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# A confirmation bias due to approximate active inference

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## Abstract

Collecting new information about the outside world is a key aspect of brain function. In the context of vision, we move our eyes multiple times per second to accumulate evidence about a scene. Prior studies have suggested that this process is goal-directed and close to optimal. Here, we show that this process of seeking new information suffers from a confirmation bias similar to what has been observed in a wide range of other contexts. We present data from a new gaze-contingent task that allows us to both estimate a participant's current belief, and compare that to their subsequent eye-movements. We find that these eye-movements are biased in a confirmatory way. Finally, we show that these empirical results can be parsimoniously explained under the assumption that the brain performs approximate, not exact, inference, with computations being more approximate in decision-making compared to sensory areas.

**Keywords:** choice bias; perceptual decision-making; eye-movements; approximate inference

## Introduction

Human decision-making is often biased, and few biases are as ubiquitous as the confirmation bias. Despite the fact that it has been documented across a wide range of cognitive and perceptual contexts, a unified understanding of its computational underpinning is currently missing (Nickerson, 1998; Michel & Peters, 2020). Two major components contribute to this bias: first, the biased *seeking* of information in supporting one's belief, and second, an *interpretation* of the observed information that is biased by one's existing beliefs. Over the past 15 years, several studies have documented evidence for a confirmation bias in perceptual decision-making tasks which have the benefit of allowing for the collection of large amounts of data using hundreds of repetitions from the same participants, and to contrast human performance to that of animals. The evidence from those studies has shed much light on the biased *interpretation* of sensory evidence (Nickerson, 1998; Michel & Peters, 2020; Lange, Chattoraj, Beck, Yates, & Haefner, 2020). However, the *seeking* of new sensory evidence, most notably by eye-movements, has so far mostly been found to be close to optimal (Najemnik & Geisler, 2005; Renninger, Verghese, & Coughlan, 2007; Navalpakkam, Koch, Rangel, & Perona, 2010; Nelson & Cottrell, 2007; Toscani, Valsecchi, & Gegenfurtner, 2013; Yang,

Lengyel, & Wolpert, 2016), but also see (Morvan & Maloney, 2012).

Our work makes two key contributions. First, it describes a new psychophysical task that requires collecting sensory information by making saccades in the context of a classic two-choice orientation discrimination task. We designed the task to be able to measure as directly as possible how saccades may be influenced by existing beliefs and found that the eye-movements of 8/10 participants exhibited a confirmation bias. Second, we show that this biased information-seeking behavior can be explained by a computational model that starts with an optimal Bayesian active sensing strategy (MacKay, 1992; Yang et al., 2016) but assumes that the required computations are implemented approximately via sampling. Importantly, such a model requires computing two terms – a sensory and a cognitive one – and it only displays the empirically observed confirmation bias when the number of samples used to compute the sensory term (and presumably implemented in sensory areas) is larger than the number of samples used to compute the cognitive term. Such a difference is compatible with previous observations on the dramatic difference in information capacity comparing sensory periphery and central processing, and suggestions that lower sensory areas act as a “high-resolution buffer” for higher-level computations (Lee & Mumford, 2003; Marois & Ivanoff, 2005).

## Gaze contingent discrimination task

### Rationale

Visual sensitivity to fine spatial structures differs greatly across the visual field. As a result, humans use saccades (as well as head and body movements) to move their eyes across a scene in order to collect information. In order to determine whether and, if so, how saccades are biased by previously collected information we designed a two-choice orientation discrimination task with a gaze-contingent stimulus display that allows for close control over the information present both at the current fixation point as well as in the periphery.

### Task/Procedure

Participants were instructed to report the dominant (most frequent) orientation on the screen encountered while moving

<sup>1</sup>equal contribution

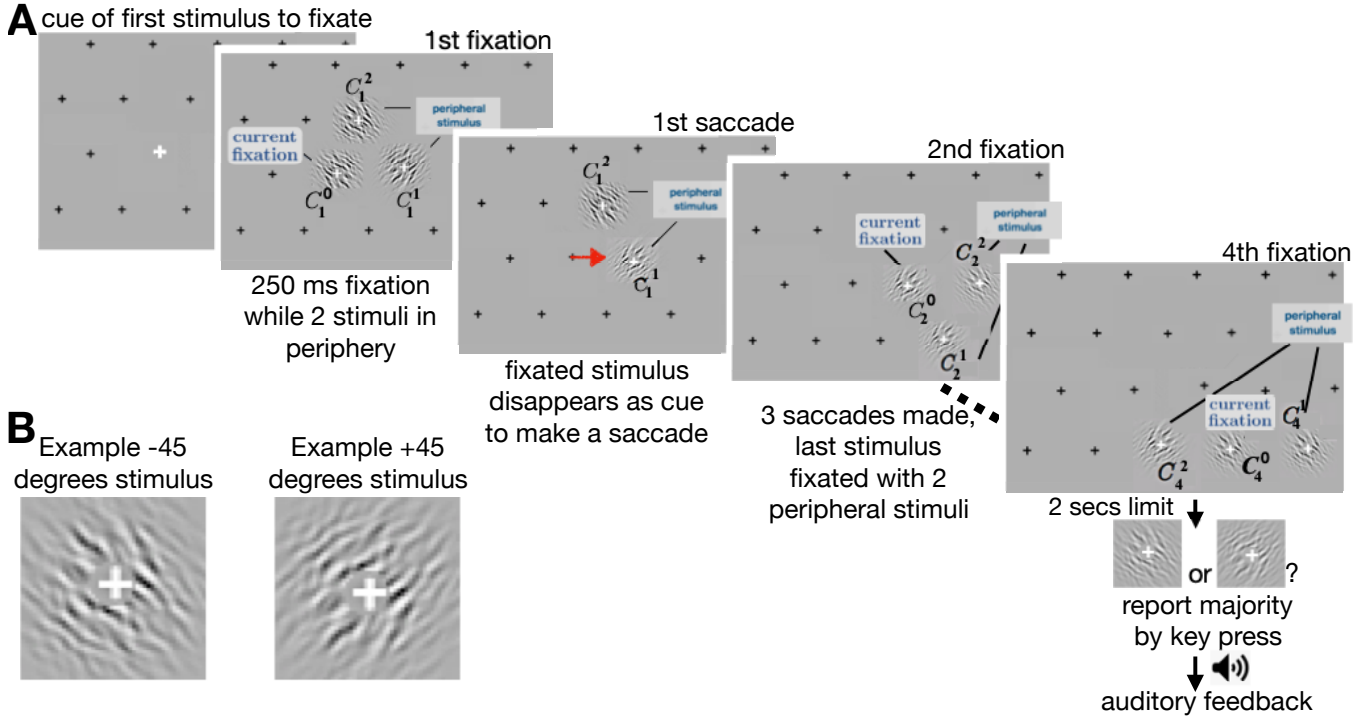


Figure 1: (A) One example trial of the gaze contingent task. A saccade is indicated by red arrow. The category on the fixated stimulus at any time  $t$  is denoted by  $C_t^0$  and the categories of the peripheral stimuli are denoted by  $C_t^1$  and  $C_t^2$ . If the first saccade is made to  $C_t^1$ , then it becomes  $C_{t+1}^0$  for the next time step  $t + 1$ . Note that the black crosses are used for illustration purposes to indicate possible locations of stimuli presentation. (B) Examples of stimulus used in the experiment.

their eyes across a screen (Fig. 1A). Each trial starts with fixation marker (white cross) in the center of a gray screen. After holding fixation for 200ms, three oriented stimuli appeared on the screen for a duration of 250ms (Fig. 1B, details below): one stimulus around the fixation marker, plus two stimuli equidistant from fixation and each other. After 250ms, the stimulus at the fixation point disappeared providing a cue to the participant to make a saccade to one of the two stimuli in the periphery. While the saccade was in progress, the non-chosen peripheral stimulus disappeared and two new stimuli appear, now peripheral with respect to the new fixation point. After 250ms, the fixated stimulus disappears again and the participant has to make another saccade to one of the two peripheral stimuli. After a total of three saccades and four fixations, the participant reports their belief about the correct stimulus category for the entire trial. The orientation of each of the nine stimuli shown in each trial is drawn from the correct orientation category with probability 0.7. This value was chosen to encourage evidence accumulation. After each choice, auditory feedback was provided to the participant on whether their choice was correct. If a participant did not move their eyes within 200ms of the fixation stimulus disappearing, or if they did not report a choice within 2s after a trial ends, then it was aborted and ignored in analysis.

Participants were trained to perform the task using one block of 20 trials on the first day. Each following session

consisted of blocks of 50 trials. Each of ten participants completed between 111 and 569 trials (median 250) across 3 sessions. The large variance in trial number resulted from the difficulty of the task, with a high fraction of aborted trials due to blinks, premature or delayed saccades, or saccades not landing at the center of one of the two peripheral stimuli. Aborted trials were excluded online, during the experiment.

### Stimulus

Each stimulus was constructed by band-pass filtering Gaussian noise in the spatial frequency and orientation domains, and masking it by a soft-edged annulus (Beaudot & Mullen, 2006; Nienborg & Cumming, 2014; Bondy, Haefner, & Cumming, 2018; Lange et al., 2020) (Figure 1B). Each annulus has a small white cross in the center which participants are instructed to foveate as shown in Fig 1B. Each stimulus subtends 2.08 degrees of visual angle around fixation. The centers of each peripheral stimuli lie at 2.88 degrees from each other and from the center of the fixated stimulus. The mean spatial frequency of the stimuli is = 6.90 cycles per degree, the spread of spatial frequency is = 3.45 cycles per degree, the (inverse) spread of orientation energy is 0.8, the image luminance =  $127 \pm 22$  and the width of the central annulus cutout is =  $0.43^\circ$ . Stimuli were generated using Matlab and Psychtoolbox and presented on a gamma-corrected 1920x1080px 120 Hz monitor (Brainard, 1997). Participants

kept a constant viewing distance of 105 cm using a chin-rest. Eye-movements were tracked using an Eyelink 1000.

Importantly, we chose the stimulus parameters and eccentricities in order to make the orientation of a foveated stimulus unambiguous for the participant, while providing some, but not perfect, information about the orientation of the stimuli in the periphery. If the information provided in the periphery is too low, then the brain will not be able to use it to decide where to move the eyes. If it is too high, then no new information is gleaned from moving the eyes and saccade plans may reflect different constraints than during natural viewing conditions. Furthermore, the design of each stimulus minimizes the effect of small fixational eye movements or variability in fixation location (within the annulus) on the information provided to the visual system

## Participants

The participants in this study consisted of 10 students at the University of Rochester (8 naive, 2 authors – highlighted in the analysis Figure 2A+B). Every naive participant was financially compensated for their time. All experiments were performed by following the guidelines and methods approved by the UR Research participants Review Board.

## Analysis

Our task design allowed us to measure whether and how a participant combined their current belief about the correct task category with the information they expected in the periphery when determining where to move their eyes next. Hence we could further test whether a participant used their current belief to make eye-movements towards confirmatory stimulus in the periphery leading to confirmation bias or not.

### Estimating a participant’s current belief within a trials:

We first performed logistic regression to determine the participant’s choice bias as well as the weights assigned to the presented stimuli: 4 foveated stimuli and 5 non-foveated stimuli. This allowed us to estimate a participant’s belief at the end of each fixation period, on each trial, by multiplying the stimuli presented so far with the corresponding weights and passing them through the logistic function yielding log odds (Figure 2C).

**Estimate saccade bias:** During each fixation period within a trial, the two peripheral stimuli were either of the same orientation (58%), or of different orientations (42%). In order to test for a confirmation bias in eye-movement strategy, we analyzed on the latter category – where saccades could be made either to a stimulus in agreement with our estimate of the participant’s current belief, or disagreement (Figure 2B).

## Findings

We found that participants could indeed successfully perform this challenging task and were consistent in their performance around threshold (Figure 2A). As expected, logistic weights

on foveated stimuli (Figure 2C, black) are larger than those on non-foveated ones (Figure 2C, magenta). This implies that for most participants the peripheral stimuli in our task contained some information that could in principle be used in planning saccades to stimuli expected to either confirm or disconfirm one’s existing belief. Furthermore, despite substantial participant-to-participant variability, most weights have a weakly increasing trend on average, in line with prior findings in comparable evidence accumulation tasks (Brunton, Botvinick, & Brody, 2013; Wyart, De Gardelle, Scholl, & Summerfield, 2012; Drugowitsch, Wyart, Devauchelle, & Koechlin, 2016; Lange et al., 2020). This means that stimuli presented later in the trial have a slightly larger influence on average on the participant’s choice than those presented earlier.

Importantly, we found that 9/10 participants were more likely to saccade to stimuli that agreed with their current belief about the trial category (8/10 statistically significant). One participant did not show any bias. It is possible that they could not extract enough information from the peripheral stimuli to guide their saccades, hence making saccades at random.

## Approximate Bayesian active sensing model

### Rationale

Maximal performance in our task is achieved by Bayesian active sensing, i.e. an observer who maximizes the gain in information about the correct choice with each saccade (MacKay, 1992; Najemnik & Geisler, 2005; Yang et al., 2016). However, it is straightforward to show that an *exact* Bayesian observer does not display any saccade selection bias since the gain in information is independent of stimulus orientation. However, what we will show below is that computing this gain in information *approximately*, in our case by sampling, will indeed induce an observer bias that matches our empirical data. The motivation for modeling the brain’s approximate computations using sampling (as opposed to a variational approximation) is based on extensive prior work showing that sampling-based representations can account for a large amount of both cognitive (Griffiths, Vul, & Sanborn, 2012; Gershman, Vul, & Tenenbaum, 2012; Sanborn, Griffiths, & Navarro, 2010) and neural data (Fiser, Berkes, Orbán, & Lengyel, 2010; Berkes, Orbán, Lengyel, & Fiser, 2011; Haefner, Berkes, & Fiser, 2016; Orbán, Berkes, Fiser, & Lengyel, 2016; Echeveste, Aitchison, Hennequin, & Lengyel, 2020). However, it is possible that a variational approximation entails the same qualitative bias as a sampling-based approximation (Lange et al., 2020).

### Model details

Figure 3A shows a simplified version of the generative model for our task from the experimenter’s perspective. Each trial is defined by a single category (45 degrees clockwise or counterclockwise),  $C$ , and consists of 4 sequential 250ms displays, indexed by  $t = 1..4$ , and represented by the plate (box) in Fig-

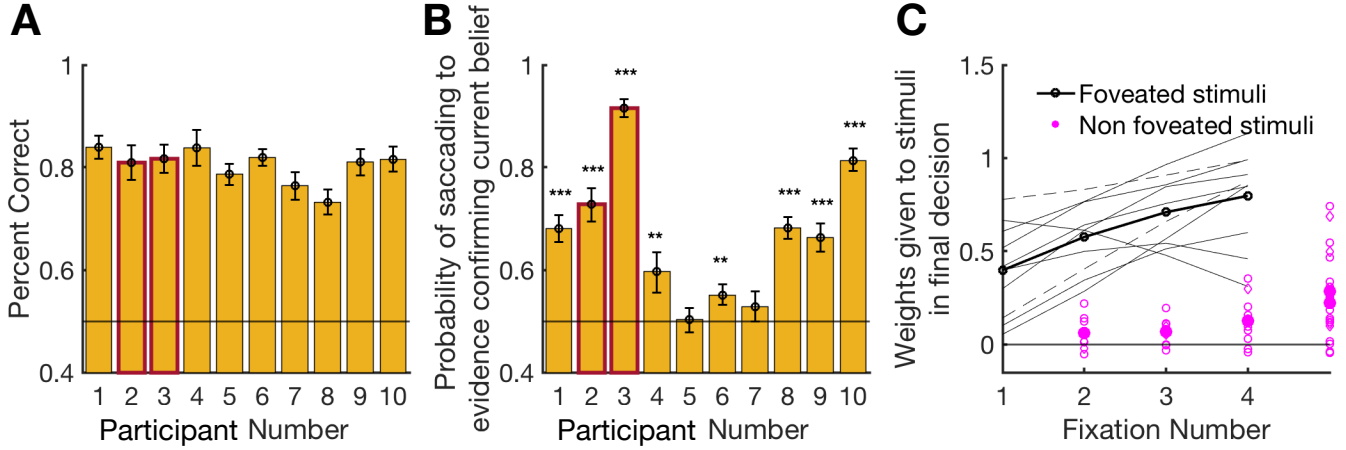


Figure 2: **Participants show bias in eye-movements** (A) Performance of 10 participants in the experiment. (B) Probability with which participants saccade to confirmatory stimulus in the periphery based on already accumulated evidence (red bordered bars are for non naive participants) in both A and B.  $p < 0.05$ ,  $p < 0.01$  and  $p < 0.001$  shown by 1, 2 and 3 stars respectively. (C) Weights given to stimuli in making the final choice. Thin black lines indicate individual participants (dashed lines for non naive). The thick black line indicates the mean weights across participants. Magenta hollow circles are for naive participants (diamonds for non-naive). The filled magenta circles indicate mean weights given to non-foveated stimuli across all participants.

ure 3A. For the first display,  $t = 1$ , the orientations for all three presented stimuli are chosen independently from each other to agree with  $C$  with probability 0.7: one at the fovea,  $C_t^0$ , and two in the periphery,  $C_t^1$  and  $C_t^2$ . The actually presented stimulus observed by the participant is then drawn as a Gaussian around the respective orientation modeling both the stochastic stimulus generation (orientation-filtered Gaussian noise) and the sensory noise of the visual system. The standard deviation of this Gaussian for the stimulus on the fovea is  $\sigma_{\text{fovea}}$ , and the variance in the periphery is  $\sigma_{\text{periphery}}$ , where  $\sigma_{\text{fovea}} < \sigma_{\text{periphery}}$ . For subsequent displays, only the two peripheral stimuli are drawn anew randomly to agree with  $C$  with 0.7 probability. The foveated stimulus, on the other hand, is identical to the peripheral stimulus saccaded to between the previous and the current display. This dependency between displays is not shown in Figure 3A for visual simplicity, but incorporated in our model. Importantly, optimal Bayesian inference over trial category  $C$  in this model requires optimally choosing saccade targets on each of the first three displays. As has been previously shown, this is accomplished by maximize the Bayesian Active Sensing (BAS) score across the two possible actions (MacKay, 1992; Najemnik & Geisler, 2005; Yang et al., 2016):

$$\max_{i=1,2} \underbrace{\mathbb{H}[C_t^i | \mathcal{D}_t]}_{\text{sensory component}} - \underbrace{\mathbb{E}_{p(C|\mathcal{D}_t)}[\mathbb{H}[C_t^i | C, \mathcal{D}_t]]}_{\text{cognitive component}} \quad (1)$$

where  $\mathcal{D}_t = \{I_{1..t}^0, I_{1..t}^1, I_{1..t}^2\}$  represents all the stimuli presented so far. Intuitively, the first term represents the participant’s uncertainty about the peripheral stimulus under consideration, and hence the information that could in principle be gleaned from saccading there. The 2nd component sub-

tracts from that the information about this location that is already known given one’s current belief about the trial category  $C$ . We call them ‘sensory component’ and ‘cognitive component’ since they are likely computed in a sensory and cognitive area, respectively.

In this work we hypothesize that the brain cannot compute this score exactly, but approximates it by sampling. We obtain:

$$\begin{aligned} &\approx \mathbb{H}[C_t^i | \mathcal{D}_t] - \frac{1}{n_{\text{cognitive}}} \sum_{j=1}^{n_{\text{cognitive}}} \mathbb{H}[C_t^i | C_{(j)}, \mathcal{D}_t] \\ &\approx \frac{1}{n_{\text{sensory}}} \sum_c \sum_{k=1}^{n_{\text{sensory}}} -p(C_t^i = c, C_{(k)} | \mathcal{D}_t) \\ &\times \log \frac{1}{n_{\text{sensory}}} \sum_{k'=1}^{n_{\text{sensory}}} p(C_t^i = c, C_{(k')} | \mathcal{D}_t) \\ &- \frac{1}{n_{\text{cognitive}}} \sum_{l=1}^{n_{\text{cognitive}}} \left[ -\sum_c p(C_t^i = c | C_{(l)}, \mathcal{D}_t) \right. \\ &\times \left. \log p(C_t^i = c | C_{(l)}, \mathcal{D}_t) \right] \end{aligned}$$

where  $n_{\text{sensory}}$  and  $n_{\text{cognitive}}$  are the number of samples used to approximate each computation. In general, these two numbers,  $n_{\text{sensory}}$  and  $n_{\text{cognitive}}$ , may potentially be different in the brain, reflecting differences in computational power and/speed in sensory and cognitive areas. It turns out that for  $n_{\text{sensory}} > n_{\text{cognitive}}$  the model exhibits a confirmation bias as described below.

## Findings

We analyzed the behavior of our model and compared it to our empirical results. The three key parameters in our

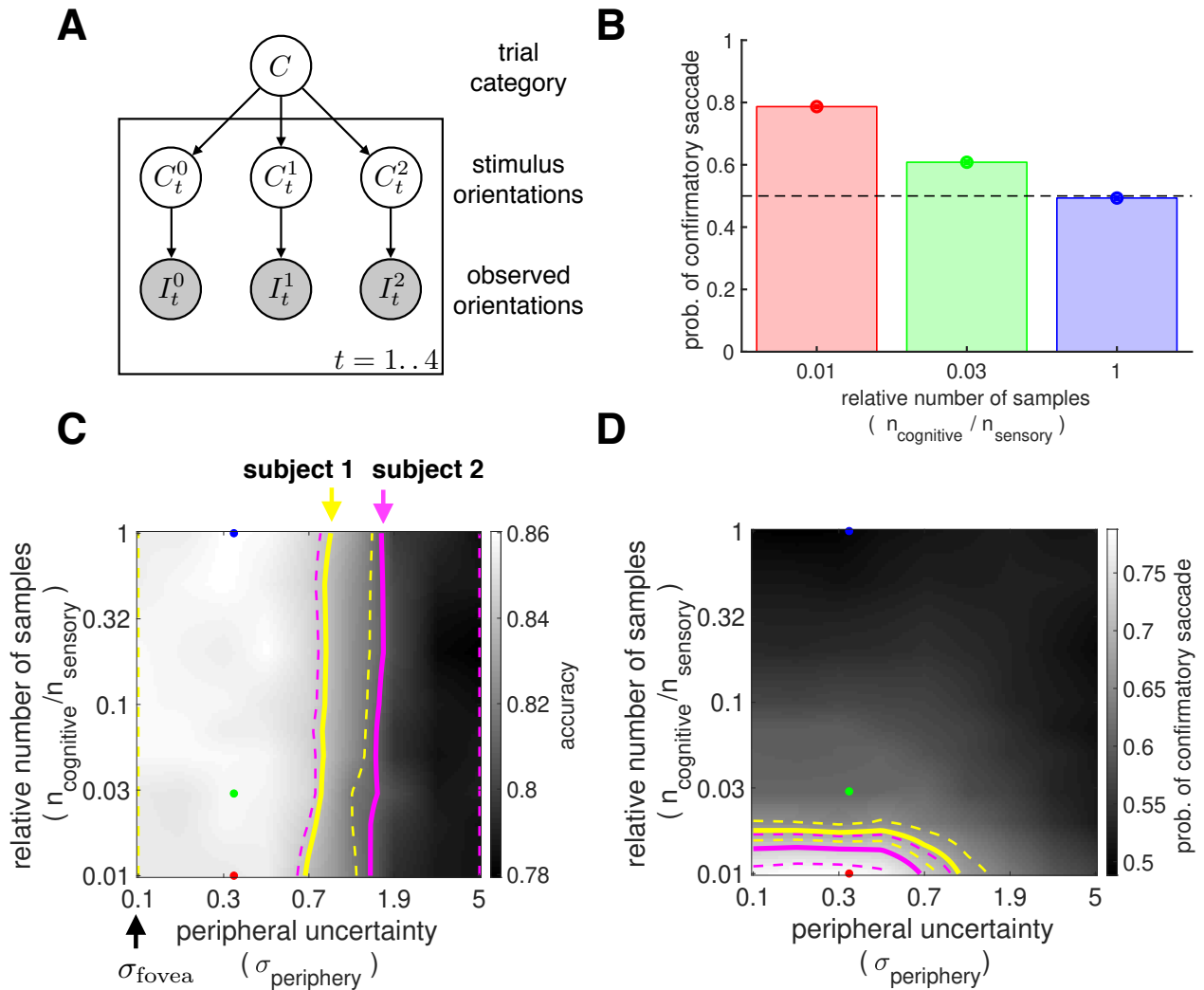


Figure 3: **Approximate inference model of saccade selection** (ref. model details for further information) (A) Simplified generative model of the task (B) Three simulated ‘participants’ who have a different degree of approximation parameterized by the relative number of samples in our model. The confirmatory saccade selection biases observed in these ‘participants’ cover the range of empirically observed biases and is inversely related to the relative number of samples (C) & (D) Performance and confirmatory saccade selection biases observed for different values of peripheral sensory uncertainty and relative number of samples. The parameters used for the three simulated ‘participants’ in (B) are shown in matching colors and the empirically observed performance and bias for two example subjects are shown as solid iso-contour lines with dashed lines indicating 68% errorbars (upper errorbar for subject 1 and lower errorbar extend outside the range of values shown in the figure)

model are the numbers of samples used for both components, and the amount of sensory uncertainty in the periphery,  $\sigma_{\text{periphery}}$ . First, we found as expected that close-to-exact inference (large number of samples for both components) induced no bias in saccade choice regardless of any other parameters. Next, based on numerical simulations, we found the same result when  $n_{\text{sensory}} = n_{\text{cognitive}}$ . However, when  $n_{\text{sensory}} > n_{\text{cognitive}}$  we found a bias for confirmatory saccades, while for  $n_{\text{sensory}} < n_{\text{cognitive}}$  we found the opposite bias: to peripheral stimuli that disagreed with the participant’s (model’s) current belief. Since none of our partic-

ipant showed the latter bias, we next focused on the case of  $n_{\text{sensory}} \geq n_{\text{cognitive}}$  by fixing  $n_{\text{sensory}} = 100$  (close to exact) while independently varying  $n_{\text{cognitive}}$  and  $\sigma_{\text{periphery}}$ . The results on accuracy and saccade bias are shown in Figure 3. We found that for our discrimination task the performance depended almost exclusively on the sensory uncertainty, and only very weakly on the degree of the approximation used to compute the BAS score. On the other hand, as long as the sensory noise in the periphery was not too large, the strength of the bias depended primarily on the degree of approximation of the cognitive computation as quantified by  $n_{\text{cognitive}}$ : the

coarser the approximation the larger the bias. Furthermore, the bias ranged from 0.5 to about 0.8 for the case  $n_{\text{cognitive}} = 1$ , covering the range of empirically observed values. The empirically observed performance and bias correspond to iso-contour lines (shown for two participants as solid lines) in the 2D parameter space shown in Figure 3C and 3D respectively. Three simulated 'participants' (models) are shown as red, green, and blue dots in Figure 3B. When the sensory noise in the periphery was too large, the bias disappeared due to the fact that regardless of approximation, the model could not infer any information to influence its saccades.

## Discussion

Our work makes three contributions. First, we present a new task design that allows for the study of perceptual decision-making across multiple goal-directed eye-movements in a highly-controlled context. Second, we provide evidence that observers' eye-movements are biased by their current beliefs when sampling new information to make perceptual judgments. Third, we showed that the empirically observed confirmation bias can be explained as the consequence of approximate computations in an ideal observer model (Yang et al., 2016).

Our gaze-contingent task was designed with the goal to tightly control the information available for both making choices in the task, and for making eye-movements to potential peripheral targets. It is therefore highly artificial, allowing for simple behavioral analyses (Rust & Movshon, 2005). It will therefore be important to verify the validity of our findings in ecologically relevant contexts by allowing for less constrained environments in non-gaze-contingent displays.

While the empirical effect that we report is very robust with an individually significant bias in 8/10 observers, the subject-to-subject variability is worth investigating in more detail. One short-coming of our current experiment is that the signal strength – eccentricity combination for the targets in the periphery is not adjusted to the peripheral sensory uncertainty for each individual observer. As a result, there may be substantial subject-to-subject variability in the uncertainty about the orientation of the peripheral targets. If the uncertainty is too large, eye movements will necessarily be unbiased as might be the case for observer 5 in our dataset. However, we did not find a significant correlation between the magnitude of the saccade bias and the regression weights for the stimuli that were never foveated. Note, however, that those weights are the result of both sensory inference and decision-making and therefore can only be expected to be indirectly related to peripheral uncertainty.

We modeled the observed biases as an ideal observer who chooses stimuli in the periphery that maximize the observer's information about the correct trial category – formalized as maximizing the mutual information between the peripheral stimulus orientation and the trial category. The mutual information can be expressed as a difference between two components: (a) the "raw" peripheral stimulus information (the

sensory component) and (b) the expected peripheral stimulus information based on the current belief about the correct trial category (cognitive component). We hypothesize that participants approximate the underlying computations by sampling to compute each of the two components. A sampling-based representation has been previously proposed for modeling behavior (Gershman et al., 2012; Griffiths et al., 2012) and neural responses (Fiser et al., 2010; Haefner et al., 2016; Orbán et al., 2016). Simulating this approximate information maximization model we found that a coarser approximation of the cognitive component than the sensory component led to a bias towards confirmatory saccades. On the other hand, we also found that a coarser approximation of the sensory than the cognitive component results in saccades to targets that are expected to disagree with the observer's current belief. Despite the model's ability to predict both types of biases in saccade selection we observed most participants having confirmatory saccades. We suggest that this is the result of the brain having a better representation of the sensory component compatible with prior proposals of higher capacity representations early in the visual hierarchy (Lee & Mumford, 2003).

The two key parameters in our model that determine its behavior – both with respect to choice accuracy, and strength of confirmation bias – are the difference (ratio) in number of samples used to evaluate the 'sensory' compared to 'cognitive' component of the Bayesian active sensing score, and the uncertainty of the stimulus in the periphery compared to that in the fovea. While visually comparing individual observers to model predictions (Figure 3C+D) suggests that the model is compatible with the data, and that the number of cognitive samples is very small, quantitative model fitting will be required to draw more reliable conclusions.

Eye movements are crucial for collecting information about the world. Our insights into how they are biased, and how approximate computations may be responsible for this bias, are not only important for our understanding of human vision but may also yield insights into potential biases and their causes in cognition and machine learning.

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