Title
THE STATISTICS OF THE FIREBALL MODEL

Author
Mantzouranis, G.

Publication Date
1978-04-01

Submitted to Physical Review C

LBL-7712
Preprint

THE STATISTICS OF THE FIREBALL MODEL

G. Mantzouranis

April 1978

Prepared for the U. S. Department of Energy
under Contract W-7405-ENG-48

THE STATISTICS OF THE FIREBALL MODEL

G. Mantzouranis

Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

[Relativistic Heavy Ions; Theory of the fireball model]

ABSTRACT

We derive the fireball model using a parametrization of the exact S-matrix. It will be shown that the fireball model is related to the normal distribution of the parameters in the S-matrix expansion. This way the fireball will be understood in the same frame as the usual statistical spectroscopy.

Work supported by the U. S. Department of Energy.

I. INTRODUCTION

Recently the fireball model has been proposed in order to calculate the inclusive spectra of light particles emitted in heavy-ion collisions at relativistic energies [1]. Methods which are similar in spirit have been applied for the study of the hadronic spectrum at very high energies [2]. Our goal is to derive the fireball model using more microscopic procedures and to connect it with the standard compound nucleus model for light-projectile-induced reactions at low energies. This way the success of the fireball in explaining data will become less mystifying.

Our program starts with a formal expansion for the scattering matrix in terms of unknown parameters. Our next observation will be that the cross sections extracted from actual experiments are averaged over a relatively broad energy interval (I) due to limited experimental resolution. Therefore, these cross sections (i.e. the bilinear expressions of the S-matrix) can only contain those parts of the nuclear dynamics which have a characteristic energy scale larger than I. This loss of information of the nuclear details will enable us to derive expressions for the averaged quantities which depend only on limited features of the parameters involved in the exact S-matrix. In this paper we adopt the point of view of the phenomenological S-matrix theory so far as averaged quantities are concerned. Indeed, it is only for these quantities that the original space of parameters can be cast into a simple and informative form. It will be shown that the particular form of the fireball corresponds to the normal distribution of theparameters in the original S-matrix and thus, the fireball has the same roots as the conventional statistical spectroscopy [3].

II. THE MATHEMATICAL PROCEDURE

A. A Parametrization of the S-Matrix

It is convenient instead of the matrix to use its matrix elements $S_{cc'}$ taken between the asymptotic wave functions of the initial and final channel. The initial channel contains two nuclei with non-vanishing relative momentum while the final one consists of several nuclear fragments and elementary particles (if the kinetic energy allows for particle production). We label the total kinetic energy with E and assume the following parametrization

$$S_{cc'} = S_{cc'}^{(o)} + \sum_{q=1}^{N} g_{qc}\, g_{qc'}\, f_q(E) \quad , \tag{1}$$

where $f_q(E)$ is a channel-independent function of E while the quantities g_{qc} are assumed to be (almost) energy-independent within a narrow energy interval around E. The label q summarizes discrete and continuous variables, but for simplicity, we confined ourselves to denoting the discrete case only. The number N is to indicate the range of variance for the variable q. Finally, the label $S_{cc'}^{(o)}$ contains the part of the scattering matrix which cannot be cast into the form of the second term on the r.h.s. of Eq. (1). In the trivial case it would contain the Kronecker symbol. We shall assume that $S_{cc'}^{(o)}$ has a smooth energy dependence as compared to the second term.

Equation (1) above is our basic starting point and it needs some additional comments. The proposed parametrization can be well-established for non-relativistic energies and two-body channels [4]. In this case q labels the discrete and continuous-intermediate states of the compound system and g_{qc} denotes the partial width for the decay of the state q into c. The function $f_q(E)$ would have the form $(E-\varepsilon_q)^{-1}$, ε_q being, in general, a complex function of q.

While a corresponding derivation of Eq. (1) from a relativistic field theory does not exist, our expansion might be supported by the arguments of the phenomenological S-matrix theory which are based upon the minimum requirements placed upon the scattering operator due to the quantum mechanical nature of the systems [5]. In fact, one is led to similar expressions containing sums over singularities [5]. The second term on the r.h.s. of Eq. (1) should contain the contribution from multistep processes, i.e. the particle's production and overlapping resonant states which are manifested as resonances in the scattering amplitude.

B. The Averaged Quantities

We now proceed with the evaluation of the energy averages of the reaction matrix $\langle|S_{cc'} - \delta_{cc'}|^2\rangle = 1/I \int_{E-I/2}^{E+I/2} dE' \, |S_{cc'} - \delta_{cc'}|^2$. One would expect that minor changes of the value of the averaging interval I should keep the averaged cross sections invariant, i.e., a re-average has no action, $\langle\langle\ \rangle\rangle \approx \langle\ \rangle$. This is experimental evidence and a physical condition in order to make averaged quantities meaningful.

Let us consider, for simplicity of notation, the inelastic case $c \neq c'$. Our procedure is a generalized version of the method proposed in Ref. [6]. Using Eq. (1), we find

$$\langle|S_{cc'}|^2\rangle = |S_{cc'}^{(o)}|^2 + 2\mathrm{Re}\, S_{cc'}^{(o)*} \sum_q g_{qc}\, g_{qc'} \langle f_q(E)\rangle + \left\langle\left|\sum_q g_{qc}\, g_{qc'}\, f_q(E)\right|^2\right\rangle$$

$$= |\langle S_{cc'}\rangle|^2 + \sum_{q,q'} g_{qc}\, g_{qc'}\, g^*_{q'c}\, g^*_{q'c'} [\langle f_q(E)\, f^*_{q'}(E)\rangle$$

$$- \langle f_q(E)\rangle \langle f^*_{q'}(E)\rangle] \qquad (2)$$

where

$$\langle S_{cc'} \rangle = S^{(o)}_{cc'} + \sum_q g_{qc}\, g_{qc'} \langle f_q(E) \rangle \quad .$$

In evaluating the double sum in Eq. (2) we observe that the interference terms ($q \neq q'$) are likely to contribute less than the terms $q = q'$ (after the average is taken). This fact can be established in the case that $f_q(E)$ has a pole singularity [4]. Generally speaking we should expect this result if the energy variation of $f_q(E)$ is independent (uncorrelated) for different q. We therefore obtain

$$\langle |S_{cc'}|^2 \rangle \approx |\langle S_{cc'} \rangle|^2 + \sum_q |g_{qc}|^2 |g_{qc'}|^2 [\langle |f_q(E)|^2 \rangle - |\langle f_q(E) \rangle|^2] \ . \tag{3}$$

The next simplification arises from the following observation. The transmission coefficients $P_{cc} = 1 - \sum_{i''} \langle S_{cc''} \rangle \langle S_{c''c} \rangle^*$ denote the probability for the formation of the intermediate state via multistep processes. We shall assume the case of *maximum absorption* for the two colliding nuclei, i.e. $P_{cc} = 1$. In low energy reactions the maximum value for the transmission coefficients corresponds to strong imaginary parts in the optical potential, i.e. strong absorption. The mathematical implication is that $S_{cc'}$ is a random number, i.e. $\langle S_{cc'} \rangle = 0$. Thus in the case of maximum absorption

$$\langle |S_{cc'}|^2 \rangle = \sum_q |g_{qc}|^2 \ |g_{qc'}|^2 \left[\langle |f_q(E)|^2 \rangle - |\langle f_q(E) \rangle|^2 \right] \quad . \tag{4}$$

Our next step is to rewrite the above expression using the distribution function for the absolute squares of parameters involved. First, we consider the number $\eta_{\varepsilon_1\varepsilon_2\varepsilon_3}(cx,c'x',\omega)$ of labels (states) q for which the following relations hold

$$|x|^2 \leqslant |g_{qc}|^2 \leqslant |x|^2 - \varepsilon_1 \tag{5a}$$

and
$$|x'|^2 \leqslant |g_{qc'}|^2 \leqslant |x'|^2 - \varepsilon_2 \tag{5b}$$

and
$$|\omega|^2 \leqslant \langle|f_q(E)|^2\rangle - |\langle f_q(E)\rangle|^2 \leqslant |\omega|^2 - \varepsilon_3 \tag{5c}$$

The distribution function is defined by

$$p(cx, c'x', \omega) = \frac{d^3}{d\varepsilon_1\, d\varepsilon_2\, d\varepsilon_3}\, \eta_{\varepsilon_1\varepsilon_2\varepsilon_3}(cx, c'x', \omega)\Big|_{\varepsilon_1=\varepsilon_2=\varepsilon_3=0} \tag{6}$$

and roughly speaking, indicates the number of labels q per value of the terms on the r.h.s. of Eq. (4). We can rewrite the sum in Eq. (4) by converting the sums into integrals

$$\langle|S_{cc'}|^2\rangle = \int d\omega \int dx \int dx'\; p(cx, c'x', \omega)\; |x|^2\, |x'|^2\, |\omega|^2 \;. \tag{7}$$

The above equation is a basic result. It indicates that averaged cross sections are sensitive to the distribution function given by Eq. (6). Even if this function were completely known, the construction of the exact $|g_{qc}|^2$ and $\langle|f_q(E)|^2\rangle$ would be impossible, not to mention the quantities $f_q(E)$, g_{qc} which appear in the exact S-matrix. The object of any dynamical model should be the determination of this function. As far as averaged data are concerned, one would consider as equivalent to each other, models which lead to the same distribution functions. Our analysis indicates the possibility of existence of such models. There is evidence that the fireball and the "row on row" model are equivalent as far as averaged nucleon inclusive spectra are concerned [7].

C. The Fireball Model

A complete theory would predict the distribution function which is needed for averaged spectra. However, we want to confine ourselves to the question, what did the existing data predict for this distribution? We shall show that the normal distribution, in addition with some geometry in calculating the level densities, led to the fireball model. By observing the success of this model in explaining the data, we should conclude that the existing data suggest a normal distribution. It is probably astonishing to discover the same distribution as in the lowest energy statistical spectroscopy [3].

Let us assume for $c \neq c'$

$$p(cx, c'x', \omega) = f(cx)\, f(c'x')\, f(\omega) \quad . \tag{8}$$

If $c = c'$, Eqs. (5a,b) become identical and thus we would write $p(cx, c'x', \omega) = f(cx)f(\omega)$. A simple examination of Eq. (5) shows that the factorization property of Eq. (8) means that the parametrization of the S-matrix in Eq. (1) contains a huge number of totally diverse contributions from several g_{qc}. This might become realistic if the wave function of the colliding ions is a very complicated superposition of asymptotic states. Low impact parameters and strong interactions might have it as sequence.

Consistent with these remarks on the meaning of the factorization is the particular choice of a Gaussian (normal) form for $f(cx)$. This enables us to express the higher moments, like $\int dx|x|^4 f(cx)$ [which occurs in the case $c = c'$], in terms of the second moment $\int dx|x|^2 f(cx)$. Thus, Eq. (7) reads

$$\langle |S_{cc'}|^2 \rangle = [\int dx\ f(cx)|x|^2]\,[\int dx'\ f(c'x')\,|x'|^2]\,[\int d\omega\ f(\omega)|\omega|^2 \quad . \tag{9}$$

Using the unitarity relation $\langle \sum_{c''} S_{cc''} S^{*}_{c''c'} \rangle = \delta_{cc'}$, the parametrization of Eq. (1) and the maximum absorption model ($\langle S \rangle = 0$), we get after some manipulations,

$$\langle |S_{cc'}|^2 \rangle = \frac{\mu_c \mu_{c'}}{\sum_{c''} \mu_{c''}} \tag{10}$$

where $\mu_c = \mu_{c'} = 1$ if $S_{cc'} \neq \delta_{cc'}$ and $\mu_c = \mu_{c'} = 0$ in the case that $S_{cc'} = \delta_{cc'}$, i.e., if the scattering between the channels c,c' vanishes due to high impact parameter or selection rules. This reminds us of the classical sharp cut-off model for the transmission coefficients.

The definition of an inclusive cross-section requires a summation over the different states of the residual system, i.e. composite system minus one nucleon. We therefore indicate in the final channel c' the quantum numbers of the outgoing nucleon by (n) and the labels of the residual system by (r), $c' \equiv (n,r)$. The relevant quantity for the inclusive cross sections reads

$$\sum_r \langle |S_{c(n,r)}|^2 \rangle = \frac{\mu_c \sum_r \mu_{(n,r)}}{\sum_{c''} \mu_{c''}} \tag{11}$$

The final step consists in incorporating into the sums of Eq. (11) the high energy geometry of the heavy-ion collision. We should assume that during the collision only the part of nucleons which are in the overlapping region participate. Thus the composite (and residual) system involves the degrees of freedom of the overlapping region. This region is defined by the impact parameter which can be determined from the labels of the initial channel c (i.e. relative angular momentum). Thus, the sums over the contributing final channels are strongly restricted due to the geometry of the collision.

Since the sum $\sum_{c''} \mu_{c''}$ means simply the number of contributing channels, one can rewrite Eq. (11) using the level density of the composite system, $\rho_c(E)$, at a given excitation E and the level density of the residual, $\rho_r(U)$, at the residual energy U, i.e.

$$\sum_r \langle |S_{cc'}|^2 \rangle = \mu_c \frac{\rho_r(U)}{\rho_c(E)} \tag{12}$$

The ratio of the level densities denotes the relative probability for the emission of a particle with the energy $\varepsilon = E-U$. The exact calculation of the level densities in an interacting system is, in general, hard enough to be practical. However, a simple model like that used in the standard fireball, i.e., the non-interacting Fermi-gas model, where all the particles share the excitation energy, might be useful. Namely, one has assumed [1] that the ratio in Eq. (12) is equal to the probability for having a particle at energy ε in a non-interacting Fermi-gas.

We recognize that Eq. (12) means that the partial cross-sections (to a fixed impact parameter) is determined by the ratio of the phase spaces in the contributing composite and residual system. The effect of the geometry is to restrict the possible open (final) channels for a given impact parameter.

So far only the absolute square of the S-matrix elements have been considered. Indeed, they are the relevant quantities for the evaluation of the angle-integrated spectra. The angular distribution involves, in general, interference terms. However, if we choose as a reference system the c.m. system of the overlapping region (c.m. of the fireball), the angular distribution becomes symmetric about 90° and thus the contribution from the interference terms vanishes. This reference system has been used in the fireball calculations [1].

III. CONCLUSION

One of our aims was to demonstrate that for averaged measurements only bulk information about the nuclear dynamics is needed to be considered. This relevant dynamical information included is a distribution function of the parameters in the expansion for the exact S-matrix. The present experimental evidence supports a normal form for this distribution. Strong collective effects, like the exotic ones, should appear as violations of the normal distribution. At low energies the collective states serve as an example for such violations [3]. The truly interesting fact will be to discover the analogue at highest energies.

ACKNOWLEDGMENTS

I would like to thank N. K. Glendenning for a reading of the manuscript and several useful suggestions.

REFERENCES

[1] J. Goset, H. H. Gutbrod, W. G. Meyer, A. M. Poskanzer, A. Sandoval, R. Stock and W. G. Westfall, Phys. Rev. C16, 629 (1977).

[2] N. K. Glendenning and Y. J. Karant, Phys. Rev. Lett. 40, 374 (1978).

[3] J. P. Draayer, J. B. French, S. S. M. Wong, Ann. of Physics 106, 472 (1977).

[4] P. A. Moldauer, Phys. Rev. 135, B642 (1964).

[5] a. G. F. Chew, S-matrix theory of strong interactions (W.A.Benjamin, New York, 1961).

b. S. Mandelstam, Phys. Rep. 13, 259 (1974).

[6] M. Kawai, A. K. Kerman, K. W. McVoy, Ann. of Physics 90, 391 (1975).

[7] J. Hüfner, J. Knoll, Nucl. Phys. A299, 460 (1977).

This report was done with support from the Department of Energy. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the Department of Energy.

TECHNICAL INFORMATION DEPARTMENT
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720