SESM 68-8a

STRUCTURES AND MATERIALS RESEARCH
DEPARTMENT OF CIVIL ENGINEERING

STEEL TUBES AND EXPANSIVE CEMENT CONCRETE, COMPOSITE COLUMNS FIGURES AND APPENDICES

by

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Problem Number	Type of Concrete Core	Load is applied at the end of	Type of loading platens at the ends
1	Solid	Concrete and steel	FRICTION
2	Solid	Concrete core only	Frictionless
3	Solid	Concrete core only	FRICTION
4	Hollow	Concrete core only	Frictionless
5	Hollow	Concrete core only	FRICTION

TABLE 2.1

Problem Number	m (Power in the shear law)	Concrete axial stress				Steel Stress			
		Top		Centre		Axial		Tang	
		Min.	Max.	Min.	Max.	Top	Center	Top	Center
1	-	- 7.03	-5.5	-6.61	-6.61	-57.3	-51.4	-17.0	-4.2
2	4	-17.1	-8.8	-5.73	-5.73	-0.81	-44.5	+8.1	-3.6
2	8	-24.37	-8.39	-6.09	-6.08	-0.51	-47.4	+6.1	-3.9
3	4	-20.6	-8.11	-6.1	-6.1	-0.36	-44.8	+0.35	-3.68
3	8	-26.08	-7.79	-6.09	-6.09	-0.7	-47.6	+0.12	-3.9
4	4	-17.7	-9.14	-5.53	-5.51	-0.4	-49.9	+7.6	-4.9
4	8	-25.1	-8.89	-5.95	-5.93	-0.86	-53.7	+5.3	-5.27
5	4	-21.07	-8.02	-5.16	-5.10	-1.3	-65.7	+0.42	-8.81
5	8	-25.7	-7.46	-5.82	-5.73	-2.6	-65.6	-0.21	-8.1

TABLE 2.2

Specimen Number	Type of Specimen	Dimensions				
			Steel		Concrete	
		(Height) Length in.	Outside Diameter in.	Thickness in.	Outside Diameter in.	Inside Diameter in.
1	Short tube A-Top	23.1	4"	0.1	3.8	0.5
2	Short tube A-Bottom	22.7	4"	0.1	3.8	0.5
3	Short tube B-Top	23.2	4"	0.1	3.8	0.5
4	Short tube B-Bottom	22.6	4"	0.1	3.8	0.5
5	Long tube A	94"	4"	0.1	3.8	0.75
6	Long tube B	94"	4"	0.1	3.8	0.75
7	Long tube C	94"	4"	0.1	3.8	0.75

TABLE 6.1

Specimen Number	Actual Buckling Load kips	Southwell's Method		Longquest Method critical load kips
		Critical Load kips	Eccentricity in.	
5	160	240	0.106	212
6	235	290	0.052	263
7	230	340	0.115	315

TABLE 6.2

Investigation Number	Type of Specimen	Specimen Number	Properties of steel tube			Proportional Limit of Composite Element			Ultimate load on composite element kips
			Outside diameter in.	Thick. in.	Yield strength ksi	Axial Strain $\times 10^6$	Circumf. Strain $\times 10^6$	Load kips	
I	steel tube + ordinary concrete	Average of 1 & 2	4	0.125	50	-750	450	82	290
	steel tube + expansive cement concrete	Average of 3 & 4	4	0.125	50	-1250	650	130	305
II	steel tube + prestressing wire + Expansive cement concrete	1	4	0.1	100	-3800	700	260	503
		2	4	0.1	100	-3900	900	255	508
		3	4	0.1	100	-4000	900	270	500
		4	4	0.1	100	-3700	800	260	500

TABLE 6.3

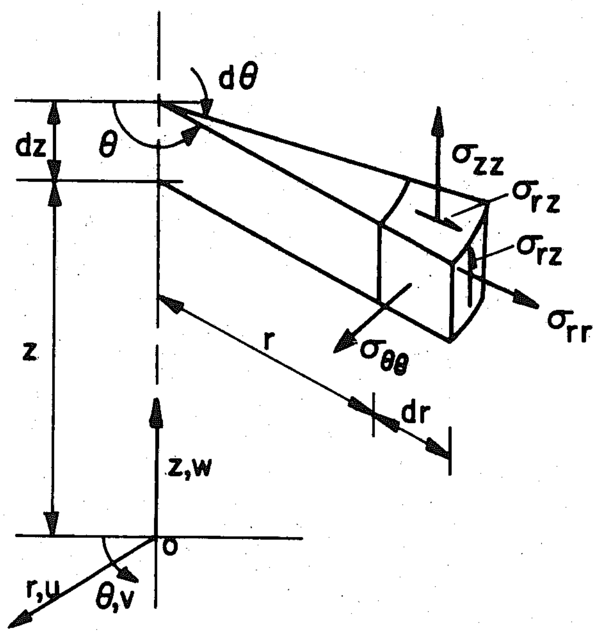


FIG. 2.1 SYSTEM OF COORDINATES

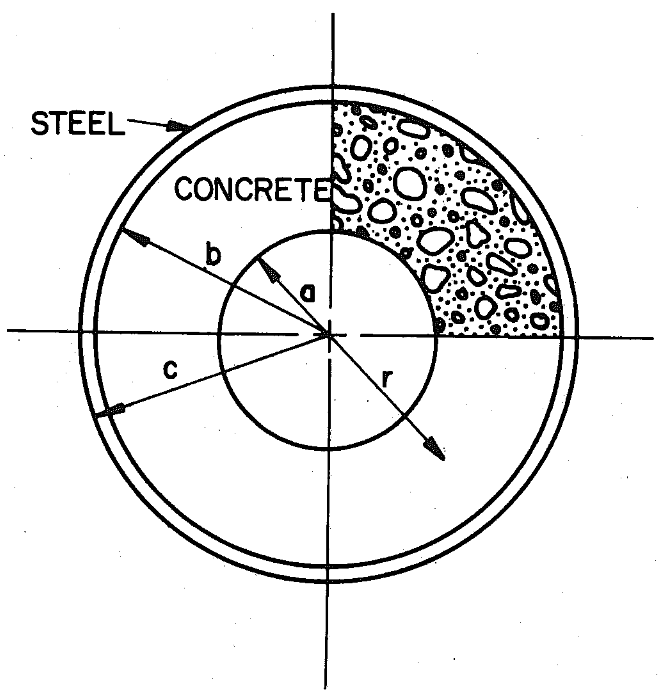


FIG. 2.2 CROSS - SECTION OF THE COMPOSITE COLUMN

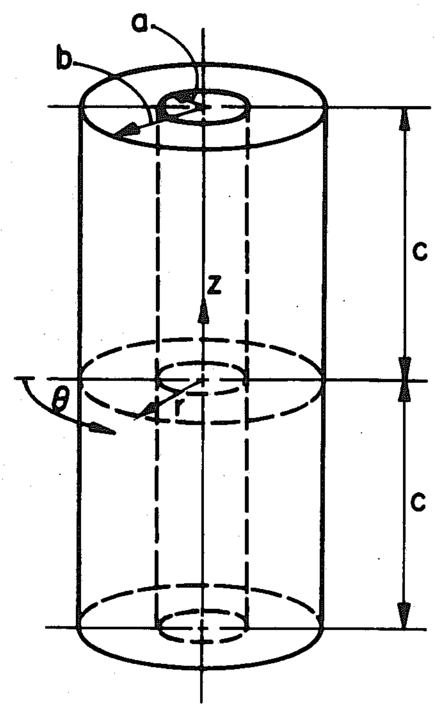


FIG. 2.3 SYSTEM OF COORDINATES AND CYLINDER

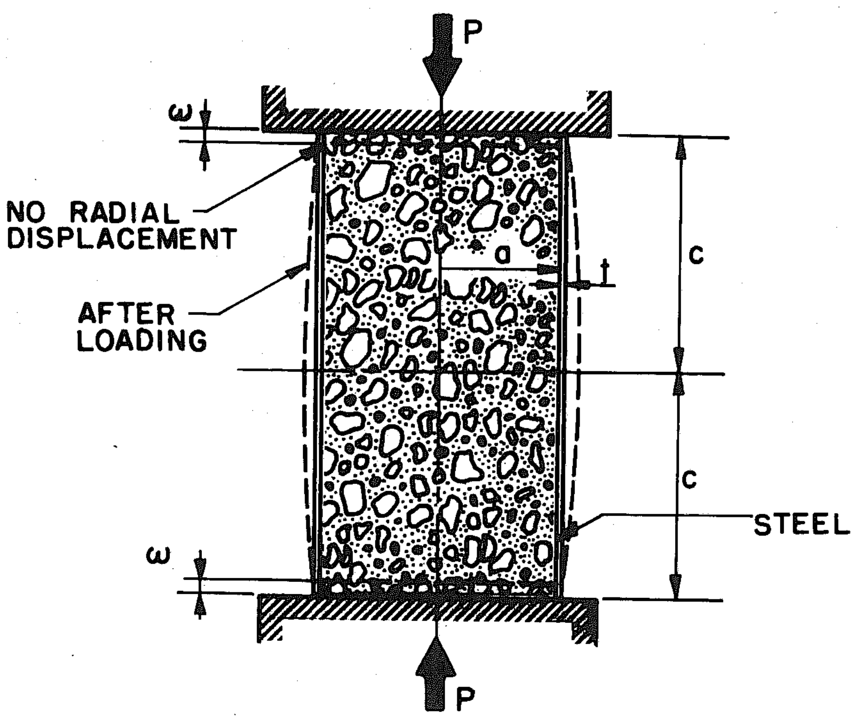
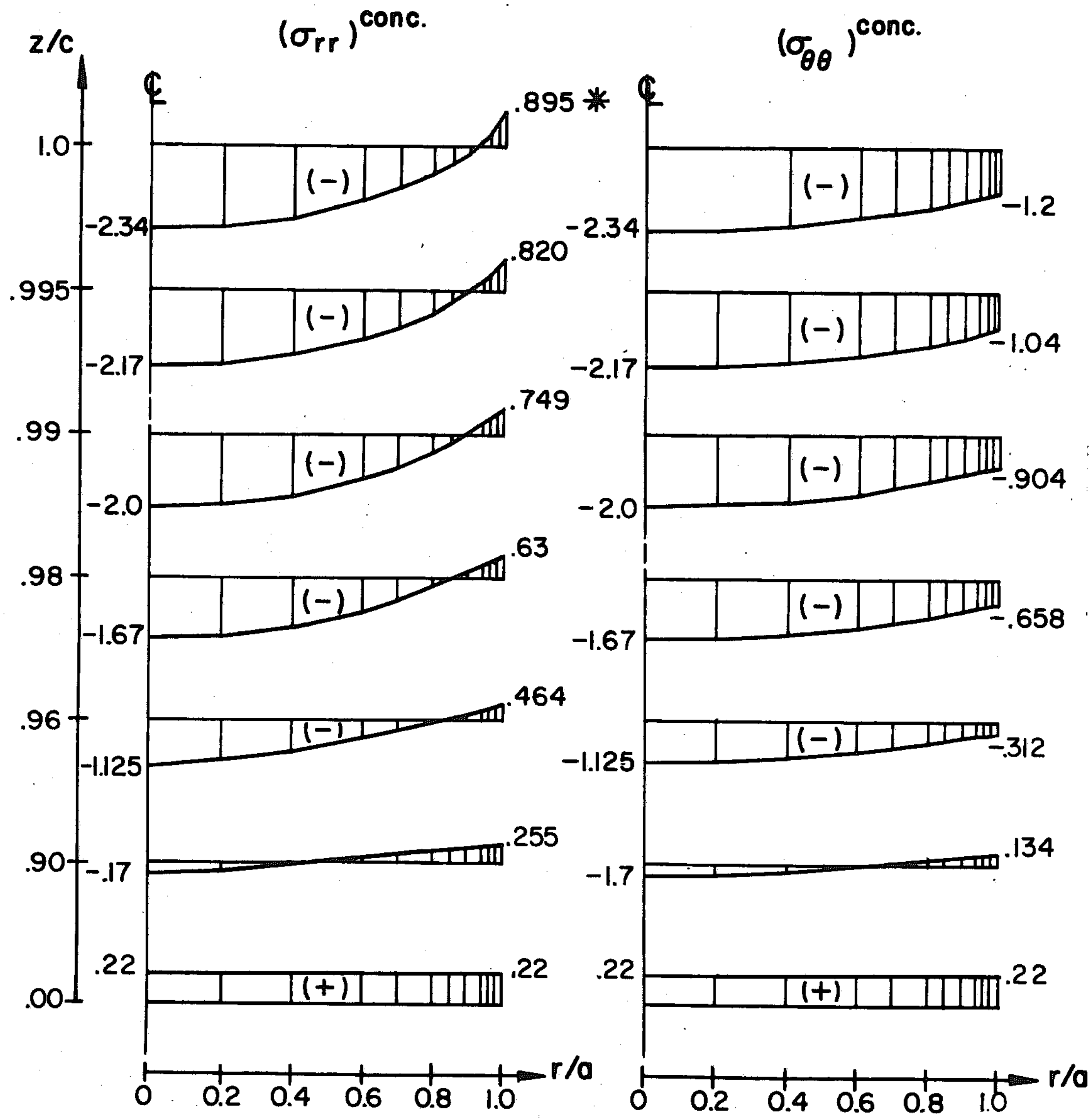


FIG. 2.4 PROBLEM NO. I, METHOD OF LOADING THE COMPOSITE ELEMENT



* NOTE: ALL STRESSES IN ksi

FIG. 2.5 PROBLEM I, EXACT SOLUTION — RADIAL AND TANGENTIAL STRESS DISTRIBUTIONS IN THE CONCRETE CORE

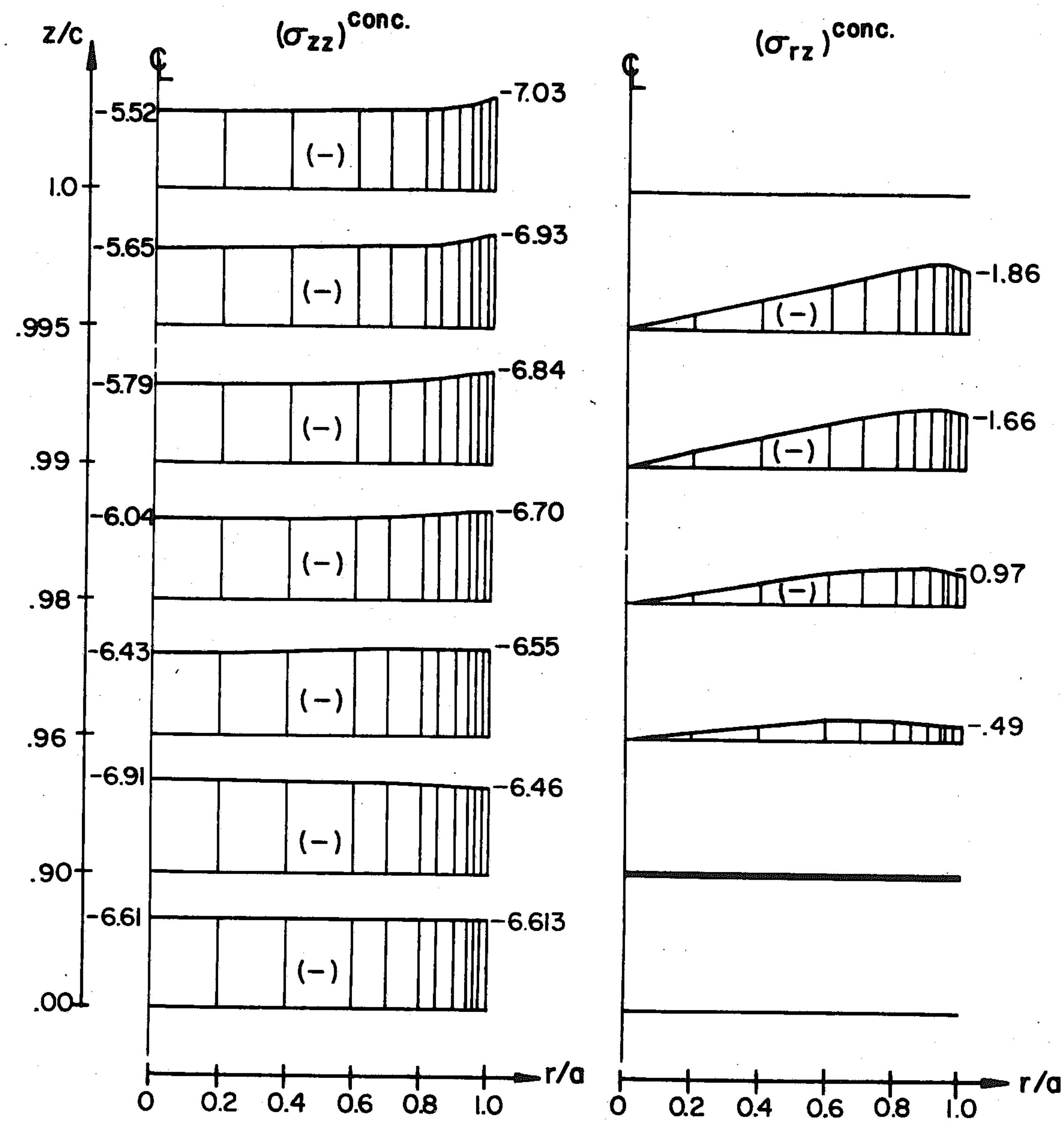


FIG. 2.6 PROBLEM I, EXACT SOLUTION — AXIAL AND SHEAR STRESS DISTRIBUTIONS IN THE CONCRETE CORE

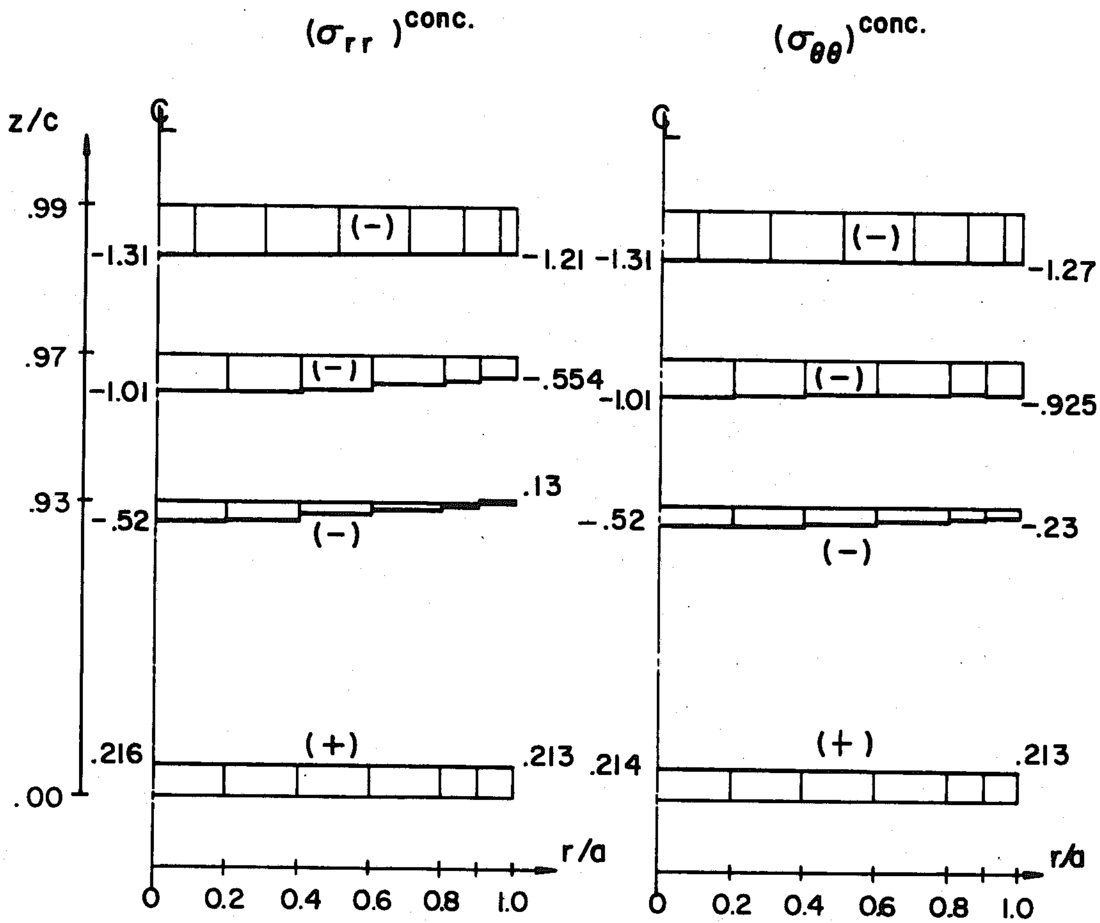


FIG. 2.7 PROBLEM I – FINITE ELEMENT METHOD. RADIAL AND TANGENTIAL STRESS DISTRIBUTIONS IN THE CONCRETE CORE

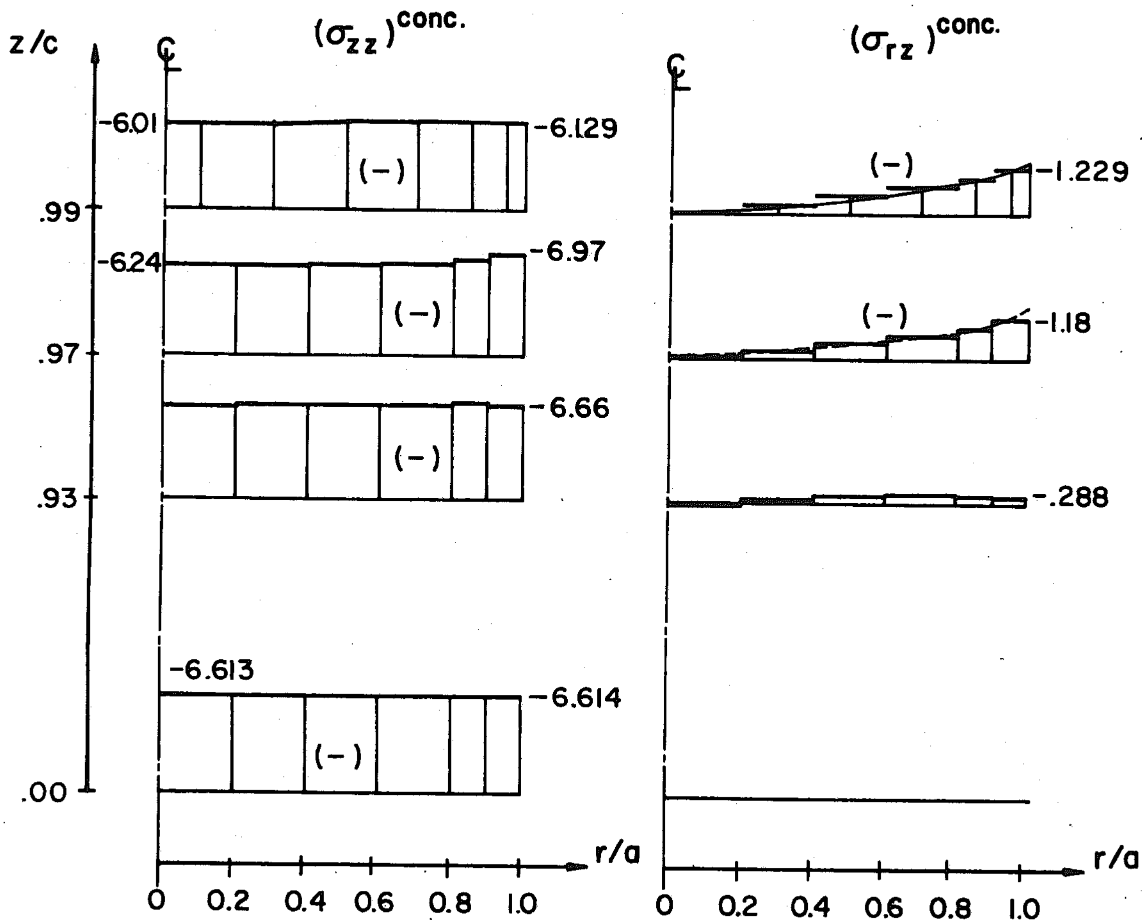


FIG. 2.8 PROBLEM I – FINITE ELEMENT METHOD. AXIAL AND SHEAR STRESS DISTRIBUTIONS IN THE CONCRETE CORE

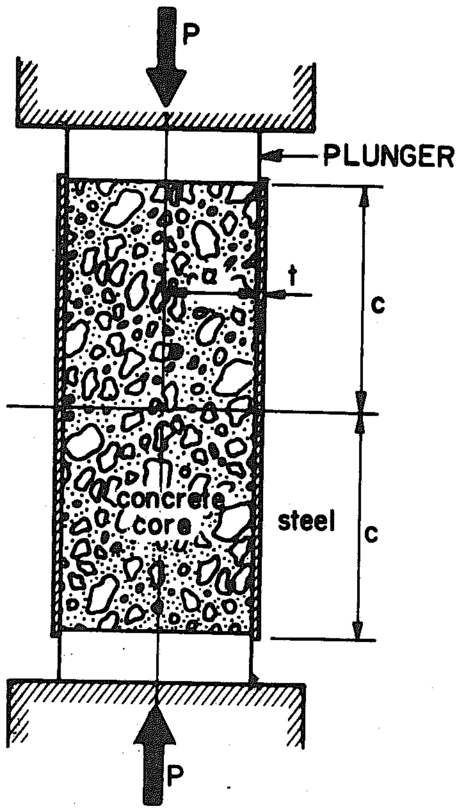


FIG. 2.9 PROBLEM 2 — METHOD OF LOADING CONCRETE CORE ALONE

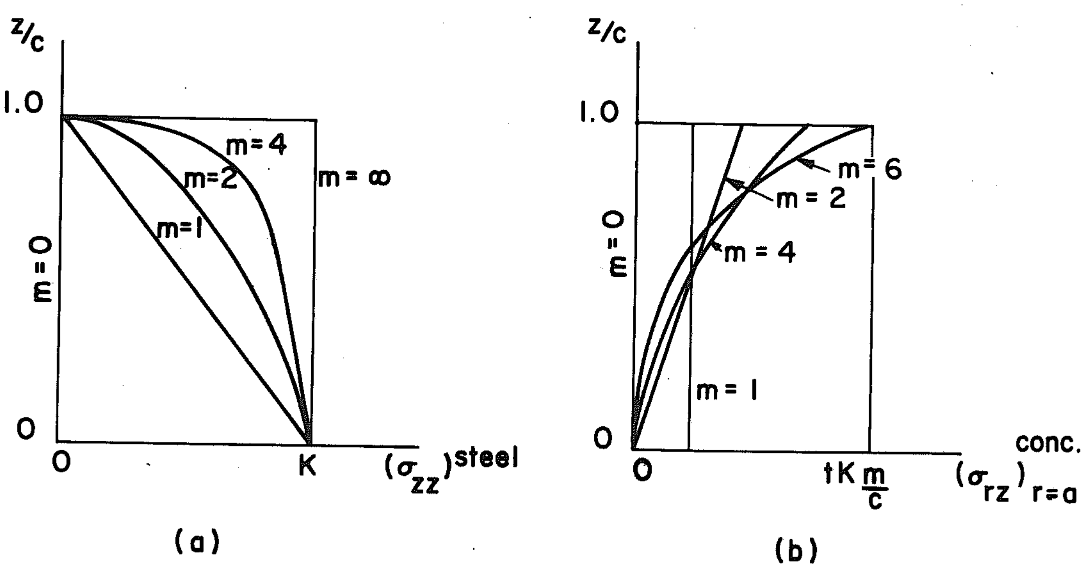


FIG. 2.10 a — AXIAL STRESS DISTRIBUTION IN THE STEEL

b — SHEAR STRESS DISTRIBUTION IN THE CONCRETE AT r = a

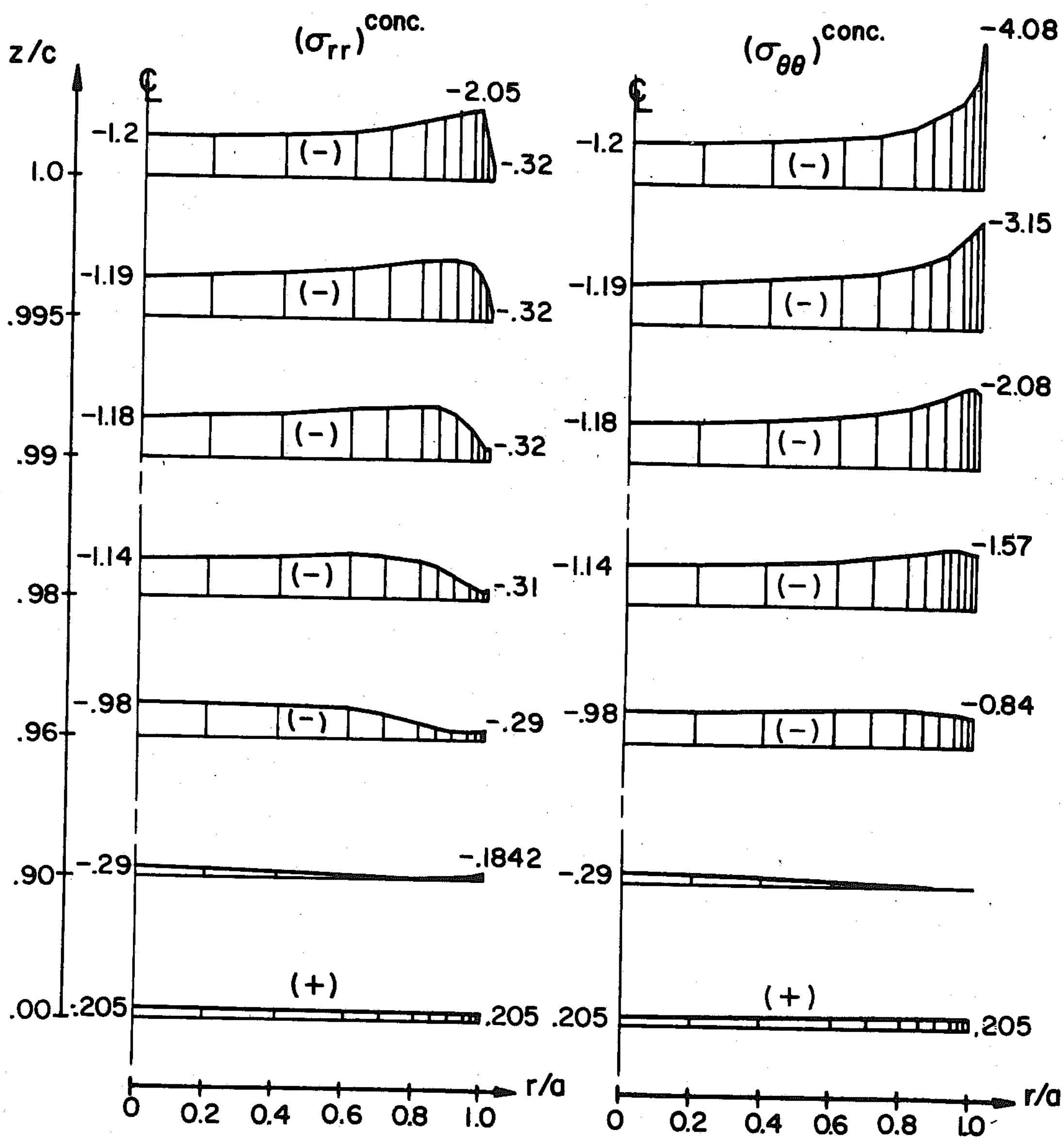


FIG. 2.11 PROBLEM 2, m = 8 – RADIAL AND TANGENTIAL STRESS DISTRIBUTIONS IN THE CONCRETE CORE

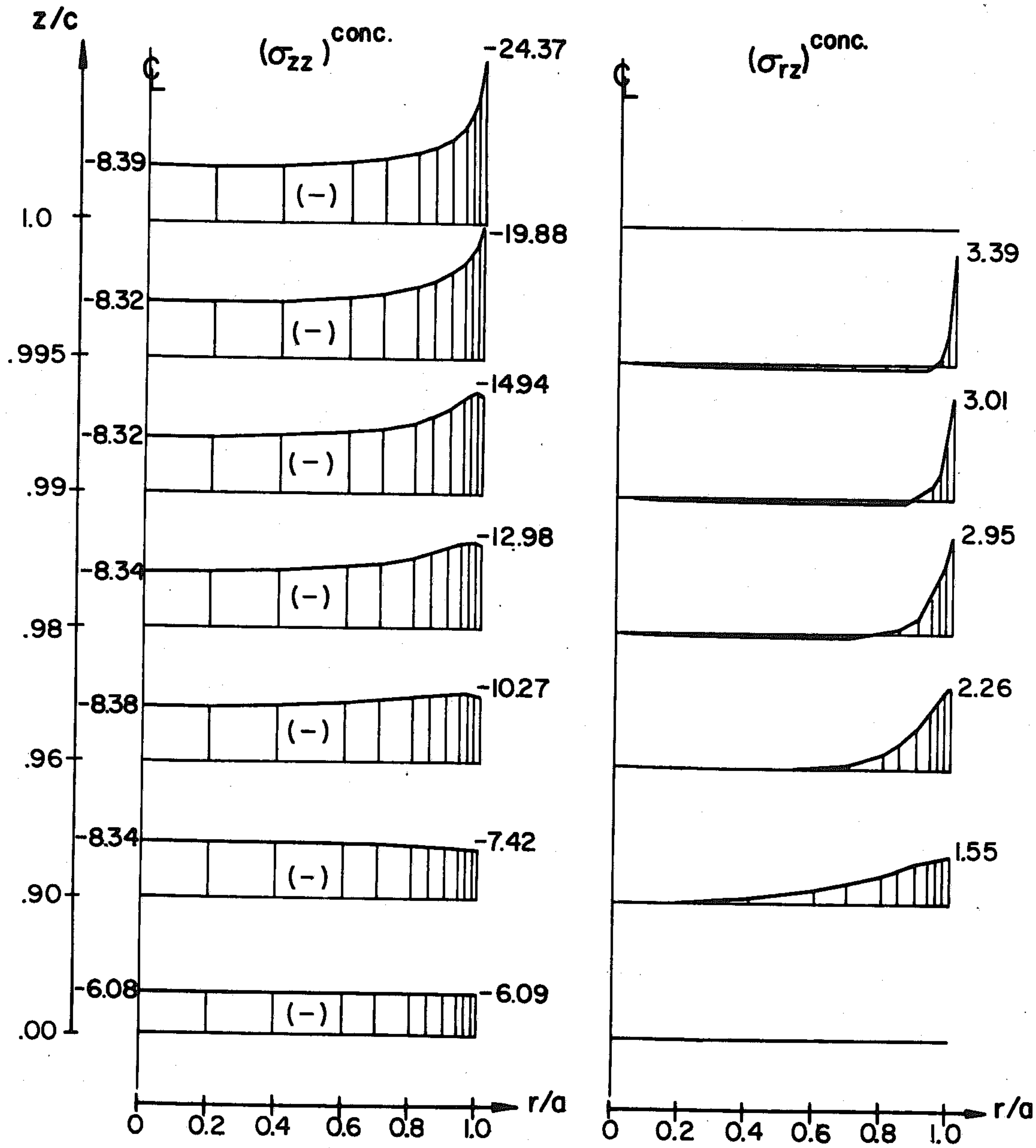


FIG. 2.12 PROBLEM 2, m = 8 — AXIAL AND SHEAR STRESS DISTRIBUTIONS IN THE CONCRETE CORE

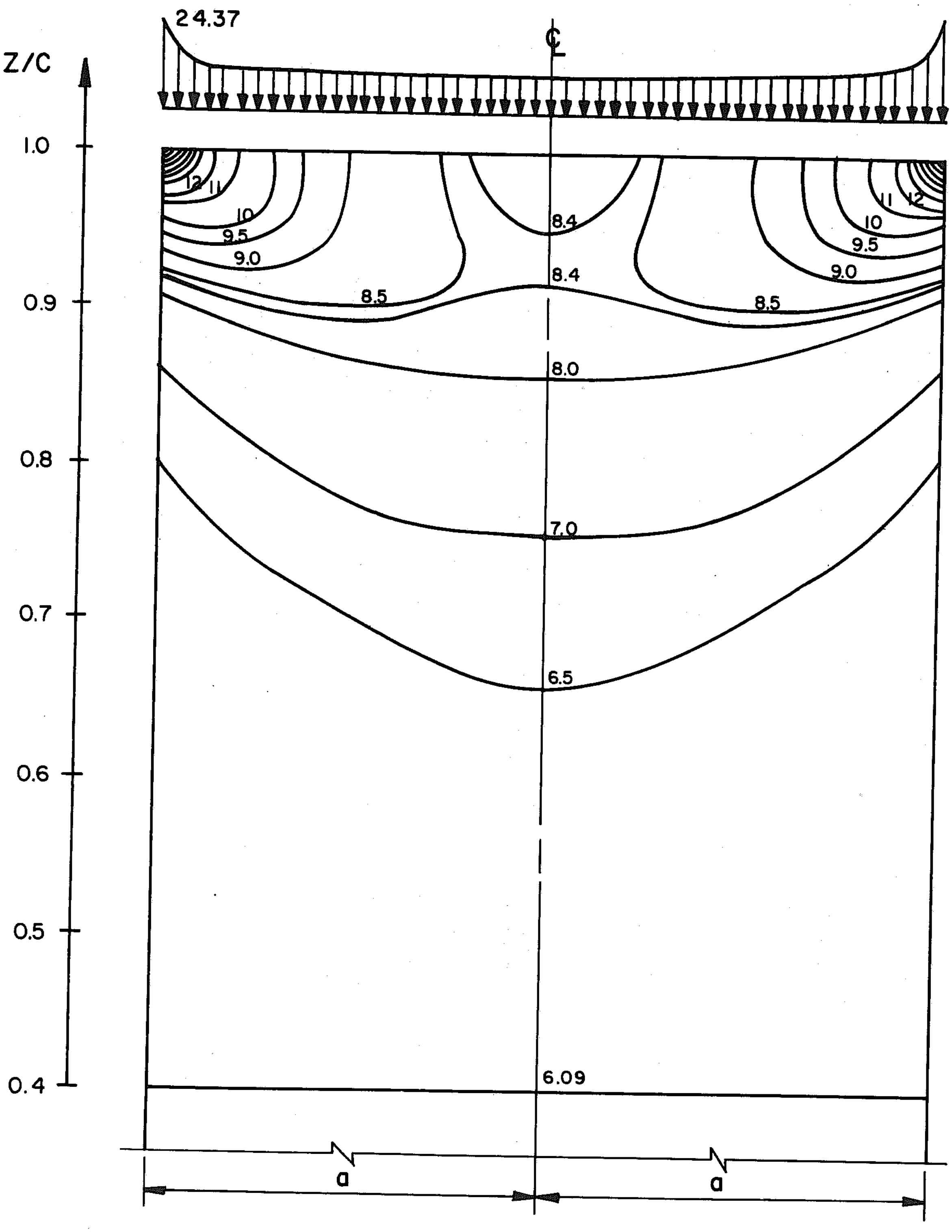


FIG. 2.13 PROBLEM 2, m = 8 - COUNTOURS OF THE AXIAL STRESS IN CONCRETE

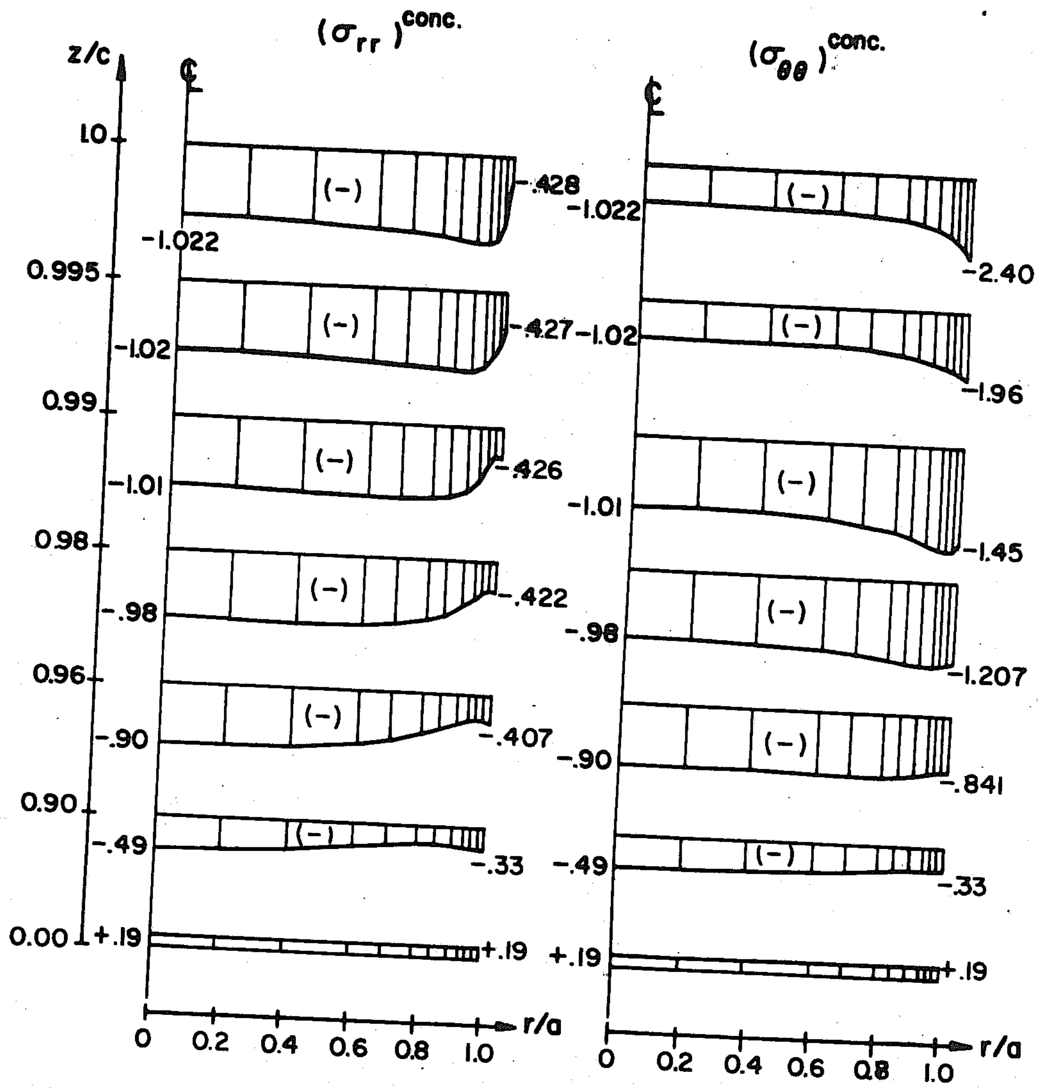


FIG. 2.14 PROBLEM 2, m = 4 — RADIAL AND TANGENTIAL STRESS DISTRIBUTIONS IN THE CONCRETE CORE

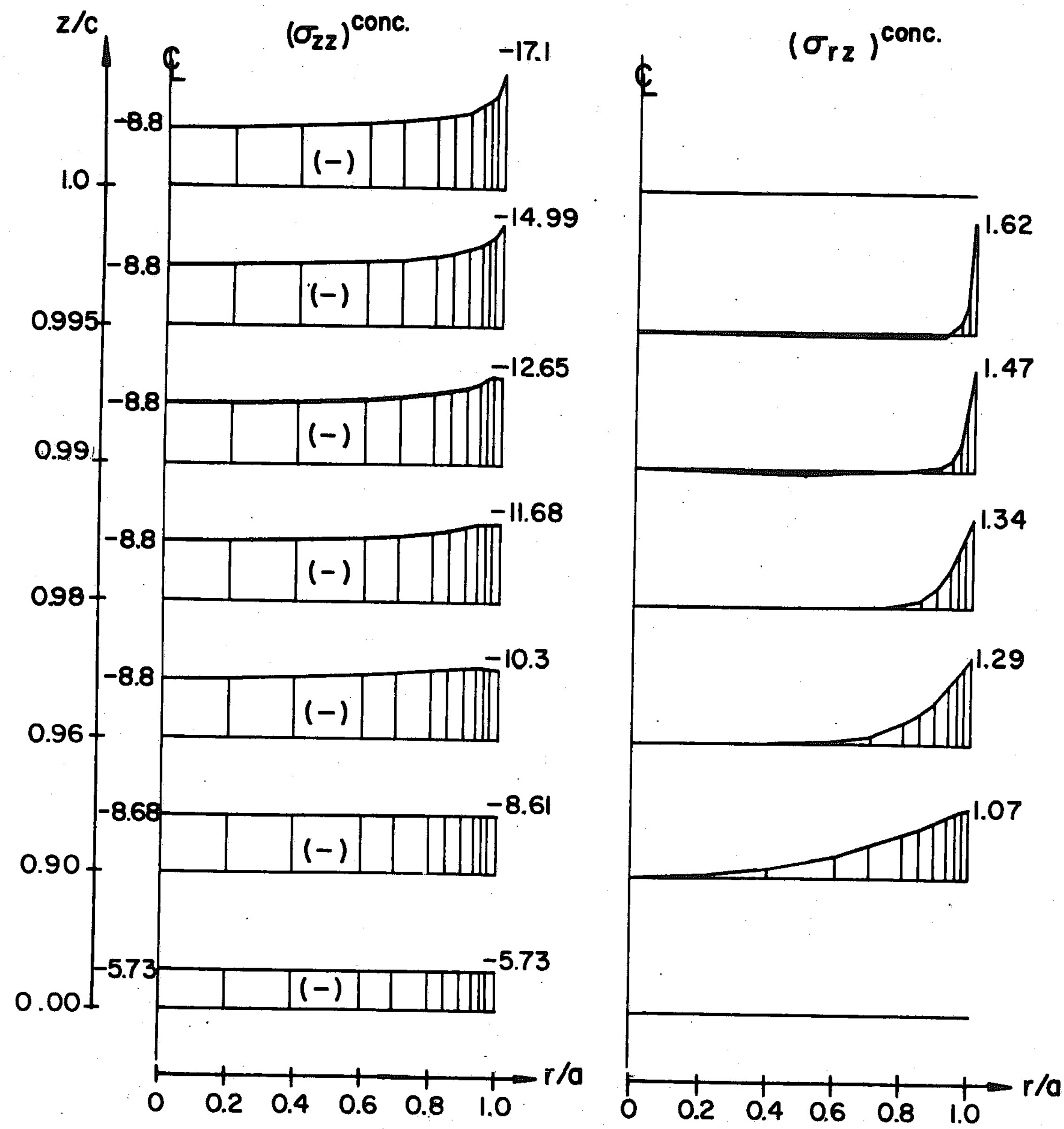


FIG. 2.15 PROBLEM 2, m = 4 — AXIAL AND SHEAR STRESS DISTRIBUTIONS IN THE CONCRETE CORE

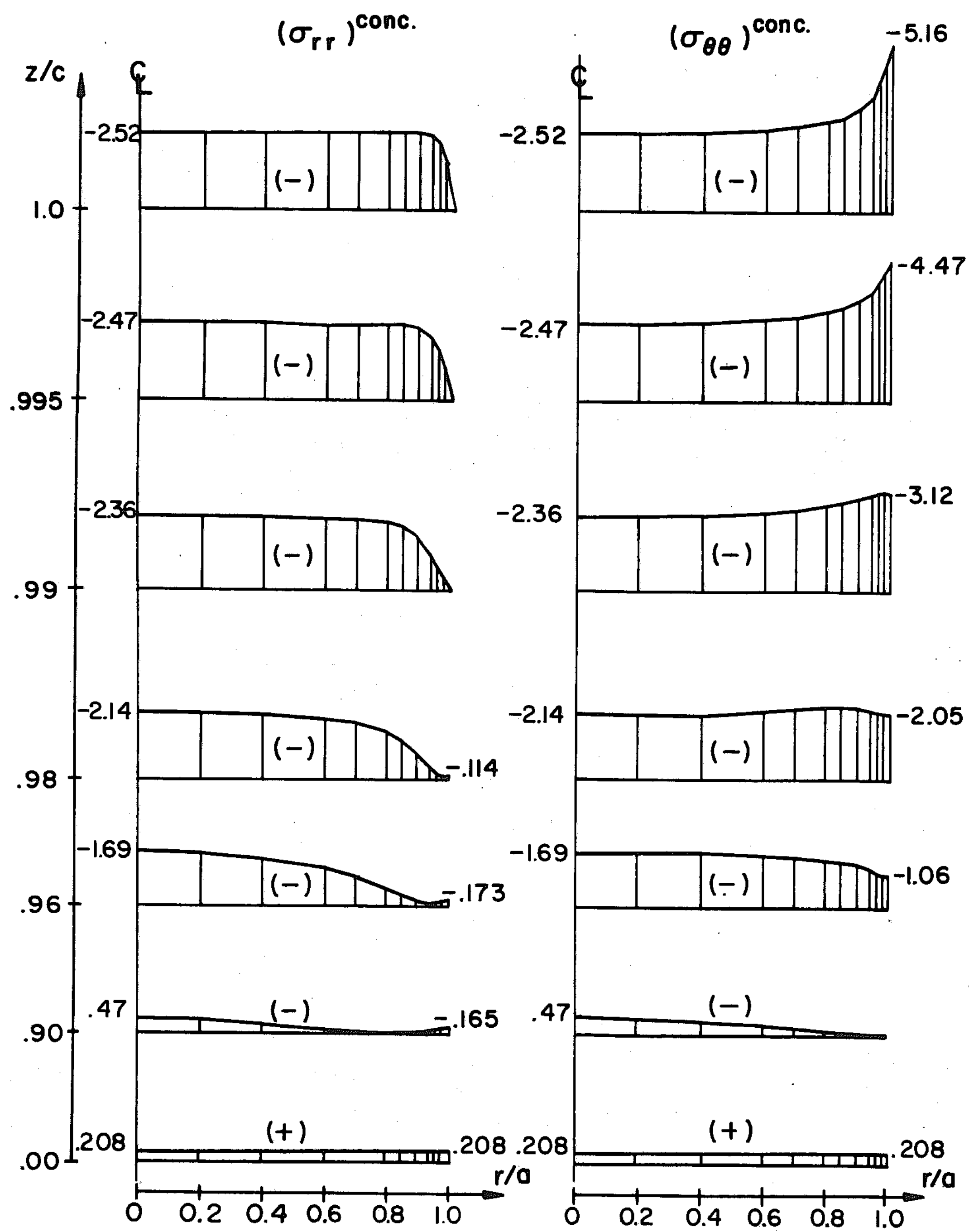


FIG. 2.16 PROBLEM 3, m = 8 — RADIAL AND TANGENTIAL STRESS DISTRIBUTIONS IN THE CONCRETE CORE

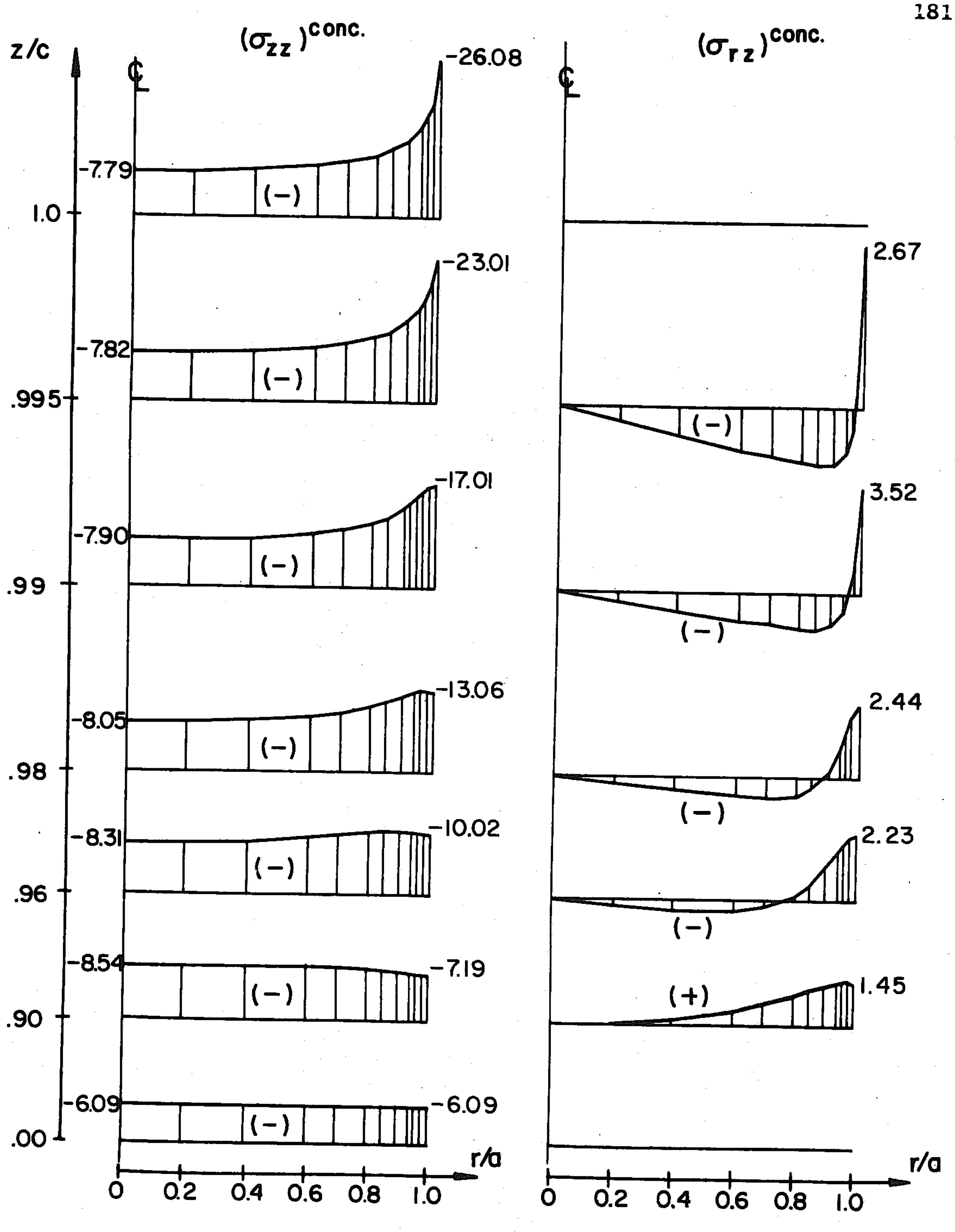


FIG. 2.17 PROBLEM 3, m = 8 — AXIAL AND SHEAR STRESS DISTRIBUTIONS IN THE CONCRETE CORE

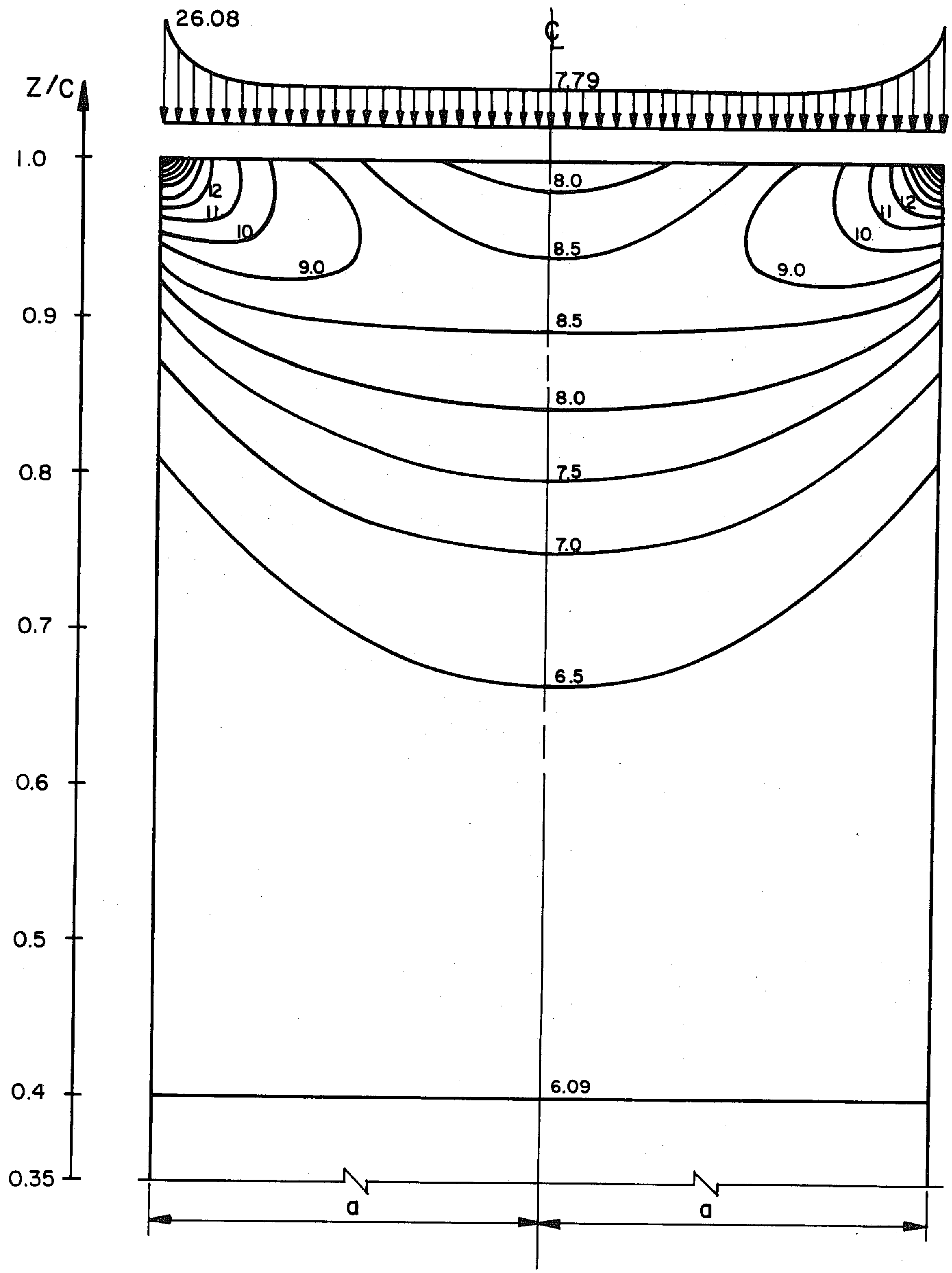


FIG. 2.18 PROBLEM 3, m = 8 — COUNTOURS OF THE AXIAL STRESS IN CONCRETE

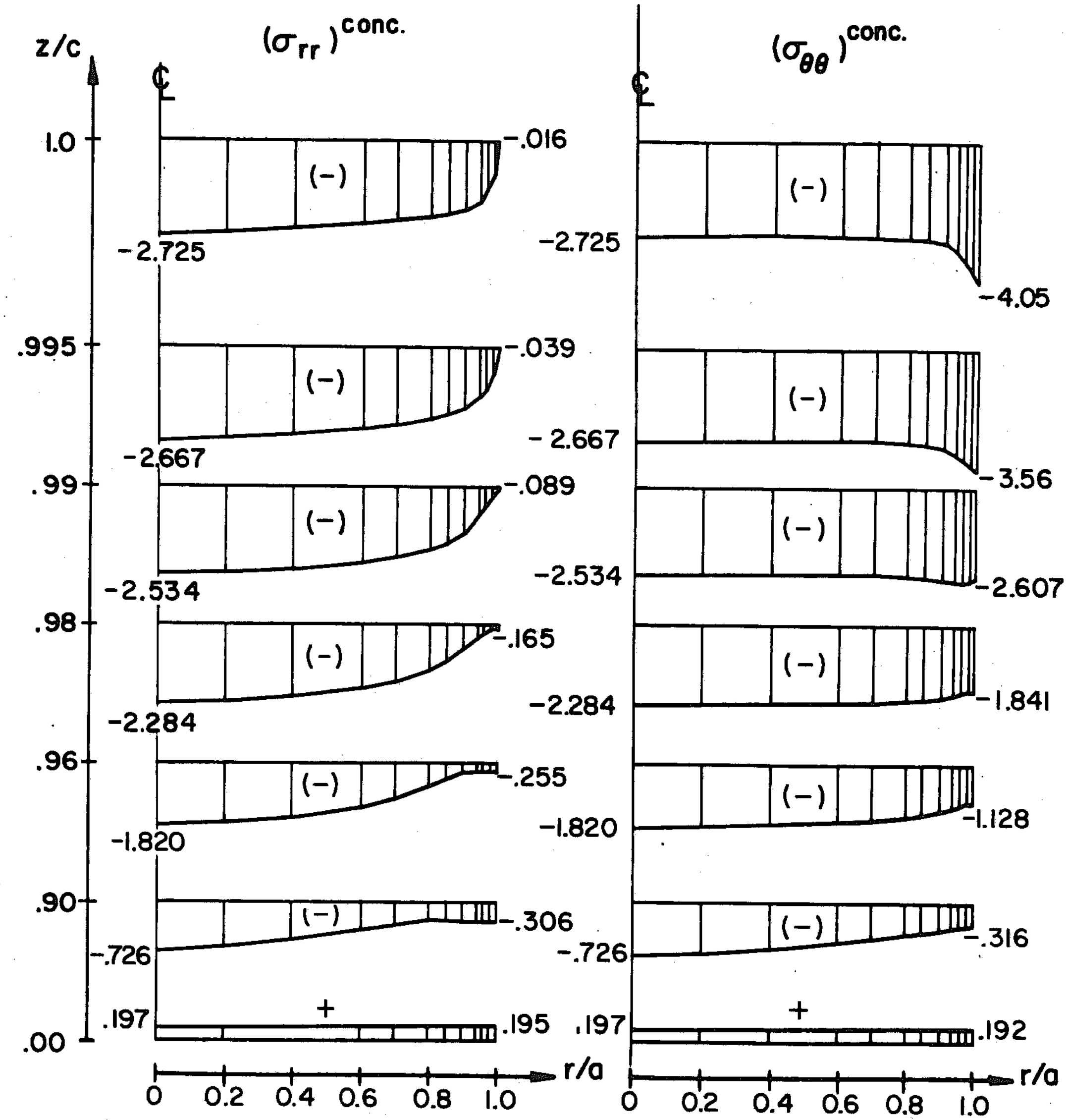


FIG. 2.19 PROBLEM 3, m = 4 — RADIAL AND TANGENTIAL STRESS DISTRIBUTIONS IN THE CONCRETE CORE

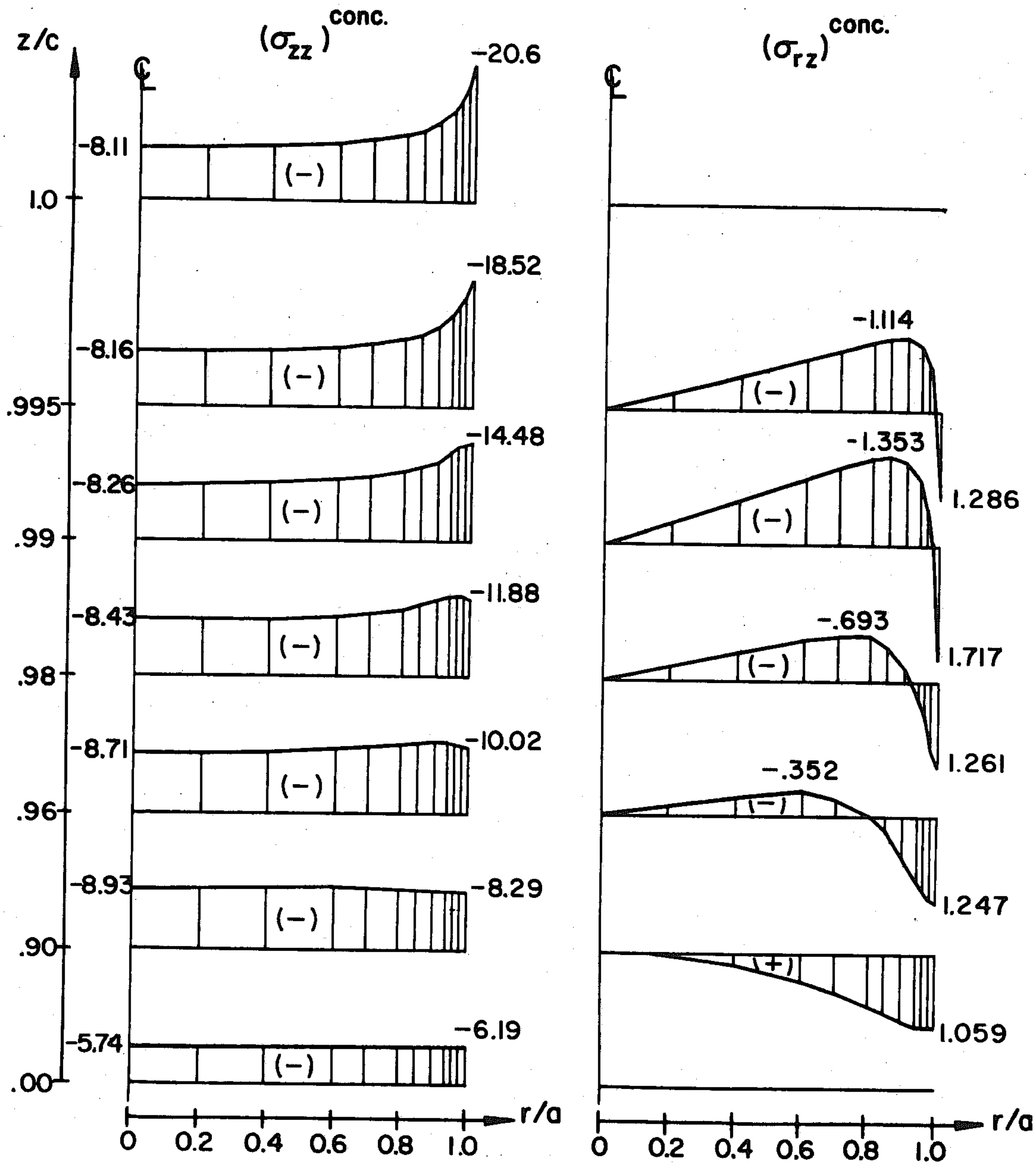


FIG. 2.20 PROBLEM 3, m = 4 — AXIAL AND SHEAR STRESS DISTRIBUTIONS IN THE CONCRETE CORE

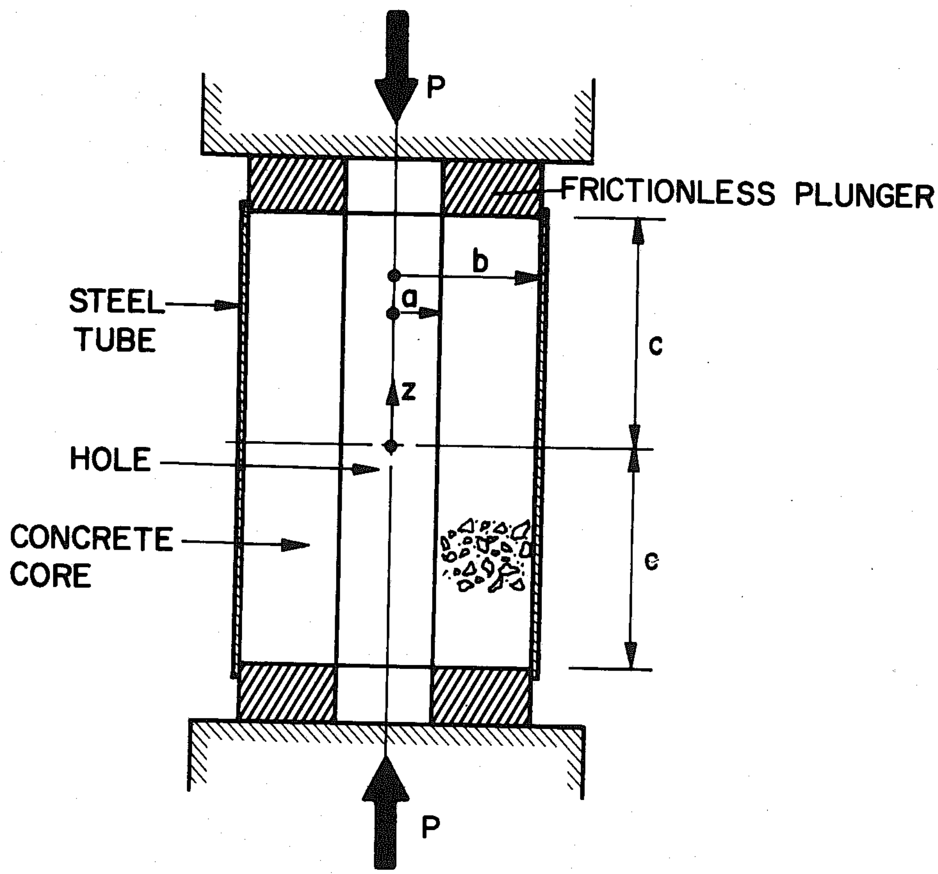


FIG. 2.21 PROBLEM 4 , COMPOSITE ELEMENT UNDER LOAD

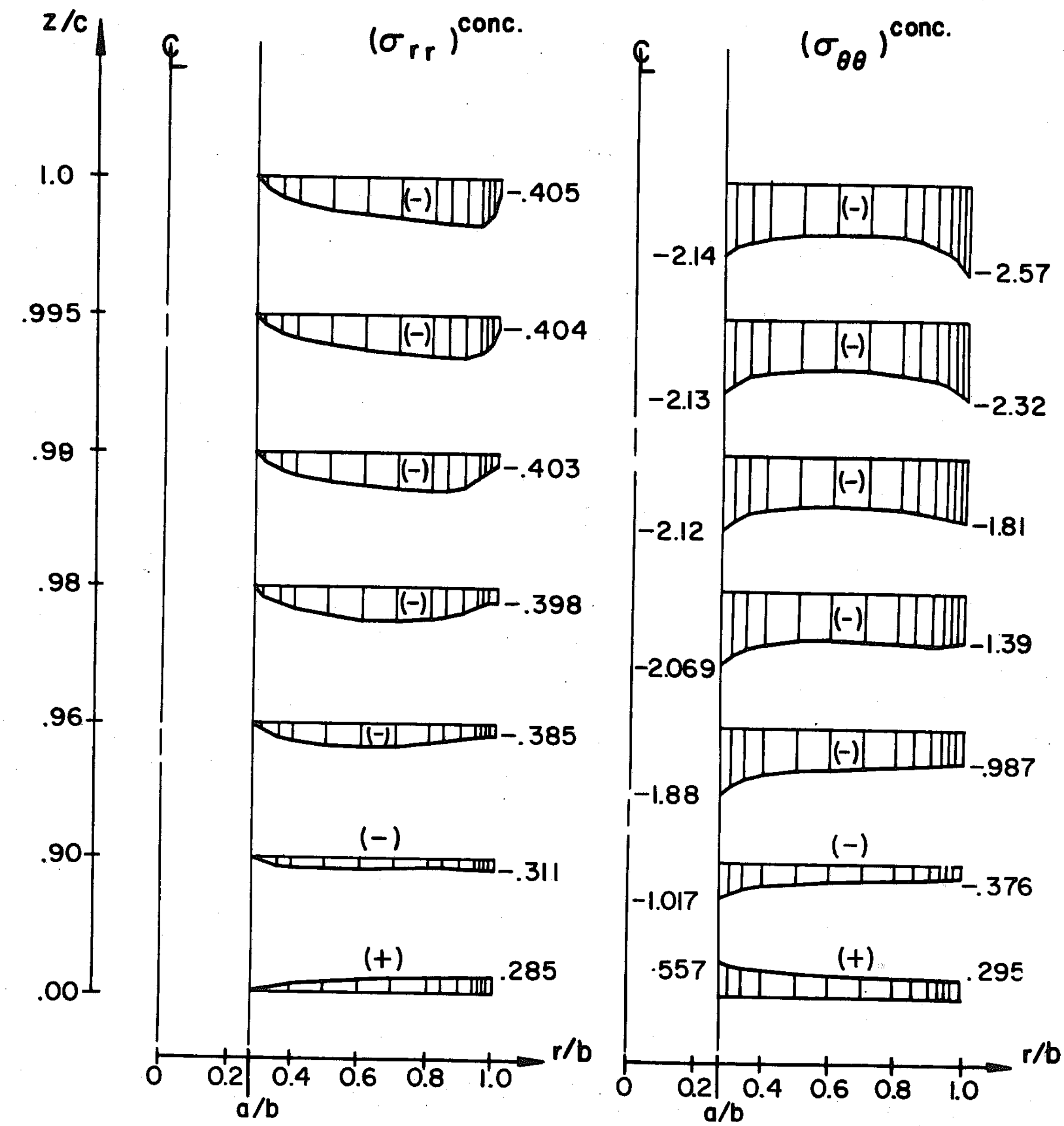


FIG. 2.22 PROBLEM 4, m = 4 — RADIAL AND TANGENTIAL STRESS DISTRIBUTIONS IN THE CONCRETE CORE (HOLLOW SECTION)

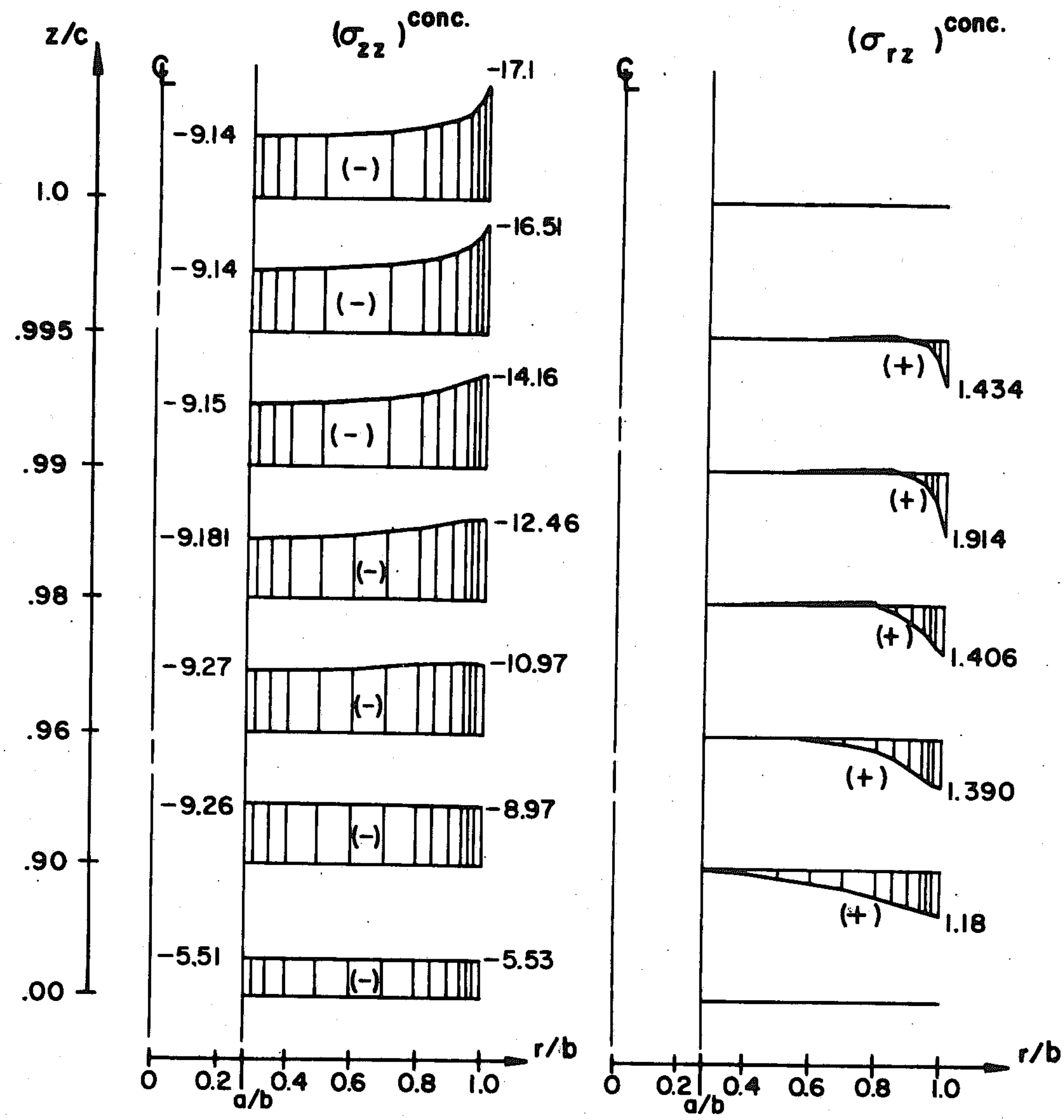


FIG. 2.23 PROBLEM 4, m = 4 — AXIAL AND SHEAR STRESS DISTRIBUTIONS IN THE CONCRETE CORE (HOLLOW SECTION)

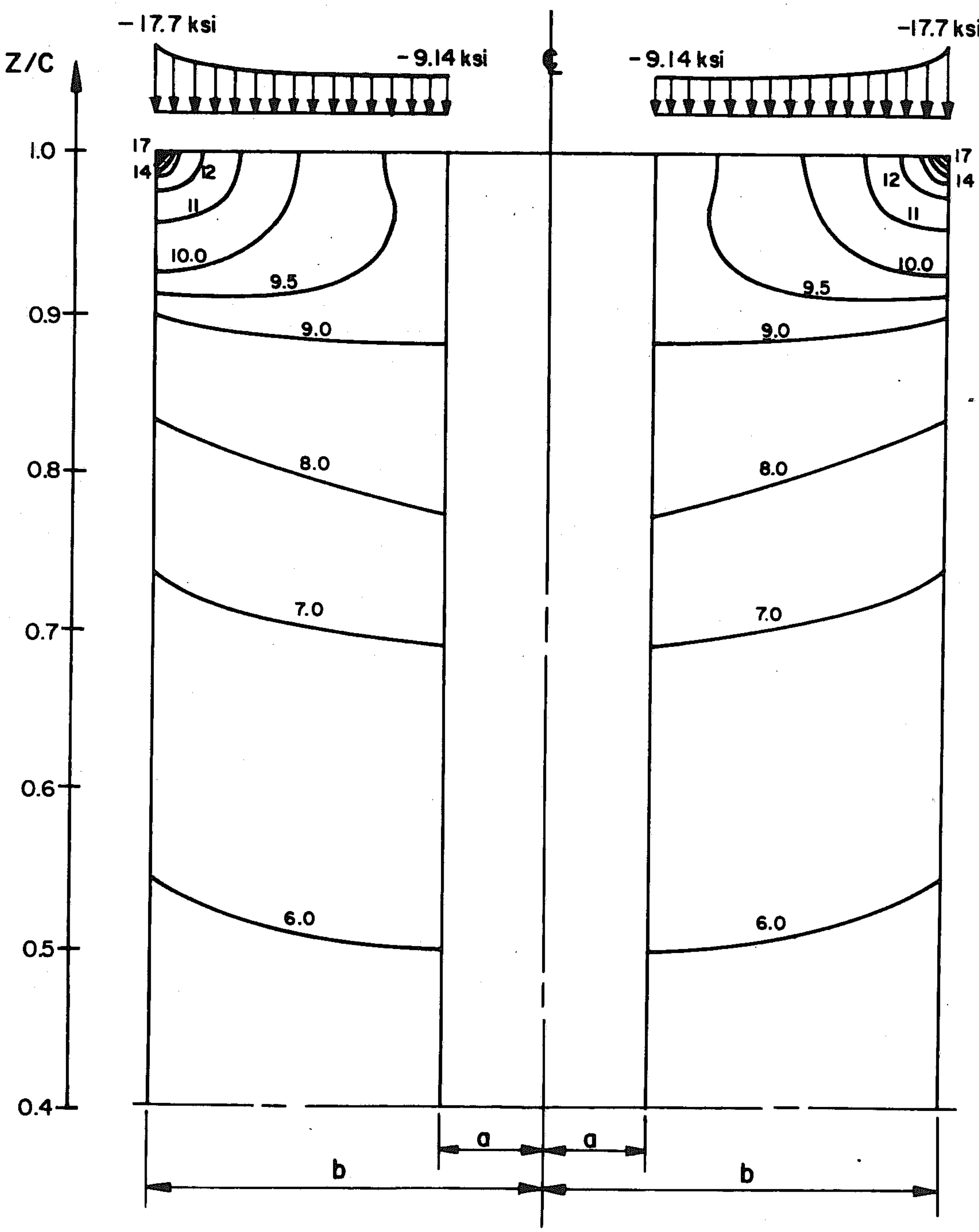


FIG. 2.24 PROBLEM 4, m = 4 – CONTOURS OF AXIAL STRESS IN THE CONCRETE (HOLLOW SECTION)

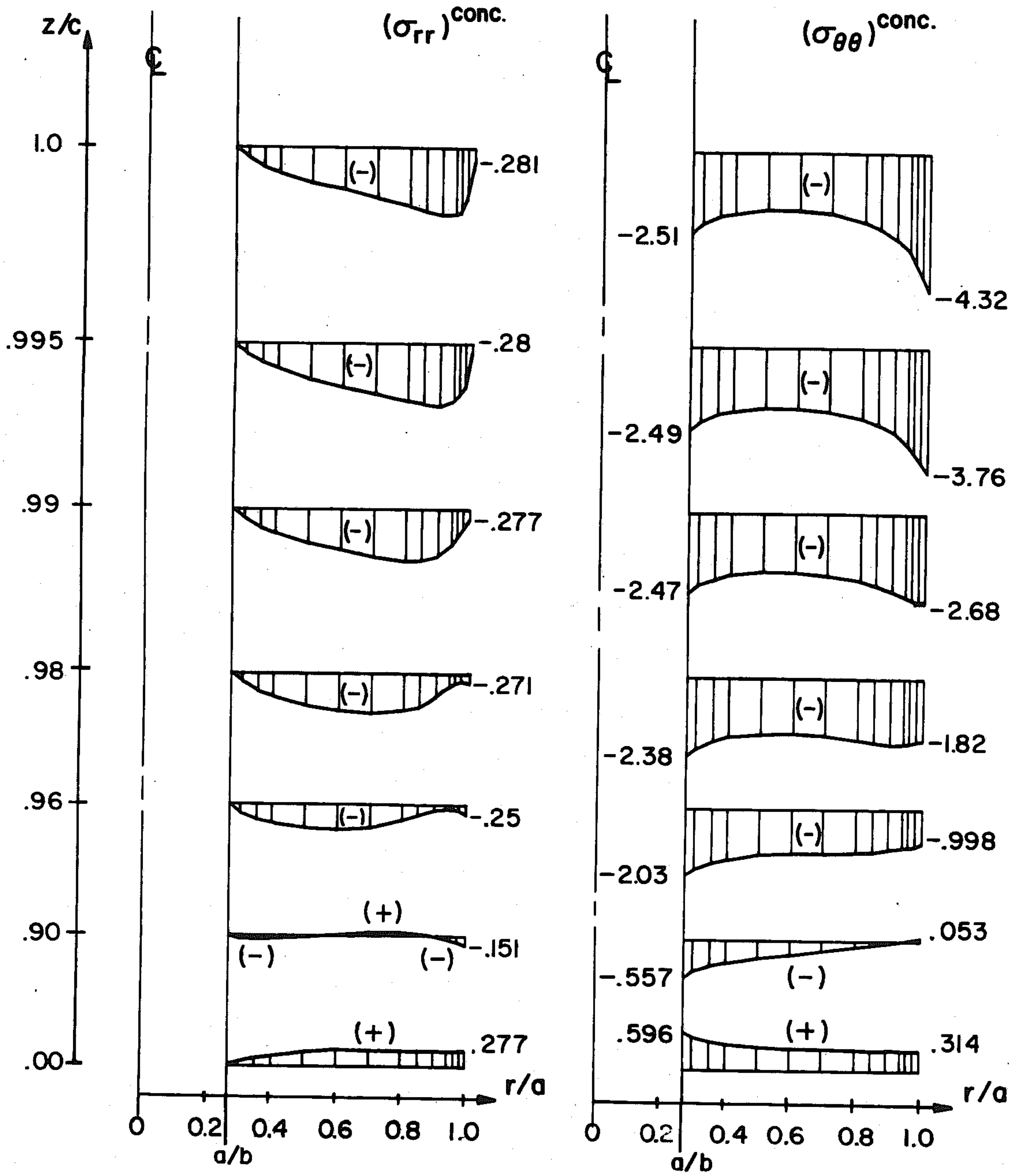


FIG. 2.25 PROBLEM 4, m = 8 — RADIAL AND TANGENTIAL STRESS DISTRIBUTIONS IN THE CONCRETE CORE (HOLLOW SECTION)

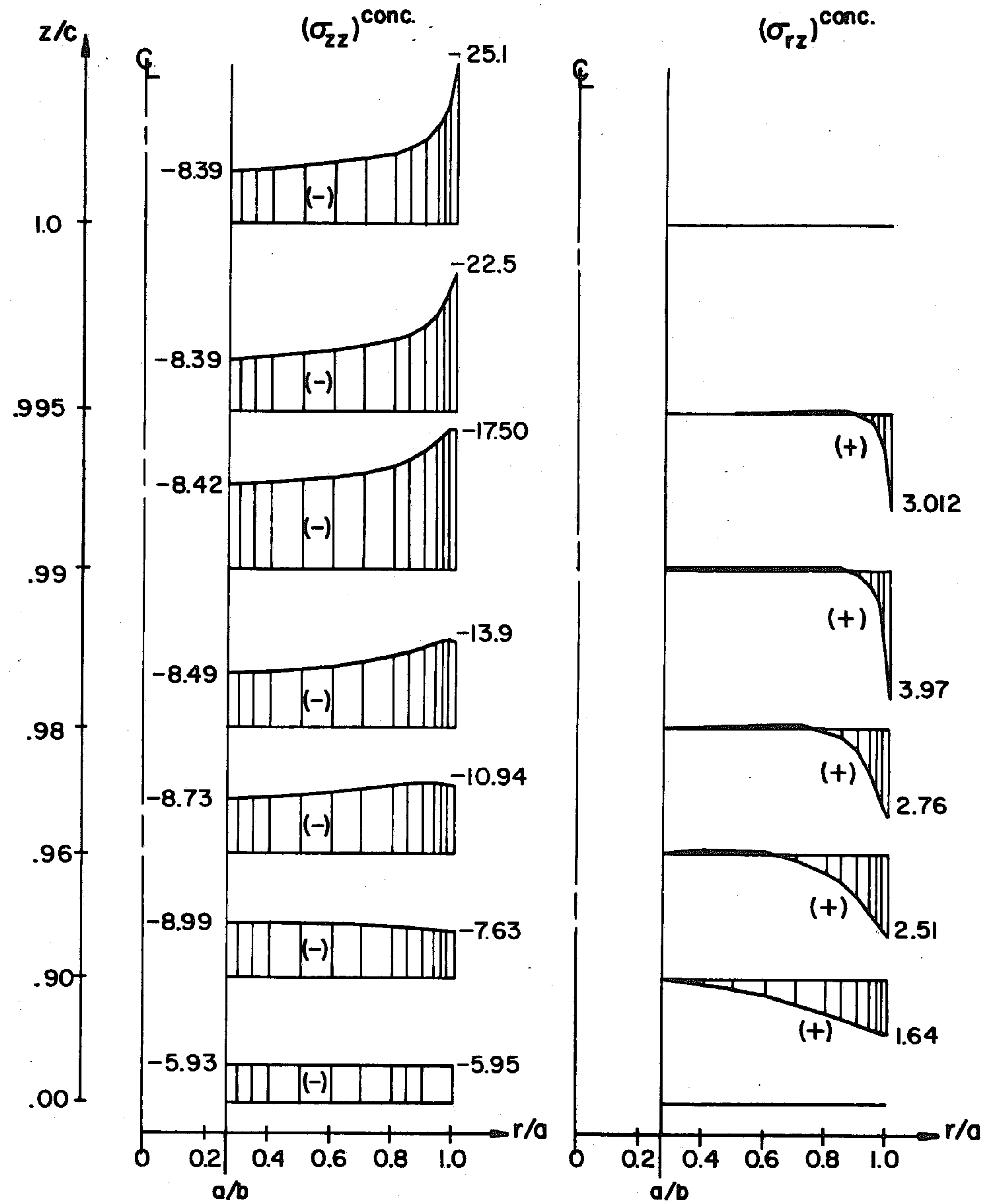


FIG. 2.26 PROBLEM 4, m = 8 — AXIAL AND SHEAR STRESS DISTRIBUTIONS IN THE CONCRETE CORE (HOLLOW SECTION)

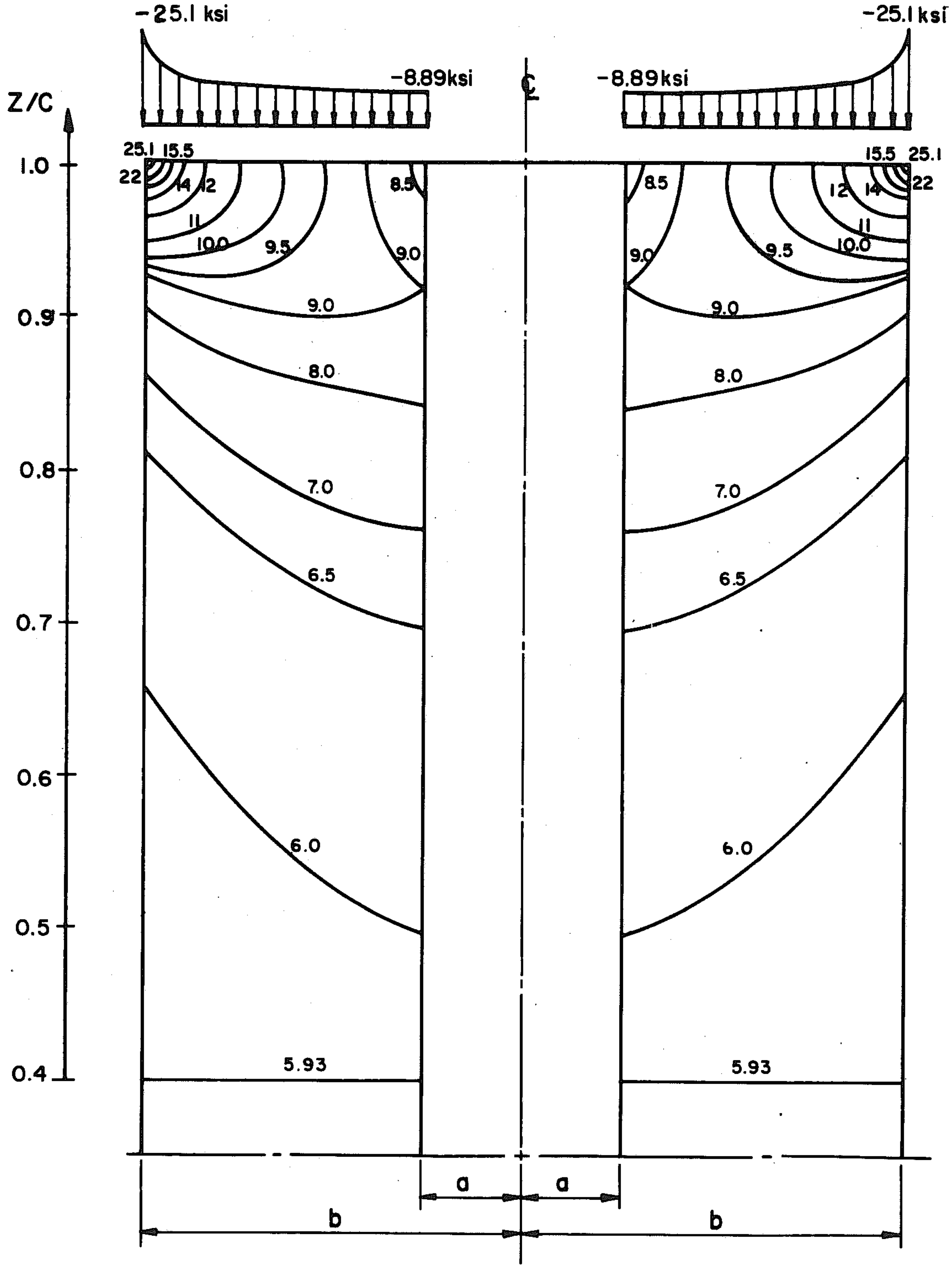


FIG. 2.27 PROBLEM 4, m=8 — COUNTOURS OF AXIAL STRESS IN CONCRETE CORE (HOLLOW SECTION)

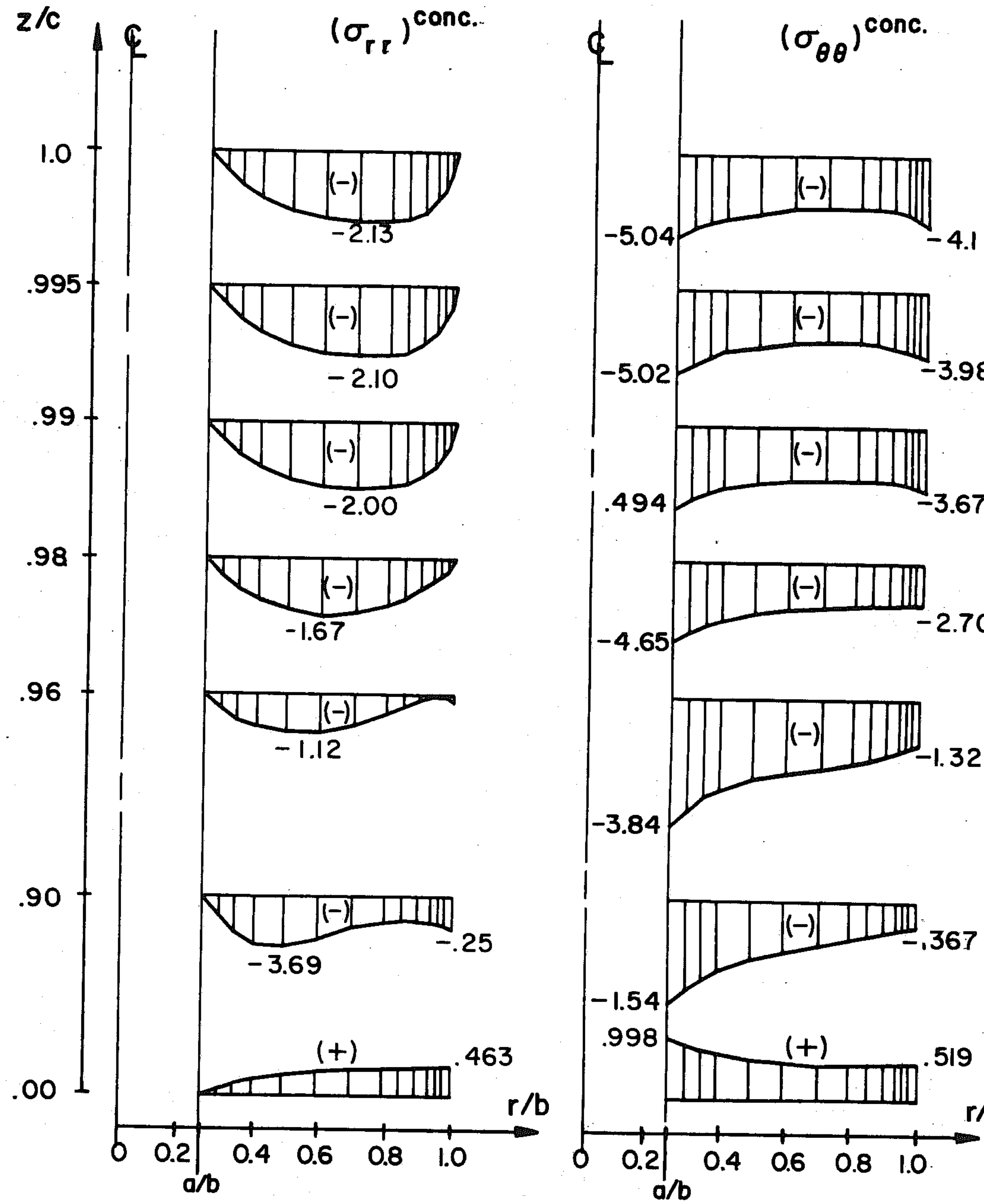


FIG. 2.28 PROBLEM 5, m = 4 — RADIAL AND TANGENTIAL STRESS DISTRIBUTIONS IN THE CONCRETE CORE (HOLLOW SECTION)

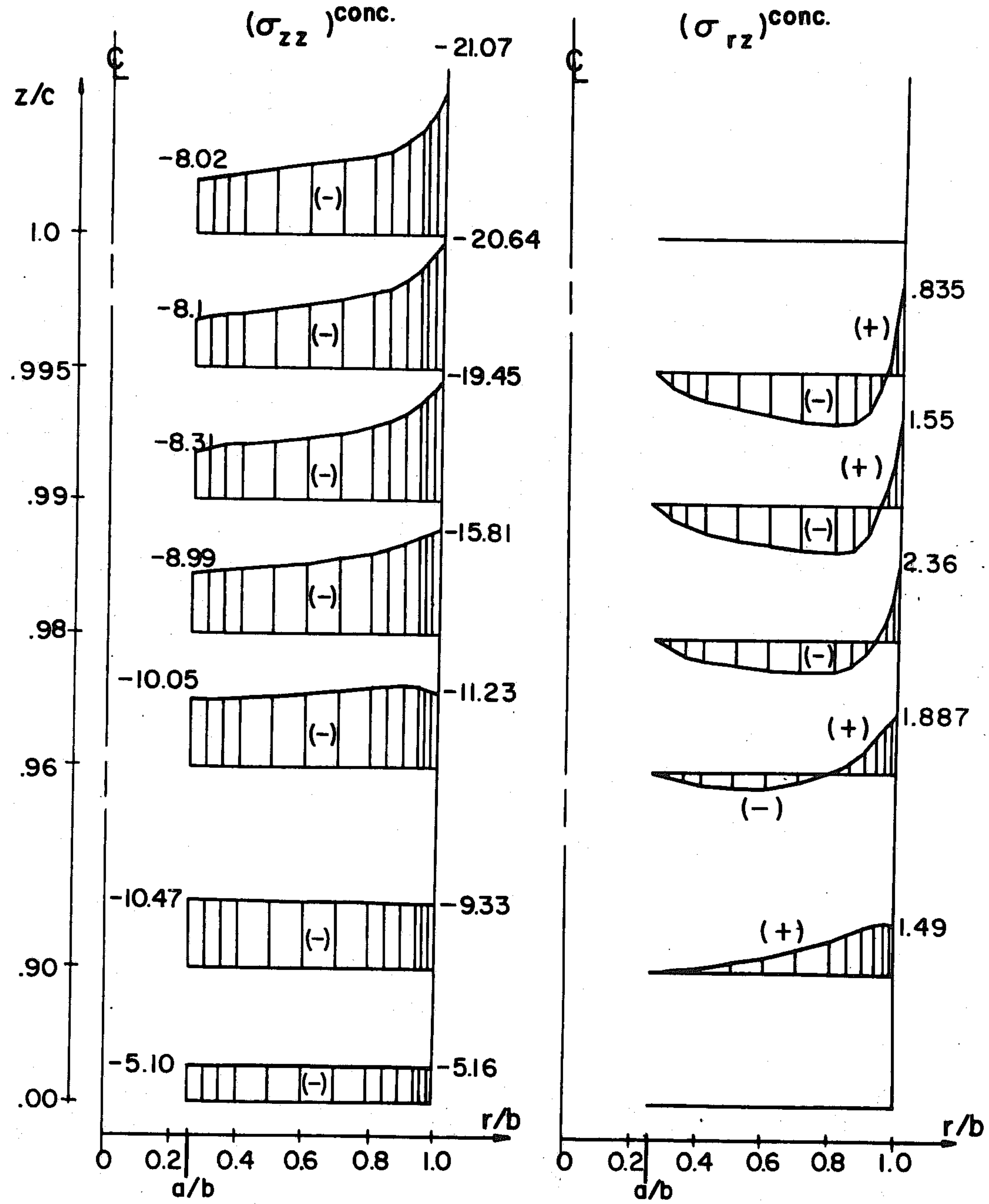


FIG. 2.29 PROBLEM 5, m = 4 — AXIAL AND SHEAR STRESS DISTRIBUTIONS IN THE CONCRETE CORE (HOLLOW SECTION)

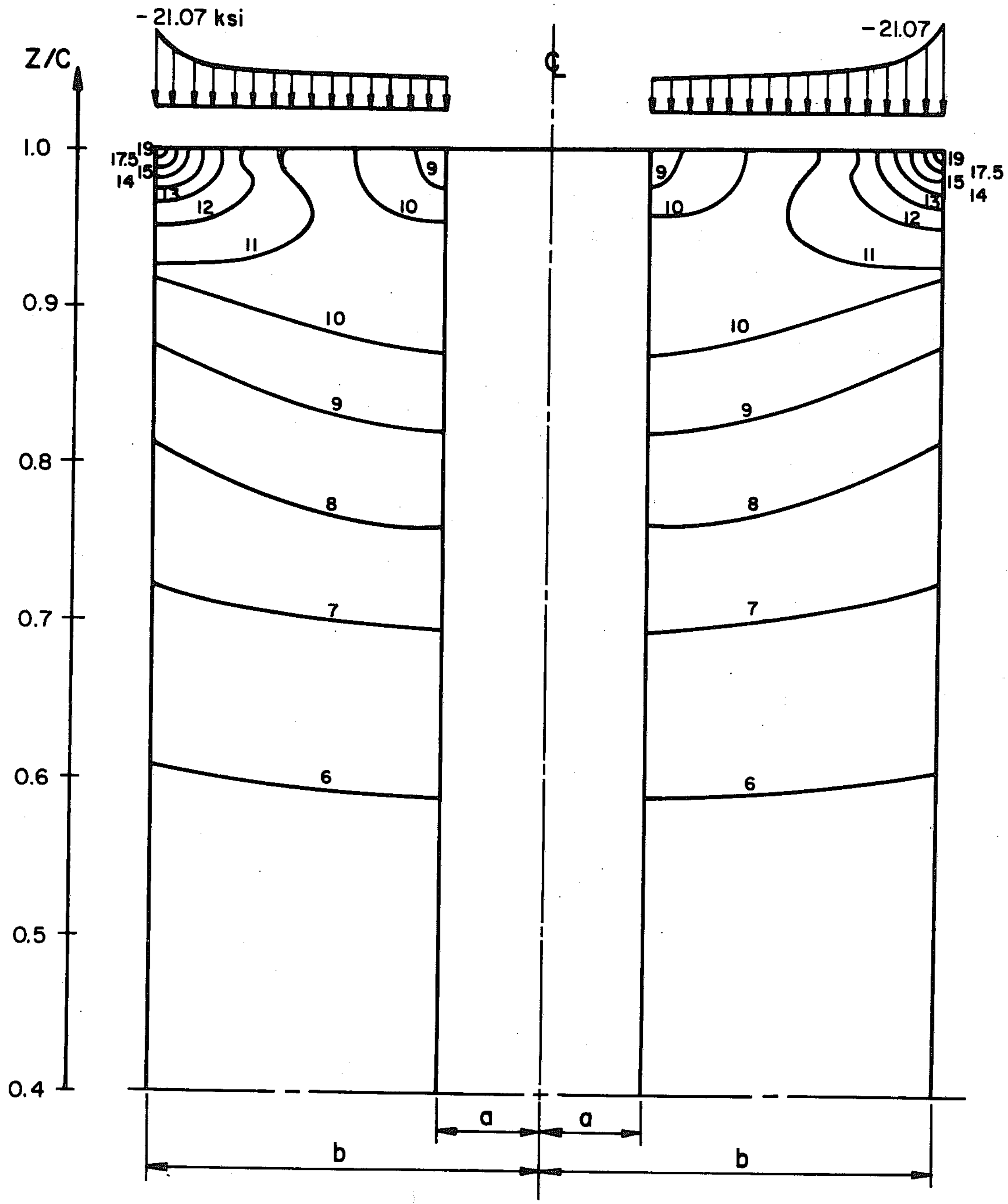


FIG. 2.30 PROBLEM 5, m = 4 – CONTOURS OF AXIAL STRESS IN THE CONCRETE CORE (HOLLOW SECTION)

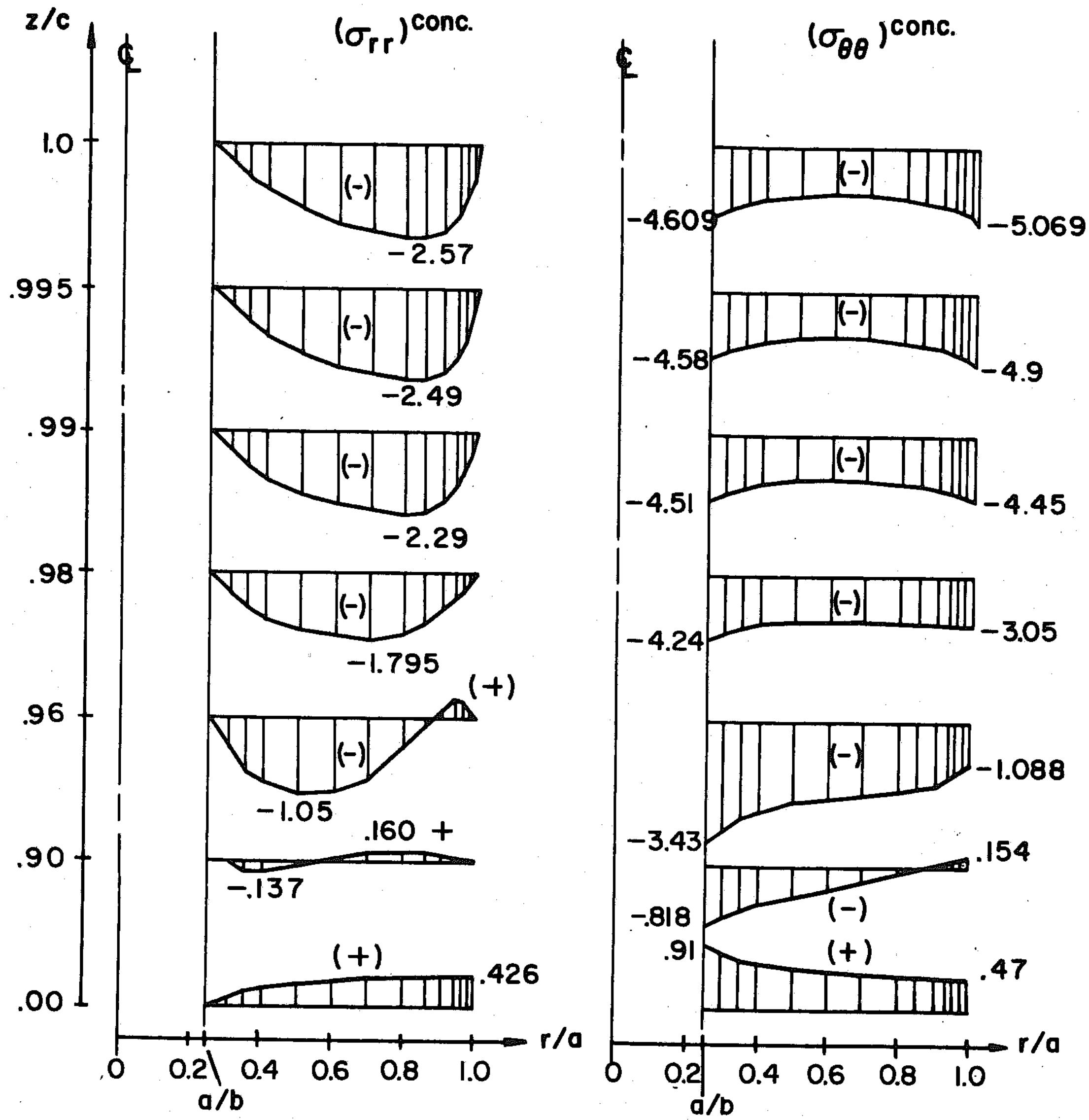


FIG. 2.31 PROBLEM 5, m = 8 — RADIAL AND TANGENTIAL STRESS DISTRIBUTIONS IN THE CONCRETE CORE (HOLLOW SECTION)

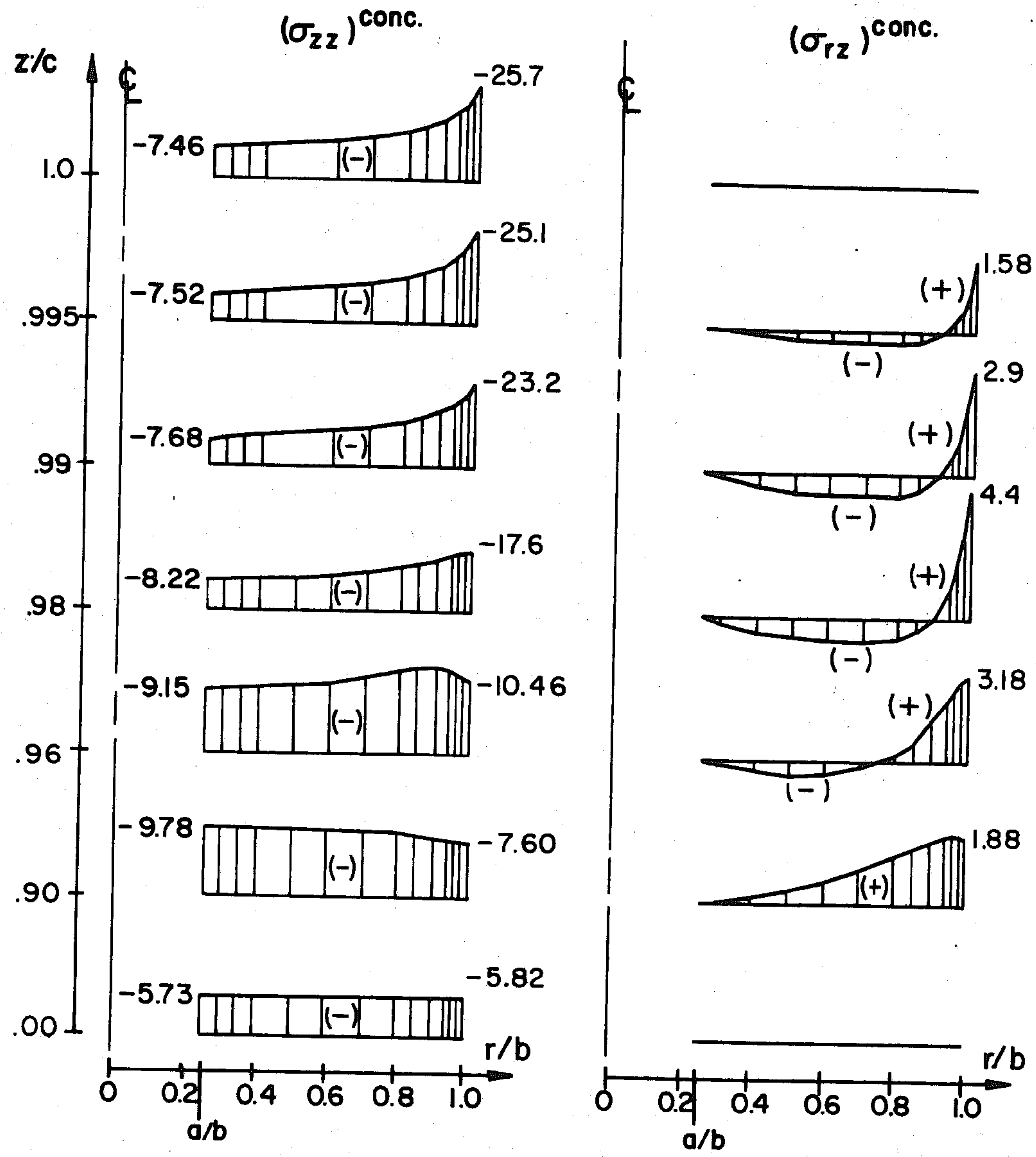


FIG. 2.32 PROBLEM 5, m = 8 — AXIAL AND SHEAR STRESS DISTRIBUTIONS IN THE CONCRETE CORE (HOLLOW SECTION)

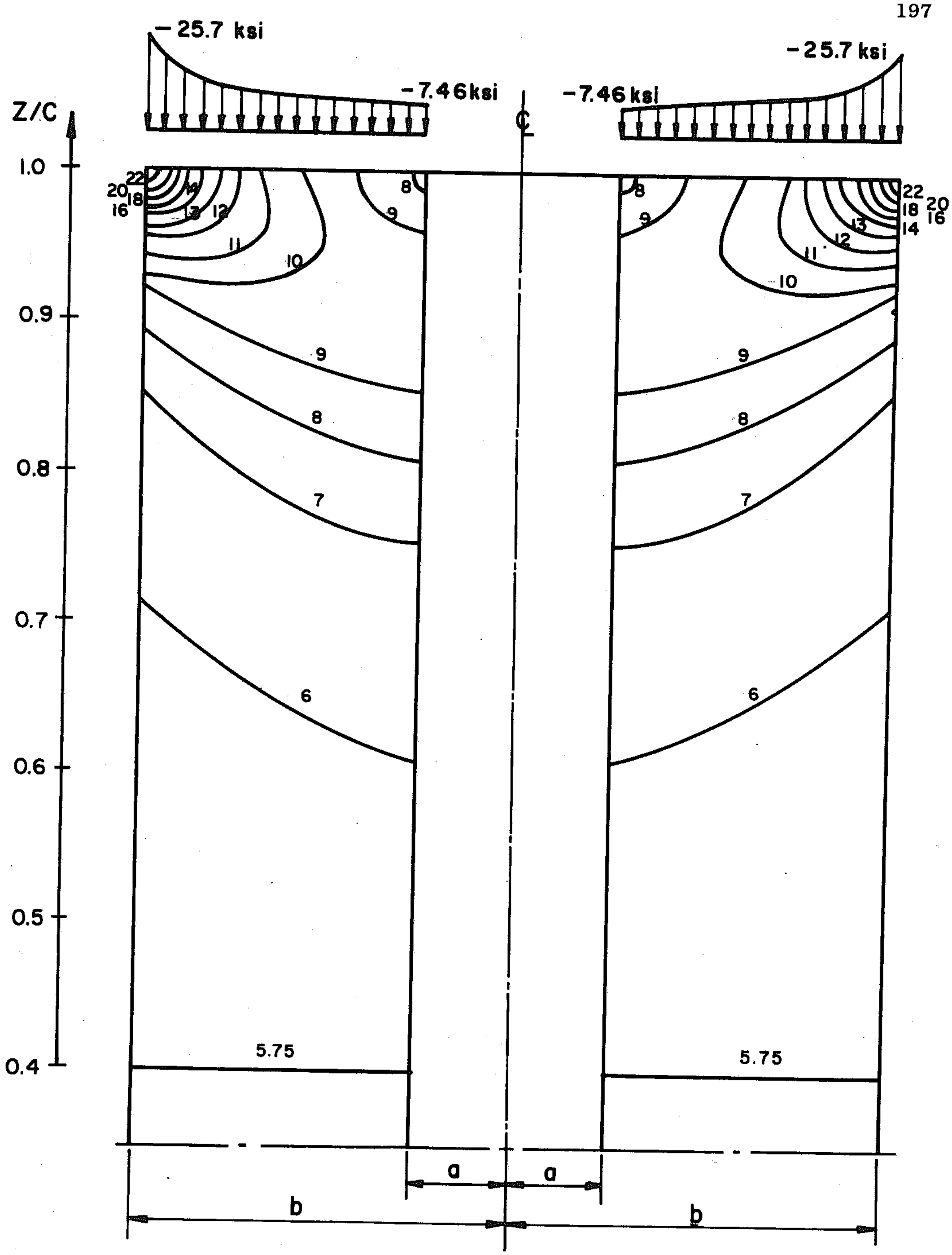


FIG. 2.33 PROBLEM 5, m=8 — CONTOURS OF AXIAL STRESS IN THE CONCRETE (HOLLOW SECTION)

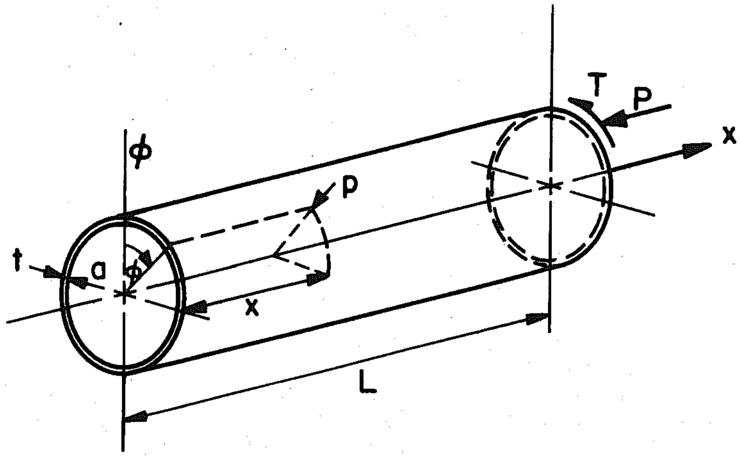


FIG. 2.34 SYSTEM OF COORDINATES

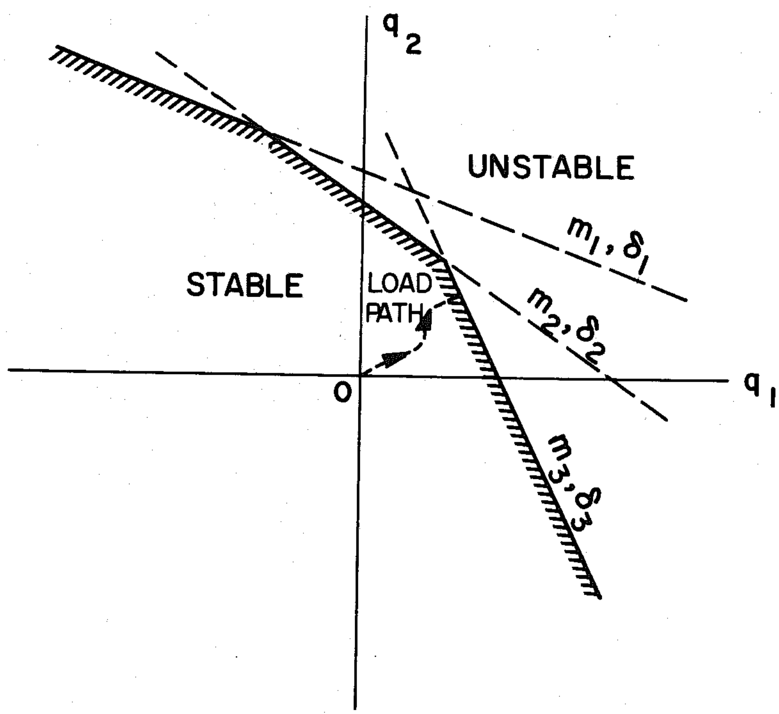


FIG. 2.35 STABLE AND UNSTABLE DOMAINS

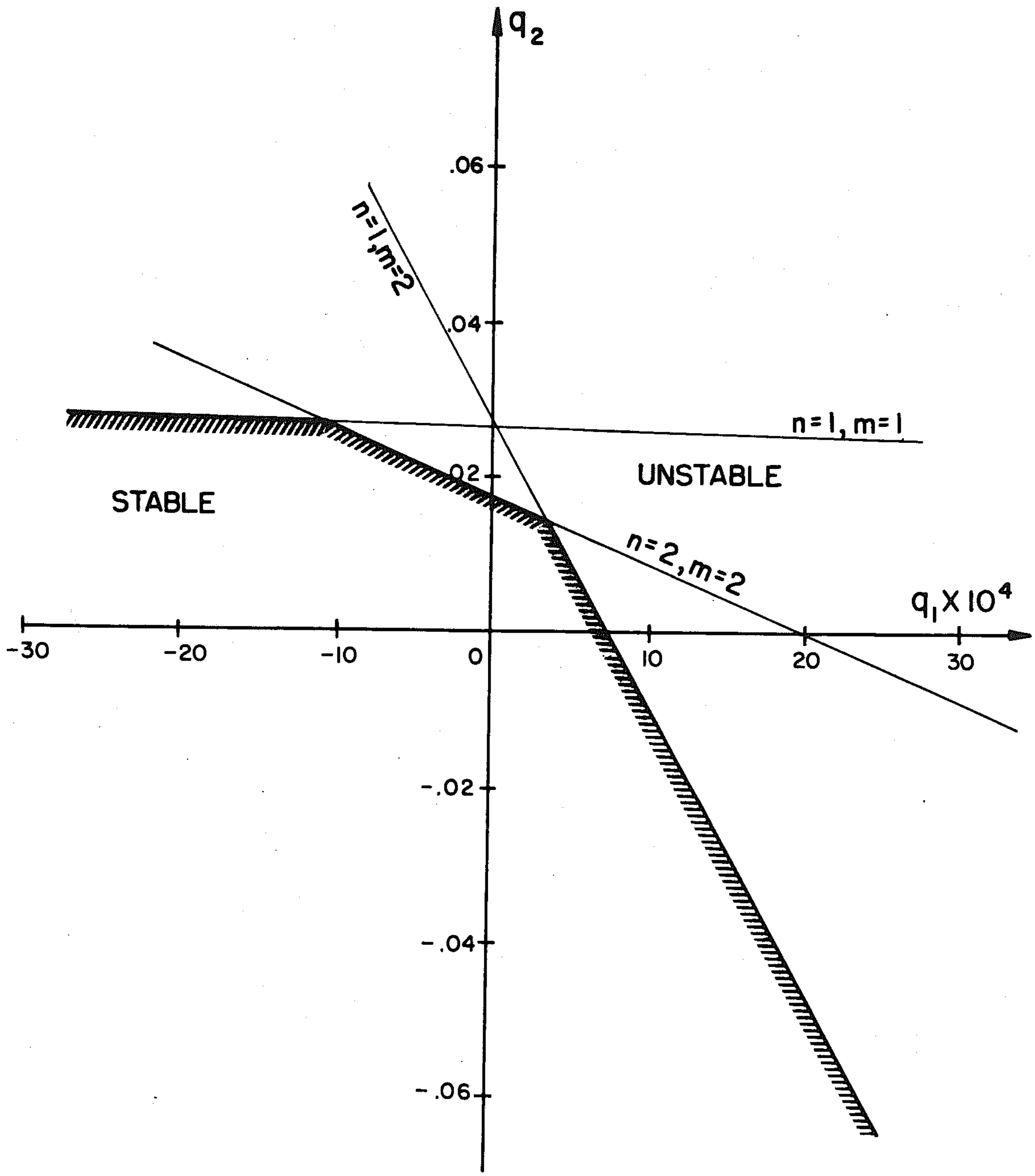


FIG. 2.36 LOCAL BUCKLING OF STEEL TUBES USED IN EXPERIMENTAL INVESTIGATION

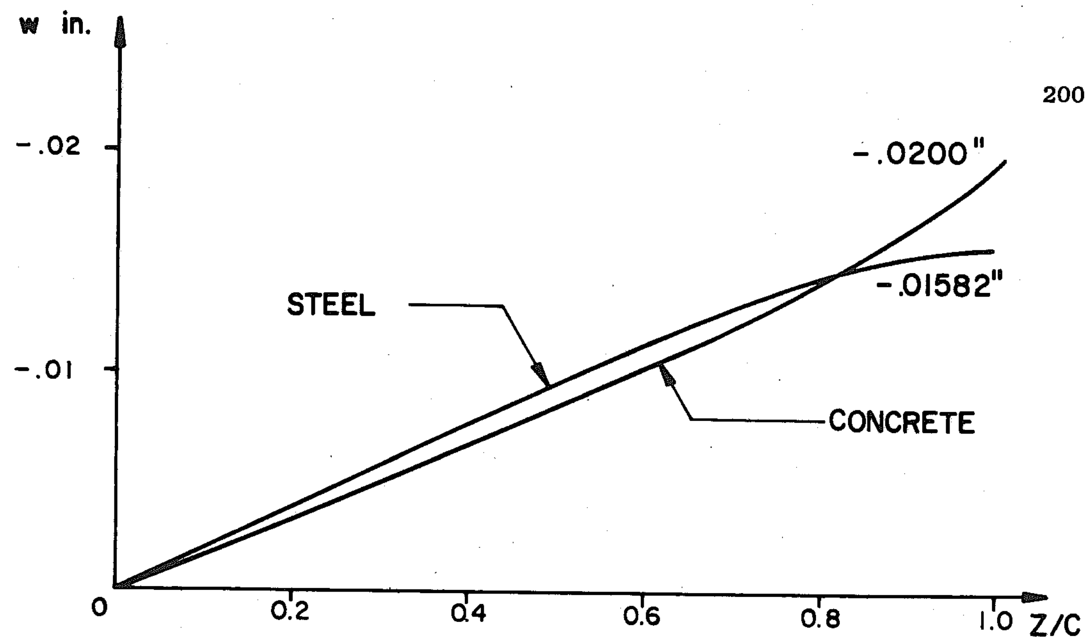


FIG. 2.37 PROBLEM 4, m = 4 - AXIAL DISPLACEMENTS AT THE INTERFACE

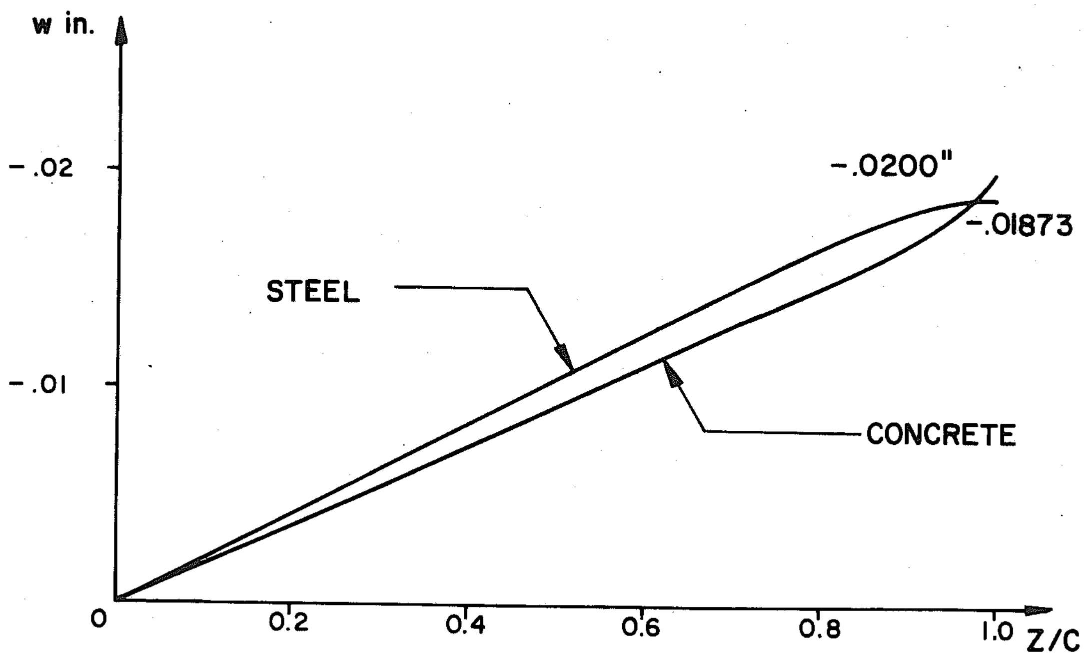
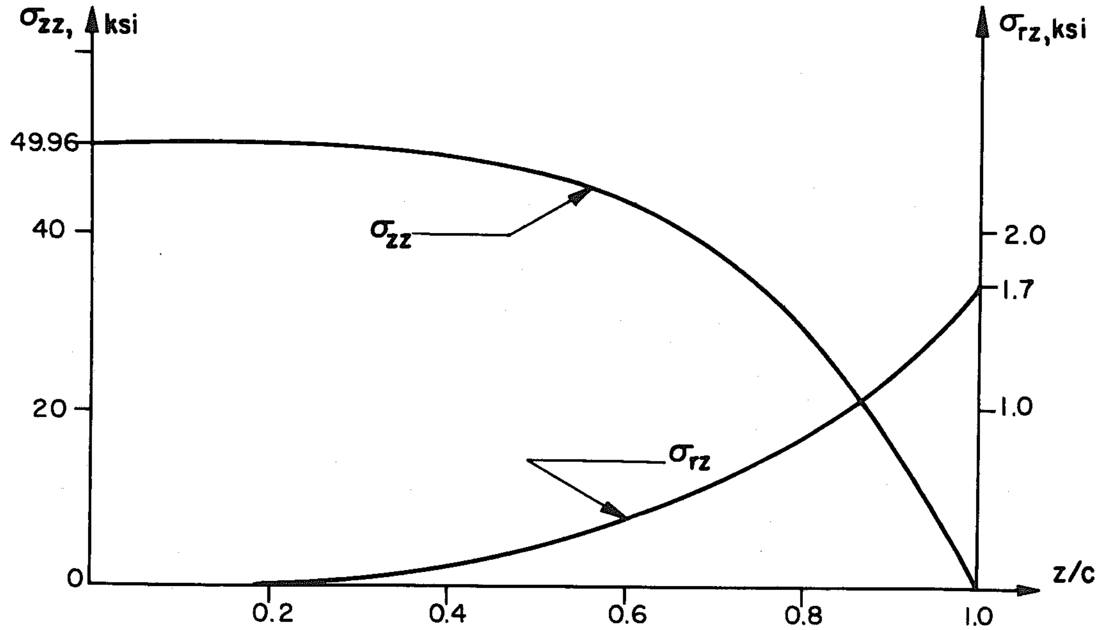


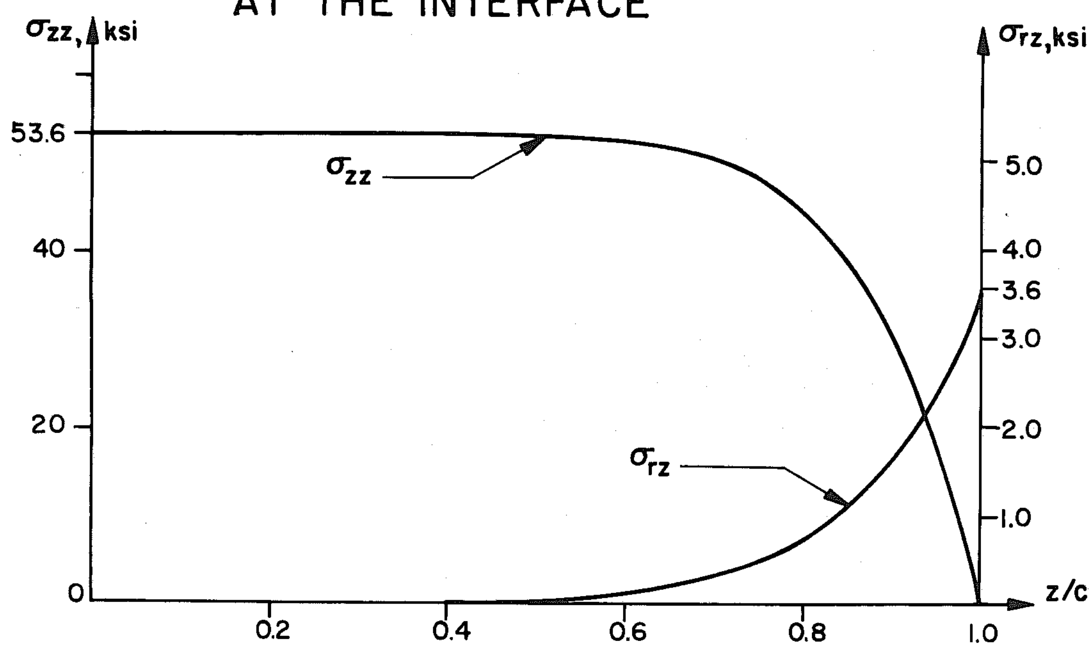
FIG. 2.38 PROBLEM 4, m = 8 - AXIAL DISPLACEMENTS AT THE INTERFACE



σ_{zz} AXIAL STRESS IN THE STEEL

σ_{rz} SHEAR STRESS AT THE INTERFACE

FIG. 2.39 PROBLEM NO. 4, m = 4, STRESSES AT THE INTERFACE



σ_{zz} AXIAL STRESS IN THE STEEL

σ_{rz} SHEAR STRESS AT THE INTERFACE

FIG. 2.40 PROBLEM NO. 4, m = 8, STRESSES AT THE INTERFACE

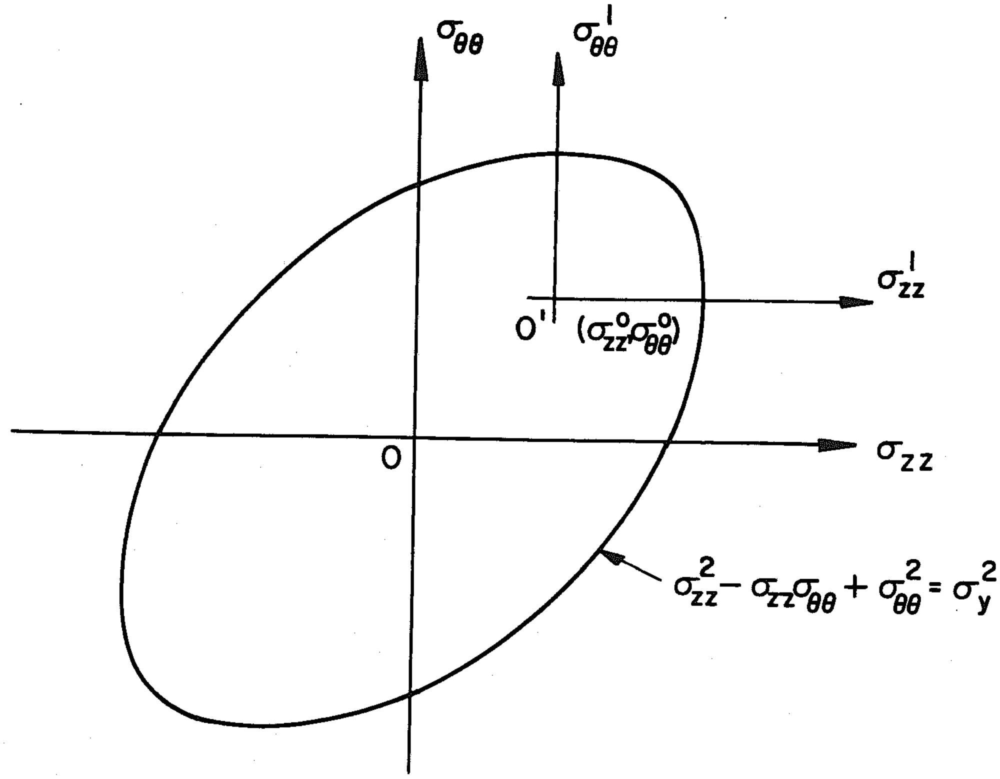


FIG. 3.1 VON MISES YIELD ENVELOPE

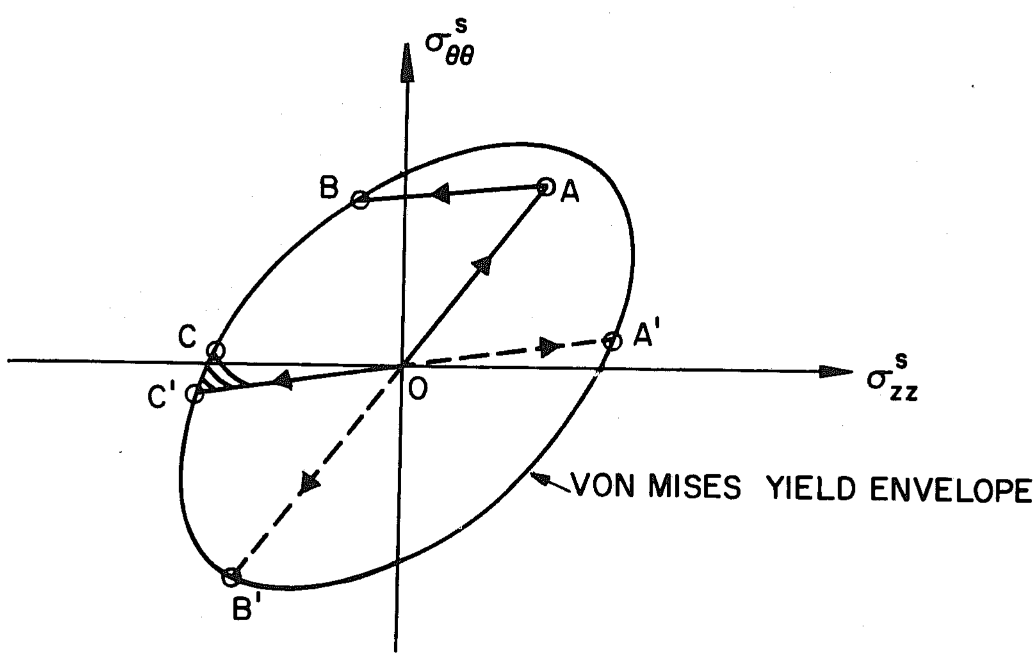


FIG. 3.2 STRESS HISTORY

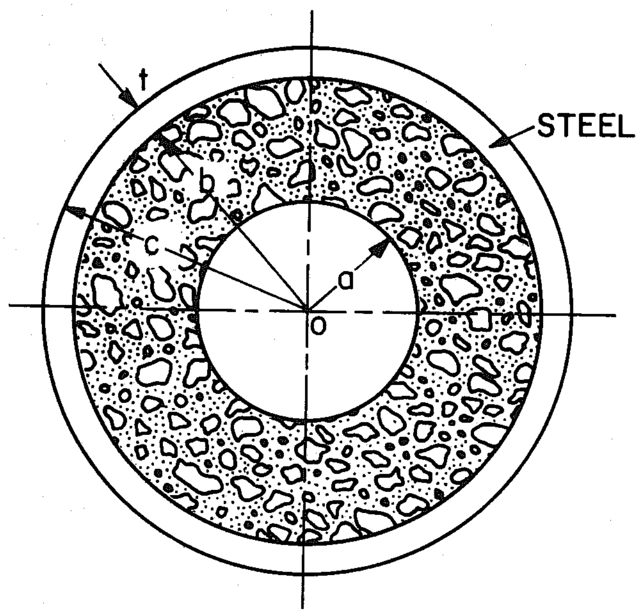


FIG. 3.3 CROSS-SECTION OF COMPOSITE COLUMN

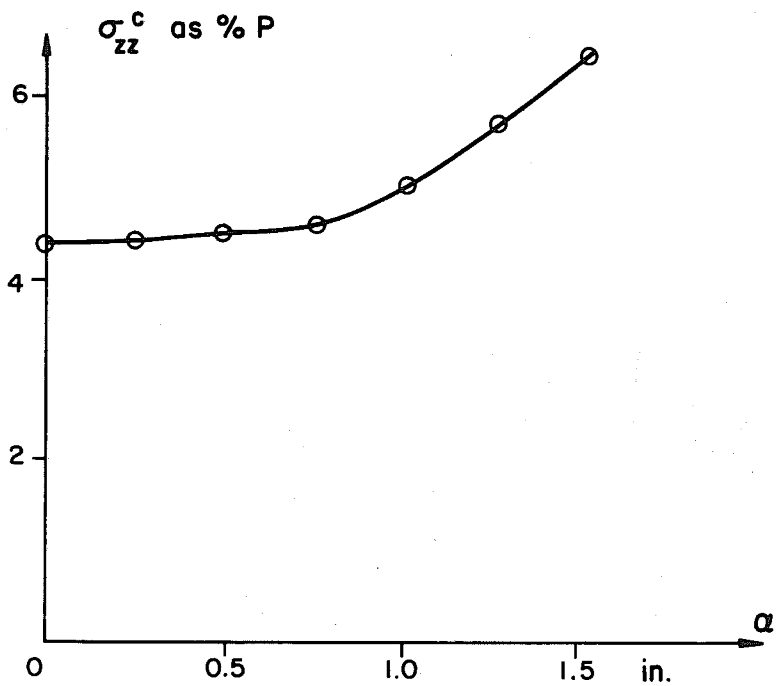


FIG. 3.4 CONCRETE AXIAL STRESS vs HOLE RADIUS

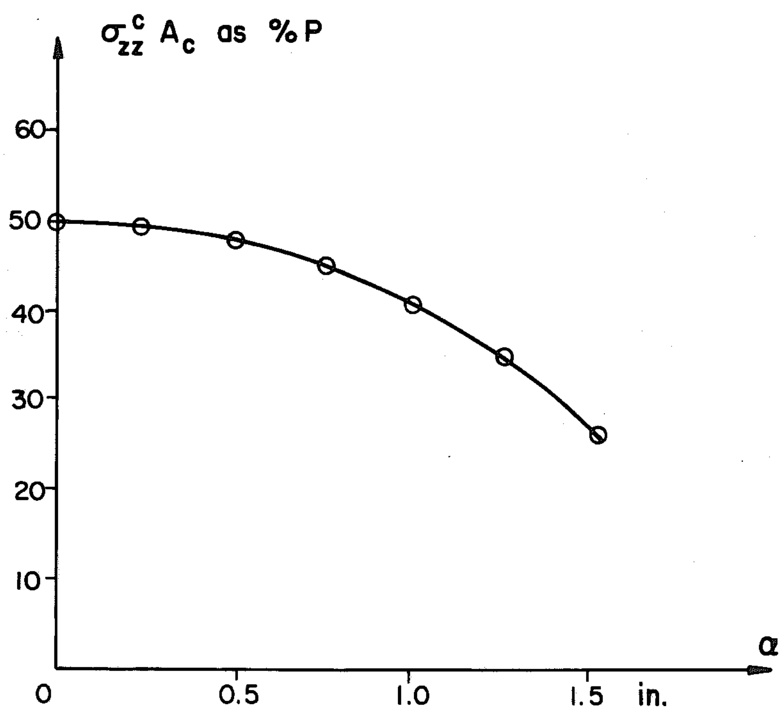


FIG. 3.5 LOAD CARRIED BY CONCRETE vs HOLE RADIUS

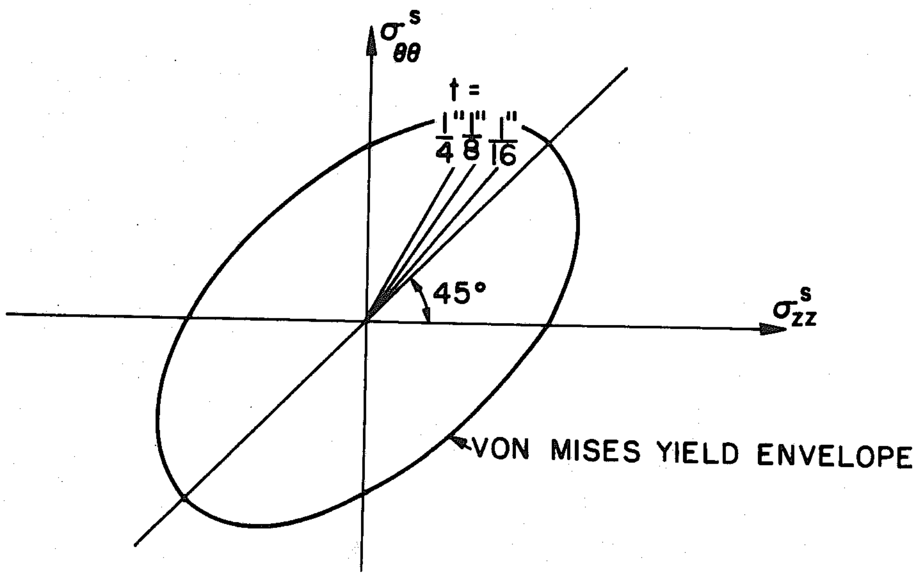


FIG. 3.6 STRESS PATHS IN STEEL DURING EXPANSION FOR DIFFERENT TUBE THICKNESSES

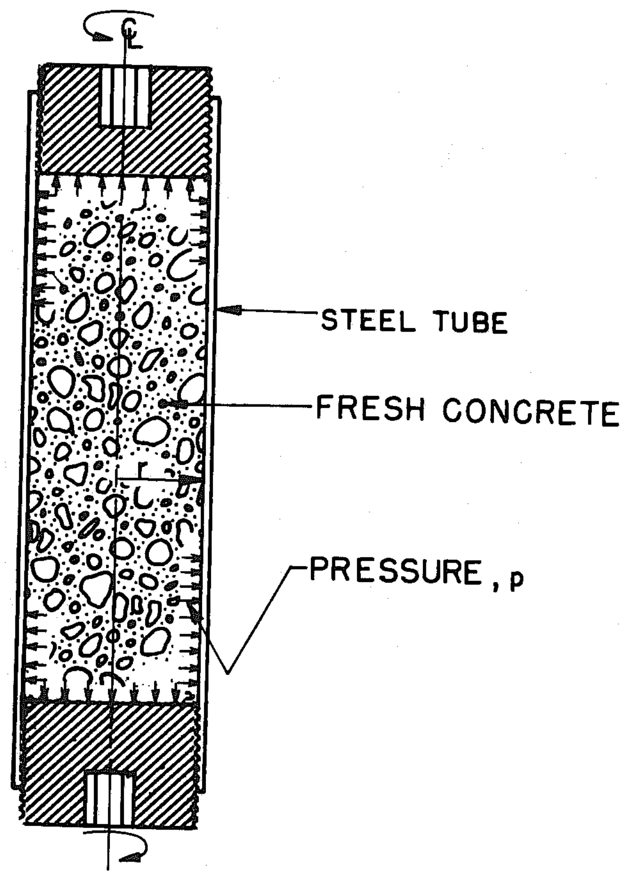
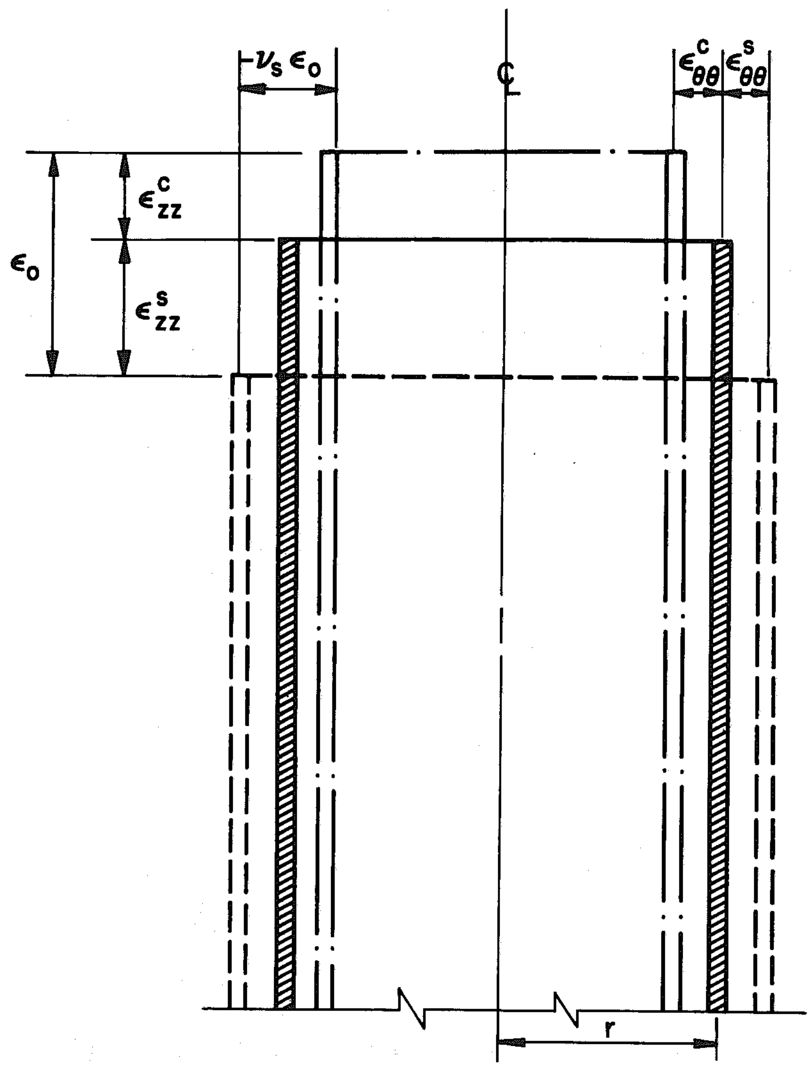


FIG. 3.7 PRESTRESSING BY MEANS OF A PLUNGER



———— TUBE BEFORE PRESTRESSING (ORIGINAL STATE)

—·— TUBE AFTER PRESTRESSING (STATE AT CASTING)

——— TUBE AFTER REALEASE OF PRESTRESSING FORCE (FINAL STATE)

FIG. 3.8 COMPATIBILITY OF STRAINS

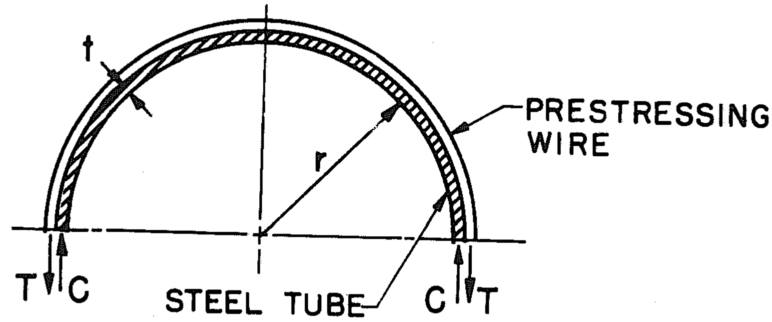


FIG. 3.9 FREE BODY DIAGRAM SHOWING EQUILIBRIUM OF FORCES IN THE STEEL TUBE AND THE PRESTRESSING WIRE

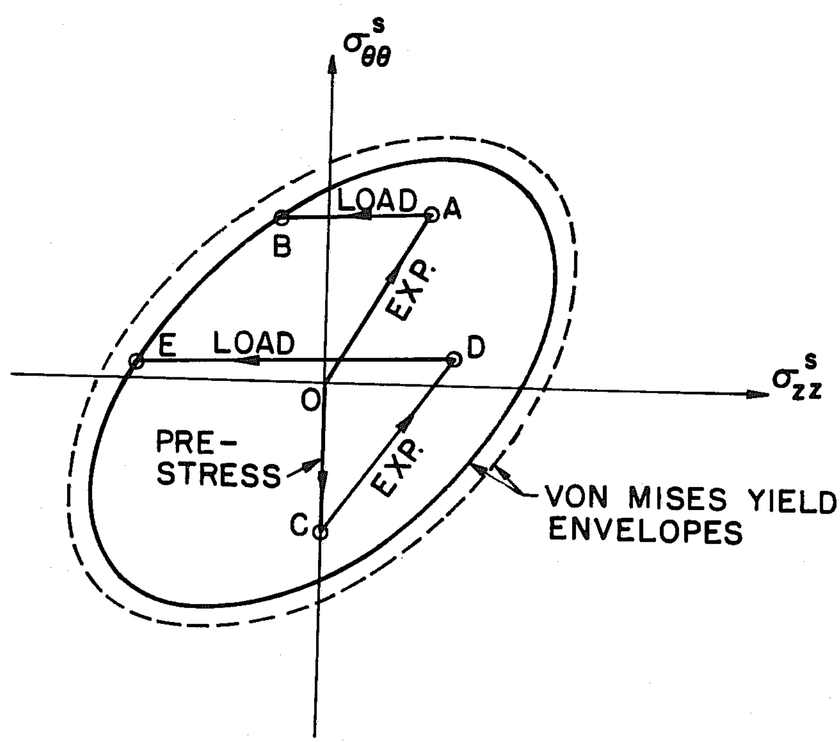


FIG. 3.10 STRESS PATH FOR PRESTRESSED AND UNPRESTRESSED TUBES

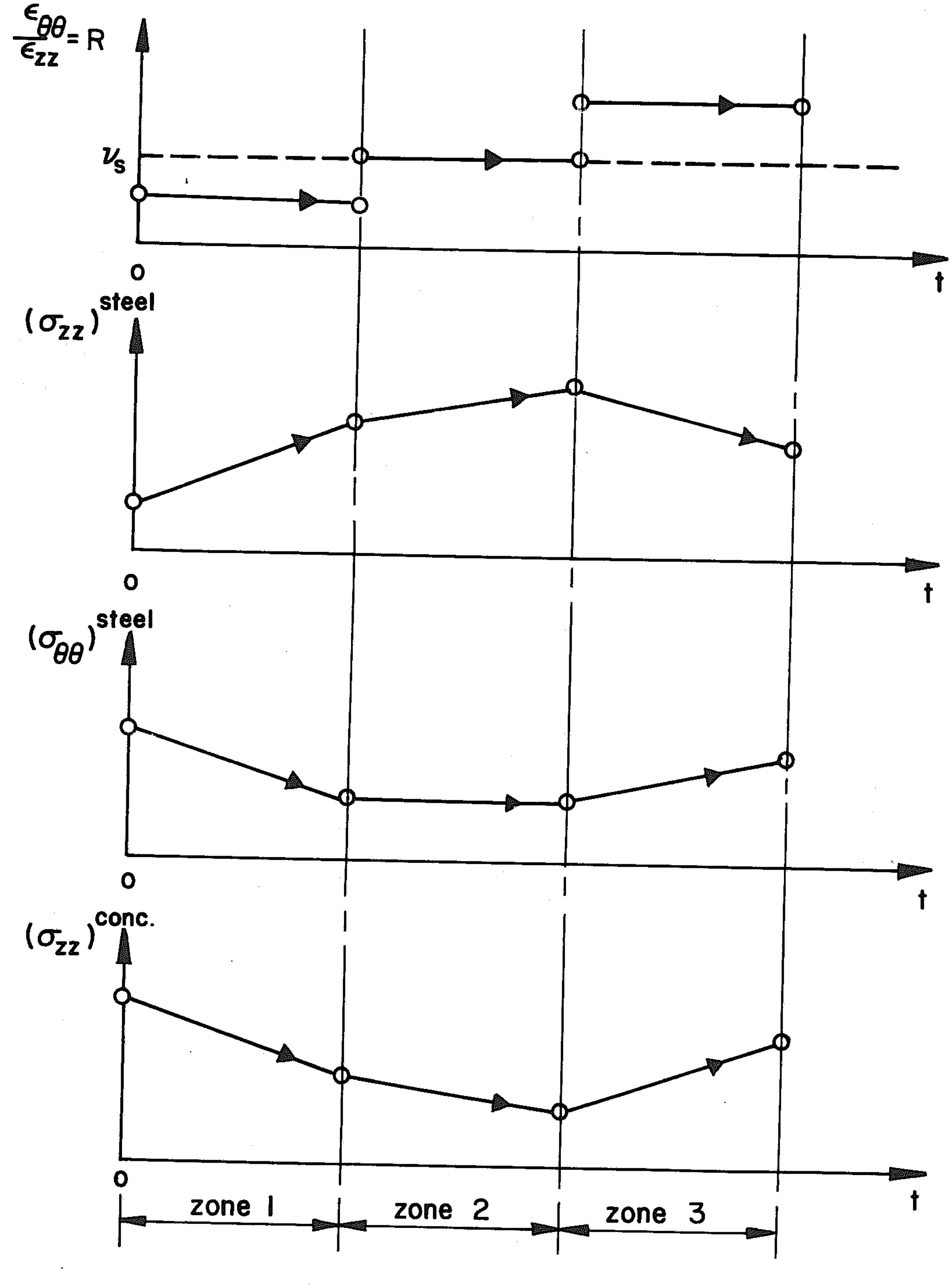
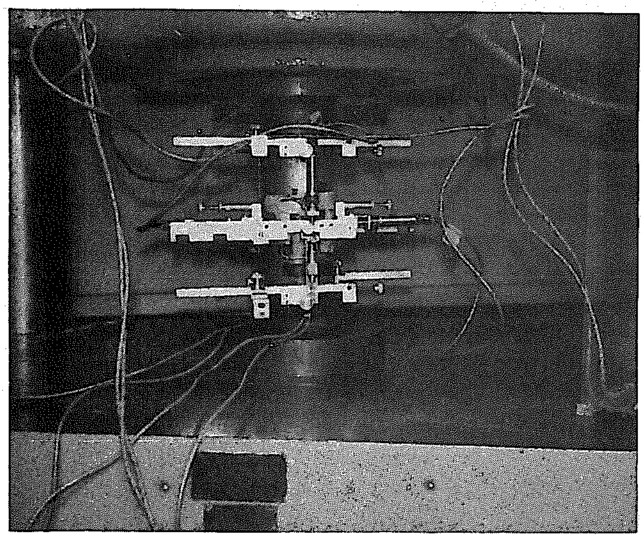
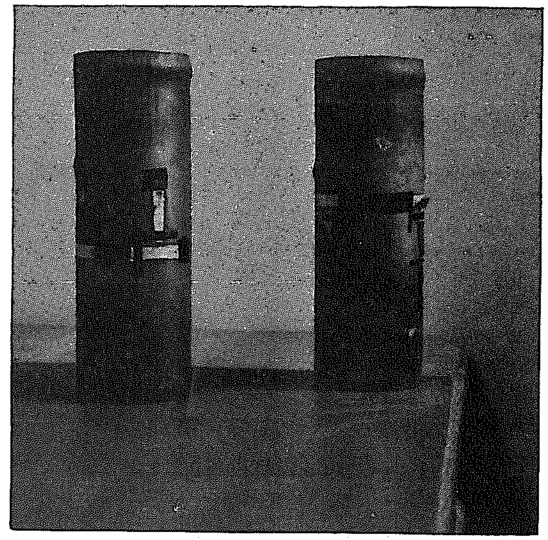


FIG. 5.1 POSSIBLE STATES OF STRESS IN THE STEEL TUBE AND CONCRETE

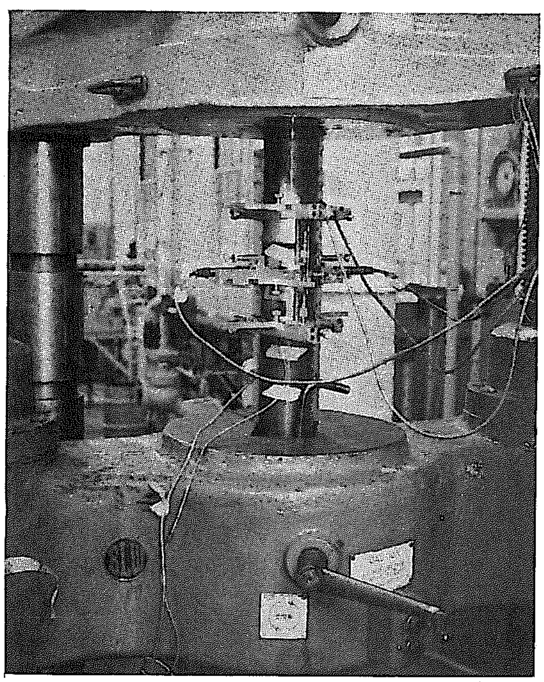


a - TESTING SETUP

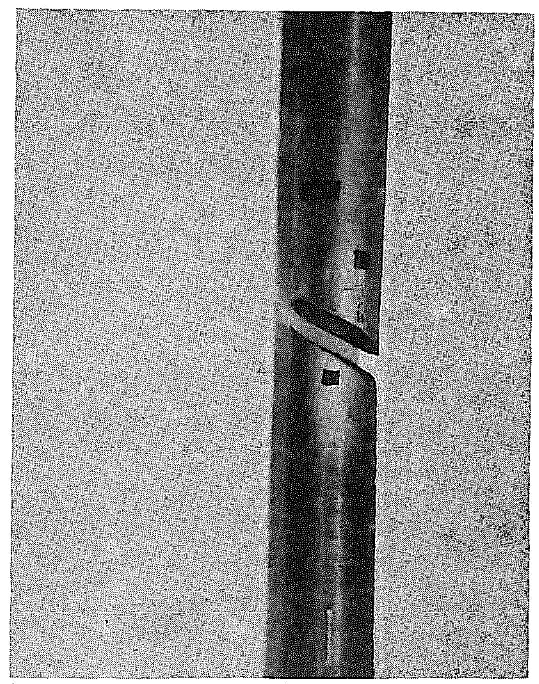


b - SPECIMEN AFTER FAILURE

FIG. 6.1 COMPRESSION TEST OF STEEL TUBE



a- TESTING SETUP



b - SPECIMEN AFTER FAILURE

FIG. 6.2 TENSION TEST OF STEEL TUBE

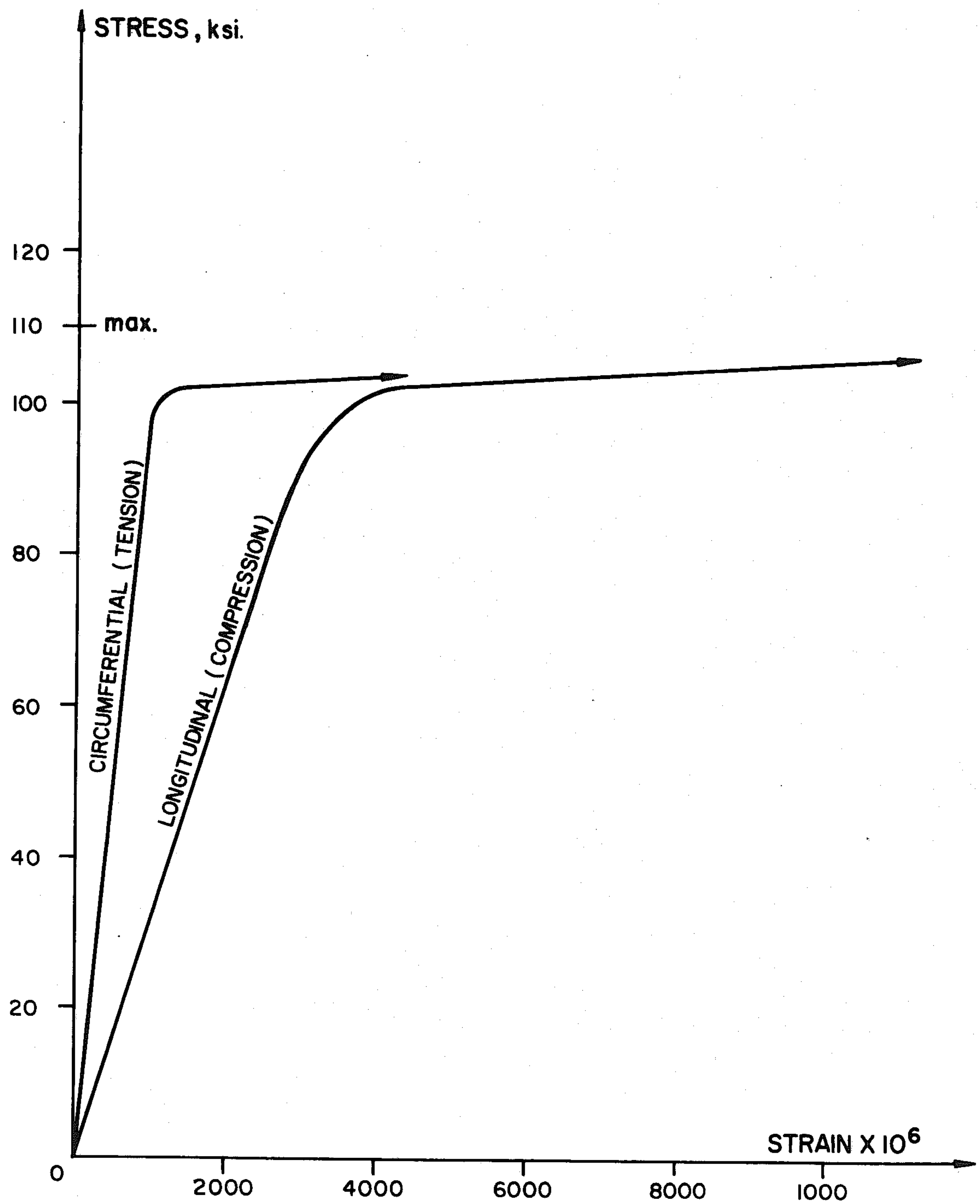


FIG. 6.3 TENSION TEST OF STEEL TUBE, STRESS - STRAIN RELATIONSHIP

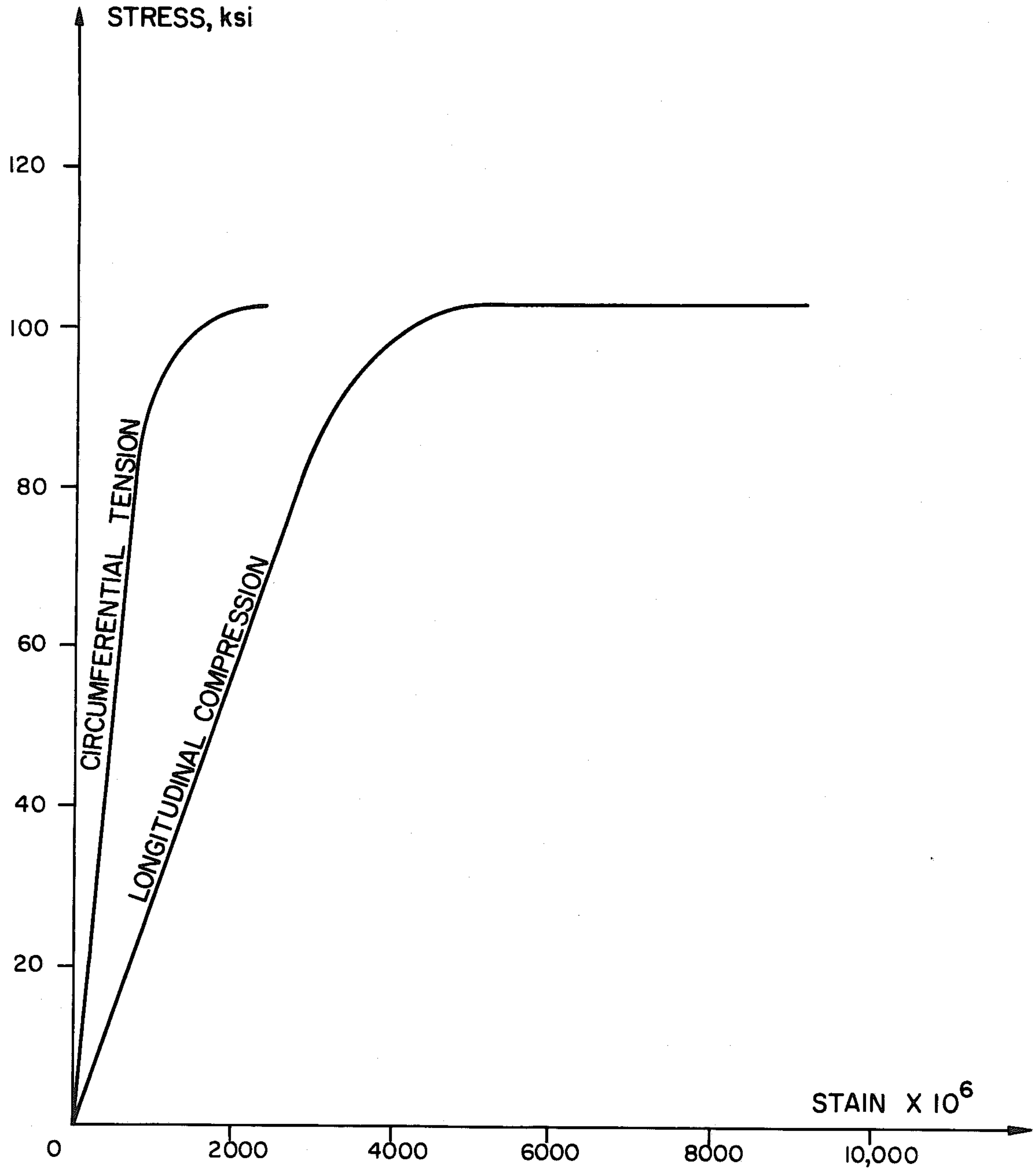
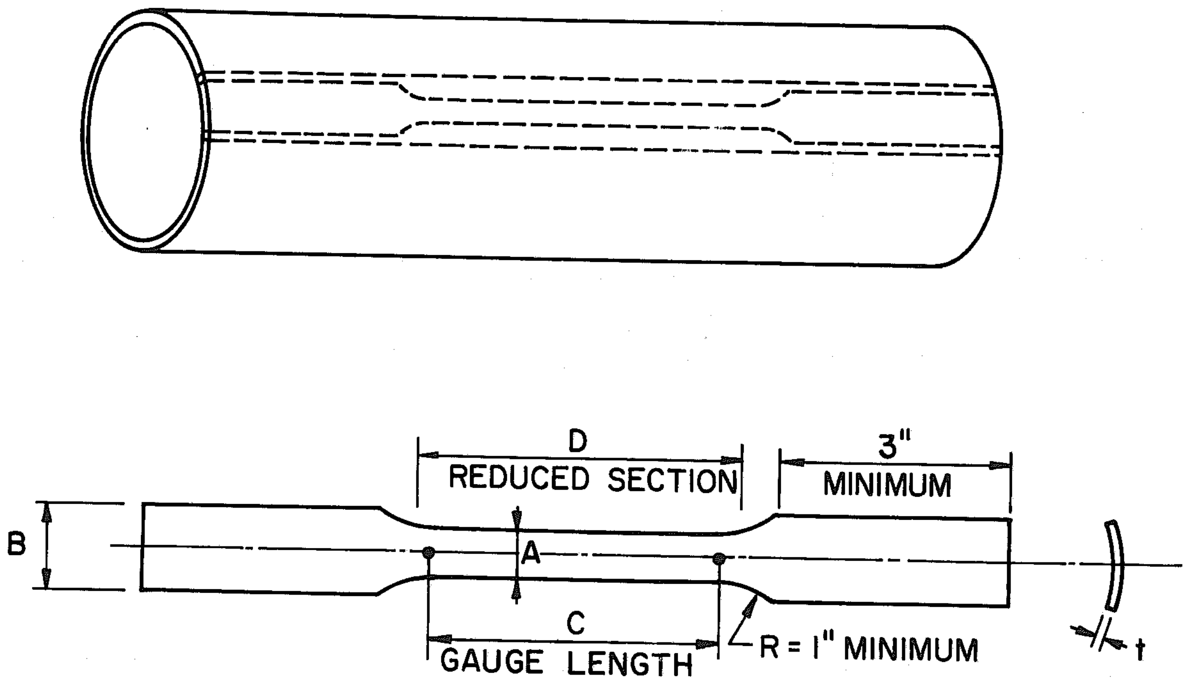


FIG. 6.4 COMPRESSION TEST OF STEEL TUBE - STRESS-STRAIN RELATIONSHIP



A = 1/2" ± 0.015
B = 1 1/16" APPROX.
C = 2" ± 0.005
D = 2 1/4" MINIMUM

CROSS SECTIONAL AREA ≃ A x t

FIG. 6.5 LOCATION AND DIMENSIONS OF LONGITUDINAL STRIPS

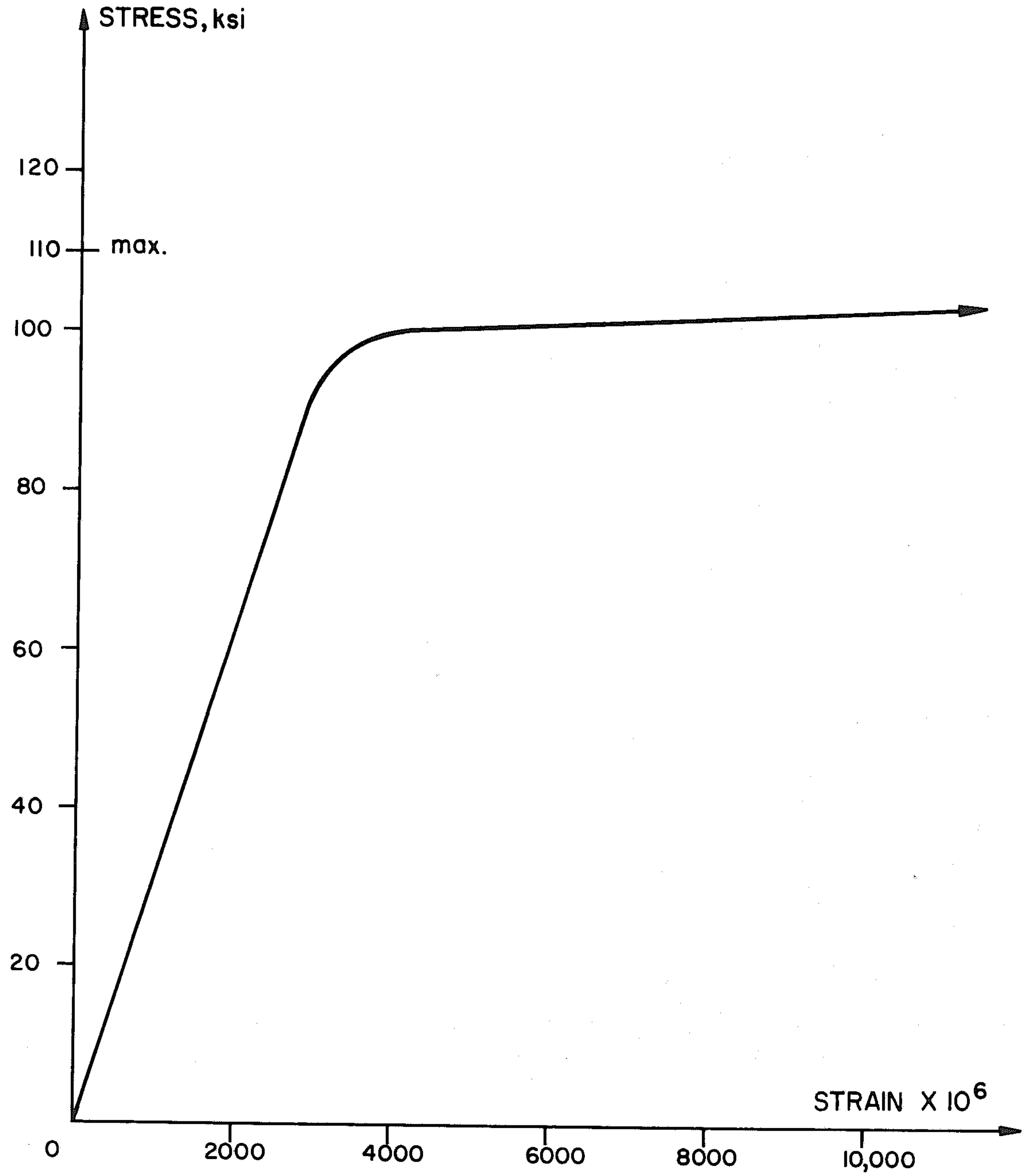


FIG. 6.6 TENSION TEST OF LONGITUDINAL STRIP
STRESS - STRAIN RELATIONSHIP

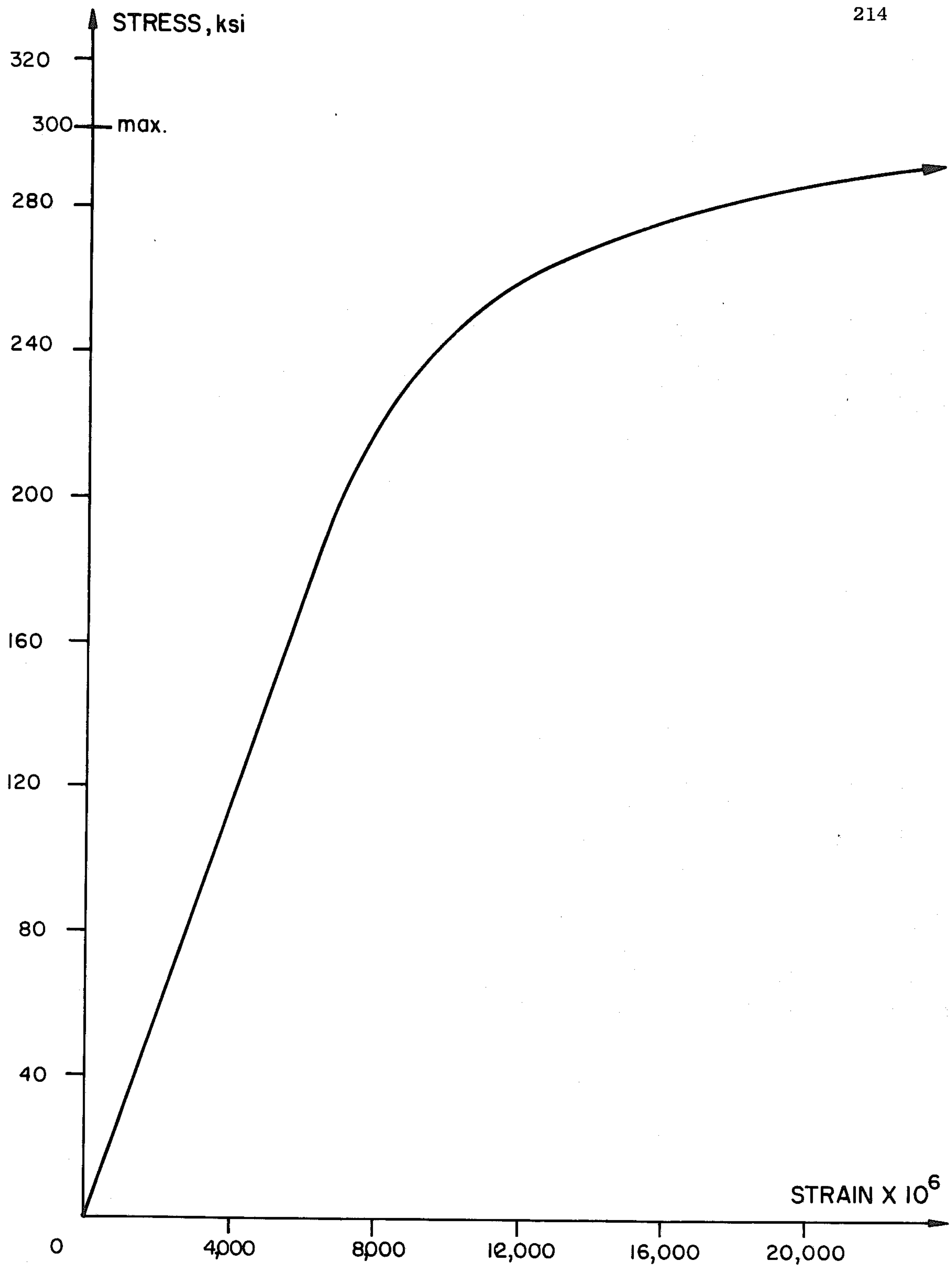
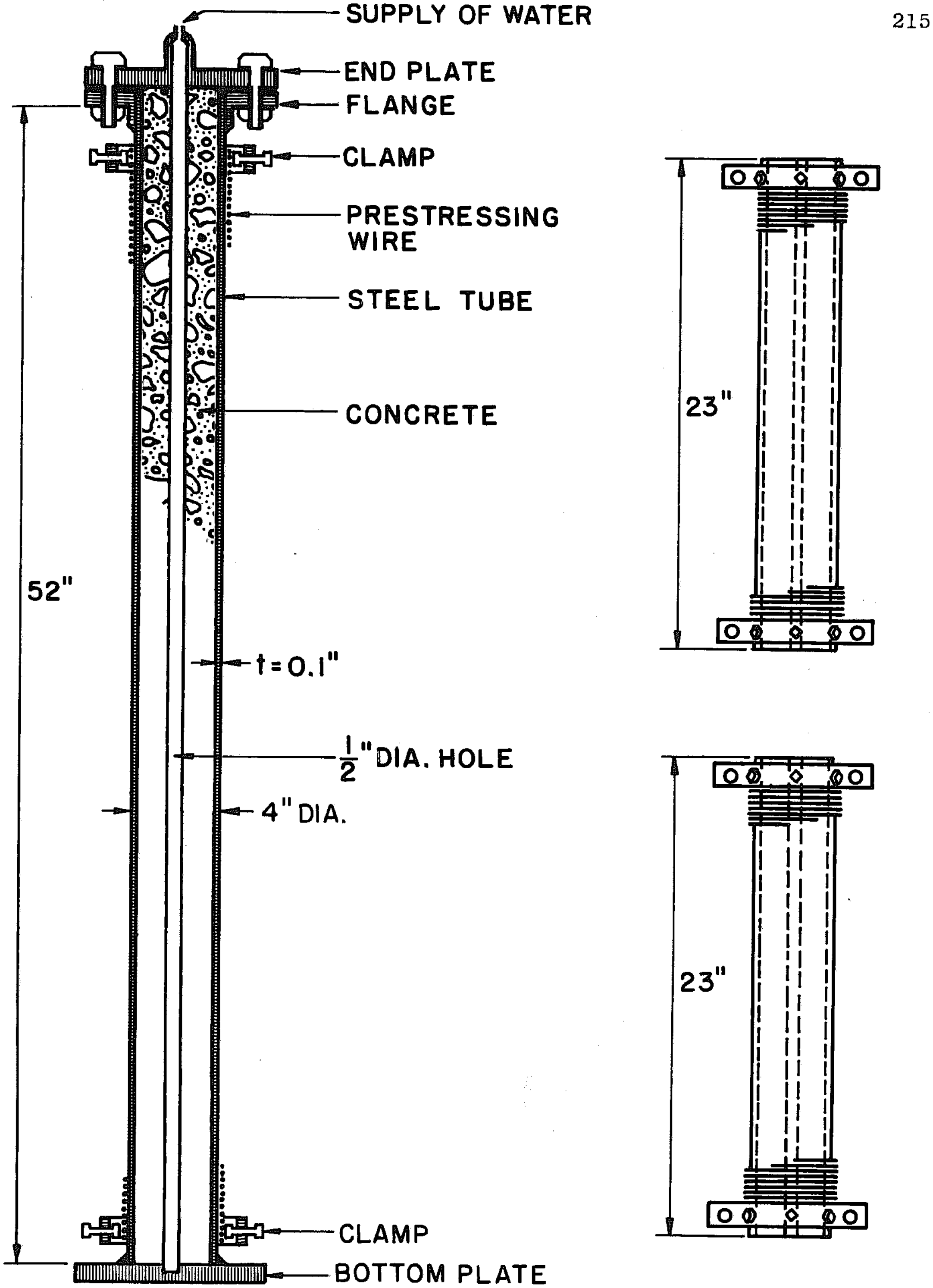


FIG. 6.7 STRESS - STRAIN RELATIONSHIP OF THE PRE-STRESSING WIRE



a) LONGITUDINAL SECTION OF THE FABRICATED SPECIMEN

b) COMPRESSION TEST SPECIMEN

FIG. 6.8 FABRICATED AND COMPRESSION TEST SPECIMENS.

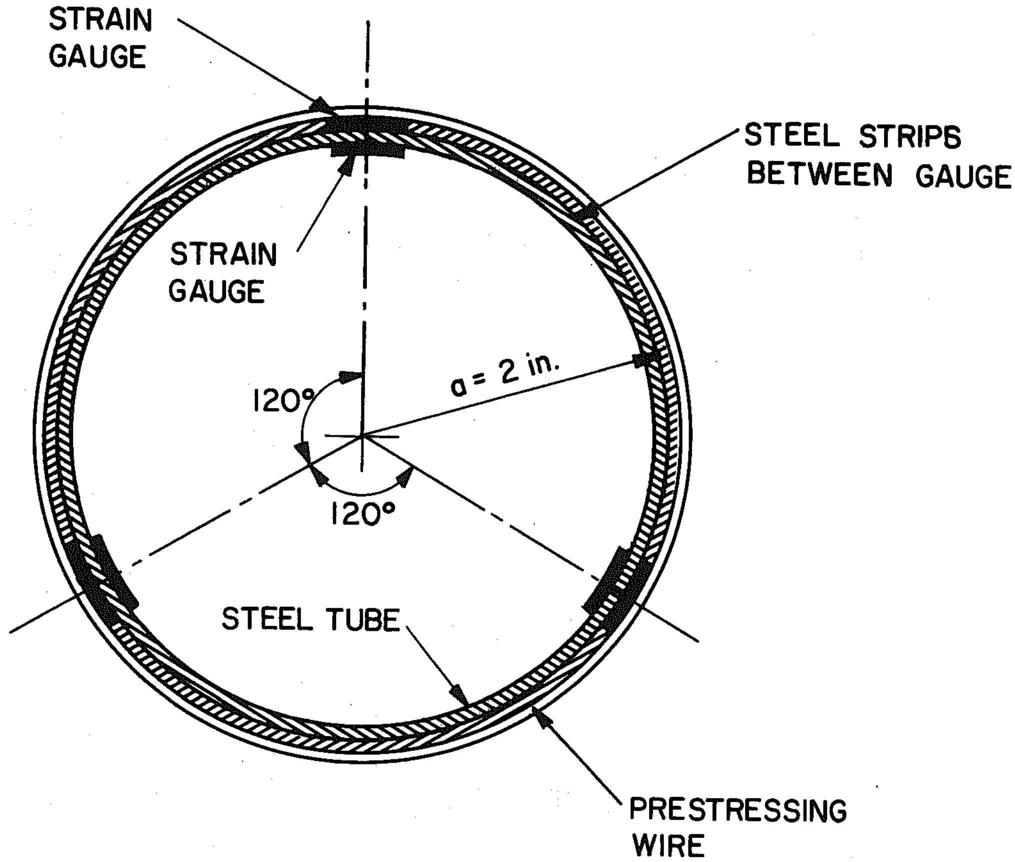


FIG. 6.9 PROTECTION AGAINST PRESTRESSING WIRE OF STRAIN GAUGES PLACED ON OUTSIDE SURFACE OF THE STEEL TUBE

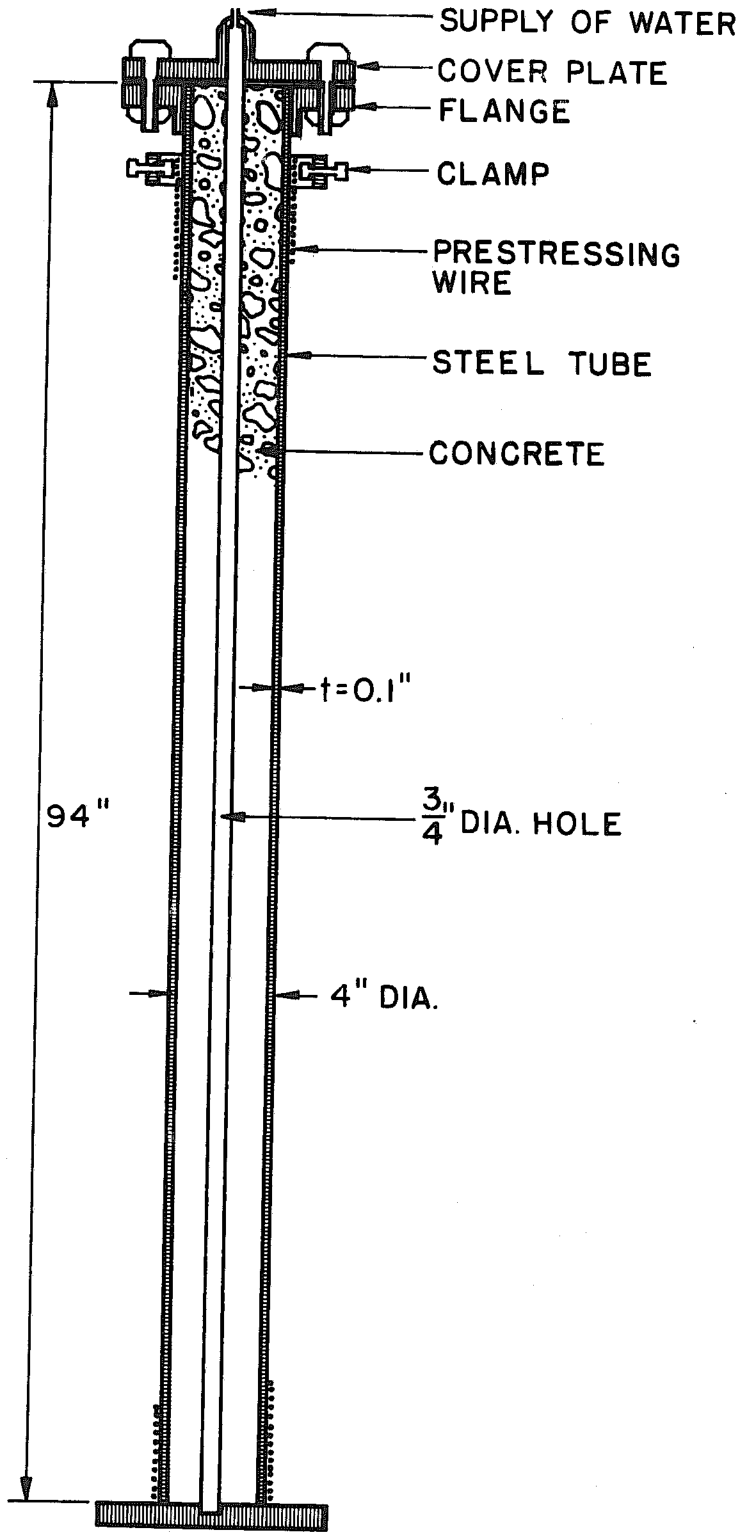


FIG. 6.10 LONG SPECIMEN

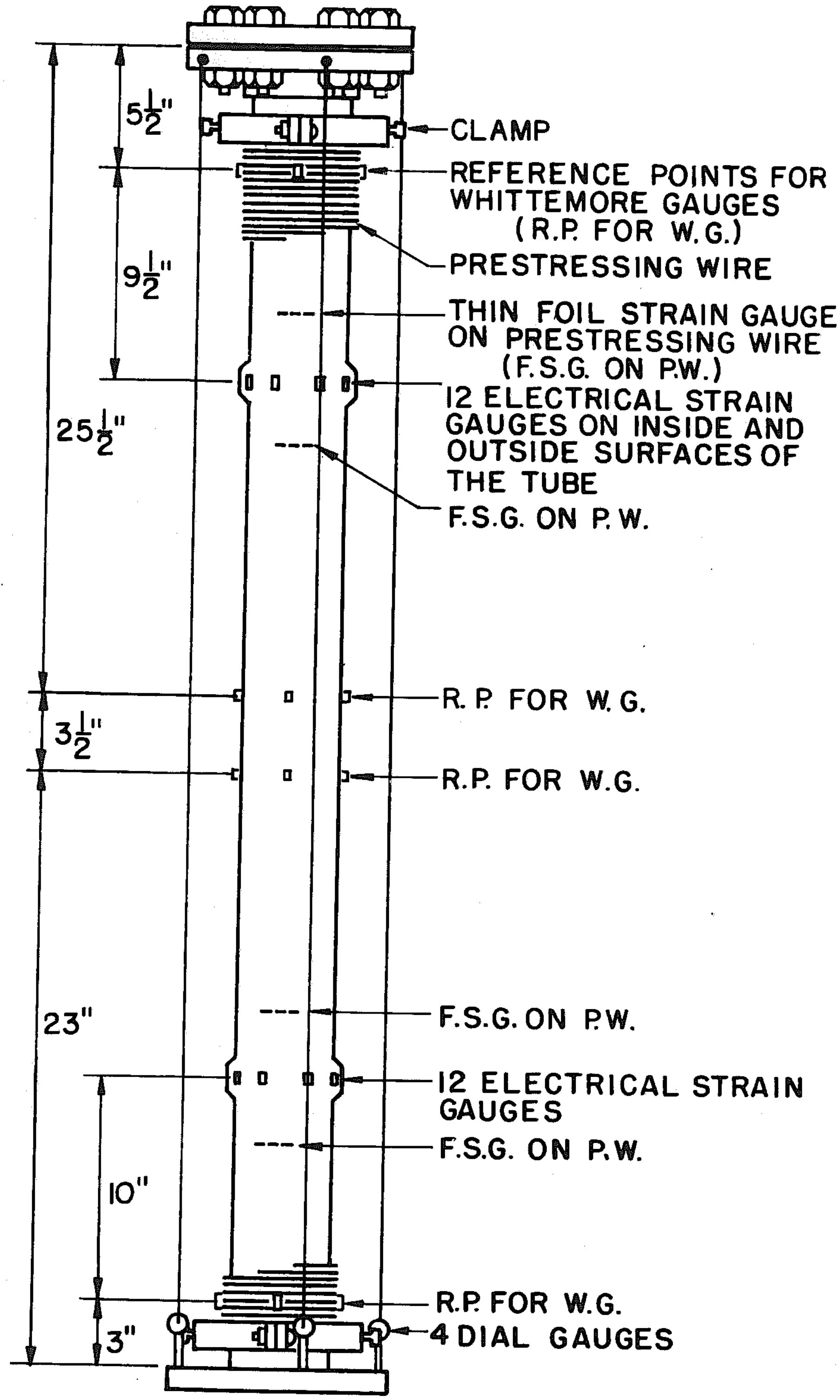


FIG. 6.11 SHORT SPECIMEN, INSTRUMENTATIONS FOR RECORDING EXPANSION STRAINS

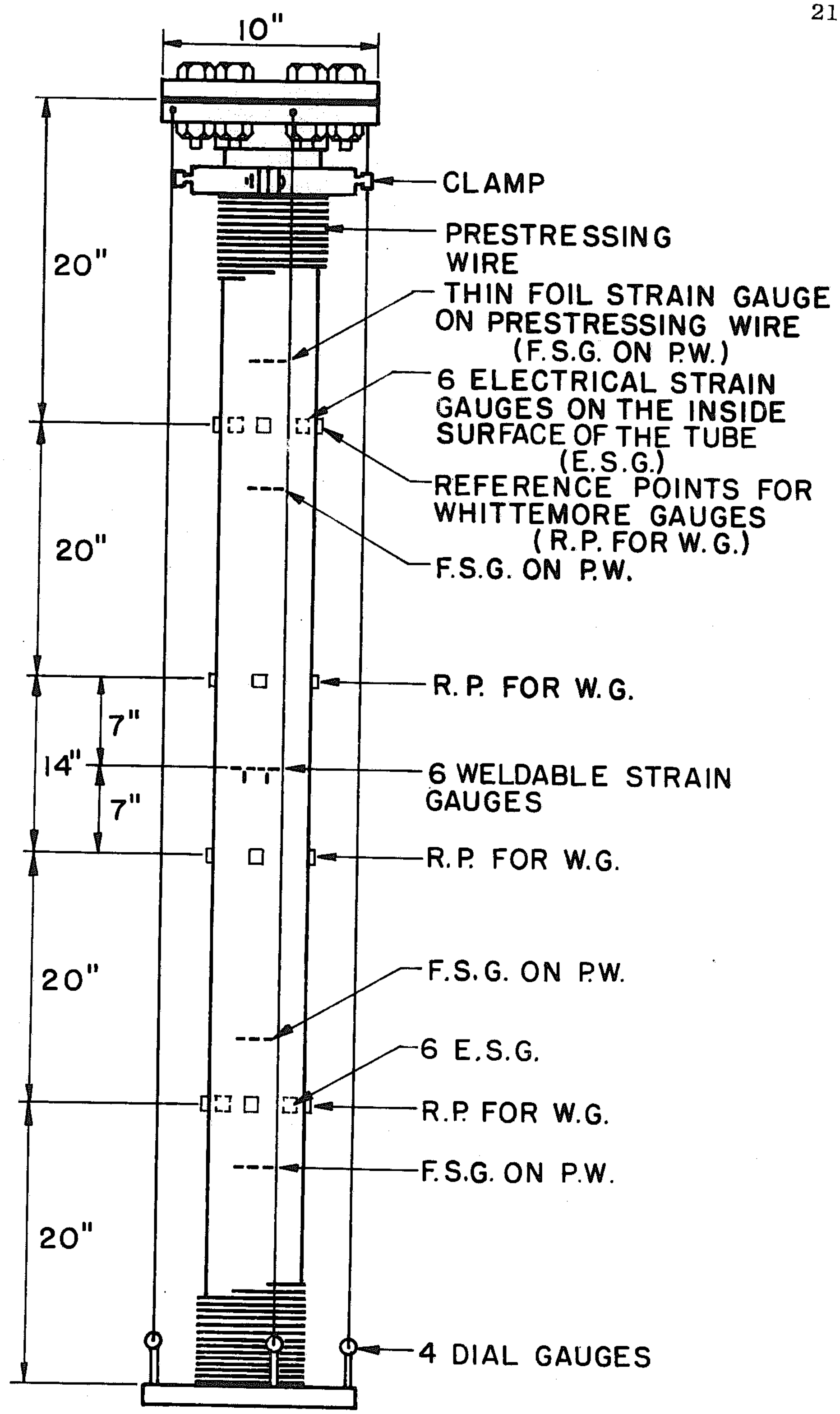
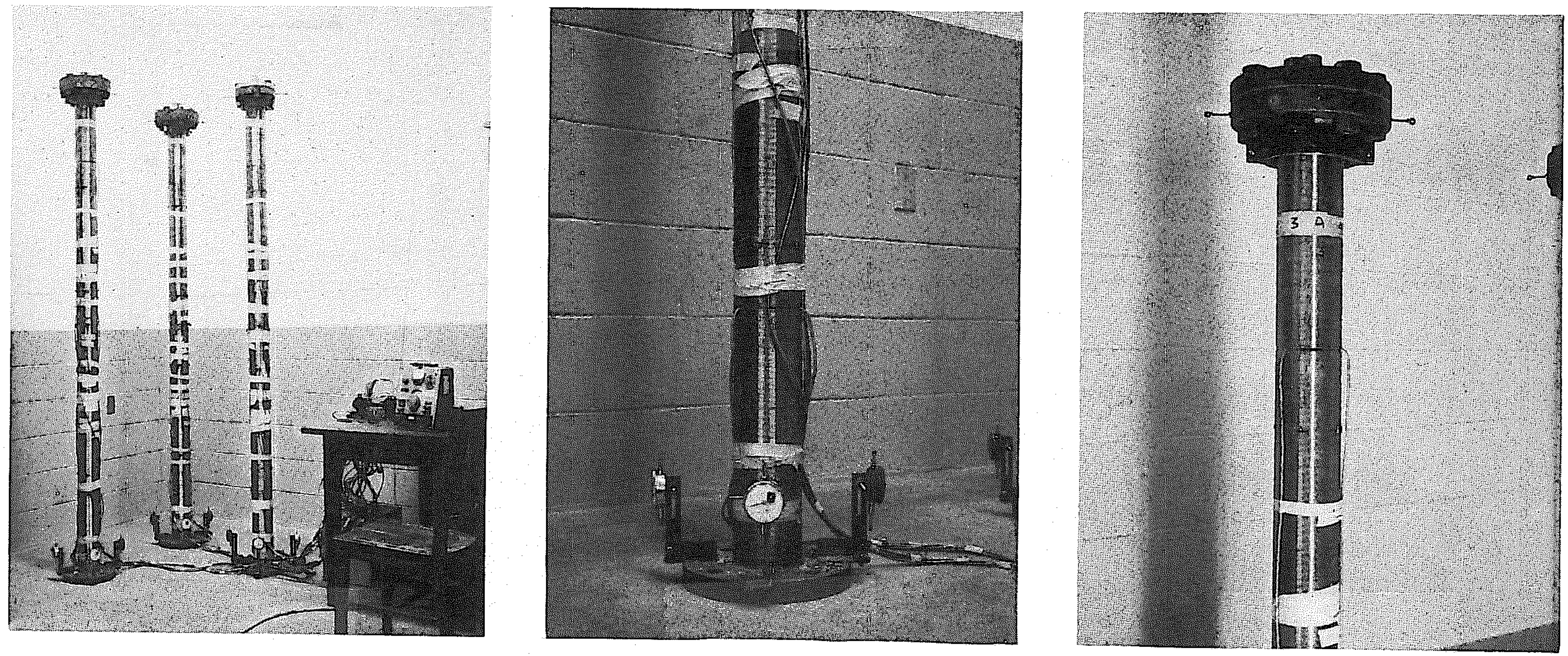


FIG. 6.12 LONG SPECIMEN, INSTRUMENTATIONS FOR RECORDING EXPANSION STRAINS



a- OVERALL VIEW

b- CLOSE-UP OF THE BOTTOM PART OF SPECIMEN 5

c- CLOSE-UP OF THE TOP PART OF SPEC-IMEN 5

FIG. 6.13 LONG SPECIMENS DURING CURING PERIODS

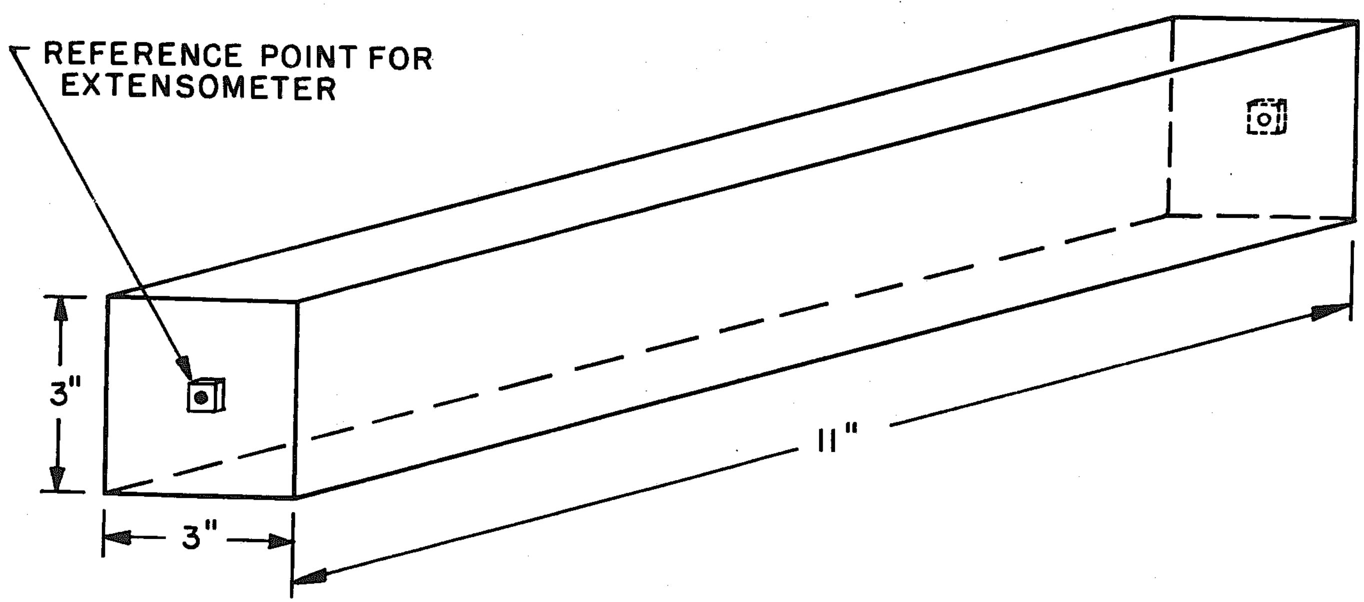


FIG. 6.14 FREE EXPANSION CONTROL SPECIMEN

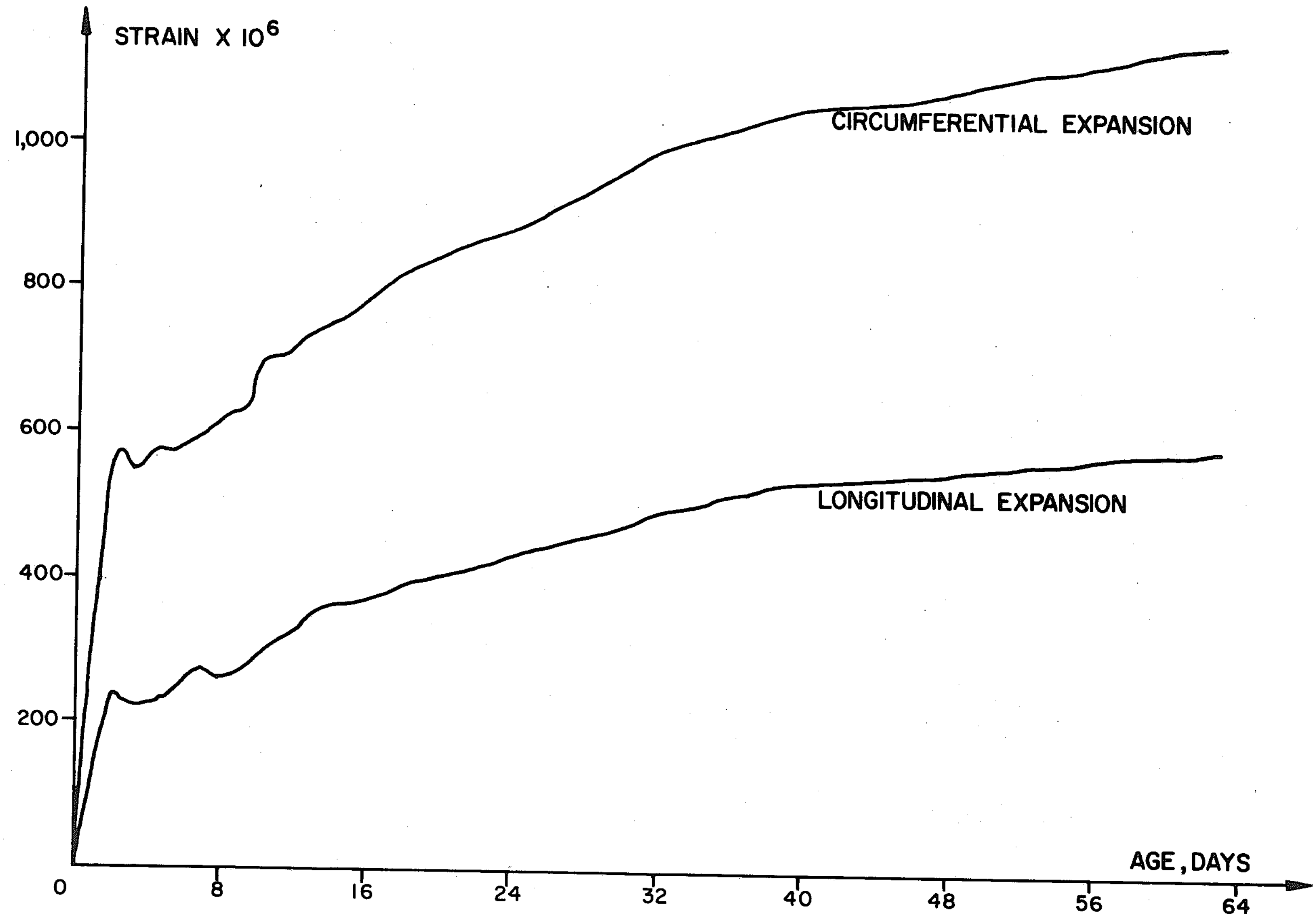


FIG. 6.15 SHORT SPECIMENS– HISTORY OF EXPANSION

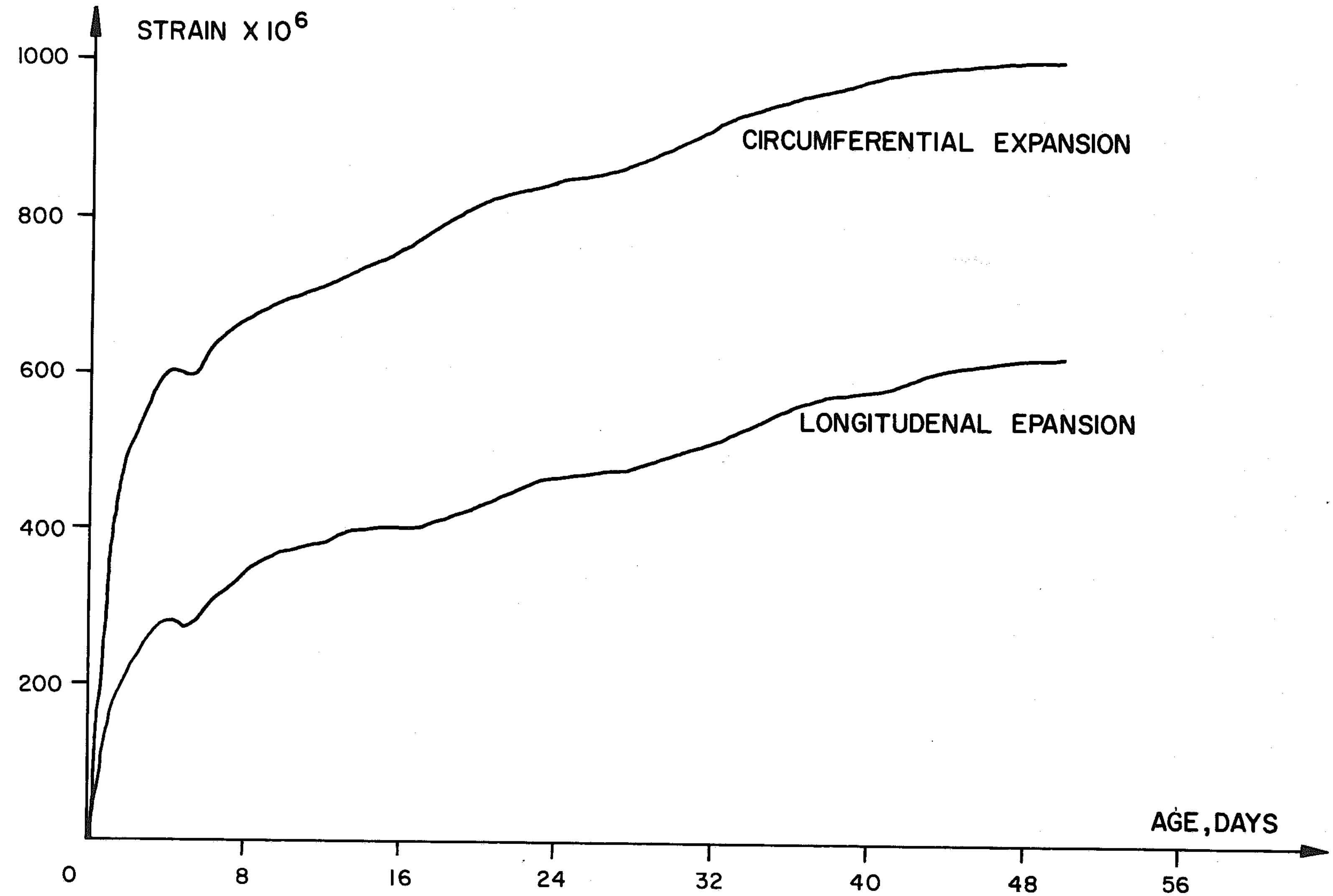


FIG. 6.16 LONG SPECIMENS – HISTORY OF EXPANSION

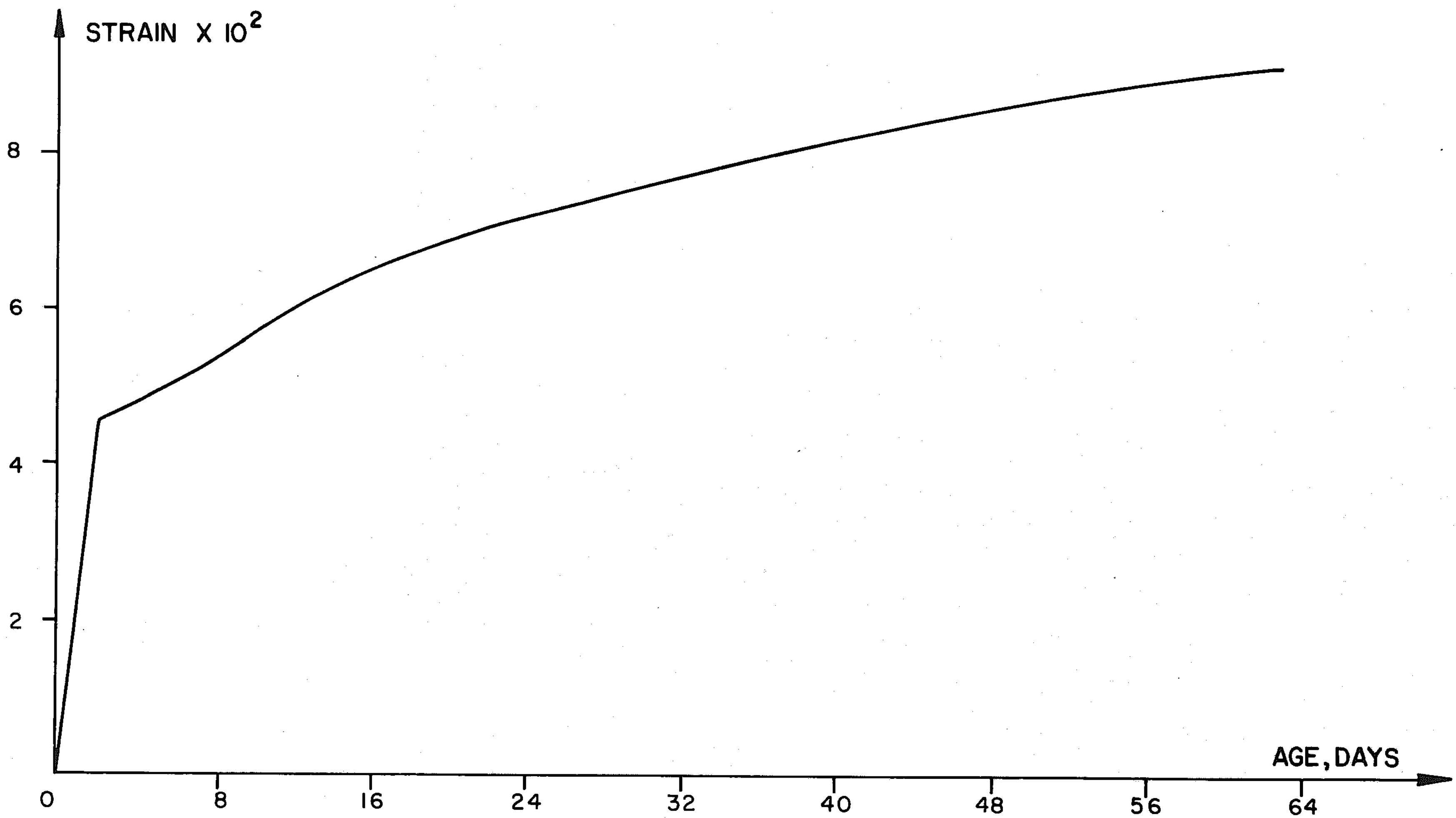


FIG. 6.17 FREE EXPANSION CONTROL SPECIMENS – HISTORY OF EXPANSION

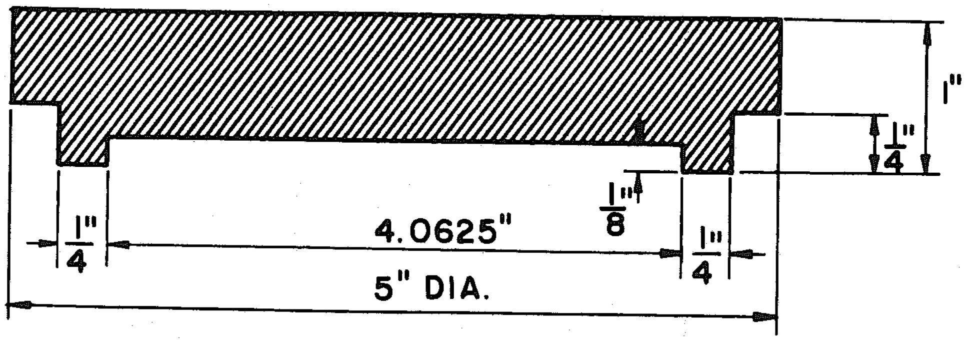


FIG. 6.18 END PLATE FOR LOADING SHORT SPECIMENS

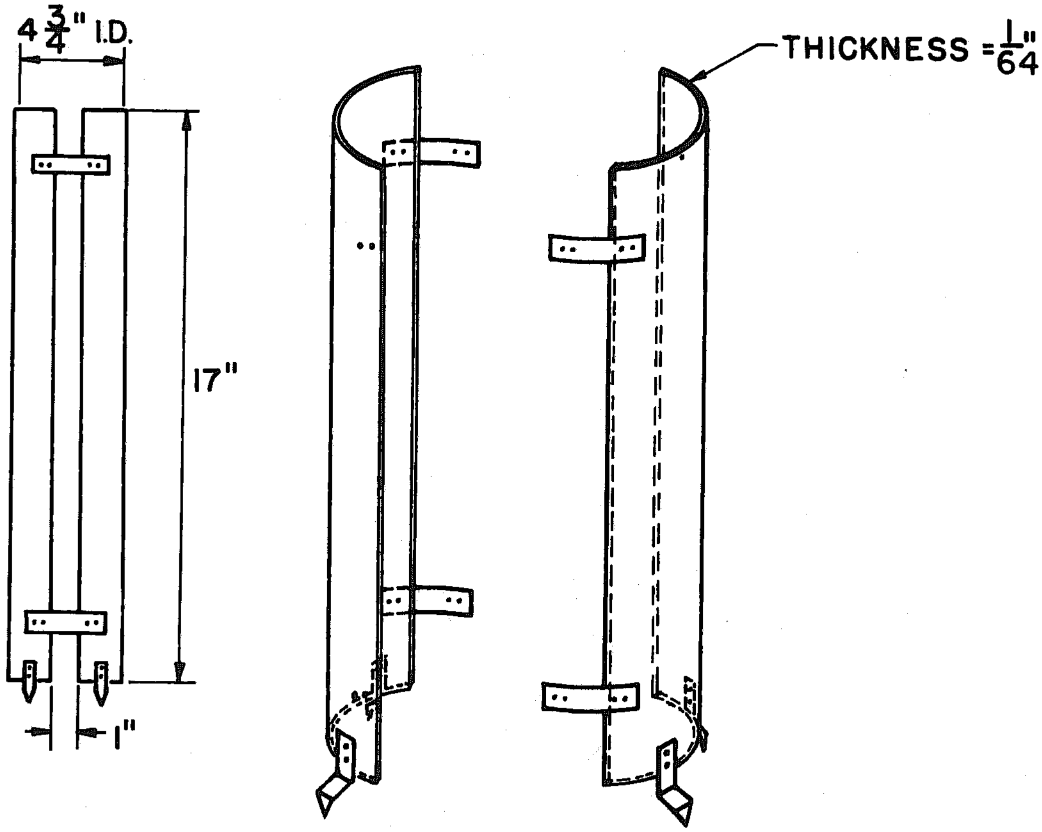


FIG. 6.19 SAFETY DEVICE AGAINST FAILURE OF PRESTRESSING WIRE

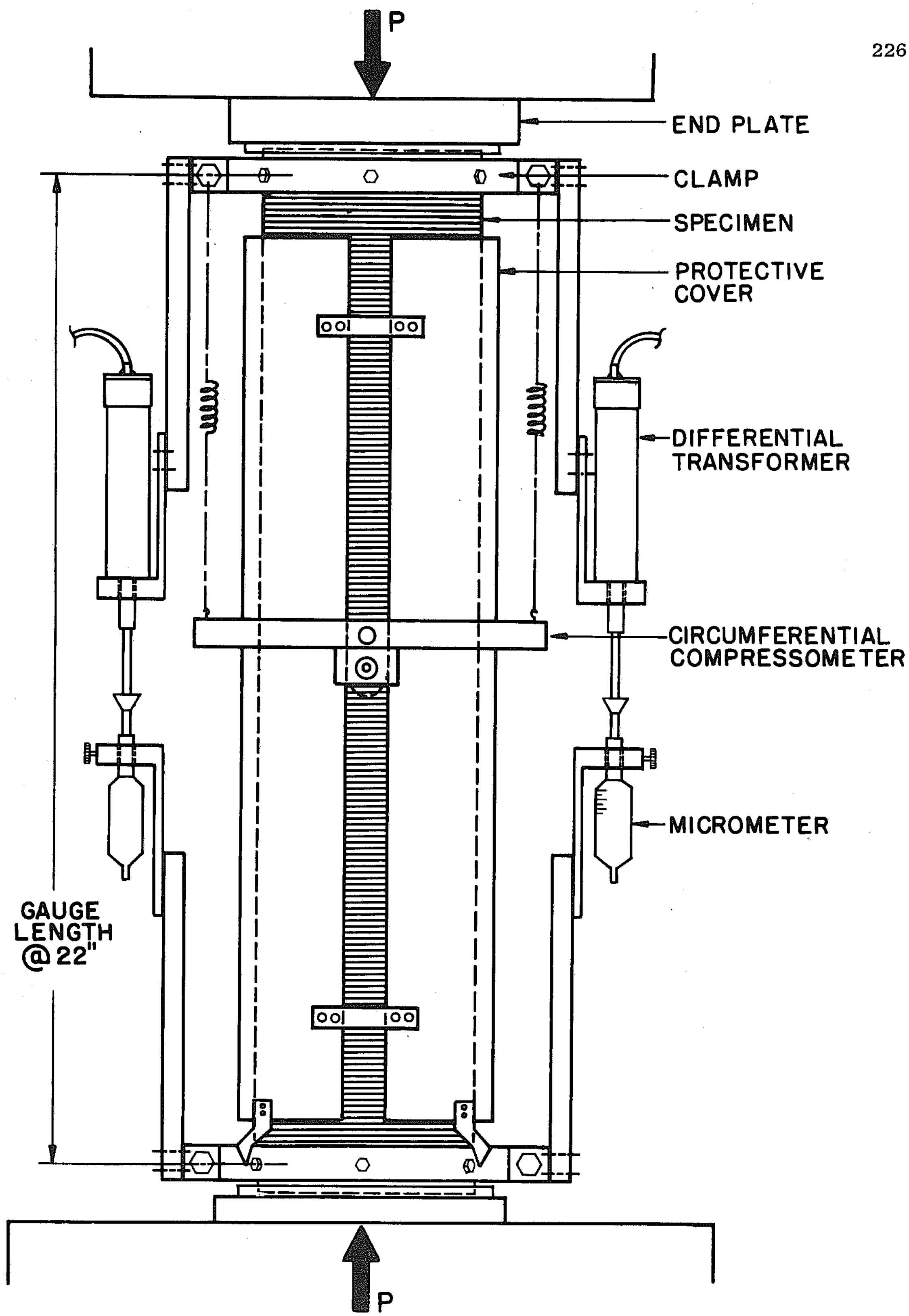


FIG. 6.20 INSTRUMENTATION OF SHORT SPECIMEN DURING LOADING TEST

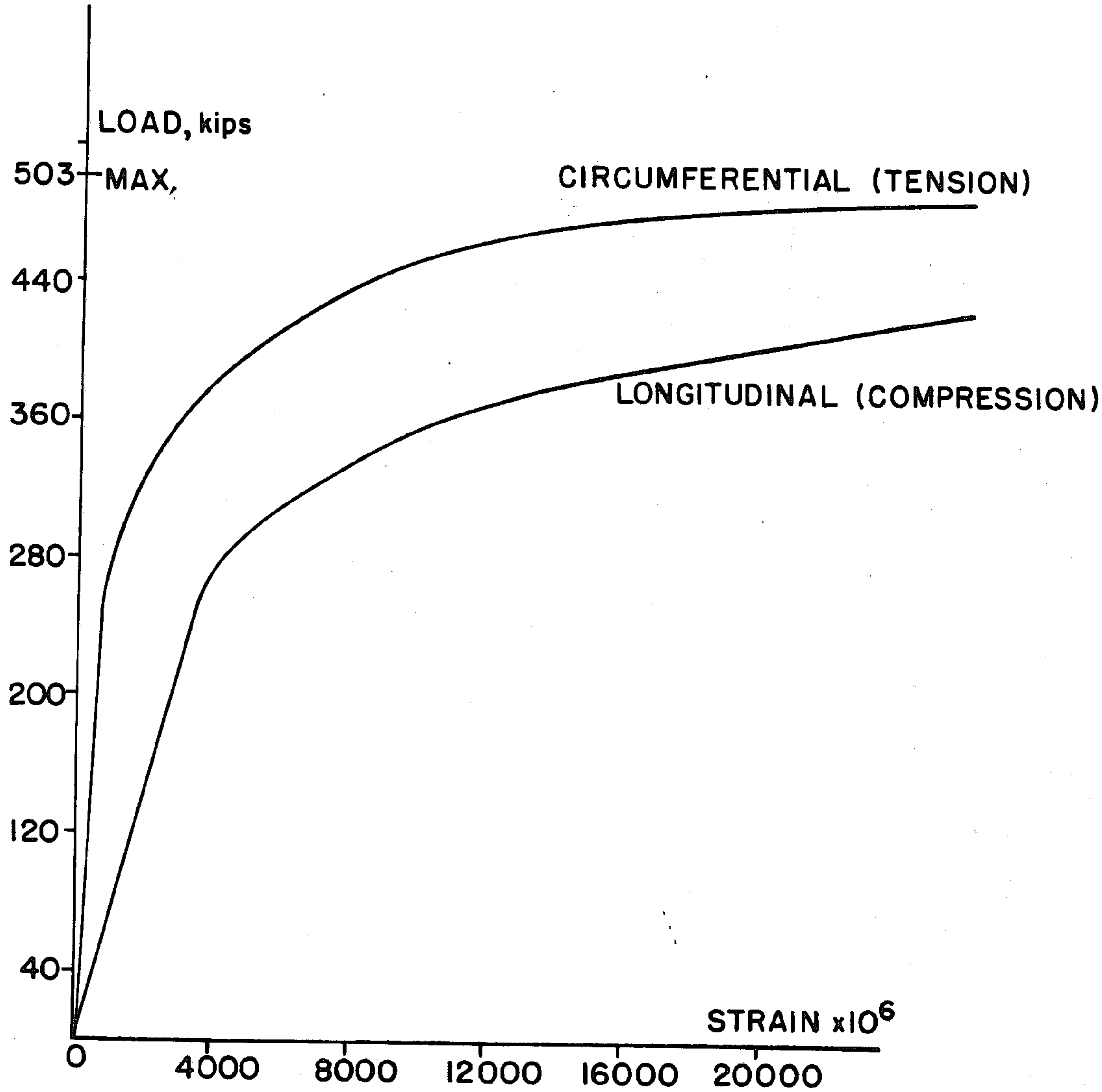


FIG. 6.21 LOAD-STRAIN DIAGRAM, SPECIMEN NO. 1

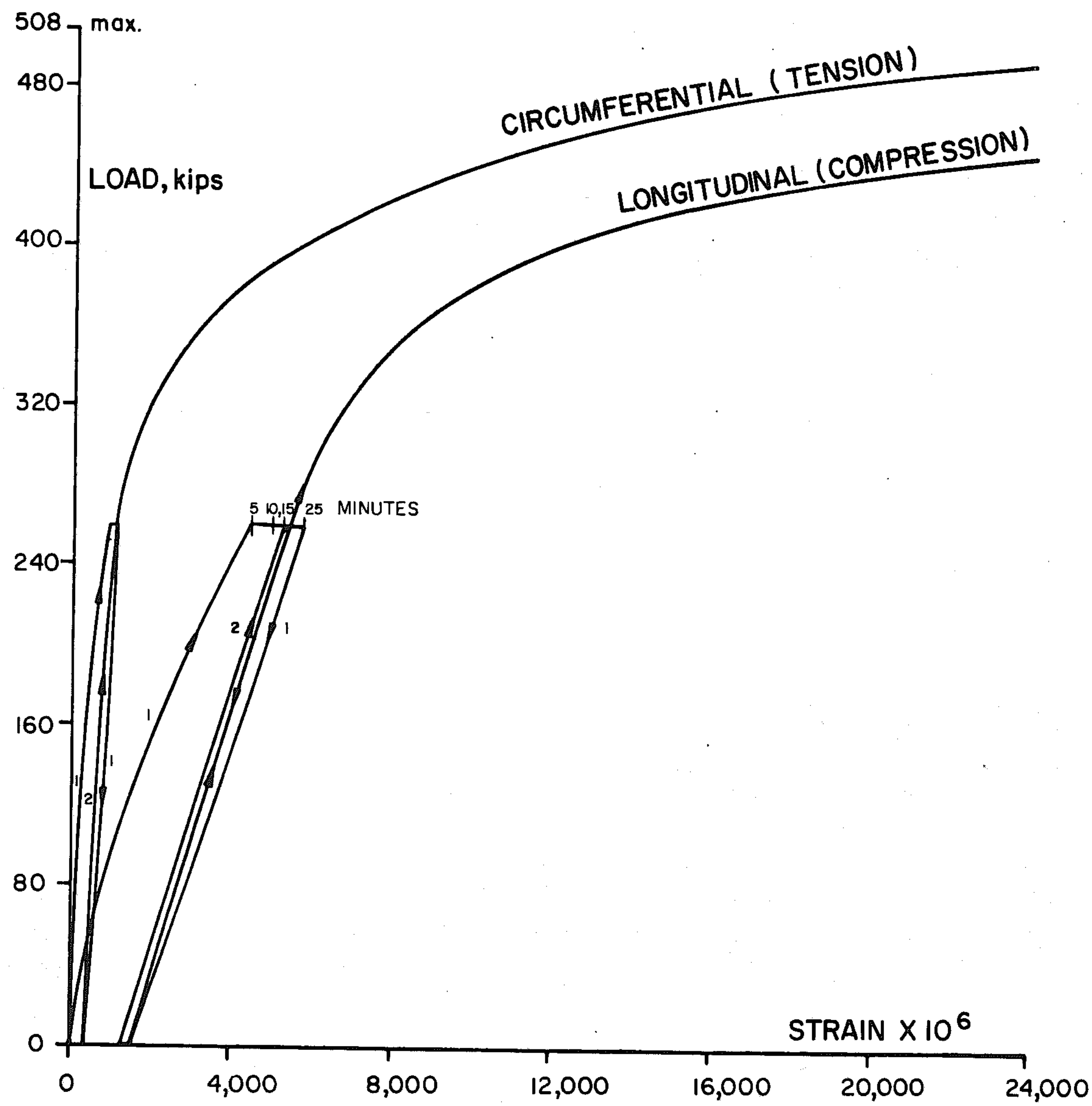


FIG. 6.22 LOAD-STRAIN DIAGRAM, SPECIMEN 2

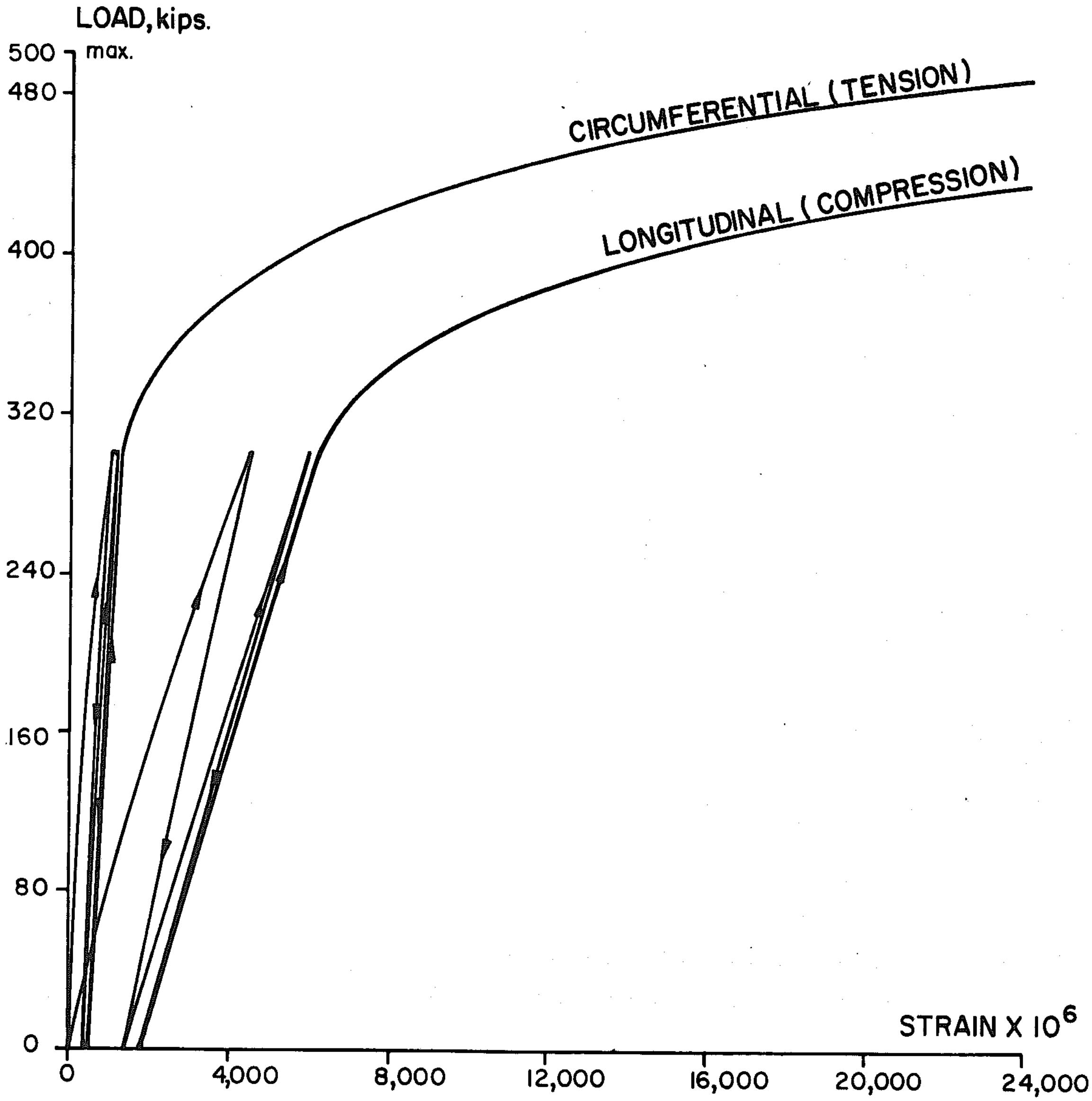


FIG. 6.23 LOAD-STRAIN DIAGRAM, SPECIMEN 3

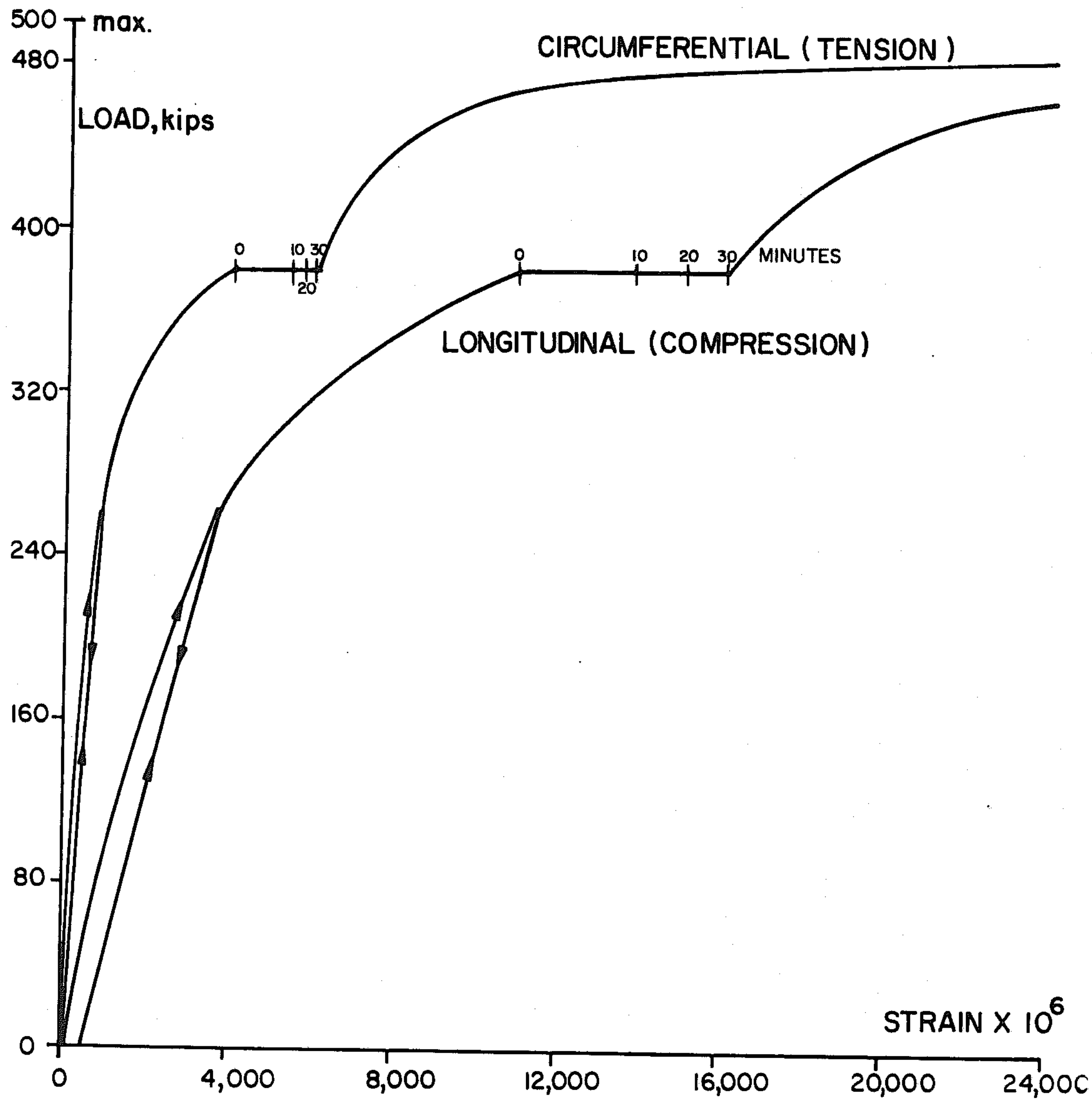
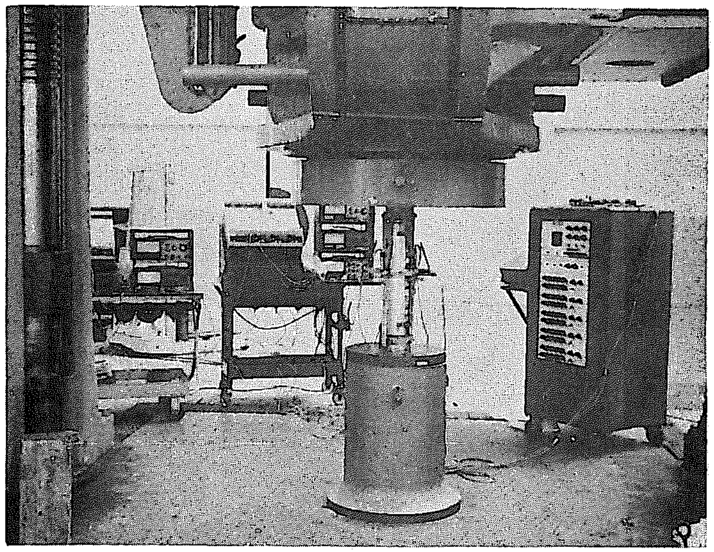
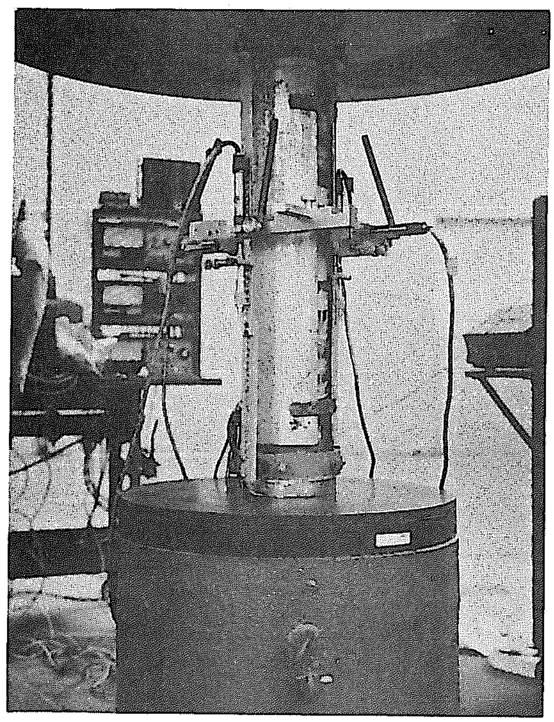


FIG. 6.24 LOAD-STRAIN DIAGRAM, SPECIMEN 4

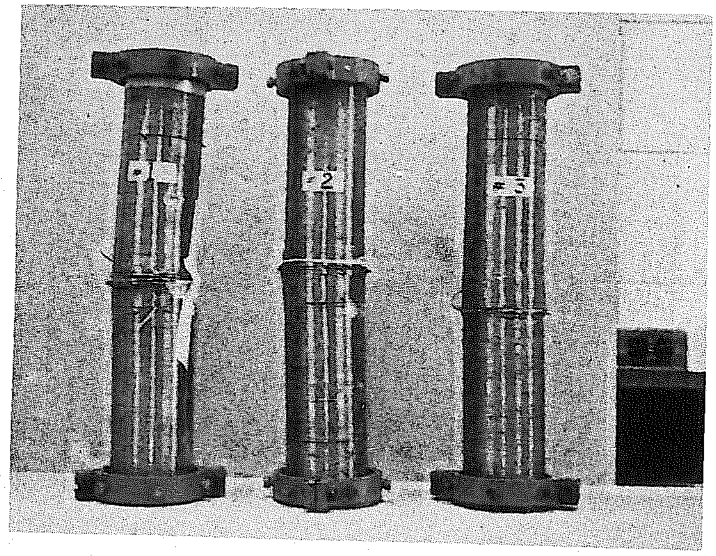


a- OVERALL VIEW OF SPECIMEN AND INSTRUMENTATION

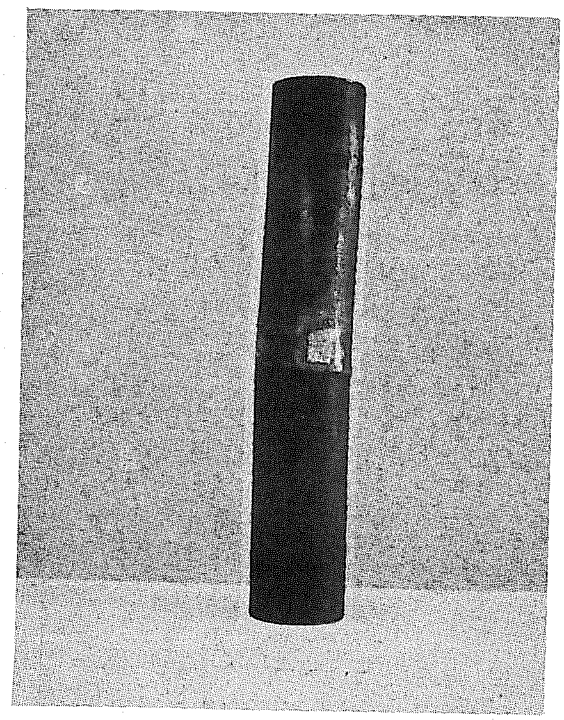


b- CLOSE-UP

FIG. 6.25 TESTING OF SHORT SPECIMENS



a – SPECIMENS 1,2& 3



b – APPEARANCE OF SPECIMEN 4 AFTER PRE-STRESSING WIRE HAS BEEN REMOVED

FIG. 6.26 SHORT SPECIMENS AFTER FAILURE

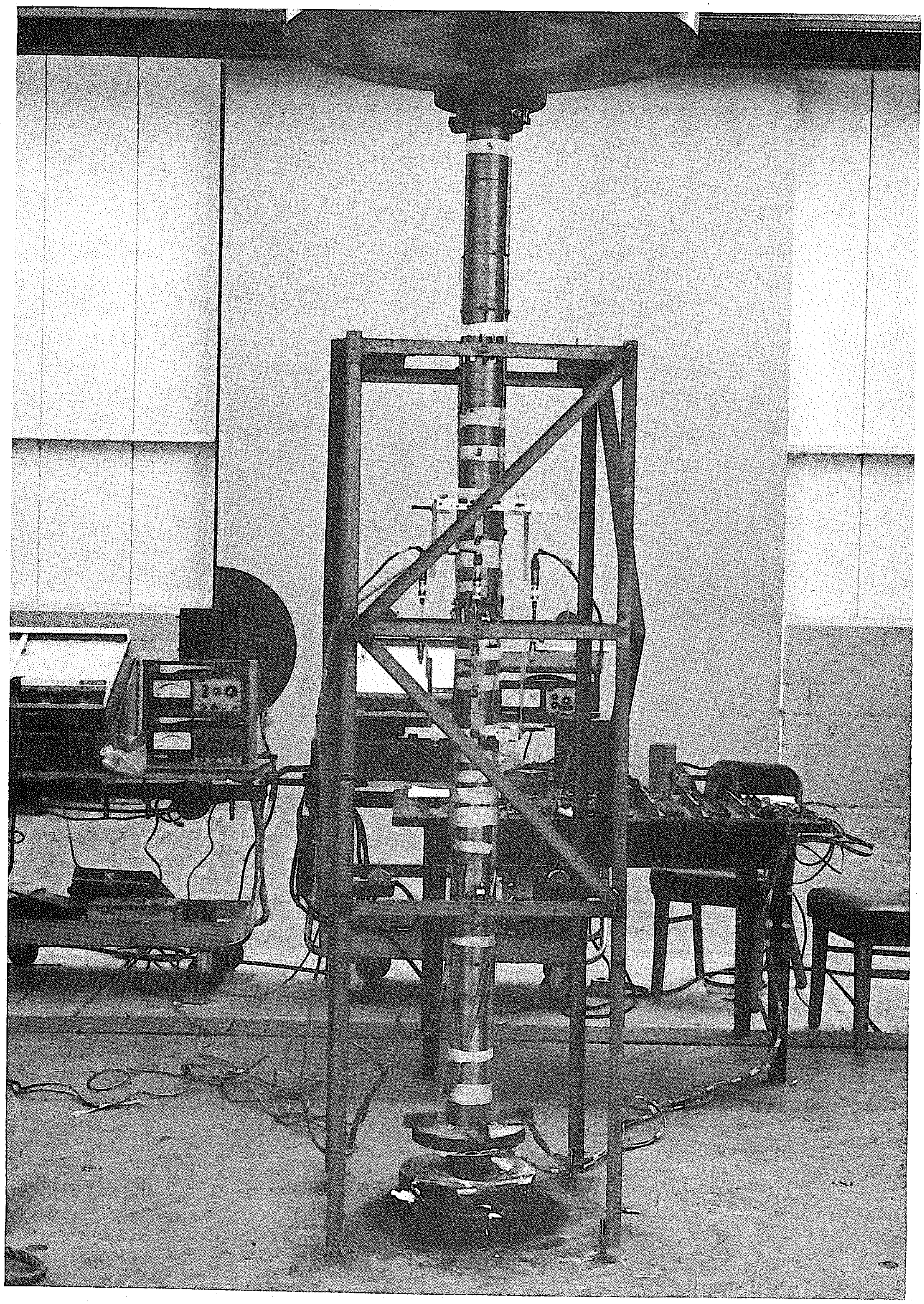


FIG. 6 27 LONG SPECIMEN DURING TESTING

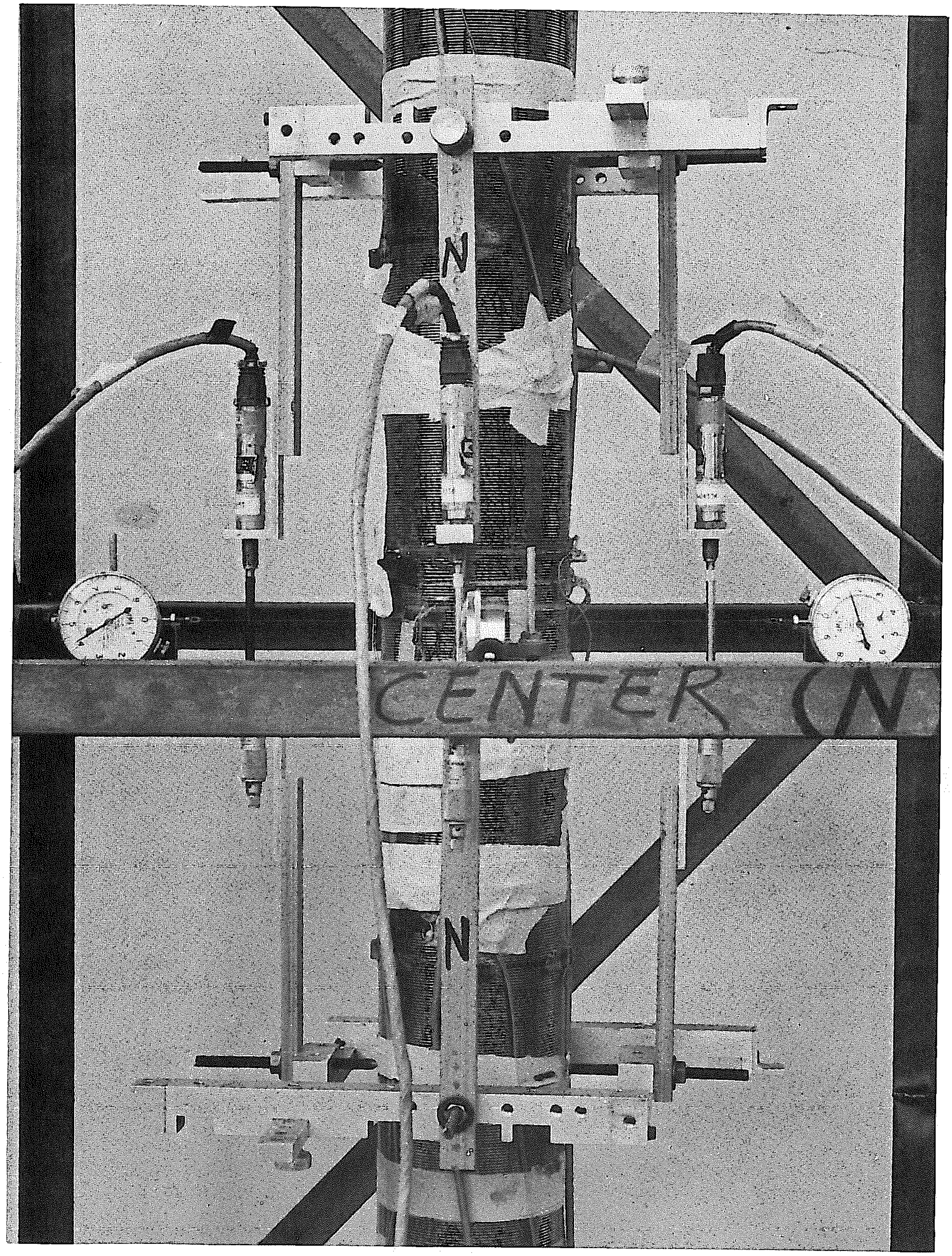


FIG. 6.28 LONG SPECIMEN, INSTRUMENTATION FOR MEASURING AXIAL STRAIN

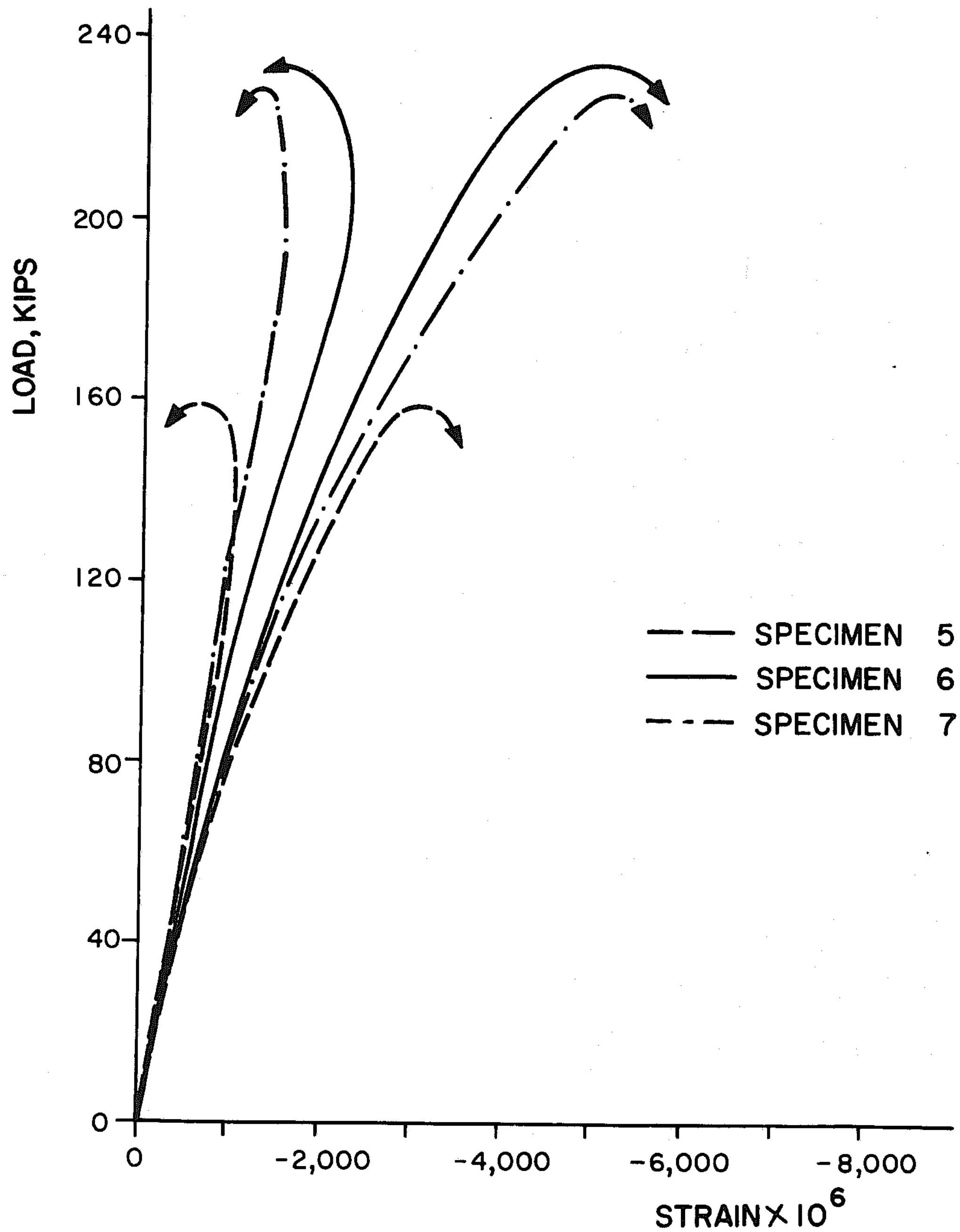


FIG. 6.29 LONGITUDINAL STRAINS IN THE EXTREME FIBERS AT MID-HEIGHT AND IN THE PRINCIPAL PLANE OF BENDING

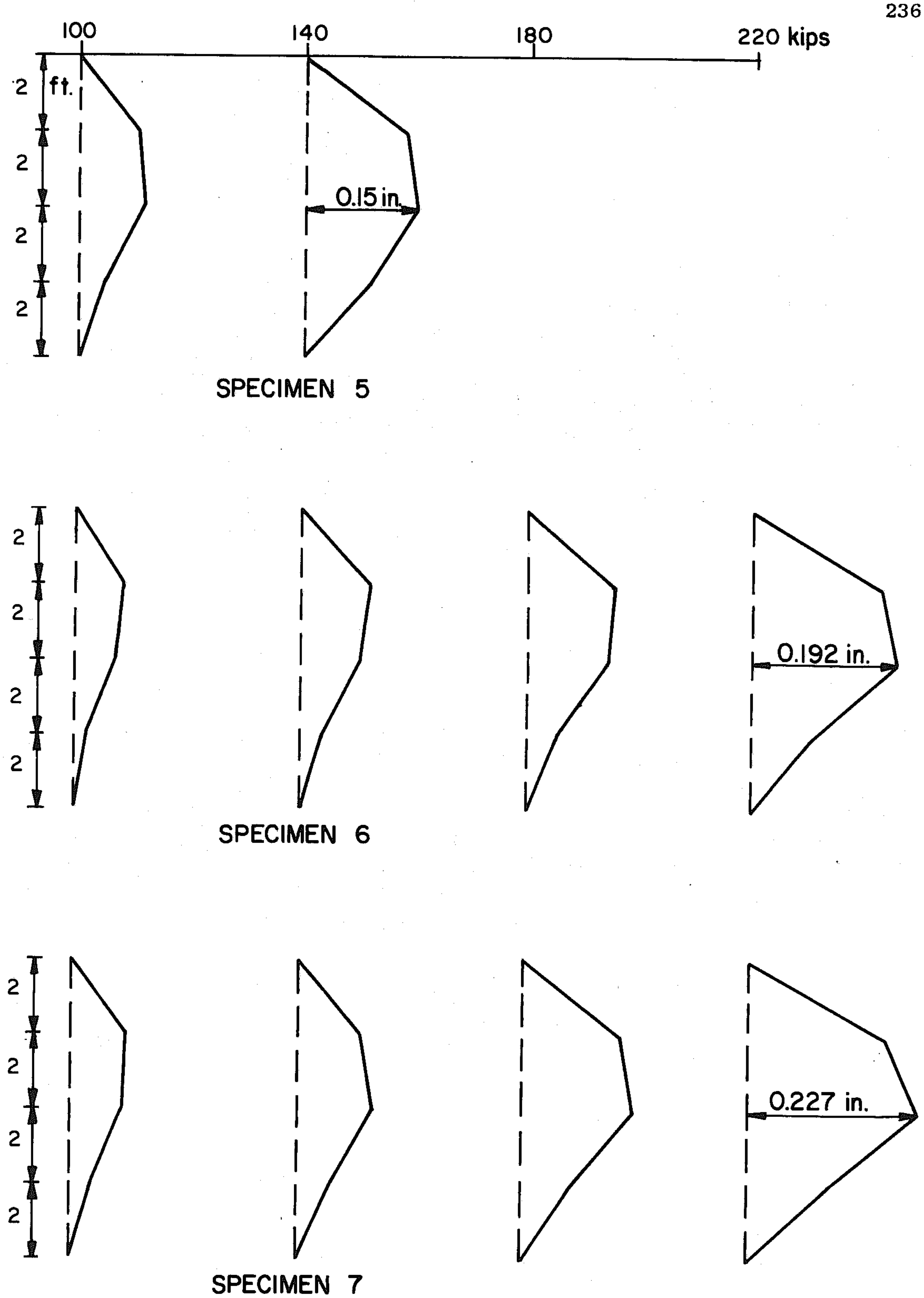


FIG. 6.30 TRANSVERSE DEFLECTIONS IN THE PRINCIPAL PLANE OF BENDING

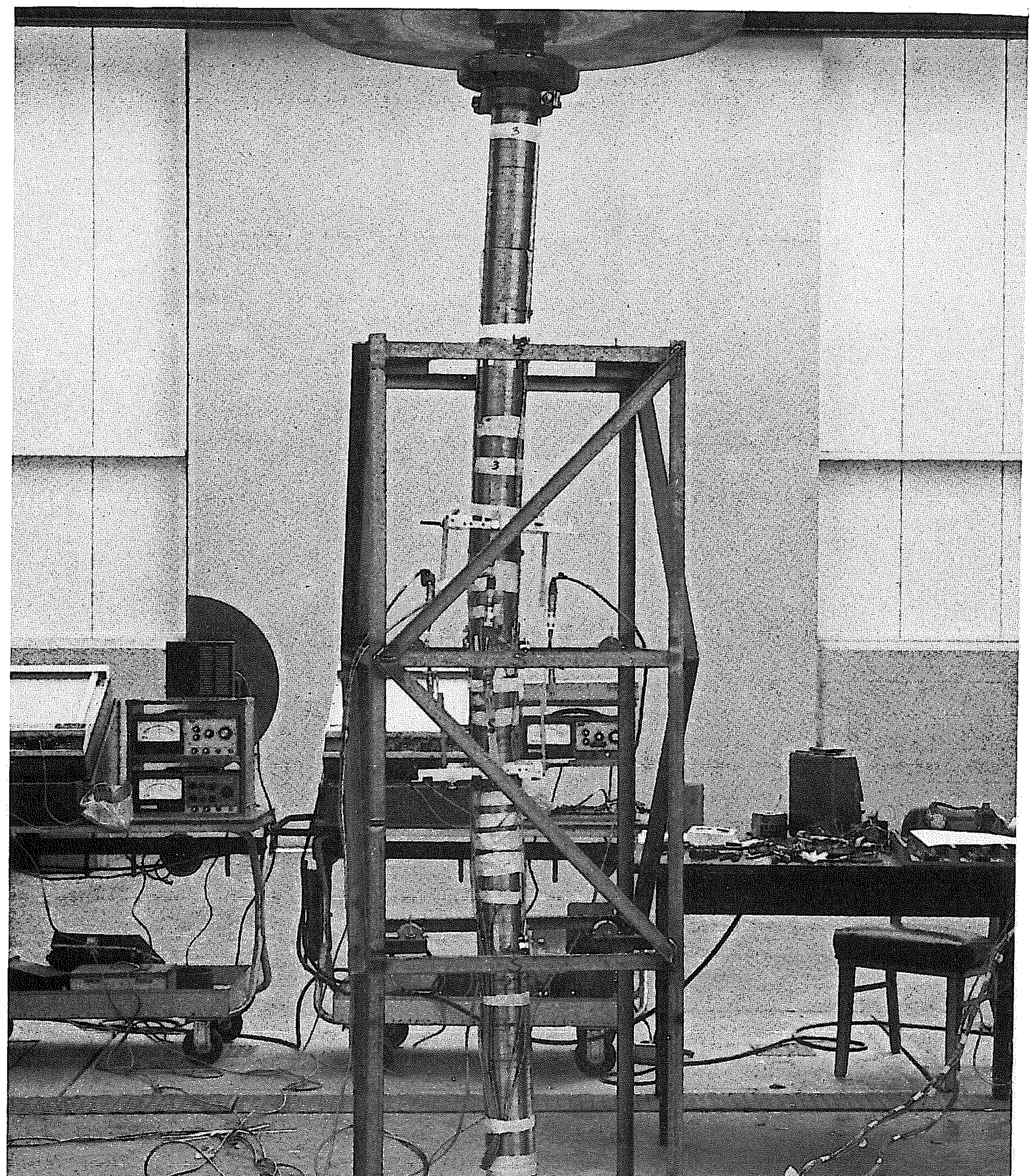
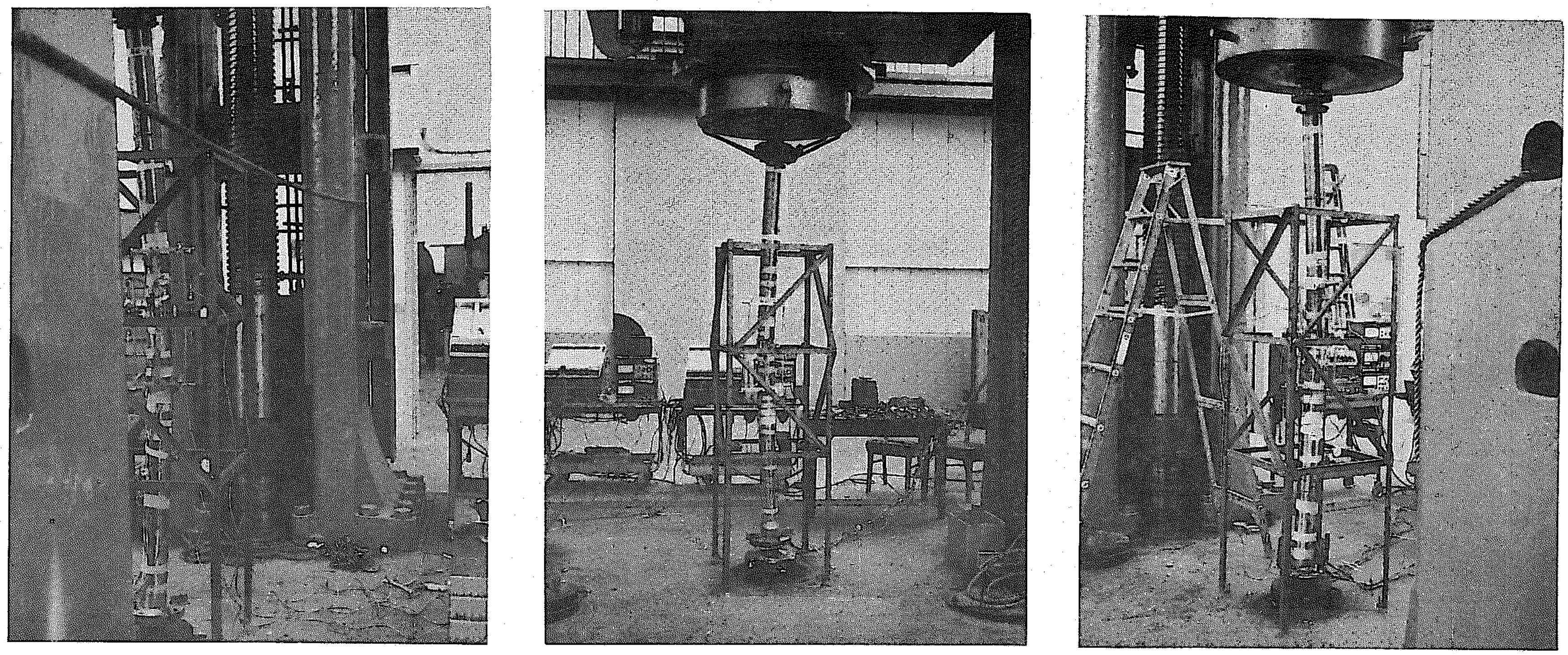


FIG.6.31 SPECIMEN 6, AFTER FAILURE



a- SPECIMEN 5   b - SPECIMEN 6   c - SPECIMEN 7

FIG. 6.32 LONG SPECIMENS AFTER FAILURE

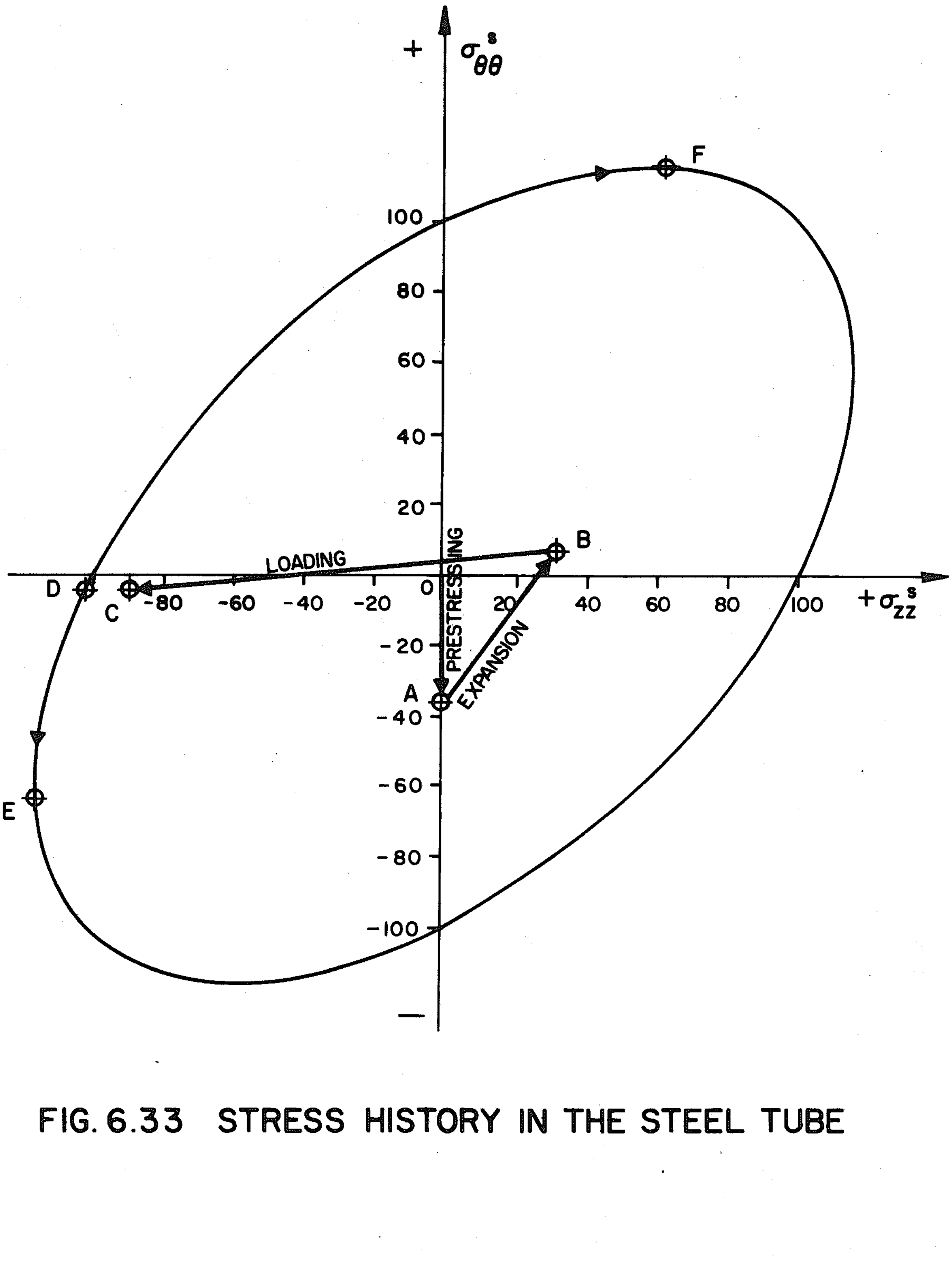


FIG. 6.33 STRESS HISTORY IN THE STEEL TUBE

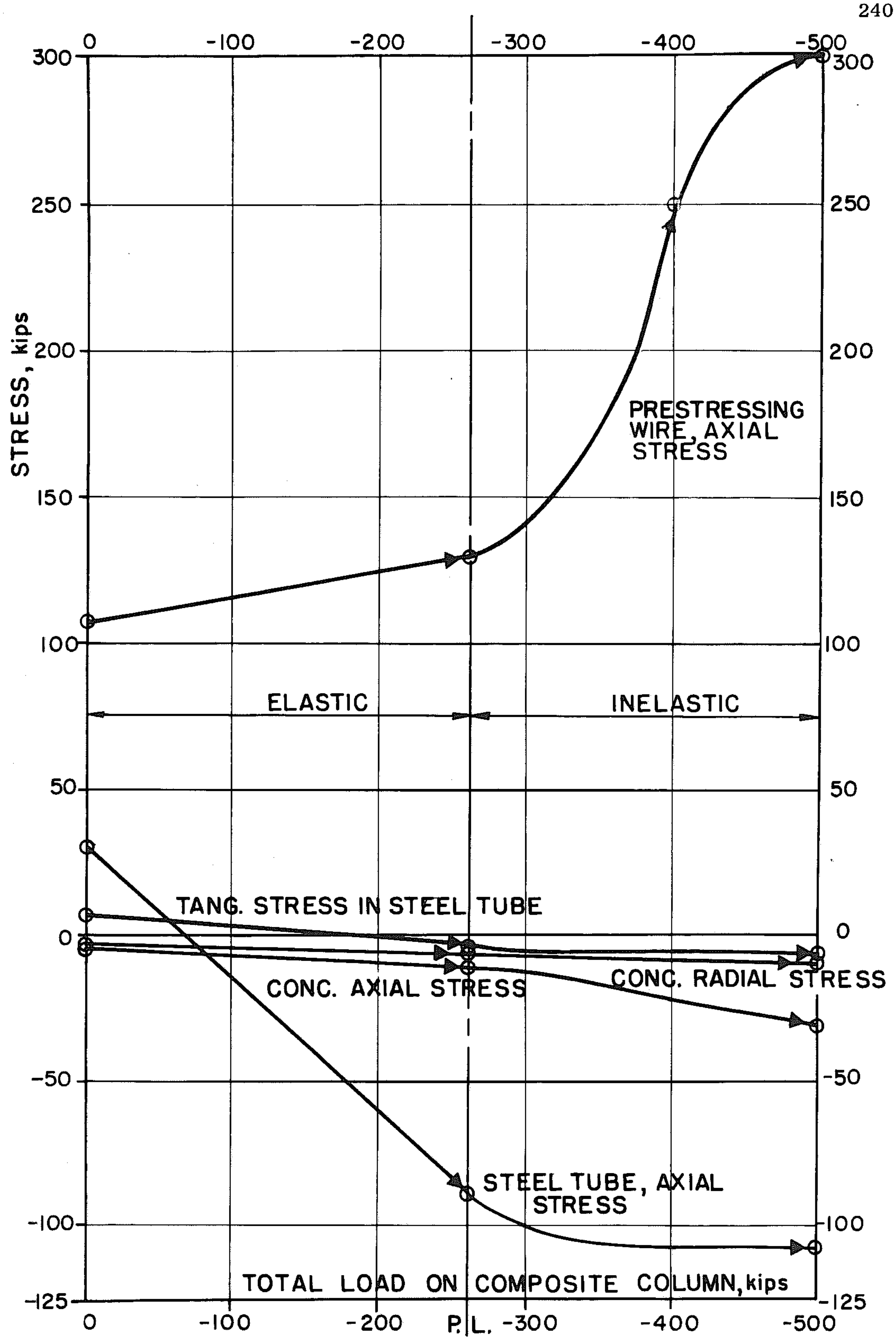


FIG. 6.34 COMPOSITE COLUMN UNDER LOAD, STRESS IN COMPONENT MATERIALS VERSUS TOTAL LOAD

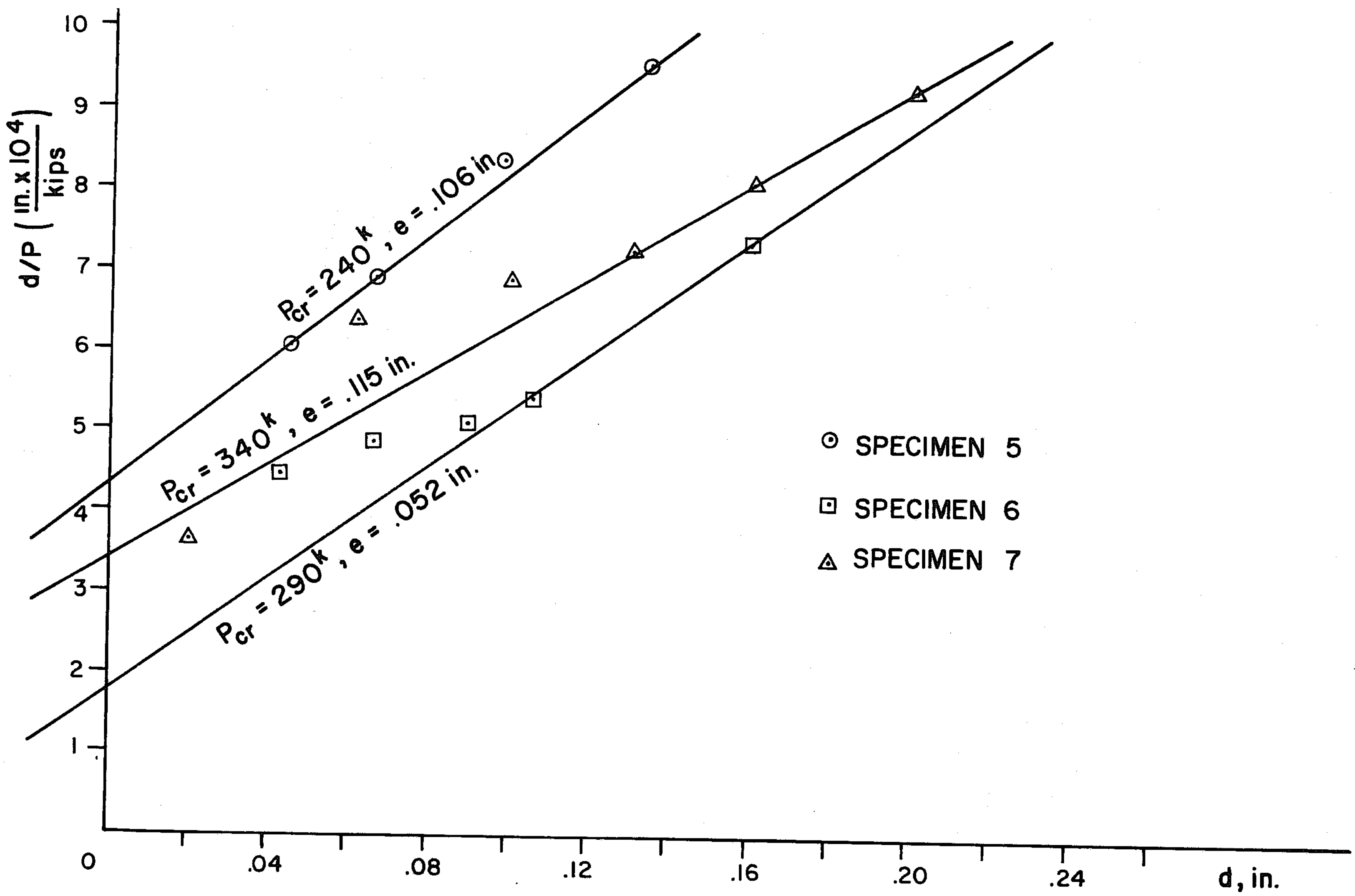


FIG. 6.35 SOUTHWELL'S PLOT

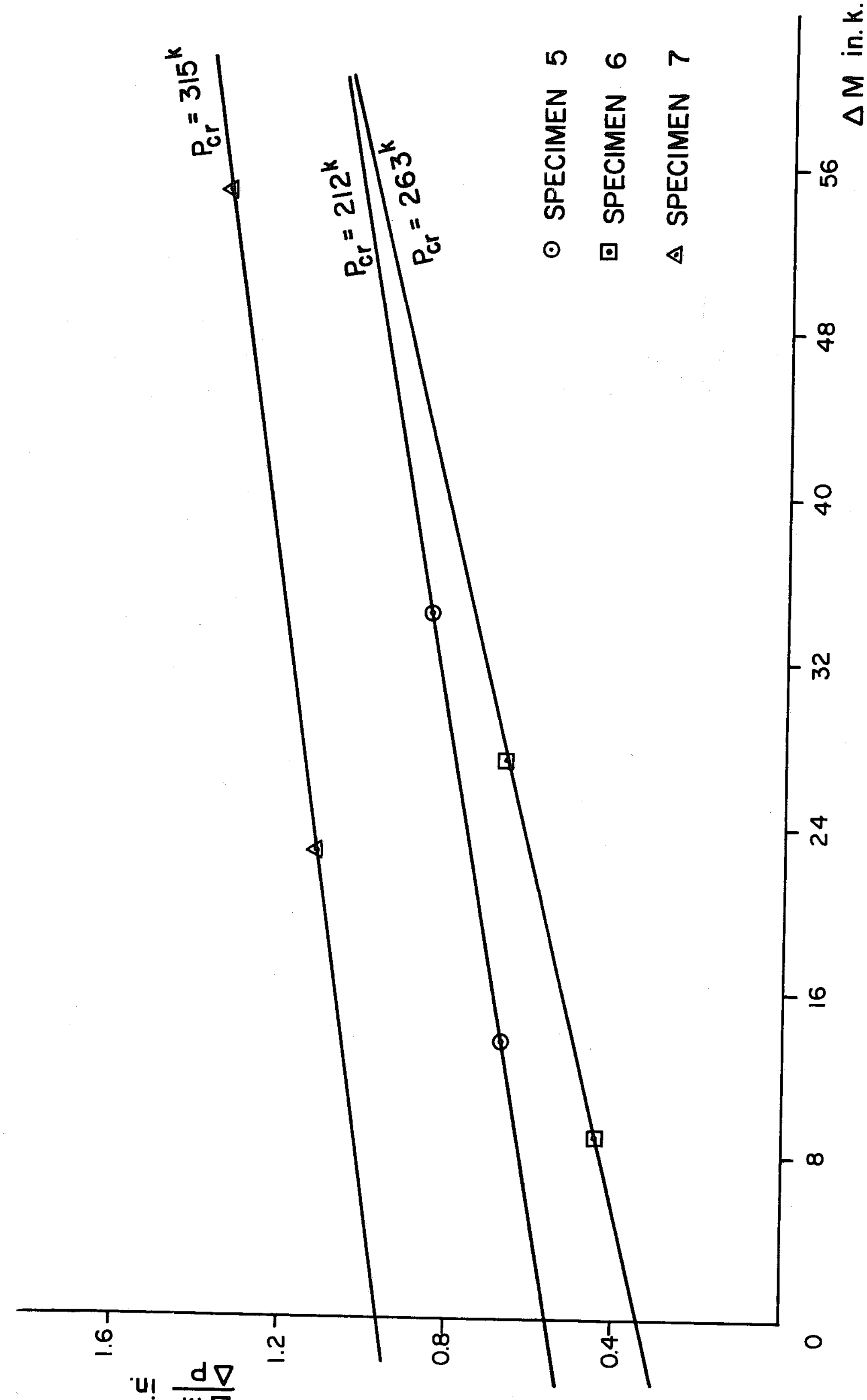


FIG. 6.36 LONDQUEST'S METHOD

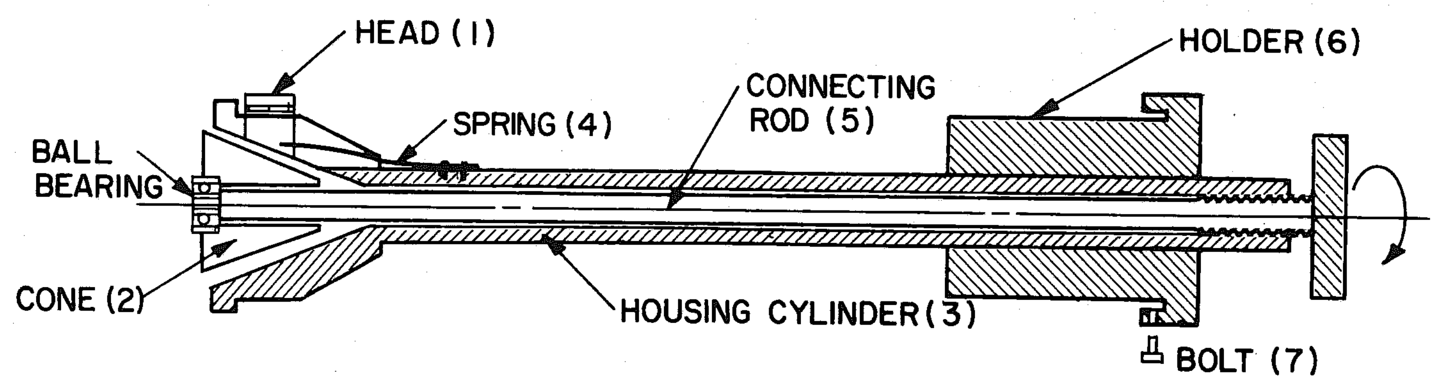


FIG. A.1 - EXPANDER (LONGITUDINAL CROSS-SECTION)

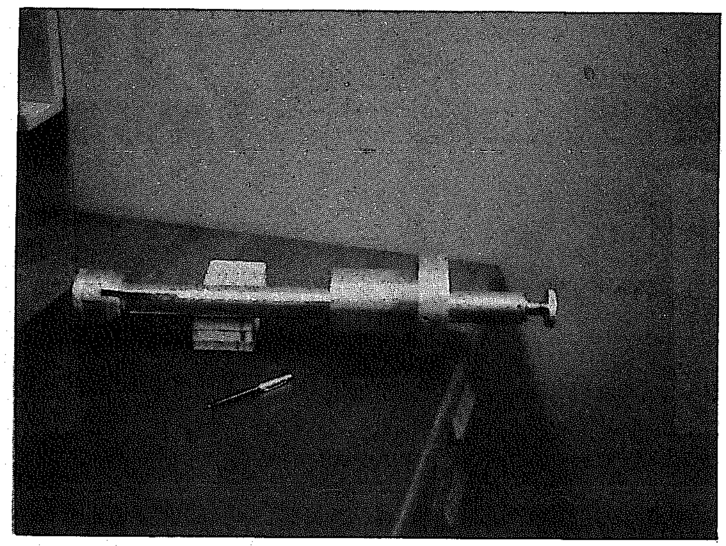


FIG. A.2 THE EXPANDER

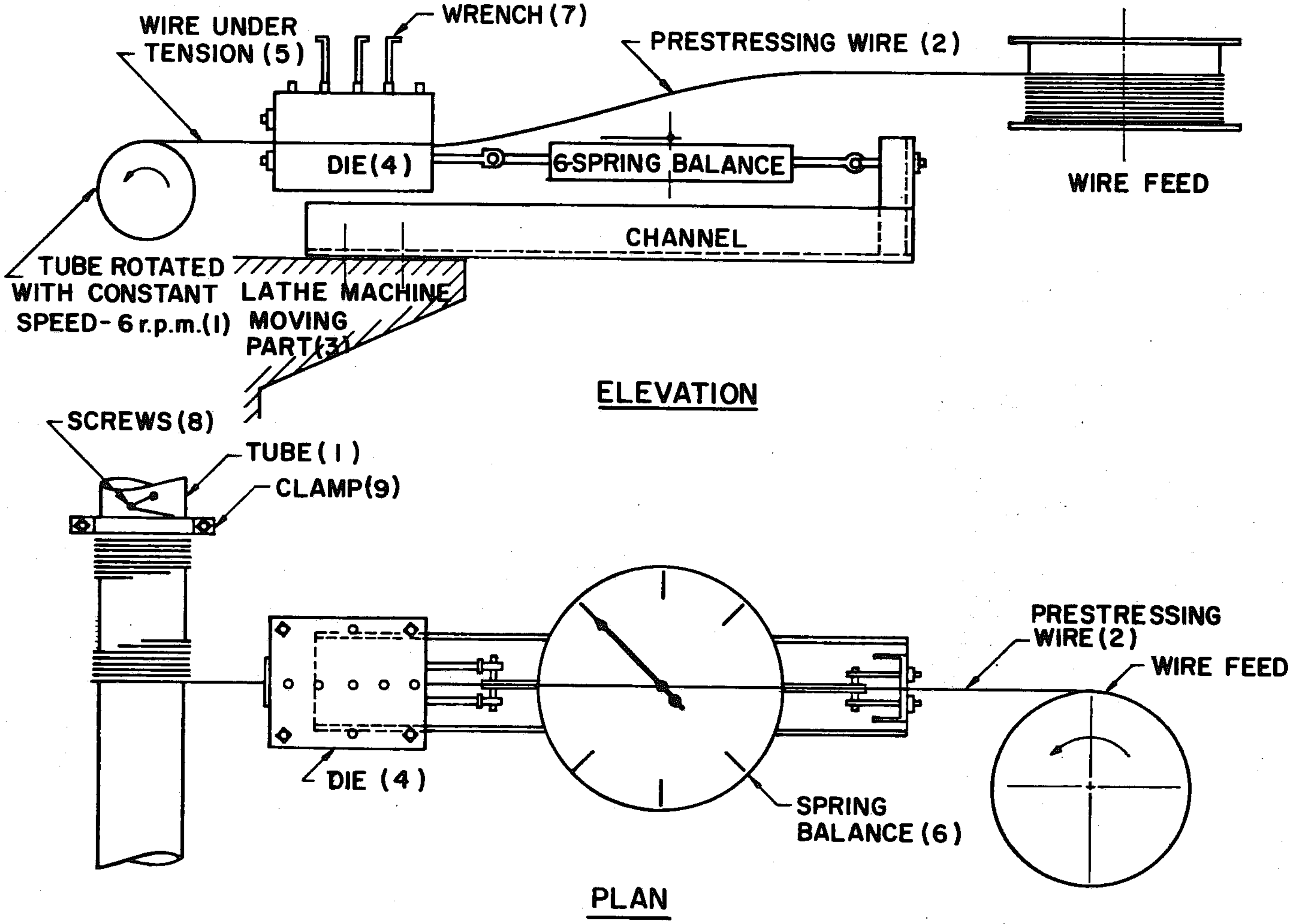


FIG. A-3 THE PRESTRESSING SYSTEM

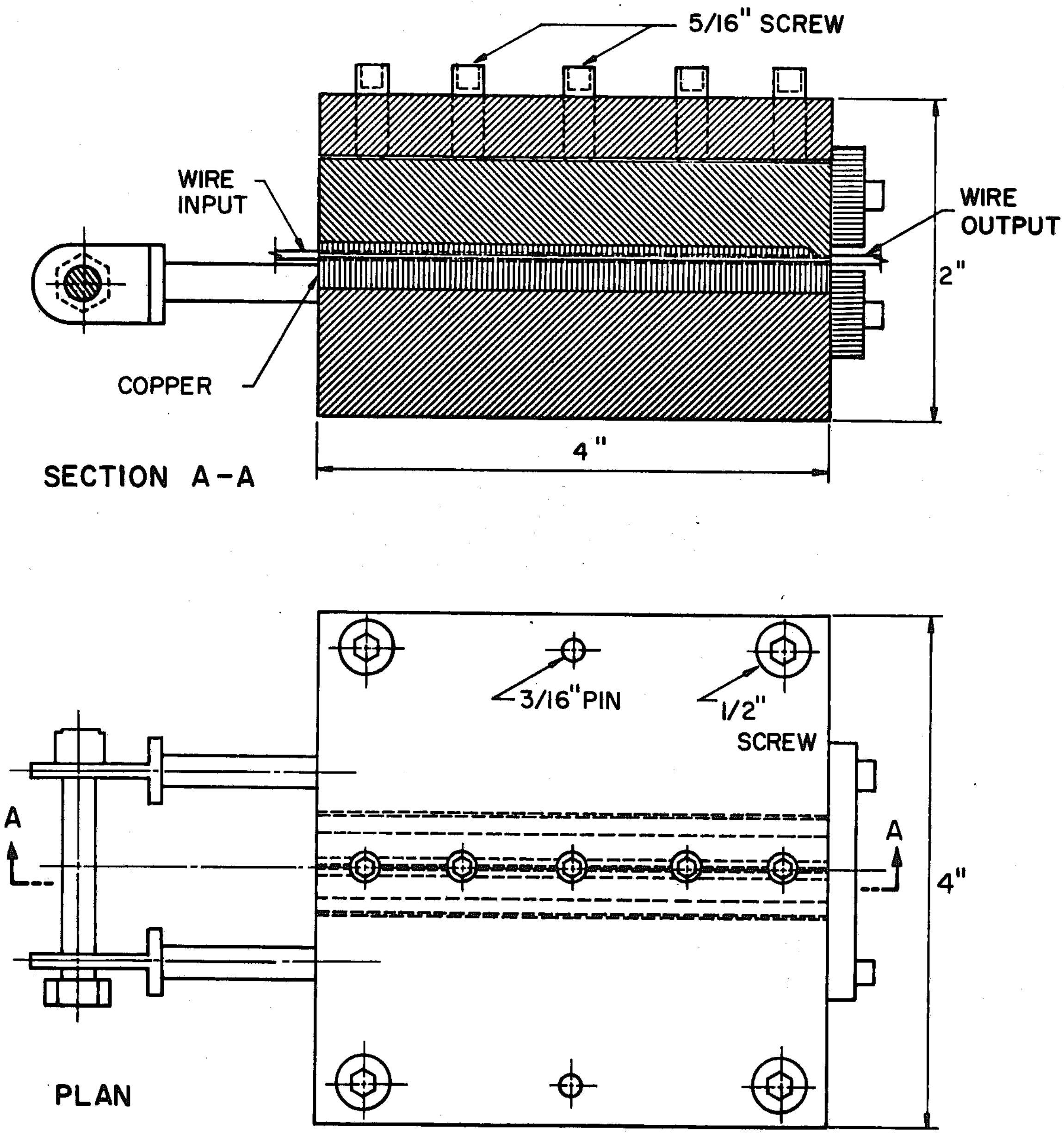


FIG. A-4 THE DIE

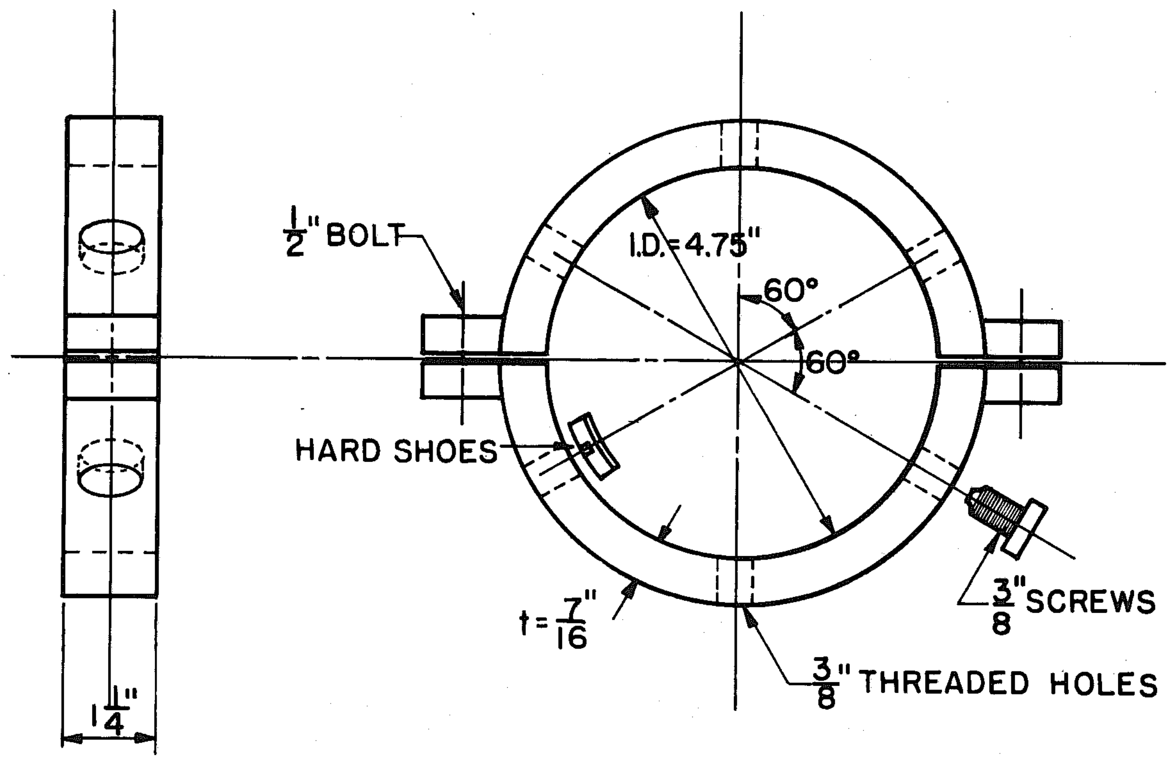
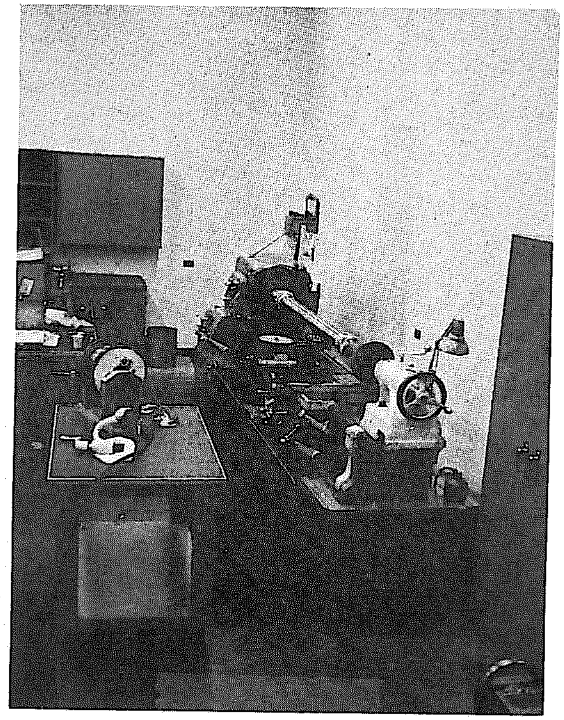
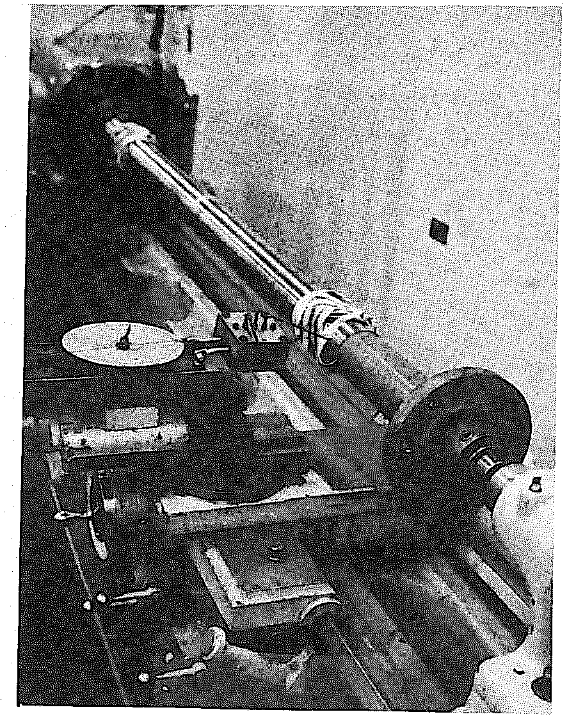


FIG. A.5 CLAMPING DEVICE



a- GENERAL VIEW



b- CLOSE-UP

FIG. A.6 PRESTRESSING TECHNIQUE

BIBLIOGRAPHY

1. Considère, A., "Resistance a la compression du beton come et du beton fretté", Le Génie Civil (Paris), 1903.

2. Considère, A., "Experimental Researches on Reinforced Concrete", English Translation, L.S. Moisseiff, 2nd Edition, McGraw-Hill Book Co., Inc., New York, 1906.

3. Von Karman, T., "Festigkeitsversuche unter allseitigen Druck", Mitteilunger auf dem Gebiete des Ingenieurweven, V. 118, 1912.

4. Richart, F.E., Brandtzaeg, A., and Brown, R.L., "A Study of the Failure of Concrete Under Combined Compressive Stresses". University of Illinois, Urbana, Engineering Experiment Station Bulletin #185, Nov. 1928.

5. Richart, F.E., Brandtzaeg, A., and Brown, R.L., "The Failure of Plain and Spirally Reinforced Concrete in Compression". University of Illinois, Urbana, Engineering Experiment Station Bulletin #190, 1929.

6. ____________, "Structural Design and Details". Lally Column Company, 1953.

7. Ostapenko, A., "Behavior of Short Concrete Filled Steel Pipe Columns". D. Sc. Thesis, Massachusetts Institute of Technology, 1957.

8. Klöppel, Von K., and Goder, W., "Collapse Load Tests on Concrete-Filled-Steel Tubes and Establishment of a Design Formula", (Traglastversuche mit Ausbetonierton Stahlrohren und Aufstellung einer Bemessungs Formel). Der Stahlbau (Berlin), Vol. 26, No. 1, Jan., 1957.

9. ____________, "The Strength of Concrete-Filled-Steel Pipes Piles". Armco Drainage and Metal Products, Inc., 1950.

10. ____________, "Standard Building Code". American Concrete Institute, 1963.

11. ____________, "Manual of Recommended Practice". American Railway Engineering Association, Chicago, Ill., 1963.

12. Gardner N.J., and Jacobson, E.R., "Structural Behavior of Concrete-Filled-Steel Tubes". Journal of the American Concrete Institute, No. 7 Proceedings, Vol. 64, July, 1967.

13. Maney, G A., "High Strength Reinforced Column Developed". ASCE-Civil Engineering, V. 14, No. 12, Dec. 1944.

14. Gitman, F.E., "Issledovanie Cylindriceskich Kolonn s predvaritelno napriazonnoi Spizalnoj armaturoj". Trudy Instituta NIZB (Moscow), 1958.

15. Gambarov, G.A., "Centralno szatie spiralno armirovanye predvaritelno napriazonnye elementy". Beton i Zhelezobeton (Moscow), April 1961.

16. Gambarov, G.A., "Polzucest i ussadka triokhossno-napriazonnovo betona". Beton i Zhelezobeton (Moscow), Jan. 1962.

17. Mikhailov, V.V., "New Developments in Self Stressed Concrete". Proceedings, World Congress on Prestressed Concrete, San Francisco, 1957.

18. Ben-Zvi, E., Muller, G., and Rosenthal, I., "Effect of Active Triaxial Stress on the Strength of Concrete Elements". Symposium on Reinforced Concrete Columns. American Concrete Institute, Publication No. SP-13.

19. Bertero, V., and Brito, J., "Behavior of Expansive Cement Concrete-Filled-Steel Tube Columns". University of California, Berkeley, Report No. SESM 66-10, September, 1966.

20. Ben-Zvi, E., "Structural Behavior of Steel Encased Short Concrete Cylinders Using Expansive Cement". M.S. Thesis, University of California, Berkeley, 1967.

21. North, O.S., "Expansive Cements-Properties-Manufacture-Research-Development-Formulas". Mineral Processing, January 1965.

22. Lossier, H., "L'Autocontrainte des Betons par les Ciments Expansifs". Memoires Societe des Ingenieurs Civil de France, Nos. 3-4, March-April 1948.

23. Mikhailov, V.V. "Stressing Cement and the Mechanism of Self-Stressing Concrete Regulation". Chemistry of Cement, Proceedings, Fourth International Symposium, Washington, D C., 1960, N.B.S. Monograph 43, 1962.

24. Budnikov, P.P., and Kosyreva, Z.S., "Sulfoaluminate, a Positive Factor in Expanding Cement". Doklady Akademii Nauk S.S.S.R., Vol. 61, 1948.

25. Klein, A., and Troxell, G.E., "Studies of Calcium Sulfoalominate Admixtures for Expansive Cements". Proceedings, American Society for Testing and Materials, Vol. 58, 1958.

26. Klein, A., Karby, T., and Polivka, M., "Properties of an Expansive Cement for Chemical Prestressing". Journal of American Concrete Institute, Proceedings 58, 1961.

27. Mehta, P.K., "A Review of Development and Chemistry of Expansive Cements". Pit and Quarry, Vol. 58, No. 5, Nov. 1965.

28. Li, Shu-Tien, "Expansive Cement Concrete--A New Idea with Exciting Possibilities". Concrete Construction, June, 1965, pp. 207-209.

29. ____________, "California Findings on an Experimental Pavement Using Compensated Shrinkage Cement". California Division of Highways, September, 1964.

30. ____________, "Findings from the Use of Compensated Shrinkage Portland Cement in the Lodi Freeway Experimental Pavement". California Division of Highways, June 1965.

31. Lin, T.Y., and Klein, A., "Chemical Prestressing of Concrete Elements Using Expanding Cements". Proceedings, American Concrete Institute, Vol. 60, 1963, pp. 1187-1218.

32. Bertero, V., and Choi, J., "Chemically Prestressed Concrete Hyperbolic Paraboloid Shell Model." Proceedings, World Conference on Shell Structures, Oct., 1962, San Francisco, California.

33. Bertero, V., and Iragorry-Montero, G.J., "Precast Chemically Prestressed Concrete Frame". University of California, Berkeley, Report No. SESM 63-4, October, 1963.

34. Dewar, J D., "Some Properties of Concrete Made with an Expansive Cement". Cement and Concrete Association, Materials Department, Departmental Note, October, 1963.

35. Bertero, V., and Polivka, M., "Effect of Degree of Restraint on Mechanical Behavior of Expansive Concrete". Proceedings, American Society of Testing and Materials, Vol. 64, pp. 797-815, 1964.

36. Bertero, V., "Curing Effects on Expansion and Mechanical Behavior of Expansive Cement Concrete". Journal of the American Concrete Institute, Vol. 64, February, 1967.

37. Bertero, V., "Effects of Type of Restraint on History of Expansion and Mechanical Behavior of Expanding Concrete". To be published.

38. Aroni, S., Polivka, M., and Bresler, B., "Expansive Cements and Expanding Concrete". Structures and Materials Research Report No. 66-7, University of California, Berkeley.

39. Timoshenko, S., and Goodier, J.N., "Theory of Elasticity". McGraw-Hill Book Company, Inc. 1951.

40. Sokolnikoff, I.S., "Mathematical Theory of Elasticity". McGraw-Hill Book Company, Incorporated, Second Edition, 1956.

41. Filon, L.N.G., "On the Elastic Equilibrium of Circular Cylinders under Certain Practical Systems of Load". Royal Society/London, Philosophical Transactions, Vol. 198, Series A, 1902.

42. Wilson, E.L., "Structural Analysis of Axisymmetric Solids". AIAA Journal, Vol. 3, No. 12, December, 1965.

43. Abramowitz, M., and Stegun, I.A., "Handbook of Mathematical Functions". Dover Publications, Inc., New York, 1965.

44. Flugge, W., "Stresses in Shells", Springer, 1960.

45. Chin, S., Zimmerman, R.M., "Behavior of Plain Concrete Under Various High Triaxial Compression Loading Conditions", University of Colorado, Technical Report No. WL TR 64-163, August 1965.

46. Snyder, J., and Lee, S L., "Buckling of Elastic-Plastic Tabular Columns". ASCE, Journal of the Structural Division, Vol. 94, No. ST1, Jan. 1968.

47. Olszak, W., and Stepień, A., "Time-Dependent Effects in Compressed Bound Elements". Preliminary Publication of the 7th Congress of the International Association for Bridge and Structural Engineering, Rio De Janeiro, 1964.

APPENDIX I

Polynomial Approximation of Bessel Functions I(x) and K(x)

For $-3.75 \leq x \leq 3.75$ and $t = \frac{x}{3.75}$

$$I_o(x) = 1 + 3.5156229\ t^2 + 3.0899424\ t^4 + 1.2067492\ t^6 + 0.2659732\ t^8 + 0.0360768\ t^{10} + 0.0045813\ t^{12} + \varepsilon$$

$$|\varepsilon| < 1.6\text{X}10^{-7}$$

$$x^{-1} I_1(x) = 0.5 + 0.87890594\ t^2 + 0.51498869\ t^4 + 0.15084934\ t^6 + 0.02658733\ t^8 + 0.00301532\ t^{10} + 0.00032411\ t^{12} + \varepsilon$$

$$|\varepsilon| < 8\text{X}10^{-9}$$

For $3.75 \leq x < \infty$ and $t = \frac{x}{3.75}$

$$x^{\frac{1}{2}} e^{-x} I_o(x) = 0.39894228 + 0.01328592\ t^{-1} + 0.00225319\ t^{-2} - 0.00157565\ t^{-3} + 0.00916281\ t^{-4} - 0.02057706\ t^{-5} + 0.02635537\ t^{-6} - 0.01647633\ t^{-7} + 0.00392377\ t^{-8} + \varepsilon$$

$$|\varepsilon| < 1.9\text{X}10^{-7}$$

$$x^{\frac{1}{2}} e^{-x} I_1(x) = 0.39894228 - 0.03988024\ t^{-1} - 0.00362018\ t^{-2} + 0.00163801\ t^{-3} - 0.01031555\ t^{-4} + 0.02282967\ t^{-5} - 0.02895312\ t^{-6} + 0.01787654\ t^{-7} - 0.00420059\ t^{-8} + \varepsilon$$

$$|\varepsilon| < 2.2\text{X}10^{-7}$$

For $0 < x \leq 2$

$$K_o(x) = -\ln(x/2)\ I_o(x) - 0.57721566 + 0.42278420\ (x/2)^2 + 0.23069756\ (x/2)^4 + 0.03488590\ (x/2)^6 + 0.00262698\ (x/2)^8 + 0.00010750\ (x/2)^{10} + 0.00000740\ (x/2)^{12} + \varepsilon$$

$$|\varepsilon| < 1\text{X}10^{-8}$$

$$x\,K_1(x) = x\ln(x/2)\,I_1(x) + 1 + 0.15443144(x/2)^2 - 0.67278579\,(x/2)^4$$
$$- 0.18156897\,(x/2)^6 - 0.01919402\,(x/2)^8 - 0.00110404\,(x/2)^{10}$$
$$- 0.00004686\,(x/2)^{12} + \varepsilon$$
$$|\varepsilon| < 8X10^{-9}$$

For $2 \leq x < \infty$

$$x^{\frac{1}{2}}e^{x}\,K_o(x) = 1.25331414 - 0.07832358\,(x/2)^{-1} + 0.02189568\,(x/2)^{-2}$$
$$- 0.01062446\,(x/2)^{-3} + 0.00587872\,(x/2)^{-4} - 0.00251540(x/2)^{-5}$$
$$+ 0.00053208\,(x/2)^{-6} + \varepsilon$$
$$|\varepsilon| < 1.9X10^{-7}$$

$$x^{\frac{1}{2}}e^{x}\,K_1(x) = 1.25331414 + 0.23498619\,(x/2)^{-1} - 0.03655620\,(x/2)^{-2}$$
$$+ 0.01504268\,(x/2)^{-3} - 0.00780353\,(x/2)^{-4}$$
$$+ 0.00325614\,(x/2)^{-5} - 0.00068245\,(x/2)^{-6} + \varepsilon$$
$$|\varepsilon| < 2.2X10^{-7}$$

APPENDIX II

Derivation of Equations 2.71, 2.72

The Governing Differential Equations Are:

$$(\lambda + 2\mu) \frac{\partial^2 u}{\partial r^2} + (\lambda + 2\mu) \frac{\partial}{\partial r} \left(\frac{u}{r} \right) + \mu \frac{\partial^2 u}{\partial z^2} + (\lambda + \mu) \frac{\partial^2 w}{\partial r \partial z} = 0 \qquad \text{(II.1)}$$

$$(\lambda + \mu) \left(\frac{\partial^2 u}{\partial r \partial z} + \frac{1}{r} \frac{\partial u}{\partial z} \right) + \mu \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right) + (\lambda + 2\mu) \frac{\partial^2 w}{\partial z^2} = 0 \qquad \text{(II.2)}$$

Have the Following General Solution:

$$u = \sum_{1}^{\infty} \left[\left\{ -A_{1n} I_1(\rho) - B_{1n} K_1(\rho) - C_{1n} r I_o(\rho) - D_{1n} r K_o(\rho) \right\} \frac{\cos(kz)}{k} + E_{1n} \left\{ \frac{I_1(\rho)}{k} \left(z \sin(kz) + \frac{\cos(kz)}{k} \right) \right\} + F_{1n} \left\{ \frac{K_1(\rho)}{k} \left(z \sin(kz) + \frac{\cos(kz)}{k} \right) \right\} \right] \qquad \text{(II.3)}$$

$$w = \sum_{1}^{\infty} \left[\left\{ A_{2n} I_o(\rho) - B_{2n} K_o(\rho) + C_{2n} r I_1(\rho) - D_{2n} r K_1(\rho) \right\} \frac{\sin(kz)}{k} + \left\{ E_{2n} I_o(\rho) - F_{2n} K_o(\rho) \right\} \frac{z \cos(kz)}{k} \right] \qquad \text{(II.4)}$$

Equations (II.3) and (II.4) should satisfy the differential equations (II.1) and (II.2). If equations (II.3) and (II.4) are substituted into equations (II.1) and (II.2), some relationships between the integration constants (A_{1n}, ---, F_{2n}) will be obtained. In order to carry out this substitution in a systematic manner and to avoid unnecessary complications, the following derivatives of equations (II.3) and (II.4) are given first. In each expression the coefficient of each one of the integration constants will be considered separately.

1 - A_{1n} coefficient in:

$$u = - I_1(\rho) \frac{\cos(kz)}{k}$$

$$u_{,r} = - \left[I_o(\rho) - \frac{I_1(\rho)}{\rho} \right] \cos(kz)$$

$$u_{,rr} = - \left[kI_1(\rho) + \frac{2}{r\rho} I_1(\rho) - \frac{1}{r} I_o(\rho) \right] \cos(kz)$$

$$u_{,z} = I_1(\rho) \sin(kz)$$

$$u_{,zz} = kI_1(\rho) \cos(kz)$$

$$u_{,rz} = \left[kI_o(\rho) - \frac{1}{r} I_1(\rho) \right] \sin(kz)$$

2 - A_{2n} coefficient in:

$$w = I_o(\rho) \frac{\sin(kz)}{k}$$

$$w_{,r} = I_1(\rho) \sin(kz)$$

$$w_{,rr} = \left[kI_o(\rho) - \frac{1}{r} I_1(\rho) \right] \sin(kz)$$

$$w_{,z} = I_o(\rho) \cos(kz)$$

$$w_{,zz} = - kI_o(\rho) \sin(kz)$$

$$w_{,rz} = kI_1(\rho) \cos (kz)$$

3 - B_{1n} coefficient in:

$$u = -K_1(\rho) \frac{\cos(kz)}{k}$$

$$u_{,r} = \left[K_o(\rho) + \frac{1}{\rho} K_1(\rho) \right] \cos(kz)$$

$$u_{,rr} = -\left[K_1(\rho) \left(k + \frac{2}{r\rho} \right) + \frac{1}{r} K_o(\rho) \right] \cos(kz)$$

$$\mathcal{U},_z = K_1(\rho)\ \sin(kz)$$

$$\mathcal{U},_{zz} = k\ K_1(\rho)\ \cos(kz)$$

$$\mathcal{U},_{rz} = -\ [k\ K_o(\rho)\ +\ \frac{1}{r}\ K_1(\rho)]\ \sin(kz)$$

4 - B_{2n} coefficient in:

$$\mathcal{W} = -K_o(\rho)\ \frac{\sin(kz)}{k}$$

$$\mathcal{W},_r = K_1(\rho)\ \sin(kz)$$

$$\mathcal{W},_{rr} = -\ [k\ K_o(\rho)\ +\ \frac{1}{r}\ K_1(\rho)]\ \sin(kz)$$

$$\mathcal{W},_z = -K_o(\rho)\ \cos(kz)$$

$$\mathcal{W},_{zz} = k\ K_o(\rho)\ \sin(kz)$$

$$\mathcal{W},_{rz} = k\ K_1(\rho)\ \cos(kz)$$

5 - C_{1n} coefficient in:

$$\mathcal{U} = -rI_o(\rho)\ \frac{\cos(kz)}{k}$$

$$\mathcal{U},_r = -\ [\rho I_1(\rho)\ +\ I_o(\rho)]\ \frac{\cos(kz)}{k}$$

$$\mathcal{U},_{rr} = -\ [\rho I_o(\rho)\ +\ I_1(\rho)]\ \cos(kz)$$

$$\mathcal{U},_z = rI_o(\rho)\ \sin(kz)$$

$$\mathcal{U},_{zz} = \rho I_o(\rho)\ \cos(kz)$$

$$\mathcal{U},_{rz} = [\rho I_1(\rho)\ +\ I_o(\rho)]\ \sin(kz)$$

6 - C_{2n} coefficient in:

$$\mathcal{W} = rI_1(\rho)\ \frac{\sin(kz)}{k}$$

$$\mathcal{W},_r = rI_o(\rho)\ \sin(kz)$$

$$\mathcal{W},_{rr} = [\rho I_1(\rho)\ +\ I_o(\rho)]\ \sin(kz)$$

$$\mathcal{W},_z = rI_1(\rho)\ \cos(kz)$$

$$\mathcal{W},_{zz} = -\rho I_1(\rho)\ \sin(kz)$$

$$\mathcal{W},_{rz} = \rho I_o(\rho)\ \cos(kz)$$

7 - D_{1n} coefficient in:

$$\mathcal{U} = -\ rK_o(\rho)\ \frac{\cos(kz)}{k}$$

$$\mathcal{U},_r = [\rho\ K_1(\rho) - K_o(\rho)]\ \frac{\cos(kz)}{k}$$

$$\mathcal{U},_{rr} = [K_1(\rho) - \rho\ K_o(\rho)]\ \cos(kz)$$

$$\mathcal{U},_z = rK_o(\rho)\ \sin(kz)$$

$$\mathcal{U},_{zz} = \rho K_o(\rho)\ \cos(kz)$$

$$\mathcal{U},_{rz} = [K_o(\rho) - \rho K_1(\rho)]\ \sin(kz)$$

8 - D_{2n} coefficient in:

$$\mathcal{W} = -rK_1(\rho)\ \frac{\sin(kz)}{k}$$

$$\mathcal{W},_r = rK_o(\rho)\ \sin(kz)$$

$$\mathcal{W},_{rr} = [K_o(\rho) - \rho K_1(\rho)]\ \sin(kz)$$

$$\mathcal{W},_z = -\ rK_1(\rho)\ \cos(kz)$$

$$\mathcal{W},_{zz} = \rho K_1(\rho)\ \sin(kz)$$

$$\mathcal{W},_{rz} = \rho K_o(\rho)\ \cos(kz)$$

9 - E_{1n} coefficient in:

$$\mathcal{U} = \frac{I_1(\rho)}{k}\ [z\ \sin(kz) + \frac{\cos(kz)}{k}]$$

$$\mathcal{U},_r = \left(I_o(\rho) - \frac{I_1(\rho)}{\rho}\right)\ [z\ \sin(kz) + \frac{\cos(kz)}{k}]$$

$$\mathcal{U},_{rr} = \left(kI_1(\rho) + \frac{2I_1(\rho)}{2\rho} - \frac{I_o(\rho)}{r}\right)\ [z\ \sin(kz) + \frac{\cos(kz)}{k}]$$

$$\mathcal{U},_z = I_1(\rho)\ z\ \cos(kz)$$

$$\mathcal{U},_{zz} = I_1(\rho)\ [\cos(kz) - kz\ \sin(kz)]$$

$$\mathcal{U},_{rz} = [kI_o(\rho) - \frac{1}{r}\ I_1(\rho)]\ z\ \cos(kz)$$

10 - E_{2n} coefficient in:

$$\mathcal{W} = \frac{I_o(\rho)}{k}\ z\ \cos(kz)$$

$$\mathcal{W},_r = I_1(\rho)\ z\ \cos(kz)$$

$$\mathcal{W},_{rr} = \left[kI_o(\rho) - \frac{I_1(\rho)}{r}\right]\ z\ \cos(kz)$$

$$\mathcal{W},_z = \frac{I_o(\rho)}{k}\ [\cos(kz) - kz\ \sin(kz)]$$

$$\mathcal{W},_{zz} = \frac{-I_o(\rho)}{k}\ [2\ k\ \sin(kz) - k^2 z\ \cos(kz)]$$

$$\mathcal{W},_{rz} = I_1(\rho)\ [\cos(kz) - kz\ \sin(kz)]$$

11 - F_{1n} coefficient in:

$$\mathcal{U} = \frac{K_1(\rho)}{k}\ [z\ \sin(kz) + \frac{\cos(kz)}{k}]$$

$$\mathcal{U},_r = -\left(K_o(\rho) + \frac{K_1(\rho)}{\rho}\right)\ [z\ \sin(kz) + \frac{\cos(kz)}{k}]$$

$$\mathcal{U},_{rr} = \left[K_1(\rho)\left(k + \frac{2}{r\rho}\right) + \frac{K_o(\rho)}{r}\right]\left[z\ \sin(kz) + \frac{\cos(kz)}{k}\right]$$

$$\mathcal{U},_z = K_1(\rho)\ z\ \cos(kz)$$

$$\mathcal{U},_{zz} = K_1(\rho)\ [\cos(kz) - kz\ \sin(kz)]$$

$$\mathcal{U},_{rz} = \left[\frac{K_1(\rho)}{\rho} - kK_o(\rho)\right]\ z\ \cos(kz)$$

12 - F_{2n} coefficient in:

$$\mathcal{W} = \frac{K_o(\rho)}{k}\ z\ \cos(kz)$$

$$\mathcal{W},_r = K_1(\rho)\ z\ \cos(kz)$$

$$\mathcal{W},_{rr} = \left[\frac{K_1(\rho)}{\rho} - kK_o(\rho)\right]\ z\ \cos(kz)$$

$$w_{,z} = \frac{K_o(\rho)}{k} [kz \sin(kz) - \cos(kz)]$$

$$w_{,zz} = \frac{K_o(\rho)}{k} [2 k \sin(kz) + k^2 z \cos(kz)]$$

$$w_{,rz} = K_1(\rho) [\cos(kz) - kz \sin(kz)]$$

Substituting these derivatives into equation (II.1), the following relations are obtained:

a) The coefficient of $I_1(\rho) \cos(kz)$:

$$(A_{2n}-A_{1n})(\lambda+\mu)k-2(\lambda+2\mu)C_{1n}+(\lambda+3\mu)E_{1n}+(\lambda+\mu)E_{2n} = o \tag{II.5}$$

b) The coefficient of $rI_o(\rho) \cos(kz)$:

$$(\lambda + \mu) (C_{2n} - C_{1n}) k = o \tag{II.6}$$

c) The coefficient of $I_1(\rho) z \sin(kz)$:

$$(\lambda + \mu) (E_{1n} - E_{2n}) k = o \tag{II.7}$$

d) The coefficient of $K_1(\rho) \cos(kz)$:

$$(B_{2n}-B_{1n}) (\lambda+\mu)k+2(\lambda+2\mu)D_{1n} + (\lambda+3\mu) F_{1n} + (\lambda+\mu)F_{2n} = o \tag{II.8}$$

e) The coefficient of $rK_o(\rho) \cos(kz)$:

$$(\lambda + \mu) (D_{2n} - D_{1n}) k = o \tag{II.9}$$

f) The coefficient of $K_1(\rho) z \sin(kz)$:

$$(\lambda + \mu) (F_{1n} - F_{2n}) k = o \tag{II.10}$$

and substitution of same derivatives into equation (II.2) gives the following relations:

g) The coefficient of $I_o(\rho) \sin(kz)$:

$$(\lambda+\mu) (A_{1n}-A_{2n})k+2(\lambda+\mu)C_{1n}+ 2\mu C_{2n}-2(\lambda+2\mu)E_{2n} = o \tag{II.11}$$

h) The coefficient of $rI_1(\rho) \sin(kz)$:

$$(\lambda + 2\mu) (C_{1n} - C_{2n}) k = o \tag{II.12}$$

i) The coefficient of $I_o(\rho)\, z \cos(kz)$:

$$(\lambda + \mu)\ (E_{1n} - E_{2n})\ k = o \qquad \text{(II.13)}$$

j) The coefficient of $K_o(\rho) \sin(kz)$:

$$(B_{2n}-B_{1n})\ (\lambda+\mu)k+2(\lambda+\mu)D_{1n}+2\mu\ D_{2n}+2(\lambda+2\mu)\ F_{2n} = o \qquad \text{(II.14)}$$

k) The coefficient of $rK_1(\rho) \sin(kz)$:

$$(\lambda+\mu)\ (D_{2n} - D_{1n})\ k = o \qquad \text{(II.15)}$$

ℓ) The coefficient of $K_o(\rho)\, z \cos(kz)$:

$$(\lambda + \mu)\ (F_{2n} - F_{1n})\ k = o \qquad \text{(II.16)}$$

From equations (II.6), (II.7), (II.9), (II.10), (II.12), (II.13), (II.15) and (II.16) the following relations are obtained:

$$\begin{aligned} C_{1n} &= C_{2n} = C_n \\ D_{1n} &= D_{2n} = D_n \\ E_{1n} &= E_{2n} = E_n \\ F_{1n} &= F_{2n} = F_n \end{aligned} \qquad \ldots \quad \ldots \quad \ldots \qquad \text{(II.17)}$$

and from equations (II.5) and (II.11):

$$(A_{1n} - A_{2n})\ (\lambda+\mu)k + 2(\lambda+2\mu)\ (C_n - E_n) = o \qquad \text{(II.18)}$$

and from equations (II.8) and (II.14):

$$(B_{1n}-B_{2n})\ (\lambda+\mu)\ k - 2\ (\lambda+2\mu)\ (D_n+F_n) = o \qquad \text{(II.19)}$$

are obtained.

APPENDIX III

Derivation of Equations (2.95) to (2.99)

The expressions for radial displacements and their derivatives are:

$$U = u_o r + u_1 \frac{r^3}{3} + D \frac{rz^2}{2} - \sum_1^\infty \left[A_{1n} \frac{I_1(\rho)}{k} - C_n \frac{rI_o(\rho)}{k} \right] \cos(kz)$$

or:

$$U = u_o r + u_1 \frac{r^3}{3} + D \frac{rc^2}{2x3} + \sum_1^\infty \left[D \frac{ra_n}{2} - A_{1n} \frac{I_1(\rho)}{k} - C_n \frac{rI_o(\rho)}{k} \right] \cos(kz)$$

and:

$$\epsilon_{\theta\theta} = u_o + u_1 \frac{r^2}{3} + D \frac{c^2}{2x3} + \sum_1^\infty \left[D \frac{a_n}{2} - A_{1n} \frac{I_1(\rho)}{\rho} - C_n \frac{I_o(\rho)}{k} \right] \cos(kz)$$

$$\epsilon_{rr} = u_o + u_1 r^2 + D \frac{c^2}{6} + \sum_1^\infty \left[D \frac{a_n}{2} + A_{1n} \left\{ \frac{I_1(\rho)}{\rho} - I_o(\rho) \right\} - C_n \left\{ \frac{I_o(\rho) + \rho I_1(\rho)}{k} \right\} \right] \cos(kz)$$

$$\frac{\partial u}{\partial z} = \sum_1^\infty [D r b_n^{(1)} + A_{1n} I_1(\rho) + C_n r I_o(\rho)] \sin(kz)$$

$$\frac{1}{r} \frac{\partial u}{\partial z} = \sum_1^\infty \left[D b_n^{(1)} + A_{1n} \frac{I_1(\rho)}{r} + C_n I_o(\rho) \right] \sin(kz)$$

and the expressions for the axial displacements and their derivatives are:

$$w = w_o z + w_1 \frac{z^3}{3} + \sum_1^\infty \left[A_{2n} \frac{I_o(\rho)}{k} + C_n r \frac{I_1(\rho)}{k} \right] \sin(kz)$$

or:

$$w = \sum_1^\infty \left[w_o b_n^{(1)} + w_1 \frac{b_n^{(3)}}{3} + A_{2n} \frac{I_o(\rho)}{k} + C_n r \frac{I_1(\rho)}{k} \right] \sin(kz)$$

and:

$$\epsilon_{zz} = w_o + w_1 \frac{c^2}{3} + \sum_1^\infty [w_1 a_n + A_{2n} I_o(\rho) + C_n r I_1(\rho)] \cos(kz)$$

$$\frac{\partial^2 w}{\partial z^2} = \sum_1^\infty [2 w_1 b_n^{(1)} - A_{2n} k I_o(\rho) - C_n \rho\, I_1(\rho)] \sin(kz)$$

$$\frac{\partial w}{\partial r} = \sum_1^\infty [A_{2n} I_1(\rho) + C_n r I_o(\rho)] \sin(kz)$$

where:

a_n = nth term in the Fourier expansion of z^2

$b_n^{(1)}$ = nth term in the Fourier expansion of z

$b_n^{(3)}$ = nth term in the Fourier expansion of z^3

$\rho = kr$

$k = \frac{n\pi}{c}$

Boundary conditions:

$$(\sigma_{rz})^{conc.}_{r=a} = \frac{\partial}{\partial z} (t \; \sigma_{zz})^{steel}$$

or:

$$\mu\left[\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \right]^{conc.}_{r=a} = \mathcal{E} \left[\frac{\partial^2 w}{\partial z^2} + \frac{\nu}{r} \frac{\partial u}{\partial z} \right]^{conc.}_{r=a}$$

or:

$$(\mu - \frac{\mathcal{E}\nu}{a}) \; [D \; a \; b_n^{(1)} + A_{1n} I_1(\alpha) + C_n a I_o(\alpha)] + \mu[A_{2n} I_1(\alpha) + C_n a I_o(\alpha)]$$

$$- \mathcal{E} \; [2w_1 \; b_n^{(1)} - A_{2n} \; k \; I_o(\alpha) - C_n \; \alpha \; I_1(\alpha)] = o$$

where:

$$\mathcal{E} = \frac{Et}{1-\nu^2}$$

$$\alpha = ka$$

$$(\sigma_{rr})^{conc.}_{r=a} = - (\frac{t}{a} \; \sigma_{\theta\theta})^{steel}$$

or:

$$[(\lambda + 2\mu) \; \epsilon_{rr} + \lambda \; \epsilon_{\theta\theta} + \lambda \; \epsilon_{zz}]^{conc.}_{r=a} = - \frac{\mathcal{E}}{a} \; [\epsilon_{\theta\theta} + \nu \; \epsilon_{zz}]^{conc.}_{r=a}$$

Therefore the constant terms are:

$$(\lambda + 2\mu) \; u_o + (\lambda + 2\mu) \; u_1 a^2 + (\lambda + 2\mu) \; D \frac{c^2}{6} + (\lambda + \frac{\mathcal{E}}{a}) \; u_o + (\lambda + \frac{\mathcal{E}}{a}) \; u_1 \frac{a^2}{3}$$

$$+ (\lambda + \frac{\mathcal{E}}{a}) \; D \frac{c^2}{6} + (\lambda + \frac{\mathcal{E}\nu}{a}) \; w_o + (\lambda + \frac{\mathcal{E}\nu}{a}) \; w_1 \frac{c^2}{3} = o$$

or:

$$[2(\lambda+\mu)+\frac{\mathcal{E}}{a}]\, u_o + [\lambda+\frac{\mathcal{E}\nu}{a}]\, w_o + \Big[\{ -\frac{3}{8}(1-\gamma)(\lambda+2\mu) - \frac{(1-\gamma)}{8}(\lambda+\frac{\mathcal{E}}{a})\}a^2$$

$$+\{ \frac{(\lambda+2\mu)}{2} + \frac{(\lambda+\frac{\mathcal{E}}{a})}{2} - \gamma(\lambda+\frac{\mathcal{E}\nu}{a})\} \frac{c^2}{3}\Big] D = o$$

and the periodic terms are:

$$(\lambda+\mu)\frac{a_n}{2} D + (\lambda+2\mu)\left(\frac{I_1(\alpha)}{\alpha} - I_o(\alpha)\right) A_{1n} - (\lambda+2\mu)\,(I_o(\alpha) + \alpha I_1(\alpha))\frac{Cn}{k}$$

$$+ (\lambda+\frac{\mathcal{E}}{a})\frac{a_n}{2} D + (\lambda+\frac{\mathcal{E}}{a})\left(\frac{-I_1(\alpha)}{\alpha}\right) A_{1n} - (\lambda+\frac{\mathcal{E}}{a})\frac{I_o(\alpha)}{k} C_n + (\lambda+\frac{\mathcal{E}\nu}{a})(-\gamma a_n)D$$

$$+(\lambda+\frac{\mathcal{E}\nu}{a})\, I_o(\alpha)\, A_{2n} + (\lambda+\frac{\mathcal{E}\nu}{a})\, aI_1(\alpha)\, C_n = o$$

or

$$\left[(\lambda+2\mu)\left(\frac{I_1(\alpha)}{\alpha} - I_o(\alpha)\right) - (\lambda+\frac{\mathcal{E}}{a})\frac{I_1(\alpha)}{\alpha}\right] A_{1n} + [(\lambda+\frac{\mathcal{E}\nu}{a})\, I_o(\alpha)]\, A_{2n}$$

$$+ [(\lambda+\frac{\mathcal{E}\nu}{a})\, aI_1(\alpha) - (\lambda+2\mu)(I_o(\alpha) + \alpha I_1(\alpha)) - (\lambda+\frac{\mathcal{E}}{a})\, I_o(\alpha)]\, C_n$$

$$+ [(\lambda+2\mu)\frac{a_n}{2} + (\lambda+\frac{\mathcal{E}}{a})\frac{a_n}{2} + (\lambda+\frac{\mathcal{E}\nu}{a})(-\gamma a_n)]\, D = o$$

APPENDIX IV

Fourier Expansions of Z^m and $Z^{(m-1)}$

1 - Fourier Expansion of Z^m, where m is even

$$z^m = \frac{a_o}{2} + \sum_{1}^{\infty} a_n \cos \frac{n\pi z}{c}$$

where:

$$a_n = \frac{1}{c}\int_{-c}^{c} z^m \cos \frac{n\pi z}{c}\, dz$$

$$a_o = \frac{1}{c}\int_{-c}^{c} z^m\, dz = \frac{2c^m}{(m+1)}$$

$$a_n = \frac{1}{c}\int_{-c}^{c} z^m \cos \frac{n\pi z}{c}\, dz$$

$$= \frac{1}{c}\left\{\left[\frac{c}{n\pi} z^m \sin \frac{n\pi z}{c}\right]_{-c}^{c} - \frac{cm}{n\pi}\int_{-c}^{c} z^{m-1} \sin \frac{n\pi z}{c}\, dz\right\}$$

$$= -\frac{m}{n\pi}\int_{-c}^{c} z^{m-1} \sin \frac{n\pi z}{c}\, dz$$

$$= -\frac{m}{n\pi}\left\{\left[\frac{-c}{n\pi} z^{m-1} \cos \frac{n\pi z}{c}\right]_{-c}^{c} + \frac{c(m-1)}{n\pi}\int_{-c}^{c} z^{m-2} \cos \frac{n\pi z}{c}\, dz\right\}$$

$$= \frac{(-1)^n\, 2c^m\, m}{(n\pi)^2} - \frac{cm(m-1)}{(n\pi)^2}\int_{-c}^{c} z^{m-2} \cos \frac{n\pi z}{c}\, dz$$

$$= \frac{(-1)^n\, 2mc^m}{(n\pi)^2} - \frac{c^2 m(m-1)}{(n\pi)^2}\left[\frac{(-1)^n\, 2(m-2)\, c^{m-2}}{(n\pi)^2}\right.$$

$$\left. - \frac{c(m-2)(m-3)}{(n\pi)^2}\int_{-c}^{c} z^{m-4} \cos \frac{n\pi z}{c}\, dz\right]$$

$$= \frac{2(-1)^n c^m}{(n\pi)^2}\left[m - \frac{m(m-1)(m-2)}{(n\pi)^2} + \frac{m(m-1)\cdots(m-4)}{(n\pi)^4} - \cdots\right]$$

2 - Fourier Expansion of Z^{m-1}, where (m-1) is odd

$$z^{m-1} = \sum_{1}^{\infty} b_n \sin \frac{n\pi z}{c}$$

where:

$$b_n = \frac{1}{c} \int_{-c}^{c} z^{m-1} \sin \frac{n\pi z}{c}\, dz$$

$$= \frac{1}{c} \left\{ \left[\frac{-c}{n\pi} z^{m-1} \cos \frac{n\pi z}{c} \right]_{-c}^{c} + \frac{c(m-1)}{n\pi} \int_{-c}^{c} z^{m-2} \cos \frac{n\pi z}{c}\, dz \right\}$$

$$= \frac{-(-1)^n 2c^{m-1}}{n\pi} + \frac{c(m-1)}{n\pi} \left[\frac{1}{c} \int_{-c}^{c} z^{m-2} \cos \frac{n\pi z}{c}\, dz \right]$$

$$= \frac{-(-1)^n 2c^{m-1}}{n\pi} + \frac{c(m-1)}{n\pi} \left[\frac{(-1)^n\ 2(m-2)\ c^{m-2}}{(n\pi)^2} \right.$$

$$\left. - \frac{c(m-2)(m-3)}{(n\pi)^2} \int_{-c}^{c} z^{m-4} \cos \frac{n\pi z}{c}\, dz \right]$$

$$= \frac{2(-1)^n c^{m-1}}{n\pi} \left[-1 + \frac{(m-1)(m-2)}{(n\pi)^2} - \frac{(m-1) \cdots (m-4)}{(n\pi)^4} + \cdots \right]$$

APPENDIX V

Derivation of Equations (2.115) to (2.120)

The displacements and strains are:

$$u = u_o r - \sum_1^{\infty} \left[A_{1n} I_1(\rho) + C_n r I_o(\rho) \right] \frac{\cos(kz)}{k}$$

$$w = w_o z + \sum_1^{\infty} \left[A_{2n} I_o(\rho) + C_n r I_1(\rho) \right] \frac{\sin(kz)}{k}$$

$$\epsilon_{rr} = u_o + \sum_1^{\infty} \left[A_{1n} \left(\frac{I_1(\rho)}{\rho} - I_o(\rho) \right) - \frac{C_n}{k} \left(I_o(\rho) + \rho I_1(\rho) \right) \right] \cos(kz)$$

$$\epsilon_{\theta\theta} = u_o - \sum_1^{\infty} \left[A_{1n} \frac{I_1(\rho)}{\rho} + C_n \frac{I_o(\rho)}{k} \right] \cos(kz)$$

$$\epsilon_{zz} = w_o + \sum_1^{\infty} \left[A_{2n} I_o(\rho) + C_n r I_1(\rho) \right] \cos(kz)$$

$$e = 2u_o + w_o + \sum_1^{\infty} \left[- A_{1n} I_o(\rho) + A_{2n} I_o(\rho) - 2 \frac{C_n}{k} I_o(\rho) \right] \cos(kz)$$

$$\epsilon_{rz} = \tfrac{1}{2} \sum_1^{\infty} \left[A_{1n} I_1(\rho) + A_{2n} I_1(\rho) + 2 C_n r I_o(\rho) \right] \sin(kz)$$

Boundary conditions:

$$(\sigma_{rz})^{conc.}_{r=a} = \frac{\partial}{\partial z} (t\sigma_{zz})^{steel}$$

or:

$$\mu \sum_1^{\infty} \left[A_{1n} I_1(\alpha) + A_{2n} I_1(\alpha) + 2 C_n a I_o(\alpha) \right] \sin(kz)$$

$$= - K \left[\frac{tm}{c^m} z^{m-1} \right] = - K \sum_1^{\infty} \left[- \frac{tm}{c^m} b_n \right] \sin(kz)$$

or:

$$\mu I_1 (\alpha) A_{1n} + \mu I_1 (\alpha) A_{2n} + 2\mu a I_o(\alpha) C_n = \left[\frac{-tm}{c^m} b_n\right] K$$

where:

b_n = nth term in the Fourier expansion of Z^{m-1}

$$(\sigma_{rr})^{conc.}_{r=a} = - \left(\frac{t}{a} \sigma_{\theta\theta} \right)^{steel}$$

or:

$$2(\lambda + \mu) u_o + \lambda w_o + \sum_1^\infty \left[A_{1n} \left\{ - I_o(\alpha) (\lambda + 2\mu) + 2\mu \frac{I_1(\alpha)}{\alpha} \right\} \right.$$

$$\left. + A_{2n} (\lambda I_o(\alpha)) - C_n \left\{ 2\lambda \frac{I_o(\alpha)}{k} + 2\mu \frac{I_o(\alpha)}{k} + 2\mu a I_1(\alpha) \right\} \right]$$

$$= - \frac{t}{a} \left[K \left\{ \nu (1 - \frac{a_o}{2c^m}) \right\} + \sum_1^\infty \frac{K\nu}{c^m} a_n \cos (kz) + Eu_o \right.$$

$$\left. + E \sum_1^\infty \left\{ - A_{1n} \frac{I_1(\alpha)}{\alpha} - \frac{C_n}{k} I_o(\alpha) \right\} \cos (kz) \right]$$

The constant terms are:

$$2(\lambda + \mu) u_o + \lambda w_o = \frac{-t}{a} \left[E u_o + K\nu \left(1 - \frac{a_o}{2c^m}\right) \right]$$

or:

$$U_o \left[2(\lambda + \mu) + \frac{t}{a} E \right] + \lambda w_o + \frac{t\nu}{a} \left(1 - \frac{a_o}{2c^m}\right) K = o$$

and the periodic terms are:

$$A_{1n} \left[- I_o(\alpha) (\lambda + 2\mu) + 2\mu \frac{I_1(\alpha)}{\alpha} \right] + \lambda A_{2n} I_o(\alpha) - C_n \left[2\lambda \frac{I_o(\alpha)}{k} + 2\mu \frac{I_o(\alpha)}{k} \right.$$

$$\left. + 2\mu a I_1(\alpha) \right] = - \frac{tE}{a} \left[-A_{1n} \frac{I_1(\alpha)}{\alpha} - C_n \frac{I_o(\alpha)}{k} \right] - \frac{t\nu}{ac^m} a_n K$$

or:

$$A_{1n} \left[- I_o(\alpha)(\lambda + 2\mu) + 2\mu \frac{I_1(\alpha)}{\alpha} - \frac{tE}{a} \frac{I_1(\alpha)}{\alpha} \right] + A_{2n} \left(\lambda I_o (\alpha) \right)$$

$$+ C_n \left[- 2 (\lambda + \mu) \frac{I_o(\alpha)}{k} - 2\mu\, a\, I_1(\alpha) - \frac{tE}{a} \frac{I_o(\alpha)}{k} \right]$$

$$= \left[\frac{-t\nu}{ac^m} a_n + \frac{t\nu}{a} \right] K \, .$$

$$(\epsilon_{zz})^{conc.}_{\substack{r=a \\ z=o}} = \frac{K}{E} (1-\nu^2) - \nu\, (\epsilon_{\theta\theta})^{conc.}_{\substack{r=a \\ z=o}}$$

or:

$$w_o + \sum_1^{\infty} \left[A_{2n}\, I_o(\alpha) + C_n\, r\, I_1(\alpha) \right] \cos (kz) = \frac{K(1-\nu^2)}{E} - \nu\, u_o$$

$$- \nu \sum_1^{\infty} \left[- A_{1n} \frac{I_1(\alpha)}{\alpha} - C_n \frac{I_o(\alpha)}{k} \right] \cos (kz)$$

or:

$$\nu u_o + w_o + K \left\{ \sum_1^{\infty} \left[A_{2n}\, I_o(\alpha) + C_n \left(a\, I_1\, (\alpha) - \frac{\nu\, I_o\, (\alpha)}{k} \right) \right.\right.$$

$$\left.\left. - A_{1n} \frac{\nu\, I_1\, (\alpha)}{\alpha} \right] - \frac{(1-\nu^2)}{E} \right\} = o \, .$$

APPENDIX VI

Derivation of Equations (2.139) to (2.141) and (2.146).

The displacements and strains are exactly the same as those given before in Appendix III.

Boundary Conditions:

$$(\sigma_{rz})^{conc.}_{r=a} = \frac{\partial}{\partial z} (t\ \sigma_{zz})^{steel} \text{ gives:}$$

$$\mu\ D\ a\ z + \mu \sum_{1}^{\infty} [A_{1n}\ I_1(\alpha) + A_{2n}\ I_1(\alpha) + 2\ C_n\ r\ I_o(\alpha)] \sin (kz)$$

$$= - K \left(\frac{tm}{c^m} z^{m-1} \right)$$

or:

$$\frac{-2(-1)^n}{k} \mu\ a\ D + A_{1n}\ (\mu\ I_1\ (\alpha)) + A_{2n}\ (\mu\ I_1(\alpha)) + C_n\ (2\mu\ a\ I_o(\alpha))$$

$$= \frac{-mt}{c^m} b_n\ K$$

where:

b_n = nth term in the Fourier expansion of z^{m-1}

$$(\sigma_{rr})^{conc.}_{r=a} = - \left(\frac{t}{a} \sigma_{\theta\theta} \right)^{steel} \text{ gives:}$$

$$2(\lambda + \mu)\ u_o + \lambda w_o + \left(\frac{4}{3} \lambda + 2\mu \right) a^2\ u_1 + (\lambda + \mu) \frac{c^2}{3} D + \lambda \frac{c^2}{3} w_1$$

$$+ \sum_{1}^{\infty} \left[A_{1n} \left\{ -I_o(\alpha)\ (\lambda + 2\mu) + 2\mu \frac{I_1(\alpha)}{\alpha} \right\} + A_{2n}\ (\lambda I_o(\alpha)) \right.$$

$$- C_n \left\{ 2\lambda \frac{I_o(\alpha)}{k} + 2\mu \frac{I_o(\alpha)}{k} + 2\mu\ a\ I_1(\alpha) \right\}$$

$$\left. + (D + w_1) \left\{ \frac{4(1-1)^n \lambda}{k^2} \right\} + D \frac{4\mu(-1)^n}{k^2} \right] \cos (kz)$$

$$= - \frac{t}{a}\left[K\nu\left(1 - \frac{a_o}{2c^m}\right) + E\,u_o + E\,u_1\,\frac{a^2}{3} + ED\,\frac{c^2}{6}\right.$$

$$+ \sum_{1}^{\infty}\left\{- \frac{a_n K}{c^m} - E\,A_{1n}\,\frac{I_1(\alpha)}{\alpha} - E\,C_n\,\frac{I_o(\alpha)}{k}\right.$$

$$\left.\left. + E\,D\,\frac{2(-1)^n}{k^2}\right\}\cos(kz)\right]$$

The constant terms give:

$$u_o\left[2(\lambda + \mu) + \frac{Et}{a}\right] + \lambda w_o + D\left[\left(\frac{4}{3}\lambda + 2\mu\right)\left\{- \frac{3}{8}(1-\gamma)\,a^2\right\}\right.$$

$$\left. + \frac{c^2}{3}(\lambda + \mu) - \gamma\,\lambda\,\frac{c^2}{3} - \frac{3}{8}(1-\gamma)\,\frac{Et}{a} + \frac{Et}{a}\left(\frac{c^2}{6}\right)\right] + K\left[\frac{t}{a}\,\nu\left(1 - \frac{a_o}{2c^m}\right)\right] = o$$

and the periodic terms give:

$$A_{1n}\left[- (\lambda + 2\mu)\,I_o(\alpha) + \frac{2\mu}{\alpha}\,I_1(\alpha) - \frac{Et}{a\alpha}\,I_1(\alpha)\right] + \lambda\,I_o(\alpha)\,A_{2n}$$

$$+ C_n\left[- \left\{2\lambda\,\frac{I_o(\alpha)}{k} + 2\mu\,\frac{I_o(\alpha)}{k} + 2\mu a\,I_1(\alpha)\right\} - \frac{Et}{a}\,\frac{I_o(\alpha)}{k}\right]$$

$$= D\left[- \left\{\frac{4\lambda\,(-1)^n}{k^2}(1-\gamma)\right\} - \frac{4\mu(-1)^n}{k^2} - \left\{\frac{2Et(-1)^n}{ak^2}\right\}\right]$$

$$+ K\left[\frac{t\nu}{ac^m}\,a_n\right]$$

where:

a_n = nth term in the Fourier expansion of z^m .

APPENDIX VII

Derivation of Equations (2.169) to (2.174) and (2.176)

Displacements and Strains:

$$u = u_o r + \frac{u_1}{r} - \sum_1^\infty \Big[A_{1n}\, I_1(\rho) + B_{1n}\, K_1(\rho) + C_n\, rI_o(\rho)$$

$$+ D_n rK_o(\rho)\Big] \frac{\cos\,(kz)}{k}$$

$$w = w_o z + \sum_1^\infty \Big[A_{2n}\, I_o(\rho) - B_{2n}\, K_o(\rho) + C_n\, rI_1(\rho) - D_n rK_1(\rho)\Big] \frac{\sin\,(kz)}{k}$$

$$\epsilon_{rr} = u_o - \frac{u_1}{r^2} + \sum_1^\infty \Big[A_{1n}\left(\frac{I_1(\rho)}{\rho} - I_o(\rho)\right) + B_{1n}\left(K_o(\rho) + \frac{K_1(\rho)}{\rho}\right)$$

$$- C_n\left(\frac{I_o(\rho)}{k} + rI_1(\rho)\right) + D_n\left(rK_1(\rho) - \frac{K_o(\rho)}{k}\right)\Big] \cos\,(kz)$$

$$\epsilon_{\theta\theta} = u_o + \frac{u_1}{r^2} - \sum_1^\infty \Big[A_{1n}\frac{I_1(\rho)}{\rho} + B_{1n}\frac{K_1(\rho)}{\rho} + C_n\frac{I_o(\rho)}{k} + D_n\frac{K_o(\rho)}{k}\Big] \cos\,(kz)$$

$$\epsilon_{zz} = w_o + \sum_1^\infty \Big[A_{2n}\, I_o(\rho) - B_{2n}\, K_o(\rho) + C_n\, rI_1(\rho) - D_n r\, K_1(\rho)\Big] \cos\,(kz)$$

$$e = 2u_o + w_o + \sum_1^\infty \Big[- A_{1n}\, I_o(\rho) + B_{1n}\, K_o(\rho) + A_{2n}\, I_o(\rho) - B_{2n}\, K_o(\rho)$$

$$- 2C_n\frac{I_o(\rho)}{k} - 2D_n\frac{K_o(\rho)}{k}\Big] \cos\,(kz)$$

$$\epsilon_{rz} = \tfrac{1}{2} \sum_1^\infty \Big[(A_{1n} + A_{2n}) I_1(\rho) + (B_{1n} + B_{2n})\, K_1(\rho) + 2\, C_n r\, I_o(\rho)$$

$$+ 2\, D_n r\, K_o(\rho)\Big] \sin\,(kz)$$

Boundary conditions:

$(\sigma_{rz})^{conc.}_{r=a} = o$ gives:

$$A_{1n}\, I_1(\alpha) + A_{2n}\, I_1(\alpha) + B_{1n}\, K_1(\alpha) + B_{2n}\, K_1(\alpha) + 2C_n\, aI_o(\alpha) + 2D_n\, a\, K_o(\alpha) = o$$

$$(\sigma_{rz})^{conc.}_{r=b} = \frac{\partial}{\partial z}(t\sigma_{zz})^{steel} \quad \text{gives:}$$

$$\mu \sum_{1}^{\infty} \Big[(A_{1n} + A_{2n}) I_1(\beta) + (B_{1n} + B_{2n}) K_1(\beta) + 2 C_n b I_o(\beta)$$

$$+ 2D_n b K_o(\beta) \Big] \sin(kz) = - K \frac{tm z^{m-1}}{c^m}$$

or:

$$A_{1n} \mu I_1(\beta) + A_{2n} \mu I_1(\beta) + B_{1n} \mu K_1(\beta) + B_{2n} \mu K_1(\beta)$$

$$+ C_n (2\mu b I_o(\beta)) + D_n (2\mu b K_o(\beta)) = \left[\frac{-t m}{c^m} b_n \right] K$$

$$(\sigma_{rr})^{conc.}_{r=a} = o \quad \text{gives:}$$

$$2(\lambda + \mu) u_o - \frac{2\mu}{a^2} u_1 + \lambda w_o + \sum_{1}^{\infty} \Big[A_{1n} \Big\{ - \lambda I_o(\alpha) + 2\mu \frac{I_1(\alpha)}{\alpha} - 2\mu I_o(\alpha) \Big\}$$

$$+ B_{1n} \Big\{ \lambda K_o(\alpha) + 2\mu K_o(\alpha) + 2\mu \frac{K_1(\alpha)}{\alpha} \Big\} + A_{2n} \lambda I_o(\alpha) - B_{2n} \lambda K_o(\alpha)$$

$$+ C_n \Big\{ - 2\lambda \frac{I_o(\alpha)}{k} - 2\mu \frac{I_o(\alpha)}{k} - 2\mu a I_1(\alpha) \Big\}$$

$$+ D_n \Big\{ - 2\lambda \frac{K_o(\alpha)}{k} - 2\mu \frac{K_o(\alpha)}{k} + 2\mu a K_1(\alpha) \Big\} \cos(kz) = o$$

The constant terms give:

$$2(\lambda + \mu) u_o - 2\mu \frac{u_1}{a^2} + \lambda w_o = o$$

and the periodic terms give:

$$A_{1n} \Big\{ - (\lambda + 2\mu) I_o(\alpha) + 2\mu \frac{I_1(\alpha)}{\alpha} \Big\} + B_{1n} \Big\{ (\lambda + 2\mu) K_o(\alpha) + 2\mu \frac{K_1(\alpha)}{\alpha} \Big\}$$

$$+ A_{2n} \lambda I_o(\alpha) - B_{2n} \lambda K_o(\alpha) + C_n \Big\{ - 2(\lambda + \mu) \frac{I_o(\alpha)}{k} - 2\mu a I_1(\alpha) \Big\}$$

$$+ D_n \Big\{ - 2(\lambda + \mu) \frac{K_o(\alpha)}{k} + 2\mu a K_1(\alpha) \Big\} = o$$

$$(\sigma_{rr})^{conc.}_{r=b} = - \left(\frac{t}{b} \sigma_{\theta\theta} \right)^{steel} \quad \text{gives:}$$

$$2(\lambda+\mu)\, u_o - 2\mu \frac{u_1}{b^2} + \lambda w_o + \sum_1^{\infty} \Big[A_{1n} \Big\{ - (\lambda+2\mu)\, I_o(\beta) + 2\mu \frac{I_1(\beta)}{\beta} \Big\}$$

$$+ B_{1n} \Big\{ (\lambda+2\mu)\, K_o(\beta) + 2\mu \frac{K_1(\beta)}{\beta} \Big\} + A_{2n}\, \lambda\, I_o(\beta) - B_{2n} \lambda\, K_o(\beta)$$

$$+ C_n \Big\{ - 2(\lambda+\mu) \frac{I_o(\beta)}{k} - 2\mu\, b\, I_1(\beta) \Big\}$$

$$+ D_n \Big\{ -2(\lambda+\mu) \frac{K_o(\beta)}{k} + 2\mu\, b\, K_1(\beta) \Big\} \Big] \cos (kz)$$

$$= - \frac{t}{b} \Big[K \Big\{ \frac{\nu m}{(m+1)} \Big\} + \sum_1^{\infty} \Big\{ \frac{K\nu}{c^m} a_n \Big\} \cos (kz)$$

$$+ E\, u_o + E \frac{u_1}{b^2} - E \sum_1^{\infty} \Big\{ A_{1n} \frac{I_1(\beta)}{\beta} + B_{1n} \frac{K_1(\beta)}{\beta}$$

$$+ C_n \frac{I_o(\beta)}{k} + D_n \frac{K_o(\beta)}{k} \Big\} \cos (kz) \Big]$$

The constant terms give:

$$u_o \Big[2(\lambda+\mu) + \frac{Et}{b} \Big] + u_1 \Big[\frac{-2\mu}{b^2} + \frac{tE}{b^2} \Big] + \lambda w_o + K \Big[\frac{t\nu m}{b(m+1)} \Big] = o$$

and the periodic terms give:

$$A_{1n} \Big[- \lambda\, I_o(\beta) + 2\mu \frac{I_1(\beta)}{\beta} - 2\mu\, I_o(\beta) - \frac{tE}{b} \frac{I_1(\beta)}{\beta} \Big] + A_{2n}\, \lambda\, I_o(\beta)$$

$$+ B_{1n} \Big[(\lambda+2\mu)\, K_o(\beta) + (2\mu - \frac{tE}{b}) \frac{K_1(\beta)}{\beta} \Big] + B_{2n} \Big\{ - \lambda\, K_o(\beta) \Big\}$$

$$+ C_n \Big[\Big\{ -2(\lambda+\mu) - \frac{tE}{b} \Big\} \frac{I_o(\beta)}{k} - 2\mu\, b\, I_1(\beta) \Big]$$

$$+ D_n \Big[\Big\{ - 2(\lambda+\mu) - \frac{tE}{b} \Big\} \frac{K_o(\beta)}{k} + 2\mu b K_1(\beta) \Big] + K \Big[\frac{t\nu}{bc^m} a_n \Big] = o$$

$$(\varepsilon_{zz})^{conc.}_{\substack{r=b\\z=o}} = (\varepsilon_{zz})^{steel}_{z=o} \quad \text{gives:}$$

$$w_o + \sum_{1}^{\infty} \left[A_{2n} \, I_o(\beta) - B_{2n} \, K_o(\beta) + C_n \, b \, I_1(\beta) - D_n \, b \, K_1(\beta) \right]$$

$$= \frac{K}{E} (1-\nu^2) - \nu u_o - \nu \frac{u_1}{b^2} - \nu \sum_{1}^{\infty} \left[- A_{1n} \frac{I_1(\beta)}{\beta} \right.$$

$$\left. - B_{1n} \frac{K_1(\beta)}{\beta} - C_n \frac{I_o(\beta)}{k} - D_n \frac{K_o(\beta)}{k} \right]$$

or:

$$- \nu u_o - \frac{\nu}{b^2} u_1 + w_o + \sum_{1}^{\infty} \left[A_{1n} \left\{ \frac{-\nu I_1(\beta)}{\beta} \right\} + B_{1n} \left\{ \frac{-\nu \, K_1 \, (\beta)}{\beta} \right\} \right.$$

$$+ A_{2n} \left\{ I_o(\beta) \right\} + B_{2n} \left\{ - K_o(\beta) \right\} + C_n \left\{ bI_1(\beta) - \frac{\nu \, I_o(\beta)}{k} \right\}$$

$$\left. + D_n \left\{ -bK_1(\beta) - \frac{\nu K_o(\beta)}{k} \right\} \right] - \frac{K}{E} (1-\nu^2) = o$$

APPENDIX VIII

Instrumentations of the Steel Tube

To be able to record history of strain (mechanical prestressing of tube and then chemical prestressing of the composite element), electrical wire gages were placed on the inside surface of the tube by the use of the expander. (Fig. (A.1) shows a diagramatic sketch of the expander. The expander is composed of three heads [1]* on one circle each is 120° apart from the other. These heads [1] can slide on a cone [2], and their motion is restricted such that when the cone [2] moves in the axial direction the 3 heads [1] can move only in a perpendicular direction with 3 equal radial distances. The heads [1] are also connected to the housing cylinder [3] by spring steel strips [4] to keep them [1] always in contact with the cone [2]. The motion of the cone [2] is controlled at the other end by screwing the connecting rod [5] against the housing cylinder [3]. The heads [1] are covered with silicon rubber which in turn is covered with teflon tape. For accurate placing of the gages on the inside surface of the steel tube, the housing cylinder [3] is graduated with marks 1 in. interval. The whole expander is positioned inside the steel tube by means of an aluminum ring [6] which is clamped at the end of the tube by three beam bolts [7].

The technique for placing the gages is as follows:

1. The lead wires are first connected to the gages.
2. Double stick scotch tape was used to fix the top face of each gage to one head [1]. The gage was carefully centered

*) Numbers in brackets refer to elements with corresponding numbers in Fig. A.1.

with a cross mark existing in the head. Three rectangular rossette gages were placed simultaneously each time by the use of the three heads.

3. Small amount of epoxy is spread on the surface of the gages, and the whole thing is introduced carefully inside the tube.
4. The holder [6] is then fixed at the end of the tube.
5. By pushing the housing cylinder of the expander the gages are introduced farther inside the tube until they reach the exact position. This position is determined from the marks on the housing cylinder.
6. The heads are then pushed against the inside surface of the wall of the tube by rotating connecting rod inside the housing cylinder.
7. The gages are left for 6-12 hrs.to allow for epoxy to harden then the heads are pulled back and the expander is finally removed from the tube.

The photo in Fig. (A.2) shows the expander and the holder.

APPENDIX IX

Technique Used in Prestressing the Steel Tube

As shown in Fig. (A.3) the tube [1] * was placed on a lathe machine, which rotated the tube with a constant angular velocity. The rotation of the tube permitted to wrap the wire [2] around it. The wire was pre-tensioned by the device shown in Fig. (A.3). The device was connected to the lathe machine moving part [3] . A die [4] is used to control the amount of prestress in the wire [5] , and a spring balance [6] is used to measure the force in the wire [5] directly. Fig. (A.4) shows a detailed sketch of the die [4] , used in stretching (tensioning) the prestressing wire. Wrenches [7] were used for adjusting the amount of tension in the wire by chaning the pressure applied by the die on the wire.

During prestressing the lathe machine rotated at its lowest speed (6 rpm), in order to be able to control the pressure applied by the die on the wire, and maintain a constant force in the wire. Before starting the prestressing, the wire is clamped to one end of the tube by a pair of screws and washers [8] . Then as soon as enough length of the pretentioned wire is wrapped, the clamp [9] -- whose detail is shown in Fig. (A.5), -- is applied. This clamp acted as a permanent anchorage for the wire. When the whole length of the tube is wrapped with the prestressed wire another clamp is applied at the other end. The photos of Figs. (A.6) and (A.7) illustrate the prestressing technique.

*) Numbers in brackets refer to elements with corresponding numbers in Fig. A.3.

APPENDIX X

```
$IBFTC AXS     DECK
C
C     ****************************************************
C     GENERAL SOLUTION OF STRESSES AND STRAINS IN THE
C     EXPANSIVE CEMENT CONCRETE-FILLED STEEL TUBE DUE TO
C     EXPANSION AND/OR CONCENTRIC AXIAL LOAD.
C     ****************************************************
C     SAAD ELDIN M. MOUSTAFA, OCTOBER 1966
C
      READ 10,E,P
      READ 10,B,C
      READ 20,A,EC,PC
      READ 20,DELA,DELE,DELP
      READ 20,AF,ECF,PCF
      READ 10,ES,PS
C
      D = 2.*C
      T = C - B
      IF (P+P) 15,15,16
   15 PRINT 2
   16 IF (E+E) 17,17,18
   17 PRINT 1
   18 PRINT 3,D
      PRINT 4,T
      PRINT 5,ES
      PRINT 6,PS
      PRINT 7
      AO  = A - DELA
      ECO = EC - DELE
      PCO = PC - DELP
      A   = AO
  100 A   = A + DELA
      PRINT 30,A
      PRINT 7
      EC = ECO
  110 EC = EC + DELE
      PRINT 8
      PRINT 40,EC
      PRINT 8
      PC = PCO
  120 PC = PC + DELP
      PRINT 50,PC
C
      CALL STRS(PC,PS,EC,ES,A,B,C,P,E)
      PRINT 7
C
      IF (PC + 0.001 - PCF) 120,210,210
  210 IF (EC + 1. - ECF) 110,220,220
  220 IF (A + 0.001 - AF) 100,230,230
```

```
  230 STOP
C
    1 FORMAT (1H1,////,30X,27HSTRESSES AND STRAINS DUE TO,
     1         13H UNIT LOADING,//)
    2 FORMAT (1H1,////,30X,27HSTRESSES AND STRAINS DUE TO,
     1         15H UNIT EXPANSION,//)
    3 FORMAT (10X,35HEXTERNAL DIAMETER OF STEEL TUBE    = ,
     1         F12.4,2X,6HINCHES)
    4 FORMAT (10X,35HTHICKNESS OF STEEL TUBE            = ,
     1         F12.4,2X,6HINCHES)
    5 FORMAT (10X,35HMODULUS OF ELASTICITY OF STEEL     = ,
     1         F12.4,2X,6HK.S.I.)
    6 FORMAT (10X,35HPOISSON'S RATIO FOR STEEL          = ,F12.4)
    7 FORMAT (10X,39H***************************************,
     1         16H****************)
    8 FORMAT (10X,39H***************************************,
     1         43H*******************************************,
     2         8H********)
   10 FORMAT (2F10.4)
   20 FORMAT (3F10.4)
   30 FORMAT (////,10X,35HINSIDE RADIUS OF CONCRETE          = ,
     1        F12.4,2X,6HINCHES)
   40 FORMAT (//,10X,35HMODULUS OF ELASTICITY OF CONCRETE= ,
     1         F12.4,2X,6HK.S.I.)
   50 FORMAT (/,10X,35HPOISSON'S RATIO FOR CONCRETE       = ,
     1         F12.4)
   61 FORMAT(/10X,21HINTEGRATION CONSTANTS)
   62 FORMAT (/,5X,2HC1,18X,2HC2,18X,2HC3,18X,2HC4,18X,3HEPZ)
   63 FORMAT (10X,5(E15.5,5X))
   70 FORMAT(/10X,27HAXIAL STRESS IN CONCRETE = ,E15.5,2X,
     1         6HK.S.I.,/,10X,27HAXIAL STRESS IN STEEL    = ,
     2         E15.5,2X,6HK.S.I.)
      END


$IBFTC STRS.   DECK
      SUBROUTINE STRS(PC,PS,EC,ES,A,B,C,P,E)
C
      AS   =  3.141593*((C*C) - (B*B))
      AC   = 3.141593*((B*B) - (A*A))
      ECS  = EC/ES
      PCS  =PC/PS
      PC2  = 1. - (2.*PC)
      PC1  = 1. -PC
      APC  = 1. + PC
      APC2= APC*PC2
      APCS= 1. - PCS
      PS1  = 1. - PS
      PS2  = 1. - (2.*PS)
      APS  = 1. +PS
      APS2= APS*PS2
      AB2  = A*A*B*B
      AC2  = (A*A) / (C*C)
      X1   = (A*A) + (B*B*PC2)
      X2   = (C*C) + (B*B*PCS*PS2)
```

```
      ALF4= AB2*APCS/X1
      BET4= AC2*X2/X1
      GAM4= -(AB2*APC/X1)
      ALF3= 1. - (ALF4/(B*B))
      BET3= (1. - BET4)/(B*B)
      GAM3= -(GAM4/(B*B))
      F    = ECS*APS2/APC2
      F1   = (AC*F)/AS
      AO1  = (ALF4*PC2)/(B*B)
      AO2  = PCS - ALF3
      A1   = (AO1 + AO2)*F
      BO1  = (PS2/(C*C)) - (PS2/(B*B))
      BO2  = BET3 - (BET4*PC2/(B*B)) + (PCS*PS2/(C*C))
      B1   = BO1 - (F*BO2)
      DO1  = GAM3 - (GAM4*PC2/(B*B)) - APC
      D1   = E*F*DO1
      AO3  = (2.*PS) - (PS1/PS)
      AO4  = (2.*PC*ALF3) - (PC1/PS)
      A2   = AO3 + (F1*AO4)
      BO3  = (PS1*PS2)/(PS*C*C)
      BO4  = (2.*PC*BET3) + ((PC1*PS2)/(PS*C*C))
      B2   = BO3 + (F1*BO4)
      DO2  = (P*APS2)/(AS*ES)
      DO3  = 1. + PC - (2.*PC*GAM3)
      D2   = DO2 + (E*F1*DO3)
      CO1  = (D1*B2) - (B1*D2)
      CO2  = (A1*B2) - (B1*A2)
      C1   = CO1/CO2
      CO3  = (A1*D2) - (D1*A2)
      C2   = CO3/CO2
      C3   = (ALF3*C1) + (BET3*C2) + (GAM3*E)
      C4   = (ALF4*C1) + (BET4*C2) + (GAM4*E)
      EPZ  = -(C1/PS) + ((C2*PS2)/(PS*C*C))
C
      IF (A+A) 763,763,764
  763 UA   = 0.
      GO TO 760
  764 UA   = (C3*A) + (C4/A)
  760 UB   = (C1*B) + (C2/B)
      UC   = (C1*C) + (C2/C)
C
      IF (A+A) 766,766,767
  766 EPRA = C3
      GO TO 761
  767 EPRA = C3 - (C4/(A*A))
  761 EPRB = C1 - (C2/(B*B))
      EPRC = C1 - (C2/(C*C))
      IF (A+A) 768,768,769
  768 EPTA = C1
      GO TO 762
  769 EPTA = C3 + (C4/(A*A))
  762 EPTB = C1 + (C2/(B*B))
      EPTC = C1 + (C2/(C*C))
C
      CEC = EC/APC2
```

```
      SES = ES/APS2
      SEGZC  = SEGZ(CEC,PC,C3,EPZ,E)
      SEGZS  = SEGZ(SES,PS,C1,EPZ,0.)
      SEGRCA= 0.
      SEGRSC = 0.
      SEGRCB = SEGR(CEC,C3,C4,PC,EPZ,E,B)
      SEGRSB = SEGR(SES,C1,C2,PS,EPZ,0.,B)
      SEGTCA = SEGT(CEC,C3,C4,PC,EPZ,E,A)
      SEGTCB = SEGT(CEC,C3,C4,PC,EPZ,E,B)
      SEGTSB = SEGT(SES,C1,C2,PS,EPZ,0.,B)
      SEGTSC = SEGT(SES,C1,C2,PS,EPZ,0.,C)
      EPTZ   = EPTC/EPZ
      SEGTZ  = SEGTSC/SEGZS
C
      PRINT 61
      PRINT 62
      PRINT 63,C1,C2,C3,C4,EPZ
      PRINT 70,SEGZC,SEGZS
      PRINT 80
      PRINT 81,EPRA,EPRB,EPRC
      PRINT 82,EPTA,EPTB,EPTC
      PRINT 71,UA,UB,UC
      PRINT 83,SEGRCA,SEGRCB
      PRINT 85,SEGRSB,SEGRSC
      PRINT 84,SEGTCA,SEGTCB
      PRINT 86,SEGTSB,SEGTSC
      PRINT 21,EPTZ,SEGTZ
   21 FORMAT (10X,8HEPTZ   = ,E15.5,10X,8HSEGTZ = ,E15.5)
   61 FORMAT(/10X,21HINTEGRATION CONSTANTS)
   62 FORMAT(/,18X,2HC1,18X,2HC2,18X,2HC3,18X,2HC4,18X,3HEPZ,/)
   63 FORMAT (10X,5(E15.5,5X))
   70 FORMAT (10X,27HAXIAL STRESS IN CONCRETE = ,E15.5,2X,
     16HK.S.I.,/,10X
     2,27HAXIAL STRESS IN STEEL    = ,E15.5,2X,6HK S I ,/)
   71 FORMAT (10X,14HRADIAL DISP.   ,3X,3(5X,E15.5))
   80 FORMAT(/10X,6HRADIUS,22X,1HA,20X,1HB,20X,1HC)
   81 FORMAT (10X,14HRADIAL STRAINS,3X,3(5X,E15.5))
   82 FORMAT (10X,14HTANG. STRAINS ,3X,3(5X,E15.5))
   83 FORMAT (10X,18HCONC. RAD. STRESS ,4X,2(E15.5,5X))
   84 FORMAT (10X,18HCONC. TANG. STRESS,4X,2(E15.5,5X))
   85 FORMAT (10X,18HSTEEL RAD. STRESS ,24X,2(E15.5,5X))
   86 FORMAT (10X,18HSTEEL TANG. STRESS,24X,2(E15.5,5X))
      RETURN
      END


$IBFTC SEGZ.   DECK
      FUNCTION SEGZ(A,B,C,D,E)
      SEGZ   = A*((2.*B*C) + (D*(1.-B)) - (E*(1.+B)))
      RETURN
      END


$IBFTC SEGR.   DECK
      FUNCTION SEGR(A,B,C,D,E,F,G)
```

```
      SEGR = A*(B - ((C/(G*G))*(1.-2.*D)) + (D*E) - (F*(1.+D)))
      RETURN
      END

$IBFTC SEGT.   DECK
      FUNCTION SEGT(A,B,C,D,E,F,G)
      SEGT = A*( B + ((C/(G*G))*(1.-2.*D)) + (D*E) - (F*(1.+D)))
      RETURN
      END
```

Input Data

Card	Column	Format	Subject
1	1-10	F10.4	Free expansion e
1	11-20	F10.4	Total applied axial load P kips.
2	1-10	F10.4	Inside radius of steel tube b
2	11-20	F10.4	Outside radius of steel tube c
3	1-10	F10.4	Initial inside radius of concrete a_o
3	11-20	F10.4	Initial modulus of elasticity of concrete E_o^{conc}.
3	21-30	F10.4	Initial Poisson's ratio of concrete ν_o^{conc} .
4	1-10	F10.4	Change in the inside ~~radius~~ of concrete Δa
4	11-20	F10.4	Change in modulus of elasticity of concrete ΔE^{conc}
4	21-30	F10.4	Change in Poisson's ratio of concrete $\Delta \nu^{conc}$
5	1-10	F10.4	Final value of inside radius of concrete a_f
5	11-20	F10.4	Final value of modulus of elasticity of concrete E_f^{conc}
5	21-30	F10.4	Final value of Poisson's ratio of concrete ν_f^{conc}
6	1-10	F10.4	Modulus of elasticity of steel E_s
6	11-20	F10.4	Poisson's ratio of steel ν_s

APPENDIX XI

```
      PROGRAM TEST(INPUT,OUTPUT)
C
C     ******************************************************
C     PROBLEM NUMBER 3
C     ******************************************************
C     SAAD ELDIN M. MOUSTAFA, AUGUST 1967
C
      DIMENSION A(100,100),A1D(200),A1K(200),A2D(200),
     1CD(200),CK(200),ZO(100),RO(100),A1N(200),A2N(200),
     2AK2(200),CN(200)
C
      READ 1,AA
      READ 1,Q6
      READ 1,Q1
      READ 1,C
      READ 1,T
      READ 1,ES
      READ 1,PS
      READ 1,W
      READ 25,M
      READ 25,NN
      READ 6,NZ,NR
      READ 1,(ZO(I),I=1,NZ)
      READ 1,(RO(J),J=1,NR)
C
      PRINT 11
      PRINT 21,AA,Q6,Q1,C,T,ES,PS,W
      PRINT 28,M
      PRINT 28,NN
      PRINT 22,(ZO(I),I=1,NZ)
      PRINT 22,(RO(J),J=1,NR)
      PRINT 2
      PRINT 3
C
      AM   = M
      GM   = (Q6 + Q1)/(Q6 + 2.*Q1)
      GMM1 = 1. - GM
      PI   = 3.1415926536
      ZET1 = 0.
      ZET2 = 0.
      ZET7 = 0.
      ZET8 = 0.
C
      DO 100 N=1,NN
      AN   = N
      AK   = AN*PI/C
      Q5   = AK*AA
      FO   = AIO(Q5)
      F1   = AI1(Q5)/FO
```

```
      EX   = EXP(Q5)
      P9   = (-1.)**N
C
      A(1,1) = 1.
      A(1,2) = -1.
      A(1,3) = 2./(AK*GM)
      A(2,1) = Q1*F1*FO
      A(2,2) = A(2,1)
      A(2,3) = 2.*Q1*AA*FO
      A(3,1) = -(Q6+2.*Q1) + F1*(2.*Q1-ES*T/AA)/Q5
      A(3,1) = A(3,1)*FO
      A(3,2) = Q6*FO
      A(3,3) = -2.*(Q1+Q6+ES*T/(AA*2.))/AK - 2.*Q1*AA*F1
      A(3,3) = A(3,3)*FO
C
      CALL INVER(A,3)
C
      Y1   = P9*2.*Q1*AA/(AK*EX)
      Y2   = -P9*AM*T*BBN(C,M,N)/((C**M)*EX)
      Y3   = -P9*(4.*Q6*GMM1+Q1*4.+ES*T*2./AA)
      Y3   = Y3/(AK*AK*EX)
      Y4   =    T*PS*P9*AAN(C,M,N)/(AA*(C**M))
      Y4   = Y4/EX
C
      A1D(N) = A(1,2)*Y1 + A(1,3)*Y3
      A1K(N) = A(1,2)*Y2 + A(1,3)*Y4
      A2D(N) = A(2,2)*Y1 + A(2,3)*Y3
      A2K(N) = A(2,2)*Y2 + A(2,3)*Y4
      CD(N)  = A(3,2)*Y1 + A(3,3)*Y3
      CK(N)  = A(3,2)*Y2 + A(3,3)*Y4
C
      ZTN7 = A2D(N) + CD(N)*AA*F1 - PS*A1D(N)*F1/Q5
     1       - PS*CD(N)/AK
      ZTN7 = ZTN7*FO*EX
      ZET7 = ZET7 + ZTN7
      ZTN8 = A2K(N) + CK(N)*AA*F1 - PS*A1K(N)*F1/Q5
     1       - PS*CK(N)/AK
      ZTN8 = ZTN8*FO*EX
      ZET8 = ZET8 + ZTN8
C
      ZTN1 = -P9*(A1D(N)*F1 + CD(N)*AA)/AK
      ZTN1 = ZTN1*FO*EX
      ZET1 = ZET1 + ZTN1
      ZTN2 = -P9*(A1K(N)*F1 + CK(N)*AA)/AK
      ZTN2 = ZTN2*FO*EX
      ZET2 = ZET2 + ZTN2
  100 CONTINUE
C
      A(1,1) = 2.*(Q6+Q1) + ES*T/AA
      A(1,2) = Q6
      A(1,3) = -(4.*Q6+6.*Q1)*GMM1*AA*AA/8.+ C*C*(Q6+Q1)/3.
     1         -GM*Q6*C*C/3. - 3.*GMM1*ES*T/(8.*AA)
     2         + ES*T*C*C/(6.*AA)
      A(1,4) = T*PS*AM/(AA*(AM+1.))
      A(2,1) = PS
```

```
      A(2,2)  = 1.
      A(2,3)  = -PS*AA*AA*GMM1/8. + ZET7
      A(2,4)  = -(1.-PS*PS)/ES + ZET8
      A(3,1)  = 0.
      A(3,2)  = 1.
      A(3,3)  = -GM*C*C/3.
      A(3,4)  = 0.
      A(4,1)  = AA
      A(4,2)  = 0.
      A(4,3)  = -AA*AA*GMM1/8. + AA*C*C/2. + ZET1
      A(4,4)  = ZET2
C
      CALL INVER(A,4)
C
      W  = W/C
      UO = A(1,3)*W
      WO = A(2,3)*W
      D  = A(3,3)*W
      DK = A(4,3)*W
      U1    = -3.*GMM1*D/8.
      W1    = -GM*D
      AZ20 = C*C/3.
      VV3 = DK*AM/(AM+1.)
      VV4 = PS*VV3
C
      DO 200 N=1,NN
      A1N(N)  = A1D(N)*D + A1K(N)*DK
      A2N(N)  = A2D(N)*D + A2K(N)*DK
      CN(N)   = CD(N)*D + CK(N)*DK
  200 CONTINUE
C
C
      DO 500 I=1,NZ
      Z    = ZO(I)
C     Z  DENOTES Z/C
C
      DO 400 J=1,NR
      R    = RO(J)
C     R  DENOTES R/A
      R    = R*AA
C
      V1   = 0.
      V3   = 0.
      V4   = 0.
      V5   = 0.
      V6   = 0.
      V7   = 0.
      V61  = 0.
      V71  = 0.
      VV1  = 0.
      VV2  = 0.
C
      DO 300 N=1,NN
      AN   = N
      AK   = AN*PI/C
```

```
      P9   =  (-1.)**N
      AKZ  =  AN*PI*Z
      ROW  =  AK*R
      FOR  =  AIO(ROW)
      FIR  =  AI1(ROW)
      EXX  =  EXP(ROW)
      CX   =  EXX*COS(AKZ)
      SX   =  EXX*SIN(AKZ)
      AAN  =  AAN(C,M,N)
C
      AZ1N  =  -P9*2.*SIN(AKZ)/AK
      AZ2N  =  P9*4.*COS(AKZ)/(AK*AK)
      AZ3N  =  -P9*2.*SIN(AKZ)*(C*C-(6./(AK*AK)))/AK
      UNA1  =  A1N(N)*FIR/AK
      UNC   =  CN(N)*R*FOR/AK
      WNA2  =  A2N(N)*FOR/AK
      WNC   =  CN(N)*R*FIR/AK
      UN    =  (-UNA1-UNC)*CX
      UN1   =  UN + D*R*AZ2N/2.
      WN    =  (WNA2+WNC)*SX
      WN1   =  WN + WO*AZ1N + W1*AZ3N/3.
      IF (R + R) 60,60,61
   60 V1N   =  (-FOR*A1N(N)/2. - FOR*CN(N)/AK)*CX
      V1N   =  V1N + D*AZ2N/2.
      GO TO 62
   61 V1N   =  (((FIR/ROW)-FOR)*A1N(N)
      V1N   =  V1N + D*AZ2N/2. - (FOR+ROW*FIR)*CN(N)/AK)*CX
   62 V3N   =  (FIR*A1N(N) + R*FOR*CN(N))*SX
      V3N   =  V3N + D*R*AZ1N
      V4N   =  (FOR*A2N(N) + R*FIR*CN(N))*CX
      V4N   =  V4N + W1*AZ2N
      V5N   =  (FIR*A2N(N) + R*FOR*CN(N))*SX
C
      VV1N  =  DK*AAN*COS(AKZ)/(C**M)
      VV1N  =  VV1N*P9
      VV2N  =  VV1N*PS
C
      V1    =   V1 + V1N
      V3    =   V3 + V3N
      V4    =   V4 + V4N
      V5    =   V5 + V5N
      V6    =   V6 + UN
      V7    =   V7 + WN
      V61   =  V61 + UN1
      V71   =  V71 + WN1
      VV1    =  VV1 + VV1N
      VV2    =  VV2 + VV2N
C
  300 CONTINUE
C
      V8   =  UO*R + U1*R*R*R/3. + D*R*Z*Z*C*C/2.
      V81  =  UO*R + U1*R*R*R/3. + D*R*AZ2O/2.
      V9   =  WO*Z*C + W1*Z*Z*Z*C*C*C/3.
      V10  =  UO + U1*R*R + D*AZ2O/2.
      V12  =  WO + W1*AZ2O
```

```
C
      UD   = V6 + V8
      UD1  = V61 + V81
      WD   = V7 + V9
      WD1  = V71
      EPRR= V1 + V10
      IF (R + R) 50,50,51
   50 EPTT= EPRR
      GO TO 52
   51 EPTT= UD1/R
   52 EPZZ= V4 + V12
      EPRZ= (V3 + V5)/2.
      EEE = EPRR+EPTT+EPZZ
C
      SS   = Q6*EEE
      STRR= SS + 2.*Q1*EPRR
      STTT= SS + 2.*Q1*EPTT
      STZZ= SS + 2.*Q1*EPZZ
      STRZ=      2.*Q1*EPRZ
C
      V15  = 0.5*(EPRR + EPZZ)
      V16  = (EPRR - EPZZ)
      V17  = V16*V16
      V18  = 0.5*SQRT(V17+4.*EPRZ*EPRZ)
      V19  = 2.*EPRZ/V16
      V19A = ATAN(V19)
      PEPR = V15 + V18
      PEPZ = V15 - V18
      SLOP = V19A*180./3.1415926536
      V20  = 0.5*(STRR + STZZ)
      V21  = (STRR - STZZ)
      V22  = V21*V21
      V23  = 0.5*SQRT(V22+4.*STRZ*STRZ)
      PSTR = V20 + V23
      PSTZ = V20 - V23
C
      R    = R/AA
      IF (R - 0.999) 8,8,9
    8 PRINT 4,Z,R,UD,WD,STRR,STTT,STZZ,STRZ,PSTZ,SLOP
      GO TO 400
    9 SEGZ =-VV1 + VV3
      SEGT =-VV2 + VV4 + ES*EPTT
      PRINT 5,Z,R,UD,WD,STRR,STTT,STZZ,STRZ,PSTZ,SLOP,SEGZ,SEGT
      PRINT 29,UD1,WD1
C
  400 CONTINUE
C
  500 CONTINUE
C
    1 FORMAT (F13.4)
    2 FORMAT (1H1,//////////,50X,7HRESULTS,/,51X,7H*******)
    3 FORMAT (//,1X,3HZ/C,3X,3HR/A,4X,7HU-DISP.,5X,7HW-DISP.,5X,
     1       8HR-STRESS,4X,8HT-STRESS,4X,8HZ-STRESS,4X,
     2       9HRZ-STRESS,3X,
     3       11HMIN. STRESS,1X,5HSLOPE,7X,11HSTEEL-Z-STR,1X,
```

```
    4       11HSTEEL-T-STR)
  4 FORMAT (1X,2(F5.3,1X),1X,8(E11.4,1X))
  5 FORMAT (1X,2(F5.3,1X),1X,10(E11.4,1X))
  6 FORMAT (2I4)
 11 FORMAT (1H1,/////////,50X,10HINPUT DATA,/,51X,
    1       10H**********)
 21 FORMAT (/10X,8(F10.2,3X))
 22 FORMAT (10X,F10.4)
 25 FORMAT (I4)
 28 FORMAT (10X,I4)
 29 FORMAT (/20X,2E14.7)
C
      STOP
      END


      SUBROUTINE INVER(A,NMAX)
C
C     *****************************************************
C     SUBROUTINE TO INVERT A MATRIX
C     *****************************************************
C
      DIMENSION A(100,100)
C
      DO 200 N=1,NMAX
      D  = A(N,N)
C
      DO 100 J=1,NMAX
  100 A(N,J) = -A(N,J)/D
C
      DO 150 I=1,NMAX
      IF (N-I) 110,150,110
  110 DO 140 J=1,NMAX
      IF (N-J) 120,140,120
  120 A(I,J) = A(I,J) + A(I,N)*A(N,J)
  140 CONTINUE
  150 A(I,N) = A(I,N)/D
      A(N,N) = 1.0/D
  200 CONTINUE
C
      RETURN
      END


      FUNCTION AIO(X)
      T = X/3.75
      IF (T-1.) 10,10,20
   10 AO = 1.0
      A1 = 3.5156229*(T**2)
      A2 = 3.0899424*(T**4)
      A3 = 1.2067492*(T**6)
      A4 = 0.2659732*(T**8)
      A5 = 0.0360768*(T**10)
      A6 = 0.0045813*(T**12)
      AIO=(AO+A1+A2+A3+A4+A5+A6 )/EXP(X)
```

```
      GO TO 30
   20 BO = 0.39894228
      B1 = 0.01328592/(T)
      B2 = 0.00225319/(T**2)
      B3 = 0.00157565/(T**3)
      B4 = 0.00916281/(T**4)
      B5 = 0.02057706/(T**5)
      B6 = 0.02635537/(T**6)
      B7 = 0.01647633/(T**7)
      B8 = 0.00392377/(T**8)
      AIO= (BO+B1+B2-B3+B4-B5+B6-B7+B8)/SQRT(X)
   30 RETURN
      END


      FUNCTION AI1(X)
      T = X/3.75
      IF (T-1.) 10,10,20
   10 AO = 0.5
      A1 = 0.87890594*(T**2)
      A2 = 0.51498869*(T**4)
      A3 = 0.15084934*(T**6)
      A4 = 0.02658733*(T**8)
      A5 = 0.00301532*(T**10)
      A6 = 0.00032411*(T**12)
      AI1= (AO+A1+A2+A3+A4+A5+A6)*(X/EXP(X))
      GO TO 30
   20 BO = 0.39894228
      B1 = 0.03988024/(T)
      B2 = 0.00362018/(T**2)
      B3 = 0.00163801/(T**3)
      B4 = 0.01031555/(T**4)
      B5 = 0.02282967/(T**5)
      B6 = 0.02895312/(T**6)
      B7 = 0.01787654/(T**7)
      B8 = 0.00420059/(T**8)
      AI1= (BO-B1-B2+B3-B4+B5-B6+B7-B8)/SQRT(X)
   30 RETURN
      END


      FUNCTION AAN(C,M,N)
C
      PI   = 3.1415926536
      AM   = M
      AN   = N
      NN   = (M/2) - 1
      MM   = M - 1
      AAN  = 0.
C
      DO 100 K=1,NN
      ANN  = 1.
      KK   = 2*K
      I    = M - KK
C
```

```
      DO 200 J=KK,MM
      AJ  = J
  200 ANN = ANN*AJ
C
      ANN = ANN/((AN*PI)**I)
      K1  = (M/2) + K
      P9  = (-1.)**K1
      ANN = ANN*P9
  100 AAN = AAN + ANN
C
      AO  = 2.*AM*(C**M)/((AN*PI)**2)
      AAN = AO*(1. + AAN)
C
      RETURN
      END


      FUNCTION BBN(C,M,N)
C
      AN  = N
      MM  = M - 1
      NN  = (M/2) - 1
      PI  = 3.1415926536
      BBN = 0.
C
      DO 100 K=1,NN
      BN  = 1.
      KK  = 2*K
      I   = M - KK
C
      DO 200 J=KK,MM
      AJ  = J
  200 BN  = BN*AJ
C
      BN  = BN/((AN*PI)**I)
      K1  = NN + K
      P9  = (-1.)**K1
      BN  = BN*P9
  100 BBN = BBN + BN
C
      BO  = 2.*(C**MM)/(AN*PI)
      BBN = BO*(BBN - 1.)
C
      RETURN
      END
```

Input Data

Card	Column	Format	Subject
1	1-13	F13.4	Inside radius of steel tube a
2	1-13	F13.4	Lame's constant for concrete λ
3	1-13	F13.4	Lame's constant for concrete μ
4	1-13	F13.4	One half the height of the composite element c
5	1-13	F13.4	Thickness of the steel tube t
6	1-13	F13.4	Modulus of elasticity of steel E
7	1-13	F13.4	Poisson's ratio of steel ν
8	1-13	F13.4	End displacement ω
9	1-4	I4	Exponent in the shear distribution law m
10	1-4	I4	Number of terms to be considered in the series NN
11	1-4	I4	Number of stations on the z-axis NZ
11	5-8	I4	Number of stations on the r-axis NR
	1-13	F13.4	z/c one card for each station (i.e., NZ cards)
	1-13	F13.4	r/a one card for each station (i.e., NR cards)

APPENDIX XII

```
      PROGRAM MAIN(INPUT,OUTPUT)
C
C     ****************************************************
C     PROBLEM NUMBER 4
C     ****************************************************
C     SAAD ELDIN M. MOUSTAFA, SEPTEMBER 1967
C
      DIMENSION A(100,100),A1N(200),A2N(200),B1N(200),
     1          B2N(200),CN(200),DN(200),ZO(100),RO(100)
C
      READ 1,BB
      READ 1,AA
      READ 1,Q6
      READ 1,Q1
      READ 1,C
      READ 1,T
      READ 1,ES
      READ 1,PS
      READ 1,W
      READ 25,M
      READ 25,NN
      READ 6,NZ,NR
      READ 1,(ZO(I),I=1,NZ)
      READ 1,(RO(J),J=1,NR)
C
      PRINT 11
      PRINT 21,BB,AA,Q6,Q1,C,T,ES,PS,W
      PRINT 28,M
      PRINT 28,NN
      PRINT 22,(ZO(I),I=1,NZ)
      PRINT 22,(RO(J),J=1,NR)
      PRINT 2
      PRINT 3
C
      AM   = M
      GM   = (Q6 + Q1)/(Q6 + 2.*Q1)
      GMM1= 1. -GM
      PI   = 3.1415926536
      ZET  = 0.
C
      DO 100 N=1,NN
      AN   = N
      AK   = AN*PI/C
      P9   = (-1.)**N
      ALF  = AK*AA
      BTA  = AK*BB
      EXA  = EXP(ALF)
      EXB  = EXP(BTA)
      FOA  = AIO(ALF)*EXA
```

```
      F1A = AI1(ALF)*EXA
      FOB = AIO(BTA)*EXB
      F1B = AI1(BTA)*EXB
      AKO = AKO(ALF)
      AK1 = AK1(ALF)
      BKO = AKO(BTA)
      BK1 = AK1(BTA)
C
      A(1,1) = 1.
      A(1,2) = -1.
      A(1,3) = 0.
      A(1,4) = 0.
      A(1,5) = 2./(GM*AK)
      A(1,6) = 0.
C
      A(2,1) = F1A
      A(2,2) = F1A
      A(2,3) = AK1
      A(2,4) = AK1
      A(2,5) = 2.*AA*FOA
      A(2,6) = 2.*AA*AKO
C
      A(3,1) = 0.
      A(3,2) = 0.
      A(3,3) = 1.
      A(3,4) = -1.
      A(3,5) = 0.
      A(3,6) = -2./(GM*AK)
C
      A(4,1) = Q1*F1B
      A(4,2) = Q1*F1B
      A(4,3) = Q1*BK1
      A(4,4) = Q1*BK1
      A(4,5) = 2.*Q1*BB*FOB
      A(4,6) = 2.*Q1*BB*BKO
C
      A(5,1) = -Q6*FOA + 2.*Q1*F1A/ALF - 2.*Q1*FOA
      A(5,2) = Q6*FOA
      A(5,3) = Q6*AKO + 2.*Q1*AKO + 2.*Q1*AK1/ALF
      A(5,4) = -Q6*AKO
      A(5,5) = -2.*Q6*FOA/AK - 2.*Q1*FOA/AK - 2.*Q1*F1A*AA
      A(5,6) = -2.*Q6*AKO/AK - 2.*Q1*AKO/AK + 2.*Q1*AA*AK1
C
      A(6,1) = -Q6*FOB + 2.*Q1*F1B/BTA - 2.*Q1*FOB
     1         -T*ES*F1B/(BB*BTA)
      A(6,2) = Q6*FOB
      A(6,3) = Q6*BKO + 2.*Q1*BKO +2.*Q1*BK1/BTA
     1         -T*ES*BK1/(BB*BTA)
      A(6,4) = -Q6*BKO
      A(6,5) = -2.*Q6*FOB/AK - 2.*Q1*FOB/AK -2.*Q1*BB*F1B
     1         -T*ES*FOB/(BB*AK)
      A(6,6) = -2.*Q6*BKO/AK - 2.*Q1*BKO/AK + 2.*Q1*BB*BK1
     1         -T*ES*BKO/(BB*AK)
C
      CALL INVER(A,6)
```

```
C
      Y4    = -T*AM*BBN(C,M,N)*P9/(C**M)
      Y6    =  T*PS*AAN(C,M,N)*P9/(BB*(C**M))
C
      A1N(N)  = A(1,4)*Y4 + A(1,6)*Y6
      A2N(N)  = A(2,4)*Y4 + A(2,6)*Y6
      B1N(N)  = A(3,4)*Y4 + A(3,6)*Y6
      B2N(N)  = A(4,4)*Y4 + A(4,6)*Y6
      CN(N)   = A(5,4)*Y4 + A(5,6)*Y6
      DN(N)   = A(6,4)*Y4 + A(6,6)*Y6
C
      ZTN   = A1N(N)*(-PS*F1B/BTA) + B1N(N)*(-PS*BK1/BTA)
     1        + B2N(N)*(-BKO) + CN(N)*(BB*F1B-PS*FOB/AK)
     2        + DN(N)*(-BB*BK1-PS*BKO/AK)
     3        A2N(N)*FOB
      ZET   = ZTN + ZET
C
  100 CONTINUE
C
      A(1,1)  = 2.*(Q6+Q1)
      A(1,2)  = -2.*Q1/(AA*AA)
      A(1,3)  = Q6
      A(1,4)  = 0.
C
      A(2,1)  = 2.*(Q6+Q1) + ES*T/BB
      A(2,2)  = -2.*Q1/(BB*BB) + T*ES/(BB**3)
      A(2,3)  = Q6
      A(2,4)  = T*PS*AM/(BB*(AM+1.))
C
      A(3,1)  = 0.
      A(3,2)  = 0.
      A(3,3)  = 1.
      A(3,4)  = 0.
C
      A(4,1)  = -PS
      A(4,2)  = -PS/(BB*BB)
      A(4,3)  = 1.
      A(4,4)  = ZET - (1.-PS*PS)/ES
C
      CALL INVER(A,4)
C
      Y3   = W/C
C
      UO   = A(1,3)*Y3
      U1   = A(2,3)*Y3
      WO   = A(3,3)*Y3
      DK   = A(4,3)*Y3
C
      AZ20= C*C/3.
      VV3 = DK*AM/(AM+1.)
      VV4 = PS*VV3
C
      DO 200 N=1,NN
      A1N(N)  = A1N(N)*DK
      A2N(N)  = A2N(N)*DK
```

```
      B1N(N)  = B1N(N)*DK
      B2N(N)  = B2N(N)*DK
      CN(N)   = CN(N)*DK
      DN(N)   = DN(N)*DK
C
  200 CONTINUE
C
      DO 500 I=1,NZ
      Z    = ZO(I)
C     Z  DENOTES Z/C
C
      DO 400 J=1,NR
      R    = RO(J)
C
      V1   = 0.
      V2   = 0.
      V3   = 0.
      V4   = 0.
      V5   = 0.
      V6   = 0.
      V7   = 0.
      V61  = 0.
      V71  = 0.
      VV1  = 0.
      VV2  = 0.
      WS   = 0.
C
      DO 300 N=1,NN
      AN   = N
      AK   = AN*PI/C
      P9   = (-1.)**N
      AKZ  = AN*PI*Z
      ROW  = AK*R
      EXX  = EXP(ROW)
      FOR  = AIO(ROW)*EXX
      FIR  = AI1(ROW)*EXX
      RKO  = AKO(ROW)
      RK1  = AK1(ROW)
      CX   = COS(AKZ)
      SX   = SIN(AKZ)
      AAN  = AAN(C,M,N)
      AZ1N= -PS*2.*SIN(AKZ)/AK
C
      UNA1  = -A1N(N)*FIR
      UNC   = -CN(N)*R*FOR
      UNB1  = -B1N(N)*RK1
      UND   = -DN(N)*R*RKO
      WNA2  = A2N(N)*FOR
      WNC   = CN(N)*R*FIR
      WNB2  = -B2N(N)*RKO
      WND   = -DN(N)*R*RK1
      UN    = (UNA1+UNC+UNB1+UND)*CX/AK
      WN    = (WNA2+WNC+WNB2+WND)*SX/AK
      UN1   = UN
      WN1   = WN + WO*AZ1N
```

```
C
      V1N   = (FIR/ROW-FOR)*A1N(N) -(FOR+ROW*FIR)*CN(N)/AK
     1        +(RKO+RK1/ROW)*B1N(N) - (RKO-ROW*RK1)*DN(N)/AK
      V1N   = V1N*CX
      V2N   = -A1N(N)*FIR/ROW - B1N(N)*RK1/ROW - CN(N)*FOR/AK
     1        - DN(N)*RKO/AK
      V2N   = V2N*CX
      V3N   = FIR*A1N(N) + RK1*B1N(N) + R*CN(N)*FOR + R*DN(N)*RKO
      V3N   = V3N*SX
      V4N   = A2N(N)*FOR - B2N(N)*RKO + CN(N)*R*FIR - DN(N)*R*RK1
      V4N   = V4N*CX
      V5N   = FIR*A2N(N) + RK1*B2N(N) + R*CN(N)*FOR + R*DN(N)*RKO
      V5N   = V5N*SX
      WSN   =-(UNA1+UNB1+UNC+UND)*SX*PS/(AK*AK)
C
      VV1N = DK*AAN*CX/(C**M)
      VV1N = VV1N*P9
      VV2N = VV1N*PS
C
      V1   =  V1 + V1N
      V2   = V2 + V2N
      V3   =  V3 + V3N
      V4   =  V4 + V4N
      V5   =  V5 + V5N
      V6   =  V6 + UN
      V7   =  V7 + WN
      V61  = V61 + UN1
      V71  = V71 + WN1
      VV1  =  VV1 + VV1N
      VV2  =  VV2 + VV2N
      WS   = WS + WSN
C
  300 CONTINUE
C
      V8   = UO*R + U1/R
      V9   = WO*Z*C
      V10  = UO - U1/(R*R)
      V12  = WO
      WST  = WS - PS*UO*Z*C
      WST  = WST + DK*(1.-PS*PS)*Z*C*(1.-(Z**M)/(AM+1.))/ES
      WST  = WST - U1*Z*C/(R*R)
C
      UD   = V6 + V8
      UD1  = V61 + V8
      WD   = V7 + V9
      WD1  = V71
      EPRR= V1 + V10
      EPTT= UD/R
      EPZZ= V4 + V12
      EPRZ= (V3 + V5)/2.
      EEE  = EPRR+EPTT+EPZZ
C
      SS   = Q6*EEE
      STRR= SS + 2.*Q1*EPRR
      STTT= SS + 2.*Q1*EPTT
```

```
      STZZ= SS + 2.*Q1*EPZZ
      STRZ=      2.*Q1*EPRZ
C
      V15  = 0.5*(EPRR + EPZZ)
      V16  = (EPRR - EPZZ)
      V17  = V16*V16
      V18  = 0.5*SQRT(V17+4.*EPRZ*EPRZ)
      V19  = 2.*EPRZ/V16
      V19A = ATAN(V19)
      PEPR = V15 + V18
      PEPZ = V15 - V18
      SLOP = V19A*180./3.1415926536
      V20  = 0.5*(STRR + STZZ)
      V21  = (STRR - STZZ)
      V22  = V21*V21
      V23  = 0.5*SQRT(V22+4.*STRZ*STRZ)
      PSTR = V20 + V23
      PSTZ = V20 - V23
C
      R   = R/BB
      IF (R - 0.999) 8,8,9
    8 PRINT 4,Z,R,UD,WD,STRR,STTT,STZZ,STRZ,PSTZ,SLOP
      GO TO 400
    9 SEGZ =-VV1 + VV3
      SEGT =-VV2 + VV4 + ES*EPTT
      PRINT 5,Z,R,UD,WD,STRR,STTT,STZZ,STRZ,PSTZ,SLOP,SEGZ,SEGT
      PRINT 29,UD1,WST
C
  400 CONTINUE
C
  500 CONTINUE
C
    1 FORMAT (F13.4)
    2 FORMAT (1H1,/////////,50X,7HRESULTS,/,51X,7H*******)
    3 FORMAT (//,1X,3HZ/C,3X,3HR/A,4X,7HU-DISP.,5X,7HW-DISP.,5X,
     1        8HR-STRESS,4X,8HT-STRESS,4X,8HZ-STRESS,4X,
     2        9HRZ-STRESS,3X,
     3        11HMIN. STRESS,1X,5HSLOPE,7X,11HSTEEL-Z-STR,1X,
     4        11HSTEEL-T-STR)
    4 FORMAT (1X,2(F5.3,1X),1X,8(E11.4,1X))
    5 FORMAT (1X,2(F5.3,1X),1X,10(E11.4,1X))
    6 FORMAT (2I4)
   11 FORMAT (1H1,/////////,50X,10HINPUT DATA,/,51X,
     1         10H**********)
   21 FORMAT (/10X,9(F10.2,3X))
   22 FORMAT (10X,F10.4)
   25 FORMAT (I4)
   28 FORMAT (10X,I4)
   29 FORMAT (14X,2(E11.4,1X),//)
   30 FORMAT (/10X,6(E11.4,3X))
C
      STOP
      END
```

```
      SUBROUTINE INVER(A,NMAX)
C
C     ******************************************************
C     SUBROUTINE TO INVERT A MATRIX
C     ******************************************************
C
      DIMENSION A(100,100)
C
      DO 200 N=1,NMAX
      D  = A(N,N)
C
      DO 100 J=1,NMAX
  100 A(N,J) = -A(N,J)/D
C
      DO 150 I=1,NMAX
      IF (N-I) 110,150,110
  110 DO 140 J=1,NMAX
      IF (N-J) 120,140,120
  120 A(I,J) = A(I,J) + A(I,N)*A(N,J)
  140 CONTINUE
  150 A(I,N) = A(I,N)/D
      A(N,N) = 1.0/D
  200 CONTINUE
C
      RETURN
      END


      FUNCTION AIO(X)
      T = X/3.75
      IF (T-1.) 10,10,20
   10 AO = 1.0
      A1 = 3.5156229*(T**2)
      A2 = 3.0899424*(T**4)
      A3 = 1.2067492*(T**6)
      A4 = 0.2659732*(T**8)
      A5 = 0.0360768*(T**10)
      A6 = 0.0045813*(T**12)
      AIO=(AO+A1+A2+A3+A4+A5+A6 )/EXP(X)
      GO TO 30
   20 BO = 0.39894228
      B1 = 0.01328592/(T)
      B2 = 0.00225319/(T**2)
      B3 = 0.00157565/(T**3)
      B4 = 0.00916281/(T**4)
      B5 = 0.02057706/(T**5)
      B6 = 0.02635537/(T**6)
      B7 = 0.01647633/(T**7)
      B8 = 0.00392377/(T**8)
      AIO= (BO+B1+B2-B3+B4-B5+B6-B7+B8)/SQRT(X)
   30 RETURN
      END
```

```
      FUNCTION AI1(X)
      T = X/3.75
      IF (T-1.) 10,10,20
   10 AO = 0.5
      A1 = 0.87890594*(T**2)
      A2 = 0.51498869*(T**4)
      A3 = 0.15084934*(T**6)
      A4 = 0.02658733*(T**8)
      A5 = 0.00301532*(T**10)
      A6 = 0.00032411*(T**12)
      AI1= (AO+A1+A2+A3+A4+A5+A6)*(X/EXP(X))
      GO TO 30
   20 BO = 0.39894228
      B1 = 0.03988024/(T)
      B2 = 0.00362018/(T**2)
      B3 = 0.00163801/(T**3)
      B4 = 0.01031555/(T**4)
      B5 = 0.02282967/(T**5)
      B6 = 0.02895312/(T**6)
      B7 = 0.01787654/(T**7)
      B8 = 0.00420059/(T**8)
      AI1= (BO-B1-B2+B3-B4+B5-B6+B7-B8)/SQRT(X)
   30 RETURN
      END


      FUNCTION AKO(X)
      T  = X/2.
      IF (T - 1.) 10,10,20
   10 AO =-ALOG(T)*AIO(X)*EXP(X)
      A1 = -.57721566
      A2 = .42278420*T*T
      A3 = .23069756*(T**4)
      A4 = .03488590*(T**6)
      A5 = .00262698*(T**8)
      A6 = .00010750*(T**10)
      A7 = .00000740*(T**12)
      AKO= AO+A1+A2+A3+A4+A5+A6+A7
      GO TO 30
   20 BO = 1.25331414
      B1 = -.07832358/T
      B2 =  .02189568/(T*T)
      B3 = -.01062446/(T**3)
      B4 =  .00587872/(T**4)
      B5 = -.00251540/(T**5)
      B6 =  .00053208/(T**6)
      B7 = BO+B1+B2+B3+B4+B5+B6
      AKO= B7/(SQRT(X)*EXP(X))
   30 RETURN
      END
```

```
      FUNCTION AK1(X)
      T   = X/2.
      IF (T - 1.) 10,10,20
   10 AO =X*ALOG(T)*AI1(X)*EXP(X)
      A1 = 1.
      A2 = 0.15443144*T*T
      A3 = -.67278579*(T**4)
      A4 = -.18156897*(T**6)
      A5 = -.01919402*(T**8)
      A6 = -.00110404*(T**10)
      A7 = -.00004686*(T**12)
      A8 = AO+A1+A2+A3+A4+A5+A6+A7
      AK1= A8/X
      GO TO 30
   20 BO = 1.25331414
      B1 =  .23498619/T
      B2 = -.03655620/(T*T)
      B3 =  .01504268/(T**3)
      B4 = -.00780353/(T**4)
      B5 =  .00325614/(T**5)
      B6 = -.00068245/(T**6)
      B7 = BO+B1+B2+B3+B4+B5+B6
      AK1= B7/(SQRT(X)*EXP(X))
   30 RETURN
      END


      FUNCTION AAN(C,M,N)
C
      PI   = 3.1415926536
      AM   = M
      AN   = N
      NN   = (M/2) - 1
      MM   = M - 1
      AAN  = 0.
C
      DO 100 K=1,NN
      ANN  = 1.
      KK   = 2*K
      I    = M - KK
C
      DO 200 J=KK,MM
      AJ   = J
  200 ANN  = ANN*AJ
C
      ANN  = ANN/((AN*PI)**I)
      K1   = (M/2) + K
      P9   = (-1.)**K1
      ANN  = ANN*P9
  100 AAN  = AAN + ANN
      AO   = 2.*AM*(C**M)/((AN*PI)**2)
      AAN  = AO*(1. + AAN)
C
      RETURN
      END
```

```
      FUNCTION BBN(C,M,N)
C
      AN  = N
      MM  = M - 1
      NN  = (M/2) - 1
      PI  = 3.1415926536
      BBN = 0.
C
      DO 100 K=1,NN
      BN  = 1.
      KK  = 2*K
      I   = M - KK
C
      DO 200 J=KK,MM
      AJ  = J
  200 BN  = BN*AJ
C
      BN  = BN/((AN*PI)**I)
      K1  = NN + K
      P9  = (-1.)**K1
      BN  = BN*P9
  100 BBN = BBN + BN
C
      BO  = 2.*(C**MM)/(AN*PI)
      BBN = BO*(BBN - 1.)
C
      RETURN
      END
```

Input Data

Card	Column	Format	Subject
1	1-13	F13.4	Inside radius of steel tube b
2	1-13	F13.4	Inside radius of concrete a
3	1-13	F13.4	Lame's constant for concrete λ
4	1-13	F13.4	Lame's constant for concrete μ
5	1-13	F13.4	One half the height of the composite element c
6	1-13	F13.4	Thickness of the steel tube t
7	1-13	F13.4	Modulus of elasticity of steel E
8	1-13	F13.4	Poisson's ratio of steel ν
9	1-13	F13.4	End displacement ω
10	1-4	I4	Exponent in the shear distribution law m
11	1-4	I4	Number of terms to be considered in the series NN
12	1-4	I4	Number of stations on the z-axis NZ
12	5-8	I4	Number of stations on the r-axis NR
	1-13	F13.4	z/c one card for each station (i.e., NZ cards)
	1-13	F13.4	r/a one card for each station (i.e., NR cards)