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RADIOFREQUENCY POWER LOSSES COMPARED FOR FOUR LINEAR ACCELERATOR CONFIGURATIONS

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Robert A. Weir

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September 30, 1955

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ABSTRACT

The rf power losses per Mev particle energy gain for four possible deuteron-accelerating devices are computed over a deuteron energy range of $1 \leq KE \leq 400$ Mev. The configurations considered are (1) the conventional drift-tube-type linear accelerator, (2) a series of "pillbox" resonators each oscillating 180° out of phase with its successor and each containing two half drift tubes, (3) a series of pillbox resonators with no drift tubes, and (4) a helical wave guide.

Values for the frequency, electric field strength, and other parameters of the various configurations considered needed for the calculation of the numerical results in this report were chosen which compare with those which presumably will be employed in further accelerator construction at Livermore.

The numerical results indicate that in the deuteron energy range l < KE < 40 Mev Cases (A) and (D) are relatively efficient, while in the range 200 < KE < 400 Case (B) is relatively efficient. Of particular interest is the point at which the ascending power loss curve of Case (A) crosses the descending curve of Case (B). This was found to be in the neighborhood of 125 Mev. Case (C) was found to be comparatively inefficient.

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Robert A. Weir

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INTRODUCTION

The object of this report is to present a set of calculated rf power losses for various possible deuteron-accelerating devices. The results should give an insight into the relative effectiveness of the following possible machines over a particle energy range of $l \leq KE \leq 400$ Mev. As a basis of comparison each of the various configurations will be assumed to impart the same energy gain to a particle per unit length of the machine, or, in other words, aside from small differences due to different transit-time factors, a length $\beta\lambda$ represents the same particle energy gain for each device.

Four configurations are considered (see Fig. 1):

(A) Conventional drift-tube-type linear accelerator. Repeat length $\beta\lambda$ and g/L = 0.25.

(B) A series of pillbox resonators, $1/2 \beta \lambda$ long and each 180° out of phase with its successor. Each pillbox contains two half drift tubes, and the pillboxes are separated from each other by walls. The distance between gap centers in two adjacent pillboxes represents half an rf cycle.

(C) A series of separate pillboxes without drift tubes. In this case a whole resonator acts as an accelerating gap.

(D) A helical wave guide. The accelerating electric field is a traveling wave, the phase velocity of which is equal to the particle velocity.

The determination of the power losses per Mev particle energy gain in Cases (A) and (B) is based on the method of Walkinshaw, Sabel, and Outram.¹ Use was made of the curves presented by these authors in their report, but additional values needed here which lay outside the range of the curves presented were calculated. The curves, when employed, were scaled to our assumed wave length of 6.2 meters. These curves used for Cases (A) and (B) are Figs. 3a and 6a respectively in Reference 1. For convenience the curves used in Reference 1 are included at the end of this paper as Figs. 2 and 3.

The various parameters assumed in this report were chosen as representative of those contemplated for possible accelerator construction at Livermore.

For Cases (A) and (B) these are:

 E_0 = gap gradient = 100 kv/in (3.94 x 10⁶ volts/meter)

 $\lambda = 6.2$ meters (free-space wave length)



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Fig. 1. Four possible configurations for a linear accelerator. (A) conventional, g/L = 0.25;

- (B) $1/2 \beta \lambda$ pillbox, g/L = 0.25;
- (B') $1/2 \beta \lambda$ pillbox, g/L = 0.5;
- (C) separate pillboxes without drift tubes;

(D) helix.

Stem diameters = 5.25 inches (case A)

Tank length = 10 wave lengths (case A)

 $d/\lambda = 0.1$ (d = drift-tube diameter)

 $\cos \phi_s = 0.9$ ($\phi_s = \text{synchronous phase angle}$) for KE ≥ 5 Mev.

The numerical results presented here will be somewhat on the optimistic side, as power dissipation caused by electron loading and imperfections in the metal surfaces are not taken into account. Experience has shown that all actual power losses are about 30% greater than those obtained theoretically.

-5 -

(A) CONVENTIONAL DRIFT-TUBE MACHINE

The formula employed to compute the power losses for the velocity range in which the shunt impedance curves were calculated as presented in Reference 1 is

$$P = \frac{\overline{E}}{T \cos \phi_s} \left\{ \frac{10^{-3}}{Q \eta} + \frac{R_s \pi b J_o^2(k\frac{d}{2}) \times 10^3}{Z_o^2 \beta \lambda} \int_{d/2}^{D/2} \left[\frac{F_1(kr)}{F_o(k\frac{d}{2})} \right]^2 dr \right\}$$

$$\frac{2\pi R_{s} J_{o}^{2} (k \frac{d}{2}) \times 10^{3}}{10 \lambda Z_{o}^{2}} \int_{d/2}^{D/2} \left[\frac{F_{1} (kr)}{F_{o} (k \frac{d}{2})} \right]^{2} r dr$$

In this formula Q is the scaling factor. The shunt impedance curves in Reference 1 are given for $d/\lambda = 0.1$ and $\lambda = 1.5$ meters. The shunt impedance scales to a good approximation as $1/\sqrt{\lambda}$. Since our assumed wave length is 6.2 meters, the scaling factor is $Q = (4.13)^{-1/2} = 1/2.03$.

Also,

P = power loss in kw/Mev particle energy gain;

 η = shunt impedance in megohm/meter as given in Fig. 3a(g/L = 0.25) of Reference 1;

b = stem diameter;

 Z_0 = impedance of free space;

 R_s = characteristic resistance of copper surface = 2.61 x 10⁻⁷/ $\overline{\nu}$, where ν = frequency;

$$\overline{\mathbf{E}}$$
 = average gradient = $\mathbf{E}_{0}/4$;

 $k = 2\pi/\lambda;$

T = transit-time factor;

 $F_{o}(kr) = J_{o}(kr)Y_{o}(k\frac{D}{2}) - Y_{o}(kr)J_{o}(k\frac{D}{2}),$

$$F_{1}(kr) = J_{1}(kr)Y_{1}(k\frac{D}{2}) - Y_{1}(kr)J_{1}(k\frac{D}{2});$$

D = tank diameter. The tank diameter can be estimated by the relation

$$\frac{J_{1}(k\frac{d}{2})}{J_{0}(k\frac{d}{2})} = \frac{g}{L} \frac{F_{1}(k\frac{d}{2})}{F_{0}(k\frac{d}{2})}$$

This is an approximation to the more exact expression for the tank diameter given in the appendix of Reference 1. It is valid for small β .

Since the power losses are presented in this report in terms of kw per Mev gain they will depend--especially in the lower-energy end of the machine--on the transit-time factor and on the synchronous phase angle. For the sake of completeness a short resumé on the derivations of these quantities as used in this paper follows.

Transit Time Factor

The drift tubes are approximated by cylinders of radius a, the electric field across the gap being assumed constant at r = a. With these boundary conditions the solution of Laplace's equation is

$$V(r, z) = E(z + \frac{L}{\pi} \Sigma \frac{T_n}{n} \sin \left(\frac{2\pi n z}{L}\right) I_0 \left(\frac{2\pi n r}{L}\right))$$

and

$$E_{z}(r, z) = 1 + 2 \Sigma T_{n} I_{o} \left(\frac{2\pi nz}{L}\right) \cos \left(\frac{2\pi nz}{L}\right),$$

where E = the design gradient.

We have

$$T_{n} = \frac{\sin (\pi ng)/L}{(\pi ng)/L} \frac{1}{I_{0}(2\pi na)/L} = \frac{\sin (\pi ng/L)}{\pi ng/L} \frac{1}{I_{0}(2\pi na/L)} ,$$

where

e a is the drift-tube bore radius.

The energy change across a gap for the synchronous particle is $\Delta W = e \int_{-L/2}^{L/2} E_z \cos (\omega t + \phi_s) dz,$

which, when we set in the expression for $\mathbf{E}_{\mathbf{z}}$ and integrate, becomes

$$\Delta W = e E L T_1 I_0 \left(\frac{2\pi r}{L}\right) \cos \phi_s.$$

At r = 0, $I_0 \left(\frac{2\pi r}{L}\right) = 1$, therefore the axial transit time factor is $\sin\left(\frac{\pi g}{L}\right)$

$$T = \frac{1}{I_o \left(\frac{2\pi a}{L}\right)} \quad \frac{\sin(L)}{\frac{\pi g}{L}}$$

Synchronous Phase Angle

The particle energy gain for the synchronous particle is

 $\Delta W_{s} = e E \beta_{s} \lambda T \cos \phi_{s}$.

The difference between ΔW_s and the energy gain for another particle in the phase-stable region is

$$\delta \Delta W = e E \beta \lambda T (\cos \phi - \cos \phi_s) \cong mc^2 \Delta(\beta_s \delta \beta)$$

and

$$\Delta \phi = - \frac{2\pi \,\delta\beta}{\beta_{\rm s}}$$

From this one obtains the phase equation for the particle by replacing Δ by d/dn, where n is the section number,

$$\frac{d}{dn} \left(\beta_{s}^{2} \frac{d\phi}{dn}\right) = \frac{2\pi e E \beta_{s} \lambda T}{mc^{2}} (\cos \phi_{s} - \cos \phi).$$

To find the limits of phase stability let $\beta_{\rm S}$ be constant and integrate once, obtaining

$$\frac{\beta_{\rm s}}{2}\left(\frac{{\rm d}\phi}{{\rm d}n}\right)^2 - \frac{2\pi e E\lambda T}{mc^2}\left(\phi\cos\phi_{\rm s}-\sin\phi\right) = c.$$

The limits of phase stability are the values of φ which cause φ cos φ_S - sin φ to be a maximum. These two values are

$$\begin{aligned} \phi_1 &= -\phi_s, \\ \phi_2 &= 2 \phi_s, \end{aligned}$$
 which means a total phase acceptance of 3 $|\phi_s|$.

As the particle energy increases in an accelerator the phase oscillations are damped in amplitude. This damping as a function of β can be ascertained approximately from the phase oscillation equations as follows:

$$\frac{\mathrm{d}}{\mathrm{dn}} \left(\beta_{\mathrm{s}}^{2} \frac{\mathrm{d}\phi}{\mathrm{dn}}\right) \cong -\beta_{\mathrm{s}}^{2} \omega^{2} \left(\phi - \phi_{\mathrm{s}}\right), \quad \omega^{2} = -\frac{2\pi e \mathrm{E}\lambda T}{\mathrm{mc}^{2}\beta_{\mathrm{s}}} \sin \phi_{\mathrm{s}},$$
$$\frac{\mathrm{d}^{2}\phi}{\mathrm{dn}^{2}} + 2\frac{\beta_{\mathrm{s}}}{\beta_{\mathrm{s}}} \frac{\mathrm{d}\phi}{\mathrm{dn}} = -\omega^{2} \left(\phi - \phi_{\mathrm{s}}\right).$$

Letting

$$B_{s}(\phi - \phi_{s}) = \psi,$$

we have

Neglecting β_s''/β_s with respect to ω^2 , we have the WKB solution:

$$\psi = \beta_{s}(\phi - \phi_{s}) \sim [\omega]^{-1/2} \exp[\pm i] \omega dn].$$

The amplitude of the phase oscillation is damped approximately as

$$\left|\phi-\phi_{s}\right|\sim\beta_{s}^{-3/4}$$
.

 $\psi'' = -\psi(\omega\omega^2 - \frac{\beta s''}{\beta s})$

To get an expression for the synchronous phase angle as a function of $\boldsymbol{\beta}$ we set

$$3 \left| \phi_{\rm s} \right| = a \beta_{\rm s}^{-3/4}.$$

The initial phase acceptance of the A tank of the A-48 linear accelerator at Livermore is 135°. This section operates in the energy range of $0.9 \leq \text{KE} \leq 3.69$. The synchronous phase angle at 0.9 MeV is therefore -45° . Since 0.9 MeV corresponds to $\beta = 0.033$ for deuterons, the constant a can be evaluated and the following equation obtained for the absolute value of the synchronous phase angle as a function of β :

$$\phi_{\rm s} = 0.0607 \ \beta_{\rm s}^{-3/4}$$
.

This estimation of the value of ϕ_s will be employed until $\cos \phi_s = 0.9$, which is the contemplated design parameter for the higher-energy range of the linear accelerator at Livermore.

Setting in the numerical values for the various parameters, we get the following for the power loss in kw per Mev particle energy gain:

$P = \frac{0.984}{T\cos\phi_s} \left[\frac{2030}{\eta} + \frac{0.51}{\beta} \right]$	¹ + 0.86 ,
$T = \frac{1}{I_o(\frac{2\pi a}{\beta\lambda})} \frac{\sin \frac{\pi}{4}}{\frac{\pi}{4}} =$	$\frac{0.9}{I_{o}(\frac{0.0386}{\beta})};$
$\phi_{\rm s} = 0.0607 \ \beta_{\rm s}^{-3/4}$,	l < KE < 5 Mev;
$\phi_{s} = 0.45,$	5 < KE < 100 Mev.

The first term in the bracket is the contribution to the power losses from the drift tubes and side walls of the tank. The second and third terms represent contributions from the drift-tube stems and end walls respectively. If the stem diameters are increased by a factor F, the second term increases by this same factor. If the tank length is decreased by a factor F, the third term increases by this same factor.

One will notice on inspection of the power-loss curve for this case that the rf power losses per Mev particle energy gain reach a minimum of particle energies of about 10 Mev. In the low-energy range of the machine the relative contribution to the total power losses from the stems is rather heavy. Moreover the efficiency of the machine is further reduced as the transit time factor is relatively low and the synchronous phase angle negatively large. All this rapidly improves, however, as the particle energy becomes greater and the bunching of the beam becomes sharper. But as the particle energy becomes greater the drift tubes must be made longer. This increased loading of the tank brings about a reduction in the ratio of the tank diameter to the drift-tube diameter. As a result of both these effects, the power losses in the conventional machine begin to increase for KE > 10 Mev, becoming serious for particle energies over 150 Mev.

(B) HALF- $\beta\lambda$ PILLBOXES

A possible alternative to the conventional machine is a series of resonant cavities, each of length $1/2 \beta \lambda$, separated from one another by walls. Each pillbox would contain a pair of half drift tubes. Two possible arrangements are compared with the standard drift-tube machine. The first is pillboxes with gap lengths $1/8 \beta \lambda (g/L = 0.25)$. The second is pillboxes with gap lengths of $1/4 \beta \lambda (g/L = 0.5)$. In both cases two pillboxes would be the equivalent of one repeat length of the conventional machine, but the gap gradient of the second would be half that of the first and of the conventional machine. The average fields are the same, so, apart from differences in transit time factors, the over-all lengths are the same for the same energy gain.

The power losses per Mev deuteron energy gain are given by $P = \frac{\overline{E}^2}{Q\eta \times 10^6} \propto \frac{\beta\lambda}{2} \propto \frac{1}{\overline{E}T \frac{\beta\lambda}{2} \cos \phi_a} \propto 10^3 \text{ kw/Mev}.$

Assuming a gap gradient in the first case of 100 kv/in. and in the second of 50 kv/in. we get, for $\lambda = 6.2$ meters and $d/\lambda = 0.1$,

P = $\frac{2370}{\eta_{0.25}}$ kw/Mev for g/L = 0.25, P = $\frac{2470}{\eta_{0.5}}$ kw/Mev for g/L = 0.5.

Here

$$= \frac{\frac{\sin \frac{\pi g}{L}}{\pi g}}{\frac{\pi g}{L}}$$

Q = 1/2.03,

т

$$\cos \phi_s = 0.9$$
,
 $\overline{E} = 25 \text{ kv/in.} = 0.984 \times 10^6 \text{ volts/meter for both cases}$

The numerical difference in the numerators between the first and second case is due to the difference in transit-time factors.

The quantity $\eta_{0.25}$ is obtained from g/L=0.25 and $\eta_{0.5}$ from g/L=0.5 from Fig. 6a in Reference 1. Values of η outside the range of β given in Reference 1 were calculated

As we can see by examining the numerical results presented at the end of this paper, the $1/2 \beta \lambda$ configuration is not indicated in the low-energy range of an accelerator as far as power losses are concerned. However, its efficiency increases with higher particle energy, surpassing that of the conventional machine at deuteron energies of about 120 Mev. The g/L = 0.5pillboxes are the best, as far as power is concerned, at high energy.

(C) SEPARATE RESONANT PILLBOXES WITHOUT DRIFT TUBES

In this case the pillboxes themselves are equivalent to accelerating gaps. Since this configuration is a possible alternate to the $1/2 \beta \lambda$ pillboxes with drift tubes (case B) we will assume the axial length of one of these pillboxes is $1/2 \beta \lambda$ and the on-axis gradient is 25 kv/in. Two such pillboxes, as in the case of the pillboxes with drift tubes, are the equivalent of one repeat length of the standard drift-tube machine apart from the difference in transit time factors. Focusing devices in this case would be outside the pillboxes.

The field equations are as follows:

$$E_{z} = E_{o}J_{o}\left(\frac{2\pi r}{\lambda}\right) e^{i\omega t},$$
$$Z_{o}H_{o} = iE_{o}J_{1}\left(\frac{2\pi r}{\lambda}\right) e^{i\omega t}.$$

From $P = \frac{R_s}{2} \int_s H^2_{tang} ds$, where P = power loss to the surface in

watts, we get, for the power loss per Mev in kw,

with
$$P = \frac{2\pi R_s E_o a J_1^2 \left(\frac{2\pi a}{\lambda}\right) \left[a + \frac{\beta \lambda}{2}\right] \times 10^3}{Z_o^2 T \beta \lambda \cos \phi_s}$$

$$T = \frac{\sin (\pi/2)}{\pi/2} , \qquad \cos \phi_s = 0.9$$

a = radius of pillbox,

$$J_{0}\left(\frac{2\pi a}{\lambda}\right) = 0$$
.

An insertion of numerical values for the various parameters gives

 $P = 44 + 33.5/\beta \text{ kw/Mev}.$

This configuration is much less efficient than the $1/2 \ \beta \lambda$ pillboxes with drift tubes.

(D) HELICAL WAVE GUIDE

In order to obtain an expression for the power losses in a helix the following approximations are made:^{2, 3}

The helix is regarded as a cylindrical surface, infinitely conducting in a direction corresponding to the actual helix, and perfectly nonconducting in a direction normal to the direction of the helix winding.

Under these assumptions the components of the vector potential are as follows (the components with the subscript 1 are exterior to the wave guide):

$$\begin{split} \mathbf{A}_{\mathbf{r}} &= -\frac{\mathbf{E}_{\mathbf{o}}\mathbf{k}}{\gamma\omega} \quad \mathbf{I}_{1}(\gamma \mathbf{r}) e^{\mathbf{i}(\omega \mathbf{t} - \mathbf{k}z)} ,\\ \mathbf{A}_{\theta} &= \frac{\mathbf{E}_{\mathbf{o}} \tan \psi}{\omega} \quad \frac{\mathbf{I}_{\mathbf{o}}(\gamma \mathbf{a})}{\mathbf{I}_{1}(\gamma \mathbf{a})} \quad \mathbf{I}_{1}(\gamma \mathbf{r}) \; e^{\mathbf{i}(\omega \mathbf{t} - \mathbf{k}z)} ,\\ \mathbf{A}_{z} &= \frac{\mathbf{i}\mathbf{E}_{\mathbf{o}}}{\omega} \quad \mathbf{I}_{\mathbf{o}}(\gamma \mathbf{r}) \; e^{\mathbf{i}(\omega \mathbf{t} - \mathbf{k}z)} ,\\ \mathbf{A}_{r\,1} &= \frac{\mathbf{E}_{\mathbf{o}}\mathbf{k}}{\gamma\omega} \quad \frac{\mathbf{I}_{\mathbf{o}}(\gamma \mathbf{a})}{K_{\mathbf{o}}(\gamma \mathbf{a})} \quad \mathbf{K}_{1}(\gamma \mathbf{r}) \; e^{\mathbf{i}(\omega \mathbf{t} - \mathbf{k}z)} ,\\ \mathbf{A}_{\theta_{1}} &= \frac{\mathbf{E}_{\mathbf{o}}^{\mathrm{tan}} \psi}{\omega} \quad \frac{\mathbf{I}_{\mathbf{o}}(\gamma \mathbf{a})}{K_{1}(\gamma \mathbf{a})} \quad \mathbf{K}_{1}(\gamma \mathbf{r}) \; e^{\mathbf{i}(\omega \mathbf{t} - \mathbf{k}z)} ,\\ \mathbf{A}_{z\,1} &= \frac{\mathbf{i}\mathbf{E}_{\mathbf{o}}}{\omega} \quad \frac{\mathbf{I}_{\mathbf{o}}(\gamma \mathbf{a})}{K_{\mathbf{o}}(\gamma \mathbf{a})} \quad \mathbf{K}_{\mathbf{o}}(\gamma \mathbf{r}) \; e^{\mathbf{i}(\omega \mathbf{t} - \mathbf{k}z)} , \end{split}$$

where $k = 2\pi/\lambda_{phase} = 2\pi\nu/V$; ν = frequency and V = velocity of the traveling wave;

$$\gamma = \frac{2\pi}{\lambda_0\beta}\sqrt{1-\beta^2},$$

 λ_0 = free-space wave length;

a = radius of the helix.

The relation between the pitch angle of the helix (designated $\psi)$ and the phase velocity is

$$\frac{\lambda_0^2 \gamma^2 \tan^2 \psi}{4\pi^2} = \frac{I_1(\gamma a) K_1(\gamma a)}{I_0(\gamma a) K_0(\gamma a)}$$

In order to compute the power loss in the helix it is assumed that the current flows along a ribbon of zero thickness and of width s cos ψ oriented in the same direction as the actual helix wire. Here s is the pitch of the helix; s = $2\pi a \tan \psi$.

To correct for the uneven current distribution in the actual case an experimentally obtained form factor is introduced.³

The quantity $s\phi/d$ is defined as follows:

 $\frac{s\Phi}{d} = \frac{\text{resistance of a given length of helix wire}}{\text{resistance of a straight wire of same length but of diameter D = s}$

The resistance per turn of the helix is then

$$R = R_s \left(\frac{s\phi}{d}\right) \frac{2\pi a \sec \psi}{\pi d} \frac{d}{s}$$
,

where d is the diameter of the helix wire. The quantity

$$R_s \frac{2\pi a \sec \psi}{\pi d} \frac{d}{s}$$

is the skin resistance of a straight wire of diameter s and of length equal to one helix turn. Values of the quantity $s\tilde{\phi}/d$ as a function of d/s are given in Reference 3. For 0.3 < d/s < 0.8, $s\tilde{\phi}/d = 4$.

Since the surface substituted for the helix is considered to be a ribbon of zero thickness and of width s $\cos \psi$, the current flowing in it is equal to the discontinuity in the magnetic field in a direction normal to the helix winding times the width of the ribbon:

I =
$$\left[H_{2}(a) - H_{2}(a)\right] \sec \psi \ s \ \cos \psi.$$

Using the relation $P = 1/2 I^2 R$, where R is the resistance per unit axial length of the helix and P is the mean power loss per unit axial length in watts, we obtain the following expression giving the power loss in kw per Mev particle energy gain in this configuration:

$$P = \frac{R_{s} \left(\frac{s\phi}{d}\right) E_{o} \lambda_{o}^{2} 10^{3}}{4\pi^{2} Z_{o}^{2} a \cos \phi_{s}} \frac{I_{o}(\gamma a)}{I_{1}(\gamma a) K_{1}(\gamma a) K_{o}(\gamma a)} \frac{\beta^{2}}{1 - \beta^{2}}$$

The value of the synchronous phase angle may be obtained in a manner analogous to that in Case A. Assuming the same initial phase acceptance for the helix as for the drift-tube configuration, we get

$$|\phi_{\rm s}| = 0.06 \ \beta_{\rm s}^{-3/4}$$
; $1 < \rm KE < 5 \ Mev.$

For comparing this configuration with the others, the following numerical values for the various parameters were assumed:

a = 1.5 inches. This number for the helix radius was chosen for comparable focusing with Case A.

Inserting the appropriate numerical values for the various parameters into the power-loss equation for this case, one obtains

5 < KE < 100 Mev;

$$P = \frac{1.29 \times 10^3}{\cos \phi_s} \frac{I_o(\gamma a)}{I_1(\gamma a)K_1(\gamma a)K_o(\gamma a)} \frac{\beta^2}{1 - \beta^2}$$

where

 $\left|\phi_{s}\right| = 0.45,$

$$\gamma a = \frac{3.87 \times 10^{-2}}{\beta} \sqrt{1 - \beta^2}.$$

A helix appears to be a relatively efficient device for accelerating deuterons of energies less than 20 Mev, provided the beam can be focused conveniently. This configuration, however, does not seem to be indicated for high deuteron energies.

ACKNOWLEDGMENTS

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Fig. 2. Power-loss curves for four configurations for a linear accelerator:

- (A) drift-tube machine;
 (B) pillboxes, g/L = 0.25;
 (B') pillboxes, g/L = 0.5;
 (C) pillboxes without drift tubes;
 (D) helix.

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Fig. 3. Curves from Walkinshaw, Sabel, and Outram, "Shunt Impedance Calculations for Proton Linear Accelerators. Part I, "AERE-T/M-104, Harwell, England. (1) Fig. 3a, g/L = 0.25; (2) Fig. 6a, g/L = 0.25; (3) Fig. 6a, g/L = 0.5. Drift-tube diameter = 15.37 cm, $\lambda = 1.5$ m. In this report η is defined $\eta = \frac{E^2}{p} \times 10^6$,

where E = electric field in volts per meter, p = power loss in watts,
$$\eta$$
 = megohms per meter.

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