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THE DETERMINATION OF TEMPERATURES WITHIN MASS CONCRETE STRUCTURES

by
EDWARD L. WILSON

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Report No. 68-17

THE DETERMINATION OF TEMPERATURES WITHIN MASS CONCRETE STRUCTURES

A Report of an Investigation

by

Edward L. Wilson
Associate Professor of
Civil Engineering

to

Walla Walla District
U.S. Engineers Office
Contract DACW-68-67-C-0049

Structural Engineering Laboratory
University of California
Berkeley, California

December 1968

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INTRODUCTION

In the construction of mass concrete structures the hydrating cement releases heat which results in an increase in temperature shortly after placement of the concrete. Because of the low modulus of elasticity and the high creep rate at the early age concrete the compressive stresses associated with this initial temperature increase are small. However, the subsequent decrease in temperature after the modulus of elasticity has increased may cause large tensile stresses and produce structural cracking.

Various construction techniques can be used in order to limit temperature variations and to minimize cracking. However, existing methods of analysis for the evaluation of temperatures in mass concrete structures are inadequate because they cannot accurately represent these complex construction procedures.

The finite element method of analysis as applied to heat transfer can accurately represent many factors which previously have been neglected. Therefore, the purpose of this investigation was to modify the finite element method as applied to heat-transfer problems and to develop a technique for the evaluation of the temperature distribution in mass concrete structures. Based on this approach a general digital computer program was developed for mass concrete structures of arbitrary geometry constructed incrementally. Also, the effects of cooling pipes and insulation forms are considered by the program.

The advantages of the method, as compared to other numerical approaches, are numerous. The method is completely general with respect to geometry and material properties; complex bodies composed of many different materials are easily represented. Inherent in most numerical procedures is the solution of a set of linear equations for unknown mesh point temperatures. In the finite element method, the linear equations produce a symmetric positive-deffinite matrix in band form which is readily solved with a minimum of computer storage and time.

Previously, the complete thermal stress analysis of mass concrete structures has involved two separate phases. First, based on certain idealizations, the heat transfer problem is solved; then the resulting temperature distribution is used in connection with certain structural idealizations to determine the thermal stresses within the structure. In general, both of these phases involve the use of digital computers, and because different idealizations are used, a separate computer input must be prepared for each problem. In the proposed finite element approach the computer input can be made compatible with existing stress analysis programs; therefore, the computer input and the overall time necessary for the complete thermal stress analysis is reduced.

METHOD OF ANALYSIS

Previous application of the finite element method to heat transfer analysis has been based on a variational approach.[1] However, this formal mathematical method will not be utilized in this presentation, but a completely physical interpretation of the heat-transfer equations for a finite element system will be given. The basic equation which is developed at each node of the discrete finite element representation of the structure is of the following form:

$$\begin{array}{l}\text{Rate at which heat}\\ \text{is stored in elements}\\ \text{adjacent to node}\end{array} + \begin{array}{l}\text{Rate at which heat}\\ \text{flows from elements}\\ \text{adjacent to node}\end{array} = \begin{array}{l}\text{Rate at which}\\ \text{external heat}\\ \text{enters node}\end{array}$$

If all nodes are considered, the above heat equilibrium can be written in matrix form as a set of first order differential equations.

$$\underline{C}\ \underline{\dot{T}(t)} + \underline{K}\ \underline{T(t)} = \underline{Q}(t)$$

where

$\underline{C}$ is defined as the heat capacity matrix

$\underline{K}$ is defined as the conductivity matrix

$\underline{T(t)}$ is a vector of the nodal point temperatures

$\underline{\dot{T}(t)}$ is a vector of the time rate of change of the nodal point temperatures

$\underline{Q(t)}$ is a vector of the external heat rates which are supplied at the nodes (heat which is generated within the elements can be considered in this vector)

The basic assumptions required to develop this heat equilibrium equation will be given in the following sections.

Temperature Gradients

The basic element (sub-division) used in the idealization of a two-dimensional body is a triangle of arbitrary shape (figure 1). Since the thickness of the triangle may be different for each element, axisymmetric bodies are a special case of this formulation. The first step in the development of the heat transfer equations is to assume the following temperature distribution within each element:

$$T(x,y) = \beta_1 + \beta_2 x + \beta_3 y \tag{2a}$$

If equation (2a) is evaluated at the three vertices and the resulting set of equations is solved, the following relationship for the constants β_1, β_2 and β_3 is obtained:

$$\begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} \cdot\cdot & \lambda & \cdot\cdot & 0 & \cdot\cdot & 0 & \cdot\cdot \\ \cdot\cdot & b_j - b_k & \cdot\cdot & b_k & \cdot\cdot & -b_j & \cdot\cdot \\ \cdot\cdot & a_k - a_j & \cdot\cdot & -a_k & \cdot\cdot & a_j & \cdot\cdot \end{bmatrix} \begin{bmatrix} \vdots \\ T_i \\ \vdots \\ T_j \\ \vdots \\ T_k \\ \vdots \end{bmatrix} \tag{2b}$$

where $\lambda = a_j b_k - a_k b_j$

or symbolically

$$\underline{\beta} = \underline{D}\,\underline{T} \tag{2c}$$

where $\underline{T}$ is a vector of the temperatures at all the nodal points in the system.

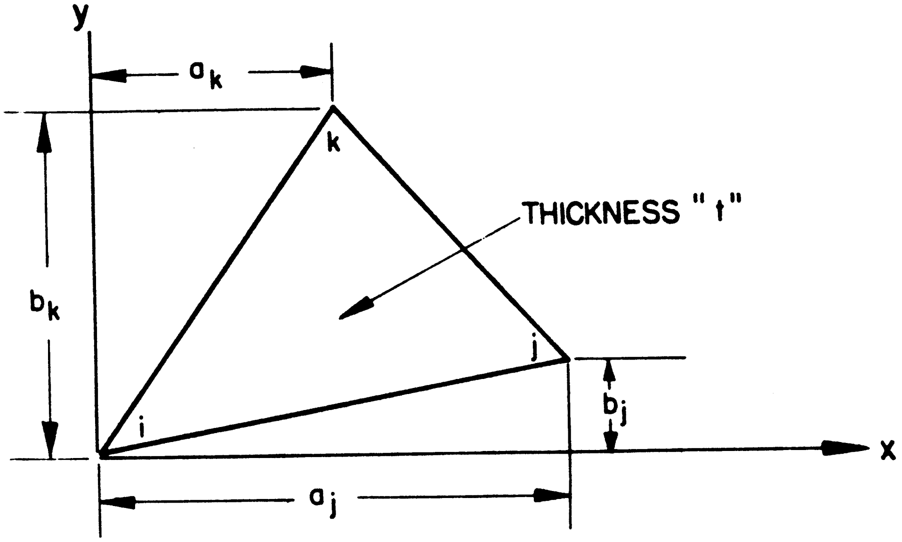


FIGURE 1 - TYPICAL TRIANGULAR ELEMENT

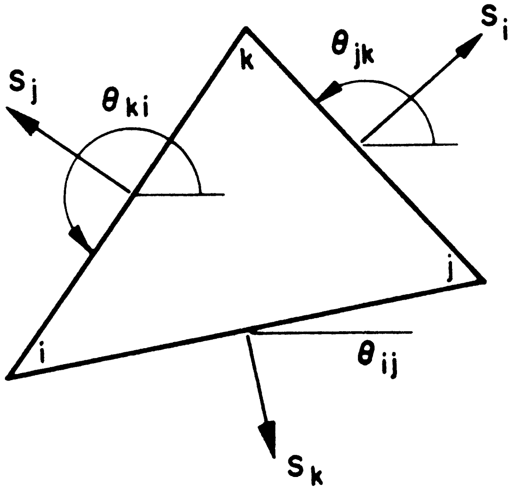


FIGURE 2 - NORMAL GRADIENTS AND REFERENCE ANGLES

Temperature Gradients (Continued)

The temperature gradients normal to the boundaries of the element are by definition

$$S_N = \frac{\partial T}{\partial N} \tag{3}$$

If equation (3) is evaluated at the three boundaries of the element, the following normal gradients are found:

$$\begin{bmatrix} S_i \\ S_j \\ S_k \end{bmatrix} = \begin{bmatrix} 0 & -\sin \Theta_{jk} & \cos \Theta_{jk} \\ 0 & -\sin \Theta_{ki} & \cos \Theta_{ki} \\ 0 & -\sin \Theta_{ij} & \cos \Theta_{ij} \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} \tag{4}$$

The angles Θ_{mn} are defined in figure 2. Equation (4) may be written in symbolic form as

$$\underline{S} = \underline{g}\ \underline{\beta} \tag{5}$$

Heat Flow

The rate of heat flow through a boundary is given by

$$q_N = k^* A S_N$$

where k* is the conductivity of the material and A is the surface area of the boundary. Consequently, the heat flow, as shown in figure 3, through the three interior boundaries, which are one half the length of the sides, is

$$\begin{bmatrix} \vdots \\ q_i \\ \vdots \\ q_j \\ \vdots \\ q_k \end{bmatrix} = \frac{k^*t}{2} \begin{bmatrix} \vdots & \vdots & \vdots \\ \ell_{jk} & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & \ell_{ki} & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & \ell_{ij} \end{bmatrix} \begin{bmatrix} S_i \\ S_j \\ S_k \end{bmatrix} \qquad (6)$$

where t is the thickness of the element and ℓ_{mn} is the length of side mn. Equation (6) written symbolically is

$$\underline{q} = \underline{f}\ \underline{s} \qquad (7)$$

The size of the vector $\underline{q}$ is equal to the number of nodal points in the system; it will contain only three non-zero rows, however, since a typical element is connected to three nodal points.

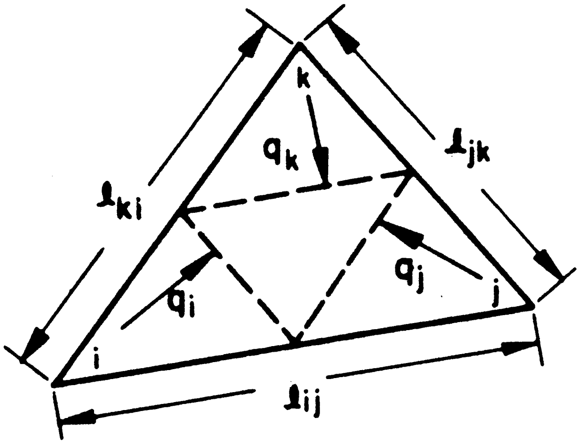


FIGURE 3 - HEAT FLOW VECTORS

Element Conductivity Matrix

Heat flow for a typical element m is expressed in terms of corner temperatures by combining equations (2), (5) and (7),

$$\underline{q}^m = \underline{K}^m \, \underline{T} \qquad (8)$$

where $\underline{k}^m$ is defined as the "element conductivity matrix" and is given by

$$\underline{k}^m = \underline{f} \, \underline{g} \, \underline{D} \qquad (9)$$

In terms of basic element dimensions and properties, equation (8) becomes

$$\begin{bmatrix} \cdot \\ q_i \\ \cdot \\ q_j \\ \cdot \\ q_k \\ \cdot \end{bmatrix} = \frac{k^*t}{2\lambda} \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & e^2 + d^2 & \cdot & b_k - a_k d & \cdot & b_j e + a_j d & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & b_k e - a_k d & \cdot & b_k^2 + a_k^2 & \cdot & b_j b_k - a_j a_k & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & b_j e + a_j d & \cdot & b_j a_k - a_j a_k & \cdot & b_j^2 + a_j^2 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} \cdot \\ T_i \\ \cdot \\ T_j \\ \cdot \\ T_k \\ \cdot \end{bmatrix} \qquad (10)$$

where $d = a_k - a_j$

$e = b_j - b_k$

Heat Flow Equilibrium Equations

For heat flow equilibrium, the rate at which heat is externally supplied to a region must be equal to the sum of the rate at which heat flows out of the region and the rate at which heat is stored within the region. For a finite element idealization the typical region in which heat equilibrium is required is shown in Figure 4.

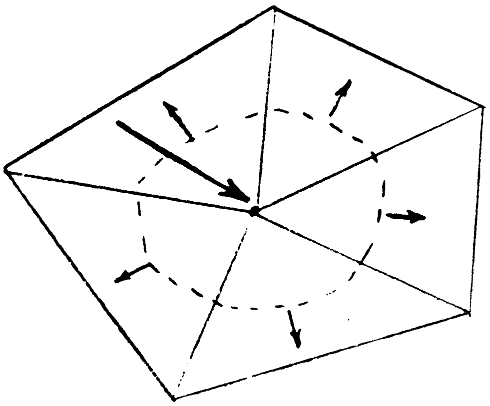


Figure 4 Region for Heat Flow Equilibrium

One-third of the volume of each triangular element is assumed to be associated with each node. Therefore, the approximate rate at which heat is stored in the region of a node will be the product of the time derivative of the temperature, the volume of the region and the specific heat of the material. The rate at which heat flows out of the region is the sum of the element heat flows which can be calculated from equation (10). The application of this procedure in the region of each nodal point yields equation (1); or

$$\underline{C}\dot{\underline{T}} + \underline{K}\underline{T} = \underline{Q} \tag{11}$$

The heat capacity matrix $\underline{C}$ is diagonal and the diagonal terms are given by

Heat Flow Equilibrium Equations (Continued)

$$C_{ii} = \Sigma \frac{1}{3} c_m V_m \tag{11b}$$

where the summation is carried out over all elements attached to nodal point i. The specific heat c_m and volume V_m are associated with a typical element m.

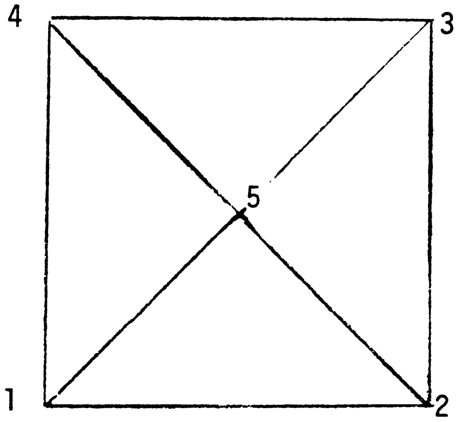
The conductivity matrix $\underline{K}$ is given by the direct summation of element conductivities $\underline{K}^m$ or

$$\underline{K} = \Sigma \underline{K}^m \tag{11c}$$

This direct summation of element conductivity matrices is similar to the combination of element stiffness matrices in the "direct stiffness" approach to stress analysis.

Quadrilateral Element

Four triangular elements can be combined to form a quadrilateral element as shown below:



The heat capacity term associated with nodal point 5 is assumed to be distributed to the four adjacent nodes. The 5 x 5 element conductivity matrix is reduced to a 4 x 4 matrix by the assumption that there is no external heat flow at node 5. Therefore, the typical term of the 4 x 4 quadrilateral conductivity matrix is given by

$$K^*_{ij} = K_{ij} - \frac{K_{i5}K_{5j}}{K_{55}}$$

This reduction procedure is similar to the "static condensation" method in structural analysis.

Insulated Boundary

The previously developed heat flow equilibrium equations can be modified to reflect surface heat transfer. The rate of heat flow across a boundary layer at the surface of the body is given by the following equation:

$$q = h(T_e - T_s)$$

where q = the rate of heat transferred to the surface element per unit area

h = the heat transfer coefficient for the surface

T_s = the temperature of the surface

T_e = the temperature of the external environment

If we consider a surface element between nodes i and j the rate at which heat is transferred to the nodes will be approximately

$$Q_i = \frac{h\ell t}{2} (T_e - T_i) \qquad (12a)$$

$$Q_j = \frac{h\ell t}{2} (T_e - T_j) \qquad (12b)$$

where t is the thickness of the element and ℓ is the distance between nodes i and j.

In order to satisfy total heat flow equilibrium at these boundary nodes equations (12a) and (12b) are added to equations (11). This will result in the following modifications to the conductivity and external heat flow matrices:

Insulated Boundary (Continued)

$$K^*_{ii} = K_{ii} + \frac{h\ell t}{2}$$

$$K^*_{jj} = K_{jj} + \frac{h\ell t}{2}$$

$$Q^*_i = Q_i + \frac{ht}{2} T_e \tag{13}$$

$$Q^*_j = Q_j + \frac{ht}{2} T_e$$

This procedure can be applied repeatedly for all insulated boundary elements.

Cooling Pipes--Normal to Plane

If a cooling pipe is placed at a node point of a finite element system equation (11) can be modified to reflect this type of element. The exact solution for the temperature distribution near a cooling pipe is shown in Fig. (5). The rate at which heat flows into the solid from the cooling pipe is given by

$$q = - 2\pi tkr\frac{\partial T}{\partial r} \tag{14}$$

where r is the distance from the center of the pipe, T is the temperature field, h is the conductivity and t is the thickness of the element. If this differential equation is integrated we find

$$q = \frac{2\ \ kt}{\text{Log}\left[\frac{a}{R}\right]} (T_w - T_a) \tag{15}$$

where T_a is the temperature at a node point at distance a from the pipe, T_w is the temperature of the cooling water and R is the radius of the pipe.

The temperature distribution within the finite elements adjacent to the pipe is assumed to be linear. Therefore the heat flow away from the pipe at distance a/2 is given by

$$q = - 2\pi tk(\frac{S}{2}) \quad (\frac{T_a - T_o}{a}) = \pi tk\ (T_o - T_a) \tag{16}$$

where T_o is the apparent temperature at the pipe in the finite element solution.

If equations (15) and (16) are combined and T_a eliminated we obtain

$$q = H(T_w - T_a) \tag{17}$$

where

$$H = \frac{2\pi kt}{\text{Log}\left[\frac{a}{R}\right] - 2}$$

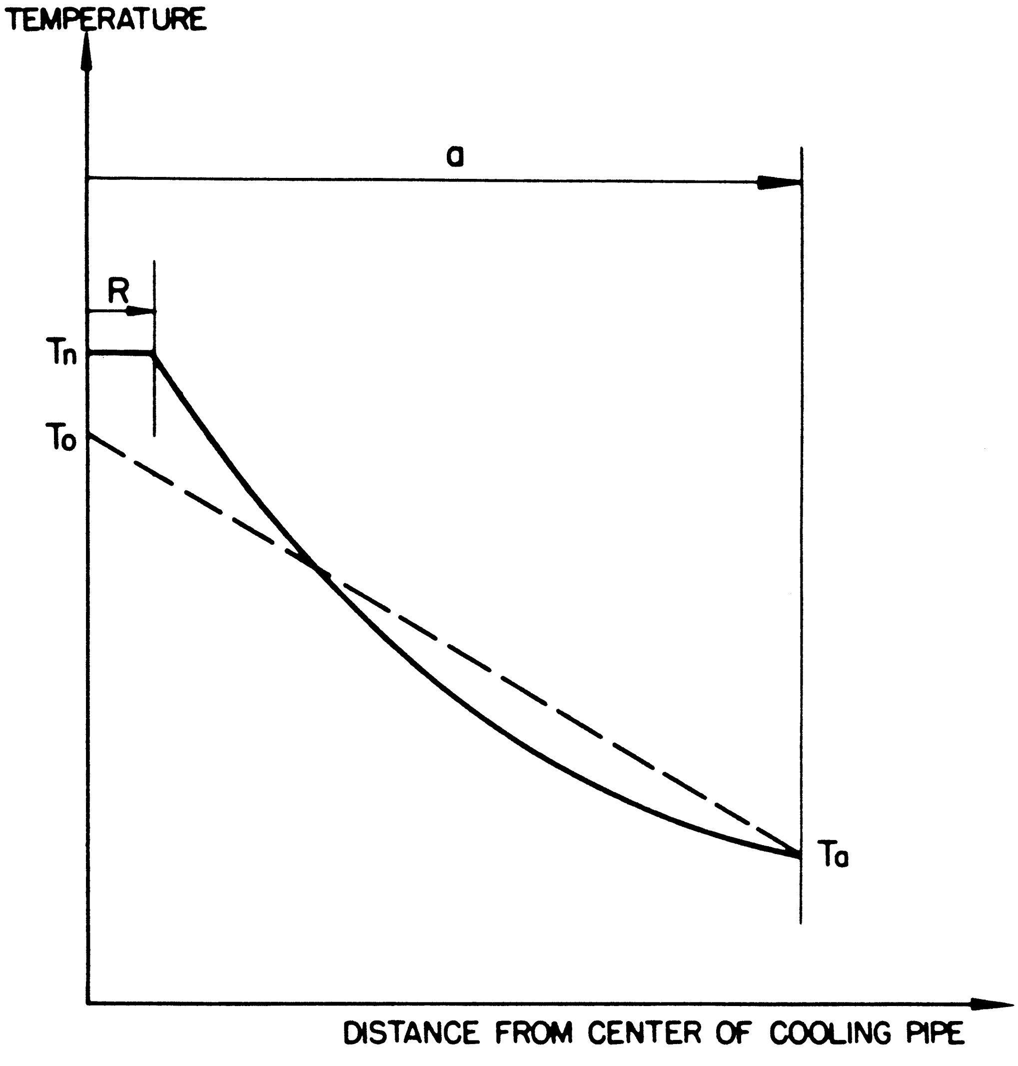


FIGURE 5 TEMPERATURE DISTRIBUTION NEAR COOLING PIPE.

It is of interest to note that for an element size "a" of 7.4 times the radius of the pipe Log $\{\frac{a}{R}\} = 2$ and the value of H is infinite; therefore, for this element size the apparent temperature of the node T_o is equal to cooling water temperature T_w. For other element size the correct value of H can be computed.

As in the case of insulated boundary the heat flow equilibrium equations can be modified to include the effect of a cooling pipe. At a typical nodal point i the following additions must be made to the conductivity and external heat flow matrices:

$$\begin{aligned} K^*_{ii} &= K_{ii} + H \\ Q^*_i &= Q_i + HT_w \end{aligned} \tag{18}$$

Temperature Boundary Conditions

At certain points in the system, the temperatures may be specified as a function of time. To allow for these boundary conditions, equation (11) is written in the following partitioned form:

$$\begin{bmatrix} \underline{Ca} & 0 \\ 0 & \underline{Cb} \end{bmatrix} \begin{bmatrix} \dot{\underline{Ta}} \\ \dot{\underline{Tb}} \end{bmatrix} + \begin{bmatrix} \underline{Kaa} & \underline{Kab} \\ \underline{Kba} & \underline{Kbb} \end{bmatrix} \begin{bmatrix} \underline{Ta} \\ \underline{Tb} \end{bmatrix} = \begin{bmatrix} \underline{Qa} \\ \underline{Qb} \end{bmatrix} \qquad (19)$$

where $\underline{Qa}$, $\underline{Tb}$, and $\dot{\underline{Tb}}$ are specified and $\underline{Qb}$ and $\underline{Ta}$ are the unknowns in the system. Equation (11a) may be rewritten as

$$\underline{Ca}\ \dot{\underline{Ta}} + \underline{Kaa}\ \underline{Ta} + \underline{Q} \qquad (20)$$

where

$$\underline{Q} = \underline{Qa} - \underline{Kab}\ \underline{Tb} \qquad (21)$$

Equation (20) is now in a form which can be solved for the unknown nodal point temperatures. The matrix $\underline{Kaa}$ is always symmetric and positive-definite, and in most cases, it can be placed in band form. Therefore, large systems can be stored in most computers.

The Step-by-Step Solution Technique

Equation (20) is a set of first order linear differential equations which can be solved by a step-by-step method. At time "t" this set of equations is of the form

$$\underline{C}\,\dot{T}_t + \underline{K}\,T_t = Q_t \tag{22}$$

If the solution is known at the previous step in time t - Δt and if the temperature at each node is assumed to vary linearly within the increment of time, the rate of change in temperature, $\dot{T}$, is given by

$$\dot{T}_t = \frac{1}{\Delta t}\ (T_t - T_{t-\Delta t}) \tag{23}$$

The substitution of equation (23) into equation (22) yields:

$$\underline{K}^*\,\underline{T}_t = \underline{Q}^*_t \tag{24}$$

where

$$\underline{K}^* = \underline{K} + \frac{1}{\Delta t}\,\underline{C}$$

$$\underline{Q}^*_t = \underline{Q}_t + \frac{1}{\Delta t}\,\underline{C}\,\underline{T}_{t-\Delta t}$$

Equation (24) can now be solved directly for the temperature at the end of the time increment. Since $\underline{K}^*$ is not a function of time it can be formed once for a given geometry and triangularized. This triangularized matrix then can be used efficiently at each increment of time with different thermal load vector Q^*_t . The complete solution procedure is summarized in Table I.

TABLE I

SUMMARY OF STEP-BY-STEP SOLUTION METHOD

INITIAL CALCULATIONS

1. Form $\underline{C}$ and $\underline{K}$ — Eq. (11)
2. Modify for insulated boundaries and cooling pipes — Eqs. (13) and (18)
3. Modify for temperature boundary conditions — Eq. (21)
4. Form $\underline{K}^*$

$$\underline{K}^* = \underline{K} + \frac{1}{\Delta t} \underline{C}$$

5. Triangularize $\underline{K}^*$

FOR EACH TIME INCREMENT

1. Calculate $\underline{Q}^*$ — Eq. (24)

$$\underline{Q}^* = \underline{Q} + \frac{1}{\Delta t} \underline{C}\, \underline{T}_{t-\Delta t}$$

2. Evaluate $\underline{T}_t$ by solving

$$\underline{K}^* \underline{T}_t = \underline{Q}^*$$

3. Repeat for next time increment.

EXAMPLES

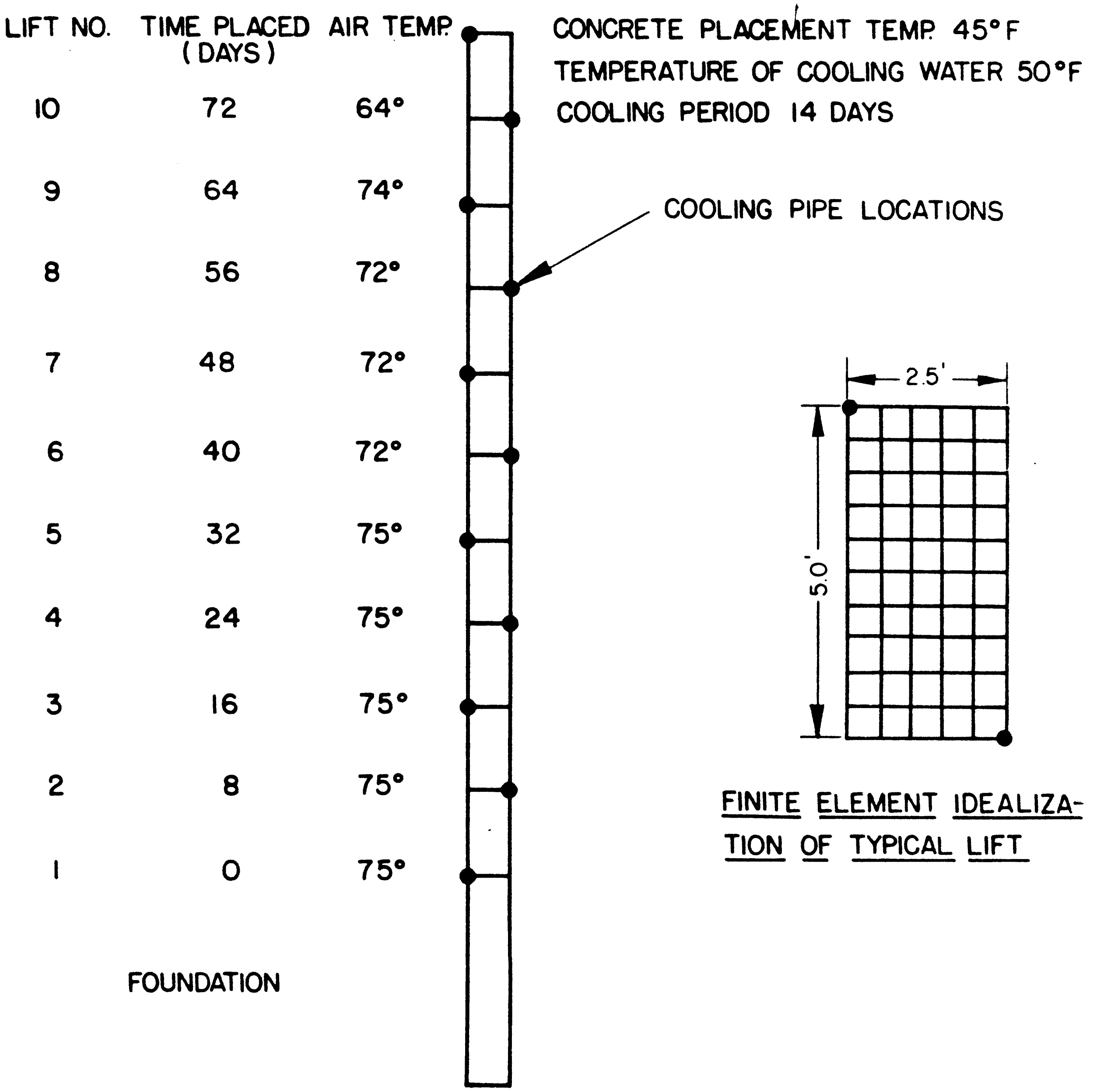
The validity of the finite element approach as applied to heat conductions has been demonstrated previously; therefore, comparison of the method with theoretical solutions will not be given. Only Examples of the method will be given.

Temperature Distribution During Incremental Construction

This example was selected to compare results of the present method with results of a finite difference method used by the Walla Walla District. The finite element idealization of the region to be studied is shown in figure 6. The temperatures at point A shown as a function of time are plotted in figure 7. The reason for the difference in the two techniques is because of the method used in the idealization of the cooling pipes. In the finite difference program the temperature of the node associated with the cooling pipe was set equal to the water temperature. This approach appears to be without theoretical justification since the diameter of the pipe is not taken into account. The results associated with the finite element program are based on a consistent idealization of the cooling pipe.

Calculation of Stresses From Temperatures

The temperature distribution obtained from the heat transfer program can be supplied to a finite element program for the evaluation of stresses as a function of time (2). This approach would be time-



LIFT NO.	TIME PLACED (DAYS)	AIR TEMP
10	72	64°
9	64	74°
8	56	72°
7	48	72°
6	40	72°
5	32	75°
4	24	75°
3	16	75°
2	8	75°
1	0	75°

<u>FINITE ELEMENT IDEALIZATION OF TYPICAL LIFT</u>

FIGURE 6 - EXAMPLE OF INCREMENTAL CONSTRUCTION

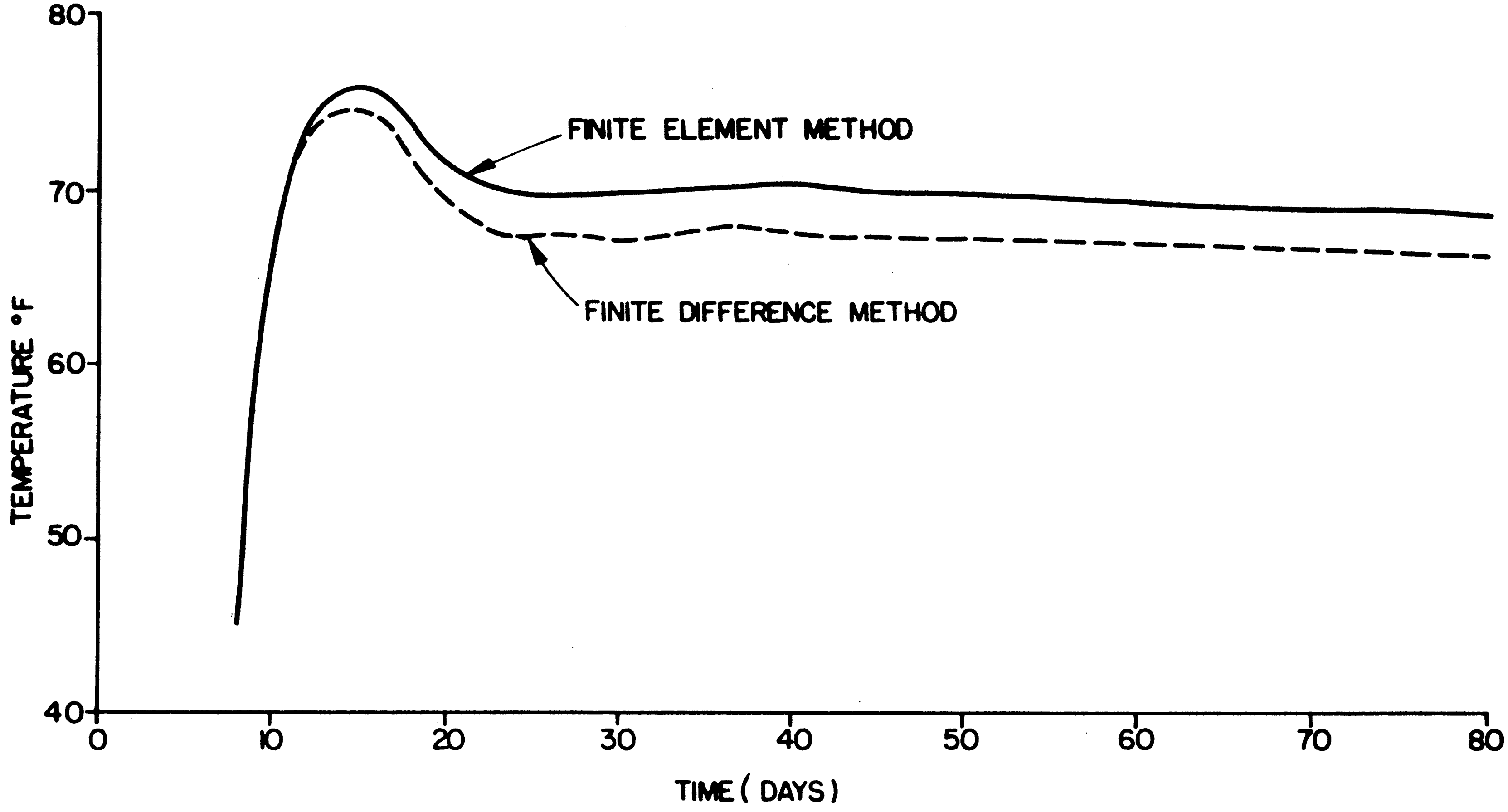


FIGURE 7 TEMPERATURE AT TOP OF LIFT NO. 2

consuming and unnecessary for most mass concrete problems. An approximate method for the calculation of thermal stresses in confined mass concrete structures has been developed (3) and will be summarized here.

The maximum tensile stresses developed in a concrete structure constructed incrementally are generally near a lift surface and are in the horizontal direction. The change in the horizontal stress $\Delta\sigma$ at a point due to an instantaneous temperature change may be approximated by

$$\Delta\sigma = E\alpha\Delta T$$

where α is the thermal coefficient of expansion and E is the instantaneous modulus of elasticity. Of course, this is true only for this type of structure and because the concrete locally is in approximately a confined state. This incremental stress will relax with time, and its magnitude can easily be predicted. Tables 2 and 3 show stress histories for a one-degree (F) temperature drop applied at different ages for two different concrete mixes. These tables were computer generated and include the effects of the variation of modulus of elasticity with time.

With the aid of a table of this type, it is possible to predict readily the horizontal stresses as a function of time when the concrete is subjected to a known temperature history. In order to calculate the stress at any time, the effect of all the daily temperature changes must be added. Thus, if ΔT_1, ΔT_2 --- and ΔT_m are the daily temperature changes which occur up to m days and $C_{1m,}$ $C_{2m,}$ --- and C_{mm} are the temperature stress influence coefficients for a concrete age of m days, then the total stress at age m is given by

TABLE 2 - Temperature Stress Influence Coefficients

(2.75 SCY Concrete)

TIME OF LOADING - ONE DEGREE F DECREASE	AGE OF CONCRETE (DAYS)																													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
0- 1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1- 2		6.349	3.998	3.078	2.737	2.592	2.513	2.457	2.408	2.362	2.318	2.276	2.234	2.194	2.155	2.118	2.081	2.046	2.013	1.980	1.949	1.919	1.891	1.863	1.837	1.811	1.787	1.764	1.742	1.720
2- 3			8.361	5.262	4.157	3.731	3.534	3.415	3.324	3.244	3.168	3.096	3.026	2.958	2.892	2.829	2.767	2.708	2.652	2.597	2.544	2.494	2.446	2.399	2.355	2.312	2.271	2.232	2.194	2.158
3- 4				9.686	6.258	5.019	4.518	4.271	4.116	3.995	3.888	3.787	3.691	3.598	3.509	3.422	3.339	3.258	3.181	3.107	3.035	2.966	2.900	2.837	2.776	2.718	2.663	2.609	2.558	2.509
4- 5					10.676	7.150	5.825	5.264	4.975	4.787	4.639	4.507	4.385	4.268	4.155	4.047	3.942	3.841	3.744	3.650	3.561	3.475	3.392	3.313	3.236	3.163	3.093	3.026	2.962	2.901
5- 6						11.464	7.899	6.512	5.903	5.580	5.367	5.198	5.049	4.910	4.777	4.650	4.528	4.410	4.296	4.187	4.083	3.982	3.885	3.793	3.704	3.619	3.537	3.459	3.383	3.312
6- 7							12.118	8.564	7.136	6.490	6.142	5.910	5.726	5.563	5.412	5.268	5.130	4.998	4.870	4.748	4.630	4.517	4.408	4.304	4.204	4.108	4.016	3.928	3.844	3.763
7- 8								12.694	9.153	7.693	7.022	6.656	6.411	6.217	6.045	5.885	5.732	5.587	5.447	5.313	5.184	5.060	4.941	4.827	4.718	4.613	4.512	4.416	4.324	4.235
8- 9									13.270	9.669	8.167	7.473	7.094	6.841	6.640	6.462	6.297	6.141	5.991	5.848	5.710	5.578	5.451	5.330	5.213	5.101	4.994	4.891	4.792	4.698
9-10										13.846	10.152	8.604	7.889	7.500	7.240	7.034	6.853	6.686	6.527	6.375	6.230	6.091	5.957	5.828	5.705	5.587	5.474	5.366	5.262	5.162
10-11											14.422	10.613	9.016	8.280	7.880	7.614	7.404	7.220	7.050	6.889	6.735	6.588	6.447	6.311	6.181	6.057	5.938	5.823	5.714	5.609
11-12												14.998	11.043	9.388	8.626	8.213	7.941	7.727	7.539	7.366	7.203	7.047	6.897	6.754	6.617	6.485	6.359	6.238	6.122	6.012
12-13													15.575	11.484	9.773	8.988	8.565	8.288	8.070	7.881	7.706	7.540	7.383	7.232	7.088	6.949	6.817	6.690	6.568	6.451
13-14														16.151	11.938	10.178	9.373	8.944	8.663	8.445	8.254	8.078	7.913	7.755	7.604	7.459	7.320	7.188	7.060	6.938
14-15															16.727	12.370	10.556	9.732	9.295	9.010	8.789	8.597	8.420	8.253	8.094	7.942	7.796	7.657	7.523	7.395
15-16																17.303	12.775	10.898	10.050	9.602	9.311	9.086	8.891	8.711	8.542	8.380	8.226	8.078	7.937	7.801
16-17																	17.879	13.185	11.248	10.378	9.918	9.623	9.395	9.197	9.015	8.844	8.680	8.524	8.375	8.232
17-18																		18.456	13.603	11.608	10.715	10.249	9.950	9.719	9.520	9.337	9.164	9.000	8.843	8.693
18-19																			19.032	14.013	11.958	11.043	10.567	10.263	10.030	9.828	9.642	9.468	9.302	9.143
19-20																				19.608	14.414	12.295	11.355	10.867	10.557	10.318	10.112	9.923	9.745	9.576
20-21																					20.184	14.820	12.639	11.674	11.175	10.858	10.615	10.405	10.213	10.032
21-22																						20.760	15.231	12.990	12.001	11.492	11.169	10.992	10.709	10.515
22-23																							21.336	15.642	13.339	12.325	11.804	11.475	11.223	11.006
23-24																								21.913	16.052	13.685	12.644	12.109	11.771	11.512
24-25																									22.489	16.467	14.038	12.970	12.421	12.074
25-26																										23.065	16.889	14.398	13.303	12.740
26-27																											23.641	17.317	14.766	13.646
27-28																												24.217	17.753	15.148

TABLE 3 - Temperature Stress Influence Coefficients

(3.25 SCY Concrete)

	AGE OF CONCRETE (DAYS)																													
TIME OF LOADING - ONE DEGREE F DECREASE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
0- 1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1- 2		7.537	4.669	3.573	3.190	3.039	2.962	2.909	2.863	2.821	2.779	2.739	2.698	2.659	2.620	2.582	2.544	2.507	2.471	2.435	2.400	2.366	2.333	2.300	2.268	2.236	2.206	2.176	2.146	2.118
2- 3			9.549	5.661	4.355	3.892	3.700	3.595	3.522	3.459	3.400	3.343	3.287	3.232	3.178	3.125	3.073	3.021	2.971	2.921	2.873	2.825	2.778	2.733	2.688	2.644	2.602	2.560	2.520	2.480
3- 4				10.874	6.582	5.128	4.586	4.346	4.210	4.111	4.025	3.944	3.867	3.791	3.716	3.643	3.571	3.500	3.430	3.362	3.295	3.229	3.165	3.102	3.040	2.980	2.921	2.864	2.808	2.754
4- 5					11.864	7.439	5.880	5.273	4.993	4.829	4.708	4.602	4.503	4.407	4.314	4.222	4.132	4.043	3.956	3.870	3.787	3.704	3.624	3.545	3.468	3.393	3.320	3.248	3.178	3.110
5- 6						12.652	8.202	6.578	5.929	5.624	5.444	5.309	5.193	5.084	4.978	4.875	4.774	4.675	4.578	4.482	4.388	4.296	4.206	4.118	4.032	3.948	3.866	3.785	3.707	3.631
6- 7							13.306	8.843	7.175	6.497	6.175	5.983	5.839	5.714	5.597	5.484	5.376	5.266	5.159	5.055	4.953	4.853	4.755	4.659	4.565	4.474	4.384	4.297	4.212	4.129
7- 8								13.882	9.320	7.595	6.888	6.551	6.351	6.202	6.073	5.952	5.835	5.721	5.609	5.500	5.393	5.288	5.185	5.084	4.986	4.889	4.795	4.704	4.615	4.528
8- 9									14.458	9.784	8.002	7.268	6.919	6.712	6.558	6.425	6.300	6.180	6.062	5.948	5.836	5.726	5.618	5.513	5.410	5.309	5.210	5.114	5.021	4.930
9-10										15.034	10.227	8.385	7.627	7.266	7.053	6.894	6.757	6.629	6.505	6.385	6.268	6.153	6.041	5.931	5.823	5.718	5.616	5.515	5.418	5.323
10-11											15.610	10.651	8.749	7.967	7.595	7.375	7.211	7.070	6.939	6.812	6.690	6.570	6.452	6.338	6.225	6.116	6.008	5.904	5.802	5.703
11-12												16.186	11.051	9.083	8.273	7.888	7.660	7.491	7.346	7.211	7.081	6.956	6.833	6.713	6.596	6.481	6.369	6.260	6.153	6.050
12-13													16.763	11.464	9.430	8.592	8.193	7.958	7.784	7.635	7.497	7.365	7.237	7.112	6.989	6.870	6.753	6.639	6.528	6.420
13-14														17.339	11.889	9.793	8.927	8.514	8.272	8.094	7.942	7.801	7.667	7.536	7.409	7.285	7.163	7.045	6.930	6.818
14-15															17.915	12.330	10.174	9.280	8.855	8.607	8.426	8.271	8.128	7.992	7.860	7.731	7.605	7.483	7.363	7.247
15-16																18.491	12.787	10.576	9.660	9.226	8.973	8.789	8.633	8.489	8.352	8.219	8.089	7.963	7.840	7.721
16-17																	19.067	13.225	10.957	10.017	9.573	9.314	9.127	8.969	8.823	8.683	8.549	8.418	8.291	8.167
17-18																		19.644	13.638	11.305	10.337	9.879	9.613	9.421	9.258	9.109	8.966	8.829	8.696	8.566
18-19																			20.220	14.061	11.665	10.669	10.198	9.925	9.728	9.561	9.409	9.264	9.124	8.989
19-20																				20.796	14.496	12.039	11.017	10.534	10.254	10.052	9.883	9.728	9.582	9.440
20-21																					21.372	14.930	12.412	11.363	10.865	10.577	10.370	10.197	10.039	9.890
21-22																						21.948	15.363	12.781	11.701	11.187	10.889	10.676	10.498	10.335
22-23																							22.524	15.807	13.163	12.053	11.523	11.216	10.997	10.813
23-24																								23.101	16.263	13.561	12.424	11.880	11.565	11.340
24-25																									23.677	16.704	13.942	12.776	12.217	11.893
25-26																										24.253	17.126	14.297	13.099	12.523
26-27																											24.829	17.557	14.662	13.431
27-28																												25.405	17.997	15.037

Calculation of Stresses From Temperatures (Continued)

$$\sigma_m = \Delta T_1 C_{1m} + \Delta T_2 C_{2m} + \text{----} \; \Delta T_m C_{mm}$$

or

$$\sigma_m = {}_{i=1}^{m} \; \Delta T_i C_{im}$$

Therefore, it is possible to calculate an approximate stress history for different placement and cooling schedules without the use of a complex digital computer program. Of course, this approximate technique may be applied only because the mass concrete is highly confined in the vicinity of the closely spaced cooling pipes.

The temperature history near the top of lift No. 2, Temperature Study No. 2, has been selected to illustrate the use of the approximate method (3). The temperature history for this case is shown in Fig. 8. The resulting approximate horizontal stresses versus time are shown in Fig. 9. For this case a typical calculation for the stress at age nine days is shown below:

DAY	TEMPERATURE (°F)	TEMP. DROP ΔT_i	C_{19}	$\Delta\sigma_9 = \Delta T_i C_{i9}$
0	45			
1	70.5	-25.5	0	0
2	73.1	- 2.6	2.863	- 7.5
3	74.4	- 1.3	3.522	- 4.6
4	75.1	- 0.7	4.210	- 2.9
5	75.5	- 0.4	4.993	- 2.0
6	75.5	0	5.929	0
7	75.5	0	7.175	0
8	75.4	+ 0.1	9.320	+ 0.9
9	62.1	+13.3	14.458	+192.3

$$\sigma_9 = \sum_{i=1}^{9} \Delta T_i C_{i9} = +176.2$$

Calculation of Stresses From Temperatures (Continued)

The approximate nine-day stress of 176 psi agrees very well with the 168 psi obtained by the finite element analysis. As indicated by Fig. 9, comparison of the two methods is reasonable for the entire temperature history.

To illustrate the approximate method further, two hypothetical temperature histories were studied and are illustrated in Fig. 10. Temperature History A represents typical behavior of surface concrete which is placed and exposed to 75°F atmospheric condition. Temperature History B represents a concrete which is surface-insulated for the first 7 days; therefore, the temperature change is essentially adiabatic. Fig. 11 illustrates the resulting horizontal stresses. In this case, the superiority of Temperature History B is clearly illustrated. Approximately one hour of hand calculation was required for this comparison. If, however, these tables were incorporated into the heat transfer program, these simple stress calculation could be conducted within a single computer program.

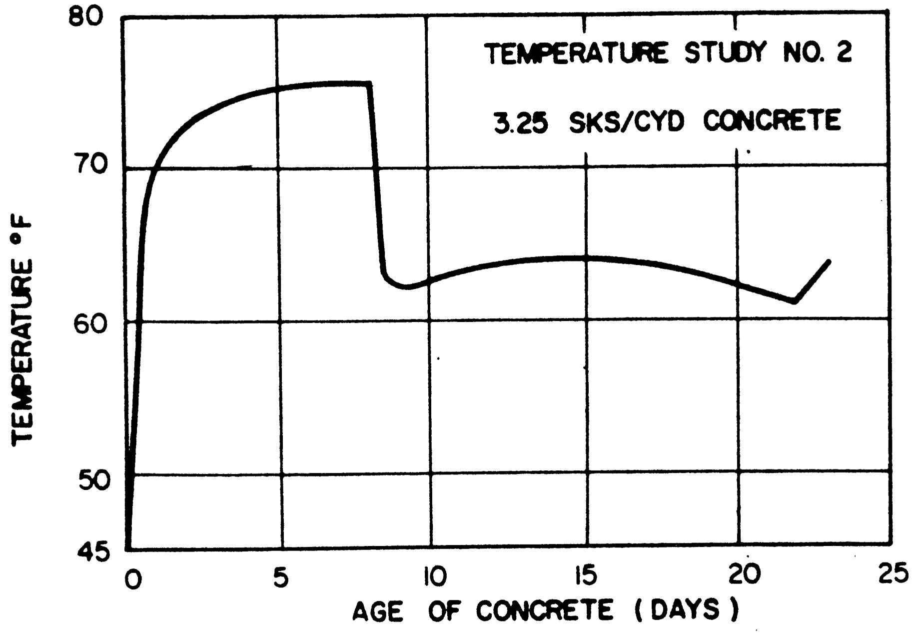


FIGURE 8 - TEMP. HISTORY NEAR TOP OF LIFT NO. 2

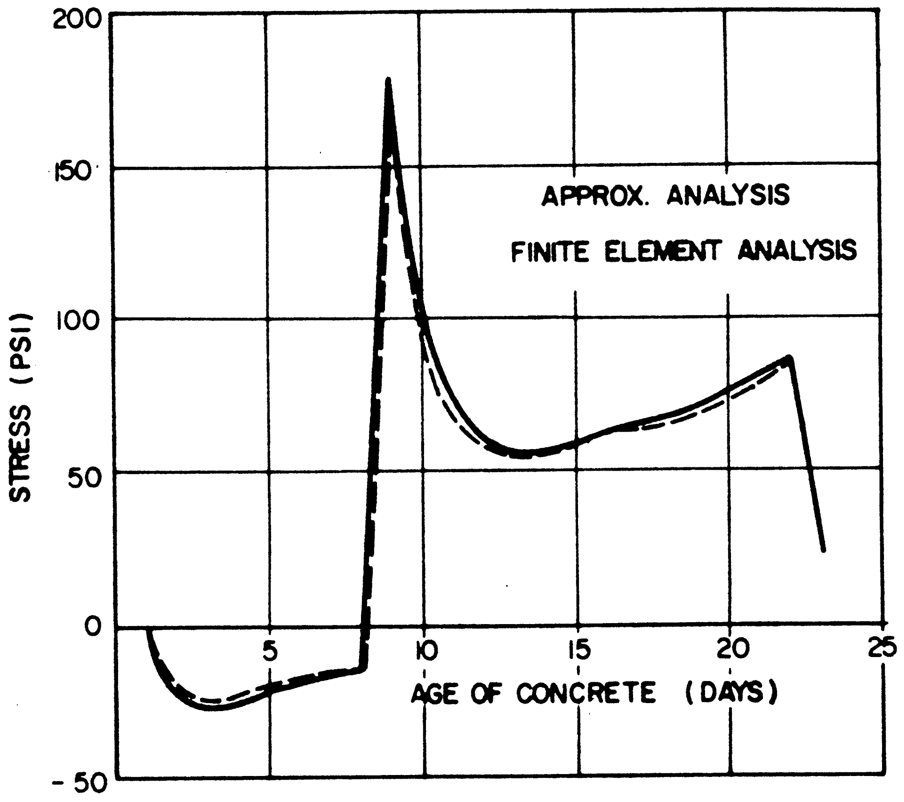


FIGURE 9 - HORIZONTAL STRESSES NEAR TOP OF LIFT NO. 2

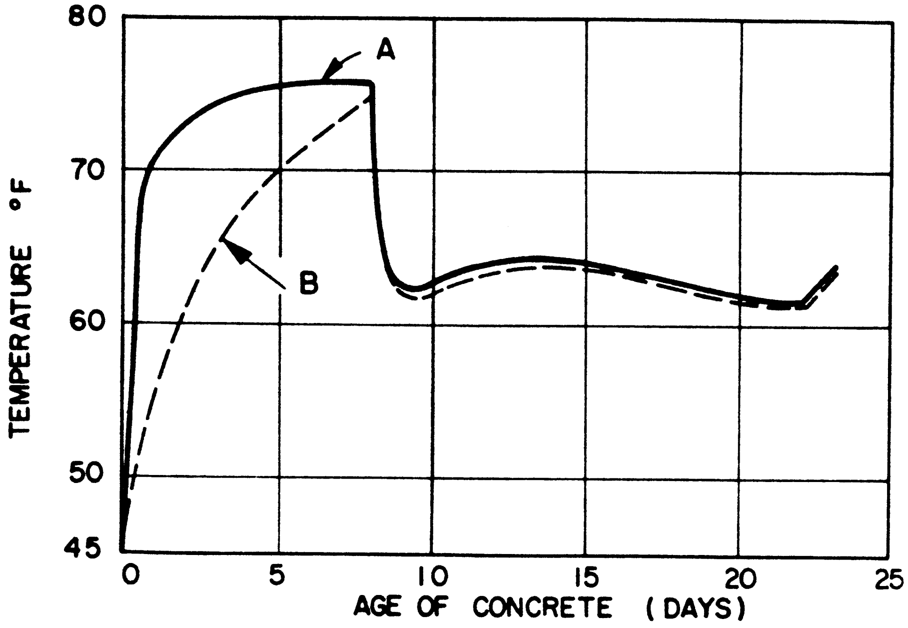


FIG. 10 HYPOTHETICAL TEMP. HISTORIES

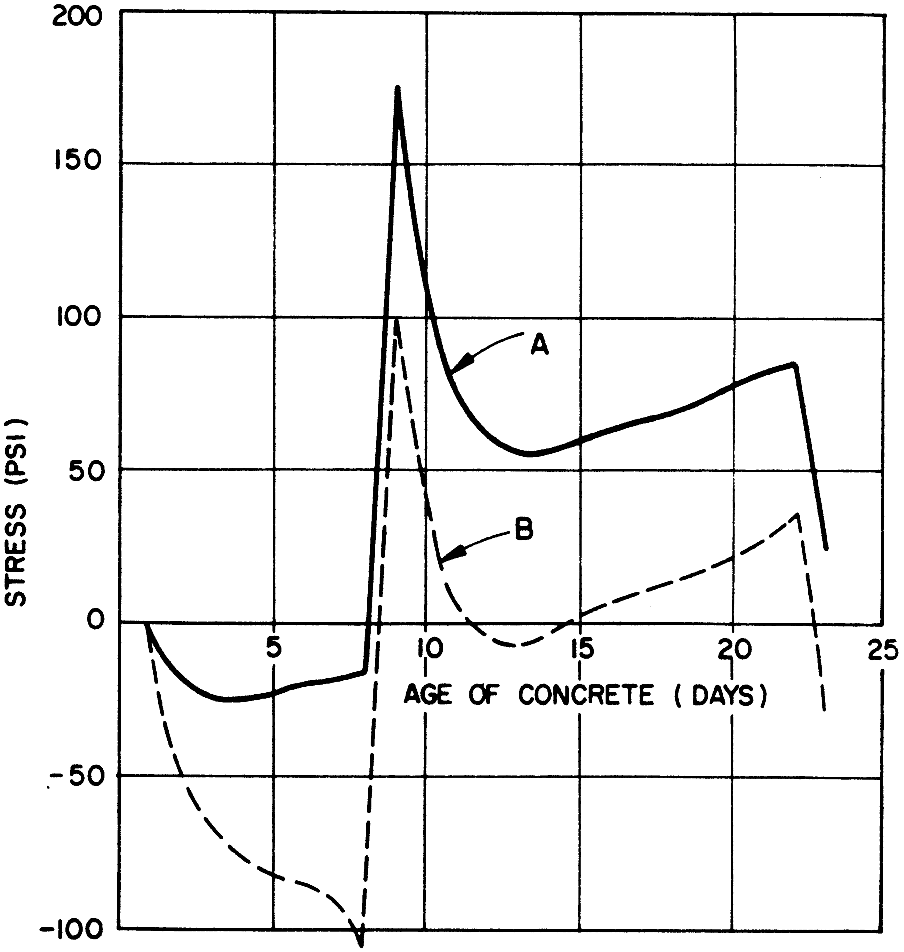


FIG. 11 STRESSES DUE TO HYPOTHETICAL TEMP. HISTORIES

FINAL REMARKS

A computer program which is based on the method of analysis presented in this report has been developed. A description on the use of this program and a Fortran IV listing is given in the Appendices of this report. The program is developed with particular reference to mass concrete structure of arbitrary geometry and construction sequence; however, it may be applied to many different types of problems involving time-dependent boundary conditions.

REFERENCES

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APPENDIX A

DESCRIPTION OF INPUT DATA FOR COMPUTER PROGRAM

DESCRIPTION OF INPUT DATA FOR COMPUTER PROGRAM

The purpose of this computer program is to determine the temperature distribution as a function of time within a concrete structure as it is being constructed. Each lift of the structure may be placed at an arbitrary time and temperature. Insulation forms may be placed or removed from the concrete surfaces at any point in time. The external air temperature and temperature of the cooling water may also vary with time.

GENERAL INPUT DATA

The first step in the analysis is to select a finite element representation for the complete structure. All elements and nodal points are then numbered in two numerical sequences each starting with one. The following group of punched cards numerically defines the complete structure to be analysed:

A. Identification Card - (72H)

Columns 1 to 72 of this card contain information to be printed with results.

B. Control Card (5I5)

Columns	1 - 5	Total number of nodal points
	6 - 10	Total number of elements
	11 - 15	Number of different materials
	16 - 20	Number of adiabatic temperature cards for each material
	21 - 25	Number of cards which describe the external temperature environment

C. Nodal Point Cards - (I5, 5X, 3F10.0)

One card for each nodal point with the following information:

Columns	1 - 5	Nodal point number
	11 - 20	X-ordinate
	21 - 30	Y-ordinate
	31 - 40	Temperature (for foundation points only)

Nodal point cards must be in numerical sequence. If cards are omitted, the omitted nodal points are generated at equal intervals along a straight line between the specified nodal points.

D. Element Cards - (6I5, F10.0)

One card for each element

Columns	1 - 5	Element number
	6 - 10	Nodal point I
	11 - 15	Nodal point J
	16 - 20	Nodal point K
	21 - 25	Nodal point L
	26 - 30	Material identification number
	31 - 40	Time of placement

Nodal point numbers (I,J, K and L) must be in counterclockwise order around each element. Maximum difference between nodal point numbers must be less than 27.

Element cards must be in element number sequence. If element cards are omitted, the program automatically generates the omitted information by incrementing the preceding I,J,K and L. The information in column 26 to 40 for the generated cards is set equal to the values given on the preceding card. The last element card must always be supplied.

Triangular elements are also possible and are identified by repeating the last nodal point number (i.e., I, J, K, K).

E. Material Property Information

The following group of cards must be supplied for each different material.

First Card (1I5, 5X, 4F10.0)

Columns	1 - 5	Material identification number
	11 - 20	Conductivity of material
	21 - 30	Specific heat of material
	31 - 40	Density of material
	41 - 50	Time after placement when heat generation ceases

Adiabatic Temperature Cards (2F10.0)

Columns	1 - 10	Time
	11 - 20	Temperature

The total number of these cards is specified in columns 16 to 20 of the control card.

F. External Temperature Environment Information (2F10.0)

The external temperature is specified at discrete points in time by a sequence of cards of the following form:

Columns	1 - 10	Time
	11 - 20	External Temperature

The number of these cards is specified in columns 21 to 25 of the control card.

LIFT DATA

It is assumed that the structure does not exist at time zero. When a change takes place in the geometry of the structure, or the number of insulating elements, or the number and temperature of the cooling pipes, new lift data must be supplied.

A. Lift Data Control Card (6I5, 3F10.0)

Columns	1 - 5	Number of nodal points to be considered
	6 - 10	Number of elements to be considered
	11 - 15	Number of insulating elements to be added
	16 - 20	Number of cooling pipes to be added
	21 - 25	Number of time increments in time span
	26 - 30	Output interval for print of temperatures
	31 - 40	Time increment to be used in time span
	41 - 50	Time at the beginning of time span
	51 - 60	Placement temperature for all elements which are placed at the beginning of time span.

B. Insulating Element Cards (2I5, 2F10.0)

One card must be supplied for each surface element which is added to the system. The behavior of the surface element is governed by the following heat transfer equation:

$$q = h(T_e - T_s)$$

where q = the rate of heat transferred to the surface element per unit of area

h = the heat transfer coefficient for the surface

T_s= the temperature of the surface

T_e= the temperature of the external environment

LIFT DATA (Continued)

Each card contains the following information:

Columns	1 - 5	Nodal point number I
	6 - 10	Nodal point number J
	11 - 20	Surface heat transfer constant h
	21 - 30	Time when insulative element is to be removed

If the surface constant h is zero the previously defined insulating element associated with points I and J will be removed.

C. Cooling Pipe Cards (1I5, 5X, 3F10.0)

One card must be supplied for each cooling pipe which is added to the system. The rate at which heat, Q , is removed by the cooling pipe is given by the following equation:

$$Q = H (T_p - T_w)$$

where

H = an empirical constant

T_p = the temperature in the concrete near the pipe

T_w = the emperature of the cooling water

Each card contains the following information:

Columns	1 - 5	Nodal point number which defines the location of the pipe
	11 - 20	Constant H
	21 - 30	Temperature of cooling water
	31 - 40	Time when cooling is to be stopped.

If the constant H is zero the previously defined cooling pipe at the specified nodal point is removed.

APPENDIX B

FORTRAN IV LISTING OF COMPUTER PROGRAM

```
      PROGRAM MAIN(INPUT, OUTPUT, TAPE5 = INPUT, TAPE6 = OUTPUT)
C
      COMMON    NUMEL,NCBH,NCPH,NUMMAT,NDT,INTER,DT,TIME,NUMNP,NUME,
     1 NUMQC,NUMET,PLTIME,B(500),X(500),Y(500),T(500),D(500),TT(500),
     2 IX(400,5),PLTM(400),VOL(400),HED(12),LM(5),E(3,3),KX(4),S(5,5),
     3 XCOND(20),SPHT(20),DENS(20),QX(20,2,20),HSTOP(20),FT(20,2),
     4 IC(20),JC(20),HC(20),TMC(20),CL(20),IP(20),HP(20),TP(20),TMP(20)
      COMMON /SYMARG/ NUMN,MBAND,A(500,27),Q(500)
C
C***********************************************************************
C     READ AND PRINT OF CONTROL INFOMATION
C***********************************************************************
   50 READ  (5,1000)  HED,NUMNP,NUMEL,NUMMAT,NUMQC,NUMET
      WRITE (6,2011)
   58 WRITE (6,2000)  HED,NUMNP,NUMEL,NUMMAT,NUMQC,NUMET
C***********************************************************************
C     READ OR GENERATE NODAL POINT INFORMATION
C***********************************************************************
      WRITE (6,2001)
      L=1
   60 READ  (5,1001)  N,X(N),Y(N),TT(N)
      DIFF=N+1-L
      IF (N-L) 65,80,70
   65 WRITE (6,2020) N
      GO TO 60
   70 DX=(X(N)-X(L-1))/DIFF
      DY=(Y(N)-Y(L-1))/DIFF
      DP=(TT(N)-TT(L-1))/DIFF
   75 X(L)=X(L-1)+DX
      Y(L)=Y(L-1)+DY
      TT(L)=TT(L-1)+DP
   80 WRITE (6,2002) L,X(L),Y(L),TT(L)
      L=L+1
      IF (N-L) 90,80,75
   90 IF (NUMNP+1-L) 100,100,60
  100 CONTINUE
C***********************************************************************
C       READ AND PRINT OF ELEMENT PROPERTIES
C***********************************************************************
      WRITE (6,2003)
      N=0
  103 READ (5,1002) M,(IX(M,I),I=1,5),PLTM(M)
  104 N=N+1
      IF (M-N) 107,107,105
  105 IX(N,1)=IX(N-1,1)+1
      IX(N,2)=IX(N-1,2)+1
      IX(N,3)=IX(N-1,3)+1
      IX(N,4)=IX(N-1,4)+1
      IX(N,5)=IX(N-1,5)
      PLTM(N)=PLTM(N-1)
  107 WRITE (6,2004) N,(IX(N,I),I=1,5),PLTM(N)
      IF (M-N) 108,108,104
  108 IF (NUMEL-N) 109,109,103
  109 CONTINUE
C
```

```
      DO 115 N=1,20
      TMC(N)=0.0
  115 TMP(N)=0.0
C***********************************************************************
C     READ AND PRINT MATERIAL PROPERTIES AND ENVIRONMENT TEMP
C***********************************************************************
      DO 120 M=1,NUMMAT
      READ (5,1001) MT,XCOND(MT),SPHT(MT),DENS(MT),HSTOP(MT)
      WRITE(6,2018) MT
      WRITE(6,2006) MT,XCOND(MT),SPHT(MT),DENS(MT),HSTOP(MT)
      WRITE(6,2007)
      READ (5,1008)  ((QX(I,J,MT),J=1,2),I=1,NUMQC)
      WRITE(6,2008)  ((QX(I,J,MT),J=1,2),I=1,NUMQC)
  120 CONTINUE
      IF(NUMET.EQ.0) GO TO 125
      READ (5,1008) ((ET(I,J),J=1,2),I=1,NUMET)
      WRITE(6,2012) ((ET(I,J),J=1,2),I=1,NUMET)
C***********************************************************************
C     FOR EACH INTERVAL OF TIME--SEVERAL TIME STEPS
C***********************************************************************
  125 TIME=0.0
      NCBH=0
      NCPH=0
      DO 130 N=1,NUMNP
  130 T(N)=0.0
C
C     READ AND PRINT OF LAYER PROPERTIES
C
  150 READ (5,1004) NUMN,NUME,NUMCB,NUMCP,NDT,INTER,DT,PLTIME,PLACET
      IF(NUMN.EQ.0) STOP
      WRITE(6,2005) NUMN,NUME,NUMCB,NUMCP,NDT,INTER,DT,PLTIME,PLACET
C
C     ELIMINATE ALL INSULATING ELEMENTS AND COOLING PIPES WITH REMOVAL
C     TIME LESS THAN PLACEMENT TIME AND READ ADDITIONAL ONES
C
      KK=0
      DO 155 N=1,NCBH
      IF(TMC(N).LE.PLTIME) GO TO 155
      KK=KK+1
      IC(KK)=IC(N)
      JC(KK)=JC(N)
      HC(KK)=HC(N)
      TMC(KK)=TMC(N)
      CL(KK)=CL(N)
  155 CONTINUE
      NCBL=KK+1
      NCBH=KK+NUMCB
      IF(NUMCB.EQ.0) GO TO 160
      READ (5,1005) (IC(N),JC(N),HC(N),TMC(N),N=NCBL,NCBH)
      WRITE(6,2013) (IC(N),JC(N),HC(N),TMC(N),N=NCBL,NCBH)
  160 KK=0
      DO 165 N=1,NCPH
      IF(TMP(N).LE.PLTIME) GO TO 165
      KK=KK+1
      IP(KK)=IP(N)
```

```
      HP(KK)=HP(N)
      TP(KK)=TP(N)
      TMP(KK)=TMP(N)
  165 CONTINUE
      NCPL=KK+1
      NCPH=KK+NUMCP
      IF(NUMCP.EQ.0) GO TO 170
      READ (5,1006) (IP(N),HP(N),TP(N),TMP(N),N=NCPL,NCPH)
      WRITE(6,2014) (IP(N),HP(N),TP(N),TMP(N),N=NCPL,NCPH)
C
C     CHECK INCONSISTENCY OF LIFT INFORMATION AND REMOVAL OF INSULATING
C     ELEMENTS AND/OR COOLING PIPES
C
  170 KK=0
      YY=DT*NDT
      IF(NCBH.EQ.0) GO TO 201
      DO 200 N=1,NCBH
      XX=TMC(N)-PLTIME
      IF(XX.LT.YY) KK=1
  200 CONTINUE
  201 IF(NCPH.EQ.0) GO TO 208
      DO 205 N=1,NCPH
      XX=TMP(N)-PLTIME
      IF(XX.LT.YY) KK=1
  205 CONTINUE
  208 IF(KK.NE.0) WRITE(6,2019)
C
C     SET ALL NEW NODES TO PLACEMENT TEMPERATURE AND CONTACT SURFACE AT
C     AVERAGE TEMPERATURES
C
      DO 210 I=1,NUMN
      B(I)=0.0
  210 Q(I)=0.0
      DO 220 N=1,NUME
      IF(PLTM(N).GT.PLTIME) GO TO 220
      DO 215 I=1,4
      II=IX(N,I)
      IF(PLTM(N).EQ.PLTIME) B(II)=B(II)+PLACET
      IF(PLTM(N).LT.PLTIME) B(II)=B(II)+T(II)
  215 Q(II)=Q(II)+1.0
  220 CONTINUE
      DO 230 N=1,NUMN
      IF(Q(I).EQ.0.0) GO TO 230
      T(N)=B(N)/Q(N)
  230 CONTINUE
C
      CALL LAYER
      GO TO 150
C***********************************************************************
C
C     FORMAT STATEMENTS
C
 1000 FORMAT (12A6/5I5)
 1001 FORMAT (I5,5X,4F10.0)
 1002 FORMAT (6I5,F10.0)
```

```
1003 FORMAT (I10,4F10.0)
1004 FORMAT(6I5,3F10.0)
1005 FORMAT(2I5,2F10.0)
1006 FORMAT(I5,5X,3F10.0)
1008 FORMAT(2F10.0)
2000 FORMAT (1H0 12A6// 25H0NUMBER OF NODAL POINTS-- I4/
    1 25H NUMBER OF ELEMENTS------ I4 /25H NUMBER OF MATERIALS----- I4/
    2 25H NUMBER OF QQ CARDS------ I4 /25H NO OF EXT TEMP CARDS---- I4)
2001 FORMAT (10H0 N.P. NO. 14X,1HX,14X,1HY,11X,4HTEMP)
2002 FORMAT (1I10,3E15.6)
2003 FORMAT (51H0    N     I     J     K     L  MATERIAL  PLACEMENT TIME)
2004 FORMAT (5I5,I10,F16.4)
2005 FORMAT (1H1 / 25H0NUMBER OF NODAL POINTS-- I4/
    1 25H NUMBER OF ELEMENTS------ I4 / 25H NUMBER OF CONVECTION BC-I4/
    2 25H NUMBER OF COOLING PIPE-- I4 / 25H NUMBER OF INCREMENTS----I4/
    3 25H OUTPUT INTERVAL--------- I4 / 20H TIME INTERVAL------ E10.3/
    4 25H BEGINNING TIME---------- F8.2/25H PLACEMENT TEMPERATURE---
    5 F8.2)
2006 FORMAT(6H0    M,11X,4HCOND,11X,4HSPHT,11X,4HDENS ,4X,26HTIME HEAT
    1GENERATION STOPS/(I6,3F15.6,F30.5))
2007 FORMAT(43H0ADIABATIC TEMPERATURE RISE OF THE MATERIAL/ 9H0    TIME
    1  4X,11HTEMPERATURE)
2008 FORMAT(F9.2,F15.6)
2009 FORMAT (4H0  M 14X 1HK 14X 1HC 14X 1HD 14X 1HQ/ (I4,4F15.6))
2011 FORMAT (27H1TWO DIMENSIONAL PLANE BODY )
2012 FORMAT(31H0TEMPERATURE OF THE ENVIRONMENT/ 9H0    TIME,4X,
    1  11HTEMPERATURE/(F9.2,F15.6))
2013 FORMAT(20H0INSULATING ELEMENTS//5H    I,5H    J,14X,1HH,15H   TIME
    1 REMOVED/(2I5,2F15.6))
2014 FORMAT(25H0DETAILS OF COOLING PIPES//5H    I,14X,1HH.15H    TEMPER
    1ATURE,15H   TIME REMOVED/(I5,3F15.6))
2018 FORMAT(1H0,4X,15HMATERIAL TYPE -,I3)
2019 FORMAT(35H0****  ERROR MESSAGE   WARNING ONLY/7X,76HNEW LIFT DATA
    1IS NOT SUPPLIED EVEN THOUGH A CHANGE HAS OCCURED IN INSULATING/
    2 7X,52HELEMENTS AND/OR COOLING PIPES.  CALCULATION PROCEEDS)
2020 FORMAT (10H0CARD NO. I4, 13H OUT OF ORDER )
2021 FORMAT (13H0BAD CARD NO. I4)
C
      END
```

```
      SUBROUTINE LAYER
      COMMON    NUMEL,NCBH,NCPH,NUMMAT,NDT,INTER,DT,TIME,NUMNP,NUME,
     1 NUMQC,NUMFT,PLTIME,B(500),X(500),Y(500),T(500),D(500),TT(500),
     2 IX(400,5),PLTM(400),VOL(400),HED(12),LM(5),E(3,3),KX(4),S(5,5),
     3 XCOND(20),SPHT(20),DENS(20),QX(20,2,20),HSTOP(20),FT(20,2),
     4 IC(20),JC(20),HC(20),TMC(20),CL(20),IP(20),HP(20),TP(20),TMP(20)
      COMMON /SYMARG/ NUMN,MBAND,A(500,27),Q(500)
C
C***********************************************************************
C     FORM CONDUCTIVITY MATRIX FOR COMPLETE BODY
C***********************************************************************
      DO 130 I=1,NUMN
      D(I)=0.0
      B(I)=0.0
      Q(I)=0.0
      DO 130 J=1,27
  130 A(I,J)=0.0
      MBAND=0
      ISTOP=0
C
      DO 200 N=1,NUME
      IF(PLTM(N).GT.PLTIME) GO TO 200
      MTYPE=IX(N,5)
      COND=XCOND(MTYPE)
C
C       2. FORM ELEMENT CONDUCTIVITY MATRIX
C
      DO 150 I=1,5
      LM(I)=IX(N,I)
      DO 150 J=1,5
  150 S(I,J)=0.0
C
      I=LM(1)
      J=LM(2)
      K=LM(3)
      L=LM(4)
      LM(5)=I
C
      XX=(X(I)+X(J)+X(K)+X(L))/4.
      YY=(Y(I)+Y(J)+Y(K)+Y(L))/4.
      VOL(N)=0.0
C
      DO 152 K=1,4
C
      I=LM(K)
      J=LM(K+1)
      IF (I-J) 135,152,135
  135 AJ=X(J)-X(I)
      AK=XX-X(I)
      BJ=Y(J)-Y(I)
      BK=YY-Y(I)
      C=BJ-BK
      DX=AK-AJ
C
      XLAM=AJ*BK-AK*BJ
```

```
      IF(XLAM.GT.0.0) GO TO 136
      ISTOP=1
      WRITE(6,2003) N
  136 VOL(N)=VOL(N)+XLAM*0.5
      COMM=.5*COND/XLAM
C
      E(1,1)=C**2+DX**2
      E(1,2)=BK*C-AK*DX
      E(1,3)=-BJ*C+AJ*DX
      E(2,1)=E(1,2)
      E(2,2)=BK**2+AK**2
      E(2,3)=-BJ*BK-AJ*AK
      E(3,1)=E(1,3)
      E(3,2)=E(2,3)
      E(3,3)=BJ**2+AJ**2
C
      KX(1)=K
      KX(2)=K+1
      IF (K-4) 145,140,145
  140 KX(2)=1
  145 KX(3)=5
C
      DO 151 I=1,3
      II=KX(I)
      DO 151 J=1,3
      JJ=KX(J)
  151 S(II,JJ)=S(II,JJ)+E(I,J)*COMM
C
  152 CONTINUE
C
      DO 155 I=1,4
      DO 155 J=1,4
  155 S(I,J)=S(I,J)-S(I,5)*S(J,5)/S(5,5)
C
C        3. ADD ELEMENT CONDUCTIVITY TO COMPLETE CONDUCTIVITY MATRIX
C
      VOL(N)=VOL(N)*SPHT(MTYPE)*DENS(MTYPE)*0.25
      DO 175 L=1,4
      I=LM(L)
      D(I)=D(I)+VOL(N)
      DO 175 M=1,4
      J=LM(M)-I+1
      IF (27-J) 157,158,158
  157 WRITE (6,2002) N
      GO TO 200
  158 IF(MBAND-J) 160,165,165
  160 MBAND=J
  165 IF(J) 175,175,170
  170 A(I,J)=A(I,J)+S(L,M)
  175 CONTINUE
C
  200 CONTINUE
      IF(ISTOP.EQ.1) STOP
C***********************************************************************
C     BOUNDARY CONDITIONS
```

```
C***********************************************************************
      IF(NCBH.EQ.0) GO TO 220
      DO 215 N=1,NCBH
      I=IC(N)
      J=JC(N)
      XL=SQRT((X(J)-X(I))**2+(Y(J)-Y(I))**2)
      H=HC(N)*XL*0.25
      A(I,1)=A(I,1)+H
      A(J,1)=A(J,1)+H
      K=J-I+1
      IF (K) 212,212,210
  210 A(I,K)=A(I,K)+H
      GO TO 215
  212 K=I-J+1
      A(J,K)=A(J,K)+H
  215 CL(N)=XL
  220 CONTINUE
C
C     COOLING PIPES
C
      IF(NCPH.EQ.0) GO TO 225
      DO 224 N=1,NCPH
      I=IP(N)
      A(I,1)=A(I,1)+HP(N)
  224 B(I)=B(I)+HP(N)*TP(N)
  225 CONTINUE
C
C     2.  TEMPERATURE BOUNDARY CONDITIONS
C
      DO 300 N=1,NUMN
C
      IF(TT(N).EQ.0.0) GO TO 300
      DO 250 M=2,MBAND
      K=N-M+1
      IF(K) 235,235,230
  230 B(K)=B(K)-A(K,M)*TT(N)
      A(K,M)=0.0
  235 L=N+M-1
      IF(NUMN-L) 245,240,240
  240 B(L)=B(L)-A(N,M)*TT(N)
  245 A(N,M)=0.0
  250 CONTINUE
      A(N,1)=1.0
      T(N)=TT(N)
  300 CONTINUE
C
C***********************************************************************
C     SOLVE FOR NODAL POINT TEMPERATURES
C***********************************************************************
C     FORM EFFECTIVE CONDUCTIVITY MATRIX FOR TIME INCREMENT
C
      DT2=1.0/DT
      DO 320 N=1,NUMN
      IF(A(N,1).EQ.0.0) A(N,1)=1.0
      IF(TT(N).NE.0.0) GO TO 320
```

```
      D(N)=DT2*D(N)
      A(N,1)=A(N,1)+D(N)
  320 CONTINUE
      CALL SYMSOL(1)
C
C     CALCULATE TEMPERATURE AT THE END OF EACH TIME INCREMENT
C
      LL=0
C
      DO 600 KK=1,NDT
C
C     DETERMINATION OF HEAT GENERATION
C
      DO 395 N=1,NUME
      IF(PLTM(N).GT.TIME) GO TO 395
      MTYPE=IX(N,5)
      TX=TIME-PLTM(N)
      IF(TX.GE.HSTOP(MTYPE)) GO TO 395
      DO 385 L=1,NUMQC
      XZ=QX(L,1,MTYPE)-TX
      IF(XZ.GT.0.0) GO TO 386
  385 CONTINUE
  386 DIFF=QX(L,1,MTYPE)-QX(L-1,1,MTYPE)
      GRAD=(QX(L,2,MTYPE)-QX(L-1,2,MTYPE))/DIFF
      QQ=GRAD*VOL(N)
      DO 390 I=1,4
      II=IX(N,I)
  390 Q(II)=Q(II)+QQ
  395 CONTINUE
C
C     CONVECTION BOUNDARY CONDITION
C
      IF(NCBH.EQ.0) GO TO 410
      DO 400 N=1,NUMET
      XZ=ET(N,1)-TIME
      IF(XZ.GT.0.0) GO TO 401
  400 CONTINUE
  401 DIFF=ET(N,1)-ET(N-1,1)
      TEMP=ET(N,2)-((ET(N,2)-ET(N-1,2))*XZ)/DIFF
      DO 405 M=1,NCBH
      I=IC(M)
      J=JC(M)
      XZ=HC(M)*CL(M)*TEMP*0.5
      Q(I)=Q(I)+XZ
  405 Q(J)=Q(J)+XZ
  410 CONTINUE
C
C       1. CALCULATE EFFECTIVE LOAD MATRIX
C
      DO 450 I=1,NUMN
      Q(I)=Q(I)+B(I)+D(I)*T(I)
      IF(TT(I).NE.0.0) Q(I)=TT(I)
  450 CONTINUE
C
C       2. SOLVE FOR TEMPERATURES
```

```
C
      CALL SYMSOL(2)
C
      DO 500 I=1,NUMN
      T(I)=Q(I)
  500 Q(I)=0.0
C
      TIME =TIME+DT
      LL=LL+1
      IF(LL-INTER) 600,550,550
  550 WRITE (6,2001) TIME,(N,T(N),N=1,NUMN)
      LL=0
C
  600 CONTINUE
      RETURN
C*********************************************************************
C
 2001 FORMAT (7H0TIME = F14.6/(I6,F14.6,I6,F14.6,I6,F14.6,I6,F14.6,
     1 I6,F14.6,I6,F14.6))
 2002 FORMAT (23H BAND TOO LARGE-EL.NO.    I4)
 2003 FORMAT(34H0ZERO OR NEGATIVE AREA  ELEMENT NO,I5)
C
      END
```

```
      SUBROUTINE SYMSOL (KKK)
C
      COMMON /SYMARG/ NN,MM,A(500,27),B(500)
C
      GO TO (1000,2000),KKK
C
C     REDUCE MATRIX
C
 1000 DO 280 N=1,NN
      DO 260 L=2,MM
      C=A(N,L)/A(N,1)
      I = N+L-1
      IF(NN-I) 260,240,240
  240 J=0
      DO 250 K=L,MM
      J=J+1
  250 A(I,J)=A(I,J)-C*A(N,K)
  260 A(N,L)=C
  280 CONTINUE
      GO TO 500
C
C     REDUCE VECTOR
C
 2000 DO 290 N=1,NN
      DO 285 L=2,MM
      I=N+L-1
      IF(NN-I) 290,285,285
  285 B(I)=B(I)-A(N,L)*B(N)
  290 B(N)=B(N)/A(N,1)
C
C     BACK SUBSTITUTION
C
      N=NN
  300 N = N-1
      IF(N) 350,500,350
  350 DO 400 K=2,MM
      L = N+K-1
      IF(NN-L) 400,370,370
  370 B(N) = B(N) - A(N,K) * B(L)
  400 CONTINUE
      GO TO 300
C
  500 RETURN
C
      END
```