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A comparative study of directive emission from photonic quasicrystals

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ABSTRACT

In this paper, we present a comparative study of the emission properties of line sources embedded in two-dimensional finite-size *aperiodically-ordered* “photonic-quasicrystal” slabs made of dielectric cylinders arranged according to representative categories of aperiodic tilings. Our study, based on a rigorous full-wave numerical method, indicates the possibility of achieving directive low-sidelobe emission at several frequencies. In this connection, parametric studies are presented, and similarities and differences with the periodic case are highlighted.

Keywords: Photonic quasicrystals, directive emission, aperiodic tilings.

1. INTRODUCTION AND BACKGROUND

The discovery of “quasicrystals”^{1–3} in solid-state physics has triggered a growing interest in the study of the physical properties of *aperiodically-ordered* structures in many branches of science and engineering.⁴ In electromagnetics and optical engineering, photonic quasicrystals (PQCs), composed of metallic or dielectric inclusions arranged according to aperiodic tilings (i.e., collections of polygonal tiles capable of covering a plane without gaps and overlaps, and yet devoid of any translational symmetry),^{5,6} have emerged as interesting alternatives to standard (periodic) configurations, in view of the additional degrees of freedom (e.g., high-order noncrystallographic rotational symmetries, inequivalent defect sites) potentially exploitable in aperiodic geometries. During the last decade, interesting applications have been suggested in a variety of scenarios, including bandgap engineering, negative refraction and superlensing, artificial magnetic conductors, enhanced transmission by subwavelength hole arrays, etc. The reader is referred to Ref. 7 for a recent review of the subject, and to Ref. 8 for a comprehensive account of the mathematical background.

In a series of recent investigations, we have been concerned with the study of the bandgap, confinement, and emission properties of two-dimensional (2-D) PQCs made of dielectric cylinders.^{9–12} In particular, in Refs. 11 and 12, we explored the possibility of achieving *directive emission* from line sources embedded in PQC slabs. While this is theoretically well-understood for periodic photonic crystals (PCs), both in defected¹³ and defect-free¹⁴ configurations, it remains a largely unexplored issue in connection with PQCs, where the lack of spatial periodicity considerably complicates the study. Starting from the preliminary studies on a Penrose-type (5-fold symmetric, quasiperiodic⁶) PQC in Ref. 11, which indicated the possibility of achieving *moderate* directivity (with moderate side-lobe levels) at three frequencies nearby the main and secondary bandgaps, we presented in Ref. 12 the first evidence of directive *low-sidelobe* radiation from a dodecagonal PQC. In the present paper, we expand upon the previous results, presenting a comparative full-wave study of the directive emission properties of line-sources embedded in PQC slabs generated according to three representative aperiodic-tiling geometries (Penrose, octagonal, dodecagonal) and a standard (square) periodic PC.

Figure 1. Problem geometry (details are explained in the text).

Figure 2. Samples of the “thick-and-thin” Penrose (a), octagonal (b), and dodecagonal (c) tilings.

Figure 3. (color online) Zoom of the central slab area for the periodic (a), Penrose (b), octagonal (c), symmetrically-cut dodecagonal (d), randomly-asymmetrically-cut dodecagonal (e). Blue bullets and red cross mark the positions of dielectric cylinders and line source, respectively.

2. PROBLEM GEOMETRY AND OBSERVABLES

Paralleling Refs. 11 and 12, we consider the 2-D geometry in Fig. 1, which involves a PQC slab made of z -invariant dielectric cylinders arranged according to three representative categories of aperiodic tilings (Penrose, octagonal, dodecagonal), whose samples are shown in Fig. 2. The reader is referred to Refs. 15–20 for a sparse sampling of previous studies on the EM properties of these types of PQCs. The “thick-and-thin” Penrose tiling (Fig. 2a) and the octagonal tiling (Fig. 2b) are generated using a standard cut-and-projection algorithm,⁶ whereas the dodecagonal tiling (Fig. 2c) is generated by iterating the Stampfli inflation rules.²¹ The PQC slabs are generated by cutting (from suitably large tilings) rectangular portions of size $L \times h$, and placing at the tile vertices (in free space) circular dielectric cylinders of relative permittivity $\epsilon_r = 12$ and radius $r = 0.138a$, with a being the lattice constant (tile sidelength). All structures are assumed to have the same lattice constant and comparable size and filling factor, to be defect-free, and to be cut symmetrically around the local center of symmetry of the tiling. As a reference configuration, we also consider a periodic (square) PC. Moreover, in order to highlight the effects of local order and symmetry, we also consider (for the sake of space, only for the dodecagonal case) a PQC slab sample cut *asymmetrically* (in a random fashion) around the tiling local symmetry center. The structures are excited by a time-harmonic ($\exp(j\omega t)$) electric line-source placed in free space nearby the slab center. Figure 3 shows the zoom around the source position for the five configurations of interest.

As meaningful observables, we consider the normalized “local density of states” (LDOS),²²

$$\rho(\mathbf{r}_s, \omega) = -4\text{Im}\{G(\mathbf{r}_s, \mathbf{r}_s; \omega)\}, \quad (1)$$

the directivity

$$D(\phi) = \frac{2\pi |E_s(\phi)|^2}{\int_0^{2\pi} |E_s(\phi)|^2 d\phi}, \quad (2)$$

and the normalized intensity

$$\bar{S}(\phi) = \frac{|E_s(\phi)|^2}{|E_{s0}(\phi)|^2}. \quad (3)$$

In (1)–(3), G denotes the PQC Green’s function, whereas E_s and E_{s0} denote the far-fields radiated by the line-source in the presence and absence, respectively, of the PQC, as a function of the ϕ angle (see Fig. 1). Note that the above observables are normalized so that they are equal to one in the absence of the PQC. As discussed in Ref. 11, in our 2-D scalar scenario, the LDOS in (1) can be intuitively interpreted as the normalized *total* power (per unit-length) radiated by the line source. This observable has already been successfully utilized as a meaningful indicator to assess the presence of bandgaps in finite-size PCs. On the other hand, the directivity and the normalized intensity are indicative of the angular confinement and power density enhancement (as compared to free-space) of the radiated far-field. In the present investigation, attention is focused on achieving *broadside* ($\phi = 90^\circ$) directivity.

In our simulations below, the above observables are efficiently computed via a well-established full-wave technique based on a multipolar Fourier-Bessel expansion.²³

3. REPRESENTATIVE RESULTS

From a comprehensive set of parametric studies, we present below the most representative results.

We start with the reference periodic case, whose response is shown in Fig. 4. Specifically, Figs. 4a–4c show the three observables in (1)–(3), as a function of the normalized frequency a/λ_0 (with λ_0 denoting the free-space wavelength). A main bandgap and several minor ones can readily be identified from the dips in the LDOS response (Fig. 4a), and potentially interesting configurations are characterized by high-amplitude peak coincidences in the broadside directivity (Fig. 4b) and normalized intensity (Fig. 4c). The vertical dashed lines mark three such representative configurations, whose radiation patterns (in the upper half-space) are displayed in Figs. 4d–4f. Note that all the potentially interesting configurations are above the main bandgap, and, with the exception of the very-low ($\sim -25\text{dB}$) sidelobe-level response at $a/\lambda_0 = 0.497$, radiation patterns display rather high secondary lobes.

Figure 4. Periodic PC slab ($L = 98.3a$, $h = 3.3a$, 396 cylinders, cf. Fig. 3a), with source at $x = y = 0$. (a), (b), (c): Normalized LDOS, broadside directivity, and broadside normalized intensity, respectively, as a function of the normalized frequency a/λ_0 . (d), (e), (f): Radiation patterns (in the upper half-space) at normalized frequencies $a/\lambda_0 = 0.497, 0.553$, and 0.749 , respectively (marked as vertical dashed lines in (a)–(c)).

We then move on to the PQC cases, starting with the Penrose one (cf. Figs. 2a and 3b), whose response (formatted as before) is shown in Fig. 5. In this case, as also observed in Ref. 11, one can identify a candidate operational frequency below the main bandgap ($a/\lambda_0 = 0.310$), and two above it ($a/\lambda_0 = 0.534, 0.558$). However, the radiation patterns exhibit rather high sidelobe levels ($\gtrsim -5$ dB).

Qualitatively similar results are obtained for the octagonal PQC, whose response is displayed in Fig. 6. In this case, two operational frequencies are found below the main bandgap ($a/\lambda_0 = 0.258, 0.327$), and one above it ($a/\lambda_0 = 0.518$).

Next, in Fig. 7, we consider the dodecagonal PCQ analyzed in Ref. 12 (cf. Figs. 2c and 3d). In this case, the lowest frequency ($a/\lambda_0 = 0.239$) is slightly smaller than those observed for the Penrose and octagonal cases above, and the corresponding sidelobe level (Fig. 7d) is comparably high (~ -6 dB). However, at the higher frequencies ($a/\lambda_0 = 0.317, 0.736$) the radiation patterns look considerably cleaner, with acceptable sidelobes (~ -10 dB). As highlighted in Ref. 12, this represents the first instance of directive emission from defect-free PQC structures. In order to highlight the role of local order and symmetry, Fig. 8 shows the response of a *randomly asymmetrically-cut* dodecagonal PQC slab (cf. Fig. 3e), with directivity and intensity observed at the main-beam direction $\phi = \phi_M$ (this time not necessarily at broadside). One can still observe moderate directivity at frequencies similar to the symmetrically-cut case in Fig. 7, but the radiation patterns look considerably poorer (sidelobe levels ~ -5 dB).

For a constant slab size, we also studied different cuts from the original tiling where only the y -symmetry was broken. In these cases, it was not always possible to find directive low-sidelobe responses, thereby suggesting that the slab *termination* (PQC-air interface) too might play a key role. We also changed the slab size and filling factor, and the source position. In particular, increasing the slab thickness, we observed similar effects as those described above in connection with the y -symmetry breaking. Indeed, unlike in periodic PCs, thickness changes in PQC slabs intrinsically involve *termination* changes, and so the two effects are not easily discernible. Concerning

Figure 5. As in Fig. 4, but for Penrose PQC slab ($L = 97.69a$, $h = 3.51a$, 450 cylinders, cf. Fig. 3b), with source at $x = 0$, $y = -0.5a$. (d): $a/\lambda_0 = 0.310$; (e): $a/\lambda_0 = 0.534$; (f): $a/\lambda_0 = 0.558$.

Figure 6. As in Fig. 4, but for octagonal PQC slab ($L = 97.74a$, $h = 3.69a$, 489 cylinders, cf. Fig. 3c), with source at $x = 0$, $y = -0.5a$. (d): $a/\lambda_0 = 0.258$; (e): $a/\lambda_0 = 0.327$; (f): $a/\lambda_0 = 0.518$.

Figure 7. As in Fig. 4, but for (symmetrically-cut) dodecagonal PQC slab ($L = 97.8a$, $h = 4a$, 435 cylinders, cf. Fig. 3d), with source at $x = 0$, $y = -0.5a$. (d): $a/\lambda_0 = 0.239$; (e): $a/\lambda_0 = 0.317$; (f): $a/\lambda_0 = 0.736$.

the cylinder radius, the value utilized throughout was found to provide the best results; moderate deterioration in the sidelobe level was observed for slight departures (e.g., $r = 0.14a$), up to the complete disappearance (e.g., for $r = 0.15a$) of the directive response at the higher frequencies. Finally, in connection with the source position, we observed that horizontal (x -symmetry-breaking) displacements are usually deleterious, while vertical ones can be beneficial for sidelobe reduction. As an example, Fig. 9 shows the source-position-optimized radiation pattern corresponding to Fig. 7d, which represents the *best* response ($D = 15.5\text{dB}$, $\bar{S} = 14.8\text{dB}$, and sidelobe level $= -12\text{dB}$) observed for a PQC in our study. It is worth emphasizing that such response is obtained at a frequency *twice as lower* than the reference periodic PC case.

Concerning the physical interpretation of the above results, we recall that isotropic sources have been shown to produce directive radiation when surrounded by grounded artificial dielectrics composed of periodic lattices of rods or grids, characterized by an effective relative permittivity less than unity.^{24–26} Such a behavior was also theoretically predicted by the leaky-wave model in Refs. 24 and 25, based on the seminal work in Ref. 27. In this framework, a parameterization capable of predicting quantitatively the directivity enhancement was presented in Ref. 28, in terms of very fast and slowly-attenuating leaky-waves excited by the source in the artificial material.

We believe that the results presented here can be interpreted within a similar framework. In this connection, we have initiated a systematic study of the properties of the modal field distributions supported by defect-free PCQ slabs.²⁹ From this study, based on the Fourier analysis of the near field distribution, we expect to “learn the rules” for a physically-incisive and computationally-effective parameterization of the underlying phenomenologies.

4. CONCLUSIONS

This paper has dealt with a study of the radiation properties of line-sources embedded in 2-D finite-size defect-free PQC slabs, composed of dielectric cylinders arranged in free space according to representative categories of aperiodic tilings. In this framework, we have carried out a comprehensive full-wave numerical study, from which we have identified certain parametric configurations which give rise to directive broadside radiation with

Figure 8. As in Fig. 7, but for randomly-asymmetrically-cut dodecagonal PQC slab ($L = 97.6a$, $h = 4a$, 418 cylinders, cf. Fig. 3e), with source at $x = 0$, $y = -0.5a$, and observation in the main beam direction $\phi = \phi_M$. (d): $a/\lambda_0 = 0.217$; (e): $a/\lambda_0 = 0.520$; (f): $a/\lambda_0 = 0.753$.

Figure 9. As in Fig. 7d, but with source at $x = 0$, $y = 1.55a$.

acceptable ($\lesssim -12$ dB) sidelobe levels. To the best of our knowledge, this represents the first observation of directive low-sidelobe radiation from PQC's.

Clearly, a deeper understanding of the underlying mechanisms, which involve complicated short-range multiple-scattering phenomena, remains crucial for the full exploitation of the above effects. Accordingly, current and future studies are aimed at studying in detail the modal structure supported by PQC slabs, as well as at the development of leaky-wave-based models for the interpretation and parameterization of the radiation phenomena.

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