Interpreting the Sign of Algebraic Expressions

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Introduction

In previous studies we found that students tend to interpret algebraic expressions with a phenomenal positive sign, such as ‘2x’, to stand for positive numbers and expressions with a phenomenal negative sign, such as ‘-2x’, to stand for negative numbers (Christou, Vosniadou, & Vamvakoussi, 2007). In the present study we further investigated the effect of the phenomenal sign misconception on students’ attempts to solve inequalities and define the domain of square root functions. This study continuous our previous investigations of inappropriate transfer of prior knowledge in the acquisition of the concept of variable in algebra, following a conceptual change approach.

Transformations in algebraic inequalities follow the following rule: when both sides of an inequality are multiplied or divided by a number, the inequality sign has to change if this number is negative. It does not change, however, if the number is positive. The most common mistake students make when solving algebraic inequalities is to multiply both sides of an inequality with an expression without taking into consideration the sign of this expression. Several researchers argue that this mistake results from students’ tendency to treat inequalities using equations as prototypes (e.g., Bazzini & Tsamir, 2004). We hypothesize that students make the above mentioned mistake not only because they treat inequalities the same way as equations, but also because they interpret algebraic expressions with positive or negative phenomenal signs as having positive or negative values respectively. The square root functions are defined as real only for positive quadrants. Students who interpret the sign of the quadrant on the basis of its phenomenal sign, may erroneously conclude that the function is real when it may not be.

Method

The participants were 110 10th graders from two public high schools in Athens. We used a questionnaire with three items related to algebraic inequalities (such as 3<1/2x) and three items related to square root functions. In inequalities, the students were presented with a series of transformations and accompanying explanations for each step. In all three items, both sides of the inequalities were multiplied by expressions such as 2x or -2x, which were regarded as having always positive or negative values, respectively. Students were asked to evaluate whether the solution process was correct or not and to justify their response. With respect to functions, the students were asked to evaluate a statement arguing that, for example, “the function f(x)= $\sqrt{-2x-1}$ could not be defined in the domain of real numbers because -2x-1 is always negative for any real number x”.

Results and Discussion

The results showed that in both the context of inequalities and in the context of square root functions the students performed better in the tasks which included a positive-like expression than a negative-like, [$t(109)=2.357$, $p<.05$; $t(109)=3.729$, $p<.001$], a finding consistent with our hypothesis, as well as noted by other researchers, such as Vlassis (2004). However, the students were better in the items with square root functions than in the items with inequalities [$F(1, 108)= 62,85$, $p<0.001$]. This could be due to the fact that in the square root function task students are more likely to concentrate on the sign of the quadrant, because this is exactly what defining the domain of these functions is about.

In a following study, not yet fully analyzed, we implemented an intervention in which we explicitly taught students how to interpret the phenomenal sign of an expression in the context of square root functions. We expected that this knowledge might transfer to the case of inequalities. Preliminary results indicate that this may actually be the case.

References

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