

UC Berkeley

UC Berkeley Electronic Theses and Dissertations

Title

Essays on Behavioral and Experimental Game Theory

Permalink

<https://escholarship.org/uc/item/9d79s492>

Author

Jin, Ye

Publication Date

2016

Peer reviewed|Thesis/dissertation

Essays on Behavioral and Experimental Game Theory

by

Ye Jin

A dissertation submitted in partial satisfaction of the

requirements for the degree of

Doctor of Philosophy

in

Economics

in the

Graduate Division

of the

University of California, Berkeley

Committee in charge:

Professor Shachar Kariv, Chair
Professor Steven Tadelis
Associate Professor David S. Ahn

Spring 2016

Essays on Behavioral and Experimental Game Theory

Copyright 2016

by

Ye Jin

Abstract

Essays on Behavioral and Experimental Game Theory

by

Ye Jin

Doctor of Philosophy in Economics

University of California, Berkeley

Professor Shachar Kariv, Chair

This dissertation consists of three chapters, which study the identification of people's reasoning abilities and belief levels in strategic interactions. The first chapter examines the methodology to identify subjects' reasoning levels in the ring games proposed by Kneeland (2015). In the second chapter, an experiment is designed to identify whether it is limited reasoning ability or incorrect belief that prevents people from playing equilibrium strategies. The experiment builds on a series of ring games similar to the ones analyzed in Chapter 1. The third chapter links people's performance in games to a measure of their cognitive ability, which also validates the findings in Chapter 2.

The ring games are first used by Kneeland (2015) to separate different orders of belief in rationality. Her experiment finds rationality levels higher than those in the literature. The first chapter points out two features in the ring games used by Kneeland (2015) that could potentially bias type classification, and reports the experimental results using two new sets of games with these features removed. Similar type distribution as seen in Kneeland (2015) is reproduced when the rows with the largest sums in the payoff matrices do not coincide with the equilibrium strategies. However, when some rows that have the lowest sums and contain zeros are set to be the best responses, a considerably larger fraction of low types are found. Additionally, the deviation rates from the theoretical prediction are higher in my data, partly due to the lesser performance of the identifying assumption.

The second chapter reports an experiment to identify the decisive factor that prevents people from using more depth of reasoning: their incorrect belief of others' rationality or their limited reasoning ability. The analysis first classifies subjects into different Lk types by the reasoning steps they use in the games. It then distinguishes between the " Lkb " players, who have high ability and best respond to Lk belief, and the " Lka " players, who could use, at most, k steps of reasoning, and thus could not respond to $L(k+1)$ or higher-order belief. The separation utilizes a combination of simultaneous and sequential ring games. In the sequential games it requires more than k reasoning steps to respond to Lk belief, so Lkb players still best respond but Lka would fail. I find that around half of the $L2$ and $L3$ subjects are best responding to $L2$ or $L3$ belief, while the rest have reached their upper

boundaries of reasoning. The findings suggest that both belief and reasoning ability could be the decisive factors of players' observed levels.

The third chapter finds positive relationship between the Cognitive Reflection Test (CRT) scores, an indicator of cognitive ability, and the reasoning levels in the ring games in Chapter 2. The CRT scores increase with levels among the lower types (from $L0$ to $L2b$), and but do not differ significantly from $L2b$ to $L4$. Striking difference is found between the CRT scores of $L2a$ and $L2b$ subjects, which suggests that cognitive ability are strongly related to reasoning skills.

To My Parents

Contents

Contents	ii
List of Figures	iv
List of Tables	vi
Introduction	1
1 Reinvestigating Level-k Behavior in Ring Games	4
1.1 Introduction	4
1.2 The Ring Games	6
1.3 Identification Strategy	9
1.4 Experimental Procedures	11
1.5 Experimental Results	12
1.6 Cluster Analysis	16
1.7 Conclusion	17
2 Does Level-k Behavior Imply Level-k Thinking?	19
2.1 Introduction	19
2.2 Related Literature	23
2.3 Experimental Design and Identification Strategy	25
2.4 Experimental Results	31
2.5 Conclusion	46
3 Cognitive Ability and Depth of Reasoning in Games	47
3.1 Introduction	47
3.2 Related Literature	49
3.3 Experiment Design and Procedures	49
3.4 Results	50
3.5 Conclusion	53
Bibliography	54

A	Additional Analysis on Ring Games	58
A.1	Type Assignment with the Assumption of Uniform Randomizing $L0$	58
A.2	Type Classification Results Allowing for 0 or 2 Deviations	64
A.3	Additional Analysis on Learning Effects	70
B	Instructions and Understanding Test	80
B.1	Instructions	80
B.2	Understanding Test	84

List of Figures

1.1	Type distribution (allowing for 1 deviation)	6
1.2	The 4-player dominance solvable ring games used in Kneeland's (2015)	7
1.3	The 4-player dominance solvable ring games used in this paper	8
1.4	Type distribution (allowing for 2 deviations)	13
2.1	G and G' : 3-player ring games	20
2.2	Decomposition of $L2$ and $L3$ subjects	23
2.3	The ring games	27
2.4	Behavior pattern of the observed $L1$ subjects	36
2.5	Behavior pattern of the observed $L2$ subjects	37
2.6	Behavior pattern of the observed $L3$ subjects	38
2.7	Behavior pattern of the observed $L4$ subjects	39
2.8	Type distributions of the subjects who have an advantage in learning	45
3.1	Distribution of CRT Scores	51
3.2	Average CRT Scores and Levels	52
A.1	Behavioral patterns of the observed $L1$ subjects with the uniform prior assumption (UP)	60
A.2	Behavioral patterns of the observed $L2$ subjects with the uniform prior assumption (UP)	61
A.3	Behavioral patterns of the observed $L3$ subjects with the uniform prior assumption (UP)	62
A.4	Behavioral patterns of the observed $L4$ subjects with the uniform prior assumption (UP)	63
A.5	Behavioral patterns of the observed $L1$ subjects (0 deviation)	65
A.6	Behavioral patterns of the observed $L1$ subjects (0 deviation)	66
A.7	Behavioral patterns of the observed $L2$ subjects (0 deviation)	67
A.8	Behavioral patterns of the observed $L3$ subjects (0 deviation)	68
A.9	Behavioral patterns of the observed $L4$ subjects (0 deviation)	69
A.10	Behavioral patterns of the observed $L1$ subjects (2 deviations)	72
A.11	Behavioral patterns of the observed $L2$ subjects (2 deviations)	73

A.12 Behavioral patterns of the observed <i>L3</i> subjects (2 deviations)	74
A.13 Behavioral patterns of the observed <i>L4</i> subjects (2 deviations)	75

List of Tables

1.1	Proportion of subjects who play the equilibrium strategies	12
1.2	(mis-)Identification rates of simulated action profiles (shown in percentages) with deviation rate 0.05	15
1.3	Clusters found in the G5G6 sessions	17
2.1	Separation of $L2b$ and $L2a$	21
2.2	Predicted action profiles of each Lkb and Lka type	29
2.3	Compliance rates of (iterative) dominating strategies	32
2.4	Type assignment from SIMUL games	34
2.5	p -values of the exact tests of goodness-of-fit	41
2.6	Behavioral patterns of the 39 $L3$ subjects in the two sequential games	42
2.7	Type assignment according to the observations from SIMUL, SEQ-P2 and SEQ-P3	43
2.8	Number of subjects who played more advantageous positions in earlier 12 games	45
3.1	Multilogit regression of types on CRT scores	53
A.1	Type assignment using the two assumptions ER and UP	59
A.2	Type assignment according to the observations from SIMUL, SEQ-P3 and SEQ-P2 (0 deviation)	70
A.3	Type assignment according to the observations from SIMUL, SEQ-P3 and SEQ-P2 (6 deviations)	71
A.4	Deviations sorted into three cases	76
A.5	Experience effect at each position ($\beta_1 + \beta_{3j}$)	77
A.6	Multilogit regression of types on Learning Effects (1)	78
A.7	Multilogit regression of types on Learning Effects (2)	79

Acknowledgments

I have received enormous help and encouragement from my advisor Shachar Kariv over the past six years. I benefit a great deal from his guidance on my research and my career. He never hesitates to provide moral support and practical solutions to help me overcome setbacks in work and life. His support helped me through the most difficult parts of my doctoral study.

The studies in this dissertation are greatly inspired by the pathbreaking work of Terri Kneeland. I am also deeply grateful to Terri for her helpful advice and for letting me use her data and online experimental interface.

I owe my gratitude to David Ahn, Ned Augenblick, Stefano Dellavigna, Yoram Halevy, Teck H. Ho, Dan Levin, John Morgan, Chris Shannon and Steve Tadelis for their invaluable advice and supports. I have also learned a lot from the discussions with Aluma Dembo, C. Matthew Leister, Weijia Li, Michèle Müller-Itten and Yang Xie.

The assistance from the Xlab at UC Berkeley had been crucial during my data collection process. I would particularly like to thank Rowilma del Castillo and Miho Tanaka for their excellent work. I also thank the Xlab for providing me with the pilot research grant.

Introduction

This dissertation studies people’s belief and reasoning ability in strategic interaction. It has been documented in the literature that people do not always choose the Nash equilibrium strategies, especially when they are playing the game for the first time¹. In his doctoral dissertation, Nash (1950) writes that in order to play the equilibrium strategies, the players should have such knowledge of “what to expect the others to do”, and that they “should be able to deduce and make use of it”. In another words, a Nash player needs to have correct belief, which is also discussed in Aumann and Brandenburger (1995) as the sufficient epistemic condition, and the reasoning ability to find the best response to his belief. Thus, the failure to play equilibrium strategies could be due to either wrong belief or limited reasoning ability.

In order to better understand and predict people’s non-equilibrium behavior, it is essential to examine their belief levels and ability levels through experiment. The studies in this dissertation take the approach of the level- k (Lk) model. The level- k model is first proposed in Nagel (1995) and Stahl and Wilson (1994, 1995). It assumes that players use limited, heterogeneous reasoning levels in games. The model starts with a specification of the decision rule of a nonstrategic or irrational type $L0$. $L1$ players best respond as if all the other opponents are $L0$. Each higher Lk type is then assumed to best respond to $L(k - 1)$ or a distribution of all the lower types (Camerer et al. (2004)). So far, the level- k theory has been widely used in a variety of settings. Crawford et al. (2013) give a detailed review. It is found that most subjects in these experiments appear to be using no more than two or three steps of reasoning when playing a game for the first time.

So far, a lot has been done towards a descriptive model of the deviations from Nash equilibrium. The question is whether predictive theories could be developed from the current findings of reasoning levels. A first step is to understand why people stop at these low levels. If it is because of their limits in reasoning ability, then it would be useful to measure people’s reasoning limits in games with different complexity and look for the links between

¹For lab experiments, see Camerer (2003) for a detailed review. There are also studies that use real world data. For examples, Goldfarb and Xiao (2011) find that not all managers are able to make the optimal decisions to enter the markets with fewer competitors; Brown et al. (2012) find that movie producers choose to withhold the reviews of bad movies before their openings, in response to consumers’s failure to deduct that cold opened movies are bad movies. Interestingly, as time passes players move to equilibrium in Goldfarb and Xiao (2011) but not in Brown et al. (2012), suggesting that non-equilibrium behavior does not always disappear with experience.

reasoning skills and IQ or other cognitive ability measures. Instead, if the observed levels are determined by their beliefs, then we would want to study how people form beliefs based on their own reasoning ability and their understanding of others' ability.

The work in this dissertation is among the earliest attempts to identify the decisive factors of the people's reasoning levels. The first chapter studies the classification of reasoning levels in a new class of games, the ring games, which provides the methodological foundation of the following analysis. The second chapter proposes a novel identification strategy that separates the subjects whose levels are determined by ability and the ones whose levels are determined by belief. I find in the experiment that over half of the subjects who exhibit two or three steps of reasoning have reached their upper bounds of reasoning. The third chapter reports experimental findings on the relationship between observed levels, reasoning abilities and cognitive abilities.

In Chapter 1, I examine the identification strategy of k -levels in the ring games proposed by Kneeland (2015). In the level- k model, the assumption on the irrational $L0$'s behavior serves as the starting point of the analysis, and all the higher Lk types' behavior are solved recursively. The model is often criticized of relying too much on the assumption of $L0$'s behavior, which might leads to the misclassification of Lk types. The main contribution of Kneeland (2015) is her identification strategy that puts a weaker restriction on $L0$'s behavior. The irrational type in her model (equivalence of $L0$) is characterized as not responding to the changes in his own payoff function. Players with k th-order belief in rationality do not respond to the payoff changes of k th- or higher-order opponents. This identifying assumption could be applied in the ring games. Kneeland (2015) finds that over 95% of her subjects could be cleanly classified to one of the $L1$ - $L4$ types.

I modify Kneeland's (2015) design to look into the validity of her identifying assumption, and the possible identification biases. I find that although over-estimation of levels does not appear to be a big concern, the proportion of subjects who follow the prediction of the identifying assumption is sensitive to the numbers in the payoff matrices. As a result, the identification using my data is not as clean as in Kneeland's (2015) original experiment. There are 15% of the subjects who are not irrational in that they obey dominance, but could not be assigned to any Lk type. Overall, the identification with ring games seems to be unbiased, but there could be a considerable amount of subjects who do not follow the identifying assumption and they should be handled with discretion.

Chapter 2 attempts to address the identification problem of the decisive factor of the observed levels in the experiments. A subject's exhibited sophistication level in a certain game depends on both his belief in the opponents' rationality levels and his ability to finish all the required reasoning steps. But belief and ability are not directly observable from choice data. In most studies, a subject is classified as Lk if he uses k steps of reasoning, which implies that both his belief level and ability level are greater than or equal to k . But few studies have provided a clear answer to whether belief or ability is the decisive factor of the observed levels.

My identification strategy utilizes a combination of sequential and simultaneous 4-player ring games. The basic idea is that, with added sequential moves, I create a situation in

which some subjects are required to use several reasoning steps to best respond, while at the same time they do not need to hold high level beliefs. Therefore a high ability subject who behaved at low levels because of his low level belief will be able to finish this task, but a low ability subject will not. I find in the experiment the existence of both the high ability subjects responding to lower-order belief and the low ability subjects who could only think two or three steps, which shows large heterogeneity in subjects' reasoning abilities. The results suggest that both subjects' beliefs and their reasoning abilities could be the decisive factor of the exhibited levels.

Chapter 3 explores the connection between subjects' performance in the games from Chapter 2 and their cognitive abilities. The Cognitive Reflection Test (CRT) is administrated in the experiment as an indicator of cognitive ability. I find that the CRT scores increase with exhibited reasoning levels among the lower types, and but do not differ significantly among the high types who uses more than three steps of reasoning. Especially, the scores of the *L2* subjects who could only think two steps is significantly lower than the ones who behave like *L2* because of their belief and could actually think more steps. The findings in this chapter suggest that cognitive ability plays an important role in reasoning skills, and support the separation of the two types in Chapter 2.

The three essays in this dissertation work on disentangling the effects of reasoning abilities and beliefs on people's performance in games, and could be regarded as a starting point of a bigger agenda to understand and predict people's strategic interactions. The special structure of the ring games provides me with a great opportunity to explore the heterogeneity in subjects' belief levels and ability levels. I show that the observed low levels in the previous studies could be explained by both the presence of low ability types and the low-order beliefs of high ability types. The main findings could also be supported by subjects' CRT scores.

Chapter 1

Reinvestigating Level-k Behavior in Ring Games

1.1 Introduction

The level-k model, proposed by Nagel (1995) and Stahl and Wilson (1994, 1995), serves as an alternative solution concept to Nash equilibrium by allowing for inconsistent beliefs and heterogeneous rationality levels. The model usually starts with a specification of the decision rule of a nonstrategic or irrational type $L0$. $L1$ players best respond to the naive belief that all the opponents are $L0$. Each higher Lk type is then assumed to believe that the opponents are $L(k - 1)$ or drawn from a distribution of the lower types (Camerer et al. (2004)) and their best responses could be solved recursively. So far, the level-k theory has been widely used in a variety of settings to explain the systematic deviations from Nash equilibrium. (For examples, see Ho et al. (1998), Costa-Gomes et al. (2001), Costa-Gomes and Crawford (2006). Crawford et al. (2013) give a detailed review.)

The $L0$ behavior, which is the key to the identification in most studies, is usually assumed to be uniform randomization over all possible actions. In some special cases, $L0$ players are assumed to play a salient strategy (e.g. Crawford and Iriberri (2007) and Arad and Rubinstein (2012)). Despite the wide use of the level-k model in different classes of games, the validity and portability of the $L0$ assumption is always questioned (e.g. the discussion between Heap et al. (2014) and Crawford (2014)).

Kneeland (2015) addresses this problem by putting a weaker restriction on the irrational type's behavior. The irrational type in her model, called $R0$, is characterized as not responding to the changes in his own payoff function. The randomly playing $L0$ could be regarded as a special case of $R0$. A player of 1st-order rationality ($R1$) is able to identify strictly dominated actions, but does not believe that his opponent responds to the changes in the payoff. So $R1$ does not respond to the changes in his opponents' payoffs. Players with k th-order belief in rationality (Rk , who have the same belief order in rationality as Lk players as defined in the level-k literature) do not respond to the payoff changes of k th- or

higher-order opponents. Kneeland’s (2015) novel identification strategy is rooted in the *ring games*, where a player’s payoff depends on one of the other players in the network, whose payoff also depends on another player’s action, and so on. Hence identification is achieved by testing whether subjects respond to the changes on the payoffs of different opponents in the ring.

In her experiment, Kneeland (2015) uses a pair of 4-player dominance solvable ring games and finds that the proportion of $R0$ - $R4$ are 7%, 22%, 27%, 22%, and 22% respectively. Although few level- k studies have been done in network games, the experiments using similar matrix games (Stahl and Wilson (1995) and Costa-Gomes et al. (2001)) could serve as a comparison. The type distribution found in Kneeland (2015) appears to be higher than these two studies. Especially the fraction of high types, who are at least 3rd-order rational, accounts for 44% in Kneeland (2015). In both Stahl and Wilson (1995) and Costa-Gomes et al. (2001), $L3$ or higher types’ choices coincide with their Equilibrium type. The Equilibrium type accounts for only 21% in Stahl and Wilson (1995), 16% in Costa-Gomes et al.’s (2001) Open Box treatment and 4.4% in Costa-Gomes et al.’s (2001) Baseline treatment.

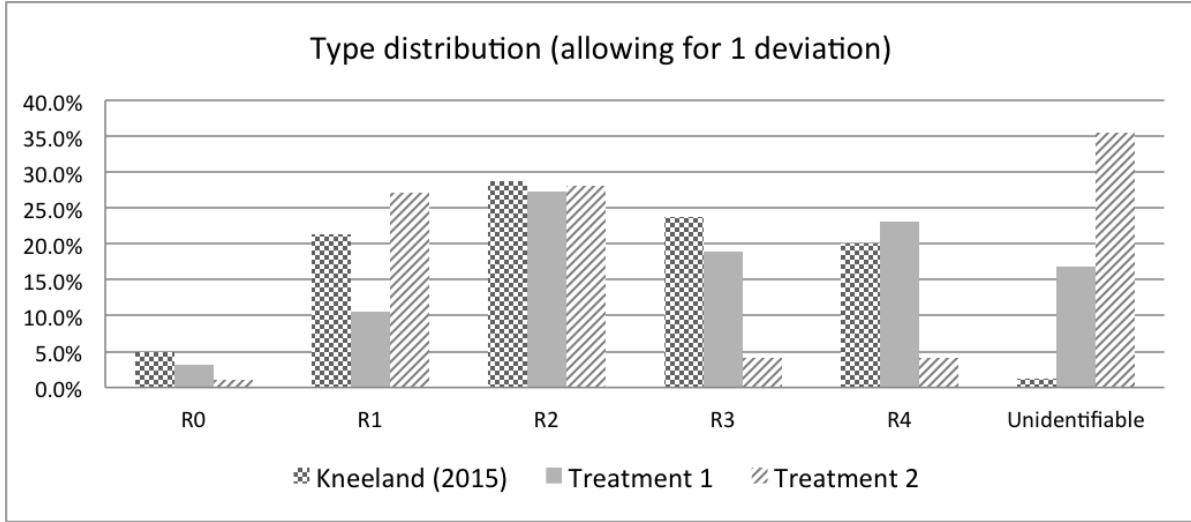
In this comment, I modify Kneeland’s (2015) design by removing two features in the payoff matrices of her ring games that could potentially bias type classification. The first treatment involves a pair of ring games close to the ones used in Kneeland (2015), but the row with the largest sum in each matrix is never set to be an equilibrium strategy, and there is not a non-equilibrium strategy that has the lowest sum of the rows and contains zero. In the second treatment, I further set some rows that have lowest sums and contain zeros to be the equilibrium strategies.

Figure 1.1 reports the type distribution from Kneeland (2015) and the two treatments in my experiment. I am able to successfully reproduce a type distribution that is not statistically different from Kneeland’s (2015) in the first treatment, with over 40% of the subjects being 3rd- or higher-order rational. However, subjects deviate from the theoretical model more times in my data and there are more subjects who satisfy at least 1st-order rationality but could not be matched to any type (the last category “unidentifiable” in the figure). The distribution is heavily skewed to the left in the second treatment, with the proportion of high types decreasing to 10%.

When looking at subjects’ off-equilibrium behavior, I find that the assumption that subjects are unresponsive to changes in higher-order payoffs does not hold as well in my data as in Kneeland’s (2015), which partly explains the larger deviation rates. Given the classified types, the subjects follow the prediction of the assumption over 80% of the time on their off-equilibrium path in Kneeland (2015), while the compliance rates are lower than 60% in both of my two treatments.

Overall, I believe that the type distributions from ring games are robust and reliable, as long as the equilibrium strategies are not set to be the ones with lowest sums and zeros. However, the deviation rates are sensitive to the payoff functions, and it would bear further investigation as to why the key assumption holds less well in some cases.

The rest of the paper is organized as follows: the next three sections present the ring games, identification strategy and the experimental procedure. Data description and type



Note: (1) The analysis uses data from Kneeland’s (2015) main treatment (80 subjects), Treatment 1 (95 subjects) and Treatment 2 (96 subjects). (2) Each subject is classified as $R1 - R4$ with no more than 1 deviation from the predicted action profiles. Otherwise they are assigned to $R0$ or unidentifiable. (3) The subjects classified as unidentifiable are able to choose dominant strategies but do not match any of the predicted patterns. They satisfy at least 1st-order rationality, which makes them different from $R0$.

Figure 1.1: Type distribution (allowing for 1 deviation)

assignment results will be reported in Section 5. Section 6 concludes.

1.2 The Ring Games

Figure 1.3 presents the two 4-player ring games (G1 and G2) used in Kneeland (2015). Each Player i chooses from three actions: a_i , b_i and c_i . In both rings, Player 1’s (P1) payoff is determined jointly by his own action and his direct opponent Player 2’s action. Player 2’s (P2) payoff is determined by his and Player 3’s actions. Player 3’s (P3) payoff depends on his and Player 4’s actions. Player 4’s (P4) payoff depends on his and Player 1’s actions. In particular, the first three payoff matrices in of G1 and G2 are identical, and Player 4 has dominant strategies in both rings. Thus, both games are dominance solvable. Each player has a unique equilibrium strategy which is highlighted as the shaded row in the payoff matrix.

Players are assumed to satisfy heterogeneous orders of rationality. A player is rational if he is able to maximize his expected payoff given a certain belief. A player satisfies 2nd-order rationality if he is rational and believes that his opponents are rational. A player is k th-order rational if he believes that his opponents satisfy $(k - 1)$ th-order rationality. Kneeland (2015) defines an Rk type as satisfying k th-order rationality but not $(k + 1)$ th-order rationality. An Rk type has the same belief order in rationality as an Lk as defined in the level- k literature.

				Player 2				Player 3				Player 4				Player 1								
				a2	b2	c2				a3	b3	c3				a4	b4	c4				a1	b1	c1
Player 1	a1	8	20	12	Player 2	a2	14	18	4	Player 3	a3	20	14	8	Player 4	a4	12	16	14	Player 1	a4	12	16	14
	b2	0	8	16		b2	20	8	14		b3	16	2	18		b4	8	12	10					
	c3	18	12	6		c2	0	16	18		c3	0	16	16		c4	6	10	8					
				P1's Payoff				P2's Payoff				P3's Payoff				P4's Payoff								

G1

				Player 2				Player 3				Player 4				Player 1								
				a2	b2	c2				a3	b3	c3				a4	b4	c4				a1	b1	c1
Player 1	a1	8	20	12	Player 2	a2	14	18	4	Player 3	a3	20	14	8	Player 4	a4	8	12	10	Player 1	a4	8	12	10
	b2	0	8	16		b2	20	8	14		b3	16	2	18		b4	6	10	8					
	c3	18	12	6		c2	0	16	18		c3	0	16	16		c4	12	16	14					
				P1's Payoff				P2's Payoff				P3's Payoff				P4's Payoff								

G2

Note: the unique equilibrium strategies are given by the highlighted rows.

Figure 1.2: The 4-player dominance solvable ring games used in Kneeland's (2015)

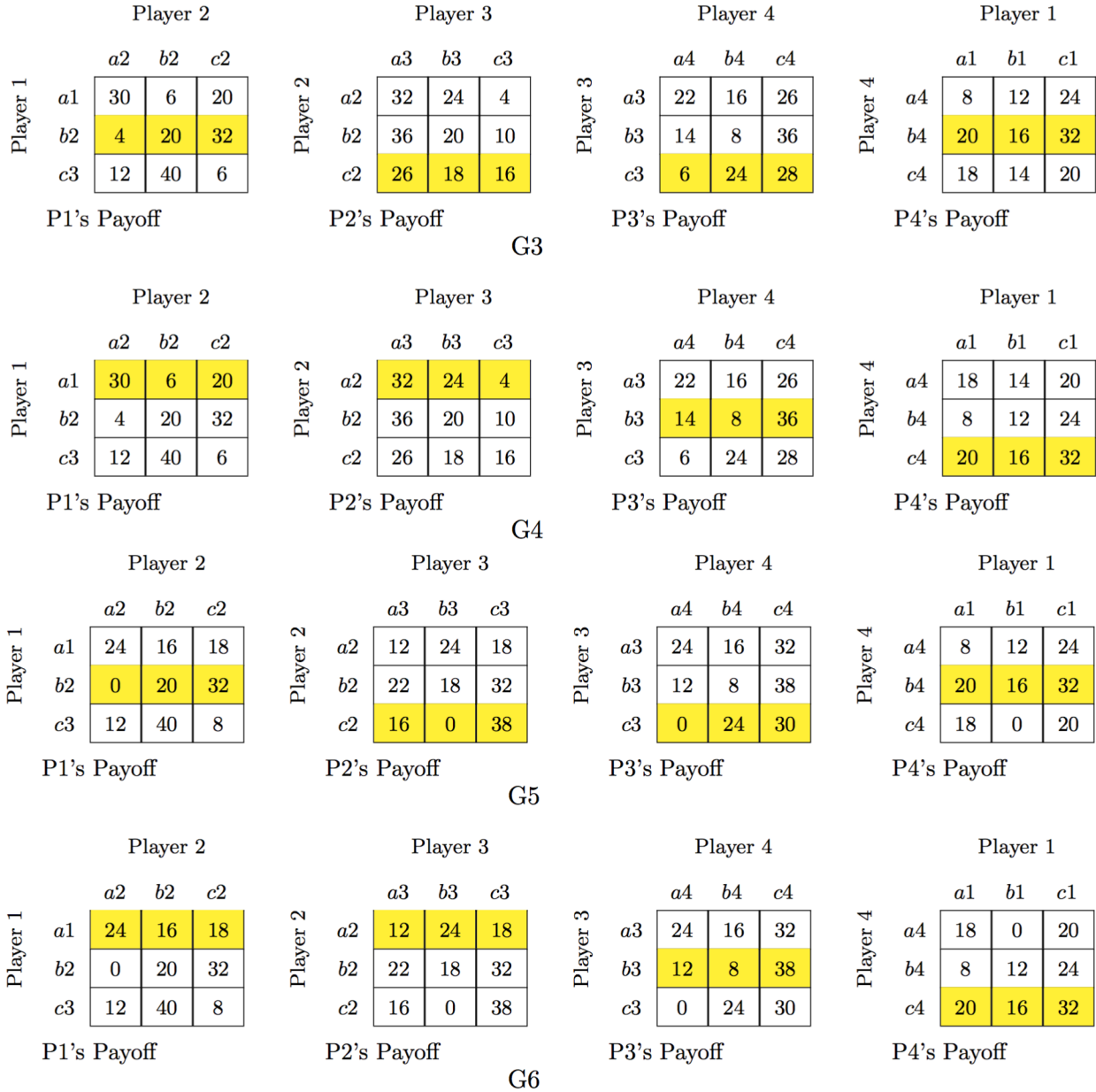
The only difference is that the Lk type's behavior depends on the model's assumption of $L0$'s actions, while Rk is allowed to hold any belief on $R0$'s behavior pattern, as long as it does not respond to payoff changes.

Since Player 4 has a best response regardless of other players' actions, he does not need to hold beliefs in others' rationality to pick out this strategy. Player 4 will choose the dominant strategy as long as he is $R1$. If Player 3 believes that Player 4 is at least $R1$, then he could best respond accordingly. By definition Player 3 will choose his equilibrium strategy as long as he is at least $R2$. The same logic applies to the other players in the ring. Player 2 needs to be at least $R3$ to best respond to $R2$ and choose the equilibrium strategy, and Player 1 needs to be at least $R4$ to choose the equilibrium strategy. Therefore we are able to observe up to $R4$ behavior if we let each subject play at all the four positions of these ring games.

Kneeland (2015) observes a mean of 2.3 steps of thinking in her main treatment, with nearly 50% of the 80 subjects being $R3$ or higher types¹. Almost 95% of the subjects fit the prediction of her theoretical model with no more than one deviation. It seems that the special structure of ring games, which emphasize the higher-order dependencies between players, is able to naturally induce higher levels of thinking. But the type distribution could also be sensitive to the numbers in the payoff matrices.

In particular, two features in Kneeland's (2015) games that could potentially affect ob-

¹At the time this paper was written, I only had access to the choice data of the 80 subjects in the main treatment of Kneeland (2015). Her robustness treatment, which includes 36 subjects, yields a type distribution that is not statistically different from the main treatment.



Note: the unique equilibrium strategies are given by the highlighted rows. G3 and G4 are used in three sessions of this experiment. G5 and G6 are used in the other three sessions.

Figure 1.3: The 4-player dominance solvable ring games used in this paper

served types. Firstly, in G1 the rows of equilibrium strategies of the first three players (a_1 , b_2 , and a_3) also happen to be the rows with the largest sums in the matrices, which are usually the first choices of non-strategic subjects. Indeed, the compliance rate of Nash equi-

librium turns out to be much higher at these positions in G1 than in G2. So some subjects' types might be overestimated with games like G1. Secondly, in both G1 and G2, several of the non-equilibrium strategies of Players 1, 2 and 3 (b_1 , c_2 , and c_3 in both rings) have the lowest sums among the three strategies and contain zeros, which makes them less likely to be chosen so the actual choices tend to be more concentrated on the other two strategies in the matrix. It could potentially raise the observed compliance rates of Nash equilibrium and make the choice data less noisy.

i modify the design by using two new pairs of ring games. In the first pair, G3 and G4, the row with the largest sum in each matrix is never set to be an equilibrium strategy. In addition, the two other strategies have an equal sum of the row. So there is not a row that has the unique lowest sum in G3 and G4. Besides, there is no zero in the payoff matrices. The G3 and G4 pair is used to test whether the high average levels of thinking and low deviation rates from the theoretical model could be successfully reproduced when the two features mentioned above are removed. In G5 and G6, the rows with the largest sums do not coincide with equilibrium choices either. Moreover, in G5 the equilibrium strategies of Players 1, 2 and 3 are the ones with the lowest sums and zeros. Therefore it could further be checked the robustness of ring game experiments when these rows, just like those unattractive non-equilibrium strategies rows in G1 and G2, are set to be the equilibrium strategies.

1.3 Identification Strategy

Kneeland (2015) models off-equilibrium behavior by assuming that the players with limited depth of reasoning will not respond to the changes in others' payoffs if they are too many steps away.

Consider an n -player ring game $\Gamma = (I = \{1, \dots, n\}, ; S_1, \dots, S_n; \pi_1, \dots, \pi_n; o)$ with a finite set of players I , a set of strategies S_i for each player i , the payoff function $\pi_i : S_i \times S_{o(i)} \rightarrow \mathbb{R}$, and the opponent mapping function $o : I \rightarrow I$ with $o(i) = 1 + i \times \text{mod}n$. The function $o(i)$ defines a player's first-order opponent in the ring, whose action would directly affect his payoff. Higher-order opponents $o^k(i)$ could be defined as $o^k(i) = o(o^{k-1}(i))$.

Kneeland (2015) defines the *payoff hierarchy* $\bar{h}^i(\Gamma)$ of ring game Γ by

$$\bar{h}^i(\Gamma) = \{\pi_{o^k(i)}\}_{k=0}^{\infty}.$$

For any player i , his own payoff is denoted by $\bar{h}_0^i(\Gamma)$. His first-order payoff, which is the payoff of his first-order opponent, is given by $\bar{h}_1^i(\Gamma)$. Similarly, his k th-order payoff, which is the payoff of his k th-order opponent, is given by $\bar{h}_k^i(\Gamma)$.

The assumption **ER** (exclusion restriction) states that an Rk player will not be affected by the changes in the payoffs of his k th- or higher-order opponents.

ER (Kneeland, 2015): A player i who satisfies k th-order rationality but not $(k + 1)$ th-order rationality ($k \geq 1$) will play the same action in any two games, Γ and Γ' , where $\bar{h}_j^i(\Gamma) = \bar{h}_j^i(\Gamma')$ for all $j < k$.

Remark ER is equivalent to assuming that higher-order players believe that $R0$ subjects do not respond to his own payoff changes. To see this, if $R0$ is unresponsive to his own payoff changes, then ER certainly holds. If higher-order subjects believe that $R0$ responds to changes in a certain way, then they should respond accordingly, which means that Rk needs to respond to the change in his k th-order payoff. Therefore, ER also implies the belief in unresponsive $R0$. The randomly choosing $L0$, as seen in a lot of level- k studies, is not responding to payoff changes and could be regarded as a special case of ER. So ER is weaker than the assumptions in these studies. However, ER does not incorporate the cases in which the irrational type non-optimally responds to payoff changes by systematically choosing some dominated strategies.

To use ER in identification, consider a pair of 4-player dominance solvable ring games that only differ in the payoff matrices of Player 4². The pairs of ring games used in Kneeland (2015) (G1 and G2) and in this paper (G3 and G4, G5 and G6) all share this feature. The payoff matrices of the first three players are exactly the same in the two ring games in each pair. The predicted action profiles for each Rk type using ER could be summarized as:

(1) $R1$ will always play Nash equilibrium strategies as P4, and choose the same actions as P3, P2 and P1 in both rings.

(2) $R2$ will always play Nash equilibrium strategies as P4 and P3, and choose the same actions as P2 and P1 in both rings.

(3) $R3$ will always play Nash equilibrium strategies as P4, P3 and P2, and choose the same action as P1 in both rings.

(4) $R4$ or higher types will always play Nash equilibrium strategies as P4, P3, P2 and P1.

In these ring games, Player 4 is the $(4 - i)$ -th order opponent of Player i ($i = 1, 2, 3$). Hence ER implies that an Rk subject will make the same choice in both rings as Player i , where $4 - i \geq k$. That is, $R1$ subjects do not respond to the change in Player 4's payoffs and make the same choices as P1, P2 and P3; $R2$ subjects make the same choices as P1 and P2; $R3$ subjects make the same choice as P1; and $R4$ or higher-order rational subjects choose the equilibrium strategies at all of the player positions.

Now consider the Player 3 positions in G3 and G4. An $R1$ or lower type player chooses the same strategy in these two positions if ER holds. But an $R2$ or higher type will be able to pick the equilibrium strategies, which are different for G3 and G4. The Nash equilibrium strategies for Player 3 are $(c3, b3)$, while the possible choices of $R1$ or lower types could only be one from the set $\{(a3, a3), (b3, b3), (c3, c3)\}$, given that ER holds. Thus, $R1$ could be separated from higher types by their behavior at Player 3 positions. Similarly, $R2$ could be separated from higher types using Player 2 positions; $R3$ can be separated from higher types using Player 1 positions.

There is no overlap between the four predicted action profiles, and the subjects who chooses these profiles exactly are guaranteed a clean identification. However, there exists a

²In most games without ring structure, the second-order opponent is usually the player himself. Hence this identification strategy can be used to identify only up to $R2$ players in these games.

considerable number of subjects who cannot be matched exactly to any of the four types. They will be assigned to the closest type based on the likelihood function that will be described in detail in Section 1.5.

1.4 Experimental Procedures

The experiment was conducted at the Experimental Social Science Laboratory (XLab) at UC Berkeley. A total of 192 subjects, who were recruited from Berkeley undergraduate classes, participated in 6 different sessions, with between 28 to 36 subjects in each session. G3 and G4 were played by 96 subjects from three sessions (hereafter “G3G4 sessions”). G5 and G6 were also played by 96 subjects from the other three sessions (hereafter “G5G6 sessions”). The data was collected through an online interface. Subjects were not allowed to interact directly, and their identities were kept confidential. After subjects read the instructions, the instructions were read aloud by an experimenter. Then the subjects were given a short quiz to test their understanding of the game structure followed by 4 unpaid practice games to become familiar with the interface.

The main experiment consisted of 8 ring games (G3 and G4, or G5 and G6) for this paper and 16 ring games with a slightly different structure, which is not analyzed in this paper. The subjects played these 24 games in a random order. In each game they were matched with a new group. Though the subjects were not allowed to write during the experiment, the online interface allowed them to mark any cell in the payoff matrices by a click of the mouse. In this way they were able to easily track the iterative dominance across matrices. The subjects were not allowed to make changes once they had confirmed their choices.

There was a time limit of 60 seconds for the subjects to complete each game. If a subject failed to choose in a game, the earnings for this game would be zero, and the system would randomly pick from the three choices for him to calculate the payoffs of his opponents³. The decision time of each game was recorded for each subject.

After the subjects had completed all 24 games, they were asked a few short questions and took a survey of their demographic characters. Each experimental session lasted about one and a half hours. At the end of the experiment, one of the 24 games was randomly chosen for payment. The average earnings were \$25, plus a \$5 participation fee, which were paid in private after each session.

³In Kneeland (2015) there is no time limit for each game, and subjects are required to spend 90 seconds before making a choice. There are 8 subjects who failed to choose in one game (out of 192×8 choice rounds in total). For 7 out of the 8 subjects, missing one choice does not affect their type identification. Only one subject (ID412 in the G3G4 sessions) is affected and is excluded from the type distribution presented in the following section.

1.5 Experimental Results

Table 1.1 breaks down the percentages of subjects who played the equilibrium strategies by each of the player positions. An evident similarity between my results and Kneeland’s (2015) is that the percentages are close to 100% at all the Player 4 positions, which indicates that almost all of the subjects satisfy the rationality condition. Besides, in all the six ring games but G6, the proportions of Nash players follow a decreasing order in the player positions Players 4, 3, 2 and 1, which is in line with the prediction of the level-k model. The equilibrium compliance rates here are higher compared with Costa-Gomes et al. (2001). In Costa-Gomes et al. (2001), who use 2×2 , 2×3 and 2×4 normal form games, the percentages of choices that comply with equilibrium are around 90% when it requires one round of elimination and 70% when it requires two rounds of elimination.

Positions	P1	P2	P3	P4	P1	P2	P3	P4
Kneeland (2015)	G1				G2			
(main treatment)	82.5%	90.0%	91.3%	95.0%	41.3%	47.5%	78.8%	98.8%
G3G4 sessions	G3				G4			
	41.7%	62.8%	85.4%	99.0%	55.8%	59.4%	84.4%	97.9%
G5G6 sessions	G5				G6			
	10.5%	16.8%	45.2%	99.0%	43.8%	35.4%	79.2%	99.0%

Note: the Kneeland (2015) data is from her main treatment, which consists of 80 subjects.

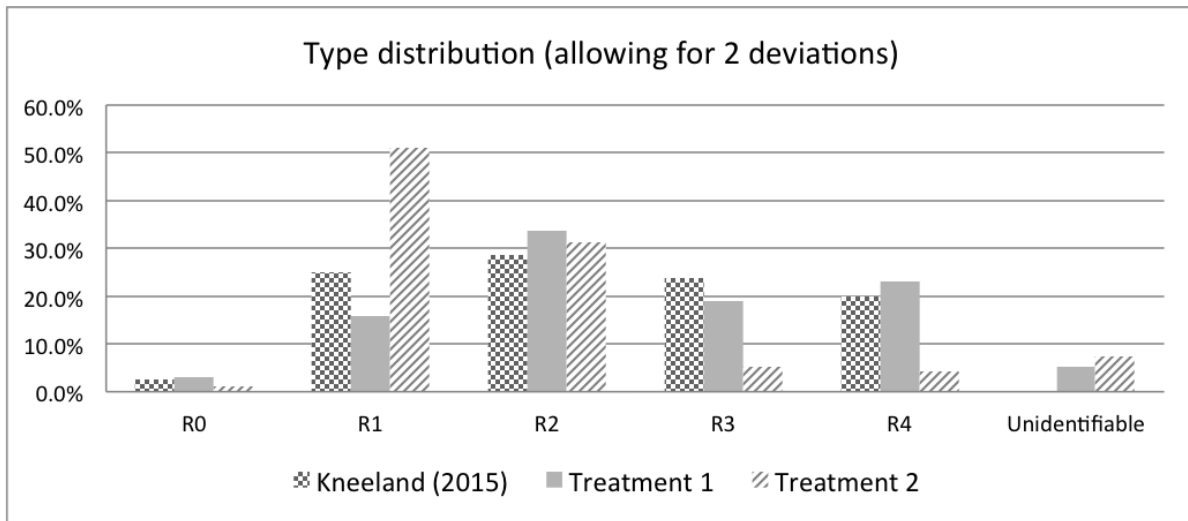
Table 1.1: Proportion of subjects who play the equilibrium strategies

I do find that there are systematic differences in the compliance rates between rings. The percentages of players who use equilibrium strategies are noticeably higher in G1 and lower in G5. In G1, even at the Player 1 positions which requires $R4$ belief, the equilibrium strategy is chosen 82.5% of the time, while in other games except G5, the compliance rates are only 40%-50%. It seems that games like G1 do not give a good separation of those who are able to solve for the equilibrium strategy and those who are simply targeting the largest sum row. In addition, in G5 a surprisingly high proportion of subjects seem to be avoiding an equilibrium strategy with low sum of the row and low minimal payoff, even when the dominant strategies are still being chosen almost 99% of the time by their opponents as Player 4.

Despite the fact that choice data could be sensitive to the numbers in one set of the payoff matrices, Kneeland’s (2015) identification strategy, which uses at least a pair of ring games, might mitigate this problem. I next perform type assignment according to the predicted profiles in Section 3.

The type assignments are done through matching a subject's action profile with the predicted profiles. Each action that does not match is counted as a deviation from that types's profile. A subject is then assigned to the type with minimum deviations. If a subject's action profile matches exactly with a type's predicted profile, he will be assigned to this type. However, if the subject deviates from the predicted profile in one or more games, there might be more than one types that have the minimum deviations. Following Kneeland (2015), this subject will be assigned to the lowest of these types.

In Kneeland (2015) a subject is assigned to $R0$ if his profile could not be matched with the predicted profiles of $R1-R4$ by one deviation or less. Figure 1.1 reports the type distributions from my data and Kneeland's (2015) main treatment allowing for 1 deviation in the matching process. 94% of the subjects in Kneeland's (2015) main treatment could be assigned to one of the $R1-R4$ types with the one-deviation criterion. However, a large fraction of the subjects in my experiment cannot be matched with any type with zero or one deviation. They do not appear to be irrational either, for they are able to choose the dominant strategies as Player 4. We call them the *unidentifiable* subjects. Kneeland (2015) does not highlight these subjects since there is only one subject in this category in her data. However, there are 16.8% and 35.4% of the subjects in our two treatment groups respectively that are unidentifiable. Simply assigning them to the irrational $R0$ type would bias type distribution to the left.



Note: (1) The analysis uses data from Kneeland's (2015) main treatment (80 subjects), Treatment 1 (95 subjects) and Treatment 2 (96 subjects). (2) Each subject is classified as $R1 - R4$ with no more than 2 deviations from the predicted action profiles. Otherwise they are assigned to $R0$ or unidentifiable. (3) The subjects classified as unidentifiable are able to choose dominant strategies as Player 4 but do not match any of the predicted patterns. They are at least rational, which makes them different from $R0$.

Figure 1.4: Type distribution (allowing for 2 deviations)

When allowing for two deviations in the type assignment process (Figure 1.4), most

previously unidentifiable subjects are moved to $R1$ or $R2$. The number of $R0$ remains unchanged in my data, but decreases in Kneeland (2015). To determine which criterion is the most appropriate in our analysis, I simulate the action profiles of different types and check the fraction of each type that could be successfully identified allowing for 0, 1 or 2 deviations (Table 1.2).

As more deviations are allowed, there is a tradeoff between more misidentified $R0$ and less misidentified $R1-R4$. The improvement occurs mostly on the identification of $R1$ and $R2$, which is also observed in the actual data. At the same time, we are risking putting 20% more of the irrational $R0$ subjects to one of the $R1-R4$ categories. Nonetheless, since we have identified the exact same groups of subjects as $R0$ using all three criteria in both of the G3G4 sessions and G5G6 sessions, it appears that the increased risk of misidentification of $R0$ is quite low. So in our case, the type classification results allowing for 2 deviations are still reliable.

Despite the higher deviation rates, the type distribution in our G3G4 sessions is quite similar to Kneeland (2015). They both give high means of thinking steps close to 2.5. The Fisher exact test comparing the distributions of Kneeland (2015) and the G3G4 sessions yields p values of 0.540 with 1-deviation criterion and 0.559 with 2-deviation, suggesting that type distribution from G3 and G4 is not statistically different from the distribution in Kneeland (2015).

The proportions of $R1$ and $R2$ types from the G5G6 sessions are considerably higher, with a much lower mean of thinking steps. There is also a larger proportion of unidentifiable types in G5G6 sessions. The Fisher exact test shows that the distribution of G5G6 sessions is significantly different from Kneeland (2015).

The type distribution in Kneeland (2015) is successfully reproduced in the G3G4 sessions, but not the G5G6 sessions, where subjects tend to be diverted from the equilibrium strategies in G5. The results indicate that the type distributions found in the ring game experiments are robust to some extent. However, the model does not fit my data as well as the data from Kneeland (2015). Over 60% of the subjects in Kneeland's (2015) data could be matched exactly to a type's predicted action profile and over 90% could be matched with no more than one deviation, while in the G3G4 sessions, only 30% of the subjects could be matched with zero deviation and 80% could be matched with no more than one deviation. The percentage is even lower in the G5G6 sessions.

One possible explanation is that compliance rates of equilibrium strategies are raised in the first ring of Kneeland's (2015) experiment because the equilibrium strategies coincide with the row of the largest sums. The higher deviation rates in my data could also be due to the low compliance rates of the assumption ER. Given the assigned types with the 2-deviation criterion, ER holds more than 80% of the time in Kneeland (2015), but only 52% and 56% of the time in our two treatment groups.

The causes of the lower performance of ER in my data could be twofold. Firstly, in G3 and G4 there does not exist a strategy with the unique lowest sum and zeros. The strategies with this feature are chosen only around 5% of the time on off-equilibrium path in the other four games, while in G3 and G4 all three strategies are chosen with similar

matched with 0 deviation, $e = 0.05$							
		$R0$	$R1$	$R2$	$R3$	$\geq R4$	UI
Simulated Types	R0	88	0.3	0.3	0	0.1	11.3
	R1	10.4	64.7	0	0	0	24.9
	R2	10.8	3.5	65.5	0.2	0	20
	R3	7.8	0.6	3.8	66	0.2	21.6
	R4	10.8	0	0.1	2.3	65.9	20.9
matched with 1 deviation, $e = 0.05$							
		R0	R1	R2	R3	R4	UI
Simulated Types	R0	87.1	4.4	1.1	0.3	0.1	7
	R1	3.1	93.7	0.3	0	0	2.9
	R2	2.2	7.5	88.6	0.2	0	1.5
	R3	2.8	3.6	7.2	84.7	0.3	1.4
	R4	2.6	0.3	4	7.5	83.6	2
matched with 2 deviations, $e = 0.05$							
		R0	R1	R2	R3	R4	UI
Simulated Types	R0	73.7	17.3	3.9	1.8	0.7	2.6
	R1	0.3	99.3	0.4	0	0	0
	R2	0.2	9	90.4	0.4	0	0
	R3	0.3	4.7	9.1	85.6	0.3	0
	R4	0.1	1.8	4.2	9.6	84.3	0

Note: Each type is simulated with 1000 action profiles according to the prediction of ER and deviation rate $e = 0.5$. $R0$ subjects are assumed to choose uniformly randomly. Allowing higher deviation rates in the simulated data would increase mis-identification of the $R1$ - $R4$ types, but similar patterns could be observed as we allow more deviations in the type assignment process.

Table 1.2: (mis-)Identification rates of simulated action profiles (shown in percentages) with deviation rate 0.05

probabilities on off-equilibrium path. When the choices are more concentrated to two out of the three available strategies, it is more likely that a subject would choose the same strategy at the same position in the two rings and thus satisfy the prediction of ER. Secondly, it is a different story in G5 and G6. At each of the Players 1, 2 and 3 positions, a large fraction of the subjects choose equilibrium strategies in G6 and the rows with the largest sums in G5. In the cluster analysis reported in the next section, 17 subjects are assigned to these clusters in which they avoid the equilibrium strategies that contain 0. The actual number of subjects who have the tendency of this kind of behavior might very well be larger than 17. These subjects are infect high reasoning ability types but are diverted from the seemingly risky equilibrium strategies in G5. Erroneously assigning them to low types leads us to observe more failures of ER.

Of course it should also be noted that the time limit of each decision round in my experiment, which is nonexistent in Kneeland (2015), might also contribute to both larger error rates in finding equilibrium strategies and the random preference shifts which could possibly lower the compliance of ER.

1.6 Cluster Analysis

To test whether there exists an omitted type or unknown heuristic that prevails in our data, I perform the cluster analysis similar to the one used by Costa-Gomes and Crawford (2006). The choice profile of each subject is treated as the profile of a pseudotype. For each set of games, I rerun the type assignment exercise by matching a subject to the four Rk types and all the pseudotypes of the rest of the subjects from the same game. For example, a subject from Kneeland (2015) will be matched to one of the $R1$ - $R4$ types and the pseudotypes of the rest of the 79 subjects. The analysis would let us know whether a subject's behavior pattern could be better explained by the Rk type profiles or a set of pseudotypes .

Following Costa-Gomes and Crawford (2006), a cluster is defined as a group such that: (1) each subject's original estimated type has smaller likelihood than the pseudotypes of all the other subjects in the group; (2) all subjects in the group are close enough, i.e. different in no more than 1 choice; (3) the choice profiles must differ from the prespecified types by more than 1 choice, otherwise it would be hard to identify whether there is a new type or they are just Rk types with 1 deviation⁴.

No group that satisfies all the three conditions above is found in the G3G4 sessions and Kneeland (2015). In the G5G6 sessions, five clusters, including 17 subjects, are identified. As shown in Table 1.3, their choices follow similar patterns, which can be described as: first identify the Nash choice, then choose the Nash choice if the row does not contain zero;

⁴Because the action profiles that cannot be matched with any type with 1 deviation or less are assigned to $R0$ or Unidentifiable types, the third condition implies that the potential member of the clusters would be $R0$ or Unidentifiable. Costa-Gomes and Crawford (2006) do not have the third requirement in their cluster definition. Rather, they use a case-by-case analysis to decide whether a group is significant enough to be a cluster

Cluster	Assigned Level	N	G5				G6			
			P1	P2	P3	P4	P1	P2	P3	P4
1	Unidentifiable	3	ms	ms	ms	ms/nc	nc	nc	nc	ms/nc
2	Unidentifiable	6	ms	ms	ms	ms/nc	ms	nc	nc	ms/nc
3	Unidentifiable	3	–	ms	ms	ms/nc	ms	ms	nc	ms/nc
4	Unidentifiable	3	ms	ms	ms	ms/nc	nc	ms	nc	ms/nc
5	Unidentifiable	2	ms	ms	ms	ms/nc	nc	nc	ms	ms/nc

Note: *ms* stands for the row with maximum sum; *nc* stands for the Nash choice.

Table 1.3: Clusters found in the G5G6 sessions

otherwise go for the row with the largest sum. Although these subjects are classified as low types or unidentifiable types, they actually show high levels of reasoning in G6, where they do not need to avoid zeros. For instance, the three subjects in the first cluster use Nash strategies in all the four positions of G6; the six subjects in cluster (2) choose Nash strategies at all positions except player 1. These subjects could have been identified as *L4* or *L3* without their zero-avoiding behavior. These clusters provide evidence on how seemingly unattractive equilibrium strategies might affect the estimated type distributions.

1.7 Conclusion

The findings in this paper suggest that the type distributions from ring games are reliable and could be replicated in a different laboratory setting, as long as the factors that divert high types from choosing equilibrium strategies are taken care of. However, the compliance rates of the key assumption ER, that subjects with limited orders of rationality do not respond to the changes too many steps away, are highly sensitive to the payoff functions. The partial failure of ER could lead to an increase in deviation rates and the rise of a group of unidentifiable subjects.

One direction of the future research could be a further exploration on the validity of ER. When using ER, it is implicitly assumed that a *Lk* player stops using the information more than *k* steps away. This could be true if *Lk* subjects always use only *k* reasoning steps. However, as suggested by Costa-Gomes and Crawford (2006) and Agranov et al. (2012), subjects could have higher cognitive abilities and they behave like *Lk* players merely because of their beliefs. In this case it would be questionable whether players are affected by the information beyond *k* steps. To shed some light on the question, an investigation on whether subjects could think more step than their exhibited levels is given in the next chapter in a ring network setting. It might also be helpful trying to monitor subjects' information

searching pattern in the ring games, with the similar methodology given by Costa-Gomes et al. (2001), Costa-Gomes and Crawford (2006), and Brocas et al. (2014). With the additional information available, it might be made clearer how many opponents' payoff matrices are relevant in decision making.

Chapter 2

Does Level- k Behavior Imply Level- k Thinking?

2.1 Introduction

The level- k theory is proposed to model player's systematic deviation from Nash equilibrium by allowing for inconsistent beliefs (Nagel, 1995; Stahl and Wilson, 1994, 1995; Ho et al., 1998). The model starts with the irrational or non-strategic $L0$ type, who is usually assumed to choose randomly or use certain salient strategies. $L1$ best responds to the belief that all the rest of the players are $L0$ and each higher Lk best responds to $L(k - 1)$.

Current literature on the level- k model mostly focuses on the identification of Lk types by looking at the number of reasoning steps used by the subjects. Most subjects in these experiments appear to be using no more than two or three steps of reasoning when playing a game for the first time. Then a natural question is why do they stop at these low levels?

This study attempts to identify whether subjects behave at these levels due to the belief that other participants do not think more than one or two steps, or due to their own lack of ability to think further. A subject's exhibited sophistication level in a certain game depends on both his belief in the opponents' rationality levels and his ability to finish all the required reasoning steps. But belief and ability are not directly observable from choice data. In most studies, a subject is classified as Lk if he uses k steps of reasoning, which implies that both his belief level and ability level are greater than or equal to k . But few studies have provided a clear answer to whether belief or ability is the decisive factor of the observed levels.

The original level- k model assumes that the heterogeneous levels are due to different orders of belief. It is argued in some studies that the observed levels are far below the subjects' upper bounds of reasoning ability (for examples, see Crawford and Costa-Gomes (2006) and Arad and Rubinstein (2012)). It implies that people have high reasoning ability but believe that others are of much lower level. This interpretation could be justified by the fact that people tend to underestimate their opponents, and that being aware of this fact, the more sophisticated players also have to respond to the belief in low-order belief.

Therefore, when given enough incentives or induced higher-order beliefs, the players will use more steps or even reach equilibrium.

The alternative story says that Lk behavior is the result of bounded reasoning ability. Subjects might have higher-order belief or even equilibrium belief, but their limited ability prevents them from using too many thinking steps. Since existing studies using various classes of games find that the average k -levels are around two, this story implies that when playing a game for the first time, the average people are only able to do two rounds of iteration, which is far from the requirement of reaching equilibrium in a lot of games. So the two stories have very different implications.

In this study, a Lk player is classified as Lkb if his observed level is determined by his belief, and as Lka if determined by his ability. The identification of Lkb and Lka players utilizes a combination of simultaneous and sequential games. The ring games are first studied in the innovative work of Kneeland (2015). She uses a set of simultaneous ring games to separate different orders of rationality. Here I provide an illustration of the separation of $L2b$ and $L2a$ with 3-player ring games.

Consider a 3-player simultaneous ring game G (Figure 2.1). Player 1's (P1) payoff depends on his and player 2's (P2) actions. Player 2's payoff depends on his and player 3's (P3) actions. Player 3's payoff depends on his and player 1's actions. In particular, Player 3 has a dominant strategy. Players 1 and 2's iterative dominant strategies could be derived from Player 3's best response, and thus the unique Nash equilibrium (c, c, b) could be solved by three rounds of iterative dominance.

		Player 2			Player 3			Player 1						
		<i>a</i>	<i>b</i>	<i>c</i>			<i>a</i>	<i>b</i>	<i>c</i>			<i>a</i>	<i>b</i>	<i>c</i>
Player 1	<i>a</i>	32	24	4	Player 2	<i>a</i>	18	20	26	Player 3	<i>a</i>	8	12	24
	<i>b</i>	36	20	10		<i>b</i>	14	8	36		<i>b</i>	20	16	32
	<i>c</i>	26	18	16		<i>c</i>	6	24	28		<i>c</i>	18	14	20
		P1's Payoff					P2's Payoff					P3's Payoff		

Note: In G , the three players move simultaneously. In G' , players 1 and 3 move in the first stage. Player 2 moves in the second stage after observing players 1's and 3's choices.

Figure 2.1: G and G' : 3-player ring games

G is similar to the simultaneous games used in Kneeland (2015). In this game, player 3 only needs to hold at least $L1$ belief and use one step of reasoning to play the dominant strategy. Player 2 needs to believe that player 3 is best responding ($L2$ belief) and use two steps of reasoning to solve for his iterative dominant strategy, and player 1 needs $L3$ belief and three steps of reasoning.

If a player chooses the (iterative) dominant strategies as players 2 and 3, but not as player 1, he would be classified as $L2$. However, it could not be inferred from these simultaneous

games whether he is responding to $L2$ belief, or whether he has a higher-level belief but could do, at most, two steps of reasoning. The difficulty of identification lies in the fact that, for a certain choice of action, the required levels of belief and the required number of reasoning steps are always the same. This is also true in most games in the existing literature.

To cope with this problem, I introduce the sequential ring games, in which one needs to use more than k steps to respond to Lk belief. The sequential game G' adopts the same payoff structure as in G , but includes two stages. Players 1 and 3 move simultaneously in the first stage. Player 2 observes their actions and then move in the second stage. The sequence of moves is common knowledge among all players. In this game, solving player 3's problem still requires one step of reasoning. Since player 2 will be able to observe player 3's choice before making his own decision, he also needs to think one step in order to be able to best respond.

It is interesting to study the decision situation of player 1 in the sequential game G' . Obviously, player 1 of G' still needs three steps to arrive at the iterative dominant strategy. But he only needs to believe that player 3 picks the dominant strategy and that, after observing player 3's move, player 2 best responds accordingly, which is $L2$ belief by definition. So as player 1 in G' , holding $L2$ belief requires one to think three steps.

Now consider a player who exhibits $L2$ behavior in the simultaneous game G . His behavior as player 1 in G' reveals whether he is Lkb or Lka (Table 2.1). If he has $L2$ belief and his ability is not binding, then he should believe that both players 2 and 3 will be able to pick the dominant strategies, and choose his iterative dominant strategy as well. However, if he behaves as $L2$ in G because he is bounded by two steps of reasoning, he is not able to use three steps and obey iterative dominance as player 1 of G' . Thus Lkb or Lka could be separated using player 1 position of 3-player ring games. Larger rings are needed to get separation of higher types.

G : simultaneous	player 1	player 2	player 3
$L2b$	×	✓	✓
$L2a$	×	✓	✓
G' : sequential	player 1	second mover	player 3
$L2b$	✓	✓	✓
$L2a$	×	✓	✓

Note: ✓ denotes choosing the (iterative) dominant strategy at this position, × otherwise.

Table 2.1: Separation of $L2b$ and $L2a$

The games used in the actual experiment include 4 players. In each game, player 4 is the

one who has a dominant strategy. The games are sorted into three sets, the simultaneous games (SIMUL) and two sets of the sequential games (SEQ-P2 and SEQ-P3). The two simultaneous rings could be used to identify Lk behavior up to $L4$. In the two rings of SEQ-P2, player 2 moves in the second stage, and in the two rings of SEQ-P3, player 3 moves in the second stage. The combination of the simultaneous and sequential games allows me to get separation on the subjects who behave as $L2$ or $L3$ in the simultaneous games. More specifically, a $L2b$ ($L3b$) player best responds to $L2$ ($L3$) belief in both the simultaneous and the sequential games. A $L2a$ ($L3a$) player could use at most two (three) steps of reasoning, and thus is not able to best respond to the same belief in the sequential games.

A total of 184 subjects participated in the experiment, and enough observations were collected from 179 of them to perform the analysis. Each subject played the 8 positions of the two simultaneous games and the 16 positions of the four sequential games, including 4 second mover positions. There are 50 and 39 subjects classified as $L2$ or $L3$ respectively in the simultaneous games. More than half of these subjects failed to best respond to the same belief in the sequential game, suggesting that a considerable amount of subjects are bounded by their ability.

I then perform a subject-by-subject type classification (Figure 2.2). Out of the 50 $L2$ subjects, 21 are confirmed to be $L2b$, who still best respond to $L2$ belief in the sequential games, and 21 appear to be $L2a$, who could not finish the one more reasoning step in the sequential games (the remaining 8 subjects are classified as $L1$, $L3$ or unidentified). Out of the 39 $L3$ subjects, 20 are classified as $L3b$ and 15 as $L3a$ (the remaining 4 subjects are classified as $L0$ or the unidentified). The results show that subjects' belief levels and ability are mostly consistent throughout the experiment, and that about half of the subjects using two or three steps have reached their upper boundaries of reasoning.

I further investigate whether the subjects' belief is based on their opponents' belief or ability. This is tested on the $L3$ subjects. In both sets of the sequential games, $L3$ players need to think four steps. They differ in that, in order to play the iterative dominant strategies as player 1, in SEQ-P3 players need to believe that the opponents are $L2b$, while in SEQ-P2 they could believe that the opponents are $L2a$ or $L2b$. The performances of the 39 $L3$ subjects are not statistically different in the two sets of games. In both SEQ-P2 and SEQ-P3, there are 16 out of these 39 subjects that best respond at player 1 positions, which supports the hypothesis that the $L3b$ players believe that their opponents are $L2b$.

Overall, I find the existence of both the high ability subjects responding to lower-order belief and the low ability subjects who could only think two or three steps, which shows large heterogeneity in subjects' reasoning ability. The results suggest that both subjects' belief and their reasoning ability could be the decisive factor of their exhibited levels.

The remainder of the paper proceeds as follows. The next section summarizes the related literature. The experimental design and detailed identification strategies are presented in Section 2.3. Section 2.4 reports the experimental results. Section 2.5 concludes.

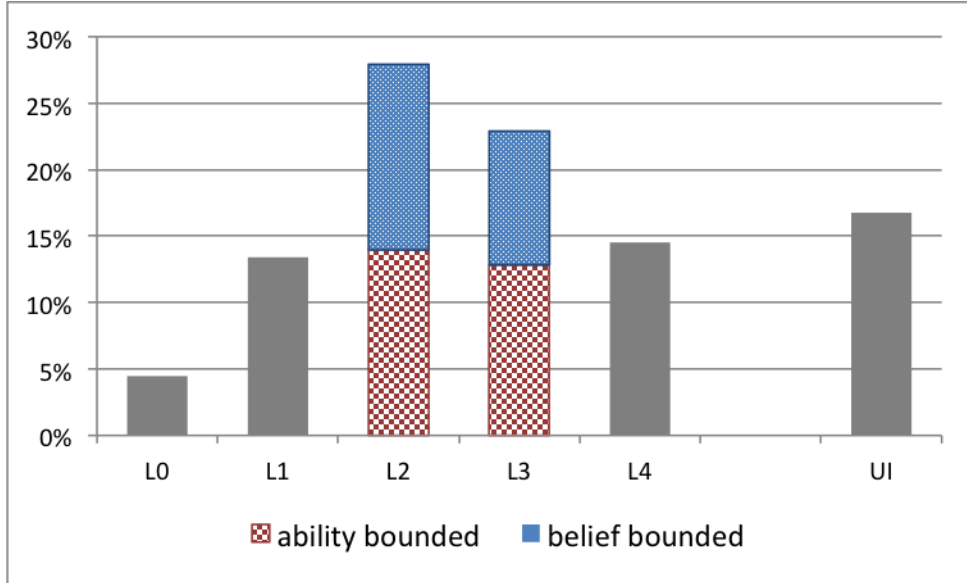


Figure 2.2: Decomposition of $L2$ and $L3$ subjects

2.2 Related Literature

The recent two decades saw the emergence of a vast literature on the level- k model. A detailed review is given by Crawford et al. (2013). This section summarizes only the studies that are most closest to this paper. The simultaneous ring games used in this study are similar to the ones proposed by Kneeland (2015). Her methodology and results will be discussed in detail in the following sections, and hence are not included here. There is also a class of literature assuming that Lk players believe that the opponents are drawn from a distribution of all the lower types, e.g. the Cognitive hierarchy model from Camerer et al. (2004). Since the games in this study do not distinguish between the level- k model and the Cognitive Hierarchy model, players in this study are assumed here to have degenerate belief, as in the original level- k model.

There are a few studies looking at subjects' reaction to the information on the opponents' types. Given the information that the opponents are higher types, the high ability Lkb subjects should be able to respond by raising their own levels, but the ability-bounded Lka ones are not able to adjust their behavior. Some studies find positive results, suggesting that the participants are Lkb . For example, Palacios-Huerta and Volji (2009) invite chess players, presumably the higher types, and college students to play the centipede game, and find that they both exhibit higher levels when playing against chess players than against students. Agranov et al. (2012) find that in a 2/3 beauty-contest game, subjects exhibit higher sophistication levels (lower average choice numbers) as the number of experienced players, some graduate students in economics, increases in the group. In Slonim (2005) experienced subjects are found to respond to the experience levels of their opponents. Alaoui

and Panta (2015) also find that subjects respond to the manipulation of their beliefs.

In some other studies, the results are mixed. For example, in Gill and Prowse (2015), who use repeated beauty contest games, the higher cognitive ability subjects respond positively to the cognitive abilities of their opponents, while the lower cognitive ability subjects do not. Their findings imply that only high types have the ability to respond. In addition, Georganas et al. (2015) find such effect in some games, but not in others. They ask the subjects to choose the strategies for the opponents randomly selected from the whole sample, the higher cognitive ability half and the lower half, and find significantly more higher levels in the responses to higher types in the undercutting games, but no such effect in the two-people guessing games.

A more direct way to identify whether subjects are best responding to their belief is to elicit their belief. Belief elicitation in strategic games has been studied in numerous papers. However, evidence is mixed on whether beliefs could be successfully elicited without altering behavior and whether subjects do act according to their stated beliefs. Some studies find that subjects' actual play in the games is not affected after belief elicitation (Nyarko and Schotter (2002), Costa-Gomes and Weizsacker (2008), Manski and Neri (2013)), while both Ruststrom and Wilcox (2009) and Gachter and Renner (2010) find that incentivized elicitation alters choices. In both Nyarko and Schotter (2002) and Manski and Neri (2013), most subjects appear to be best responding to their first-order belief. However, in Bhatt and Camerer (2005), only 66% of the choices match with the stated first-order belief. Costa-Gomes and Weizsacker (2008) also find that most choices are one step below the stated first-order belief.

Additional strategies are developed in order to show that high ability subjects are responding to low-order belief. Costa-Gomes et al. (2001) and Costa-Gomes and Crawford (2006) train subjects to play against robots programed to use certain Lk or equilibrium decision rules. They find that the subjects have no problem responding to any of these rules, which implies that they have the enough reasoning ability to reach equilibrium. Arad and Rubinstein (2012) use a very simple undercutting game, but still find that subjects mostly use no more than three steps of reasoning. They conclude that this could not be due to obstacles in thinking, and thus must be due to non-equilibrium beliefs. Other methods of identifying belief and strategic thinking processes include tracking players' information search (Costa-Gomes et al. (2001), Costa-Gomes and Crawford (2006) and Brocas et al. (2014)), or translating their beliefs from communication records between players (Burchardi and Penczynski, 2014). Bhatt and Camerer (2005) use brain image to search for the connection between the brain activities of making choices and stating belief.

The mixed findings in the previous studies suggest heterogeneity in the population. The results in this paper, that there exist both belief-bounded and ability-bounded subjects, are in line with the literature. Since the identification in this paper only requires choice data, it circumvents the challenge of belief manipulation or belief elicitation, which makes it easier to achieve a within-subjects design.

This paper is also related to the literature that studies level- k models in sequential games. For examples, see the two stage games in Stahl and Haruvy (2008), and the club game in Breitmoser et al. (2014). Ho and Su (2013) provide a dynamic level- k model to study the

centipede games. To the best of my knowledge, the design in this paper is the first to create more than k reasoning steps for a Lk belief using sequential moves.

2.3 Experimental Design and Identification Strategy

Some Notations

In the dominance-solvable ring games, $L0$ is assumed to be unable to identify a strictly dominated strategy. $L1$ best responds to the belief that others do not obey strict dominance. $L2$ best responds to the belief that others are $L1$. For each $k \geq 2$, Lk holds the belief that the opponents are $L(k-1)$ and best respond accordingly.

In previous studies, a player is classified as Lk if he exhibits Lk behavior. That is, he plays best responses to the Lk type's belief. This study distinguishes between the ones who hold Lk belief and have the ability to best respond, and the ones who might believe that others are higher than $L(k-1)$, but behave like Lk because they could not identify the best responses to higher-order beliefs.

I assume that each player is characterized by his belief level b and his ability level a ($b, a = 1, 2, 3, \dots$). Similar setup could be found in Strzalecki (2014), Alaoui and Panta (2015) and Georganas et al. (2015) $b = k$ if the player holds Lk type's belief. $a = k$ if the player is able to do at most k steps of iterative thinking. The belief level b and ability level a are not directly observable from the choice data. Since in most games used in previous studies, it requires k steps of reasoning to respond to Lk belief. Therefore when a player chooses a best response to Lk belief, it could only be inferred that he has $\min\{a, b\} = k$.

In this study I try to separate the two cases. If a player exhibits Lk behavior due to his belief and has much higher ability, he is called Lkb and has $b = k$ and $a > k$. In the other case, a player behaves as Lk because he could think at most k steps in this game. He is called Lka and has $b \geq k$ and $a = k$. Non-strict inequality is used here, because it is indistinguishable whether his belief level is also k or he has higher level belief but is not able to best respond.

Note that here b is defined by the belief in others' belief. A player has $b = k$ if he believes that others best respond to $b = k - 1$. In this experiment, I also test whether there exist players who hold the belief in others' ability (that others have $a = k - 1$) instead of others' belief.

The Games

The main part of the experiment consists of three sets of 4-player ring games, which are called SIMUL (the simultaneous games), SEQ-P2 and SEQ-P3 (the sequential games) (see Figure 2.3). The SIMUL set contains two ring games, G1 and G2, with simultaneous moves. The two sets of sequential games are labeled by the second movers. In the games of SEQ-P2, G3 and G4, player 2 is the second mover. Players 1, 3 and 4 move simultaneously in the first

stage and player 2 moves in the second stage after observing their actions. In the SEQ-P3 games, G5 and G6, player 3 moves in the second stage. The sequence of move and the information structure are common knowledge among all the players.

Each player has three actions to choose from, and the payoffs are represented by 3×3 matrices. Player 1's (P1) payoff is determined jointly by his own action and his direct opponent player 2's (P2) action. Player 2's payoff is determined by his and player 3's (P3) actions. Player 3's payoff depends on his and player 4's (P4) actions. Player 4's payoff depends on his and player 1's actions.

In these ring games, player 4 always has strictly dominant strategies. Therefore in each game there is a unique Nash equilibrium, which could be solved from four rounds of iterated elimination of strictly dominated strategies.

In addition, in each pair of rings (G1 and G2, G3 and G4, G5 and G6), the payoff matrices of the two players 1, 2 and 3 positions are identical. The two player 4 positions have the same three strategies, but the strategies are labeled in different sequences. This feature, combined with the dominant strategies at player 4 positions, is designed specifically for type classification, which will be discussed in the next subsection.

The payoffs in these games differ from the original ring games in Kneeland (2015) in two ways. Firstly, the row with the largest sum in each matrix is never an iterative dominant strategy. It was shown in Chapter 1 that the rows with the largest sums are usually the first choices of the non-strategic subjects. If these rows also happen to be the iterative dominant strategies, subjects' types could be overestimated. In addition, none of the payoffs contains the salient number 0. Chapter 1 provides the evidence that the 0s in the payoffs could divert the players from choosing these actions and bias type classification. Thus it would help to get a better separation of the Lk types by taking care of these two issues.

***Lk* Types Classification**

Subjects' Lk behavior could be inferred from their choices in the simultaneous games. The identifying assumption **ER** (the *exclusive restriction* in Kneeland (2015)) is applied to separate subjects with different levels. The assumption ER says that a player with Lk but not higher order belief does not respond to the changes in $(k + 1)$ th- or higher-order payoffs. It is weaker than the assumption that $L0$ chooses all the possible actions with equal probability. The latter is a special case of ER. The type classification using the uniformly randomizing $L0$ assumption appears to be less robust among lower types, but the pattern in the main result holds for higher types. These results will be reported in the appendix.

A player's k th-order payoff is defined as his k th-order belief about payoffs. Therefore a player's 1st-order payoff is his own payoff. In the simultaneous ring games, each player only forms belief on his direct opponent. Therefore the player's 2nd-order payoff is his opponent's payoff, and his 3rd-order payoff is the payoff of his opponent's opponent, and so on.

ER could be used to model players' off-equilibrium behavior in ring games¹. It implies

¹In most other games without ring structure, the second-order opponent is usually the player himself.

		Player 2				Player 3				Player 4				Player 1					
		a2	b2	c2		a3	b3	c3		a4	b4	c4		a1	b1	c1			
Player 1	a1	30	6	20	Player 2	a2	32	24	4	Player 3	a3	18	20	26	Player 4	a4	8	12	24
	b1	4	20	32		b2	36	20	10		b3	14	8	36		b4	20	16	32
	c1	12	40	6		c2	26	18	16		c3	6	24	28		c4	18	14	20
		P1's Payoff				P2's Payoff				P3's Payoff				P4's Payoff					

SIMUL: G1

		Player 2				Player 3				Player 4				Player 1					
		a2	b2	c2		a3	b3	c3		a4	b4	c4		a1	b1	c1			
Player 1	a1	30	6	20	Player 2	a2	32	24	4	Player 3	a3	18	20	26	Player 4	a4	18	14	20
	b1	4	20	32		b2	36	20	10		b3	14	8	36		b4	8	12	24
	c1	12	40	6		c2	26	18	16		c3	6	24	28		c4	20	16	32
		P1's Payoff				P2's Payoff				P3's Payoff				P4's Payoff					

SIMUL: G2

		Player 2*				Player 3				Player 4				Player 1					
		a2	b2	c2		a3	b3	c3		a4	b4	c4		a1	b1	c1			
Player 1	a1	16	32	8	Player 2*	a2	6	30	28	Player 3	a3	22	8	24	Player 4	a4	24	20	32
	b1	28	16	14		b2	22	28	8		b3	28	12	20		b4	16	12	28
	c1	14	10	32		c2	4	20	36		c3	32	4	18		c4	22	18	8
		P1's Payoff				P2*'s Payoff				P3's Payoff				P4's Payoff					

SEQ-P2: G3

		Player 2*				Player 3				Player 4				Player 1					
		a2	b2	c2		a3	b3	c3		a4	b4	c4		a1	b1	c1			
Player 1	a1	16	32	8	Player 2*	a2	6	30	28	Player 3	a3	22	8	24	Player 4	a4	16	12	28
	b1	28	16	14		b2	22	28	8		b3	28	12	20		b4	22	18	8
	c1	14	10	32		c2	4	20	36		c3	32	4	18		c4	24	20	32
		P1's Payoff				P2*'s Payoff				P3's Payoff				P4's Payoff					

SEQ-P2: G4

		Player 2				Player 3*				Player 4				Player 1					
		a2	b2	c2		a3	b3	c3		a4	b4	c4		a1	b1	c1			
Player 1	a1	18	30	10	Player 2	a2	20	10	26	Player 3*	a3	8	18	36	Player 4	a4	8	24	16
	b1	36	8	14		b2	34	8	14		b3	28	32	6		b4	24	28	40
	c1	20	14	34		c2	16	32	10		c3	16	40	8		c4	18	6	32
		P1's Payoff				P2's Payoff				P3*'s Payoff				P4's Payoff					

SEQ-P3: G5

		Player 2				Player 3*				Player 4				Player 1					
		a2	b2	c2		a3	b3	c3		a4	b4	c4		a1	b1	c1			
Player 1	a1	18	30	10	Player 2	a2	20	10	26	Player 3*	a3	8	18	36	Player 4	a4	18	6	32
	b1	36	8	14		b2	34	8	14		b3	28	32	6		b4	8	24	16
	c1	20	14	34		c2	16	32	10		c3	16	40	8		c4	24	28	40
		P1's Payoff				P2's Payoff				P3*'s Payoff				P4's Payoff					

SEQ-P3: G6

Note: * denotes second movers.

Figure 2.3: The ring games

that a low type player's choice will not be affected by the changes in the other players' decision situations, if the changes do not look so relevant to his own payoff.

In G1 and G2, the payoff matrices are identical in the players 1, 2 and 3 positions respectively. The only difference is in the payoff matrices of player 4. Hence ER implies that an $L1$ subject chooses the dominant strategies as player 4 but makes the same choice in both rings as players 1, 2 and 3. The behavior pattern of other Lk types could also be predicted. An $L2$ subject makes the same choices as players 1 and 2 in both G1 and G2, and chooses the (iterative) dominant strategies as players 3 and 4. An $L3$ subject makes the same choice at the two player 1 positions and chooses the (iterative) dominant strategies as players 2, 3 and 4. $L4$ or higher types choose the (iterative) dominant strategies at all player positions.

Separation of Lkb and Lka

Since in the simultaneous games, responding to Lk belief requires k steps of reasoning, Lkb and Lka subjects should behave like the same Lk type. Separation could be done through comparing their behavior between simultaneous and sequential games. But in order to make meaningful comparisons, it is essential to assume that a subject's belief level and ability level remain constant in these games.

A1: As a first mover, a subject's upper boundary of reasoning steps stays the same in simultaneous and sequential ring games.

A2: A subject's belief about first mover opponents stays the same in simultaneous and sequential ring games.

Assuming that a subject's type remains constant over similar games is standard in previous studies that involve within-subjects design (Stahl and Wilson, 1995; Costa-Gomes et al., 2006; Costa-Gomes and Crawford, 2006). In this study, I need to additionally assume that adding sequential moves does not change a subject's belief. This assumption is crucial, for one's belief and reasoning ability are likely to vary with the complexity of the games (Georganas et al., 2015). I would like to argue that this is not the case for the first movers.

Although the games in SEQ-P2 and SEQ-P3 involve sequential moves, they share the same payoff structure and reasoning process with the simultaneous games. Thus, as a first mover, one needs to go through the same thought process to obey iterative dominance in both the simultaneous and sequential games. So it is assumed in A1 that a subject's ability level should not change as a first mover.

Due to the same reason, a subject's belief in the first movers' rationality should not change either. For example, if a subject believes in the simultaneous games that his opponent is

Hence this identification strategy can be used to identify up to $L2$ players in these games. In ring games, by increasing the number of players in a ring, a player's high-order opponent's payoff functions could be perturbed without affecting his own payoff and lower-order payoffs. Thus ER could be used to separate higher-orders of rationality as long as enough players are included in the rings.

capable of choosing the strictly dominant action from a 3×3 matrix, he should hold the same belief in the sequential games on first movers. Similarly, the higher-order beliefs on first movers also stay the same.

As for the second movers, although it also takes $L1$ belief and one step of reasoning to best respond, the task complexity is not exactly the same as the $L1$ task in the simultaneous game. Thus, I only assume that the task for the second mover is easier, if not the same, than the task for player 4 in the simultaneous games.

A3: If a subject believes that the opponents obey strict dominance in the simultaneous games, then he believes that they are capable of best responding as the second movers in the sequential games.

With these three assumptions, behavioral patterns of Lkb and Lka subjects in the sequential games could be predicted, as shown in Table 2.2.

		P1	P2	P3	P4			P1	P2	P3	P4
$L1b$	SEQ-P2	×	2nd	×	✓	$L1a$	SEQ-P2	×	2nd	×	✓
	SEQ-P3	×	×	2nd	✓		SEQ-P3	×	×	2nd	✓
$L2b$	SEQ-P2	×	2nd	✓	✓	$L2a$	SEQ-P2	×	2nd	✓	✓
	SEQ-P3	×	✓	2nd	✓		SEQ-P3	×	×	2nd	✓
$L3b$	SEQ-P2	✓	2nd	✓	✓	$L3a$	SEQ-P2	×	2nd	✓	✓
	SEQ-P3	✓	✓	2nd	✓		SEQ-P3	×	✓	2nd	✓
$L4b$	SEQ-P2	✓	2nd	✓	✓	$L4a$	SEQ-P2	✓	2nd	✓	✓
	SEQ-P3	✓	✓	2nd	✓		SEQ-P3	✓	✓	2nd	✓

Note: ✓ denotes choosing the (iterative) dominant strategy at this position, × otherwise. “2nd” denotes second movers. All the types in this table should play the dominant strategies in the subgame as second movers.

Table 2.2: Predicted action profiles of each Lkb and Lka type

$L2b$ and $L2a$ could be separated using the player 2 positions of SEQ-P3. In SEQ-P3, if player 2 has at least $L2$ belief, he would believe that player 4 chooses the dominant strategy. In addition, given A3, he should also believe that player 3 best responds after observing player 4’s action. Therefore, it requires $L2$ belief and three steps of reasoning to play the iterative dominant strategies at the player 2 positions of SEQ-P3. An $L2b$ subject ($b = 2, a > 2$) will be able to solve for the iterative dominant strategies at these positions, while an $L2a$ subject ($b \geq 2, a = 2$) is not able to think three steps and thus not able to best respond.

Similarly, $L3b$ and $L3a$ could be separated using the player 1 position of SEQ-P2 and SEQ-P3. In SEQ-P2, a player 1 with at least $L3$ belief would believe that player 3 chooses the iterative dominant strategy, and that the second mover, player 2, best responds to his observation of player 3's action. In SEQ-P3, a player 1 with at least $L3$ belief should also believe that the opponent best responds as player 2, which requires $L2$ belief. Therefore, it requires $L3$ belief and four steps of reasoning to play the iterative dominant strategies at the player 1 positions in both SEQ-P2 and SEQ-P3. So the $L3b$ subjects ($b = 3, a > 3$) are able to best respond as player 1, but the $L3a$ subjects ($b \geq 3, a = 3$) are not.

In addition, by comparing $L3$ subjects' behavior in SEQ-P3 and SEQ-P2, it could be tested whether $L3b$ subjects believe that their opponents hold $L2$ beliefs or they believe that their opponents are only able to think two steps. In SEQ-P2, $L2b$ or $L2a$ are not distinguishable. A subject classified as $L3b$ might believe that the opponents are $L2b$ or $L2a$. But in SEQ-P3, $L3b$ needs to believe that the opponents are $L2b$. Therefore if there exists any subject who believes that the opponents are $L2a$, he will be classified as $L3b$ in SEQ-P3, but $L3a$ in SEQ-P2.

With this design, $L1b$ and $L1a$ cannot be separated. $L4b$ and $L4a$ could be separated with 5-player ring games, which are not used in this experiment.

Experimental Procedures

The experiment was conducted at the Experimental Social Science Laboratory (XLab) at UC Berkeley. A total of 184 subjects, who were recruited from Berkeley undergraduate classes, participated in 6 different sessions, with between 20 to 36 subjects in each session. Each experimental session lasted about one and a half hours. The average earnings were \$25, plus a \$5 participation fee, which were paid in private after each session.

The data was collected through an online interface. Subjects were not allowed to interact directly, and their identities were kept confidential. After subjects read the instructions, the instructions were read aloud by an experimenter. Then the subjects were given a short quiz to test their understanding of the game structure, followed by 4 unpaid practice games to help them get familiar with the interface.

In the main part of the experiment, the subjects played at the 24 positions of the games in a random order. In each game they were matched with a new group. Though the subjects were not allowed to write during the experiment, our online interface allowed them to mark any cell in the payoff matrices by a click of the mouse. In this way they were able to easily track the equilibrium strategies across matrices. The subjects were not allowed to make changes once they had confirmed their choices.

There was a time limit of 60 seconds for the subjects to complete each game. If they failed to choose in one game, the earnings for this game would be zero, and the system would randomly pick from the three choices for them to calculate the payoffs of their opponents. The second movers were given an additional 30 seconds after the first movers had submitted

their choices². The decision time of each game was recorded for each subject.

After the subjects had completed all 24 games, they took the Cognitive Reflection Test. The CRT is composed of three short questions and is designed to measure subjects' cognitive ability (Frederick, 2005).

At the end of each session, subjects finished a survey of their demographic characters. One of the 24 games was randomly chosen for payment. One out of the three questions in the CRT were chosen for payoff and the subjects got \$0.25 if their answers were correct.

2.4 Experimental Results

This section starts by reporting the descriptive statistics and the identification of *Lk* behavior from subjects' choices in the simultaneous games. Then I analyze the behavioral patterns of each type in the sequential games, and separate the *Lkb* and *Lka* subjects. In addition, a robustness test of learning effects is provided at the end of the section.

Data Description

The percentage of subjects who choose the (iterative) dominant strategies at each of the 24 positions is reported in Table 2.3. For the second movers, I look at whether they choose the dominant strategies in the subgames. Out of the 184 participants, 172 finished all the choices within the time limit; 7 failed to choose in one game but the missing choices do not affect their type classification. The following analysis is based on the 179 subjects. The 5 subjects are excluded because they failed to choose in one game and the missing choices affect their type classification.

The compliance rates are quite high at all of the player 4 positions and the second mover positions (player 2 of SEQ-P2 and player 3 of SEQ-P3). Over 95% of the subjects choose the dominant strategies at these positions, which suggests that the majority of the participants understand the payoff structure and are capable of best responding to strict dominance. The compliance rates of iterative dominance decrease as the required reasoning steps go up. In the simultaneous games, compliance rates are the highest at player 4 positions, followed by players 3, 2 and 1. Similar patterns are observed in the sequential games. This is consistent with the prediction of the level-k model in that fewer players achieve higher rounds of iterative thinking.

A trace of the treatment effects of the sequential moves could be found in the differences in compliance rates. For example, player 2 positions of both SIMUL and SEQ-P3 require three steps of reasoning, but the player 2 position requires only *L2* belief in SIMUL and *L3* belief in SEQ-P3. If subjects' behavior is solely determined by their beliefs, higher compliance rates are expected at the player 2 position of SEQ-P3, as more people have

²There are 12 subjects who failed to choose in one game (out of 184×24 games in total). For 7 out of the 12 subjects, missing one choice does not affect their type identification. Five subjects (406, 412, 610, 814, 1017) are affected and are excluded from the type distribution presented in the following section.

2nd-order belief in rationality compared to 3rd-order belief. Such a pattern could be found in the data. However, the compliance rates at player 2 positions of SEQ-P3 are lower than player 3 of SIMUL, which also requires $L2$ belief. It might imply that not every subject who exhibits $L2$ behavior in the simultaneous games is able to proceed to three steps of thinking. A similar pattern is found for $L3$ subjects. That is, the compliance rates of the player 1 positions in SEQ-P2 and SEQ-P3 are higher than the player 1 positions in SIMUL, but lower than player 2 of SIMUL. Of course, it requires further investigation to tell whether these changes do come from the $L2$ and $L3$ subjects.

		player 1	player 2	player 3	player 4
SIMUL	G1	34.2%	56.0%	77.7%	97.3%
	G2	46.7%	54.3%	79.3%	97.8%
		player 1	player 2 (2nd)	player 3	player 4
SEQ-P2	G3	47.3%	96.7%	80.4%	96.7%
	G4	42.9%	97.3%	77.7%	94.6%
		player 1	player 2	player 3 (2nd)	player 4
SEQ-P3	G5	49.5%	71.2%	96.2%	98.4%
	G6	51.6%	65.2%	97.8%	97.3%

Note: $N = 179$. (2nd) denotes second stage movers.

Table 2.3: Compliance rates of (iterative) dominating strategies

Observed Lk Behavior in Simultaneous Games

A subject would exhibit Lk behavior in the simultaneous games if he holds at least Lk belief and has the ability to perform at least k steps of reasoning. Up to $L4$ behavior could be identified from subjects' choices in the simultaneous games.

The type classification method is from Kneeland (2015). It is assumed that each subject's behavior is determined by a single type, which remains constant throughout the experiment. A subject i deviates from his Lk type's choice profile with probability ϵ_{ik} , which is i.i.d. across games. When a subject deviates from his own type, it is assumed that he chooses the other two strategies with equal probability. The likelihood of a player i being type k given his action profile can be defined as

$$d_{ik}(\epsilon_{ik}, x_{ik}) = (1 - \epsilon_{ik})^{G-x_{ik}} \left(\frac{\epsilon_{ik}}{2}\right)^{x_{ik}}, \quad (2.1)$$

where G is the number of games and x_{ik} is the number of observations that do not match the predicted profile of type k .

A subject is assigned to the type k with the highest likelihood d_{ik} , which is equivalent to finding the lowest number of deviations x_{ik} . If a subject's action profile matches exactly with a type's predicted profile, he will be assigned to this type. However, if $\min_k(x_{ik}) > 0$, there might be more than one minimum x_{ik} . Following Kneeland (2015), this subject will be assigned to the lowest type that has the minimum number of deviations.

If an action profile deviates too much from the predicted profiles of $L1$ - $L4$, he would be labeled as the irrational $L0$ or the *unidentifiable* type. The unidentifiable subjects are defined as deviating from Lk predictions, but picking the dominant strategies as player 4 in both two rings. Hence they are at least capable of best responding and should be distinguished from the irrational, unpredictable $L0$ type. The unidentified types might be using some rules of their own that cannot be captured by our model.

Therefore a cutoff point is needed so that those subjects with $\min_k(x_{ik})$ larger than the cutoff will be assigned to $L0$ or the unidentifiable. Table 2.4 reports the type assignment results with the cutoffs being 0, 1 or 2 deviations. In the first row, when a subject cannot be matched exactly to a Lk type, he is assigned to $L0$ or unidentifiable. This seems to be too strict as there are over 60% of the subjects left unidentified. When allowing for 1 deviation, the share of the unidentified subjects drops down to 15%, with more subjects being assigned to one of the four Lk types. If the 2-deviation cutoff is used, there is a further drop in the number of the unidentified subjects and an increase in $L1$ and $L2$. The numbers of higher types do not change.

To determine which cutoff is the most appropriate in this study, a sample of 10,000 random choosing subjects is simulated and analyzed through the type assignment process. The analysis focuses on how many of these subjects could be correctly assigned to $L0$ and how many are wrongly assigned to one of the Lk types. When allowing for 1 deviation, over 86% of them are classified as $L0$, and around 5% go to the Lk types. It does not differ too much from the 0-deviation cutoff. However, when allowing for 2 deviations, over 25% of the random choosing subjects are assigned as Lk , which is too high to be acceptable. So it appears that the 1-deviation cutoff is the most appropriate, for it gives reliable results and provides enough observations of the Lk types for the following analysis. The results using the 0-deviation and 2-deviation cutoffs, from which a similar pattern could be found as in the main results, are reported in the appendix.

The type classification in Kneeland (2015) also uses the 1-deviation cutoff. The 1-deviation cutoff type distribution in my data is very close to the distribution found in Kneeland (2015). If the unidentified subjects are excluded, the Fisher's exact test comparing these two categorical distributions yields a p -value of 0.926, suggesting that they are statistically not different. However, there is a much larger proportion of unidentifiable of 15.1%, compared to 1.2% in Kneeland's (2015) data³, which is due to larger deviation rates

³Kneeland (2015) does not include a category of unidentifiable subjects, since there is only one such subject in her main treatment. This subject is assigned to $L0$ in her original paper.

Level	$L0$	$L1$	$L2$	$L3$	$L4$	UI
0 deviation	8	5	17	17	19	113
	4.5 %	2.8 %	9.5 %	9.5 %	10.6 %	63.1 %
random choice	89.0 %	0.4 %	0.1 %	0.1 %	0.0 %	10.4 %
1 deviation	7	26	50	39	30	27
	3.9 %	14.5 %	27.9 %	21.8 %	16.8 %	15.1 %
random choice	86.4 %	4.4 %	1.0 %	0.3 %	0.2 %	7.7 %
2 deviations	5	40	56	39	30	9
	2.8 %	22.3 %	31.3 %	21.8 %	16.8 %	5.0 %
random choice	72.5 %	18.9 %	4.2 %	1.5 %	0.7 %	2.2 %

Note: $N = 179$. Each subject is classified as $L1 - L4$ with no more than 0, 1 or 2 deviations from the predicted action profiles. Otherwise they are assigned to $L0$ or unidentifiable. The subjects classified as unidentifiable are able to choose dominant strategies as Player 4 but do not match any of the predicted patterns. They are at least rational, which makes them different from $L0$. The random choices are simulated with 10,000 randomly choosing subjects.

Table 2.4: Type assignment from SIMUL games

of my subjects. It should be noted that even with the 2-deviation cutoff and thus only 5% unidentifiable subjects, Fisher's exact test still does not reject that the distribution in this study is different from Kneeland's (2015), though with a lower p -value of 0.853.

Just as the one in Kneeland (2015), this type distribution is relatively higher than the literature (for example, Stahl and Wilson (1995) and Costa-Gomes et al. (2001)), which might be attributed to the special features of ring games. There are 27.9% and 21.8% of the subjects classified as $L2$ and $L3$ respectively with the 1-deviation cutoff, which serves as the starting point of the following analysis on the separation of Lkb and Lka .

Behavioral Pattern of Each Lk Type in the Sequential Games

In this section, the behavioral pattern of each Lk type, especially $L2$ and $L3$, is analyzed to determine the existence of Lkb and Lka subjects. I first show that most subjects' behavior falls into the predicted categories, which supports the consistency of subjects' belief levels and ability levels across three sets of games. I then provide evidence on the existence of both Lkb and Lka subjects by a closer look at the patterns of the $L2$ and $L3$ subjects. Finally, I show that the $L3b$ subjects hold the belief that their opponents are $L2b$, by comparing the $L3$ subjects' behavior in the two sets of sequential games.

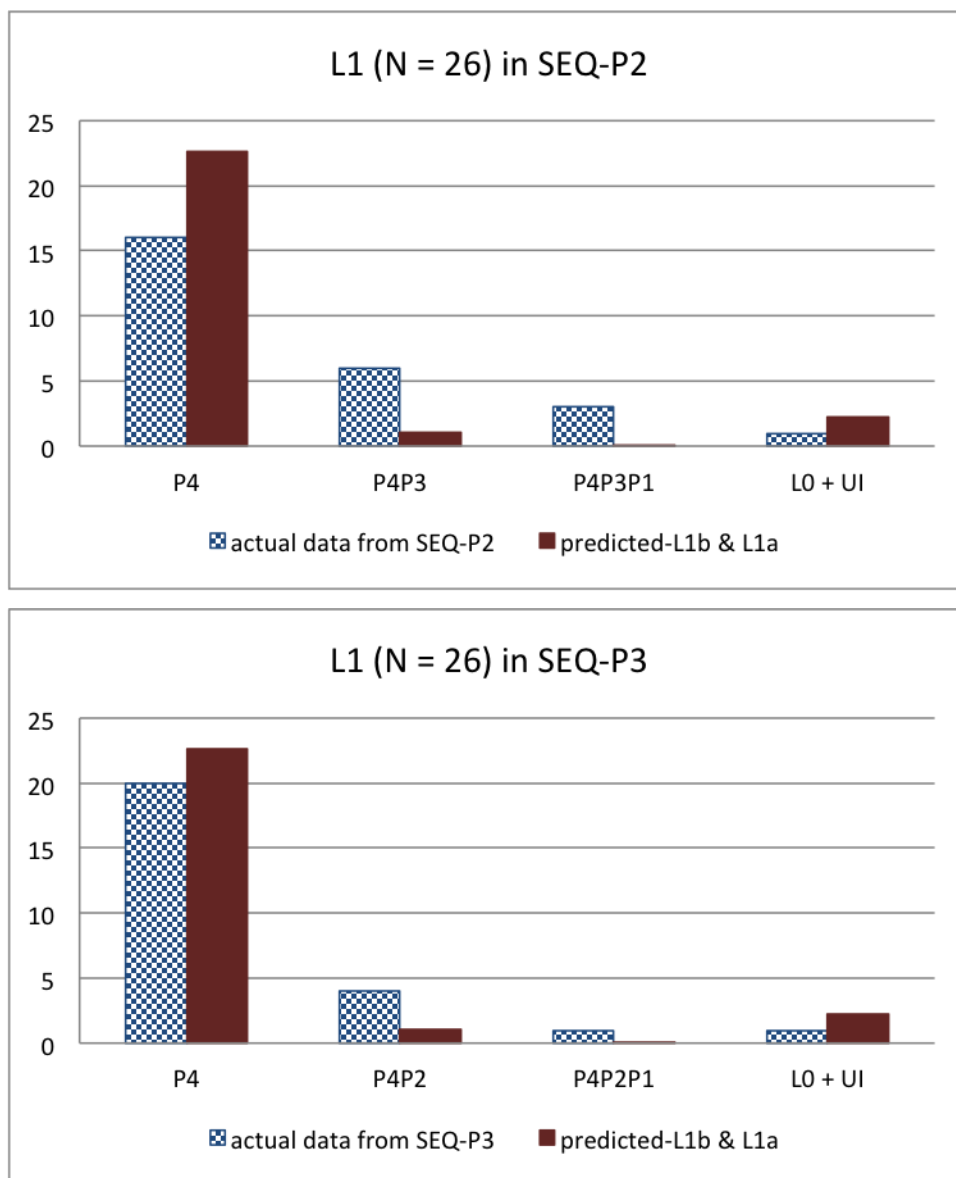
Result 1 Subjects' behavioral patterns in the sequential games are close to the theoretical predictions assuming that their belief levels and ability levels are consistent across simultaneous and sequential games. Most subjects fall into the predicted categories.

The dark bars of Figure 2.4, Figure 2.5, Figure 2.6 and Figure 2.7 describes the behavioral pattern of each type in the sequential games. According to Table 2.2, subjects' behavior could be sorted into three categories based on their choices as first movers. In SEQ-P2, the subjects could be playing the (iterative) dominant strategies only at P4 position, at P4 and P3 positions, or at all of the P4, P3 and P1 positions. In SEQ-P2, the three categories are playing the (iterative) dominant strategies at P4 position, at P4 and P2 positions, or at P4, P2 and P1 positions. The three categories correspond to the subjects who are still best responding to $L1$, $L2$ and $L3$ belief in the sequential games. Each subject could be assigned to one of the categories by the method in Table 2.3 and the 1-deviation cutoff⁴. If a subject deviates too much from these categories, e.g. best responds as player 1 but not as player 4, it means that either he is irrational or his behavior could not be explained by the theoretical model. Such a subject will be assigned to $L0$ or remain unidentified if he chooses the dominant strategies as player 4.

In theory, these subjects' behavior should follow Table 2.2 if their belief levels and ability levels remain the same in both simultaneous and sequential games. However, since subjects sometimes deviate from their own types, one could be misidentified if he has too many deviations. In order to determine how many deviations from the predicted categories could occur due to misidentification, for each of the $L1$ - $L4$ types, I simulate the choices of 10,000 pseudo-subjects in sequential games, assuming that all of them are either Lkb or Lka . The average deviation rate of each type used in the simulation is from the type classification results of the simultaneous games in Table 2.3. In ??, the lighter bars of each category give the predicted behavior pattern of Lkb or Lka obtained from simulation. According to the simulation, around 85% of the subjects should fall into the predicted categories if their belief and ability do not change.

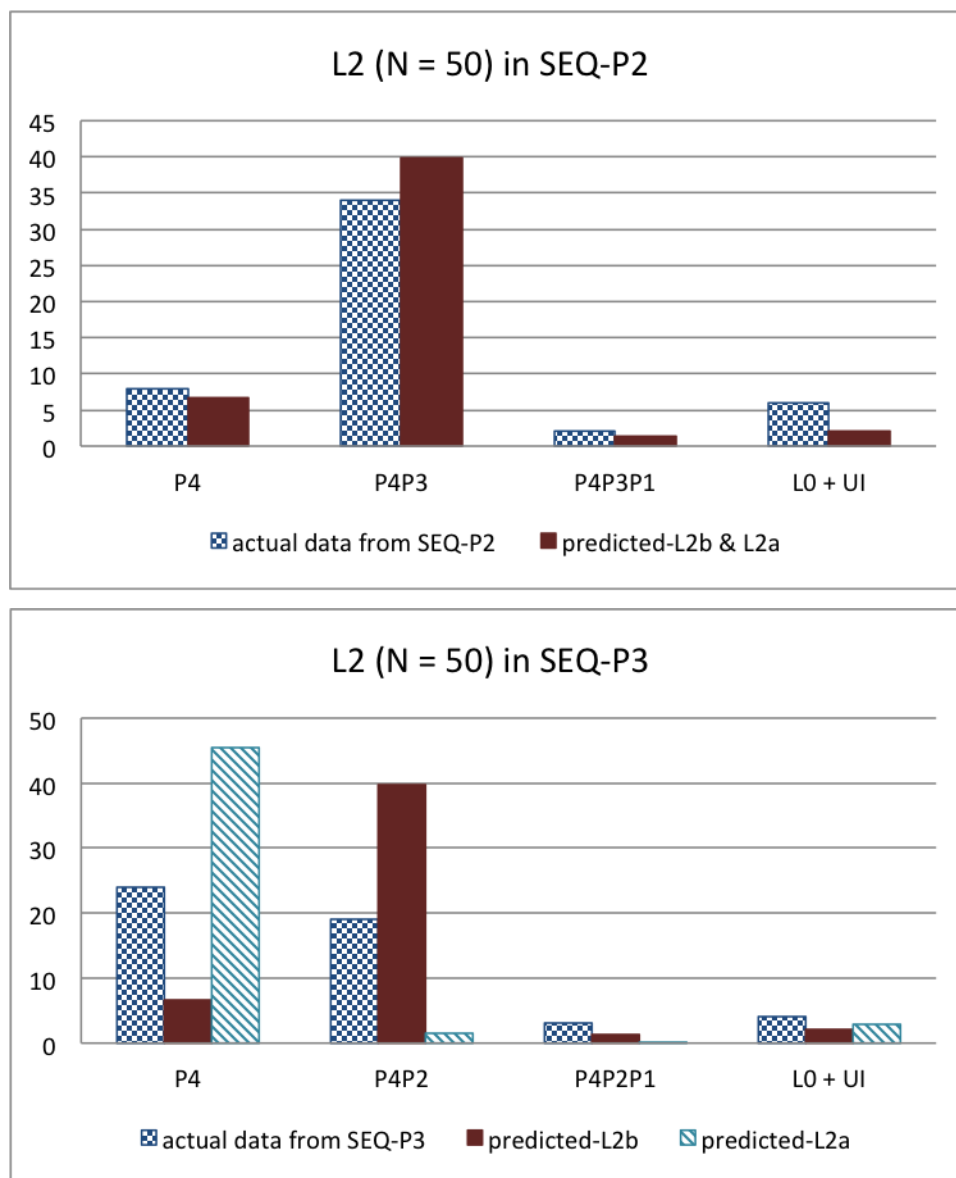
Let's first take a look at $L1$, $L4$ and $L2$ in SEQ-P2. The predictions of Lkb or Lka behavior are the same for these subjects. Their actual choices in SEQ-P2 and SEQ-P3, which are represented by the first bar (the dark one) in each category, share similar patterns with the simulated distributions. The actual distributions are less concentrated on the theoretically predicted categories, possibly as a result of higher deviation rates or less consistency across games. According to Table 2.5, exact tests of goodness-of-fit show that in three of these five cases ($L1$ in SEQ-P3, $L2$ in SEQ-P2 and $L4$ in SEQ-P3) the actual type distributions are

⁴According to ER, a subject best responding to $L1$ belief in SEQ-P2 plays dominant strategies as player 4, but chooses the same actions as players 1 and 3. This gives the predicted action profile for the "P4" category in SEQ-P2, which includes $L1b$ and $L1a$. Similarly, a subject responding to $L2$ belief in SEQ-P2 should play the (iterative) dominant strategies as players 3 and 4 and choose the same action as player 1, which gives the action profile for the "P4P3" category in SEQ-P2, including $L2a$, $L2b$ and $L3a$. A subject responding to $L3$ belief in SEQ-P2 should play the (iterative) dominant strategies as players 1, 3 and 4, which is the "P4P3P1", including $L3b$, $L4a$ and $L4b$. The actions of the three categories in SEQ-P3 could be predicted in the same way.



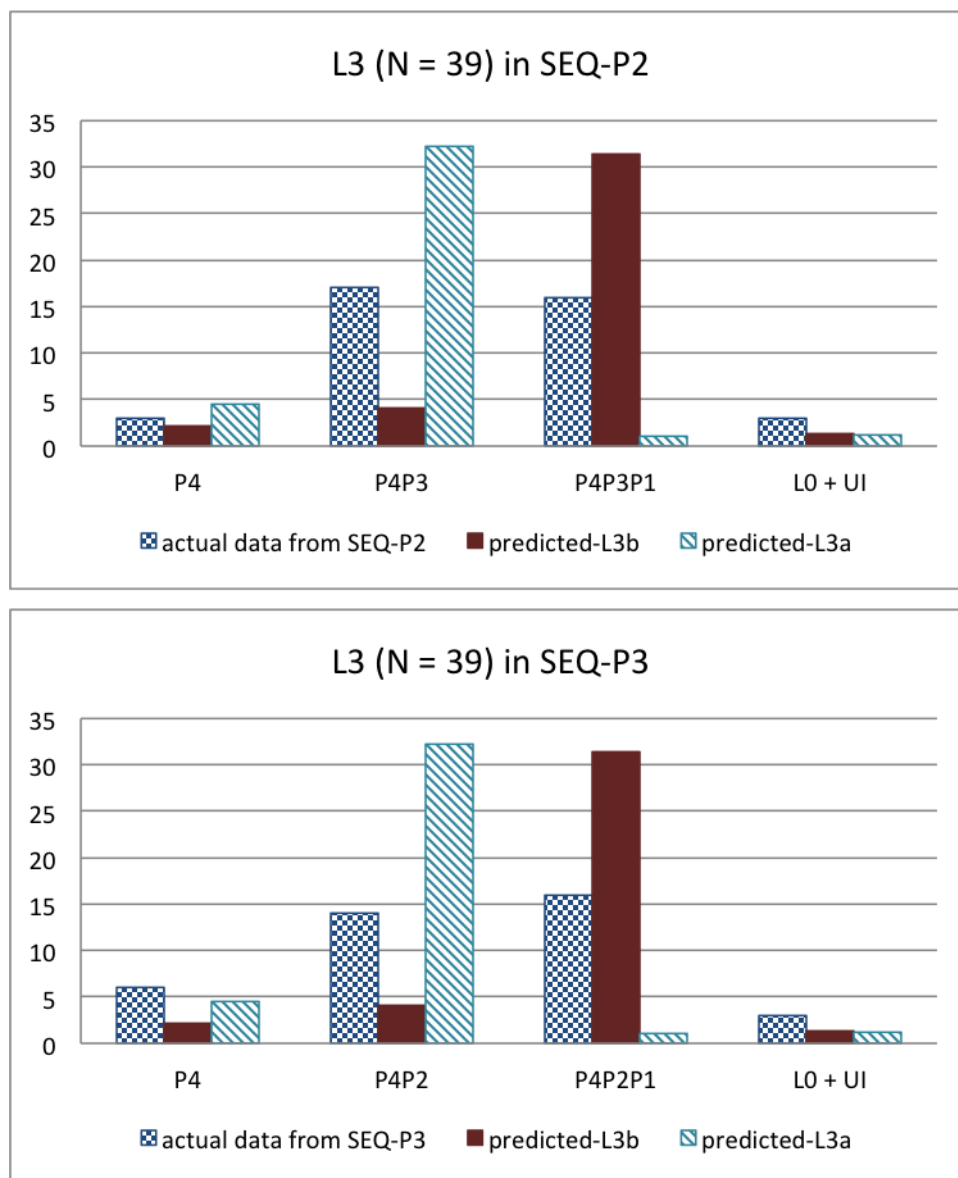
Note: In SEQ-P2, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 (“P4”), at P4 and P3 (“P4P3”), and at P4, P3 and P1 (“P4P3P1”). In SEQ-P3, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 (“P4”), at P4 and P2 (“P4P2”), and at P4, P2 and P1 (“P4P2P1”). Each subject is assigned to a category with ER and 1-deviation cutoff. The dark blue bars give the actual type distribution. The light blue and medium blue bars give the simulated type distribution assuming that these subjects are *Lkb* or *Lka*. In certain cases the behaviors of *Lkb* and *Lka* are not distinguishable, and they are represented by the same light blue bar.

Figure 2.4: Behavior pattern of the observed *L1* subjects



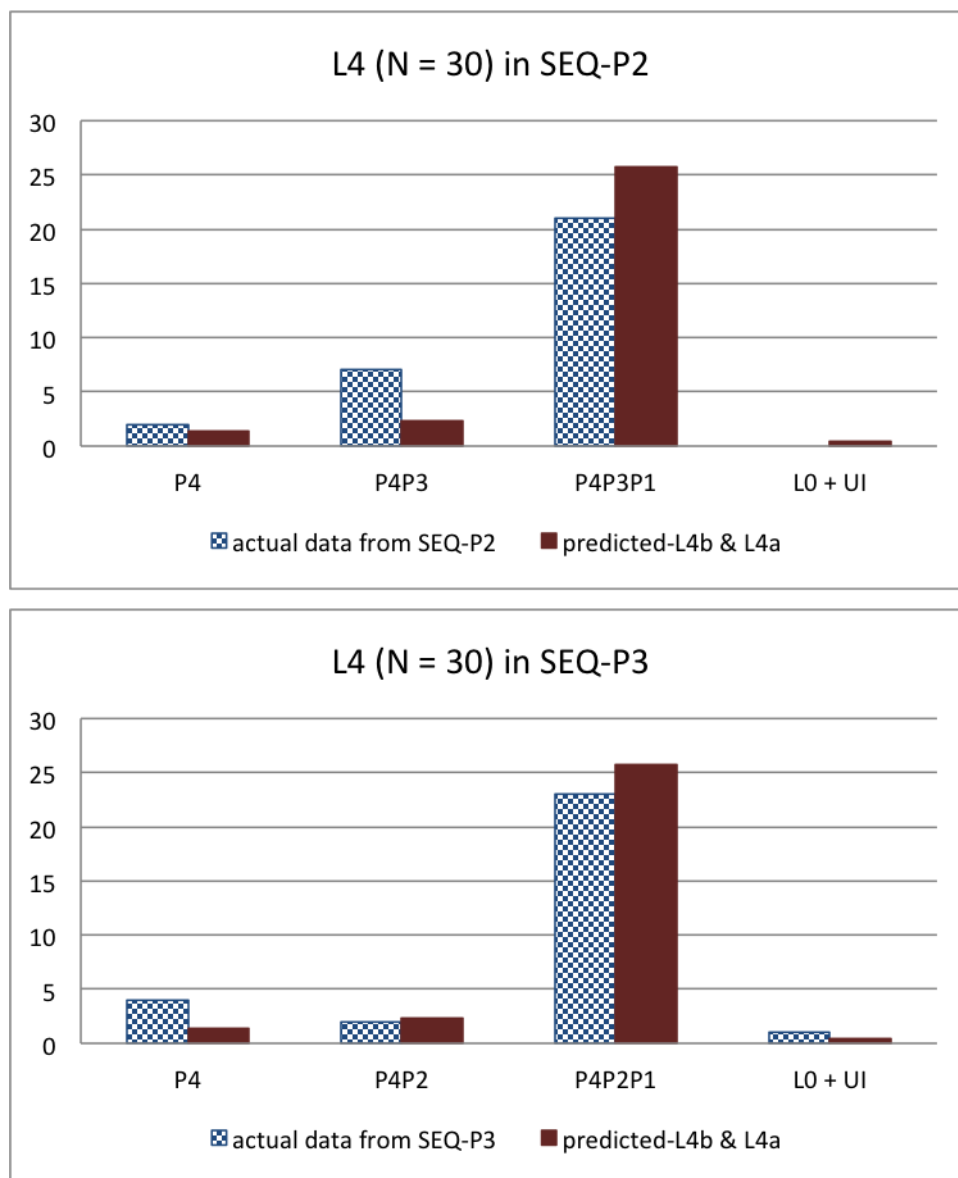
Note: In SEQ-P2, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 (“P4”), at P4 and P3 (“P4P3”), and at P4, P3 and P1 (“P4P3P1”). In SEQ-P3, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 (“P4”), at P4 and P2 (“P4P2”), and at P4, P2 and P1 (“P4P2P1”). Each subject is assigned to a category with ER and 1-deviation cutoff. The dark blue bars give the actual type distribution. The light blue and medium blue bars give the simulated type distribution assuming that these subjects are *Lkb* or *Lka*. In certain cases the behaviors of *Lkb* and *Lka* are not distinguishable, and they are represented by the same light blue bar.

Figure 2.5: Behavior pattern of the observed *L2* subjects



Note: In SEQ-P2, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 (“P4”), at P4 and P3 (“P4P3”), and at P4, P3 and P1 (“P4P3P1”). In SEQ-P3, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 (“P4”), at P4 and P2 (“P4P2”), and at P4, P2 and P1 (“P4P2P1”). Each subject is assigned to a category with ER and 1-deviation cutoff. The dark blue bars give the actual type distribution. The light blue and medium blue bars give the simulated type distribution assuming that these subjects are *Lkb* or *Lka*. In certain cases the behaviors of *Lkb* and *Lka* are not distinguishable, and they are represented by the same light blue bar.

Figure 2.6: Behavior pattern of the observed *L3* subjects



Note: In SEQ-P2, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 (“P4”), at P4 and P3 (“P4P3”), and at P4, P3 and P1 (“P4P3P1”). In SEQ-P3, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 (“P4”), at P4 and P2 (“P4P2”), and at P4, P2 and P1 (“P4P2P1”). Each subject is assigned to a category with ER and 1-deviation cutoff. The dark blue bars give the actual type distribution. The light blue and medium blue bars give the simulated type distribution assuming that these subjects are *Lkb* or *Lka*. In certain cases the behaviors of *Lkb* and *Lka* are not distinguishable, and they are represented by the same light blue bar.

Figure 2.7: Behavior pattern of the observed *L4* subjects

not different statistically from the predicted ones at the significance level of 0.05 (Table 2.5). In the case of $L4$ subjects in SEQ-P2, the difference is at the 0.05 level but not the 0.01 level.

Since there might exist both $L2b$ and $L2a$ in SEQ-P3, and both $L3b$ and $L3a$ in SEQ-P2 and SEQ-P3, the type distributions of the $L2$ and $L3$ subjects could be different from the simulated ones. However, the difference should only be reflected in the predicted directions. Specifically, for $L2$'s behavior in SEQ-P3, the difference from the simulated distribution should only be in the proportions of best responses at player 2 position. Therefore small p -values should only occur in the "P4" and "P4P2" categories and the proportions of the other categories should not differ too much from the simulated ones. Similarly, for $L3$'s behavior in SEQ-P2 and SEQ-P3, the difference between actual data and simulated data should only be in the proportions of subjects best responding at player 1 position. As shown in columns (2)-(5) Table 2.5, the follow-up test of each category confirm that very few subjects fall outside of the predicted categories.

Thus it could be concluded that the simulated type distributions are good predictions of the aggregate pattern, which provides strong evidence that most subjects are consistent in belief and ability levels.

Result 2 About half of the $L2$ - and $L3$ -behaving subjects in the simultaneous games are best responding to the $L2$ or $L3$ belief, the rest are Lka types who could perform at most two or three steps of reasoning.

According to Figure 2.5, although there are a few $L2$ subjects playing the iterative dominant strategies as player 2 in SEQ-P3, which requires $L2$ belief and three steps of reasoning, more than half fail at this position. Especially, most of them best respond only as player 4, suggesting that they are bounded by two steps. The exact tests show that this group's behavior is significantly different from the simulated distribution assuming that they are all $L2b$ or $L2a$, with p -values less than 0.0001. The follow-up tests confirm that the deviations from the simulated distribution are confined to "P4" and "P4P2", but not to the rest of the categories, suggesting that such a pattern could not be due to deviations and misidentification.

A similar pattern could be found on the $L3$ subjects (Figure 2.5). The proportions of $L3b$, who best respond at all positions, and $L3a$, who could not best respond at player 1 position, appear to be half-half, and few subjects fall into other categories. The exact tests support that both distributions are significantly different from the simulated ones, mainly due to the differences in the "P4P3"/"P4P2" and "P4P3P1"/"P4P2P1" categories.

Overall, the evidence suggests that only half of the $L2$ - and $L3$ -behaving subjects in the simultaneous games are best responding to their belief. The rest of them use two or three steps of reasoning because this is the most they could do. Since there is evidence of both Lkb and Lka types, a model with only players' belief levels or their ability levels could not explain the behavior pattern.

Result 3 No difference has been found on $L3$'s behavior in SEQ-P2 and SEQ-P3, sug-

SEQ-P2		all categories	each category vs. the rest			
			P4	P4P3	P4P3P1	L0 + UI
<i>L1</i>	vs. predicted- <i>L1b</i> & <i>L1a</i>	0.0012	0.0040	0.0121	0.0485	0.6249
<i>L2</i>	vs. predicted- <i>L2b</i> & <i>L2a</i>	0.1702	0.7124	0.0704	0.5396	0.0648
<i>L3</i>	vs. predicted- <i>L3b</i>	0.0000	0.7669	0.0000	0.0000	0.2674
	vs. predicted- <i>L3a</i>	0.0000	0.5337	0.0000	0.0000	0.2674
<i>L4</i>	vs. predicted- <i>L4b</i> & <i>L4a</i>	0.0320	0.7208	0.0329	0.0093	0.4522
SEQ-P3		all categories	each category vs. the rest			
			P4	P4P2	P4P2P1	L0 + UI
<i>L1</i>	vs. predicted- <i>L1b</i> & <i>L1a</i>	0.0824	0.1752	0.1111	0.4373	0.6249
<i>L2</i>	vs. predicted- <i>L2b</i>	0.0000	0.0000	0.0000	0.2676	0.3278
	vs. predicted- <i>L2a</i>	0.0000	0.0000	0.0000	0.0547	0.6334
<i>L3</i>	vs. predicted- <i>L3b</i>	0.0000	0.0791	0.0006	0.0000	0.2674
	vs. predicted- <i>L3a</i>	0.0000	0.5058	0.0000	0.0000	0.2674
<i>L4</i>	vs. predicted- <i>L4b</i> & <i>L4a</i>	0.1842	0.1181	1.0000	0.0913	0.4453

Note: the numbers in bold fonts represents the test results at the 0.05 significance level. The first column shows the original tests including all four categories. Columns (2)-(5) show the follow-up tests of each category vs. the sum of all the other categories. Since there are four follow-up tests at the same time, the significance level is corrected by $0.05/4 = 0.0125$.

Table 2.5: p -values of the exact tests of goodness-of-fit

gesting that the *L3b* subjects hold belief in their opponents' belief not ability.

Now that there are traces of both *Lkb* and *Lka* types, it brings the question whether subjects' belief is in the opponents' belief or ability. That is, whether an *Lkb* subject believes that the opponents are bounded by $(k - 1)$ th-order belief or bounded by $k - 1$ steps of reasoning.

It could be tested on the *L3* subjects. In SEQ-P2, in order to play the iterative dominant strategy as player 1, *L3b* needs to believe that the opponents think two steps, while in SEQ-P3 *L3b* needs to believe that the opponents use three steps of thinking. If all *L3b* subjects hold the belief that their opponents best respond to *L2* belief, then SEQ-P3 and SEQ-P2 should make no difference for them, which serves as the null hypothesis in this test. If this is the case, there should be the same proportion of *L3* subjects identified as *L3b* in SEQ-P2

and SEQ-P3. Otherwise, if a subject believes that the opponents are $L2a$, he will not choose the iterative dominant strategies as player 1 in SEQ-P3. In this case, less $L3$ subjects would be identified as $L3b$ in SEQ-P3 than in SEQ-P2.

In the actual data, no difference could be found statistically in the type distributions from the two sets of sequential games (Figure 2.5). A closer look at the behavior patterns of the $L3$ subjects (Table 2.6) has confirmed that most of these subjects show the same $L3b$ or $L3a$ behavior in both SEQ-P3 and SEQ-P2. The pattern is consistent with the simulated distribution assuming that the subjects remain as the same $L3b$ or $L3a$ type in both sets of the sequential games. Therefore no evidence is found to reject the hypothesis that $L3b$'s belief is in the opponents' belief, not their reasoning steps.

		behavior in SEQ-P2			
		P4	P4P3	P4P3P1	UI
in SEQ-P3	P4	3	2	1	0
	P4P2	0	9	4	1
	P4P2P1	0	5	10	1
	UI	0	1	1	1

Table 2.6: Behavioral patterns of the 39 $L3$ subjects in the two sequential games

It should be noted that with a sample size of 39 $L3$ subjects, to get a power higher than 0.8, it requires at least 25% of them responding to the belief that the opponents are bounded by two steps of thinking⁵. So there may very well exist a small number of such subjects, who could not be detected in this experiment due to lack of power.

Separation of Lkb and Lka Subjects

In this section each subject is assigned a type using the 20 first-mover choices from SIMUL and the two sequential games (excluding the 4 choices as second movers) with a likelihood function similar to (2.1). $L2b$ and $L3b$ could be separated from $L2a$ and $L3a$ using these three sets of games.

Six types ($L1$, $L2a$, $L2b$, $L3a$, $L3b$, $L4$) are included in the assignment process. There are four types who are always best responding to the Lk belief in all games. For the $L2b$ and $L3b$ types, by combining their behavior in the simultaneous and sequential games, they could be identified as having $b = 2$ and $b = 3$ respectively and strictly higher ability than the observed levels. For the $L1$ and $L4$ types, it could only be inferred $\min\{b, a\} \geq 1$ or $\min\{b, a\} \geq 4$ respectively, but it is not clear which factor is binding.

⁵This result is calculated by G*Power. See Faul et al. (2007).

In addition, there are the $L2a$ and $L3a$ types, whose observed levels are determined by their ability. They behave like $L2b$ or $L3b$ in the simultaneous games, but could be separated from $L2b$ and $L3b$ by their behavior in the sequential games. $L2a$ subjects have $a = 2$ and $b \geq 2$, and choose the (iterative) dominant strategies only when it requires no more than two steps of reasoning. They differ from $L2b$ by not choosing the iterative dominant strategies at the player 2 positions of SEQ-P3. $L3a$ players have $a = 3$ and $b \geq 3$, and choose the (iterative) dominant strategies only when it requires no more than 3 steps of reasoning. They differ from $L3b$ by not best responding at the player 1 positions of SEQ-P2 and SEQ-P3.

Based on the evidence from the last subsection, I do not include a type who believes that the opponents are $L2a$, i.e. chooses the iterative dominant strategies at player 1 positions of SEQ-P2 but not SEQ-P3.

Subjects are assigned to a type with no more than 3 deviations from the type's predicted action profile. The results with 0-deviation or 6-deviation cutoffs are reported in the appendix. When a subject deviates too much from a predicted profile, he is assigned to the unidentifiable category if he chooses 5 out of the 6 dominating strategies at player 4 positions correctly, otherwise he is classified as $L0$. A randomly choosing subject has only 0.2% of a chance to correctly choose 5 dominant strategies.

		Baseline + Sequential Types								
		$L0$	$L1$	$L2a$	$L2b$	$L3a$	$L3b$	$\geq L4$	UI	sum
Baseline Types	$L0$	6	0	0	0	0	0	0	1	7
	$L1$	0	22	1	1	0	1	0	1	26
	$L2$	0	3	21	21	0	1	0	4	50
	$L3$	1	0	0	0	20	15	0	3	39
	$L4$	0	0	0	1	1	0	26	2	30
	UI	1	1	1	1	0	1	0	22	27
	sum	8	26	23	24	21	18	26	33	179

Note: $N = 179$. Subjects are assigned to a type with no more than 3 deviations. Otherwise they are assigned to $L0$ or unidentifiable. The subjects classified as unidentifiable are able to choose 5 out of 6 dominant strategies as player 4 but do not match any of the predicted patterns.

Table 2.7: Type assignment according to the observations from SIMUL, SEQ-P2 and SEQ-P3

Table 2.7 shows how the identified types change from using the 8 positions in the simultaneous games to using the 20 first-mover positions in the simultaneous plus sequential games. Most of the $L1$ and $L4$ subjects stay as the same level when more observations are included, and all of the $L0$ subjects in the simultaneous games are still identified as $L0$. Only

approximately 15% of the subjects fall into a different category, but overall the majority of the subjects appear to be quite consistent across the three sets of games.

21 out of the 50 subjects who exhibit *L2* behavior in the simultaneous games are identified as the *L2a* type, who could do at most two steps of reasoning, compared with the other 21 *L2b*, who have higher ability and are always responding to *L2* belief. Of the 39 *L3* subjects in the simultaneous games, more than half are identified as *L3a*, and 15 are identified as the higher ability *L3b* type. The result suggests that around half of the *L2* and *L3* behavior observed in the simultaneous games is due to lack of ability to think further, which validates the findings in Subsections 2.4 and 2.4.

Robustness Check: Learning Effect

An essential assumption in the above analysis is that each subject's belief level and ability level remain constant throughout the experiment, with a few deviations due to random preference shift or trembling hand. However, if certain learning effects prevailed, the subjects became more proficient in locating iterative dominant strategies as the experiment proceeded. Then the observed pattern could be driven by the order of games in the experiment. For example, the subjects who played the more difficult positions, such as player 1 or player 2 positions, in the later periods would have a larger chance of figuring them out. If this is the case, the higher types would not be the group with higher ability, but the group with more opportunity to learn.

A detailed analysis of learning effects will be reported in the appendix. In summary, I find mild learning effects on a couple of player positions, but the type classification results do not appear to be affected by learning. This subsection compares the type distributions of the whole sample and the groups who might have an advantage in learning, showing that learning has almost no effect in shaping the type distribution.

Given the special structure of the ring games, it might be advantageous to play the player 4 positions first, for it could help the subjects to figure out early in the experiment that the games could be solved by iterative dominance. In addition, playing as second movers might also help, for it motivates the subject to look into the dependency relationships between him and the opponents. If this is true, then the subjects who played more player 4 positions or second movers in the earlier stage would be more likely to behave like higher types. To address this concern, I check whether the performances of these subjects differ from the whole sample.

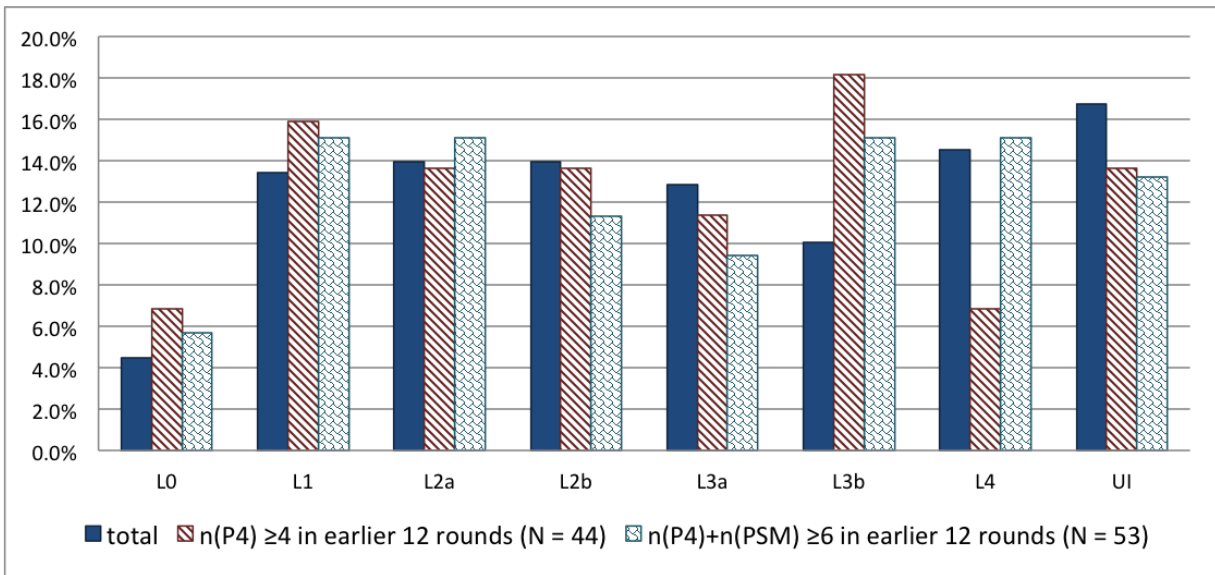
In each session, subjects played the 24 games in different random orders. Table 2.8 gives the number of subjects who played more player 4 positions and second mover positions in the earlier 12 games. The numbers of player 4 positions are denoted by $n(P4)$, and the number of second mover positions by $n(PSM)$. There are 44 subjects who played at more than four player 4 positions in the earlier 12 games, and 53 subjects who played at more than six player 4 or second mover positions. These subjects are regarded as having advantages in learning. The following results still hold if other cutoffs are used to determine advantages.

$n(P4)$	≥ 1	≥ 2	≥ 3	≥ 4	≥ 5	≥ 6
N	179	155	124	44	31	0
$n(P4) + n(PSM)$	≥ 3	≥ 4	≥ 5	≥ 6	≥ 7	≥ 8
N	179	154	108	53	36	7

Note: $n(P4)$ denotes the number of player 4 positions played by the subject in the earlier 12 games. $n(PSM)$ denotes the number of second mover positions played by the subject in the earlier 12 games. The highlighted cells represent the cutoffs used in the following analysis.

Table 2.8: Number of subjects who played more advantageous positions in earlier 12 games

As shown in Figure 2.8, no evidence has been found that they performed better than the rest of the subjects. The distributions of these groups are not shifted toward the higher types. The goodness-of-fit tests show that the type distribution of the subjects with $n(P4) + n(PSM) \geq 6$ is not statistically different from the distribution of the whole sample, and that for the group with $n(P4) \geq 4$ the differences only occur in the *L3b* and *L4* categories. Therefore it is safe to say that the identified patterns of the main results are not affected by subjects' learning of iterative dominance.



Note: $n(P4)$ denotes the number of player 4 positions played by the subject in the earlier 12 games. $n(PSM)$ denotes the number of second mover positions played by the subject in the earlier 12 games.

Figure 2.8: Type distributions of the subjects who have an advantage in learning

2.5 Conclusion

This paper reports an experiment to separate the high ability subjects (Lkb) who behave as Lk due to their beliefs and the low ability subjects (Lka) who could think at most k steps. The separation happens at certain first mover positions of the sequential ring games, where it requires three or four steps to respond to $L2$ or $L3$ belief. The $L2b$ or $L3b$ subjects are still able to best respond to the same $L2$ or $L3$ belief as they did in the simultaneous games. But the $L2a$ or $L3a$ ones could do at most two or three steps of reasoning, and thus could not handle the one more step.

I find that most subjects behave consistently, in the sense of belief levels and ability levels, across the three sets of ring games. Their behavioral patterns in the sequential games fit theoretical predictions. Out of the 50 and 39 subjects classified as $L2$ and $L3$ from their choices in the simultaneous games, around half have reached their upper boundaries of reasoning. In addition, evidence on $L3$ subjects supports that their beliefs are on their opponents' belief levels but not reasoning steps.

The findings suggest large heterogeneity in subjects' abilities to best respond to even low order belief. The observed low levels in the previous studies could be explained by both the presence of low ability types and the low-order beliefs of high ability types. Although the high types have incorrect beliefs, their low-order beliefs are not entirely unfounded, given the large proportion of the cognitively bounded subjects.

The existing literature has demonstrated the descriptive power of the level- k model. To make it also an explanatory and predictive model, it requires a better understanding of why people behave as certain levels, or where people get their belief from. For example, the existence of ability-bounded subjects in this study shows that a lot of people might not start with a clear idea of the opponents' levels. Rather, their belief could be formed through the anchoring and adjusting process suggested by Brandenburger and Li (2015), and this process would stop when they have reached their cognitive boundaries.

Chapter 3

Cognitive Ability and Depth of Reasoning in Games

3.1 Introduction

This paper explores the connection between people's depth of reasoning in the games from Chapter 2 and their cognitive ability. Recent literature on the level- k model extends our understanding on people's non-equilibrium behavior in games. It is found that a lot of subjects consistently use specific reasoning rules, the level- k thinking, and have heterogeneous reasoning abilities. For each of these subjects, with his choice data in a series of games, strategies have been developed to identify whether this subject is a high level player who use several steps of reasoning, or a low level one who does not think strategically (see the first two chapters for detailed reviews). Since such a model implies a relationship between reasoning levels in games and cognitive ability, a critical next step in this line of research is to check whether a subject's performance in game theory experiments does correlate with measures of his cognitive ability.

A positive correlation between identified reasoning levels and cognitive ability could help validate the level- k model, and rule out some alternative explanations that does not involve differences in cognitive ability (e.g. the subjects use certain heuristics). It also provides the external validity of game theory experiments, since cognitive ability has been found to correlate with real world attainments, such as education levels, incomes and career decisions (Heckman et al, 2006; Burks et al., 2011). In addition, while current models in behavioral game theory are more on the descriptive side, understanding the influence of cognitive ability on the performance in games could lead to a model with more predictive power.

In this experimental work, I use the Cognitive Reflection Test (CRT) as a measure of subjects' cognitive ability. First proposed by Frederick (2005), it is designed to test people's cognitive ability in decision making. The CRT is composed of three short questions with intuitive but erroneous answers. One needs to suppress the impulsion of giving instant answers, and reflect on these questions for a brief moment to get them right. Below is a

sample question from Frederick (2005):

If it takes 5 machines 5 minutes to make 5 widgets, how long would it take 100 machines to make 100 widgets? ___ minutes.

For the participants, the first response that comes to their mind is very likely to be 100. It takes a bit more thinking for them to realize that the correct answer is 5 instead of 100.

It is found that CRT scores correlate with other measures of cognitive ability, such as the Wonderlic IQ test and SAT scores (Frederick, 2005). Correlations have also been found with patience level, risk preference (Frederick, 2005) and the performance on heuristics-and-biases tasks (Toplak et al., 2011). Thus, the CRT is a preferred measure in the context of game theory experiment, not only because it is easy to administrate, but also because it is an indicator of both cognitive ability and the tendency to seek rational thinking and avoid heuristics.

Existing studies present the correlation between cognitive ability and the observed reasoning levels in games (see next section for a summary of the results). But it remains unclear through which channel does cognitive ability affect observed levels. Higher cognitive ability suggests larger working memory, and thus stronger deduction and computation skills, which is vital for the search of optimal strategies. On the other hand, people with higher cognitive ability might be better at forming accurate beliefs on others' actions.

These games analyzed in Chapter 2 are of particular interest for the study of subjects' cognitive ability and their performance in games, because they allow me to separate the subjects who are bounded by two or three reasoning steps (the *Lka* types, level determined by ability) and the ones who could actually think more steps but stop at low levels because of their belief (the *Lkb* types, level determined by belief). According to the model in Chapter 2, the *Lkb* subjects have better reasoning skills in games than the *Lka* subjects for each $k = 2, 3$. If cognitive ability is related to reasoning skills, we should expect these *Lkb* subjects to score higher in the CRT than the *Lka* subjects. In addition, the *L2b* (*L3b*) subjects have at least the same reasoning abilities as the *L3a* (*L4*) subjects but lower level beliefs. If cognitive ability also affects belief formation, it might be reflected in the differences in their CRT scores.

I find that the CRT scores increase with levels among the lower types (from *L0* to *L2b*), and then flatten out (from *L2b* to *L4*). Especially, I find that the belief-bounded higher-ability *L2b* subjects performed much better in the CRT than the ability-bounded lower-ability *L2a* ones, which supports the hypothesis that cognitive ability affects reasoning ability in games. But among the players who use at least three steps of reasoning in the ring games (*L2b*, *L3a*, *L3b* and *L4*), the differences in CRT scores are not significant. Thus, the second hypothesis that cognitive ability affects belief formation could not be tested.

The remainder of the paper proceeds as follows. The next section summarizes the related literature. The experimental design and procedure are presented in Section 3. Section 4 reports the experimental results. Section 5 concludes.

3.2 Related Literature

The CRT is proposed by Frederick (2005) as a measure of cognitive ability, especially in the context of decision making. Although it is a short test composed of three questions, researches have found over 0.4 correlation between CRT scores and other cognitive ability measure, such as the SAT, the Wonderlic Personnel Test, and the Vocabulary and Matrix Reasoning subtests (Frederick, 2005; Obrecht et al., 2009; Toplak et al., 2011).

The CRT is reported to relate to some important decision making characteristics. The subjects who score high in the CRT are more patient, and are more willing to take risk (Frederick, 2005). In addition, because the CRT tasks separate impulsive and reflective thinkers, it suggests that the ones who score higher in the test tend to do more rational thinking and are less likely to succumb to heuristics and bias. For example, Oechssler et al. (2009) find that higher CRT scores are correlated with lower conjunction fallacy and lower biases in updating probabilities. Toplak et al. (2011) discover that the CRT is a better predictor of the performance in a series of heuristics-and-biases tasks than other cognitive ability measures.

There is an emerging literature on the relationship between people's behavior in strategic interaction and indicators of their cognitive ability, including the CRT. Rydval et al. (2009) find that higher working memory, need for cognition, and premeditation are associated with a higher likelihood of obeying dominance. Works on the Beauty Contest Games also show that the subjects who score high in the CRT or other tests of cognitive ability play the strategies closer to equilibrium (Burnham et al., 2009; Schnusenberg and Gallo, 2011; Branas-Garza et al., 2012). Georganas et al. (2015) find no monotone relationship between CRT scores and k -levels, but they find that CRT scores are able to predict earnings in the games. Gill and Prowse (2015) use the Raven test as an indicator of cognitive ability. They find that in the repeated Beauty Contest Games, high ability subjects choose the numbers closer to equilibrium, learn faster and respond positively to the cognitive ability of their opponents.

3.3 Experiment Design and Procedures

The CRT from Frederick (2005) is composed of three questions as follows:

- (a) A bat and a ball cost 1.10 in total. The bat costs a dollar more than the ball. How much does the ball cost? ___ cents.
- (b) If it takes 5 machines 5 min to make 5 widgets, how long would it take 100 machines to make 100 widgets? ___ min.
- (c) In a lake, there is a patch of lily pads. Every day, the patch doubles in size. If it takes 48 days for the patch to cover the entire lake, how long would it take for the patch to cover half of the lake? ___ days.

The quick, intuitive response to the three questions are 10 cents, 100 minutes and 24 days respectively, while the correct answers are 5 cents, 5 minutes and 47 days. Subjects could score 0, 1, 2 or 3 out of 3.

The games used in the analysis here are the three sets of simultaneous and sequential ring games presented in Chapter 2.

The experiment tries to answer two questions. First, does cognitive ability relate to the reasoning ability in these games? It is done by comparing the performance of *L2a (L3a)* and *L2b (L3b)* subjects in the CRT. *L2a (L3a)* has at least the same belief level as *L2b (L3b)*, but is inferior in reasoning ability. If higher cognitive ability is associated with higher reasoning ability in games, we should expect that *L2b (L3b)* score higher in the CRT than *L2a (L3a)*.

Second, does cognitive ability relate to belief formation? It requires the comparison between *L2b (L3b)* and *L3a (L4)* subjects in the CRT. *L2b (L3b)* has at least the reasoning ability as *L3a (L4)*, but has lower order belief. If higher cognitive ability is associated with more accurate belief formation, we should expect them to differ in CRT scores.

The experiment was conducted at the Experimental Social Science Laboratory (XLab) at UC Berkeley. A total of 184 subjects, who were recruited from Berkeley undergraduate classes, participated in 6 different sessions, with between 20 to 36 subjects in each session.

The subjects first went through instructions and the 24 games described in Chapter 2. They then were informed that they would take the Cognitive Reflection Test. The instructions shown on the screen were: *Below are three problems that vary in difficulty. Try to answer as many as you can. At the end of this session, one question will be randomly selected for payment at the end of the experiment. The payment is \$0.25 if your answer is correct*¹.

Each experimental session lasted about one and a half hours. The average earnings were \$25, plus a \$5 participation fee, which were paid in private after each session.

3.4 Results

The participants in this experiment obtain an average CRT score of 1.64. The percentages of subjects who score 0, 1, 2 and 3 are 20.7%, 22.3%, 28.3% and 28.8% respectively (Figure 3.1). To serve as a comparison, Fredrick (2005) conducts the CRT with 3,428 subjects, primarily college students, and finds a mean score of 1.24. The mean scores from top universities like Princeton and Harvard are around 1.4-1.6. Only MIT students get a high mean of over 2. Fredrick (2005) also reports a large gender difference. In his experiment, men score 1.47 on average and women score 1.03, and the difference is significant at 0.0001 level. I also find a similar pattern, that the average score of male subjects is 1.76 and the average of female subjects is 1.60. But the difference is not significant. I will disregard gender difference in the following analysis.

¹In Frederick's (2005) experiments, the subjects were paid a constant participation fee to answer various questions including the CRT.

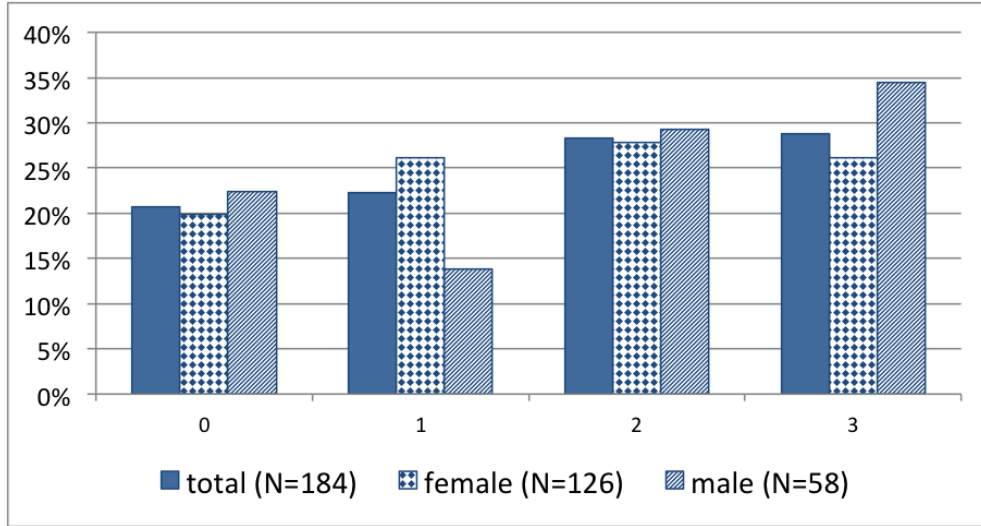
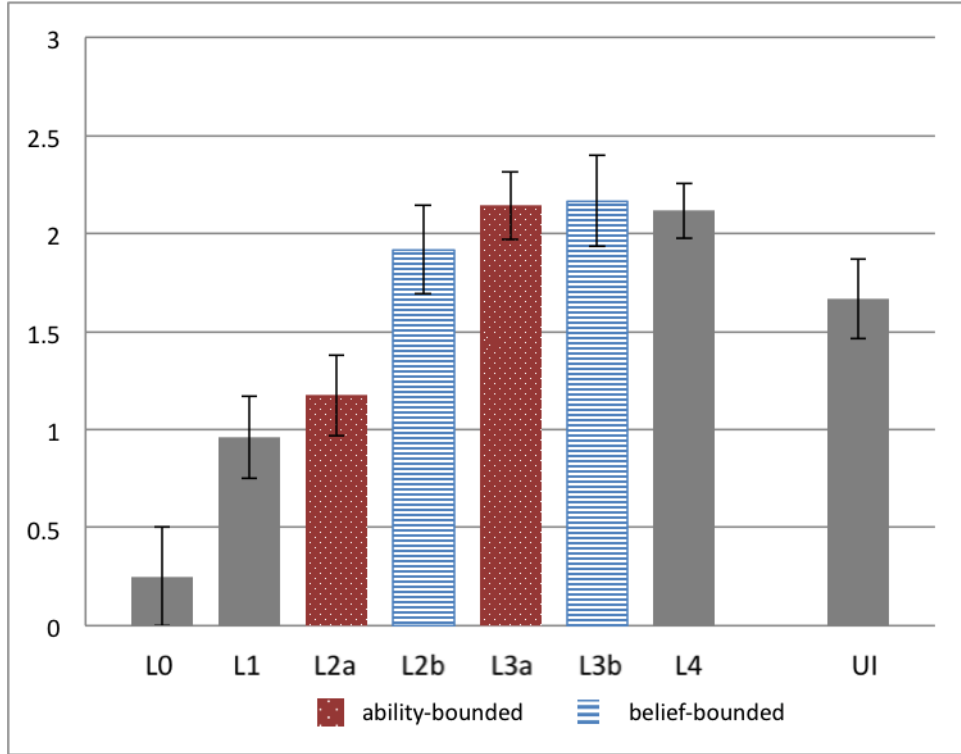


Figure 3.1: Distribution of CRT Scores

In Figure 3.2 the average CRT scores are displayed by types. The data is from 179 subjects who do not have missing choices that would affect type classification. As discussed in Chapter 2, $L1$ and $L4$ always use one step or four steps of reasoning, defined as in the traditional level- k model. $L2a$ and $L3a$ are identified to be bounded by, at most, two or three steps of reasoning. $L2b$ and $L3b$ could actually think more than two or three steps, but act like $L2a$ and $L3a$ in the simultaneous games because of their low order belief. $L0$ acts irrationally. The last category, the unidentifiable subjects (UI), are the ones who obey dominance, but do not follow the predicted actions of any Lk type.

Among the lower types (from $L0$ to $L2b$), subjects' CRT scores increase with their identified levels, which mirrors the findings in the previous studies. Two-sided t-tests show that the differences are significant between these groups, except for $L1$ and $L2a$. The most striking result is that the $L2a$ subjects, who are identified to be of lower reasoning abilities, did much worse in CRT than the $L2b$ subjects, although both types exhibited the same behavior in the simultaneous games. Because $L2a$ subjects have at least the same belief level as $L2b$, but are identified to have lower reasoning ability, this result points to a clear relationship between cognitive ability and reasoning ability in games.

The differences between high types ($L2b$ - $L4$) are not that clear. T-tests show that the means are not different for these four groups. All of these subjects are able to do at least three steps of iterative thinking in the ring games, and their mean scores are pretty high. Therefore, one possible explanation of why higher types are not separable by CRT scores could be that the CRT results do not distinguish between the subjects whose reasoning abilities have reached a certain high level. Because of this, the fact that $L3a$ and $L3b$ subjects do not differ in CRT scores does not necessarily mean that cognitive ability does not matter here. Similarly, the fact that $L2b$ ($L3b$) and $L3a$ ($L4$) are indistinguishable in



Note: Standard errors are reported in the error bars.

Figure 3.2: Average CRT Scores and Levels

the CRT does not mean that cognitive ability has no effect on belief formation.

An additional interesting observation is the difference between $L0$ and unidentifiable subjects. The mean CRT score of $L0$ subjects is close to 0. Actually, 7 out of 8 subjects in this group scored 0 in the test. Contrastingly, the unidentifiable subjects perform much better and their average score is close to the mean of the whole sample. It supports the separation of these unidentifiable subjects from the irrational $L0$. The unidentifiable subjects could be of high cognitive ability, but do not follow the prediction of the level- k model. Or it could be that they are inconsistent and act as different types across different games.

I further run multilogit regressions to determine whether subjects' levels could be explained by their performances in the CRT². Table 3.1 reports 7 regressions of subjects' identified types on CRT scores, with different types as base outcomes. For example, the first row shows the coefficients of CRT scores on the probability of being each type versus being $L0$. The regressions show that the lower types ($L0$, $L1$ and $L2a$) are distinguishable from the higher types ($L2b$, $L3a$, $L3b$ and $L4$) using their CRT scores. However, the differences within these two groups are not significant.

²Similar analysis was done by Georganas et al. (2015). They do not find a monotone relationship between levels and CRT scores as seen in this study. But their results point to a link between CRT scores and earnings, which could also be observed in my data.

Independent Variable: CRT scores							
base outcome	<i>L0</i>	<i>L1</i>	<i>L2a</i>	<i>L2b</i>	<i>L3a</i>	<i>L3b</i>	<i>L4</i>
vs <i>L0</i>		1.440 (1.028)	1.570 (0.966)	2.471** (1.042)	2.717*** (1.024)	2.814*** (1.043)	2.867*** (1.019)
vs <i>L1</i>	-1.440 (1.028)		0.130 (0.349)	1.031*** (0.385)	1.277*** (0.339)	1.373*** (0.362)	1.426*** (0.333)
vs <i>L2a</i>	-1.570 (0.966)	-0.130 (0.349)		0.901** (0.353)	1.147*** (0.340)	1.243*** (0.365)	1.297*** (0.310)
vs <i>L2b</i>	-2.471** (1.042)	-1.031*** (0.385)	-0.901** (0.353)		0.246 (0.325)	0.342 (0.360)	0.396 (0.319)
vs <i>L3a</i>	-2.717*** (1.024)	-1.277*** (0.339)	-1.147*** (0.340)	-0.246 (0.325)		0.0962 (0.324)	0.150 (0.286)
vs <i>L3b</i>	-2.814*** (1.043)	-1.373*** (0.362)	-1.243*** (0.365)	-0.342 (0.360)	-0.0962 (0.324)		0.0534 (0.328)
vs <i>L4</i>	-2.867*** (1.019)	-1.426*** (0.333)	-1.297*** (0.310)	-0.396 (0.319)	-0.150 (0.286)	-0.0534 (0.328)	

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Standard errors are reported in the parentheses. Each row shows a regression with a different type being the base outcome. Session fixed effects are included in the multilogit regressions. Unidentifiable subjects are excluded.

Table 3.1: Multilogit regression of types on CRT scores

3.5 Conclusion

This experimental study finds positive relationship between the CRT, an indicator of cognitive ability, and reasoning levels in the games. The CRT scores increase with levels among the lower types (from *L0* to *L2b*), but do not differ significantly from *L2b* to *L4*. The striking difference between the CRT scores of *L2a* and *L2b* subjects suggests that cognitive ability plays an important role in reasoning skills. However, since the CRT is a rough measure that only takes the scale of 0 to 3, the higher ability subjects are not distinguishable by this indicator. Therefore, it calls for future study to explore better measures of cognitive ability and thinking dispositions that could be a good predictor of people's performance in games.

Bibliography

- [1] Agranov, Marina, Elizabeth Potamites, Andrew Schotter and Chloe Terigiman. 2012. "Beliefs and Endogenous Cognitive Levels: An Experimental Study." *Games and Economic Behavior* 75: 449-63.
- [2] Alaoui, Larbi and Antonio Panta. 2015. "Endogenous Depth of Reasoning." *The Review of Economic Studies* advance access published December 4, 2015, doi:10.1093/restud/rdv052
- [3] Arad, Ayala and Ariel Rubinstein. 2012. "The 11-20 Money Request Game: A Level- k Reasoning Study". *American Economic Review* 102(7): 3561-73.
- [4] Aumann, Robert and Adam Brandenburger. 1995. "Epistemic Conditions for Nash Equilibrium." *Econometrica* 63(5):1161-80.
- [5] Bhatt, Meghana and Colin F. Camerer. 2005. "Self-referential Thinking and Equilibrium as States of Mind in Games: fMRI Evidence." *Games and Economic Behavior* 52: 424-59.
- [6] Branas-Garza, Pablo, Teresa Garcia-Munoz and Roberto Hernan Gonzalez. 2012. "Cognitive Effort in the Beauty Contest Game." *Journal of Economic Behavior and Organization* 83: 254-260.
- [7] Brandenburger, Adam and Xiaomin Li. 2015. "Thinking about Thinking and Its Cognitive Limits." Working paper.
- [8] Breitmoser, Yves, Jonathan H.W. Tan and Daniel John Zizzo. 2014. "On the Beliefs off the Path: Equilibrium Refinement due to Quantal Response and Level- k ." *Games and Economic Behavior* 86: 102-25.
- [9] Brocas, Isabelle, Juan D. Carriloo, Stephanie W. Wang and Colin F. Camerer. 2014. "Imperfect Choice or Imperfect Attention? Understanding Strategic Thinking in Private Information Games." *Review of Economic Studies* 81: 944-70.
- [10] Brown, Alexander L. , Colin F. Camerer, and Dan Lovallo. 2012. "To Review or Not to Review? Limited Strategic Thinking at the Movie Box Office." *American Economic Journal: Microeconomics* 4: 1-26.

- [11] Burchardi, Konrad B. and Stefan P. Penczynski. 2014. "Out of Your Mind: Eliciting Individual reasoning in One Shot Games." *Games and Economic Behavior* 84: 39-57.
- [12] Burks, Stephen V., Jeffrey P. Carpenter, Lorenz Goette and Aldo Rustichini. 2009. "Cognitive skills affect economic preferences, strategic behavior, and job attachment." *Proceedings of the National Academy of Sciences* 106: 7745-7750.
- [13] Burnham, Terence C., David Cesarini, Magnus Johannesson, Paul Lichtenstein and Bjorn Wallace. 2009. "Higher Cognitive Ability is Associated with Lower Entries in a p-Beauty Contest." *Journal of Economics Behavior and Organization* 72: 171-175.
- [14] Camerer, Folin F.. 2003. *Behavioral Game Theory: Experiments in Strategic Interaction*. Princeton University Press.
- [15] Camerer, Colin F., Teck-Hua Ho, and Juin Kuan Chong. 2004. "A Cognitive Hierarchy Model of Games." *Quarterly Journal of Economics*, 119(3): 861-898.
- [16] Costa-Gomes, Miguel A., Vincent P. Crawford and Bruno Broseta. 2001. "Cognition and Behavior in Normal-Form Games: An Experimental Study." *Econometrica* 69(5): 1193-1235.
- [17] Costa-Gomes, Miguel A., and Vincent P. Crawford. 2006. "Cognition and Behavior in Two-Person Guessing Games: An Experimental Study." *American Economic Review* 96(5): 1737-68.
- [18] Costa-Gomes, Miguel A., and Georg Weizsacker. 2008. "Stated Beliefs and Play in Normal-Form Games" *Review of Economic Studies* 75: 729-62.
- [19] Crawford, Vincent P., Miguel A. Costa-Gomes, and Nagore Iriberri. 2013. "Structural Models of Nonequilibrium Strategic Thinking: Theory, Evidence, and Applications." *Journal of Economic Literature* 51(1):5-62.
- [20] Crawford, Vincent P.. 2014. "A Comment on 'How Portable is Level-0 Behavior? A Test of Level-k Theory in Games with Non-neutral Frames' by Heap, Rojo-Arjona, and Sugden." Unpublished Manuscript.
- [21] Crawford, Vincent P. and Nagore Iriberri. 2007. "Fatal Attraction: Saliency, Naivete, and Sophistication in Experimental Hide-and-Seek Games." *American Economic Review*, 97(5): 1731-50.
- [22] Faul, Franz, Edgar Erdfelder, Albert-Georg Lang, and Axel Buchner. 2007. "G*Power 3: A flexible statistical power analysis program for the social, behavioral, and biomedical sciences." *Behavior Research Methods* 39: 175-191.
- [23] Frederick, Shane. 2005. "Cognitive Reflection and Decision Making." *Journal of Economic Perspectives* 19: 25-42.

- [24] Gchter, Simon and Elke Renner. 2010. "The Effects of (Incentivized) Belief Elicitation in Public Goods Experiments." *Experimental Economics* 13: 364-377
- [25] Georganas, Sotiris, Paul J. Healy and Roberto A. Weber. 2015. "On the Persistence of Strategic Sophistication." *Journal of Economic Theory* 159: 369-400..
- [26] Gill, David and Victoria Prowse. 2015. "Cognitive Ability, Character Skills, and Learning to Play Equilibrium: A Level-k Analysis." *Journal of Political Economy*: forthcoming.
- [27] Goldfarb, Avi and Mo Xiao. 2011. "Who Thinks about the Competition? Managerial Ability and Strategic Entry in US Local Telephone Markets." *American Economic Review* 101: 3130-3161.
- [28] Heap, S. H., D. R. Arjona, and R. Sudden. 2014. "How portable is Level-0 Behavior? A Test of Level-k Theory in Games with Non-Neutral Frames." *Econometrica*, 82(3), 1133-51.
- [29] Heckman, James J., Jora Stixrud and Sergio Urzua. 2006. "The effects of cognitive and non-cognitive abilities on labor market outcomes and social behavior." *Journal of Labor Economics* 24: 411-482.
- [30] Ho, Teck-Hua, Colin F. Camerer, and Keith Weigelt. 1998. "Iterated Dominance and Iterated Best Response in Experimental 'p-Beauty Contests'." *American Economic Review*, 88(4): 947-69.
- [31] Ho, Teck-Hua, and Xuanming Su. 2013. "A Dynamic Level-k Model in Sequential Games." *Management Science* 59(2):452-69.
- [32] Kneeland, Terri. 2015. "Identifying Higher-order Rationality". *Econometrica* 83: 2065-2079.
- [33] Manski, Charles F. and Claudia Neri. 2013. "First- and Second-Order Subjective Expectations in Strategic Decision-Making: Experimental Evidence." *Games and Economic Behavior*. 81: 232-54.
- [34] Nagel, Rosemarie. 1995. "Unraveling in Guessing Games: An Experiment Study." *American Economic Review*, 85(5): 1313-1326.
- [35] Nash, John F. 1950. "Non-cooperative Games." Doctoral dissertation.
- [36] Nyarko, Yaw and Andrew Schotter. 2002. "An Experimental Study of Belief Learning Using Elicited Beliefs." *Econometrica* 70: 971-1005.
- [37] Obrecht, Natalie A., Gretchen B. Chapman and Rochel Gelman. 2009. "An encounter frequency account of how experience affects likelihood estimation." *Memory & Cognition* 37: 632-643.

- [38] Oechssler, Jorg, Andreas Roeder and Patrick W. Schmitz. 2009. "Cognitive abilities and behavioral biases." *Journal of Economic Behavior and Organization* 72: 147-152.
- [39] Palacios-Huerta, Ignacio, and Oscar Volij. 2009. "Field Centipedes." *American Economic Review*, 99(4): 1619-35.
- [40] Rutstrom, E. Elisabet and Nathaniel T. Wilcox. 2009. "Stated Beliefs versus Inferred Beliefs: A Methodological Inquiry and Experimental Test." *Games and Economic Behavior* 67: 616-32.
- [41] Rydval, Ondrej, Andreas Ortmann and Michal Ostadnický. 2009. "Three very simple games and what it takes to solve them." *Journal of Economic Behavior and Organization* 72: 589-601.
- [42] Schnusenberg, Oliver and Andres Gallo. 2011. "On cognitive ability and learning in a beauty contest." *Journal for Economic Educators* 11: 13-24.
- [43] Slonim, Robert L. 2005. "Competing Against Experienced and Inexperienced Players." *Experimental Economics* 8:55-75.
- [44] Stahl, Dale O., and Ernan Haruvy. 2008. "Level-n Bounded Rationality in Two-player Two-stage Games." *Journal of Economic Behavior and Organization* 65: 41-61.
- [45] Stahl, Dale O., and Paul W. Wilson. 1994. "Experimental Evidence on Players' Models of Other Players." *Journal of Economic Behavior and Organization* 25(3): 309-27.
- [46] Stahl, Dale O., and Paul W. Wilson. 1995. "On Players' Models of Other Players: Theory and Experimental Evidence." *Games and Economic Behavior* 10(1): 218-54.
- [47] Strzalecki, Tomasz. 2014. "Depth of Reasoning and Higher Order Beliefs". *Journal of Economic Behavior and Organization* 108: 108-22.
- [48] Toplak, Maggie. E., Richard F. West, and Keith E. Stanovich. 2011. "The Cognitive Reflection Test as a predictor of performance on heuristics-and-biases tasks." *Memory & Cognition* 39: 1275-1289.

Appendix A

Additional Analysis on Ring Games

This appendix chapter provides additional robustness analysis on the ring games studied in Chapter 2.

A.1 Type Assignment with the Assumption of Uniform Randomizing $L0$

The assumption used to predict Lk behavior in the ring games is that an Lk player does not respond to the changes in $(k + 1)$ - or higher-order payoffs (**ER** in Kneeland (2015)). An alternative assumption, which is widely used in Lk experiments, is that the irrational $L0$ type uniformly randomize on all the possible actions (**UP**: uniform prior on $L0$).

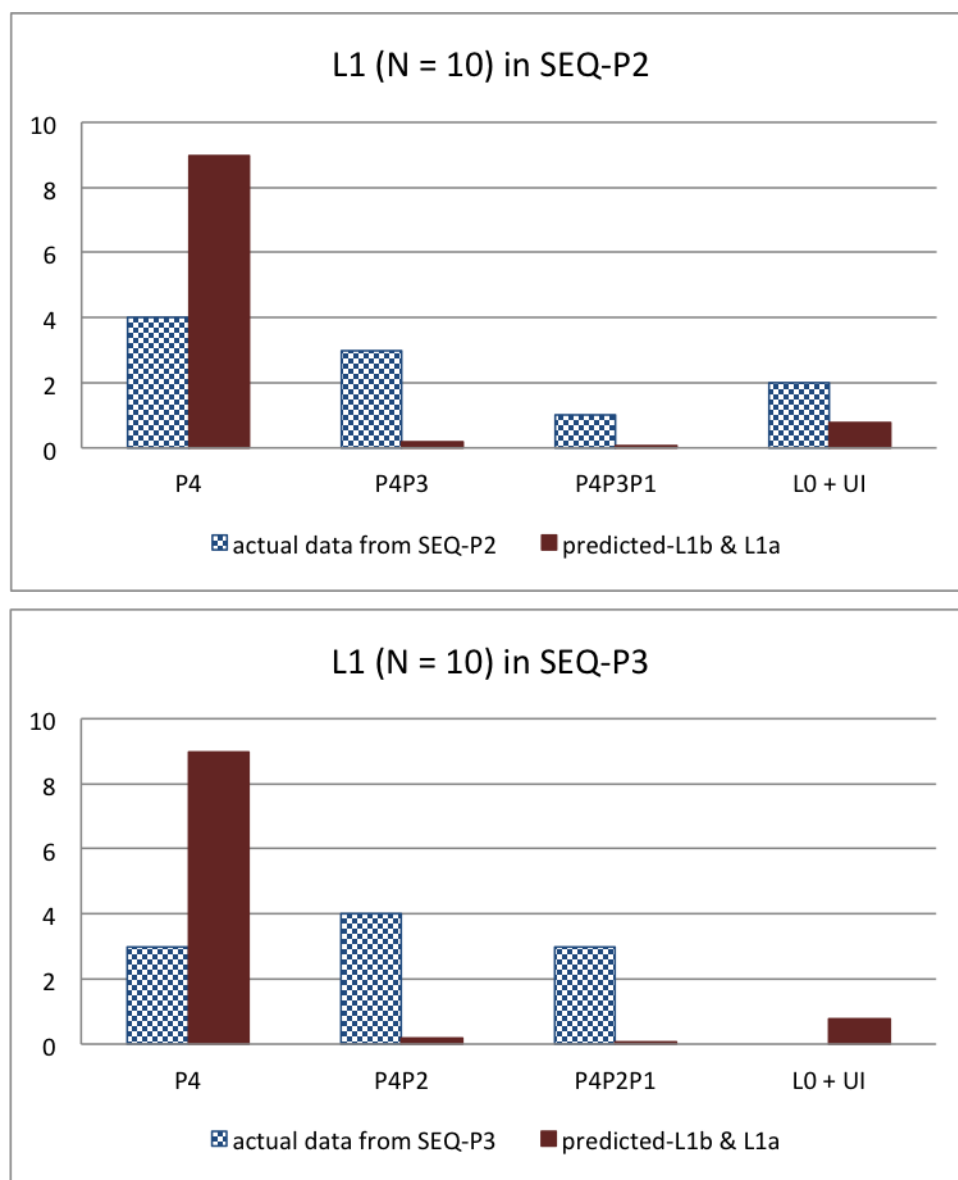
The distribution of assigned types using the predicted Lk behavior of UP is given in Table A.1. The 2-deviation cutoff is used here, so a random choosing subject only has a probability of less than 5% of being assigned to one of the Lk types, which is comparable to the main results. The distributions shift to the right in all three sets of games. In the simultaneous games, the number of $L4$ subjects almost doubles, and the number of $L1$ drops by more than a half. The shift is even larger in SEQ-P3. The number of subjects assigned to “P4” category under UP is less than a third of the number under ER. And the number in “P4P2P1” category increases by more than 100%. A closer look at the type changes confirms that everyone’s level rises or at least stays the same. Nobody goes to a lower level under UP. Nevertheless, the numbers of $L0$ and the unidentifiable are quite close. There are 6 subjects classified as $L0$ and 14 classified as the unidentifiables under both assumptions.

I try to perform the same analysis of each type’s behavioral pattern in the sequential games (Figure A.1, Figure A.2, Figure A.3 and Figure A.4). However, the overestimation of lower types in the two sequential games makes the treatment effects less clear. A large number of $L1$ - and $L2$ -behaving subjects in SIMUL are classified as “P4P2P1” in the sequential games, which is inconsistent with the theoretical prediction. It is impossible to tell how much subjects are Lka with such large inconsistency across the three sets of games. The

		<i>L0</i>	<i>L1</i>	<i>L2</i>	<i>L3</i>	<i>L4</i>	<i>UI</i>
SIMUL	ER	7	26	50	39	30	27
		3.9%	14.5%	27.9%	21.8%	16.8%	15.1%
	UP	6	10	43	39	53	28
		3.4%	5.6%	24.0%	21.8%	29.6%	15.6%
		<i>L0</i>	P4	P4P3	P4P3P1	-	<i>UI</i>
SEQ-P2	ER	7	43	69	46	-	14
		3.9%	24.0%	38.5%	25.7%	-	7.8%
	UP	7	32	60	78	-	2
		3.9%	17.9%	33.5%	43.6%	-	1.1%
		<i>L0</i>	P4	P4P2	P4P2P1	-	<i>UI</i>
SEQ-P3	ER	5	68	46	45	-	15
		2.8%	38.0%	25.7%	25.1%	-	8.4%
	UP	5	22	46	94	-	12
		2.8%	12.3%	25.7%	52.5%	-	6.7%

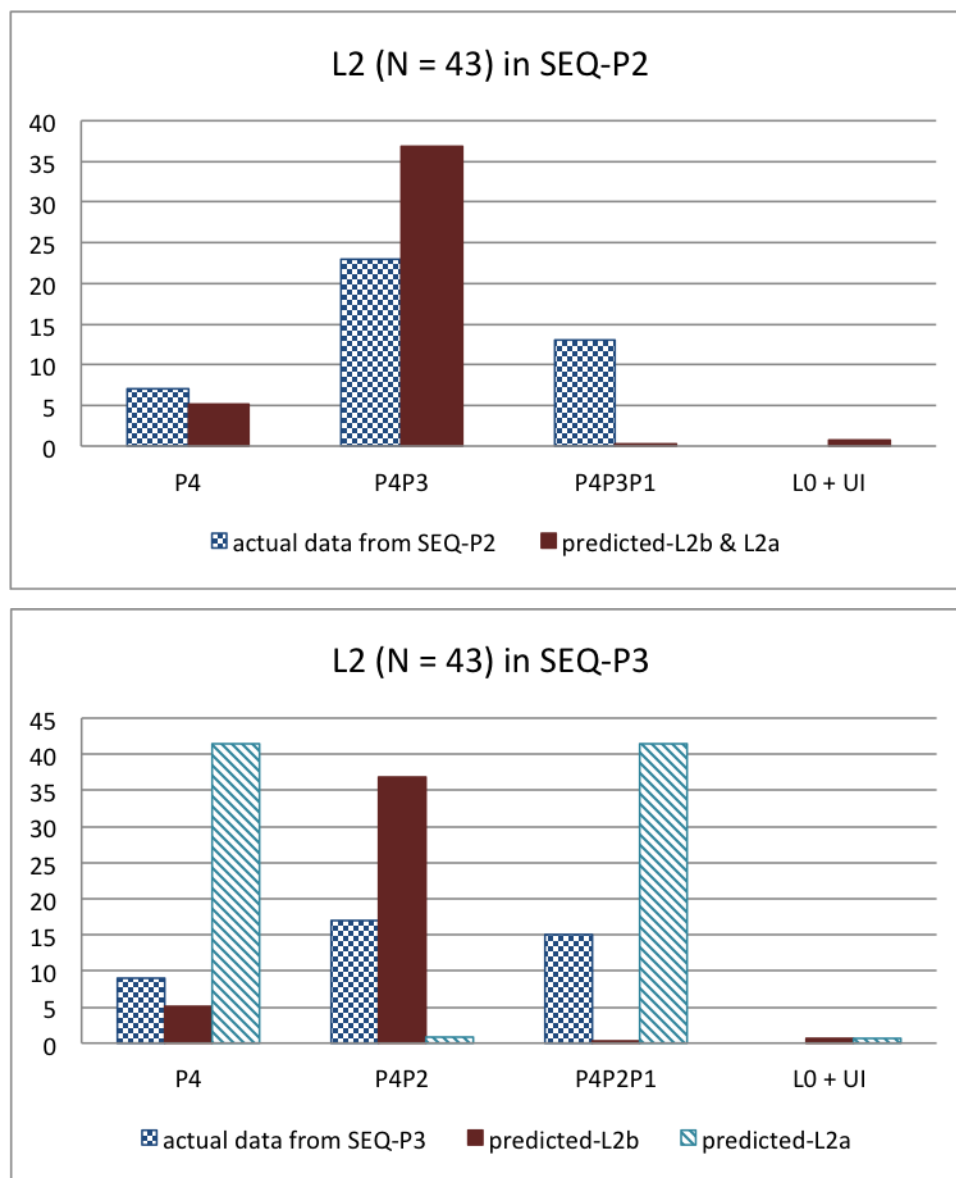
Note: $N = 179$. Subjects are assigned to a type with no more than 1 deviation when using ER, with no more than 2 deviations when using UP. Otherwise they are assigned to *L0* or unidentifiable. The subjects classified as unidentifiable are able dominant strategies as Player 4 but do not match any of the predicted patterns.

Table A.1: Type assignment using the two assumptions ER and UP



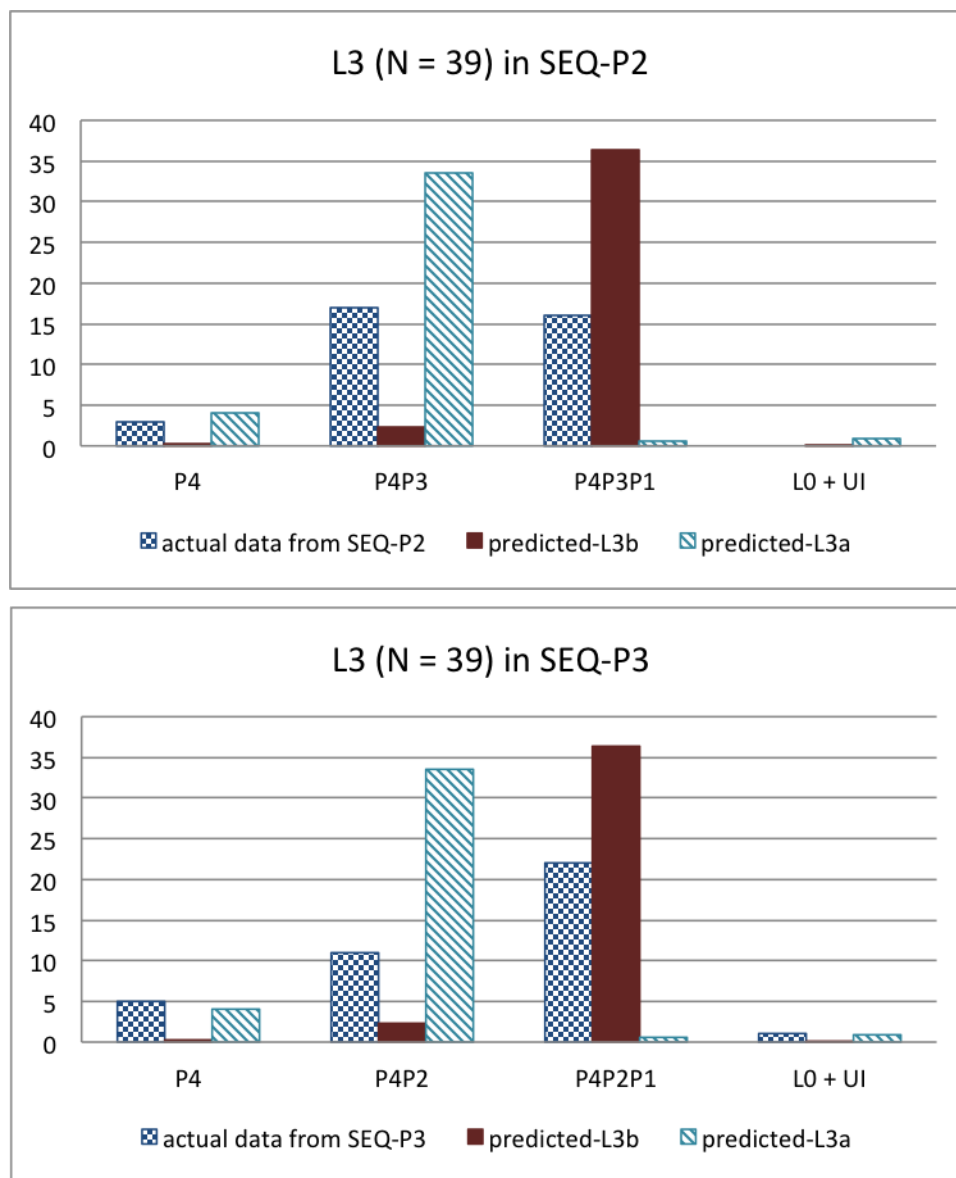
Note: In SEQ-P2, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 (“P4”), at P4 and P3 (“P4P3”), and at P4, P3 and P1 (“P4P3P1”). In SEQ-P3, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 (“P4”), at P4 and P2 (“P4P2”), and at P4, P2 and P1 (“P4P2P1”). Each subject is assigned to a category with UP and 2-deviation cutoff. The dark blue bars give the actual type distribution. The light blue and medium blue bars give the simulated type distribution assuming that these subjects are *Lkb* or *Lka*.

Figure A.1: Behavioral patterns of the observed *L1* subjects with the uniform prior assumption (UP)



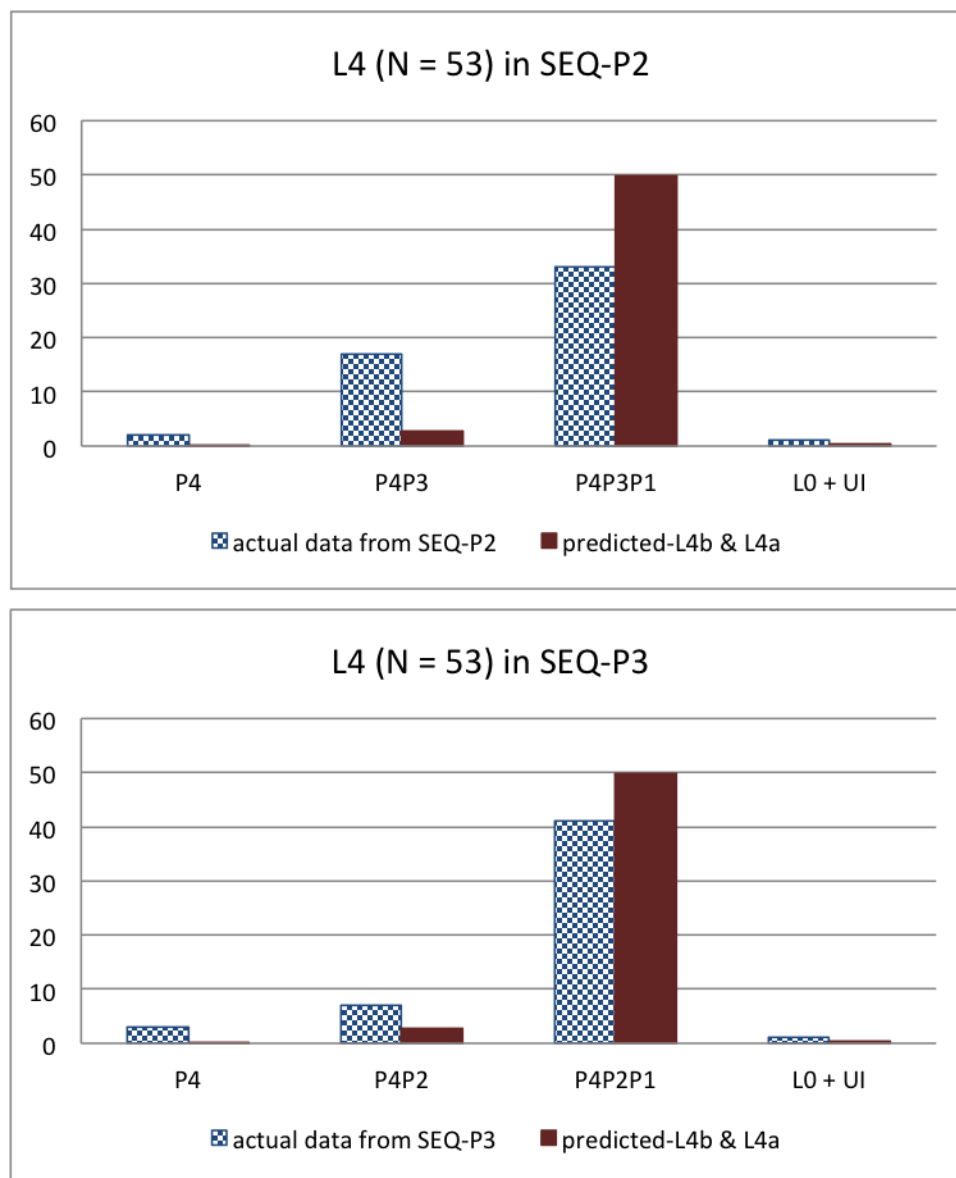
Note: In SEQ-P2, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 (“P4”), at P4 and P3 (“P4P3”), and at P4, P3 and P1 (“P4P3P1”). In SEQ-P3, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 (“P4”), at P4 and P2 (“P4P2”), and at P4, P2 and P1 (“P4P2P1”). Each subject is assigned to a category with UP and 2-deviation cutoff. The dark blue bars give the actual type distribution. The light blue and medium blue bars give the simulated type distribution assuming that these subjects are *Lkb* or *Lka*.

Figure A.2: Behavioral patterns of the observed *L2* subjects with the uniform prior assumption (UP)



Note: In SEQ-P2, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 (“P4”), at P4 and P3 (“P4P3”), and at P4, P3 and P1 (“P4P3P1”). In SEQ-P3, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 (“P4”), at P4 and P2 (“P4P2”), and at P4, P2 and P1 (“P4P2P1”). Each subject is assigned to a category with UP and 2-deviation cutoff. The dark blue bars give the actual type distribution. The light blue and medium blue bars give the simulated type distribution assuming that these subjects are *Lkb* or *Lka*.

Figure A.3: Behavioral patterns of the observed *L3* subjects with the uniform prior assumption (UP)



Note: In SEQ-P2, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 (“P4”), at P4 and P3 (“P4P3”), and at P4, P3 and P1 (“P4P3P1”). In SEQ-P3, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 (“P4”), at P4 and P2 (“P4P2”), and at P4, P2 and P1 (“P4P2P1”). Each subject is assigned to a category with UP and 2-deviation cutoff. The dark blue bars give the actual type distribution. The light blue and medium blue bars give the simulated type distribution assuming that these subjects are *Lkb* or *Lka*.

Figure A.4: Behavioral patterns of the observed *L4* subjects with the uniform prior assumption (UP)

overestimation is less severe among higher types, and a similar pattern could be observed on $L3$ and $L4$ subjects as in the main analysis.

Type classification with UP tends to overestimate the lower types. This is because the predicted action profile of each type on the off-equilibrium path using UP is a special case of that using ER. Since UP puts stronger restrictions on the off-equilibrium path, it is more difficult to match a subject to a low type. The overestimation would be more severe if fewer subjects follow the prediction of UP, as observed in SEQ-P3. Hence I use ER in the main analysis, which I believe provides more robust and reliable results.

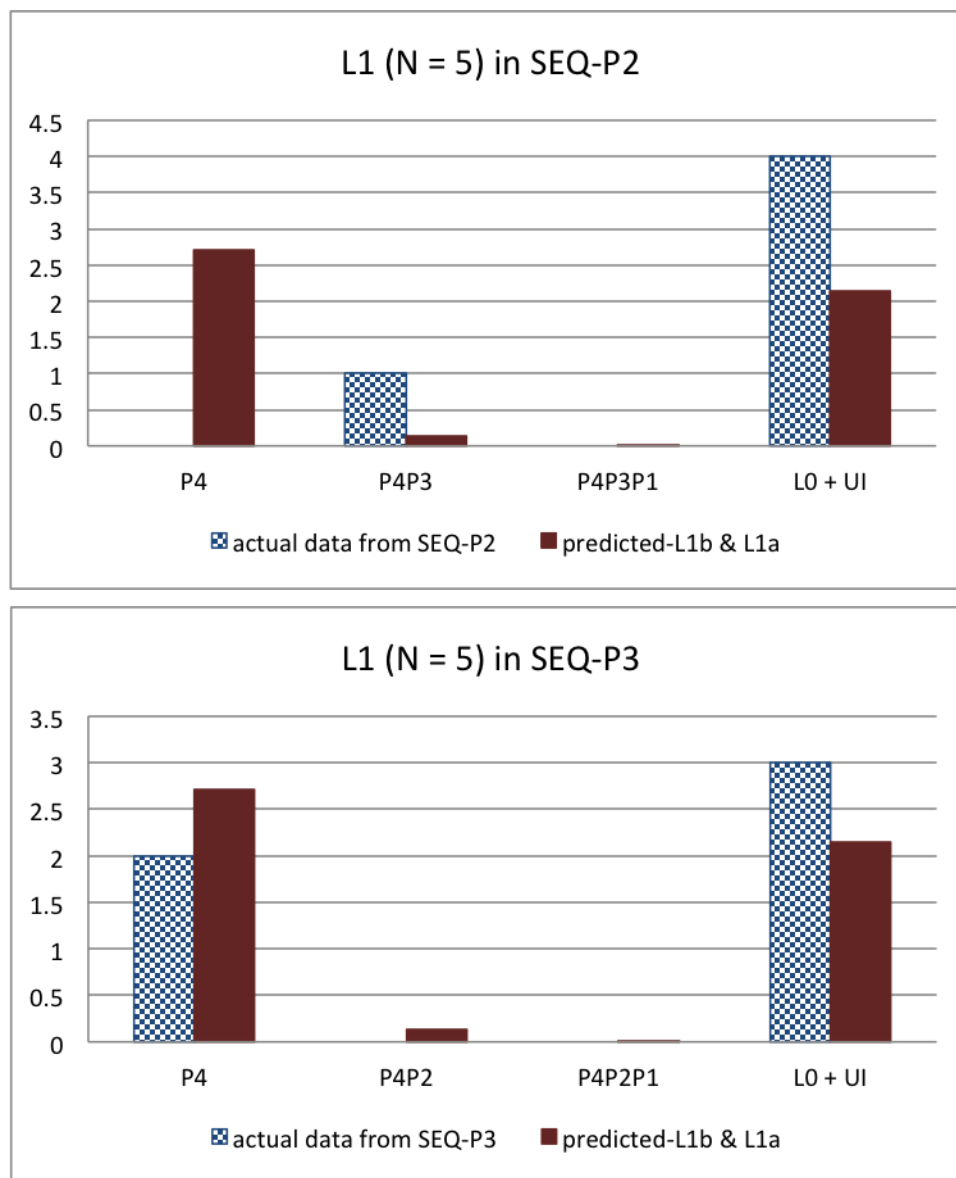
A.2 Type Classification Results Allowing for 0 or 2 Deviations

Table 2.4 has shown what the type distribution looks like allowing for 0 or 2 deviations. If no deviation is allowed for during the type assignment process, there will be a large fraction of subjects (113 out of 184) who could not be put into any of the $L0$ - $L4$ categories. With the 0-deviation cutoff, since few subjects' behavior profile could be matched to a certain type with 0 deviation in all three sets of games, it is hard to determine a clear pattern of each type's behavior in the sequential games. As shown in the following graphs, although most $L4$ subjects behave as predicted in both sequential games, half of the $L1$, $L2$ and $L3$ subjects fall into the unidentifiable category. So it is less clear what proportion of the subjects are best responding to their belief and what proportion are bounded by reasoning ability in the sequential games.

In the type classification using all three sets of games, most subjects remain unidentified with the 0-deviation cutoff (Table A.2).

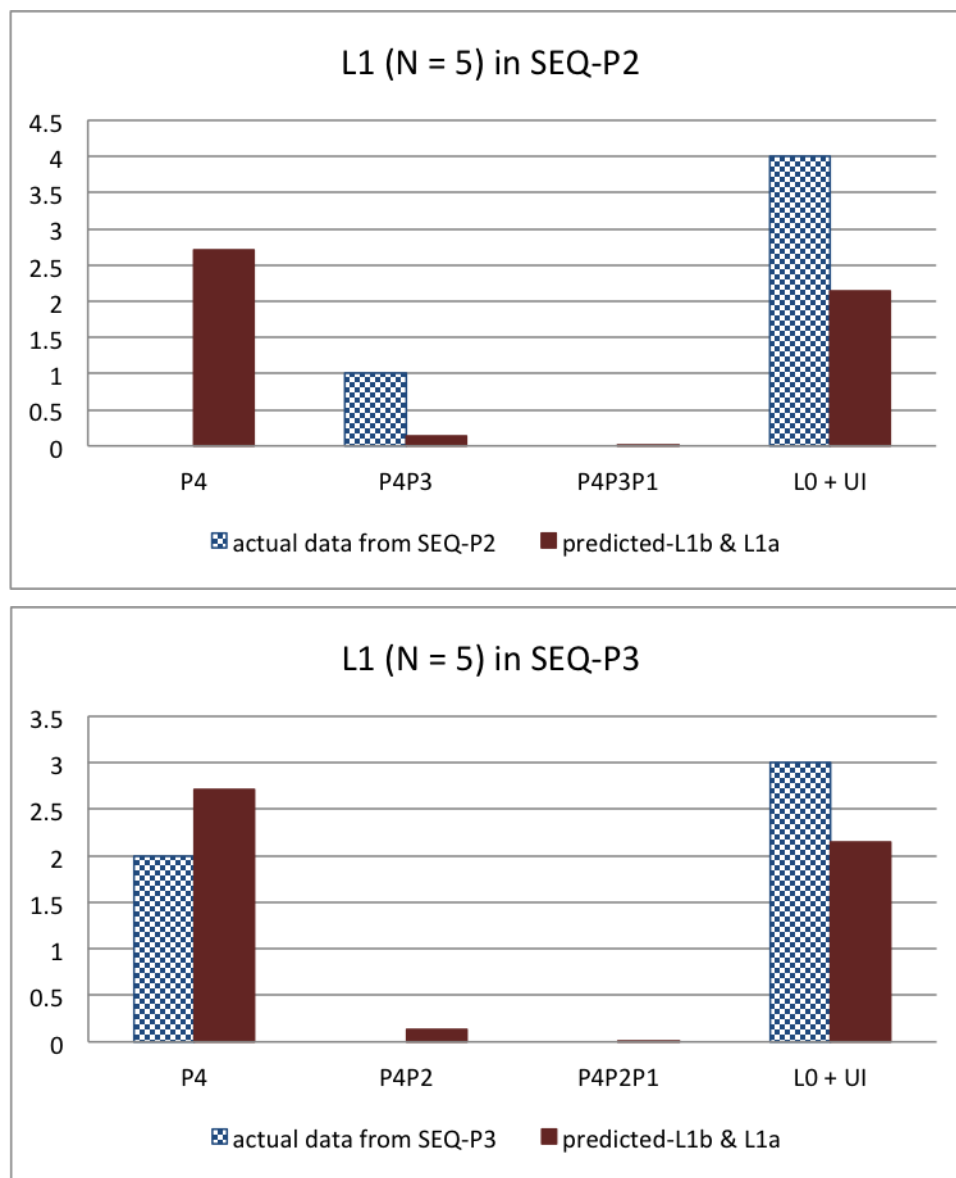
When allowing for 2 instead of 1 deviation in the matching process, the risk of misidentifying the randomly choosing subjects to an Lk type increases. More specifically, according to Table 2.4, 15% of the $L0$ players who used to be classified as $L0$ and 5% who used to be classified as unidentifiable are now assigned to one of the $L1$ - $L4$ categories. When turning to the actual data, a decrease in the number of unidentifiables could be observed. But it is hard to tell how many of the subjects leaving the unidentifiable group are the misidentified $L0$ or the real Lk types with larger deviation rates. The change in the $L0$ category might provide some clues. There are only 2 subjects leaving the $L0$ category when switched to the 2-deviation cutoff. So the increase in the misidentification of $L0$ appears not to be a big problem in my data. Therefore it is helpful to take a look at the results with the 2-deviation cutoff, in which almost all the subjects could be classified.

Most previously unidentified subjects in the simultaneous games are moved to one of the lower types ($L1$ and $L2$) when allowing for 2 deviations, and there is no change in the number of $L3$ and $L4$ subjects. The behavioral pattern of each type in sequential games appears to be similar as in the main analysis (??). But it should be noted that since it identifies more lower types from the previously unidentifiable pool by using the 2-deviation cutoff, the use



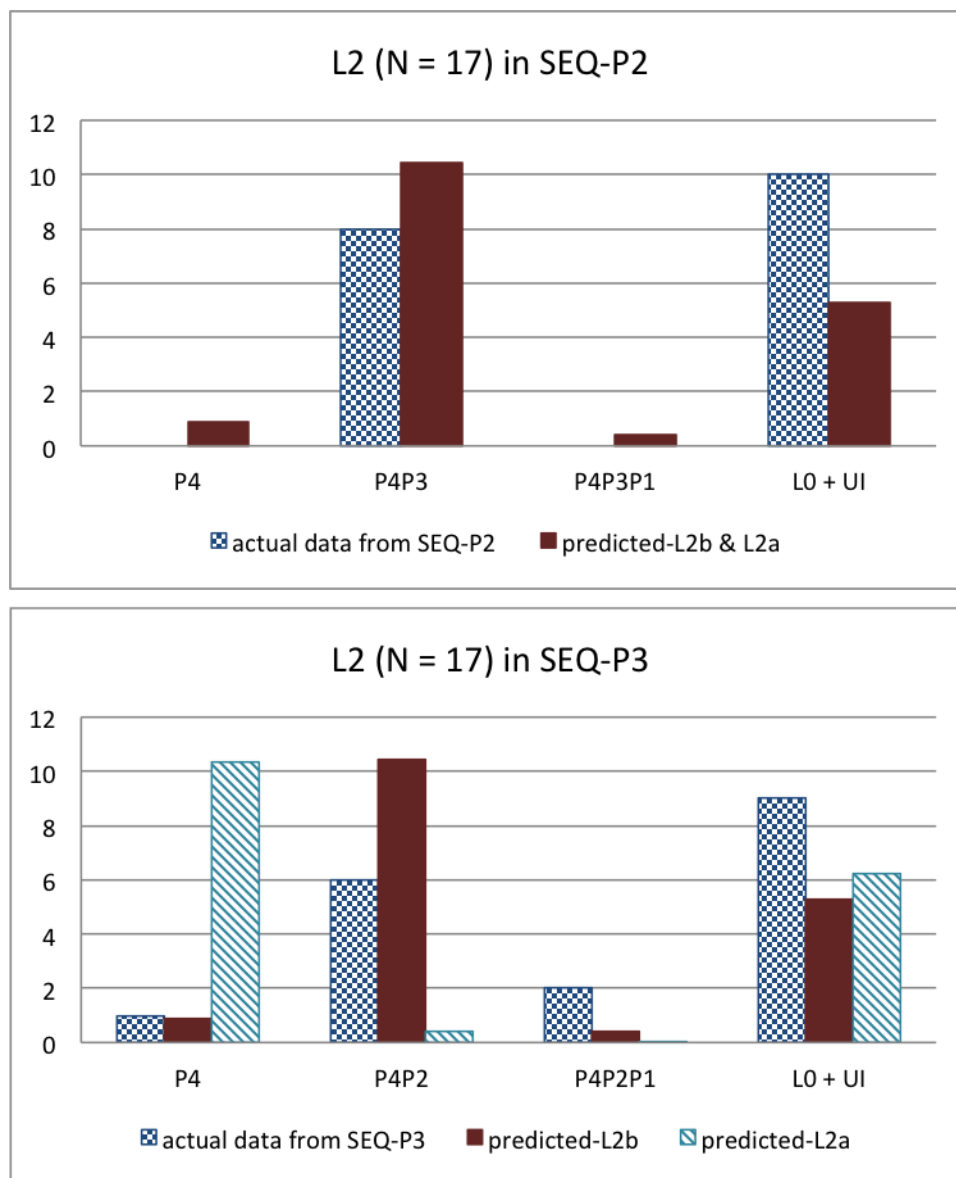
Note: In SEQ-P2, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 (“P4”), at P4 and P3 (“P4P3”), and at P4, P3 and P1 (“P4P3P1”). In SEQ-P3, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 (“P4”), at P4 and P2 (“P4P2”), and at P4, P2 and P1 (“P4P2P1”). Each subject is assigned to a category with ER and 0-deviation cutoff. The dark blue bars give the actual type distribution. The light blue and medium blue bars give the simulated type distribution assuming that these subjects are *Lkb* or *Lka*.

Figure A.5: Behavioral patterns of the observed *L1* subjects (0 deviation)



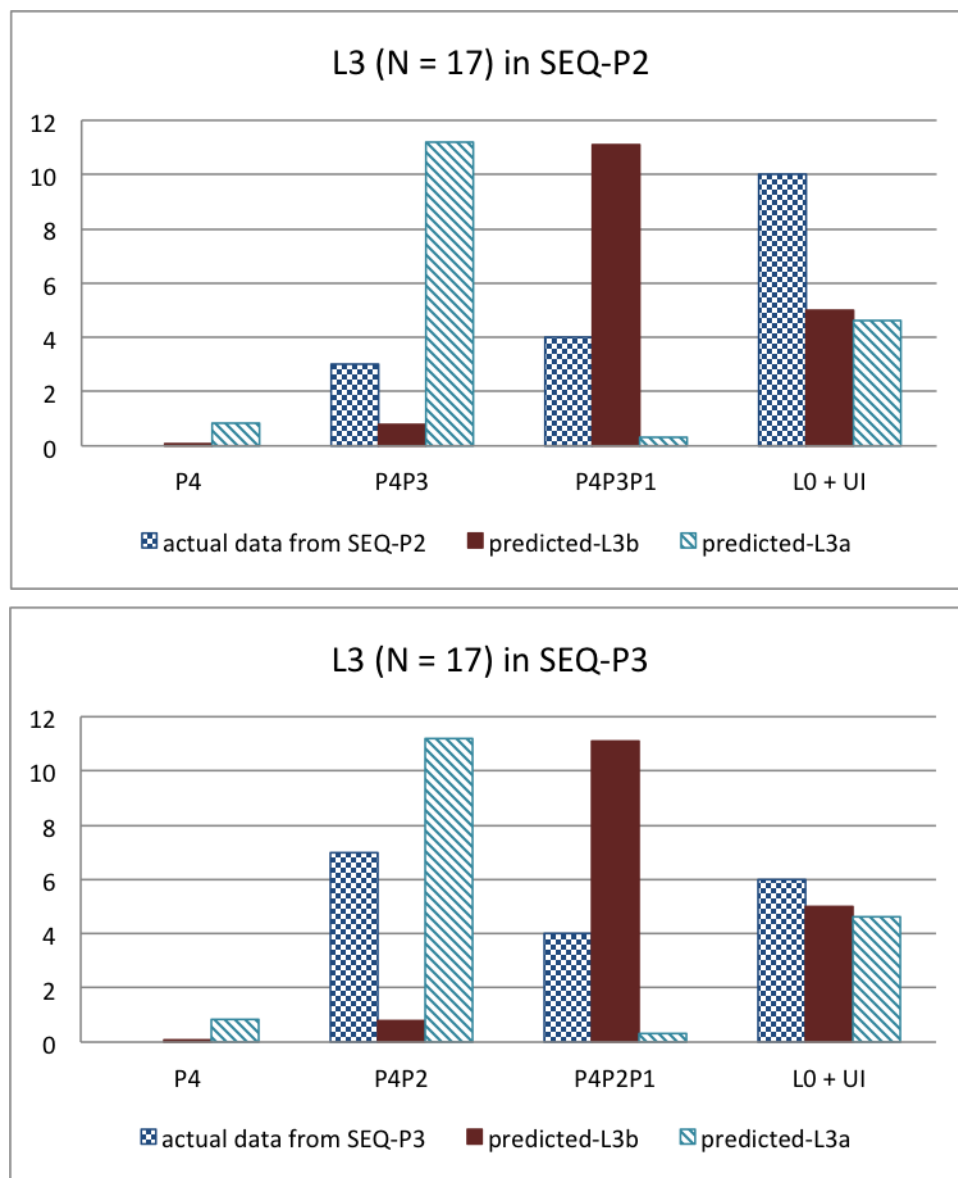
Note: In SEQ-P2, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 (“P4”), at P4 and P3 (“P4P3”), and at P4, P3 and P1 (“P4P3P1”). In SEQ-P3, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 (“P4”), at P4 and P2 (“P4P2”), and at P4, P2 and P1 (“P4P2P1”). Each subject is assigned to a category with ER and 0-deviation cutoff. The dark blue bars give the actual type distribution. The light blue and medium blue bars give the simulated type distribution assuming that these subjects are *Lkb* or *Lka*.

Figure A.6: Behavioral patterns of the observed *L1* subjects (0 deviation)



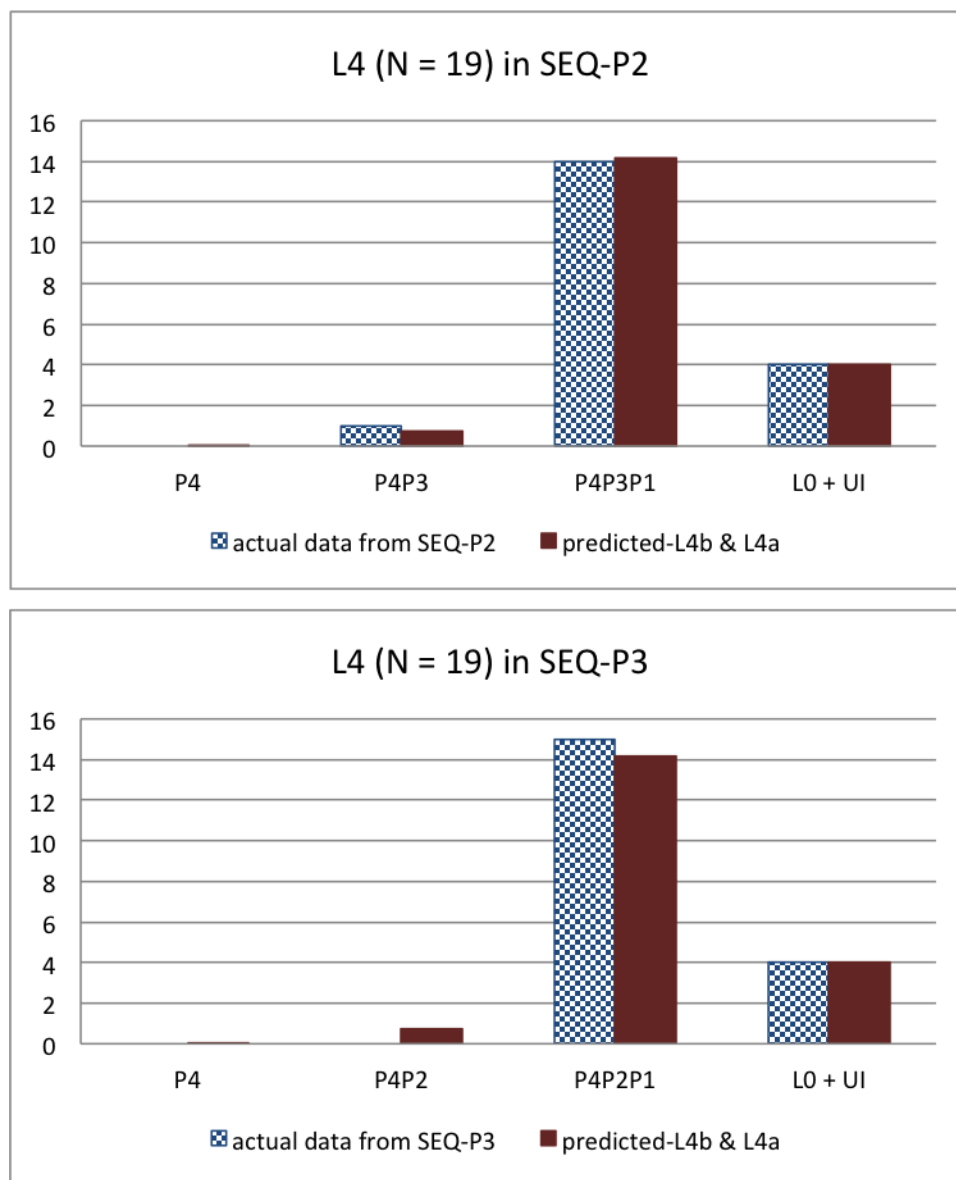
Note: In SEQ-P2, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 (“P4”), at P4 and P3 (“P4P3”), and at P4, P3 and P1 (“P4P3P1”). In SEQ-P3, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 (“P4”), at P4 and P2 (“P4P2”), and at P4, P2 and P1 (“P4P2P1”). Each subject is assigned to a category with ER and 0-deviation cutoff. The dark blue bars give the actual type distribution. The light blue and medium blue bars give the simulated type distribution assuming that these subjects are *Lkb* or *Lka*.

Figure A.7: Behavioral patterns of the observed *L2* subjects (0 deviation)



Note: In SEQ-P2, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 (“P4”), at P4 and P3 (“P4P3”), and at P4, P3 and P1 (“P4P3P1”). In SEQ-P3, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 (“P4”), at P4 and P2 (“P4P2”), and at P4, P2 and P1 (“P4P2P1”). Each subject is assigned to a category with ER and 0-deviation cutoff. The dark blue bars give the actual type distribution. The light blue and medium blue bars give the simulated type distribution assuming that these subjects are *Lkb* or *Lka*.

Figure A.8: Behavioral patterns of the observed *L3* subjects (0 deviation)



Note: In SEQ-P2, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 (“P4”), at P4 and P3 (“P4P3”), and at P4, P3 and P1 (“P4P3P1”). In SEQ-P3, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 (“P4”), at P4 and P2 (“P4P2”), and at P4, P2 and P1 (“P4P2P1”). Each subject is assigned to a category with ER and 0-deviation cutoff. The dark blue bars give the actual type distribution. The light blue and medium blue bars give the simulated type distribution assuming that these subjects are *Lkb* or *Lka*.

Figure A.9: Behavioral patterns of the observed *L4* subjects (0 deviation)

		Baseline + Sequential Types								
		<i>L0</i>	<i>L1</i>	<i>L2a</i>	<i>L2b</i>	<i>L3a</i>	<i>L3b</i>	$\geq L4$	<i>UI</i>	sum
Baseline Types	L0	7	0	0	0	0	0	0	1	8
	L1	0	0	0	0	0	0	0	5	5
	L2	0	0	1	3	0	0	0	14	18
	L3	0	0	0	0	2	2	0	13	17
	L4	0	0	0	0	0	0	12	7	19
	UI	1	0	0	0	0	0	0	111	112
	sum	8	0	1	3	2	2	12	151	179

Note: $N = 179$. Subjects are assigned to a type with 0 deviation.

Table A.2: Type assignment according to the observations from SIMUL, SEQ-P3 and SEQ-P2 (0 deviation)

of this cutoff picks up slightly more of the ability-bounded $L2a$ and $L3a$ types versus the belief-bounded $L2b$ and $L3b$ types in Table A.3.

A.3 Additional Analysis on Learning Effects

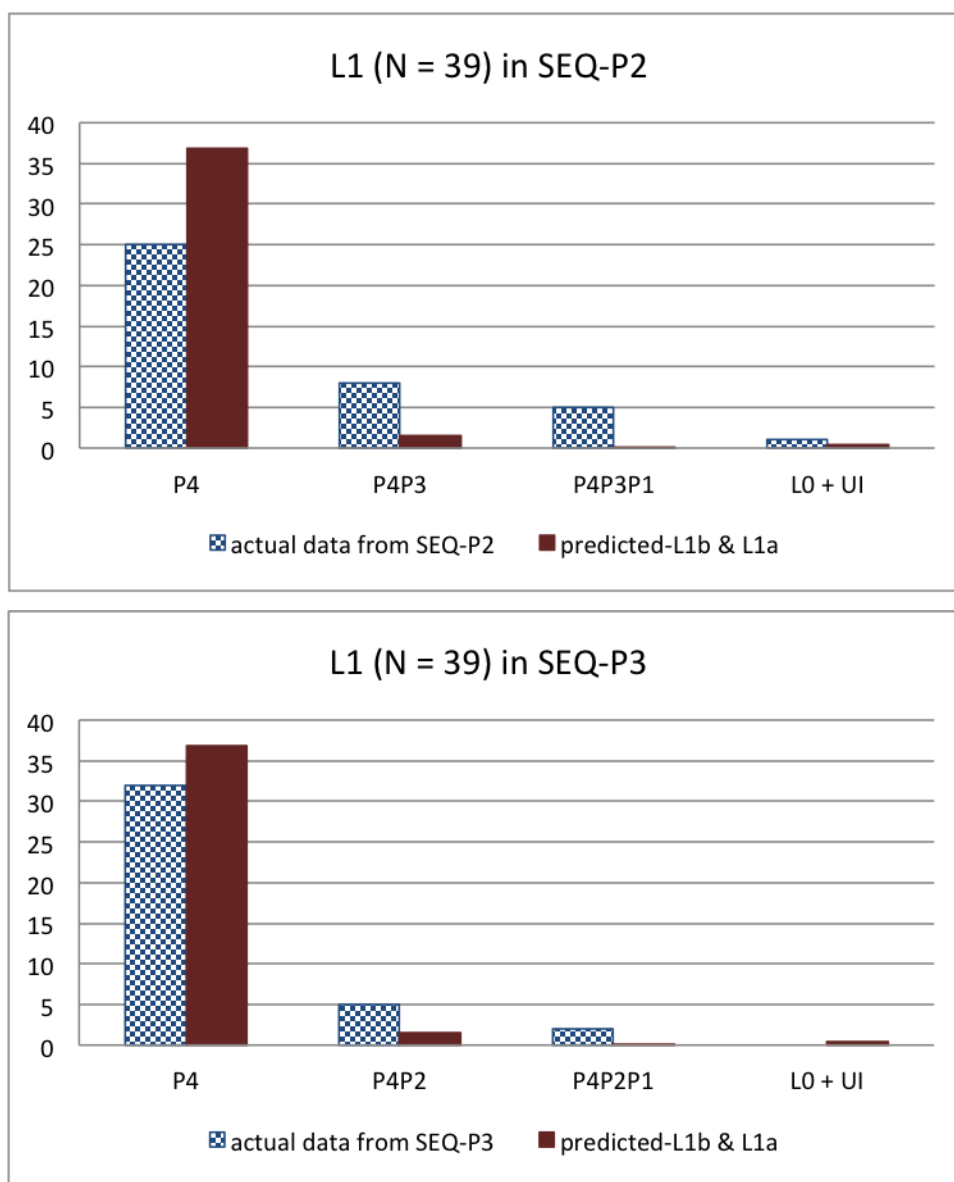
A thorough analysis on learning effects is reported in this section. I start by checking whether there exist systematic type shifts in the data. That is, whether subjects are more likely to shift to a higher (lower) type in the later (earlier) period of the experiment. Since the choices in all 20 first mover positions are needed to pin down a subject's type, it is not possible to estimate one's type in the early and late parts of the experiment separately. Instead, I look at the deviations from the choice pattern of one's assigned type. The deviation observed in the type assignment process could be sorted into one of three cases: (1) a non-equilibrium strategy is chosen at a position where the type should have chosen an equilibrium strategy. (2) two different strategies are chosen at the same position of the two paired rings where the type should be on off-equilibrium path and choose the same strategy, and at least one of the two strategies chosen is an equilibrium strategy. (3) two different strategies are chosen at the same position of the two paired rings where the type should be on off-equilibrium path and choose the same strategy, and neither of the two strategies chosen is an equilibrium strategy.

Case (1) implies that the subject might shift to a lower level when playing that game and Case (2) corresponds to the shift to a higher level. If learning affects subjects' behavior, they should be more likely to deviate to a higher level in the later half of the experiment and more likely to a lower level in the earlier half. If, however, the growth of fatigue plays

		Baseline + Sequential Types								
		<i>L0</i>	<i>L1</i>	<i>L2a</i>	<i>L2b</i>	<i>L3a</i>	<i>L3b</i>	$\geq L4$	<i>UI</i>	sum
Baseline Types	L0	0	5	0	0	0	0	0	0	5
	L1	1	30	4	2	0	2	0	0	39
	L2	0	2	29	24	0	2	0	0	57
	L3	1	0	0	0	23	15	0	0	39
	L4	0	1	1	1	1	0	26	0	30
	UI	1	7	0	1	0	0	0	0	9
	sum	3	45	34	28	24	19	26	0	179

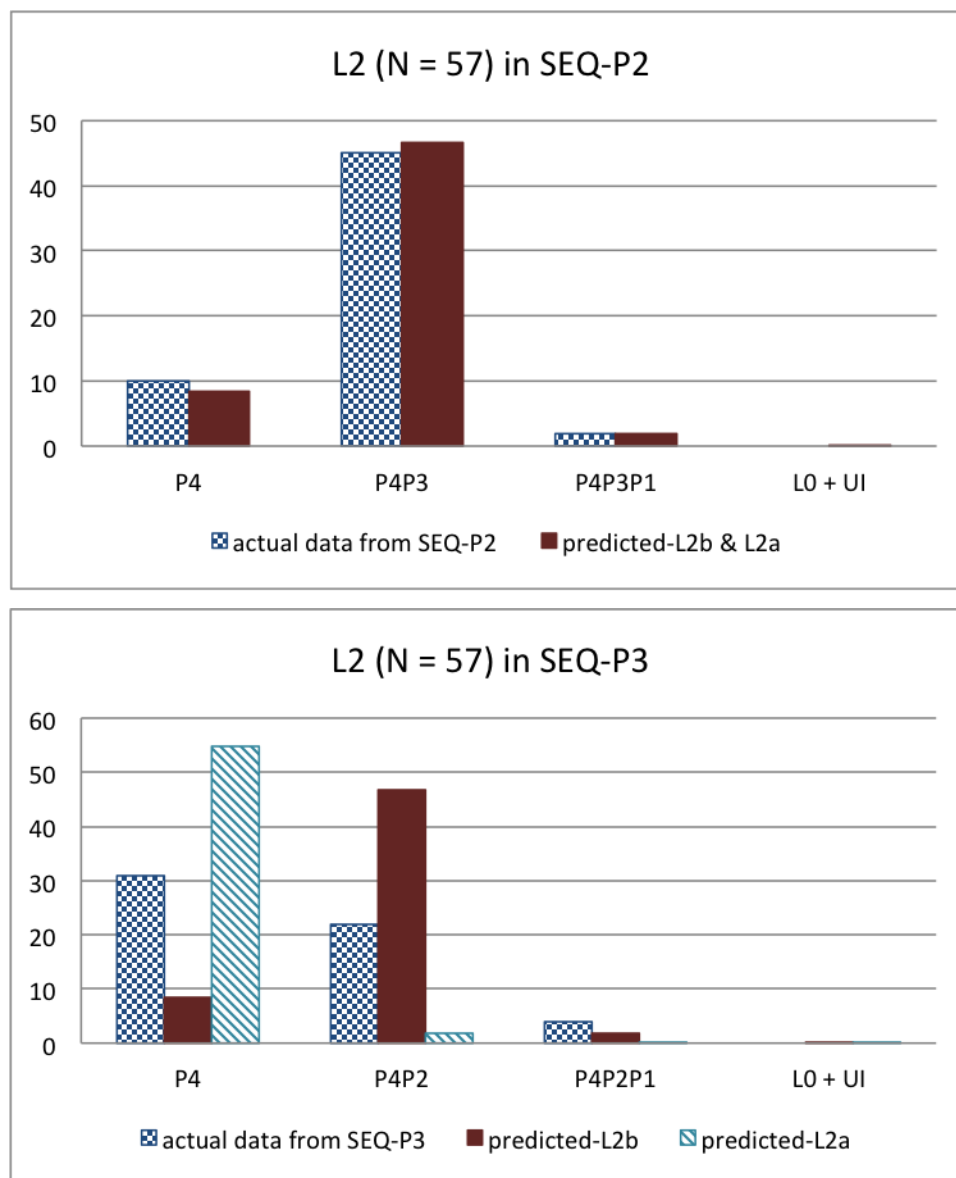
Note: $N = 179$. Subjects are assigned to a type with no more than 6 deviations. Otherwise they are assigned to *L0* or unidentifiable. The subjects classified as unidentifiable are able to choose 5 out of 6 dominant strategies as player 4 but do not match any of the predicted patterns.

Table A.3: Type assignment according to the observations from SIMUL, SEQ-P3 and SEQ-P2 (6 deviations)



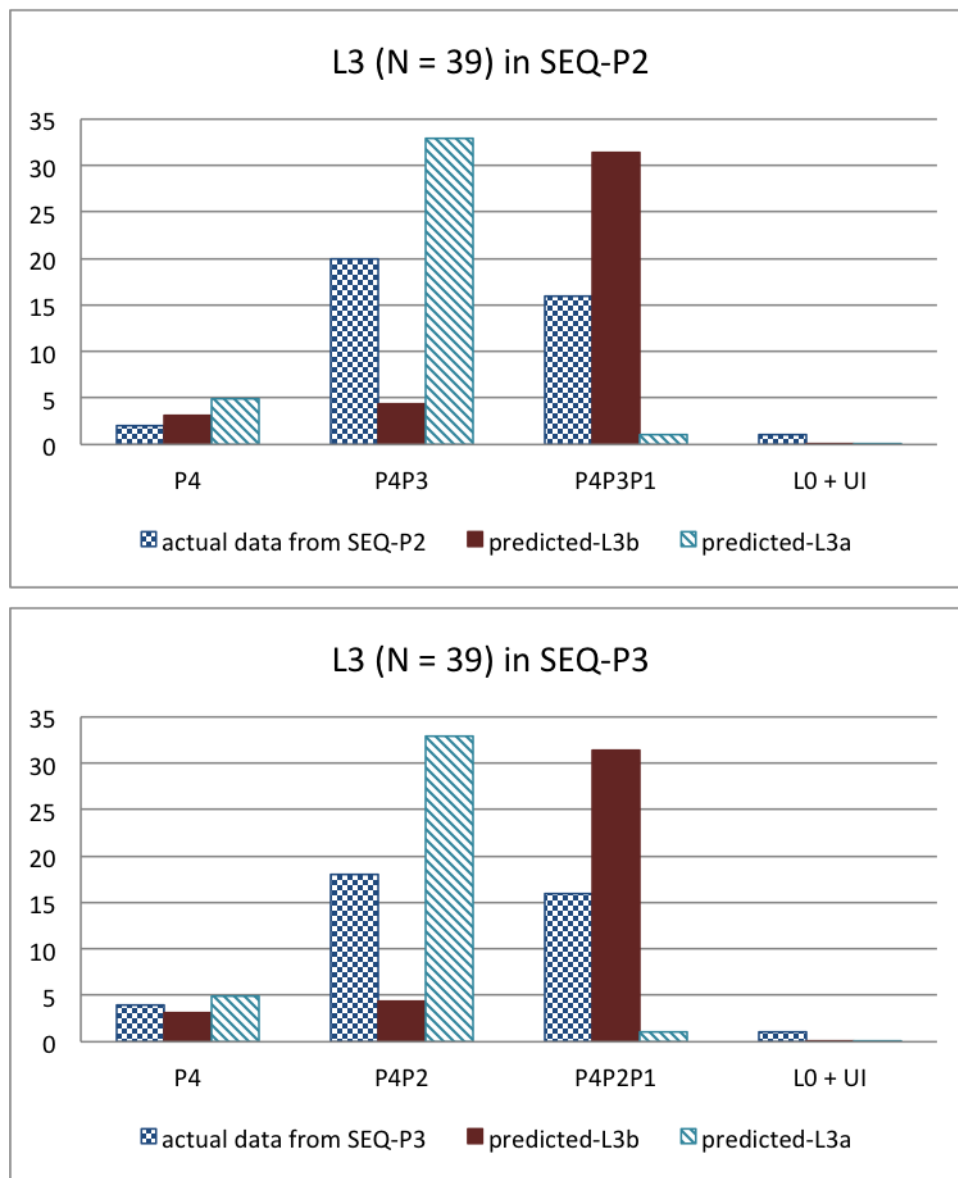
Note: In SEQ-P2, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 (“P4”), at P4 and P3 (“P4P3”), and at P4, P3 and P1 (“P4P3P1”). In SEQ-P3, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 (“P4”), at P4 and P2 (“P4P2”), and at P4, P2 and P1 (“P4P2P1”). Each subject is assigned to a category with ER and 2-deviation cutoff. The dark blue bars give the actual type distribution. The light blue and medium blue bars give the simulated type distribution assuming that these subjects are *Lkb* or *Lka*.

Figure A.10: Behavioral patterns of the observed *L1* subjects (2 deviations)



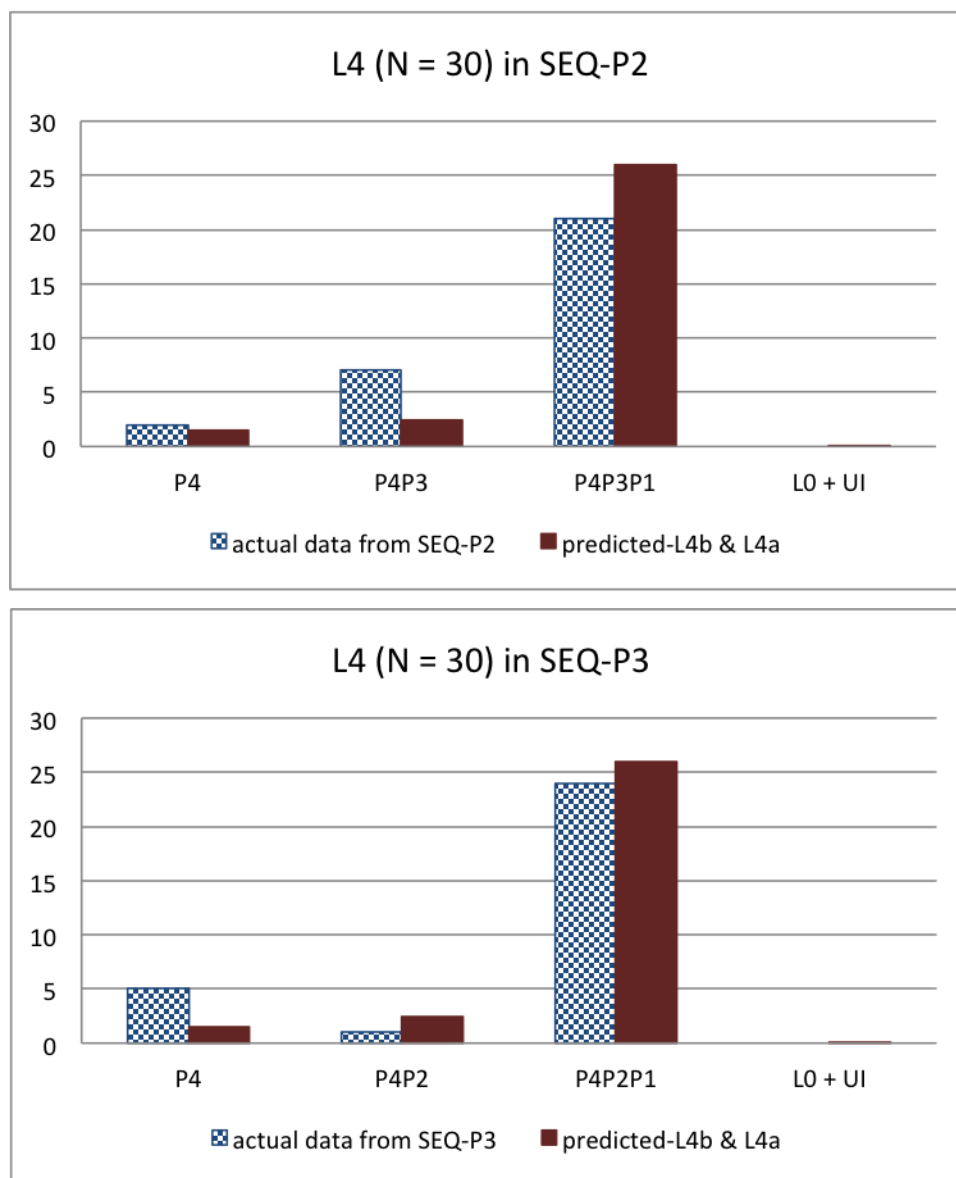
Note: In SEQ-P2, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 (“P4”), at P4 and P3 (“P4P3”), and at P4, P3 and P1 (“P4P3P1”). In SEQ-P3, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 (“P4”), at P4 and P2 (“P4P2”), and at P4, P2 and P1 (“P4P2P1”). Each subject is assigned to a category with ER and 2-deviation cutoff. The dark blue bars give the actual type distribution. The light blue and medium blue bars give the simulated type distribution assuming that these subjects are *Lkb* or *Lka*.

Figure A.11: Behavioral patterns of the observed *L2* subjects (2 deviations)



Note: In SEQ-P2, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 (“P4”), at P4 and P3 (“P4P3”), and at P4, P3 and P1 (“P4P3P1”). In SEQ-P3, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 (“P4”), at P4 and P2 (“P4P2”), and at P4, P2 and P1 (“P4P2P1”). Each subject is assigned to a category with ER and 2-deviation cutoff. The dark blue bars give the actual type distribution. The light blue and medium blue bars give the simulated type distribution assuming that these subjects are *Lkb* or *Lka*.

Figure A.12: Behavioral patterns of the observed *L3* subjects (2 deviations)



Note: In SEQ-P2, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 (“P4”), at P4 and P3 (“P4P3”), and at P4, P3 and P1 (“P4P3P1”). In SEQ-P3, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 (“P4”), at P4 and P2 (“P4P2”), and at P4, P2 and P1 (“P4P2P1”). Each subject is assigned to a category with ER and 2-deviation cutoff. The dark blue bars give the actual type distribution. The light blue and medium blue bars give the simulated type distribution assuming that these subjects are *Lkb* or *Lka*.

Figure A.13: Behavioral patterns of the observed *L4* subjects (2 deviations)

a more important role, it should be opposite. Of course it could not be ruled out that some deviations in Case (1) and (2) are caused by preference shifts or mistakes. But if the occurrences of preference shifts or mistakes are assumed to be time-invariant, then they could be canceled out when only the differences of the earlier and later halves are examined.

	8 Baseline Games			20 First Mover Games		
	Case (1)	Case (2)	Case (3)	Case (1)	Case (2)	Case (3)
Shifts	downward	upward	-	downward	upward	-
Earlier	12	26	25	42	80	53
Later	2	27		24	79	

Note: the first two cases are counted in the earlier and later 12 games respectively. *L0* and unidentifiable subjects are excluded.

Table A.4: Deviations sorted into three cases

Table A.4 reports the deviations of the classified subjects, from both the 8 SIMUL games and the 20 first mover games type assignments. *L0* and unidentifiable subjects are excluded from this analysis, because according to the definition *L0*s could be using any combination of strategies, and since I could not identify the decision rules of the unidentifiable it would be hard to determine which choices are deviations from their rules. The 24 games were played in a random order and the orders of play were different for each subject. Cases (1) and (2) could therefore be put into two categories, that is whether these deviations occur in the first or later 12 games. In Case (3) it could only be observed that subjects are choosing two different strategies at the same position, but it is impossible to tell which one is the deviation (or both could be deviations). So only the total numbers of deviations in Case (3) are reported.

In Case (1), players deviate to a lower type. This kind of deviation is more likely to happen in the first half of the experiment, suggesting some sort of learning effects. However, the occurrences of Case (2) deviations, which imply a shift to a higher level, are quite close between earlier and later periods of the experiment.

I next run a probit regression to determine the learning effect specifically at each position.

$$\text{Probit}(Y_i) = \alpha + \beta_1 L12_i + \beta_2 POS_i + \beta_{3j} POS_{ij} \times L12_i + \epsilon_i, \quad (\text{A.1})$$

where $Y_i = 1$ when an equilibrium strategy is chosen, and $Y_i = 0$ otherwise; $L12_i = 1$ if that choice is made in the later 12 games, and $L12_i = 0$ otherwise; POS_{ij} denotes the position dummy at position j . The session fixed effects are also controlled.

Table A.5 reports the coefficients $\beta_1 + \beta_{3j}$ of each position j . One position dummy, player 4 of G5, is dropped because of collinearity. Significant positive learning effects are found at only 3 of the 23 positions.

		player 1	player 2	player 3	player 4
Baseline	G1	0.215	0.009	0.287	0.127
		(0.193)	(0.189)	(0.210)	(0.389)
	G2	0.267	0.055	0.235	0.010
		(0.190)	(0.188)	(0.211)	(0.417)
		player 1	player 2 (2nd)	player 3	player 4
Seq-P2	G3	0.210	0.108	0.570**	-0.226
		(0.193)	(0.331)	(0.234)	(0.365)
	G4	0.232	-0.683	-0.103	0.272
		(0.199)	(0.427)	(0.227)	(0.315)
		player 1	player 2	player 3 (2nd)	player 4
Seq-P3	G5	0.028	0.492**	0.142	-
		(0.187)	(0.200)	(0.344)	-
	G6	0.393**	-0.144	0.237	0.010
		(0.193)	(0.197)	(0.369)	(0.395)

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Standard errors are reported in the parentheses. (2nd) denotes second stage movers. Player 4 of G5 is dropped due to collinearity.

Table A.5: Experience effect at each position ($\beta_1 + \beta_{3j}$)

Since the identification uses at least a pair of ring games, the learning effects at three positions is unlikely to affect the type distribution. It is reported in Table 2.7 that the performance of the subjects who could have better opportunities to learn is not different from the whole sample. Here I further show with multilogit regressions that playing more player 4 positions or second mover positions in the earlier periods does not affect the probability of being assigned to a high type (Table A.6 and Table A.7).

Independent Variable: $I(n(P4) > 4$ in earlier 12 games)							
base outcome	$L0$	$L1$	$L2a$	$L2b$	$L3a$	$L3b$	$L4$
vs $L0$		-0.398 (0.883)	-0.713 (0.897)	-0.770 (0.887)	-0.843 (0.893)	0.182 (0.884)	-1.713* (0.999)
vs $L1$	0.398 (0.883)		-0.315 (0.720)	-0.372 (0.670)	-0.445 (0.715)	0.581 (0.656)	-1.315 (0.823)
vs $L2a$	0.713 (0.897)	0.315 (0.720)		-0.0564 (0.682)	-0.129 (0.703)	0.896 (0.713)	-1.000 (0.832)
vs $L2b$	0.770 (0.887)	0.372 (0.670)	0.0564 (0.682)		-0.0730 (0.697)	0.952 (0.676)	-0.943 (0.811)
vs $L3a$	0.843 (0.893)	0.445 (0.715)	0.129 (0.703)	0.0730 (0.697)		1.025 (0.723)	-0.870 (0.835)
vs $L3b$	-0.182 (0.884)	-0.581 (0.656)	-0.896 (0.713)	-0.952 (0.676)	-1.025 (0.723)		-1.895** (0.801)
vs $L4$	1.713* (0.999)	1.315 (0.823)	1.000 (0.832)	0.943 (0.811)	0.870 (0.835)	1.895** (0.801)	

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Standard errors are reported in the parentheses. Each row shows a regression with a different type being the base outcome. Session fixed effects are included in the multilogit regressions. Unidentifiable subjects are excluded.

Table A.6: Multilogit regression of types on Learning Effects (1)

Independent Variable: $I(n(P4) + N(PSM)) > 6$ in earlier 12 games)							
base outcome	$L0$	$L1$	$L2a$	$L2b$	$L3a$	$Lb3$	$L4$
vs $L0$		0.381 (0.893)	-0.320 (0.907)	-0.633 (0.926)	-0.740 (0.948)	0.689 (0.951)	-0.761 (0.922)
vs $L1$	-0.381 (0.893)		-0.700 (0.690)	-1.014 (0.681)	-1.121 (0.746)	0.308 (0.704)	-1.142* (0.674)
vs $L2a$	0.320 (0.907)	0.700 (0.690)		-0.314 (0.699)	-0.420 (0.740)	1.009 (0.758)	-0.441 (0.697)
vs $L2b$	0.633 (0.926)	1.014 (0.681)	0.314 (0.699)		-0.107 (0.758)	1.322* (0.757)	-0.128 (0.696)
vs $L3a$	0.740 (0.948)	1.121 (0.746)	0.420 (0.740)	0.107 (0.758)		1.429* (0.819)	-0.0208 (0.741)
vs $L3b$	-0.689 (0.951)	-0.308 (0.704)	-1.009 (0.758)	-1.322* (0.757)	-1.429* (0.819)		-1.450* (0.742)
vs $L4$	0.761 (0.922)	1.142* (0.674)	0.441 (0.697)	0.128 (0.696)	0.0208 (0.741)	1.450* (0.742)	

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Standard errors are reported in the parentheses. Each row shows a regression with a different type being the base outcome. Session fixed effects are included in the multilogit regressions. Unidentifiable subjects are excluded.

Table A.7: Multilogit regression of types on Learning Effects (2)

Appendix B

Instructions and Understanding Test

B.1 Instructions

You are about to participate in an experiment in decision making. The experiment includes 24 decision problems and 9 short questions. You will get separate instructions for the short questions after the decision making part. If you follow the instructions and make good decisions, you may earn a considerable amount of money. Besides that, you will also get a \$5 participation fee. All the money that you earn will be paid to you in private after today's session.

You may not write during the experiment. It is also important to remain silent throughout the experiment and not to look at other people's work. If you have any questions or need assistance of any kind, please raise your hand, and the experimenter will come to you. Otherwise, if you write, laugh, talk with other participants, exclaim out loud, etc., YOU WILL BE ASKED TO LEAVE. Thank you.

Your earnings will be determined by your decisions and the decisions of other participants in the experiment. Before making your decisions, you will be able to gather information about how your earnings and other participants' earnings depend on your and their decisions. Then you need to take an understanding test and have the opportunity to practice on 4 problems. You will NOT be paid for the practice rounds.

The experiment has 24 rounds. In each round, you will be matched with a RANDOMLY selected group of the other participants. A new group will be formed and matched with you in each round. You will not know which ones of the other participants you are matched with, and your identity and the identities of the other participants will never be revealed.

Each round concerns a DECISION SITUATION in which you and the other participants in your group separately and independently make decisions called CHOICES. Together, your and their choices determine your and their earnings in this round, which may be different for each member in the group.

Neither your nor their choices in a round will affect how you or the other participants are matched or the decision situations you and they face in the rest of the experiment.

In each round, you will choose among three alternatives. The other members in your group will also have three alternatives to choose from. Your earnings will depend on the combination of your choice and one of the other group member's choice. These earnings possibilities will be represented in a table like the one below. Your choices will determine the row of the table. The choice of one of the other members in your group, whom we call Participant X, will determine the column of the table. You may choose from "a, b, c" and Participant X may choose from "d, e, f". The cell corresponding to this combination of choices will determine your earnings.

Time left: 26 seconds

Your Earnings				Participant X's Earnings			Participant Y's Earnings			Participant Z's Earnings														
		X's Choices					Y's Choices					Z's Choices												
		d	e	f			d	e	f			g	h	i			j	k	l			a	b	c
Your Choices	a	14	8	4	X's Choices	d	20	14	8	Y's Choices	g	4	16	14	Z's Choices	j	8	20	12	Your Choices	a	8	20	12
	b	20	8	14		e	16	2	18		h	14	12	10		k	0	8	6		b	0	8	6
	c	0	16	18		f	0	16	16		i	6	10	8		l	18	12	6		c	18	12	6

Your earnings are given by the blue numbers. You can highlight any cell by clicking on them.

Participant Z is the observer in this group.

Your earnings depend on your and Participant X's choices.

Participant X's earnings depend on Participant X's and Participant Y's choices.

Participant Y's earnings depend on Participant Y's and Participant Z's choices.

Participant Z's earnings depend on Participant Z's and your choices.

Please choose: a b c

You will not be able to change your choice after you have confirmed it.

CONFIRM

For example, given the above earnings tables, if you choose *b* and Participant X chooses *e*, then you will earn 8 dollars for this round. If instead Participant X chooses *f*, you will earn 14 dollars.

Your earnings will always be listed in the FIRST table. Participants X, Y and Z's earnings are listed in the other three tables. X's earnings depend upon his/her choice and Y's choice. Y's earnings depend upon his/her choice and Z's choice. Z's earnings depend upon his/her choice and your choice.

For example, if you choose *a*, Participant X chooses *e*, Participant Y chooses *h*, Participant Z chooses *l*, then you will earn 8 dollars, X will earn 2 dollars, Y will earn 10 dollars and Z will earn 18 dollars.

The earnings tables will be given to every member of the group. After you have made the decision in that round, you need to click the button to confirm your choice and wait to

proceed to the next problem after all the other group members have finished their choices. The earnings tables may change from round to round, so you should always look at the earnings carefully at the beginning of each round.

Besides, you may highlight any cell by clicking on it to help you making the decisions (see screenshot below).

There is a time limit of 60 seconds for you to make choice in each round. There will be a counting down clock showing how much time is left for this round. The clock is located on the upper right corner, as shown in the screenshot below. If you fail to make a decision within the time limit, you will earn \$0 in this round. If any group member fails to make a choice within the time limit, the system will randomly pick a choice out of his three alternatives when calculating the earnings of other group members. For example, given the earnings tables below, if you choose *b*, X chooses *f*, Y fails to choose, Z chooses *l*, and the system chooses *g* for Y, then you will earn \$8, X will earn \$8, Y will earn \$0 and Z will earn \$10.

Time left: 26 seconds

Your Earnings				Participant X's Earnings				Participant Y's Earnings				Participant Z's Earnings							
		X's Choices					Y's Choices					Z's Choices					Your Choices		
		d	e	f			g	h	i			j	k	l			a	b	c
Your Choices	a	20	4	10	X's Choices	d	2	18	4	Y's Choices	g	16	6	0	Z's Choices	j	12	14	0
	b	0	18	8		e	6	10	12		h	10	4	20		k	16	4	18
	c	14	12	16		f	8	0	14		i	12	14	2		l	2	10	20

Your earnings are given by the blue numbers. You can highlight any cell by clicking on them.

Participant X is the observer in this group.

Your earnings depend on your and Participant X's choices.

Participant X's earnings depend on Participant X's and Participant Y's choices.

Participant Y's earnings depend on Participant Y's and Participant Z's choices.

Participant Z's earnings depend on Participant Z's and your choices.

Please choose: a b c

You will not be able to change your choice after you have confirmed it.

CONFIRM

In certain rounds, one of the group members will be RANDOMLY selected to be the OBSERVER. An observer will be able to look at the other group members' choices before making his/her choice. At the beginning of each round, We will inform every member in the group which member has been assigned to be the observer. (But, of course, his/her true identify will always remain anonymous.)

For example, if Participant X is selected to be the observer, you and Participants Y and Z will see this sentence on the screen:

“Participant X is the observer.”

And participant X will see this on the screen:

“You are the observer.”

Below is a screenshot of the interface for an observer. After the other participants have made their choices, these choices will appear in the box.

Time left: 26 seconds

Your Earnings				Participant X's Earnings				Participant Y's Earnings				Participant Z's Earnings												
		X's Choices				Y's Choices				Z's Choices				Your Choices										
		a	d	e	f			g	h	i			j	k	l			a	b	c				
Your Choices	a	6	16	4	X's Choices	d	14	12	16	Y's Choices	g	4	16	14	Z's Choices	j	20	8	4	Your Choices	a	20	8	4
	b	8	12	18		e	18	6	8		h	20	12	0		k	14	0	12		b	14	0	12
	c	14	0	10		f	10	0	20		i	18	2	8		l	18	16	10		c	18	16	10

Your earnings are given by the blue numbers. You can highlight any cell by clicking on them.

You are the **observer** in this group. Your group mates' choices will appear in the box below.

Participant X chose f
Participant Y chose i
Participant Z chose l

Your earnings depend on your and Participant X's choices.

Participant X's earnings depend on Participant X's and Participant Y's choices.

Participant Y's earnings depend on Participant Y's and Participant Z's choices.

Participant Z's earnings depend on Participant Z's and your choices.

Please choose: a b c

You will not be able to change your choice after you have confirmed it.

CONFIRM

The observer will be given an extra of 30 seconds to make his/her choice, which means the observer will have 90 seconds in that round. If you are the observer, please wait patiently until your group-mates' choices show up in the boxes.

Whether you or the other members will be selected as the observer does NOT depend on your or their previous choices nor the group formation. You will be randomly matched with a new group in each round and the identities of all the group members will always remain anonymous.

Once you have confirmed your choice for a round, you will not be able to change your choice for that round. After you have made your choices for all of the 24 rounds, one round

will be randomly selected for payment at the end of the experiment. Every participant in this room will be paid based on his/her choices and the choices of his/her randomly selected group-mates in this round. Any of the rounds could be selected. So you should treat each round like it will be the one determining your payment.

You will be informed of your total earnings, the round chosen for payment, what choice you made in that round and the action of your randomly matched group-mates only at the end of the experiment. You will not learn any other information about the actions of other participants' in the experiment. Your final payment will be the earnings calculated by the system plus the \$5 participation fee. All the money that you earn will be paid to you in private after today's session.

B.2 Understanding Test

You will now take an understanding test. There are 7 questions and you will have 5 minutes in total to finish all the questions. The main experiment will begin after all the participants have finished the understanding test. Now consider the earnings tables below.

Your Earnings				Participant X's Earnings				Participant Y's Earnings				Participant Z's Earnings							
		X's Choices					Y's Choices					Z's Choices					Your Choices		
		d	e	f			g	h	i			j	k	l			a	b	c
Your Choices	a	8	20	12	X's Choices	d	14	8	4	Y's Choices	g	20	14	8	Z's Choices	j	4	16	14
	b	0	8	6		e	20	8	14		h	16	2	18		k	14	12	10
	c	18	12	6		f	0	16	18		i	0	16	16		l	6	10	8

Your earnings are given by the blue numbers. You can highlight any cell by clicking on them.

Participant Y is the observer in this group.

- Your earnings depend on your choice and the choice of which other group member?
 - Participant X.
 - Participant Y.
 - Participant Z.
- Suppose you choose c , Participant X chooses e , Participant Y chooses h , and Participant Z chooses j . What will your earnings be?
 - 16.
 - 8.
 - 12.
 - 18.

3. Suppose Participant X chooses d , Participant Y chooses g , and Participant Z chooses k . Which choice would give you the highest earning?
 - (a) a .
 - (b) b .
 - (c) c .
4. Suppose you choose a . What is your highest possible earning?
 - (a) 20.
 - (b) 18.
 - (c) 16.
 - (d) 12
5. Suppose Participant Z choose k . What is his/her lowest possible earning?
 - (a) 12.
 - (b) 0.
 - (c) 8.
 - (d) 10.
6. What does Participant Y know about your decision situation?
 - (a) Your earnings table.
 - (b) Your choice.
 - (c) (a) and (b).
 - (d) None of the above.
7. What does Participant Z know about your decision situation?
 - (a) Your earnings table.
 - (b) Your choice.
 - (c) (a) and (b).
 - (d) None of the above.