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Performance Analysis of n-Channel Symmetric FEC-Based Multiple Description Coding for OFDM Networks

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***Abstract*—Recently, multiple description source coding has emerged as an attractive framework for robust multimedia transmission over packet erasure channels. In this paper, we mathematically analyze the performance of n-channel symmetric FEC-based multiple description coding for a progressive mode of transmission over orthogonal frequency division multiplexing (OFDM) networks in a frequency-selective slowly-varying Rayleigh faded environment. We derive the expressions for the bounds of the throughput and distortion performance of the system in an explicit closed form, whereas the exact performance is given by an expression in the form of a single integration. Based on this analysis, the performance of the system can be numerically evaluated. Our results show that at high SNR, the multiple description encoder does not need to fine-tune the optimization parameters of the system due to the correlated nature of the subcarriers. It is also shown that, despite the bursty nature of the errors in a slow fading environment, FEC-based multiple description coding without temporal coding provides a greater advantage for smaller description sizes.**

***Index Terms*—Cross-layer design, multimedia communications, multiple description coding, orthogonal frequency division multiplexing (OFDM), progressive transmission, wireless video.**

I. Introduction

In recent years, there has been significant interest in the transmission of multimedia services over wireless channels, and it has invoked intense research for cross-layer optimization design [1], [2], which is particularly important for the transmission over mobile radio channels exhibiting time-variant channel-quality fluctuations.

Progressive image or scalable video coders [3]–[7] employ a progressive mode of transmission such that as more bits are transmitted, the source can be reconstructed with better quality at the receiver. Such coders are usually sensitive to channel impairments. Early studies [8], [9] considered the transmission of a progressively compressed bitstream using rate-compatible punctured convolutional codes. However, channel coding becomes less effective in a slow fading channel where prolonged deep fades often result in the erasure of the whole packet [9].

Multiple description source coding has recently emerged as an attractive framework for robust multimedia transmission over packet erasure channels [10]. The basic idea is to generate multiple descriptions of the source such that each independently describes the source with a certain fidelity. When more than one description is available at the decoder, they can be synergistically combined to enhance the quality [11]. Due to the individually decodable nature of the descriptions, the loss of some of them will not jeopardize the decoding of correctly received descriptions. Earlier studies of multiple description source coding concentrated on information-theoretic bounds for specific input source models [12]–[14]. Recently, practical implementation of multiple description source coding has received attention [10]. For progressive bitstreams under deep fades in a mobile channel, n-channel symmetric FEC-based multiple description coding [15]–[18] is employed [15], [19]–[21]. In this scheme, contiguous information symbols from a progressive bitstream are spread across multiple packets (i.e., descriptions) instead of being packed in the same packets [8], [22]. The information symbols are then protected against channel errors using systematic maximum distance separable (MDS) erasure codes, and the level of error protection depends on the relative importance of the information symbols. This FEC-based multiple description coding has become popular [20], [21], [23], [24] since it is flexible in generating arbitrary numbers of descriptions from a progressive bitstream.

Orthogonal frequency division multiplexing (OFDM) is being considered in a large number of current applications. OFDM differentiates itself from single carrier systems in many ways such as robustness against frequency-selective fading. The use of FEC-based multiple description coding over OFDM systems was considered in [23], [25] for progressive images and scalable video, in a frequency-selective slow Rayleigh fading channel. It was demonstrated that the multiple description coding in [23] using the SPIHT image coder provides superior performance over the approach in [26] which does not use multiple description coding over OFDM systems. The FEC-based multiple description coding in OFDM systems since then has been of much interest [27]–[30].

In this paper, we mathematically analyze the performance of n-channel symmetric FEC-based multiple description coding for a progressive mode of transmission over OFDM networks in a frequency-selective slowly Rayleigh fading channel. Based

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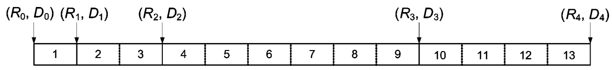


Fig. 1. Progressive description from the source coder partitioned into five quality levels of rate R_g and distortion $D(R_g) = D_g\ (g = 0, 1, \ldots, 4)$.

on this analysis, the performance of the system can be numerically evaluated, and system parameters such as channel code rates can be determined without a Monte Carlo simulation. The rest of this paper is organized as follows. In Section II, we provide some technical preliminaries, and the system model is described in Section III. The analysis of the throughput and distortion performance is derived in Section IV. In Section V, numerical results and discussions are provided, and we conclude our work in Section VI.

II. Preliminaries

A. Orthogonal Frequency Division Multiplexing (OFDM)

OFDM splits a high-rate data stream into a number of lower-rate data streams that are transmitted over orthogonal subcarriers. Based on the frequency-selectivity of a channel, frequency diversity can be exploited by adding redundancy across the subcarriers. Generally, the maximum achievable diversity gain of an OFDM system is proportional to the number of independent fading channels, N. Note that $N = 1$ corresponds to a flat-fading environment, while $N > 1$ corresponds to a frequency-selective environment.

B. FEC-Based Multiple Description Coding

We provide a brief overview of the FEC-based multiple description coding [15], [19], [20] in which MDS erasure codes are used. An (n, k) erasure code with minimum distance $d_{\min}$ refers to a construction where k information symbols are encoded into n channel symbols such that the reception of any $(n - d_{\min} + 1)$ of the n channel symbols enables k information symbols to be recovered. Channel codes with $d_{\min} = n - k + 1$ are referred to as MDS codes, which implies that the k information symbols can be recovered if any k channel symbols are correctly received. Reed–Solomon (RS) codes have this property. Fig. 1 shows a typical progressive bitstream, in which the source can be reconstructed progressively from the prefixes of the bitstream, while an error generally renders the subsequent bits useless. Fig. 2 illustrates a practical realization of n-channel symmetric FEC-based multiple description coding [15], [16], [19] by applying unequal FEC to different parts of a progressive bitstream. A progressive bitstream from a source encoder is converted into multiple descriptions in which contiguous information symbols are spread across the multiple descriptions. The information symbols are protected against channel errors using systematic $(n = 4, k)$ MDS codes, with the level of protection depending on the relative importance of the information symbols. If any g out of n descriptions are received, those codewords with information symbols less than or equal to g can be decoded. As a result, decoding is guaranteed at least up to distortion $D(R_g)$ which is the distortion achieved with R_g information symbols. For example, in codeword 1, the erasure

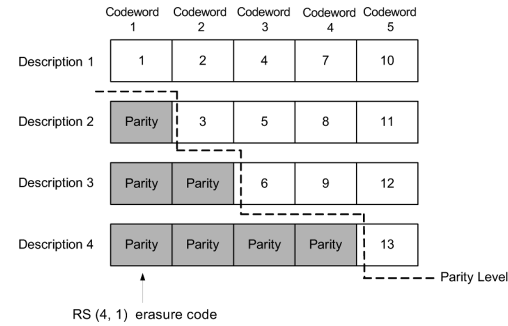


Fig. 2. n-channel symmetric FEC-based multiple description coding technique for a progressive bitstream.

of any three descriptions still allows the decoder to reconstruct information symbol 1 and achieve a delivered quality equal to $D(R_1)$.

III. System Model

We briefly describe the system model considered in [23], [25]. The total number of subcarriers of the OFDM system is denoted by N_t. A frequency-selective fading environment has N independent fading channels and each of the N channels consists of M highly correlated subcarriers ($N_t = NM$). Let $s[n, u, v]$ be the vth input modulated symbol of a description at the uth subcarrier in the nth channel. Let V denote the block length of a description in terms of the modulated symbols. At the receiver, the output signal $r[n, u, v]$ can be expressed as

$$r[n, u, v] = \alpha[n, u, v]s[n, u, v] + w[n, u, v]$$
$$\text{for } 1 \le n \le N,\ 1 \le u \le M,\ 1 \le v \le V \qquad (1)$$

where $w[n, u, v]$ is a zero-mean complex Gaussian random variable. It is assumed that $w[n, u, v]$ is independent for different n's, u's and v's. Due to the highly correlated nature of the subcarriers within a channel, we have

$$\alpha[n, u, v] \approx \alpha[n, v] \qquad (2)$$

where α is a zero-mean complex valued Gaussian random variable with Rayleigh-distributed envelope. This corresponds to the widely used block fading channel model [31]–[34] in the frequency domain. In the time domain, the channel is assumed to experience slow Rayleigh fading (i.e., the channel symbol duration is much smaller than the coherence time) such that the fading coefficients are nearly constant over a description, and hence we have $\alpha[n, u, v] \approx \alpha[n]$.

Fig. 3 shows n-channel symmetric FEC-based multiple description coding for a progressive bitstream transmission over OFDM systems. The bitstream is converted into NM descriptions using the FEC-based multiple description encoder [15], [16], [19]. Due to the assumption of slow Rayleigh fading, channel coding plus interleaving in the time domain is not considered [35], [36]. Each RS code symbol consists of eight bits (four QPSK symbols). Cyclic redundancy check (CRC) codes are appended to each description for error detection. We assume that the size of the CRC codes appended to a description is negligible compared to the description size itself. The NM

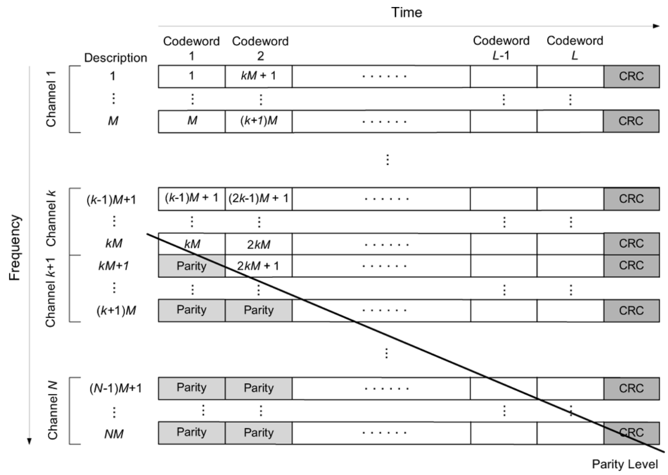


Fig. 3. n-channel symmetric FEC-based multiple description coding for progressive bitstream transmission over an OFDM system.

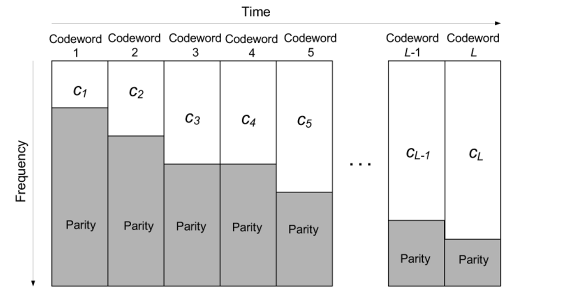


Fig. 4. UEP techniques employing decreasing level of error protection for the codewords $(c_1 \leq c_2 \leq \cdots \leq c_L)$.

independent descriptions are mapped to $N_t = NM$ subcarriers and transmitted through the OFDM system. The description size in terms of code symbols is denoted by L. Since each RS code symbol contains four QPSK symbols, the description size in terms of modulated symbols is $V = 4L$.

IV. Throughput and Distortion Analysis

We first derive the average throughput and distortion in terms of the probability of n description errors in an OFDM frame $(0 \leq n \leq N_t)$. N_t is the total number of descriptions in an OFDM frame. Let c_l denote the number of RS code symbols assigned to information symbols for codeword l $(1 \leq l \leq L)$. As the compressed bitstream from an image/video encoder has different sensitivities toward channel errors, the overall system performance is improved by employing unequal error protection (UEP). Error protection decreases for the codewords on the right (i.e., $c_1 \leq c_2 \leq \cdots \leq c_L$) [23], as shown in Fig. 4.

Let $E[R]$, $E[D]$, and $P_f(n)$ denote the average throughput, average distortion, and the probability of n description errors in a frame. $D(x)$ denotes the operational rate-distortion curve, and $D(0)$ is the distortion when the decoder reconstructs the source without any transmitted information. Since an RS code is used for each codeword, k information symbols can be recovered if any k channel symbols are correctly received. It can be shown that the average throughput in terms of the number of bits is given by

$$E[R] = \sum_{l=1}^{L} \left(N_{cs} c_l \sum_{n=0}^{N_t - c_l} P_f(n) \right) \tag{3}$$

where N_{cs} $(= 8)$ is the number of bits of an RS code symbol. It can be shown that the average distortion is given by

$$E[D] = D(0)\,\Pr(0) + \sum_{l=1}^{L} \left\{ D\left(\sum_{k=1}^{l} N_{cs} c_k \right) \Pr\left(\sum_{k=1}^{l} N_{cs} c_k \right) \right\} \tag{4}$$

where $\Pr(R)$ is the probability that the throughput is R bits, and $\Pr(0)$ is the probability that no information bits are successfully decoded at the receiver. It can be shown that $E[D]$ is expressed as (5) at the bottom of the page, where $\sum_{i=I_1}^{I_2} f(i) \triangleq 0$ for $I_2 < I_1$, and $c_{L+1} \triangleq N_t + 1$.

We next derive the probability of n description errors in a group of M subcarriers which have the same fading coefficients. Since we assume slow Rayleigh fading, the conditional probability of a description error for a given Rayleigh fading coefficient h, denoted by $P_d(e|h)$, is

$$P_d(e|h) = 1 - (1 - P_{cs}(e|h))^L \tag{6}$$

where $P_{cs}(e|h)$ is the conditional probability of an RS code symbol error given h. It equals

$$\begin{aligned} P_{cs}(e|h) &= 1 - (1 - P_s(e|h))^4 \\ &= 1 - \left(1 - Q\left(h\sqrt{\gamma_s}\right)\right)^8 \end{aligned} \tag{7}$$

$$\begin{aligned} E[D] &= D(0) \sum_{n=N_t-c_1+1}^{N_t} P_f(n) + D\left(\sum_{k=1}^{1} N_{cs} c_k\right) \sum_{n=N_t-c_2+1}^{N_t-c_1} P_f(n) + D\left(\sum_{k=1}^{2} N_{cs} c_k\right) \sum_{n=N_t-c_3+1}^{N_t-c_2} P_f(n) \\ &\quad + \cdots + D\left(\sum_{k=1}^{L-1} N_{cs} c_k\right) \sum_{n=N_t-c_L+1}^{N_t-c_{L-1}} P_f(n) + D\left(\sum_{k=1}^{L} N_{cs} c_k\right) \sum_{n=0}^{N_t-c_L} P_f(n) \\ &= D(0) \sum_{n=N_t-c_1+1}^{N_t} P_f(n) + \sum_{l=1}^{L} \left\{ D\left(\sum_{k=1}^{l} N_{cs} c_k\right) \sum_{n=N_t-c_{l+1}+1}^{N_t-c_l} P_f(n) \right\} \end{aligned} \tag{5}$$

where $P_s(e|h) = 2Q(h\sqrt{\gamma_s}) - Q^2(h\sqrt{\gamma_s})$ is the conditional probability of a modulated QPSK symbol error for a given h, and γ_s is the signal-to-noise ratio (SNR) per modulated QPSK symbol. We define a group to be M correlated subcarriers with the same fading coefficients. The conditional probability of n description errors in a group $(0 \leq n \leq M)$ for a given h, $P_g(n|h)$, is given by

$$P_g(n|h) = \binom{M}{n} P_d^n(e|h)\left(1 - P_d(e|h)\right)^{M-n}. \quad (8)$$

From (6) and (7), for $0 \leq n \leq M$, $P_g(n|h)$ can be expressed as

$$P_g(n|h) = \binom{M}{n}\left\{1 - \left(1 - Q\left(h\sqrt{\gamma_s}\right)\right)^{8L}\right\}^n \times \left(1 - Q\left(h\sqrt{\gamma_s}\right)\right)^{8L(M-n)}. \quad (9)$$

The probability of n description errors in a group, $P_g(n)$, can be calculated by taking the expectation of $P_g(n|h)$ with regard to h representing a Rayleigh probability distribution:

$$P_R(h) = \frac{2h}{\Omega}\exp\left(-\frac{h^2}{\Omega}\right), \quad h \geq 0 \quad (10)$$

where Ω is the second moment of h. From (9) and (10), for $0 \leq n \leq M$, $P_g(n)$ is given by (11) at the bottom of the page. Note that for an integer value of $q \geq 5$, there is no closed-form expression for the integral in (11) [37]. To avoid numerical integration in $P_g(n)$, instead, we can use exponential-type upper and lower bounds on $Q(x)$ [38]. From [38, eqs. (8), (9),and (26)], the upper and lower bounds on $Q(x)$ for $x \geq 0$, denoted by $f_u(x)$ and $f_l(x)$, respectively, are given by

$$f_u(x) = \sum_{i=1}^{B} a_i \exp\left(-b_i x^2\right)$$
$$f_l(x) = \sum_{i=2}^{B} a_i \exp\left(-b_{i-1} x^2\right) \quad (12)$$

where $a_i = (\theta_i - \theta_{i-1})/\pi$, $b_i = \csc^2\theta_i/2$, and $\theta_1, \theta_2, \ldots, \theta_{B-1}$ are arbitrary values satisfying $0 = \theta_0 \leq \theta_1 \leq \theta_2 \leq \cdots \leq \theta_{B-1} \leq \theta_B = \pi/2$ [38, eq. (8)]. By increasing B, the bounds $f_u(x)$ and $f_l(x)$ converge to the exact $Q(x)$ [38]. Using (12), the upper bound of the integration in (11) is

$$\begin{aligned}\int_0^{\infty} Q^q\left(h\sqrt{\gamma_s}\right)P_R(h)dh &\leq \int_0^{\infty} f_u^q\left(h\sqrt{\gamma_s}\right)P_R(h)dh \\ &= \int_0^{\infty}\left(\sum_{i=1}^{B} a_i \exp\left(-b_i\gamma_s h^2\right)\right)^q \frac{2h}{\Omega}\exp\left(-\frac{h^2}{\Omega}\right)dh.\end{aligned} \quad (13)$$

We have

$$\begin{aligned}\left(\sum_{i=1}^{B} a_i \exp\left(-b_i\gamma_s h^2\right)\right)^q = \sum_{k_1,k_2,\ldots,k_B}\Bigg\{&\binom{q}{k_1,k_2,\ldots,k_B}a_1^{k_1}a_2^{k_2}\cdots a_B^{k_B} \\ &\times\exp\left(-\left(b_1k_1 + b_2k_2 + \cdots + b_Bk_B\right)\gamma_s h^2\right)\Bigg\}\end{aligned} \quad (14)$$

where the summation is taken over all sequences of nonnegative indexes $k_1, k_2, \ldots, k_B$ such that the sum of all $k_1, k_2, \ldots, k_B$ is equal to q, and $\binom{q}{k_1,k_2,\ldots,k_B} = q!/k_1!k_2!\cdots k_B!$ are the multinomial coefficients. It can be readily shown that

$$\int_0^{\infty}\exp\left(-\alpha h^2\right)\frac{2h}{\Omega}\exp\left(-\frac{h^2}{\Omega}\right)dh = \frac{1}{\alpha\Omega + 1}. \quad (15)$$

From (14) and (15), (13) can be rewritten as

$$\begin{aligned}\int_0^{\infty} Q^q\left(h\sqrt{\gamma_s}\right)P_R(h)dh \leq \sum_{k_1,k_2,\ldots,k_B}\Bigg\{&\binom{q}{k_1,k_2,\ldots,k_B} \\ &\times\frac{a_1^{k_1}a_2^{k_2}\cdots a_B^{k_B}}{(b_1k_1 + b_2k_2 + \cdots + b_Bk_B)\gamma_s\Omega + 1}\Bigg\}\end{aligned} \quad (16)$$

Therefore, $P_g(n)$, given by (11), is upper bounded as (17) on the next page. In a similar way, from (12), it can be shown that

$$\begin{aligned}P_g(n) &= \int_0^{\infty}\binom{M}{n}\left\{1 - \left(1 - Q\left(h\sqrt{\gamma_s}\right)\right)^{8L}\right\}^n\left(1 - Q\left(h\sqrt{\gamma_s}\right)\right)^{8L(M-n)}P_R(h)dh \\ &= \int_0^{\infty}\binom{M}{n}\sum_{p=0}^{n}\left[\binom{n}{p}(-1)^p\left(1 - Q\left(h\sqrt{\gamma_s}\right)\right)^{8L(p+M-n)}\right]P_R(h)dh \\ &= \binom{M}{n}\sum_{p=0}^{n}\left[\binom{n}{p}(-1)^p\sum_{q=0}^{8L(p+M-n)}\left\{(-1)^q\binom{8L(p+M-n)}{q}\int_0^{\infty}Q^q\left(h\sqrt{\gamma_s}\right)P_R(h)dh\right\}\right]\end{aligned} \quad (11)$$

$P_g(n)$, given by (11), is lower bounded as (18) at the bottom of the page.

We next derive the probability of n description errors in an OFDM frame. An erroneous group is defined as a group which has at least one description error. For n description errors in a frame $(0 \leq n \leq N_t = NM)$, the number of erroneous groups, m, is in the range of

$$\left\lceil \frac{n}{M} \right\rceil \leq m \leq \min(N, n) \tag{19}$$

where $\lceil x \rceil$ denotes the smallest integer which is greater than or equal to x. To see this, note the following.

1) The total number of descriptions in all erroneous groups, mM, should be larger than or equal to the number of description errors in a frame, that is $mM \geq n$. Since m is an integer, the infimum of m is given by $\lceil n/M \rceil$.
2) The number of erroneous groups, m, should be smaller than or equal to both the number of description errors in a frame, n, and the number of groups in a frame, N. Hence, the supremum of m is given by min (N, n).

Next, we will show that for n description errors in a frame $(0 \leq n \leq NM)$, the probability of m erroneous groups, $P_f(n, m)$, is given by (20) at the bottom of the page, where $P_g(l)$ is given by (11), and it is assumed that for $i = 1, 2, \ldots, m$, the ith group from the top in a frame has the ith largest number of description errors among m erroneous groups. After proving (20), we will generalize (20) without the assumption.

Proof: See Appendix A. □

We have proved (20) under the assumption that for $i = 1, 2, \ldots, m$, the ith group from the top in a frame has the ith largest number of description errors. From (20), it follows that k_i is the ith largest number of description errors which a group has since

$$k_m \geq k_{m-1} \geq \cdots \geq k_2 \geq k_1 \tag{21}$$

where $k_1 \triangleq n - \sum_{i=2}^{m} k_i$. Next, we will generalize (20) without the assumption. We will show that the number of ways of assigning $k_m, k_{m-1}, \ldots, k_1$ satisfying (21) to N groups in a frame is given by (22) on the next page, where $m \geq 1$, $\delta(x)$ is the Kronecker delta function and

$$\sum_{i=I_1}^{I_2} f(i) \triangleq 0 \text{ and } \prod_{i=I_1}^{I_2} f(i) \triangleq 1 \text{ for } I_1 > I_2. \tag{23}$$

Proof: See Appendix B. □

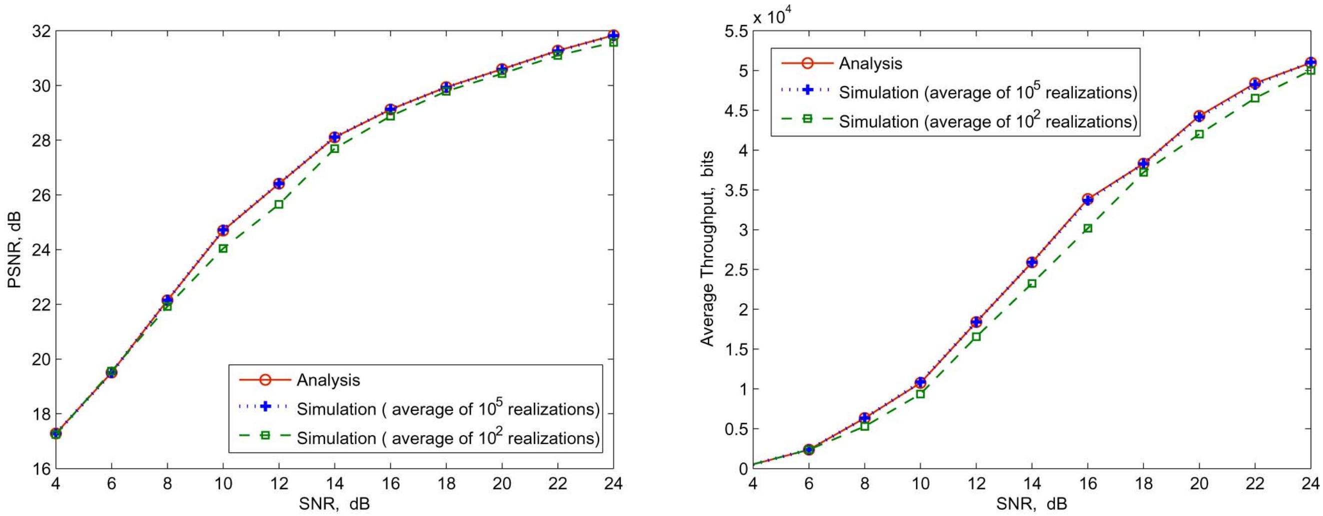
Note that $C(m)$, given by (22), is defined for $m \geq 1$. If we let $m = 0$ in (22), however, $C(0)$ yields

$$C(0) = \frac{N!}{(1+0)!\,1\,(N-0)!} = 1 \tag{24}$$

$$P_g(n) \leq \binom{M}{n} \sum_{p=0}^{n} \left[\binom{n}{p} (-1)^p \sum_{q=0}^{8L(p+M-n)} \left\{ (-1)^q \binom{8L(p+M-n)}{q} \times \sum_{k_1, k_2, \ldots, k_B} \left(\binom{q}{k_1, k_2, \ldots, k_B} \frac{a_1^{k_1} a_2^{k_2} \cdots a_B^{k_B}}{(b_1 k_1 + b_2 k_2 + \cdots + b_B k_B)\gamma_s \Omega + 1} \right) \right\} \right] \tag{17}$$

$$P_g(n) \geq \binom{M}{n} \sum_{p=0}^{n} \left[\binom{n}{p} (-1)^p \sum_{q=0}^{8L(p+M-n)} \left\{ (-1)^q \binom{8L(p+M-n)}{q} \times \sum_{k_1, k_2, \ldots, k_{B-1}} \left(\binom{q}{k_1, k_2, \ldots, k_{B-1}} \frac{a_2^{k_1} a_3^{k_2} \cdots a_B^{k_{B-1}}}{(b_1 k_1 + b_2 k_2 + \cdots + b_{B-1} k_{B-1})\gamma_s \Omega + 1} \right) \right\} \right] \tag{18}$$

$$P_f(n, m) = \begin{cases} \sum_{k_m = \lceil n/m \rceil}^{\min(M,\, n-(m-1))} \sum_{k_{m-1} = \lceil (n-k_m)/(m-1) \rceil}^{\min(k_m,\, n-k_m-(m-2))} \cdots \sum_{k_2 = \lceil (n-k_m-k_{m-1}\cdots-k_3)/2 \rceil}^{\min(k_3,\, n-k_m-k_{m-1}\cdots-k_3-1)} \\ \quad P_g(k_m) P_g(k_{m-1}) \cdots P_g(k_2) P_g(n - \sum_{i=2}^{m} k_i) P_g^{N-m}(0), \quad \text{for } m \geq 2 \\ P_g^m(n) P_g^{N-m}(0), \quad \text{for } m = 0, 1 \end{cases} \tag{20}$$



(a) PSNR (b) Average Throughput

Fig. 5. PSNR and average throughput of the FEC-based multiple description coding for image transmission over an OFDM system.

where the first equality follows from (23). From (19), (20), (22) and (24), the probability of n $(0 \leq n \leq NM)$ description errors in a frame, $P_f(n)$, is given by the following:

$$P_f(n) = \sum_{m=\lceil n/N \rceil}^{\min(N,n)} P'_f(n,m) \tag{25}$$

where $P'_f(n,m)$ is given by (26) at the bottom of the page. $P_g(l)$ is given by (11), and $C(m)$ is given by (22). Note that the expression of $P_f(n)$, given by (25), holds for the case $n = 0$ since we have $C(0) = 1$ from (24).

Finally, from (3), (5), and (25), the average throughput, $E[R]$, and the average distortion, $E[D]$, are obtained in explicit expressions for given parameters such as the number of descriptions in a frame (N_t), the size of a description (L), the number of groups (N), the number of information symbols for codeword l $(c_l,\ l = 1, \ldots, L)$, SNR per modulated symbol (γ_s), and the operational rate-distortion curve $(D(x))$. Note that the expressions of $E[R]$ and $E[D]$ are not closed forms due to a single integration in $P_g(l)$ given by (11), while the upper and lower bounds on the average throughput or distortion are expressed in closed forms using (17) and (18), respectively. Note that these bounds

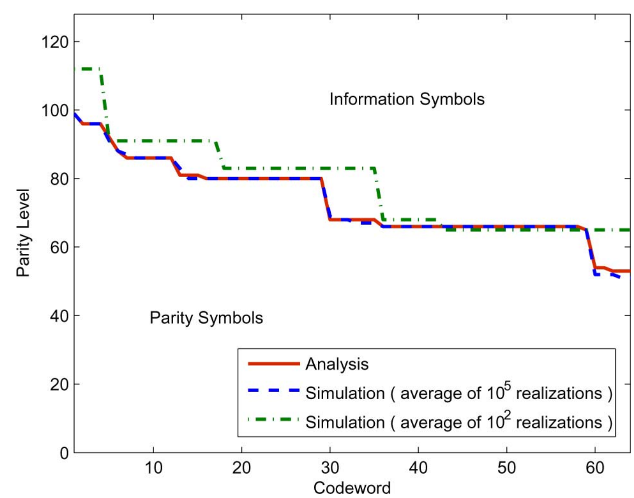
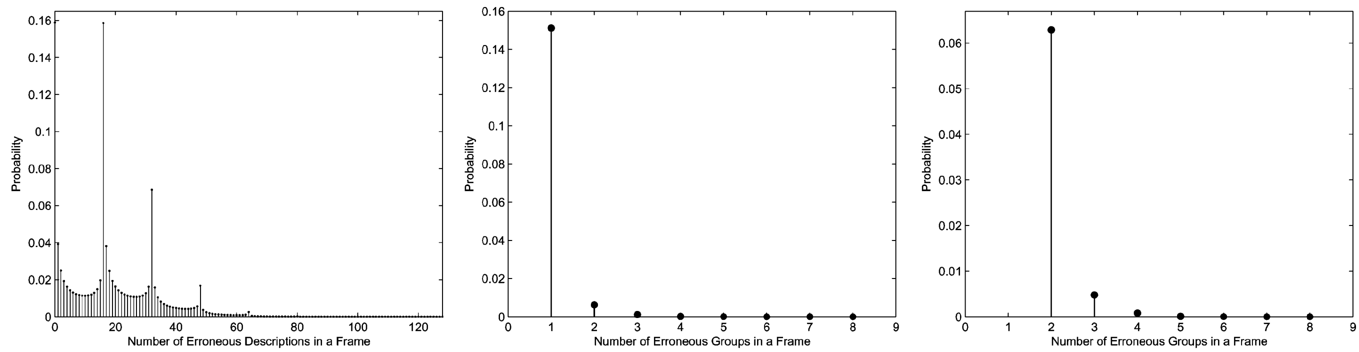


Fig. 6. Optimal allocation of parity symbols for RS codewords at an SNR of 14 dB.

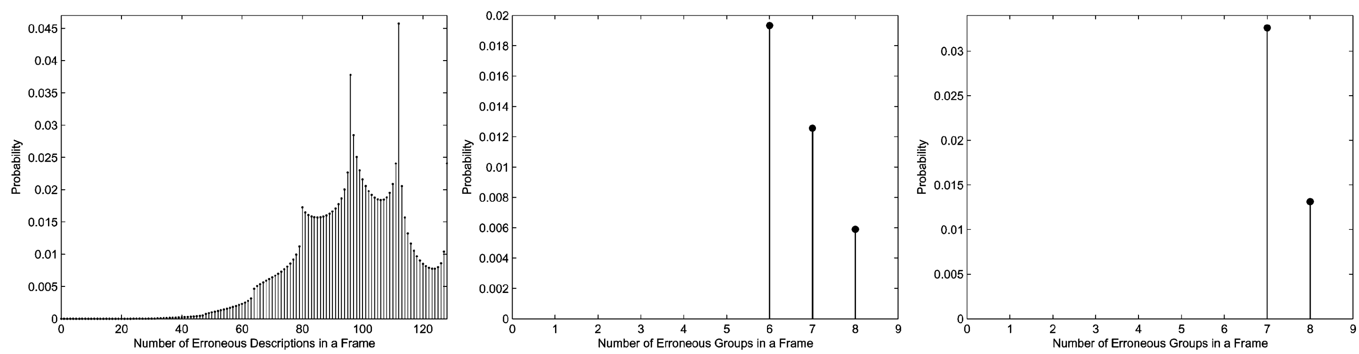
$$C(m) = \frac{N!}{\left(1 + \sum_{i=1}^{m-1} \delta(k_m - k_{m-i})\right)! \prod_{j=1}^{m-2} \left\{1 + (1 - \delta(k_{m-j} - k_{m+1-j})) \sum_{i=j+1}^{m-1} \delta(k_{m-j} - k_{m-i})\right\}! (N-m)!} \tag{22}$$

$$P'_f(n,m) = \begin{cases} \sum_{k_m=\lceil n/m \rceil}^{\min(M,\, n-(m-1))} \sum_{k_{m-1}=\lceil (n-k_m)/(m-1) \rceil}^{\min(k_m,\, n-k_m-(m-2))} \cdots \sum_{k_2=\lceil (n-k_m-k_{m-1}\cdots-k_3)/2 \rceil}^{\min(k_3,\, n-k_m-k_{m-1}\cdots-k_3-1)} \\ \quad C(m)\, P_g(k_m) P_g(k_{m-1}) \cdots P_g(k_2) P_g(n - \sum_{i=2}^{m} k_i) P_g^{N-m}(0), \quad \text{for } m \geq 2 \\ C(m)\, P_g^m(n) P_g^{N-m}(0), \quad \text{for } m = 0, 1 \end{cases} \tag{26}$$



(a) $P_f(n)$ (b) $P_f'(n=16,m)$ (c) $P_f'(n=32,m)$

Fig. 7. (a) The probability of n description errors in a frame, $P_f(n)$, given by (25) at $\mathrm{SNR} = 18$ dB. (b)(c) The probability of m erroneous groups, $P_f'(n,m)$, given by (26) for n description errors in a frame at $\mathrm{SNR} = 18$ dB.



(a) $P_f(n)$ (b) $P_f'(n=96,m)$ (c) $P_f'(n=112,m)$

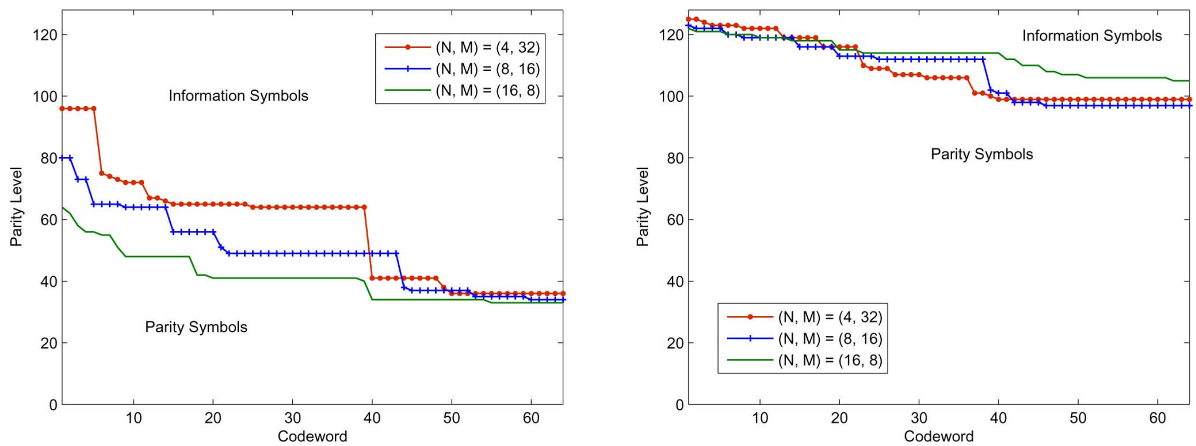
Fig. 8. (a) Probability of n description errors in a frame, $P_f(n)$, given by (25) at $\mathrm{SNR} = 8$ dB. (b)(c) The probability of m erroneous groups, $P_f'(n,m)$, given by (26) for n description errors in a frame at $\mathrm{SNR} = 8$ dB.

can become arbitrarily close to the exact average throughput or distortion by increasing the number of terms, B, in (17) and (18) [38].

V. Numerical Evaluation and Discussion

Based on the analysis in the previous section, we evaluate the peak-signal-to-noise ratio (PSNR) performance of the FEC-based multiple description coding technique for progressive image transmission in OFDM systems. The performance is evaluated for the standard 8 bits per pixel (bpp) 512×512 Lena image with a transmission rate of 0.25 bpp using the progressive source coder SPIHT [3]. Using (5) and (25), we evaluate the average distortion, $E[D]$, for $N_t = 128$, $N = 8$, and $L = 64$ as an example, and convert $E[D]$ into PSNR using the relation of $\mathrm{PSNR} = 10\log\left(255^2/E[D]\right)$. To find the optimal FEC assignment for the RS codewords in the context of multiple description coding, the hill climbing approach was proposed in [15] by Mohr *et al.*, and this algorithm was widely adopted in the previous literature [23], [25], [27], [28]. This approach is described in [15, Sec. IV] in detail. The number of parity symbols for codeword $l(1 \le l \le L)$, $N_t - c_l$, is optimized according to the procedure shown in [15, Fig. 5]. During the optimization, the computation of PSNR is required for a given number of parity symbols (see line 15 of [15, Fig. 5]), and for this computation, we use the average distortion expression given by (5) and (25) without requiring a Monte Carlo simulation. The resultant PSNR for various channel SNRs is depicted in Fig. 5(a), and the average throughput given by (4) and (25) is evaluated in Fig. 5(b). The optimal parity symbol allocation for RS codewods at a specific SNR of 14 dB is shown in Fig. 6. Monte Carlo simulation results are also depicted in Figs. 5 and 6, and the simulation results are almost the same as the analytical results. From here onwards, all the curves in this section will be obtained solely by the results of the analysis in the previous section, i.e., Monte Carlo simulation will not be used.

For a high SNR of 18 dB, in Fig. 7, we show the probability of n description errors in a frame, $P_f(n)$, and the probability of m erroneous groups, $P_f'(n,m)$, which is given by (26). Note that for the block fading channel model in the frequency domain, 16 subcarriers within a group (i.e., $M = N_t/N = 16$) are highly correlated as shown in (2). Therefore, it is expected that most description errors occur as a bundle of $16, 32, 48, \ldots, 128$ (i.e., kM for $k = 1, 2, \ldots, N$), as shown in Fig. 7(a). Moreover, for $n = kM (k = 1, 2, \ldots, N)$ description errors in a frame, the numbers of erroneous groups are the most likely to be k, as verified in Fig. 7(b) and (c). This indicates that when deciding the number of parity symbols of RS codewords, the multiple description encoder can include M more or M fewer descriptions, instead of attempting to fine-tune by including one more or one fewer description, because the correlated nature



(a) SNR = 18 dB (b) SNR = 8 dB

Fig. 9. Optimal allocation of parity symbols for RS codewords for various numbers of subcarriers in a group ($M = 8, 16, 32$).

of the subcarriers makes it unlikely that only one more or one fewer description would be received. This will be discussed in detail later in this section. For a low SNR of 8 dB, we also evaluate $P_f(n)$ and $P'_f(n,m)$ in Fig. 8. In Fig. 8(a), the probability of $kM(k = 1, 2, \ldots, N)$ description errors in a frame is not very dominant compared to a high SNR case. Moreover, Fig. 8(b) and (c) show that for $n = kM$ description errors in a frame, the probability of k erroneous groups is not dominant either. This result implies that at low SNR, multiple description errors do not occur in a highly correlated manner. For low SNR, the probability of error is not dominated by the fading channel effect of $\alpha[n,u,v]$ in (1) which is highly correlated for different u's, but is very affected by the additive Gaussian noise of $w[n,u,v]$ which is independent for different n's and u's. Hence, we note that at low SNR, the encoder should decide whether to include one more or one fewer description (i.e., fine-tune) for error protection levels. As an example, Fig. 9 shows the optimal parity levels at SNRs of 18 dB and 8 dB, respectively, after the multiple description encoder fine-tuned the allocation of parity symbols (note that the curves of Figs. 5 and 6 are also the results from fine-tuning). It is seen that at a high SNR of 18 dB, the parity level exhibits stepwise behavior compared to a low SNR case, and the step size becomes greater as M increases. This is due to the correlated nature of subcarriers shown in Fig. 7.

We now observe the PSNR performance for the case when the encoder decides whether to include M more or M fewer descriptions (i.e., coarse-tune) rather than fine-tune for parity symbol levels. As an example, for $(N, M) = (8, 16)$, the PSNR performance from coarse-tuning at various channel SNRs is depicted in Fig. 10. It is seen that for SNRs which are greater than or equal to 10 dB, the performance of coarse-tuning is roughly the same as that of fine-tuninig. However, for SNRs lower than 10 dB, coarse-tuning degrades the performance. For reference, the allocation of parity symbols for the cases of fine-tuning and coarse-tuning at SNRs of 18 dB and 8 dB is shown in Fig. 11. For the coarse optimization, the search step size of the hill-climbing approach [15] was set to M rather than to one (see line 5 of [15, Fig. 5]). From Figs. 7–11, we have observed that due to the correlated nature of the subcarriers, the multiple description encoder can coarse-tune the system optimization at high SNRs, and thus can reduce the computational complexity of the optimization.

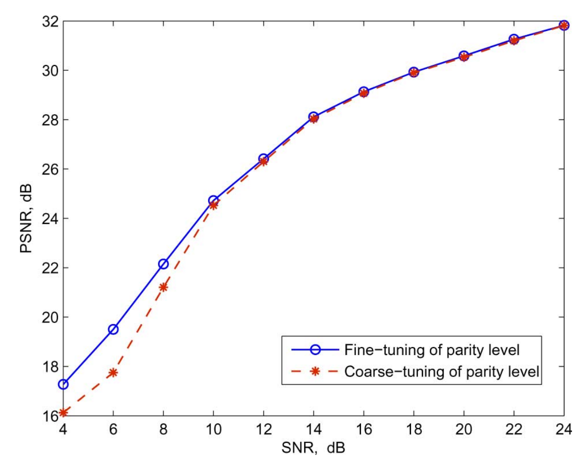
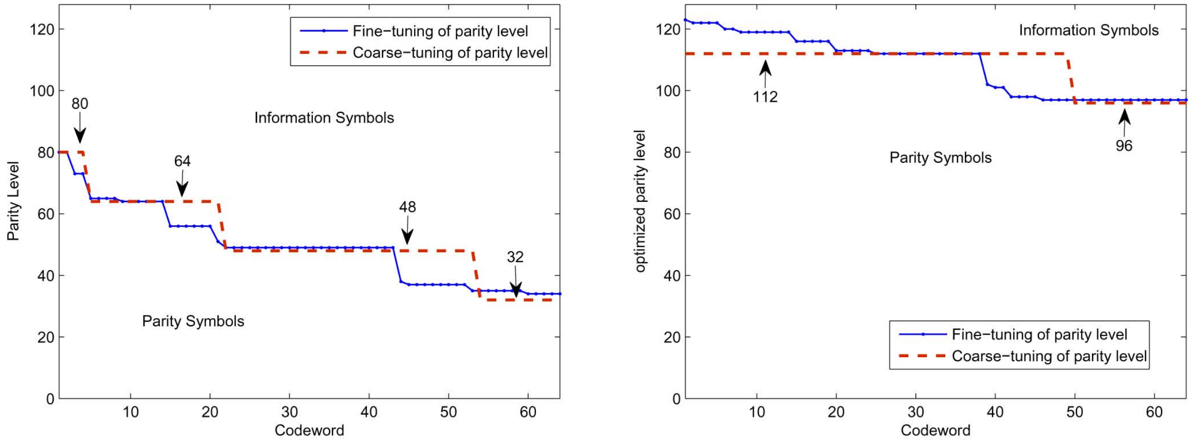


Fig. 10. PSNR performance when the encoder decides whether to include M more or M fewer descriptions (i.e., coarse-tune) for $(N, M) = (8, 16)$.

We next observe the probability of a code symbol error of RS codewords for various description sizes. From (6) and (7), the probability of a description error, $P_d(e)$, is derived as

$$
\begin{aligned}
P_d(e) &= \int_0^\infty \left(1 - (1 - P_{cs}(e|h))^L\right) P_R(h)dh \\
&= 1 - \int_0^\infty (1 - Q(h\sqrt{\gamma_s}))^{8L} P_R(h)dh \\
&= 1 - \sum_{q=0}^{8L} \left\{ (-1)^q \binom{8L}{q} \right. \\
&\qquad \left. \times \int_0^\infty Q^q(h\sqrt{\gamma_s}) P_R(h)dh \right\} \qquad (27)
\end{aligned}
$$



(a) SNR = 18 dB (b) SNR = 8 dB

Fig. 11. Allocation of parity symbols for the cases of fine-tuning and coarse-tuning with $(N, M) = (8, 16)$. a) $\mathrm{SNR} = 18$ dB b) $\mathrm{SNR} = 8$ dB.

where $P_R(h)$ is given by (10). From (7), the probability of a RS code symbol error, $P_{cs}(e)$, is expressed as

$$\begin{aligned} P_{cs}(e) &= 1 - \int_0^\infty \left(1 - Q(h\sqrt{\gamma_s})\right)^8 P_R(h)dh \\ &= 1 - \sum_{q=0}^{8} \left\{ (-1)^q \binom{8}{q} \right. \\ &\qquad \left. \times \int_0^\infty Q^q(h\sqrt{\gamma_s}) P_R(h)dh \right\}. \end{aligned} \tag{28}$$

Note that from Fig. 2, codewords are encoded by RS codes, and the probability of a channel symbol error for RS codewords is the same as the probability of a description error given by (27). On the other hand, consider the case where multiple description coding is not employed, but modulated QPSK symbols are directly encoded by RS codes in the same way as in Fig. 2. For this case, there is no information about whether a channel symbol is erroneous or not, and thus we do not have erasure channels for RS codes. Note that (n, k) RS codes can correct up to $n - k$ channel symbol erasures, while it can correct only up to $\lfloor (n-k)/2 \rfloor$ channel symbol errors. However, if multiple description coding is not employed, the probability of a channel symbol error for RS codewords becomes lower since it is the same as $P_{cs}(e)$ given by (28). To see this, in Fig. 12, we evaluated the probability of a description error given by (27) and the probability of a code symbol error given by (28). Fig. 12 shows that the probabilities of a description error for description sizes $L = 4$, 16, 64, 256, 1024 are about 2, 3, 4, 5, 6 times greater than the probability of a code symbol error in almost all SNRs. From this, it is seen that, despite the bursty nature of the errors associated with a slow fading environment, FEC-based multiple description coding without temporal coding has a greater advantage for smaller description sizes when the size of the CRC codes appended to a description is negligible compared to the description size itself.

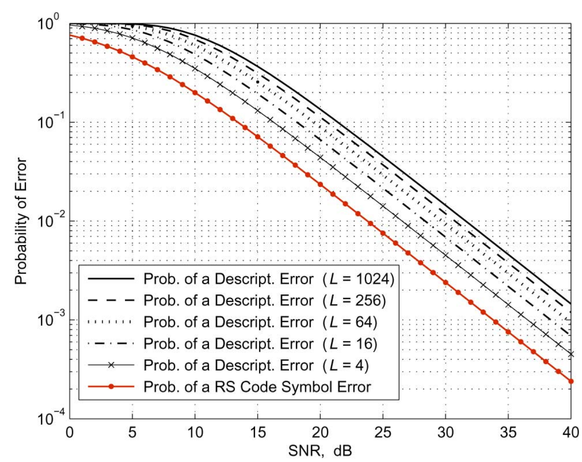


Fig. 12. Probabilities of a description error for various description sizes ($L =$ 4, 16, 64, 256, 1024) and the probability of a RS code symbol error.

VI. Conclusion

Multiple description source coding has emerged as an attractive framework for robust multimedia transmission over packet erasure channels. In this paper, we mathematically analyzed the performance of n-channel symmetric FEC-based multiple description coding for transmission of progressive bitstreams over OFDM networks in a frequency-selective slowly-varying Rayleigh faded environment. Using induction, we derived the average throughput and distortion performance in an explicit expression for given parameters such as the number of descriptions in an OFDM frame, the size of a description, and the channel conditions. While these exact expressions are in the form of a single integration, the upper and lower bounds of the performance were derived in a closed-form expression. Using the derived analysis, the performance for the transmission of a progressive image was numerically evaluated, and it was shown to be the same as the computationally intensive simulation results. We also numerically evaluated the number of description errors and the number of erroneous groups in a frame. This indicated that at high SNR, the multiple description encoder does not have to fine-tune the system optimization, such as the error

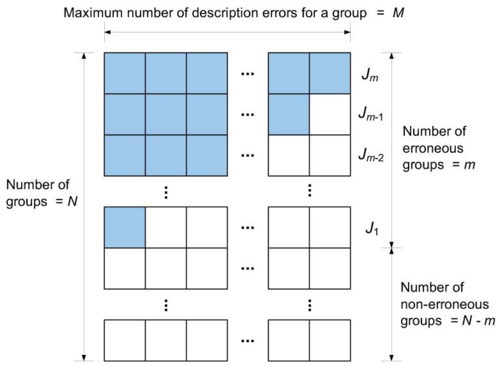


Fig. 13. OFDM frame with m erroneous groups.

protection level for the RS codewords, and thus can reduce the computational complexity of the optimization. We also evaluated the probability of a description error, which showed that, despite the bursty nature of the errors in a slow fading environment, FEC-based multiple description coding without temporal coding in a wireless environment has a greater advantage for smaller description sizes when the overhead due to the CRC codes is negligible compared to the description size.

APPENDIX A

PROOF OF *(20):* Note that the maximum number of description errors which one group can have is M. Let J_i denote the $m+1-i$th largest number of description errors which a group among m erroneous groups has $(i=1,2,\ldots,m)$. That is

$$M \geq J_m \geq J_{m-1} \geq \cdots \geq J_1 \geq 1, \\ J_m + J_{m-1} + \cdots + J_1 = n. \tag{29}$$

By the assumption below (20), the ith group from the top in a frame has J_{m+1-i} description errors $(i=1,2,\ldots,m)$. Fig. 13 shows an OFDM frame with m erroneous groups, where each subcarrier (i.e., description) is denoted by a small square box. Dark and bright boxes indicate erroneous and non-erroneous descriptions, respectively.

The Case Where $m \geq 2$: First, we will prove (20) for $m \geq 2$ by induction on the number of erroneous groups in a frame. Consider two erroneous groups (i.e., $m=2$), shown in Fig. 14. If we let $m=2$ in (29), we have

$$M \geq J_2 \geq J_1 \geq 1 \text{ and } J_2 + J_1 = n. \tag{30}$$

From (30), it follows that $J_2 \geq J_1 = n - J_2$ or $J_2 \geq n/2$. Since J_2 is an integer, we have

$$J_2 \geq \left\lceil \frac{n}{2} \right\rceil. \tag{31}$$

From (30), it follows that

$$J_1 = n - J_2 \geq 1 \Leftrightarrow J_2 \leq n - 1. \tag{32}$$

From (30)–(32), we have

$$\left\lceil \frac{n}{2} \right\rceil \leq J_2 \leq \min(M, n-1). \tag{33}$$

From (33) and Fig. 14, it follows that

$$\begin{aligned} P_f(n,2) &= \sum_{J_2=\lceil n/2 \rceil}^{\min(M,n-1)} P_g(J_2) P_g(J_1) P_g^{N-2}(0) \\ &= \sum_{J_2=\lceil n/2 \rceil}^{\min(M,n-1)} P_g(J_2) P_g(n-J_2) P_g^{N-2}(0) \end{aligned} \tag{34}$$

where the second equality follows from (30). If we let $m=2$ in (20), we have

$$P_f(n,2) = \sum_{k_2=\lceil n/2 \rceil}^{\min(M,n-1)} P_g(k_2) P_g(n-k_2) P_g^{N-2}(0). \tag{35}$$

It is seen that (35) is identical to (34).

Consider the case where there are r erroneous groups (r is some integer in the range of $2 \leq r \leq N-1$). If we let $m=r$ in (29), we have

$$M \geq J_r \geq J_{r-1} \geq \cdots \geq J_1 \geq 1 \\ J_r + J_{r-1} + \cdots + J_1 = n. \tag{36}$$

By the assumption below (20), the ith group from the top in a frame has J_{r+1-i} description errors $(i=1,2,\ldots,r)$, which is shown in Fig. 15(a). Suppose that (20) holds for this case. That is, we have (37) at the bottom of the page.

Consider the case where there are $r+1$ erroneous groups (r is some integer in the range of $2 \leq r \leq N-1$). If we let $m=r+1$ in (29), we have

$$M \geq J_{r+1} \geq J_r \geq \cdots \geq J_1 \geq 1, \\ J_{r+1} + J_r + \cdots + J_1 = n. \tag{38}$$

$$P_f(n,r) = \\ \sum_{k_r=\lceil n/r \rceil}^{\min(M,\, n-(r-1))} \sum_{k_{r-1}=\lceil (n-k_r)/(r-1) \rceil}^{\min(k_r,\, n-k_r-(r-2))} \cdots \sum_{k_2=\lceil (n-k_r-k_{r-1}\cdots-k_3)/2 \rceil}^{\min(k_3,\, n-k_r-k_{r-1}\cdots-k_3-1)} P_g(k_r) P_g(k_{r-1}) \cdots P_g(k_2) P_g\left(n - \sum_{i=2}^{r} k_i\right) P_g^{N-r}(0) \tag{37}$$

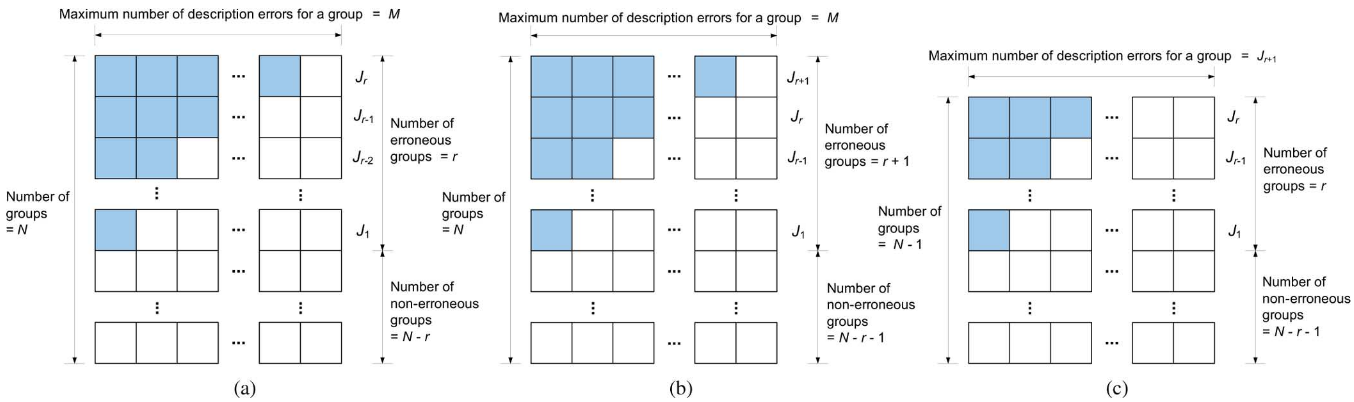


Fig. 15. (a) The case where there are r erroneous groups for n description errors in a frame. (b) The case where there are $r+1$ erroneous groups for n description errors in a frame. (c) The case where there are r erroneous groups for $n - J_{r+1}$ description errors in a frame except the first group which has J_{r+1} description errors.

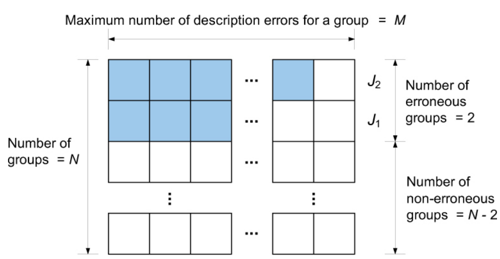


Fig. 14. The case where there are two erroneous groups (i.e., $m = 2$) for n description errors in a frame.

By the assumption below (20), the ith group from the top in a frame has J_{r+2-i} description errors ($i = 1, 2, \ldots, r+1$), which is shown in Fig. 15(b).

The range of the largest number of description errors which a group among $r+1$ erroneous groups can have is given by

$$\left\lceil \frac{n}{r+1} \right\rceil \le J_{r+1} \le \min(M, n-r). \tag{39}$$

To see this, note that

1) Since J_{r+1} is the largest number of description errors in an erroneous group, J_{r+1} should be larger than or equal to $n/(r+1)$, which is the average number of description errors in an erroneous group. Since J_{r+1} is an integer, we have $J_{r+1} \ge \lceil n/(r+1) \rceil$.
2) J_{r+1} should be less than or equal to $n-r$ since each of the other r erroneous groups should have at least one description error.
3) From (38), we have $J_{r+1} \le M$.

From (39), the probability of $r+1$ erroneous groups in a frame can be expressed as

$$P_f(n, r+1) = \sum_{J_{r+1}=\lceil n/(r+1) \rceil}^{\min(M, n-r)} P_g(J_{r+1}) P_{(N-1)g}(n - J_{r+1}, r) \tag{40}$$

where $P_g(J_{r+1})$ is the probability of J_{r+1} description errors in the first group from the top in a frame, and $P_{(N-1)g}(n - J_{r+1}, r)$ is the probability that for $n - J_{r+1}$ description errors in a frame except the first group, there are r erroneous groups. Fig. 15(c) shows $N-1$ groups except the first group having J_{r+1} description errors. From Fig. 15(a) and (c), note the following.

1) The maximum number of description errors which one group can have is J_{r+1} instead of M.
2) The number of groups is $N-1$ instead of N.
3) There are $n - J_{r+1}$ description errors instead of n description errors.

Therefore, $P_{(N-1)g}(n - J_{r+1}, r)$ in (40) can be derived from the induction hypothesis given by (37) if M, N, and n are replaced by J_{r+1}, $N-1$, and $n - J_{r+1}$, respectively. That is, we have (41) at the bottom of the page.

From (41), $P_f(n, r+1)$, given by (40), can be expressed as (42) on the next page. If we let $m = r+1$ in (20), we have (43)

$$\begin{aligned} P_{(N-1)g}(n - J_{r+1}, r) = &\sum_{k_r=\lceil (n-J_{r+1})/r \rceil}^{\min(J_{r+1},\, n-J_{r+1}-(r-1))} \; \sum_{k_{r-1}=\lceil (n-J_{r+1}-k_r)/(r-1) \rceil}^{\min(k_r,\, n-J_{r+1}-k_r-(r-2))} \\ &\cdots \sum_{k_2=\lceil (n-J_{r+1}-k_r-k_{r-1}\cdots-k_3)/2 \rceil}^{\min(k_3,\, n-J_{r+1}-k_r-k_{r-1}\cdots-k_3-1)} P_g(k_r) P_g(k_{r-1}) \\ &\cdots P_g(k_2) P_g\Big(n - J_{r+1} - \sum_{i=2}^{r} k_i\Big) P_g^{N-1-r}(0) \end{aligned} \tag{41}$$

at the bottom of the page. It is seen that (42) and (43) are the same. We have proved (20) for $m \geq 2$ by induction.

The Case Where $m = 1$ or 0: Next, we will prove (20) for $m = 1$, which is the case where there is only one erroneous group. If we let $m = 1$ in (29), we have

$$M \geq J_1 \geq 1 \text{ and } J_1 = n. \tag{44}$$

By the assumption below (20), the first group from the top in a frame has $J_1 = n$ description errors, and thus

$$P_f(n, 1) = P_g(n) P_g^{N-1}(0) \tag{45}$$

which is identical to (20) for $m = 1$. Lastly, we will prove (20) for $m = 0$, which is the case where there is no erroneous group (i.e., $n = 0$). It is obvious that for this case, $P_f(0, 0) = P_g^N(0)$, which is identical to (20) for $m = 0$. □

APPENDIX B

PROOF OF *(22):* We define the following:

i) B_i: a set of erroneous groups each of which has the same number of non-zero description errors ($i = 1, 2, \ldots, l$, where l is the number of distinct sets). We assume that for $1 \leq j < k \leq l$, the number of description errors which an erroneous group in a set B_k has is greater than that in a set B_j. As a result, for $k_m \geq k_{m-1} \geq \cdots \geq k_1$ given by (21), B_l is a set of erroneous groups each of which has k_m description errors.

ii) a_i: the cardinality of set B_i $(i = 1, 2, \ldots, l)$. Since the total number of erroneous groups in a frame is m, we have

$$a_l + a_{l-1} + \cdots + a_1 = m. \tag{46}$$

From i) and ii), the number of ways of assigning $k_m, k_{m-1}, \ldots, k_1$ to N groups in a frame, $C(m)$, is given by

$$C(m) = \frac{N!}{a_l!\, a_{l-1}! \ldots a_1!\, (N-m)!}. \tag{47}$$

For example, when $N = 16$, $m = 7$, and $k_7 = k_6 > k_5 > k_4 = k_3 = k_2 > k_1$, we have

$$l = 4,\ a_4 = 2,\ a_3 = 1,\ a_2 = 3,\ a_1 = 1 \tag{48}$$

and (47) becomes $16!/2!\,1!\,3!\,1!\,9!$.

Note that for given $k_m \geq k_{m-1} \geq \cdots \geq k_1$ in (21), a_l can be expressed as

$$a_l = 1 + \sum_{s=1}^{m-1} \delta(k_m - k_{m-s}). \tag{49}$$

Fig. 16(a) shows an OFDM frame with m erroneous groups, where each group is denoted by a rectangular box. Dark and bright rectangular boxes indicate erroneous and non-erroneous groups.

The Case Where $N \geq 2$: First, we will prove (22) for $N \geq 2$. Consider the case where there is one erroneous group (i.e., $m = 1$). From (46), we have $l = 1$ and $a_1 = 1$. From (47), the number of ways of assigning k_1 to N groups is given by

$$C(1) = \frac{N!}{1!\,(N-1)!} = N. \tag{50}$$

If we let $m = 1$ in (22), we have (51) on the next page, where the second equality follows from (23). It is seen that (51) is identical to (50).

Consider the case where there are $m = r$ erroneous groups (r is some integer in the range of $1 \leq r \leq N-1$), which is shown

$$P_f(n, r+1) = \sum_{J_{r+1}=\lceil n/(r+1) \rceil}^{\min(M,\, n-r)} \sum_{k_r=\lceil (n-J_{r+1})/r \rceil}^{\min(J_{r+1},\, n-J_{r+1}-(r-1))} \sum_{k_{r-1}=\lceil (n-J_{r+1}-K_r)/(r-1) \rceil}^{\min(k_r,\, n-J_{r+1}-k_r-(r-2))} \cdots \sum_{k_2=\lceil (n-J_{r+1}-k_r-k_{r-1}\cdots-k_3)/2 \rceil}^{\min(k_3,\, n-J_{r+1}-k_r-k_{r-1}\cdots-k_3-1)} P_g(J_{r+1}) P_g(k_r) P_g(k_{r-1}) \cdots P_g(k_2) P_g\left(n - J_{r+1} - \sum_{i=2}^{r} k_i\right) P_g^{N-1-r}(0) \tag{42}$$

$$P_f(n, r+1) = \sum_{k_{r+1}=\lceil n/(r+1) \rceil}^{\min(M,\, n-r)} \sum_{k_r=\lceil (n-k_{r+1})/r \rceil}^{\min(k_{r+1},\, n-k_{r+1}-(r-1))} \cdots \sum_{k_2=\lceil (n-k_{r+1}-k_r\cdots-k_3)/2 \rceil}^{\min(k_3,\, n-k_{r+1}-k_r\cdots-k_3-1)} P_g(k_{r+1}) P_g(k_r) \cdots P_g(k_2) P_g\left(n - \sum_{i=2}^{r+1} k_i\right) P_g^{N-r-1}(0) \tag{43}$$

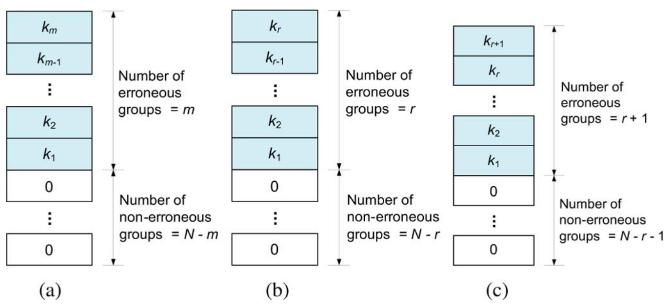


Fig. 16. (a) An OFDM frame with m erroneous groups (b) The case where there are r erroneous groups (c) The case where there are $r+1$ erroneous groups.

in Fig. 16(b). From (46), we have $a_l + a_{l-1} + \cdots + a_1 = r$, and from (47), the number of ways of assigning $k_r, k_{r-1}, \ldots, k_1$ satisfying $k_r \geq k_{r-1} \geq \cdots \geq k_1$ to N groups is given by

$$C(r) = \frac{N!}{a_l!\, a_{l-1}! \, \ldots \, a_1!\,(N-r)!}. \tag{52}$$

From (49), for given $k_r \geq k_{r-1} \geq \cdots \geq k_1$, the cardinality of a set of erroneous groups with k_r description errors, a_l, can be expressed as

$$a_l = 1 + \sum_{s=1}^{r-1} \delta(k_r - k_{r-s}). \tag{53}$$

Suppose that (22) holds for this case. Then, $C(r)$ is given by (54) at the bottom of the page.

Consider the case where there are $m = r+1$ erroneous groups (r is some integer in the range of $1 \leq r \leq N-1$), which is the case shown in Fig. 16(c). From (21), we have

$$k_{r+1} \geq k_r \geq \cdots \geq k_1. \tag{55}$$

1) For $k_{r+1} > k_r$: From (52), and Fig. 16(b) and (c), it follows that $C(r+1)$ can be expressed as

$$\begin{aligned} C(r+1) &= \frac{N!}{a_{l+1}!\, a_l!\, a_{l-1}! \, \ldots \, a_1!\,(N-r-1)!} \\ &= \frac{N!}{1!\, a_l!\, a_{l-1}! \, \ldots \, a_1!(N-r-1)!} \end{aligned} \tag{56}$$

where a_{l+1} is the cardinality of B_{l+1}, a set of erroneous groups having k_{r+1} description errors, and it is obvious that $a_{l+1} = 1$ due to $k_{r+1} > k_r$. From (52) and (56), $C(r+1)$ can be expressed in terms of $C(r)$.

$$C(r+1) = \frac{(N-r)!}{(N-r-1)!}\, C(r). \tag{57}$$

2) For $k_{r+1} = k_r$: From (52), and Fig. 16(b) and (c), it follows that $C(r+1)$ can be expressed as

$$C(r+1) = \frac{N!}{(a_l+1)!\, a_{l-1}! \, \ldots \, a_1!\,(N-r-1)!}. \tag{58}$$

From (52) and (58), $C(r+1)$ can be expressed in terms of $C(r)$:

$$C(r+1) = \frac{a_l!}{(a_l+1)!} \cdot \frac{(N-r)!}{(N-r-1)!}\, C(r). \tag{59}$$

Using (53), $C(r+1)$, given by (59), can be rewritten as

$$\begin{aligned} &C(r+1) \\ &\quad = \frac{\left(1 + \sum_{s=1}^{r-1} \delta(k_r - k_{r-s})\right)!}{\left(2 + \sum_{s=1}^{r-1} \delta(k_r - k_{r-s})\right)!} \frac{(N-r)!}{(N-r-1)!}\, C(r). \end{aligned} \tag{60}$$

$$\begin{aligned} C(1) &= \frac{N!}{\left(1 + \sum_{i=1}^{0} \delta(k_1 - k_{1-i})\right)! \prod_{j=1}^{-1} \left\{1 + (1 - \delta(k_{1-j} - k_{2-j})) \sum_{i=j+1}^{0} \delta(k_{1-j} - k_{1-i})\right\}!\,(N-1)!} \\ &= \frac{N!}{(1+0)!\, 1\, (N-1)!} \end{aligned} \tag{51}$$

$$C(r) = \frac{N!}{\left(1 + \sum_{i=1}^{r-1} \delta(k_r - k_{r-i})\right)! \prod_{j=1}^{r-2} \left\{1 + (1 - \delta(k_{r-j} - k_{r+1-j})) \sum_{i=j+1}^{r-1} \delta(k_{r-j} - k_{r-i})\right\}!(N-r)!} \tag{54}$$

From the results of 1) and 2) (i.e., (57) and (60)), for $k_{r+1} \geq k_r$, $C(r+1)$ can be expressed as

$$C(r+1) = \frac{\left(1+\sum_{s=1}^{r-1}\delta(k_r-k_{r-s})\right)!}{\left(1+\delta(k_{r+1}-k_r)+\sum_{s=1}^{r-1}\delta(k_r-k_{r-s})\right)!} \times \frac{(N-r)!}{(N-r-1)!}\,C(r). \tag{61}$$

Using (54), $C(r+1)$, given by (61), can be rewritten as (62) at the bottom of the page. Let $p = j+1$ and $q = i+1$. Then, $C(r+1)$ can be expressed as (63) at the bottom of the page. If we let $m = r+1$ in (22), we have (64) at the bottom of the page. It can be shown that the ratio of (64) to (63), γ, is given by

$$\gamma = \frac{\left(1+\delta(k_{r+1}-k_r)+\sum_{s=1}^{r-1}\delta(k_r-k_{r-s})\right)!}{\left(1+\sum_{i=1}^{r}\delta(k_{r+1}-k_{r+1-i})\right)!} \times \frac{1}{\left\{1+(1-\delta(k_r-k_{r+1}))\sum_{i=2}^{r}\delta(k_r-k_{r+1-i})\right\}!}. \tag{65}$$

It is clear that

$$\sum_{i=1}^{r}\delta(k_{r+1}-k_{r+1-i}) = \delta(k_{r+1}-k_r)+\sum_{i=1}^{r-1}\delta(k_{r+1}-k_{r-i}) \tag{66}$$

and

$$\sum_{i=2}^{r}\delta(k_r-k_{r+1-i}) = \sum_{i=1}^{r-1}\delta(k_r-k_{r-i}). \tag{67}$$

Using (66) and (67), γ, given by (65), can be rewritten as (68) at the top of the following page.

1) For $k_{r+1} = k_r$, γ is given by

$$\gamma = \frac{\left(2+\sum_{s=1}^{r-1}\delta(k_r-k_{r-s})\right)!}{\left(2+\sum_{i=1}^{r-1}\delta(k_{r+1}-k_{r-i})\right)!} = \frac{\left(2+\sum_{s=1}^{r-1}\delta(k_r-k_{r-s})\right)!}{\left(2+\sum_{i=1}^{r-1}\delta(k_r-k_{r-i})\right)!} = 1 \tag{69}$$

where the second equality follows from the fact that $k_{r+1} = k_r$.

$$C(r+1) = \frac{N!}{\left(1+\delta(k_{r+1}-k_r)+\sum_{s=1}^{r-1}\delta(k_r-k_{r-s})\right)!\prod_{j=1}^{r-2}\left\{1+(1-\delta(k_{r-j}-k_{r+1-j}))\sum_{i=j+1}^{r-1}\delta(k_{r-j}-k_{r-i})\right\}!\,(N-r-1)!} \tag{62}$$

$$C(r+1) = \frac{N!}{\left(1+\delta(k_{r+1}-k_r)+\sum_{s=1}^{r-1}\delta(k_r-k_{r-s})\right)!} \times \frac{1}{\prod_{p=2}^{r-1}\left\{1+(1-\delta(k_{r+1-p}-k_{r+2-p}))\sum_{q=p+1}^{r}\delta(k_{r+1-p}-k_{r+1-q})\right\}!(N-r-1)!} \tag{63}$$

$$C(r+1) = \frac{N!}{\left(1+\sum_{i=1}^{r}\delta(k_{r+1}-k_{r+1-i})\right)!\prod_{j=1}^{r-1}\left\{1+(1-\delta(k_{r+1-j}-k_{r+2-j}))\sum_{i=j+1}^{r}\delta(k_{r+1-j}-k_{r+1-i})\right\}!\,(N-r-1)!} \tag{64}$$

$$\gamma = \frac{\left(1+\delta(k_{r+1}-k_r)+\sum_{s=1}^{r-1}\delta(k_r-k_{r-s})\right)!}{\left(1+\delta(k_{r+1}-k_r)+\sum_{i=1}^{r-1}\delta(k_{r+1}-k_{r-i})\right)!\left\{1+(1-\delta(k_r-k_{r+1}))\sum_{i=1}^{r-1}\delta(k_r-k_{r-i})\right\}!} \tag{68}$$

$$\gamma = \frac{\left(1+\sum_{s=1}^{r-1}\delta(k_r-k_{r-s})\right)!}{\left(1+\sum_{i=1}^{r-1}\delta(k_{r+1}-k_{r-i})\right)!\left(1+\sum_{i=1}^{r-1}\delta(k_r-k_{r-i})\right)!} = \frac{1}{\left(1+\sum_{i=1}^{r-1}\delta(k_{r+1}-k_{r-i})\right)!} = 1 \tag{70}$$

2) For $k_{r+1} > k_r$, γ is given by (70) at the top of the page, where the third equality follows from the fact that $k_{r+1} > k_r \geq k_{r-1} \geq \cdots \geq k_1$ which is derived from both (55) and $k_{r+1} > k_r$.

From (69) and (70), it is seen that γ is always 1, and thus (63) and (64) are the same. We have proved (22) for $N \geq 2$.

The Case Where $N = 1$: For this case, we have $m = 1$, and thus the number of ways of assigning k_1 to a group is one (i.e., $C(1) = 1$). If we let $m = 1$ in (22), we have

$$C(1) = \frac{N!}{(1+0)!\,1\,(N-1)!} = 1 \tag{71}$$

where the first equality follows from (23). Therefore, (22) holds for $N = 1$. □

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