger than about 200 GeV so that their production at LEP II is suppressed.

6.2. Pedestrian dark matter

Because of the unbroken P parity, the lightest pedestrian field is absolutely stable. It may therefore constitute the dark matter of the universe, depending on the parameters of the model. Here we discuss this issue for the simplest case of a single Planck-brane pedestrian field, discussed in Section 4.2 (without additional fields P'').

Since the bulk pedestrian fields receive a large supersymmetric mass of order 10 TeV on the TeV brane, these fields are much heavier than the Planck-brane pedestrian multiplet P' . The lightest pedestrian field is thus a component of P' . In the model of Section 4.2, the scalar component of P' obtains a soft supersymmetry-breaking mass of about 600–800 GeV (see Eq. (79)) while the mass of the fermionic component is given by $h\langle S \rangle + M_{P'}$. Since $\langle S \rangle \simeq 200$ GeV and $h \lesssim 0.4$ (from the RG analysis; see Section 5.1), we expect that the lightest pedestrian field is the fermionic component of P' , $\psi_{P'}$, for a wide range of $M_{P'}$, which is essentially a free parameter of the theory.

In the minimal case considered here, the annihilation of $\psi_{P'}$ occurs through s -channel exchange of the S scalar, with mass about 700–800 GeV, or through t -channel exchange of $\psi_{P'}$ using the mixing between the S and Higgs scalars. Let us assume, for simplicity, that the mass of $\psi_{P'}$ is larger than the Higgsino mass so that the annihilation into two Higgsinos are kinematically allowed. In this case, the dominant contribution to the annihilation comes from the diagram with the s -channel S -scalar exchange, giving the thermally averaged cross section of order $\langle \sigma v \rangle \simeq ((\lambda h)^2 / 8\pi) (m_{\psi_{P'}}^2 / m_S^4)$. This can easily give the correct abundance for dark matter in our generic parameter region, $\lambda \simeq 0.8$, $h \lesssim 0.4$ and $m_S \simeq 700$ –800 GeV with $m_{\psi_{P'}}$ essentially a free parameter in a range $m_{\psi_{P'}} < 600$ –800 GeV. Note that $\psi_{P'}$ annihilation into the two Higgsinos is not subject to the p -wave suppression, so that we can naturally obtain the correct dark matter abundance with the masses of $O((100 \text{ GeV}) - (1 \text{ TeV}))$. This implies that the relic abundance does not change much even in the case that the pedestrian dark matter is a Dirac fermion. The annihilation rate, however, can be enhanced for $M_{\psi_{P'}} \simeq m_S/2 \simeq 400$ GeV due to the s -channel S -scalar pole, resulting in a significant reduction of the relic abundance.

7. Purely 4D realizations

In this section we present an outline for constructing purely 4D theories with reduced fine-tuning. First, we make the logarithm in Eq. (3) smaller by requiring a low mediation scale of supersymmetry breaking. In particular, we consider theories in which the fundamental scale of supersymmetry breaking is of order 100 TeV. Such theories, with supersymmetry breaking mediated by standard-model gauge interactions, were constructed, for example, in Refs. [45,46]. Here, we adopt the basic construction of [46] to illustrate our point.

The DSB sector consists of an $SP(2)$ gauge theory with 6 chiral superfields $\tilde{Q}_i$ ($i = 1, \dots, 6$) in the fundamental 4-dimensional representation, together with 15 singlets Z^a ($a = 1, \dots, 14$) and Z . With the tree-level superpotential $W = \lambda' Z^a (\tilde{Q}\tilde{Q})_a + \lambda Z (\tilde{Q}\tilde{Q})$, where $(\tilde{Q}\tilde{Q})_a$ denotes a flavor 14-plet of $SP(3)_{\text{flavor}}$ unbroken after the inclusion of the superpotential, supersymmetry is broken. For a certain parameter region, the supersymmetry breaking VEV is given by $F_Z \simeq \lambda \Lambda^2$, where Λ is the dynamical scale of $SP(2)$ gauge interactions. We assume throughout that the Z field does not have a VEV, $\langle Z \rangle = 0$. In fact, this point is (at least) a local minimum of the potential [47].

Supersymmetry breaking is mediated to the SSM sector both by vector-like mediator fields, which are charged under both 321 and $SP(2)$ gauge interactions, and by vector-like messenger fields, which are charged only under 321. In particular, we introduce mediator fields $\mathcal{D}(\mathbf{3}^*, \mathbf{1})_{1/3}$, $\bar{\mathcal{D}}(\mathbf{3}, \mathbf{1})_{-1/3}$, $\mathcal{L}(\mathbf{1}, \mathbf{2})_{-1/2}$ and $\bar{\mathcal{L}}(\mathbf{1}, \mathbf{2})_{1/2}$ that are in the fundamental representation of the supersymmetry-breaking $SP(2)$ gauge group. Here, the numbers in parentheses denote quantum numbers under 321. We also introduce messenger fields $\mathcal{D}'(\mathbf{3}^*, \mathbf{1})_{1/3}$, $\bar{\mathcal{D}}'(\mathbf{3}, \mathbf{1})_{-1/3}$, $\mathcal{L}'(\mathbf{1}, \mathbf{2})_{-1/2}$ and $\bar{\mathcal{L}}'(\mathbf{1}, \mathbf{2})_{1/2}$ that are singlet under the $SP(2)$ gauge group. The superpotential for these fields is given by

$$W = m_{\mathcal{D}} \mathcal{D} \bar{\mathcal{D}} + m_{\mathcal{L}} \mathcal{L} \bar{\mathcal{L}} + (m'_{\mathcal{D}} + k_{\mathcal{D}} Z) \mathcal{D}' \bar{\mathcal{D}}' + (m'_{\mathcal{L}} + k_{\mathcal{L}} Z) \mathcal{L}' \bar{\mathcal{L}}', \quad (92)$$

where $m_{\mathcal{D}}$, $m_{\mathcal{L}}$, $m'_{\mathcal{D}}$, $m'_{\mathcal{L}}$ are mass parameters that are assumed to be of order $4\pi\Lambda$.¹⁸ An important point is that we take $m_{\mathcal{D}} \neq m_{\mathcal{L}}$ ($m_{\mathcal{D}} > m_{\mathcal{L}}$) and $m'_{\mathcal{D}}/k_{\mathcal{D}} \neq m'_{\mathcal{L}}/k_{\mathcal{L}}$ ($m'_{\mathcal{D}}/k_{\mathcal{D}} > m'_{\mathcal{L}}/k_{\mathcal{L}}$), so that the mediator/messenger sector does not respect an approximate $SU(5)$ symmetry. As we will see, this breaks the unwanted mass relation of Eq. (5). The successful prediction for gauge coupling unification, on the other hand, is preserved because the mediator and messenger fields fill complete multiplets of $SU(5)$. Breaking the $SU(5)$ structure in the messenger sector of gauge mediation models was also considered in [48] to reduce fine-tuning. Note that our choice of mediator fields corresponds to 4 pairs of $\mathbf{5} + \mathbf{5}^*$ under $SU(5)$, so that we have 5 pairs of $\mathbf{5} + \mathbf{5}^*$ in total. This makes 321 strongly coupled, $g_a \sim 4\pi$, at the unification scale, as in the models of Section 4.

¹⁸ These mass parameters may be generated by the nonperturbative dynamics of some new gauge interaction. The dynamical scale of this new gauge theory may be related to that of the supersymmetry-breaking $SP(2)$ gauge interaction if there are massive fields that are charged under both gauge groups and which decouple at a scale somewhat larger than $4\pi\Lambda$. A simple example of such behavior arises if the couplings of both gauge groups are rather large but their beta functions small above the scale where the massive fields decouple.

The masses of the MSSM gauginos are generated when the messenger fields are integrated out at the scale $4\pi\Lambda$. The 321 gaugino masses, M_a , are given by

$$\begin{aligned} M_1 &\simeq \frac{g_1^2}{16\pi^2} \left(\frac{2}{5} \frac{k_{\mathcal{D}} F_Z}{m'_{\mathcal{D}}} + \frac{3}{5} \frac{k_{\mathcal{L}} F_Z}{m'_{\mathcal{L}}} \right), \\ M_2 &\simeq \frac{g_2^2}{16\pi^2} \frac{k_{\mathcal{L}} F_Z}{m'_{\mathcal{L}}}, \quad M_3 \simeq \frac{g_3^2}{16\pi^2} \frac{k_{\mathcal{D}} F_Z}{m'_{\mathcal{D}}}. \end{aligned} \quad (93)$$

The sfermion masses, on the other hand, receive contributions both from mediator and messenger fields and are given by

$$m_{\tilde{f}}^2 \simeq 2 \sum_{a=1,2,3} \frac{g_a^4 C_a^{\tilde{f}}}{(16\pi^2)^2} \left(\frac{2|\lambda|^2 |F_Z|^2}{\tilde{m}_a^2} + \tilde{m}_a'^2 \right), \quad (94)$$

where $\tilde{f} = \tilde{q}, \tilde{u}, \tilde{d}, \tilde{l}, \tilde{e}$, and $C_a^{\tilde{f}}$ are given by $(C_1^{\tilde{f}}, C_2^{\tilde{f}}, C_3^{\tilde{f}}) = (1/60, 3/4, 4/3), (4/15, 0, 4/3), (1/15, 0, 4/3), (3/20, 3/4, 0)$ and $(3/5, 0, 0)$ for $\tilde{f} = \tilde{q}, \tilde{u}, \tilde{d}, \tilde{l}$ and $\tilde{e}$, respectively. The mass parameters $\tilde{m}_a^2$ and $\tilde{m}_a'^2$ in Eq. (94) are defined by

$$\frac{1}{\tilde{m}_1^2} \equiv \frac{2}{5} \frac{1}{|m_{\mathcal{D}}|^2} + \frac{3}{5} \frac{1}{|m_{\mathcal{L}}|^2}, \quad \frac{1}{\tilde{m}_2^2} \equiv \frac{1}{|m_{\mathcal{L}}|^2}, \quad \frac{1}{\tilde{m}_3^2} \equiv \frac{1}{|m_{\mathcal{D}}|^2}, \quad (95)$$

and

$$\tilde{m}_1'^2 \equiv \frac{2}{5} \left| \frac{k_{\mathcal{D}} F_Z}{m'_{\mathcal{D}}} \right|^2 + \frac{3}{5} \left| \frac{k_{\mathcal{L}} F_Z}{m'_{\mathcal{L}}} \right|^2, \quad \tilde{m}_2'^2 \equiv \left| \frac{k_{\mathcal{L}} F_Z}{m'_{\mathcal{L}}} \right|^2, \quad \tilde{m}_3'^2 \equiv \left| \frac{k_{\mathcal{D}} F_Z}{m'_{\mathcal{D}}} \right|^2. \quad (96)$$

We find that for $m_{\mathcal{D}} > m_{\mathcal{L}}$ and $m'_{\mathcal{D}}/k_{\mathcal{D}} > m'_{\mathcal{L}}/k_{\mathcal{L}}$, the masses of the colored particles are suppressed relative to those obtained from the masses of non-colored particles using the unified mass relations.

The Higgs sector of our model contains the two Higgs doublets of the MSSM, H_u and H_d , and a singlet S . We assume the presence of a superpotential term of the form $W = \lambda S H_u H_d$, and possibly of an additional term $\delta W = (\kappa/3) S^3$. The other terms in the Higgs-sector superpotential are effectively generated through couplings to fields in the mediator and the DSB sectors. Suppose there are tree-level superpotential interactions

$$W = h \tilde{Q} \mathcal{L} H_u + \bar{h} \tilde{Q} \tilde{\mathcal{L}} H_d. \quad (97)$$

Then, the effective μ -term is generated after integrating out the mediator fields as $W \simeq -h\bar{h} \langle ((\tilde{Q} \tilde{\mathcal{Q}}) / m_{\mathcal{L}}) H_u H_d \rangle \equiv \mu H_u H_d$. Since $\mu \simeq h\bar{h}\Lambda/4\pi$, we obtain the weak-scale μ -term for $h\bar{h} = \mathcal{O}(0.01\text{--}0.1)$. With these values of h and $\bar{h}$ (and taking $h \sim \bar{h}$), radiatively generated soft supersymmetry-breaking Higgs masses, $m_{H_u, H_d}^2 \simeq (h^2/16\pi^2)(|\lambda|^2 \Lambda^4 / m_{\mathcal{L}}^2) \simeq (h\Lambda/16\pi^2)^2$, are sufficiently small. The μB term is zero at tree level, but it is generated at radiative level.

We now introduce a mediator field $\mathcal{N}(\mathbf{1}, \mathbf{1})_0$, which is in the fundamental representation of $SP(2)$, and the superpotential

$$W = \frac{m_{\mathcal{N}}}{2} \mathcal{N}^2 + k \tilde{Q} \mathcal{N} S, \quad (98)$$

in parallel with Eqs. (92), (97), where $m_{\mathcal{N}}$ is a mass parameter of order $4\pi\Lambda$. Integrating out the $\mathcal{N}$ field, we obtain an effective supersymmetric mass for S : $W \simeq -(k^2/2)((\tilde{Q}\tilde{Q})/m_{\mathcal{N}})S^2 \equiv (\mu_S/2)S^2$. By choosing the values of k and $m_{\mathcal{N}}$ appropriately, μ_S can be made to be of order the weak scale. The superpotential of our Higgs sector is thus given by $W = \lambda S H_u H_d + \mu H_u H_d + (\mu_S/2)S^2 + (\kappa/3)S^3$. By shifting the S field such that the coefficient of the $H_u H_d$ term becomes zero, we can write the superpotential in the form of

$$W = \lambda S H_u H_d + L_S^2 S + \frac{M_S}{2} S^2 + \frac{\kappa}{3} S^3, \quad (99)$$

where L_S and M_S are mass parameters of order the weak scale. This is the general form of the Higgs-sector superpotential we used in our analysis of electroweak symmetry breaking.

The soft supersymmetry-breaking operator of the form $\mathcal{L} = |S|^2$ is also generated through loops of $\mathcal{N}$ and $\tilde{Q}$, with the coefficient of order $(k^2/16\pi^2)(|\lambda F_Z|^2/m_{\mathcal{N}}^2) \simeq (k\Lambda/16\pi^2)^2$ (before the shift of S). The sign of this coefficient is incalculable due to strong $SP(2)$ gauge interactions. After shifting S , we obtain soft supersymmetry-breaking interactions of the form

$$\mathcal{L}_{\text{soft}} = -m_{H_u}^2 |H_u|^2 - m_{H_d}^2 |H_d|^2 - m_S^2 |S|^2 - (L_S^2 C_S S + \text{h.c.}). \quad (100)$$

We thus almost reproduce the general supersymmetry-breaking terms in the Higgs sector used in our previous analysis.¹⁹

We have seen that all the essential ingredients needed to reduce fine-tuning can be reproduced in our 4D theory described here. In this theory, some of the parameters, especially those in the Higgs sector, are incalculable because of strong gauge interactions in the DSB sector. While we do not perform a complete analysis of electroweak symmetry breaking here, we expect that there is a parameter region in which fine-tuning is reduced in this theory or a modified/extended version.

8. Conclusions

In this paper we have constructed supersymmetric theories that do not suffer from significant fine-tuning in obtaining realistic electroweak symmetry breaking. In these theories, supersymmetry is dynamically broken at relatively low scale of order 10–100 TeV, which is then transmitted to the SSM sector through standard-model gauge interactions. The spectrum of superparticles does not respect unified mass relations because of the breaking or absence of unified symmetry in the supersymmetry breaking sector. The Higgs sector of our theories contains a singlet field S in addition to the MSSM two Higgs doublets H_u and H_d , with general superpotential interactions given in Eq. (1). Such a superpotential can naturally arise through singlet fields that interact both with the S field and the supersym-

¹⁹ If we could somehow induce the supersymmetry-breaking operator of the form $\mathcal{L} = F_S^\dagger S + \text{h.c.}$ with the weak-scale coefficient, we would completely reproduce the structure of the Higgs sector discussed in the previous sections.

metry breaking sector. The lightest of these singlets may be stable due to an unbroken Z_2 symmetry, and thus may constitute the dark matter of the universe.

We have constructed an explicit model in warped space, and studied its properties. We have analyzed electroweak symmetry breaking of the model in detail, performing a renormalization group analysis and a minimization of the Higgs potential. This allowed us explicitly to demonstrate that the model allows parameter regions in which the fine-tuning associated with electroweak symmetry breaking, defined as a fractional change of the weak scale in response to fractional changes of fundamental parameters of the theory, is reduced to the 20% level. This is a significant improvement over conventional supersymmetry breaking scenarios, which typically require fine-tuning of order 2–3% or even worse. The parameter region with reduced fine-tuning requires the superparticles to be relatively light, so that these particles must be seen at the LHC. We have explicitly calculated the spectra of superparticles in a few sample points in parameter space, and discussed some of their phenomenological aspects, including neutralino phenomenology and pedestrian dark matter.

We have also presented a theory constructed purely in 4D, which reproduces structures for the superparticle spectrum and the Higgs sector similar to those of our warped-space model. This theory possesses all the essential features necessary to reduce fine-tuning, and thus can potentially give natural electroweak symmetry breaking.

While we have worked in the context of explicit models, some of our analysis and considerations are general. We hope that these results are useful for advancing our understanding of electroweak symmetry breaking in supersymmetric theories.

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