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August 3, 1965

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ABSTRACT

The phenomenon called "line-tying" in magnetized plasmas requires unimpeded electron flow along the magnetic field and into the bounding surfaces. We discuss here the nature of the impedance that a low-density plasma and its wall sheath offers to such electron flows. The result is used to analyze the stabilizing effect of a tenuous background plasma on the growth of flutes in mirror-confined plasmas in the guiding-center approximation. Only a reduction in growth rate is found, and strong electron emission from the boundaries may be required if the effect is to be pronounced.

I. INTRODUCTION

The interchange instability of hydromagnetic equilibria in open-ended systems (such as mirror machines) can be suppressed to some extent if the field lines on leaving the medium (plasma) are "frozen" in highly conducting metal surfaces /1-3/. In its ideal form, this phenomenon, which has been called "line-tying," requires not only excellent conductivity of both plasma and metal boundary, but also perfect contact between the plasma and the wall. Although such conditions are obviously unrealistic, several experimentally observed stabilizing effects allegedly due to some plasma contact with metal walls have been reported in the literature /4-8/. The precise nature of this contact has, however, not been thoroughly explored in these reports. We, therefore, describe more fully the relevant properties of the transition zone (or zones) between a confined plasma and a metal boundary, and draw some conclusions regarding the resulting effects on plasma stability.

For clarity, only the classic example of the long-flute interchange mode of a collisionless plasma in a uniform gravitational field is treated formally in the guiding-center approximation /9/. Extension to more complete stability analyses is of course possible, but leads to very complicated if not intractable forms from which it is difficult to extract the pertinent information. These complications are well illustrated in a recent analysis by Chang et al. /10/ of the stability of the earth's radiation belts, including contact with the ionosphere. Similarly, studies by Chen /11/ of the effect of the end plates on the drift instabilities in thermal alkali-vapor plasmas, and by Furth /12/ and by Hartman /13/ on such effects in mirror-confined plasmas, produced rather complex relations even though their models for the contact are quite simple.

The stabilizing effect under discussion is due mainly to electron flow along the magnetic field lines under the influence of the perturbed potential $\phi(\underline{r}, t) = \phi_0(\underline{r}) + \phi_1(\underline{r}, t)$. We expect for the perturbation $\phi_1 \ll \phi_0$, where the unperturbed potential ϕ_0 exists to equalize the mean electron and ion loss rates, as calculated for mirror-confined plasmas, for instance, by BenDaniel /14/. Based on BenDaniel's model, one would estimate for the variation in electron removal from the center of a mirror region

$$e \frac{\partial n_1}{\partial t} \approx - \frac{en_0}{\tau_i} \left(\frac{e}{kT_e} + \frac{3}{2\phi_0} \right) \phi_1, \quad (1)$$

where n_0 and n_1 are the unperturbed and perturbed densities and τ_i is the mean ion-scattering loss time. This assumes prompt removal of all charges once they pass through the mirror surface. In reality the charge drain from a solitary plasma would be even smaller than indicated by Eq. (1), because the space between a magnetic-mirror surface and a metal end wall always represents a finite impedance to the charge flow.

2. EFFECT OF A BACKGROUND PLASMA

It has been pointed out repeatedly that a tenuous and relatively cold background plasma may have a pronounced stabilizing effect. In fact, in many experiments such a plasma has been observed and may perhaps be unavoidable. It may, for instance, exist transiently--although on a long time scale--simply because it has been produced simultaneously with the hot plasma. Alternatively, it may be continuously regenerated by ionization of a tenuous background of neutral gas originating at the vacuum-chamber walls.

Regardless of their origin, the background ions can be assumed to have low energies and thus most of them are excluded from the central region if the potential there is high, i. e., if the primary plasma has a high electron temperature. The background electrons, on the other hand, are free to travel

through the main plasma, and thus represent an anisotropic contribution to the principal electron population. Inasmuch as, by definition, they are not confined electrostatically to the region between the mirrors, these electrons can establish excellent communication between the main plasma and the surrounding space.

Obviously the background electrons cannot remain "cold" for very long, since their scattering rate in the plasma body may be substantial, particularly if they excite nonthermal oscillations because of their initial anisotropy. Of course, this heat transfer to the background represents an energy drain for the main plasma, but this is immaterial in the present context. The principal function of these electrons is to provide an adequate supply of free current carriers that can adjust the charge on a magnetic flux tube and thus prevent the growth of transverse electric fields more effectively than the rate given by Eq. (1). It would be erroneous, however, to assume that the effective impedance of the space between the mirror region and the end plates is necessarily negligible if the external plasma is virtually collision-free. For the purposes of our discussion the ohmic resistance of the background plasma may frequently be ignored, but since we are interested in the response to electric fields that vary in time, we may have to include the effect of the finite inertia of the electrons.

In order to keep the derivations general, for the net charge motion along the magnetic field we write

$$j_{\parallel} = -\frac{\omega_p^2}{4\pi(\nu^2 + \omega^2)} (\nu + i\omega) \nabla_{\parallel} \phi_1, \quad (2)$$

where we have assumed that $\phi_1(\underline{r}, t) \propto e^{-i\omega t}$, and the resistivity is expressed in terms of an effective collision frequency ν . Cases where $\nu \gg \omega$ have been investigated by Chen /11/ and by Furth /12/; the opposite limit, $\omega \gg \nu$, was considered by Babykin et al. /6/. Inasmuch as we are here primarily concerned with the action of a very low-density background, we also focus our attention on the case $\omega \gg \nu$. For simplicity we assume that the external plasma is essentially at rest, fairly cold, and uniform in space, and therefore completely passive as far as instabilities are concerned. If the length of this plasma is denoted by l , we can thus represent its effect as an inductive susceptance per unit area given by $\omega_p^2/4\pi l\omega$, where ω_p refers to the plasma frequency of the external plasma only. Note that the corresponding impedance inside the main plasma is much lower, since ω_p there is much larger.

The picture must be completed by a consideration of the sheath that will in general establish itself in the transition region between the external plasma and the metal end plates. In other words, the space is really divided into three zones, as indicated in the qualitative sketch of the potential and density

distributions along a magnetic flux line envisioned in this discussion (Fig. 1).

3. EFFECT OF THE PLASMA SHEATH

It has already been demonstrated /11/, /13/, and /15/ that the effect of the sheath characteristics on the line-tying phenomenon and therefore on plasma stability cannot usually be ignored. As is well known, the potential difference across the sheath adjusts itself in such a way that the electron and ion currents exactly cancel each other in the unperturbed state. Therefore, unless the end plates are emitting more electrons than the plasma, most of the plasma electrons are still confined in a potential well, much as they were in BenDaniel's model. The gain in electrical contact between a hot plasma and the walls by means of a cold background is thus caused principally by the fact that the background ions are not confined and stream freely towards the end plates where they recombine. The contact can be improved much beyond this natural limitation only if the plates are made to emit electrons more copiously than the cold plasma emits ions.

The impedance of the sheath has two parallel components that are readily estimated if the frequencies involved are much less than the background plasma frequency. The latter is not a serious restriction, of course. The first component arises from the fact that the net current collected by the wall is a function of the potential drop across the sheath; this impedance is resistive and leads to a direct analog of relation (1). The second component is due to the dependence of the sheath's space-charge content on the potential drop; this impedance is capacitive and matters only at relatively high frequencies. Formally, the net current $j_{||}$ leaving the plasma is related to the perturbation ϕ_{s1} of the sheath drop ϕ_s by

$$j_{||} = j_w + \frac{1}{4\pi} \frac{\partial E_w}{\partial t} \approx \left[\left(\frac{\partial j_w}{\partial \phi_s} \right)_{\phi_s = \phi_{s0}} - \frac{i\omega}{4\pi} \left(\frac{\partial E_w}{\partial \phi_s} \right)_{\phi_s = \phi_{s0}} \right] \phi_{s1}, \quad (3)$$

where j_w and E_w denote the current and the electric field at the wall and the sheath edge is assumed to be at a potential $\phi_s(t) \equiv \phi_{s0} + \phi_{s1}e^{-i\omega t}$ with respect to the wall.

To be specific, let us assume that the external electron distribution can be represented by a Maxwellian with temperature T_{ext} and density n_{ext} , the ion-flux density toward the end plates is given by $n_{ext} \langle v \rangle_i = j_i/e$, and the electron current emitted by the end plate is $j_0 < j_e - j_i$, where j_e is the random electron current density in the external plasma. In that case we can use the standard sheath model and obtain

$$\left(\frac{\partial j_w}{\partial \phi_s} \right)_{\phi_s = \phi_{s0}} = \frac{e^2 n_{\text{ext}} \langle v \rangle_i (1 + j_0/j_i)}{kT_{\text{ext}}} = \frac{\langle v \rangle_i (1 + j_0/j_i)}{4\pi \lambda_D^2} \quad (4)$$

where λ_D is the Debye length of the external plasma.

The capacitive term depends on the sheath structure and is therefore more difficult to evaluate. However, if we approximate the model by neglecting the electron density in the sheath and use Child's law for the ion current, we find

$$\left(\frac{\partial E_w}{\partial \phi_s} \right)_{\phi_s = \phi_{s0}} \approx \frac{1}{2} \lambda_D \left[\ln \frac{j_e}{j_0 + j_i} \right]^{-1/2} \quad (5)$$

It is clear that the capacitive contribution in Eq. (3) can be neglected if $\omega \lambda_D / \langle v \rangle_i \ll 1$, i. e., for frequencies well below the ion plasma frequency of the background medium. This is the approximation used by Chen /11/ and by Hartman /13/.

4. TOTAL ADMITTANCE BETWEEN MIRROR AND END PLATES

For our evaluation of the incomplete line-tying process, (henceforth called "line-slip"), we must relate the parallel current $j_{\parallel}$ to the total perturbed potential drop $\phi_1 = \phi_{s1} + \ell \nabla_{\parallel} \phi_1$ between the mirror region and the end plates. The relation is best expressed in terms of an admittance per unit area Y , i. e., $j_{\parallel} = Y \phi_1$; the function $Y(\omega)$ then follows in an obvious way from Eqs. (2) to (5). For the collision-free case in which we are interested here, we find

$$Y(\omega) = Y(0) \frac{1 - i\omega\tau_i}{1 - \omega^2 \tau_l \tau_e - [i\omega \tau_l \tau_e / \tau_i]} \quad (6)$$

where the dc conductance $Y(0)$ is given by Eq. (4), the constants τ_i and τ_e are very nearly the mean transit times of ions and electrons through the sheath, and τ_l is the average electron transit time through the external plasma: $\tau_i \equiv \lambda_D [2 \langle v \rangle_i (1 + j_0/j_i) \sqrt{\ln j_e / (j_i + j_0)}]^{-1}$

$$\tau_e \equiv \lambda_D [(2kT_{\text{ext}}/m_e) \ln j_{e0} / (j_0 + j_i)]^{-1/2}, \quad \tau_l \equiv \ell (2kT_{\text{ext}}/m_e)^{-1/2}.$$

Thus these times are related roughly as $\tau_e / \tau_i \approx \sqrt{m_e / m_i}$ and $\tau_e / \tau_l \approx \lambda_D / \ell$, so that $\tau_l \tau_e / \tau_i^2 \approx m_e \ell / m_i \lambda_D$.

The low-frequency limit $\omega \tau_i \ll 1$ results in the simplified relation

$$Y(\omega) \approx Y(0) / (1 - i\omega \tau_l \tau_e / \tau_i). \quad (7)$$

Babykin et al. /6/ assumed that the imaginary term in (7) dominated, i. e., that $\omega \tau_l \gg \sqrt{m_i / m_e}$. The two approximations are reconcilable only in the

long-plasma limit $l/\lambda_D \gg m_i/m_e$ so that the plasma's inductive impedance dominates over the sheath resistance in spite of the low frequency.

In the opposite extreme, where the plasma column is so short that its inductive impedance can be neglected, even for relatively large ω , Eq. (6) simplifies to $Y(\omega) = Y(0)(1 - i\omega\tau_i)$, provided that $l \ll \lambda_D m_i/m_e (1 + \omega^2\tau_i^2)$.

In our application to the example of the simple interchange instability, we initially keep our expression for the external admittance $Y(\omega)$ unspecified.

5. INTERCHANGE STABILIZATION

For reasons stated in the introduction we restrict ourselves to an analysis in the guiding-center approximation, i. e., we ignore finite-gyroradius effects as well as nonadiabatic processes. We are thus limited to frequencies $\omega \ll \Omega_i \equiv eB/m_i c$ and to appropriately gentle gradients. For simplicity we replace the driving force by a uniform gravitational acceleration $\underline{g}$ directed along the positive x axis while the plasma with density $n = n_0(x) + n_1(x, y, t)$ is embedded in a uniform magnetic field $\underline{B}$ aligned with the z axis of a Cartesian coordinate system. The perturbed quantities are then assumed to vary as $\exp i(ky - \omega t)$. A uniform nonzero electric field $\underline{E}_0$ in the x direction can be included in the derivation as it simply adds a constant to the zero-order drifts $\underline{u}_e$ and $\underline{u}_i$.

The derivation of the dispersion relation is standard /9/. The guiding center drifts due to $\underline{g}$, $\underline{E}$, and $\partial \underline{E}/\partial t$ are used to evaluate $n_1(x, y, t)$ for both ions and electrons with the help of their continuity equations. The electron equation contains the drain term, so that after carrying out the time differentiation we find that

$$-i\omega n_{e1} + \frac{\partial (u_{ex} n_e)_1}{\partial x} + \frac{\partial (u_{ey} n_e)_1}{\partial y} = \frac{Y(\omega)\phi_1}{eL} \quad (8)$$

Here L is the length of the plasma subjected to $\underline{g}$ and $Y(\omega)$ is the total admittance between this plasma and the highly conducting boundary. With neglect of $\nabla \times \underline{E}$ and $\nabla \times \underline{B}$, Poisson's equation then yields

$$\nabla^2 \phi_1(x, y) = Z(k, \omega)\phi_1, \quad (9)$$

where

$$Z(k, \omega) = \frac{k^2 \alpha (u_i - u_e) + i\beta_\omega (\omega - ku_i)}{(K\omega - ku_i)(\omega - ku_i)},$$

in which the following abbreviations have been used: $K = 1 + \omega_p^2/\Omega_i\Omega_e$, $\alpha = \omega_p^2 (\partial n_0/\partial x)/n_0\Omega_e$, and $\beta_\omega = 4\pi Y/L$.

If we now impose boundary conditions on ϕ_1 , the eigenvalues of Eq. (9) determine our dispersion relation. For instance, if Z does not depend on x

and if $\phi_1(0, y) = \phi_1(h, y) = 0$, we obtain the algebraic form

$$Z(k, \omega) = -k^2 [1 + (N\pi/kh)^2] \equiv -k^2 a_N,$$

where N is an integer.

The stabilizing effect of the term containing β becomes most transparent when we consider cases in which $K \gg 1$ and that have sufficiently short wavelengths so that $kh \gg N\pi$; we describe them in a frame in which $E_0 = 0$, so that $|u_e| \ll |u_i| = |g/\Omega_i|$. Our dispersion relation can then be approximated by

$$Kk^2(\omega^2 + \omega_0^2) + i\beta_\omega(\omega - ku_i) \approx 0, \quad (10)$$

provided $kg \ll K\Omega_i\omega$, which is not very restrictive. Here $\omega_0^2 \equiv -(g/n_0)(\partial n_0/\partial x)$ denotes the solution for the unimpeded Taylor instability, i. e., for $\beta = 0$.

If the complete form of Eq. (6) is used in the expression for β_ω , Eq. (10) is of the fourth degree in ω . The problem becomes more tractable if we limit ourselves to sufficiently low frequencies so that Eq. (7) can be used, and if we investigate only cases of good contact, i. e., those with large magnitudes of β_0 . It is then instructive to rewrite (10) as follows:

$$\omega = \frac{ku_i + i(\omega^2 + \omega_0^2)Kk^2/\beta_0}{1 - (\omega^2 + \omega_0^2)KLl k^2/\omega_{p(\text{ext})}^2}. \quad (11)$$

In the approximation treated by Babykin et al. /6/, the numerator is considered negligible so that the dispersion relation consists of equating the denominator to zero. Stability, $\omega^2 > 0$, would then be predicted from (11) if

$$\frac{n_{\text{ext}}}{n_0} > k^2 l l \omega_0^2 / \Omega_e \Omega_i, \quad (12)$$

which agrees with the result in Rev. /6/ if we replace g by $\rho_H^2 / L\Omega_i^2$, where ρ_H is the hot-ion gyroradius. We see, however, that in reality a finite growth rate persists as long as β_0 is not infinite.

An estimate of the persisting growth rate due to line-slip in the sheath is most easily obtained by consideration of a very short external plasma, so that (12) is very easily satisfied and the denominator in (11) can be replaced by unity. We then obtain

$$\omega \approx ku_i + i\omega_0^2 Kk^2/\beta_0,$$

provided that the reduction in growth rate is drastic, i. e., $Kk^2\omega_0/\beta_0 \ll 1$.

Substitution of the physical parameters for β_0 and g yields for this condition

$$\left(1 + \frac{j_0}{j_i}\right) \frac{n_{\text{ext}}}{n_0} \gg k^2 \rho_H^2 \left(\frac{T_{\text{ext}} L}{T_i n_0} \frac{\partial n_0}{\partial x} \right)^{1/2} \quad (13)$$

where ρ_H and T_i refer to the high-temperature ions of the confined plasma. We thus see that the stabilizing effect is proportional to the external plasma density. If the latter is to be kept low, strong electron emission, j_0 , from the end plates could be substituted.

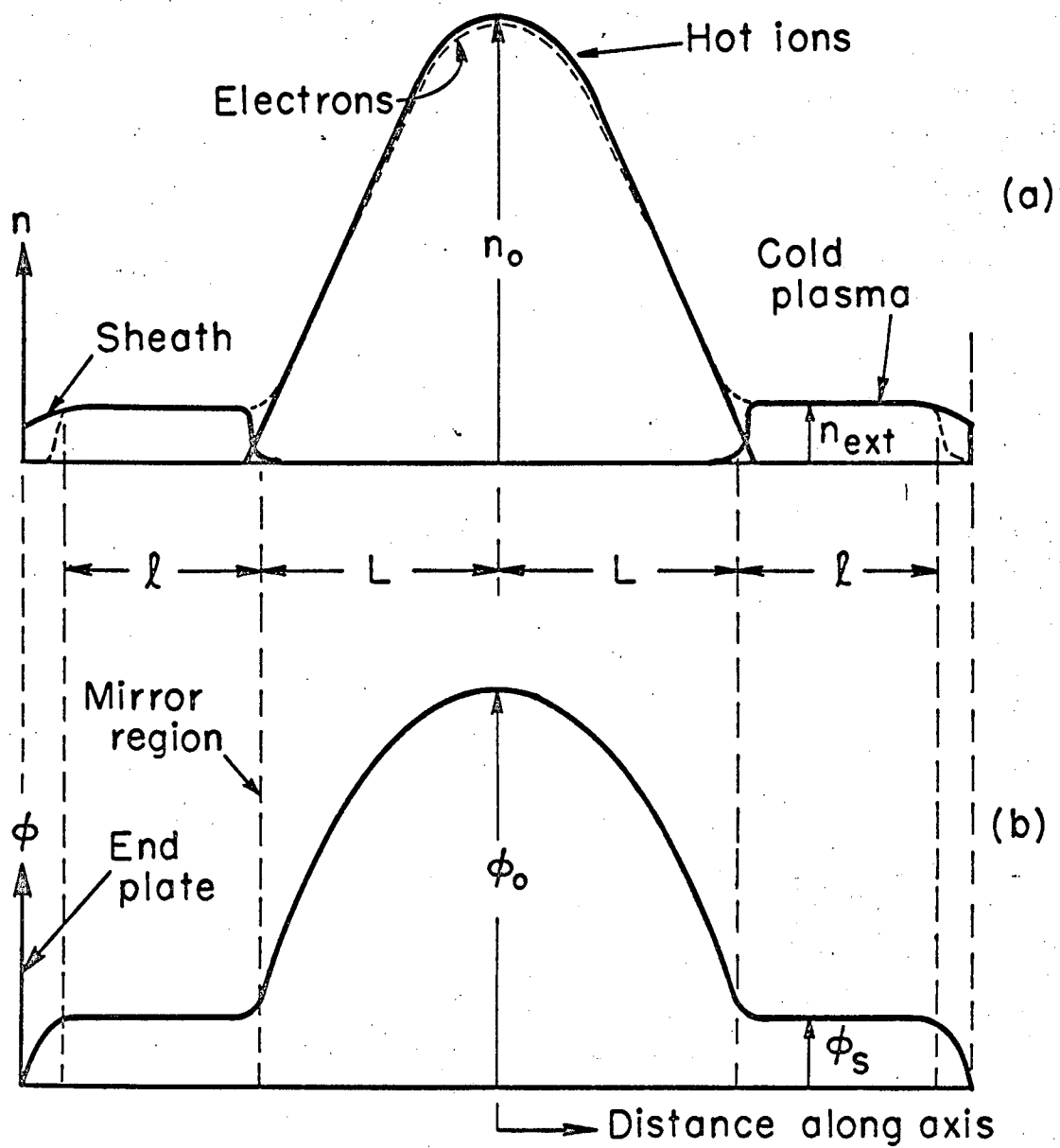
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FIGURE CAPTION

- Fig. 1. (a) Ion and electron densities near axis of mirror-confined plasma with cold low-density plasma background and sheath at nonemitting end plates.
(b) Corresponding potential distribution (schematic).



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