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EXACT VLASOV-MAXWELL EQUILIBRIA WITH SHEARED MAGNETIC FIELDS*

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ABSTRACT

We present a theoretical formalism which allows the generation of a large class of exact Vlasov-Maxwell equilibria with sheared magnetic fields. All quantities are assumed to vary only in one spatial direction, x, and the magnetic field has components only in the y and z directions. The Vlasov equations are solved by making the distribution functions depend only on constants of the motion. The Maxwell equations are then reduced to finding the motion of a pseudo-particle in a two dimensional potential. Three examples corresponding to sheet-like, sheath-like, and wave-like equilibria are presented.

I. INTRODUCTION

Knowledge of the exact Vlasov-Maxwell equilibrium is often necessary when analyzing the stability of a plasma, expecially when the inhomogeneity scale length is not large compared to the ion gyroradius. Examples of such equilibria with unidirectional, i.e., unsheared, magnetic fields have been constructed previously.[1-6] However, for reasons of plasma stability or particle containment, devices are often built with sheared magnetic fields; for example the toroidal stuffed cusp, Tormac,[7-9] in which the width of the sheath

is on the order of an ion gyroradius. In this paper we present a theoretical formalism which allows us to generate a large class of exact Vlasov-Maxwell equilibria with sheared magnetic fields.

For simplicity we consider a situation in which all quantities vary only in the x direction, and the magnetic field has components B_y and B_z in the y and z directions. The equilibrium is characterized by a zero electric field. To find a self-consistent equilibrium, we must solve the coupled Vlasov-Maxwell equations. The Vlasov equations are easily satisfied by making the distribution functions depend only on constants of the motion. Maxwell's equations are then a coupled set of nonlinear integro-differential equations. We will find a large, but not complete, class of solutions to these equations.

II. GENERAL FORMALISM

Since the electric field is taken to be zero, we require exact charge neutrality:

$$N_i(x) = N_e(x) \quad . \tag{1}$$

The magnetic field can be derived from a vector potential, $\vec{A}$, and

$$B_z = \frac{dA_y}{dx} \qquad B_y = -\frac{dA_z}{dx} \quad . \tag{2}$$

The Maxwell's equations for the magnetic field become

$$\frac{d^2A_y}{dx^2} = -\frac{4\pi J_y}{c} \tag{3}$$

$$\frac{d^2A_z}{dx^2} = -\frac{4\pi J_z}{c} \qquad (4)$$

where $\vec{J}(x)$ is the current density.

The constants of the motion for particles of species, s (s = i or e), are the Hamiltonian

$$H_s = \frac{m_s}{2}\left(v_x^2 + v_y^2 + v_z^2\right) , \qquad (5)$$

and the y and z components of momentum,

$$P_{ys} = m_s v_y + \frac{q_s A_y}{c} , \qquad (6)$$

$$P_{zs} = m_s v_z + \frac{q_s A_z}{c} , \qquad (7)$$

where m_s, q_s are the mass and charge of particles of species s.

In order to satisfy the Vlasov equation for species s, the distribution function must be a function of the constants of the motion. We assume it is of the form

$$f_s = e^{-\beta_s H_s} g_s(p_{ys}, P_{zs}) , \qquad (8)$$

where β_s are constants and g_s are functions to be determined. This form for f_s is arbitrary but is motivated by physical reasonableness and by the considerable mathematical simplicity which follows from the chosen dependence on H_s.

The number densities of ions and electrons are easily seen to be given by

$$N_s(x) = \frac{1}{m_s^2}\sqrt{\frac{2\pi}{m_s\beta_s}} \int e^{-\frac{\beta_s}{2m_s}\left[\left(P_y - \frac{q_s A_y}{c}\right)^2 + \left(P_z - \frac{q_s A_z}{c}\right)^2\right]} \times g_s(P_y, P_z)\, dP_y\, dP_z . \qquad (9)$$

The current density can be written

$$J_{ys} = \frac{q_s}{m_s^3}\sqrt{\frac{2\pi}{m_s\beta_s}} \int \left(P_y - \frac{q_s A_y}{c}\right) e^{-\frac{\beta_s}{2m_s}\left[\left(P_y - \frac{q_s A_y}{c}\right)^2 + \left(P_z - \frac{q_s A_z}{c}\right)^2\right]} \times g_s(P_y, P_z)\, dP_y\, dP_z , \qquad (10)$$

$$J_{zs} = \frac{q_s}{m_s^3}\sqrt{\frac{2\pi}{m_s\beta_s}} \int \left(P_z - \frac{q_s A_z}{c}\right) e^{-\frac{\beta_s}{2m_s}\left[\left(P_y - \frac{q_s A_y}{c}\right)^2 + \left(P_z - \frac{q_s A_z}{c}\right)^2\right]} \times g_s(P_y, P_z)\, dP_y\, dP_z . \qquad (11)$$

Let us observe that

$$J_{ys} = \frac{c}{\beta_s}\frac{\partial N_s}{\partial A_y}$$

$$J_{zs} = \frac{c}{\beta_s}\frac{\partial N_s}{\partial A_z} . \qquad (12)$$

Let us now assume that N_i and N_e are equal not only as functions of x but also as functions of A_y and A_z; i.e.,

$$N_i(A_y, A_z) = N_e(A_y, A_z) . \qquad (13)$$

This is a restrictive assumption but despite it we are able to find many equilibria. The total current can now be written as

$$J_y = c\left(\frac{1}{\beta_i} + \frac{1}{\beta_e}\right)\frac{\partial N_i}{\partial A_y}$$

$$J_z = c\left(\frac{1}{\beta_i} + \frac{1}{\beta_e}\right)\frac{\partial N_i}{\partial A_z} \quad . \tag{14}$$

Note, by the way, that the ratio of ion current to electron current is

$$\frac{|J_i|}{|J_e|} = \frac{\beta_e}{\beta_i} \quad . \tag{15}$$

Since β_s is the inverse temperature, Equation (15) is what we would expect to be the case.

If we define

$$U(A_y,A_z) = 4\pi\left(\frac{1}{\beta_i} + \frac{1}{\beta_e}\right)N_i \quad , \tag{16}$$

then Equations (3) and (4) become

$$\frac{d^2A_y}{dx^2} = -\frac{\partial U}{\partial A_y} \quad , \qquad \frac{d^2A_z}{dx^2} = -\frac{\partial U}{\partial A_z} \quad . \tag{17}$$

These are just the Hamiltonian equations for a pseudo-particle with coordinates (A_y,A_z) moving in the potential $U(A_y,A_z)$. Equations (17) can be derived from the Hamiltonian

$$H_A = \frac{{P_{A_y}}^2 + {P_{A_z}}^2}{2} + U(A_y,A_z) \quad . \tag{18}$$

Since

$$B_z = \frac{dA_y}{dx} = \frac{\partial H_A}{\partial P_{A_y}} = P_{A_y} \quad \text{and} \quad -B_y = \frac{dA_z}{dx} = \frac{\partial H_A}{\partial P_{A_z}} = P_{A_z} \quad , \tag{19}$$

we note that the constancy of H_A in x is just the equation of total pressure balance.

We have reduced the equations for the fields to a two dimensional potential problem. Typically, however, instead of knowing the distribution functions from which we can derive the fields we have some idea of what the fields are and want to find the distribution functions. Thus, usually we know the fields and can, by solving Equations (2), find the trajectory of the pseudo-particle in the (A_y,A_z) plane. We then want to find a potential, $U(A_y,A_z)$, which will produce this trajectory, a problem which, in many cases, is easy to solve qualitatively. Given the potential, $U(A_y,A_z)$, we must then find the distribution functions. Using Equations (9), (13), and (16), we find that the distribution functions satisfy

$$\frac{1}{m_s^2}\sqrt{\frac{2\pi}{m_s\beta_s}}\int e^{-\frac{\beta_s}{2m_s}\left[\left(P_y-\frac{q_sA_y}{c}\right)^2 + \left(P_z-\frac{q_sA_z}{c}\right)^2\right]} g_s(P_y,P_z)dP_ydP_z = \frac{\beta_e\beta_i\,U(A_y,A_z)}{4\pi(\beta_e+\beta_i)} \quad . \tag{20}$$

Equations (20) are integral equations for g_s; the distribution functions are then given by Equation (8).

Once the trajectory of the pseudo-particle, i.e., the fields, is known, the potential, $U(A_y,A_z)$, can be changed, without changing the magnetic fields, arbitrarily on any set which does not intersect the trajectory. However, the distribution functions, given by Equation (20), depend on $U(A_y,A_z)$ for all (A_y,A_z) and thus, there are arbitrarily many distribution functions which produce a given set of fields. This arbitrariness in the potential can be used to produce a variety of features, such as asymmetric momentum distributions, in the distribution functions.

Note also that an overall constant can be added to the potential without changing the fields. The freedom to add this constant must sometimes be used to insure that the distribution functions, which are solutions of Equation (20), are everywhere non-negative. For convenience, the potential can be translated arbitrarily in the (A_y,A_z) plane without changing the fields.

Motivated by the much wider class of situations to which it might be applied, we have attempted to extend this formalism to cylindrical geometry, but have found that a straightforward extension isn't possible.

III. EXAMPLES

In this section we give three examples, each of which illustrates a different way of solving Equation (20).

(a) Unsheared Sheath

Consider a situation in which the magnetic field is unidirectional; we can take $A_z = 0$. Equation (20) then becomes

$$\frac{2\pi}{m_s{}^2\beta_s}\int e^{-\frac{\beta_s}{2m_s}\left[\left(P_y-\frac{q_sA_y}{c}\right)^2\right]} g_s(P_y)dP_y = \frac{\beta_e\beta_i\ U(A_y)}{4\pi(\beta_e+\beta_i)} , \qquad (21)$$

where we have assumed

$$g_s = g_s(P_y) , \qquad (22)$$

and

$$U = U(A_y) . \qquad (23)$$

Let us now assume that

$$U(A_y) = D\, e^{-\gamma A_y{}^2} \qquad (24)$$

where D and γ are constants, so that the potential now resembles a "hill". We can easily choose the velocity (i.e., magnetic field) at $-\infty$ to be such that the pseudo-particle just manages to roll to the top of the hill; i.e., we choose

$$B_z(-\infty) = \frac{dA_y(-\infty)}{dx} = \sqrt{2D} = B_0 . \qquad (25)$$

Thus, the magnetic field and, from the constancy in x of H_A (see Eq. (18)), particle density are as shown in Figure 1; this is a sheath.

To find the distribution function we must solve Equation (21) with $U(A_y)$ given by Equation (24). We Fourier transform Equation (21) and, denoting transformed functions by a tilde, obtain

$$\frac{2\pi}{m_s{}^2\beta_s}\sqrt{\frac{2\pi m_s}{\beta_s}}\,\tilde{g}_s(w)\, e^{-\frac{m_s w^2}{2\beta_s}} = \frac{\beta_e\beta_i D}{(\beta_e+\beta_i)8\pi^2}\sqrt{\frac{\pi}{\gamma}}\; e^{-\frac{w^2}{4\gamma}} . \qquad (26)$$

Solving for $\tilde{g}_s$ we find

$$\tilde{g}_s(w) = \frac{\beta_e\beta_i D m_s{}^2\beta_s}{16\pi^3(\beta_e+\beta_i)}\sqrt{\frac{\beta_s}{2\gamma m_s}}\; e^{-\frac{w^2}{4\gamma}+\frac{m_s w^2}{2\beta_s}} . \qquad (27)$$

In order to be able to invert the Fourier transform we must require

$$\gamma < \min\left(\frac{\beta_e}{2m_e}, \frac{\beta_i}{2m_i}\right) \quad . \tag{28}$$

This simply says that if the sheath is too narrow then charge neutrality can not be maintained. If we let

$$\delta_s = \frac{1}{4\gamma} - \frac{m_s}{\beta_s} \quad , \tag{29}$$

then the distribution functions are given by

$$f_s(H_s, P_{ys}) = \frac{m_s^2 \beta_s N_0}{4\pi^2} \sqrt{\frac{\pi \beta_s}{2\gamma \delta_s m_s}} \, e^{-\frac{P_{ys}^2}{4\delta_s} - \beta_s H_s} \quad , \tag{30}$$

where N_0 is the density at $x = +\infty$.

(b) Sheared Sheet

Let us assume that

$$U(A_y, A_z) = D\, e^{\gamma(A_y + A_z)} \tag{31}$$

where D and γ are constants. Then Equation (17) becomes

$$\frac{d^2 A_y}{dx^2} = -D\gamma\, e^{\gamma(A_y + A_z)} \quad , \tag{32}$$

$$\frac{d^2 A_z}{dx^2} = -D\gamma\, e^{\gamma(A_y + A_z)} \quad . \tag{33}$$

We observe that

$$\frac{d^2 A_y}{dx^2} = \frac{d^2 A_z}{dx^2} \quad . \tag{34}$$

Equation (34) can be immediately integrated twice to give

$$A_y = A_z + E_1 x + E_2 \quad , \tag{35}$$

where E_1 and E_2 are constants. Combining Equations (33) and (35) gives

$$\frac{d^2 A_z}{dx^2} = -D'\, e^{2\gamma A_z + \gamma E_1 x} \quad , \tag{36}$$

where

$$D' = D\gamma e^{\gamma E_2} \quad . \tag{37}$$

If we define

$$G = 2\gamma A_z + \gamma E_1 x \quad , \tag{38}$$

then Equation (36) becomes

$$\frac{d^2 G}{dx^2} = -2\gamma D' e^{G} \quad . \tag{39}$$

Multiplying by $\frac{dG}{dx}$ and integrating gives

$$\left(\frac{dG}{dx}\right)^2 = -4\gamma D' e^{G} + E_3 \quad , \tag{40}$$

where E_3 is a constant. Equation (40) can be easily integrated to give

$$G(x) = \ln\left\{\frac{-E_3}{4\gamma D' \cosh^2 \frac{\sqrt{E_3}x}{2}}\right\} . \qquad (41)$$

Using Equation (38) we find

$$A_z(x) = -\frac{1}{\gamma}\ln\left(\cosh\frac{\sqrt{E_3}x}{2}\right) + \frac{1}{2\gamma}\ln\left(\frac{-E_3}{4\gamma D'}\right) - \frac{E_1 x}{2} . \qquad (42)$$

Combining Equations (35) and (42), we get

$$A_y(x) = -\frac{1}{\gamma}\ln\left(\cosh\frac{\sqrt{E_3}x}{2}\right) + \frac{E_1 x}{2} + E_2 + \frac{1}{2\gamma}\ln\left(\frac{-E_3}{4\gamma D'}\right) . \qquad (43)$$

If we require

$$B_y(-\infty) = 0$$

$$B_y(+\infty) = B_0 , \qquad (44)$$

then, dropping additive constants, we find

$$A_z(x) = -\frac{1}{\gamma}\ln\left(\cosh\frac{\gamma B_0 x}{2}\right) - \frac{B_0 x}{2} \qquad (45)$$

$$A_y(x) = -\frac{1}{\gamma}\ln\left(\cosh\frac{\gamma B_0 x}{2}\right) + \frac{B_0 x}{2} . \qquad (46)$$

The trajectory of the pseudo-particle is shown in Fig. 2. We can now find the magnetic fields from Equation (2);

$$B_z = \frac{B_0}{2} - \frac{B_0}{2}\tanh\frac{\gamma B_0 x}{2} \qquad (47)$$

$$B_y = \frac{B_0}{2} + \frac{B_0}{2}\tanh\frac{\gamma B_0 x}{2} . \qquad (48)$$

In order to find the distribution functions, we must solve Equation (20) with the potential given by Equation (31). Although we cannot use Fourier transforms in this case, the solution is easily seen to be, by inspection,

$$g_s(P_y,P_z) = \frac{\beta_e\beta_i D}{4\pi(\beta_e+\beta_i)}\left(\frac{m_s\beta_s}{2\pi}\right)^{\frac{3}{2}} e^{\frac{\gamma^2 c^2 m_s}{\beta_s q_s}} e^{\frac{\gamma c(P_y+P_z)}{q_s}} . \qquad (49)$$

The distribution function is now given by Equation (8). The number density and magnetic fields for this equilibrium are shown in Figure 3. This is a plasma sheet in a sheared magnetic field.

(c) Wave-like Solution

In our previous examples the pseudo-particle's trajectory went to infinity. If the potential, $U(A_y,A_z)$, increases as A_y,A_z go to infinity then the pseudo-particle will be confined and periodic motion can result. Thus, let us assume that the potential is

$$U(A_y,A_z) = D_1 + \frac{D_2}{2}(A_y^2 + A_z^2) . \qquad (50)$$

Equation (17) is then

$$\frac{d^2A_y}{dx^2} = -D_2A_y , \qquad (51)$$

$$\frac{d^2A_z}{dx^2} = -D_2A_z . \qquad (52)$$

The solutions of Equations (51) and (52) are clearly

$$A_y = A_{y_0} \sin\left(\sqrt{D_2}x + \delta_1\right) \qquad (53)$$

$$A_z = A_{z_0} \sin\left(\sqrt{D_2}x + \delta_2\right) , \qquad (54)$$

where A_{y_0}, A_{z_0}, δ_1, δ_2 are real constants. The magnetic fields are

$$B_z = \sqrt{D_2}\, A_{y_0} \cos\left(\sqrt{D_2}x + \delta_1\right) \qquad (55)$$

$$B_y = -\sqrt{D_2}\, A_{z_0} \cos\left(\sqrt{D_2}x + \delta_2\right) . \qquad (56)$$

We have found a stationary wave solution. By transforming to a moving frame of reference, so that the magnetic field becomes both an electric and a magnetic field, we produce a travelling electromagnetic wave that is an exact solution of the Vlasov-Maxwell equations. (Note that by choosing a potential, $U(A_y,A_z)$, which depends on higher powers of A_y and A_z we could produce waves with nonsinusoidal shapes.)

We can solve Equation (20) for the distribution function by inspection, but we choose instead to illustrate another technique. Note that[10]

$$e^{-\frac{\beta_s}{2m_s}\left(P-\frac{q_sA}{c}\right)^n} = \sum_m e^{-\frac{\beta_s P^2}{2m_s}} \frac{H_n\left(\sqrt{\frac{\beta_s}{2m_s}}\,P\right)\left(\sqrt{\frac{\beta_s}{2m_s}}\,\frac{q_sA}{c}\right)^2}{n!} , \qquad (57)$$

where H_n is the nth hermite polynomial. Using expansion (57) in both variables in Equation (20), we find

$$\frac{\beta_e\beta_i\, U(A_y,A_z)}{4\pi(\beta_e+\beta_i)} = \frac{1}{m_s^2}\sqrt{\frac{2\pi}{m_s\beta_s}}\sum_{m,n}\frac{1}{m!n!}\left(\frac{\beta_s}{2m_s}\right)^{\frac{m+n}{2}}$$

$$\times\left(\frac{q_sA_y}{c}\right)^m\left(\frac{q_sA_z}{c}\right)^n \int e^{-\frac{\beta_s}{2m_s}\left(P_y^2+P_z^2\right)} H_m\left(\sqrt{\frac{\beta_s}{2m_s}}\,P_y\right)$$

$$\times H_n\left(\sqrt{\frac{\beta_s}{2m_s}}\,P_z\right) g_s(P_y,P_z)\,dP_y\,dP_z . \qquad (58)$$

Let us also expand g_s as

$$g_s(P_y,P_z) = \sum_{k,\ell} C_{k\ell}^{(s)} H_k\left(\sqrt{\frac{\beta_s}{2m_s}}\,P_y\right) H_\ell\left(\sqrt{\frac{\beta_s}{2m_s}}\,P_z\right) . \qquad (59)$$

Using Equation (59) in Equation (58) we find

$$U(A_y,A_z) = \frac{4\pi^2(\beta_e+\beta_i)}{\beta_e\beta_i m_s^2}\sqrt{\frac{2\pi}{m_s\beta_s}}\sum_{m,n}\left(\frac{2\beta_s}{m_s}\right)^{\frac{m+n}{2}}$$

$$\times\; C_{mn}^{(s)}\left(\frac{qA_y}{c}\right)^m\left(\frac{qA_z}{c}\right)^n . \qquad (60)$$

This equation determines $C_{mn}^{(s)}$ in terms of the (m,n) coefficient of the Taylor series for $U(A_y,A_z)$. Equation (60) is particularly useful when, as is the case in Equation (50), $U(A_y,A_z)$ is a polynomial. Thus Equations (50) and (60) yield:

$$C_{00}^{(s)} = \frac{\beta_e \beta_i m_s^2}{4\pi^2(\beta_e + \beta_i)} \sqrt{\frac{m_s \beta_s}{2\pi}}\, D_1 , \qquad (61)$$

$$C_{02}^{(s)} = C_{20}^{(s)} = \frac{\beta_e \beta_i m_s^3 c^2}{16\pi^2(\beta_e + \beta_i)\beta_s q_s^2} \sqrt{\frac{m_s \beta_s}{2\pi}}\, D_2 . \qquad (62)$$

Inserting the expressions for the hermite polynomials,[10] we find

$$g_s(P_y,P_z) = \frac{\beta_e \beta_i m_s^2}{4\pi^2(\beta_e + \beta_i)} \sqrt{\frac{m_s \beta_s}{2\pi}} \left[D_1 + \frac{m_s c^2}{\beta_s q_s^2} D_2 \left(P_y^2 + P_z^2 - 1 \right) \right] . \qquad (63)$$

Requiring $g_s \geq 0$ implies

$$D_1 \geq D_2 \max_s \left(\frac{m_s c^2}{\beta_s q_s^2} \right) . \qquad (64)$$

Equation (64) simply says that there must be enough particles present to produce the required currents.

The hermite polynomial expansion can be used to solve Equation (20) whenever the potential can be expanded in a convergent power series; in fact, our second example could have been solved in this fashion.

We could easily construct other examples of Vlasov-Maxwell equilibria. Because of the intuitive nature of two dimensional potential problems, choosing a potential, $U(A_y,A_z)$, that will produce the desired magnetic fields is generally easy, even though

simple analytic solutions of Equations (3) and (4) do not, in general, exist. The solution of Equation (20) for the distribution functions is more difficult, but, if the potential, $U(A_y,A_z)$, can be chosen to be a real analytic function the hermite polynomial expansion method can be used to find the distribution functions.

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FOOTNOTES AND REFERENCES

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FIGURE CAPTIONS

Figure 1: Ratio of magnetic field to maximum magnetic field, B/B_0, and ratio of particle density to maximum particle density, N/N_0, as a function of x for the unsheared sheath of section III(a). We have taken $\gamma = .25$ in Equation (24), with $A_y(0) = -2$.

Figure 2: Trajectory of the pseudoparticle with coordinates (A_y, A_z) as given by Equations (45) and (46). The components of velocity of the pseudoparticle are related to the magnetic field by Equation (2).

Figure 3: Ratio of B_z to maximum B_z, B_z/B_0, ratio of B_y to maximum B_y, B_y/B_0, and ratio of particle density to maximum particle density, N/N_0, as a function of x for the equilibrium given by Equations (47), (48), and (49). In Equations (47) and (48), we have taken $\gamma B_0 = 2$. As can be seen, this corresponds to a sheared sheet.

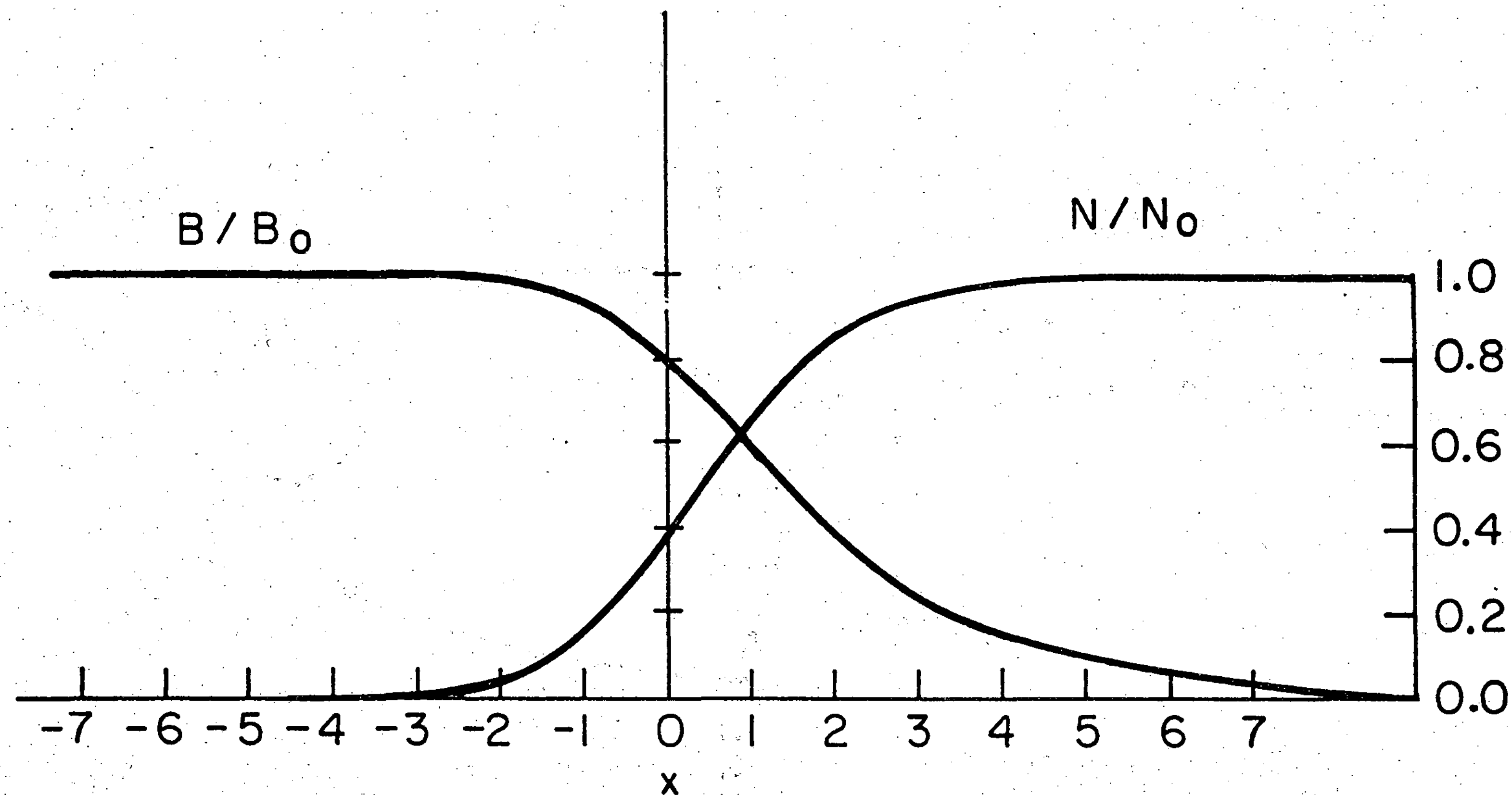

Fig. 1

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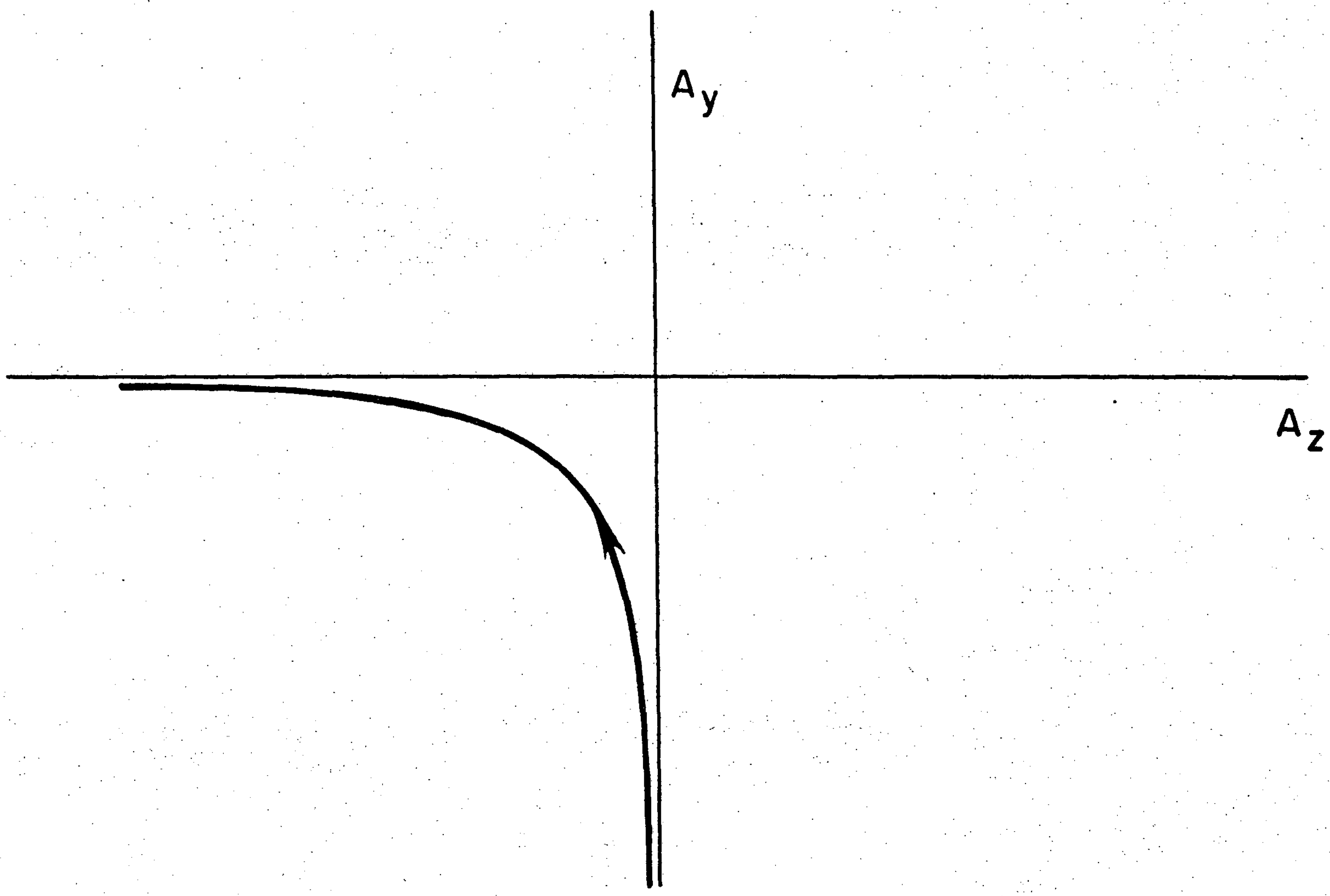

Fig. 2

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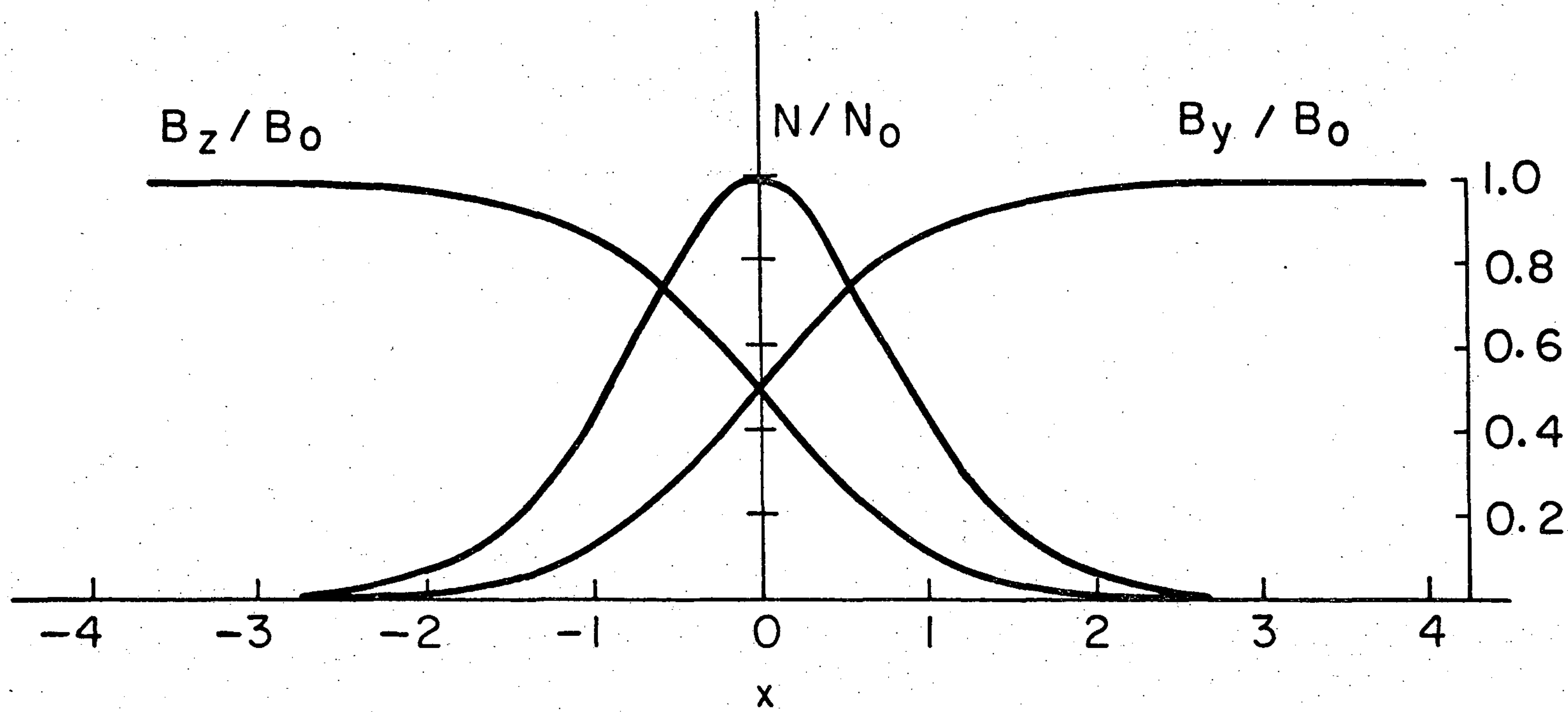

Fig. 3

XBL 761-2173

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