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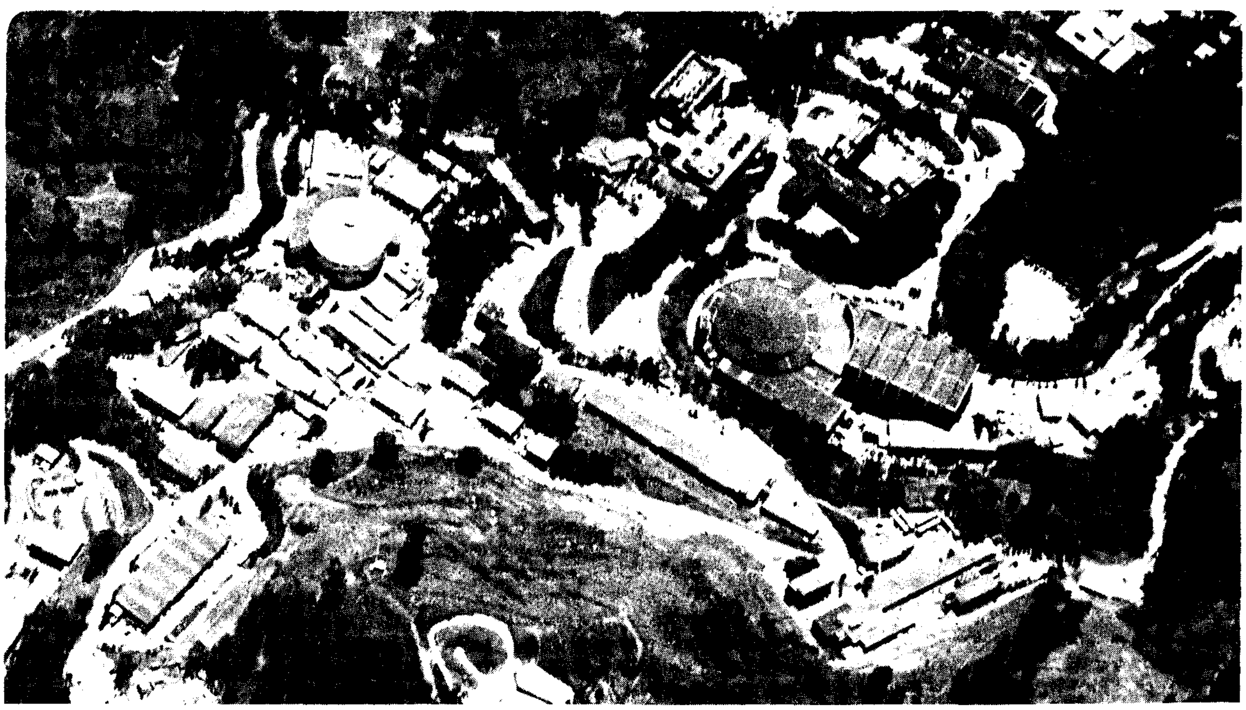

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M_s - M_u QUARK MASS DIFFERENCE AND THE SCALAR FORM FACTOR OF $K^0_{\mu 3}$ REACTION *

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ABSTRACT

We study through QCD sum rules the connection between the invariant quark mass difference $\hat{m}_s - \hat{m}_u$ and the scalar form factor of the reaction $K^0 \rightarrow \pi^- \mu^+ \nu_\mu$ in the physical region. We use both theoretical information, (the value of $f_+(0)$ and the Callan-Treiman relation, including m_π^2/m_k^2 corrections) and experimental one (the value of λ_0 from a linear fit) to give a lower bound for $\hat{m}_s - \hat{m}_u$. Taking the world most recent fitted value for λ_o, $\lambda_o = .025$, which may be reasonably identified with the slope at $t = 0$, and $f_+(0) \sim .98$, we obtain $\hat{m}_s - \hat{m}_u \geq 250$ MeV for $\Lambda_{\overline{MS}} = 150$ MeV. The relevant hypothesis and experimental trends are discussed.

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I. INTRODUCTION

Despite the absence of any fundamental understanding of their origin, the masses of quarks as parameters appearing in the QCD lagrangian have been extensively studied in the last decade.[1,3] As their sum or difference factorizes at leading order in the divergences of hadronic currents, they may be related to physical observables via dispersive intregrals of the propagators of those divergences. This sum rule approach[2] has proved to be very successful and led to a deeper quantitative estimate of chiral symmetry breaking. In the particular case of the $m_s - m_u$ mass difference, a recent study,[3] based on Laplace QCD sum rules and the $K\pi$ $I = 1/2$ s wave experimental phase shift,[4] has given the bound $\hat{m}_s - \hat{m}_u \geq 210$ MeV for $\Lambda_{\overline{MS}} = 150$ MeV. We want here to add another piece of information to the estimate of this quantity, based on the available experimental data on the $K^0\mu_3$ scalar form factor in the physical domain of momentum transfer. This is the most direct approach, which could provide, in the limit of high precision measurements, very useful information on the mass parameters. However, due to the experimental uncertainties, we shall have to use in addition theoretical constraints deduced from low energy theorems and PCAC relations for the pion; they are the value of $f_+(0)$ and that of $d(t)$ at the unphysical point $t = M_{K0}^2$ (the so called Callan-Treiman relation). We shall use the most refined versions of those estimates, and, consequently take them as fixed. The experimental information will be incorporated through the value of the slope in a linear fit of $d(t)$, proportional to the parameter λ_0. We shall identify it with the slope at $t = 0$. Indeed, as far as experiment is concerned, there is no evidence for a deviation from a linear fit. On the other side, the phase shift analysis done in Refs. [3, 4] and the reasonable assumptions made therein give only a very small deviation from linearity in the physical domain of t. We are thus left with two sources of uncertainty, the value of λ_0, which has been rather unstable along the history of $K\ell_3$

decays, and the scale Q^2 inherent to any QCD sum rule computation. As far as the second is concerned, the poor convergence shown at the two loop-level by the α_s series for the propagator of the divergence of the hadronic current makes us cautious and not go down to Q^2 lower than 2 GeV^2. About the first incertitude, we shall consider the following choices of λ_0:

- λ_0 best statistics,
- λ_0 world present fit,
- λ_0 world average,
- λ_0 most recent,

which order in the sense of higher and higher values, giving higher and higher bounds for $\hat{m}_s - \hat{m}_u$. While the order of magnitude obtained here agrees with that of precedent work[3] if one keeps to lower values of λ_0, this study shows that the interplay between mass parameters and experimental data is very sensitive. In particular, if the value of λ_0 keeps on at a high value (and still better measurements would be desired), a reasonably low quark mass difference would necessitate a slight modification of the theoretical constraints.

II. OUTLINE OF THE METHOD

We just sketch out here the main steps of the derivation. More detailed information may be found in Ref. [5]. Let $\psi(q^2)$ be the propagator of the divergence of the strangeness changing current V_μ^{4-i5}:

$$\psi(q^2) = i\int d^4x \, e^{iq\cdot x} \langle 0 | T \, \partial_\mu V^\mu(x) \, \partial_\nu V^{\nu+}(0) | 0 \rangle \tag{2.1}$$

Recall

$$V^{\mu \; 4-i5}(x) = \bar{s}(x) \gamma^\mu u(x) \tag{2.2}$$

$$\partial_\mu V^{\mu \; 4-i5}(x) = -i(m_u - m_s) \, \bar{s}(x) u(x) \; . \tag{2.3}$$

Saturating its absorptive part with the $K^0\pi^+$ intermediate state, using crossing analyticity and the positivity of the spectral function, we end up with the inequality:[5]

$$\psi''(q^2) \equiv \frac{\partial^2}{(\partial q^2)^2}\psi(q^2) \geq \frac{3}{2}\frac{1}{16\pi^2}\int_{t_0}^{\infty} dt \, \frac{2\sqrt{(t-t_1)(t-t_0)}}{t\,(t+Q^2)^3} \, |d(t)|^2 \tag{2.4}$$

where the notations are the following: $Q^2 = -q^2, \; q^2 < 0$,

$$t_0 = (M_{K^0} + M_{\pi^-})^2 \quad , \quad t_1 = (M_{K^0} - M_{\pi^-})^2 ; \tag{2.5}$$

$$d(t) = (M_{K^0}^2 - M_{\pi^-}^2) \, f_+(t) + t \, f_-(t) \tag{2.6}$$

is the scalar form factor of the $K^0\mu_3$ decay, with f_+ and f_- defined by:

$$\langle \pi^-(p') | V^\mu(0) | K^0(p) \rangle = (p+p')^\mu f_+(t) + (p-p')^\mu f_-(t) \; , \tag{2.7}$$

$$t = (p-p')^2 = q^2 .$$

The 3/2 in the r.h.s. of Eq. (2.4) is an isospin Clebsh Gordan coefficient.

Taking the second derivative with respect to q^2 makes $\psi''(q^2)$ a convergent quantity in QCD. We shall use its 2-loop expression in the $\overline{MS}$ renormalization scheme:[3,6]

$$\psi''(q^2) = \frac{3}{8\pi^2} \frac{\left(\bar{m}_s(Q^2) - \bar{m}_u(Q^2)\right)^2}{Q^2} \times$$
$$\Big\{ 1 - \frac{\bar{m}_s^2(Q^2) + \bar{m}_u^2(Q^2) + \left(\bar{m}_s(Q^2) + \bar{m}_u(Q^2)\right)^2}{Q^2} + O\left(\frac{m^2}{Q^2} \ln \frac{Q^2}{m^2}\right)$$
$$+ \frac{\bar{\alpha}_s(Q^2)}{\pi} \left(\frac{11}{3} + O\left(\frac{m^2}{Q^2} \ln \frac{Q^2}{m^2}\right)\right)$$
$$+ \frac{16\pi^2}{3} \frac{(m_u + \frac{1}{2} m_s)\langle \bar{s} s\rangle + (m_s + \frac{1}{2} m_u)\langle \bar{u} u\rangle}{Q^4}$$
$$+ \frac{2\pi}{3} \frac{\alpha_s \langle F_{\mu\nu} F^{\mu\nu}\rangle}{Q^4} + O\left(\frac{1}{Q^6}\right) \Big\} \quad (2.8)$$

where we have incorporated the non perturbative corrections up to the power Q^{-4}.

The $\bar{m}_{i(Q^2)}$ are the running QCD masses:

$$\bar{m}_i(Q^2) = \frac{\hat{m}_i}{\left(\frac{1}{2} \ln \frac{Q^2}{\Lambda^2}\right)^{-\gamma_1/\beta_1}} \left[1 - \frac{\gamma_1 \beta_2}{\beta_1^3} \frac{\ln \ln Q^2/\Lambda^2}{\frac{1}{2} \ln Q^2/\Lambda^2} + \frac{1}{\beta_1^2} \left(\gamma_2 - \frac{\gamma_1 \beta_2}{\beta_1} \right) \frac{1}{\frac{1}{2} \ln Q^2/\Lambda^2} \right] \quad (2.9)$$

The $\hat{m}_i$'s are the invariant masses under the renormalization group equations; β_1, β_2, γ_1, γ_2 are the first 2 coefficients of the expansion of the renormalizatón group functions $\beta(a_s)$ and $\gamma(a_s)$ respectively.

For 3 colors and 3 flavors we have [7a,b]:

$$\beta_1 = -\frac{9}{2} , \quad \gamma_1 = 2 , \quad \beta_2 = -8 , \quad \gamma_2 = \frac{91}{12} . \quad (2.10)$$

$a_s(Q^2)$ is the QCD running coupling constant:

$$\bar{\alpha}_s(Q^2) = \frac{2\pi}{-\beta_1 \ln Q^2/\Lambda^2} . \quad (2.11)$$

The non perturbative terms are calculated using the PCAC estimates:

$$(m_u + m_d)(\langle \bar{u} u\rangle + \langle \bar{d} d\rangle) = -2 f_\pi^2 m_\pi^2$$
$$(m_d + m_s)(\langle \bar{d} d\rangle + \langle \bar{s} s\rangle) = -2 f_K^2 m_K^2 , \quad (2.12)$$

and taking the SU(3) symmetry relation $<uu> = <dd> = <ss>$. For the term $a_s<FF>$ we use the recent corrected estimate:[8]

$$\alpha_s \langle F_{\mu\nu} F^{\mu\nu} \rangle = .1 \ (\mathrm{GeV})^4 . \quad (2.13)$$

The inequality (2.4) and a technique originally due to Okubo[9] is at the origin of the bounds that we get for the quark mass difference $\hat{m}_s - \hat{m}_u$. (See Ref. 5 for more details.) We use the conformal mapping:

$$\frac{1+z}{1-z} = \left(\frac{t - t_0}{t_0}\right)^{1/2} , \quad (2.14)$$

which projects the cut $[\lambda_0, +\infty[$ of d(t) into the unit circle in the complex z plane. It is a simple matter to incorporate as constraints the value of d(t) at any fixed point outside the cut and also its derivative at one or several of those same points. We shall take here as constraints:

- the value $d(0) = (M_{K^0}^2 - M_{\pi^-}^2)\, f_+(0)$
- the value $d(M_K^2)$ given by the Callan Treiman relation, .
- the slope $d'(0)$. at $t = 0$.

The final condition is given by the positivity of the determinant:[5]

$$\begin{vmatrix} \Psi''(q^2) & d(0)\varphi(0) & d(M_K^2)\,\varphi(z_K) & d(0)\varphi'(0)-4t_0\,d'(0)\,\varphi(0) \\ d(0)\,\varphi(0) & 1 & 1 & 0 \\ d(M_K^2)\,\varphi(z_K) & 1 & 1/(1-z_K^2) & z_K \\ d(0)\varphi'(0)-4t_0 d'(0)\varphi(0) & 0 & z_K & 1 \end{vmatrix}$$

(2.15)

where z_k means

$$z(M_K^2) = \frac{\sqrt{t_0 - M_{K^0}^2} - \sqrt{t_0}}{\sqrt{t_0 - M_{K^0}^2} + \sqrt{t_0}} . \qquad (2.16)$$

The function $\phi(z)$ is computed by standard technique and is in this case given by:

$$\varphi(z) = \frac{1}{4}\sqrt{\frac{3}{\pi}}\ \frac{1}{t_0}\ \frac{1+z}{1-z}\ \frac{\left(\sqrt{\frac{t_0 - t_1}{t_0}} + \frac{1+z}{1-z}\right)^{1/2}}{\left(\sqrt{\frac{t_0 + Q^2}{t_0}} + \frac{1+z}{1-z}\right)^{3}} .$$

(2.17)

The equation (2.15) may be rewritten as:

$$(\hat{m}_s - \hat{m}_u)^2 \geq F(Q^2, \hat{m}_s, \hat{m}_u) \times \left\{ \frac{1-z_K^2}{z_K^2}\left[d(0)\varphi(0) - d(M_K^2)\,\varphi(z_K) + z_K\left(d(0)\varphi'(0) - 4t_0 d'(0)\,\varphi(0)\right)\right]^2 + d^2(0)\,\varphi^2(0)\left[d(0)\varphi'(0) - 4t_0 d'(0)\varphi(0)\right]^2 \right\},$$

(2.18)

where the function $F(Q^2, \hat{m}_s, \hat{m}_u)$ is

$$F = \left(\frac{1}{2}\ln Q^2/\Lambda^2\right)^{-2\gamma_1/\beta_1} \left(1 - \frac{\gamma_1\beta_2}{\beta_1^3}\ \frac{\ln\ln Q^2/\Lambda^2}{\frac{1}{2}\ln Q^2/\Lambda^2} + \frac{1}{\beta_1^2}\left(\gamma_2 - \frac{\gamma_1\beta_2}{\beta_1}\right)\frac{1}{\frac{1}{2}\ln Q^2/\Lambda^2}\right)^{-1}$$

$$\times \left(1 - \frac{\hat{m}_s^2 + \hat{m}_u^2 + (\hat{m}_s + \hat{m}_u)^2}{Q^2\left(\frac{1}{2}\ln Q^2/\Lambda^2\right)^{-2\gamma_1/\beta_1}} + \frac{11}{3}\ \frac{\bar{\alpha}_s(Q^2)}{\pi} + \frac{16\pi^2}{3}\ \frac{(m_s + \frac{1}{2}m_u)\langle\bar{u}u\rangle + (m_u + \frac{1}{2}m_s)\langle\bar{s}s\rangle}{Q^4} + \frac{2\pi}{3}\ \frac{\alpha_s\langle FF\rangle}{Q^4}\right)^{-1}$$

(2.19)

From Eq. (2.18) the bound on $\hat{m}_s - \hat{m}_u$ is obtained by an iterative procedure similar to that of Ref. [3]: we neglect in Eq. (2.19) $\hat{m}_u$ with respect to $\hat{m}_s$, start with, $\hat{m}_s = 0$, then plug in the r.h.s. of Eq. (2.18) the successive values of $m_s - m_u$ so obtained. The procedure converges for small enough values of $\hat{m}_s - \hat{m}_u$ (The limit, given by the pole of F, is above 600 MeV for $Q^2 \geq 1\ \text{GeV}^2$ and so doesn't constitute a limitation of the method here).

III. PHENOMENOLOGICAL INPUTS

1. $f_+(0)$ and the Callan-Treiman relation

The ideal implementation of the method would undoubtedly rely on high precision measurements of d(t). As this is not yet the case, we shall supplement our lack of practical knowledge by constraints that one expects to be valid at a high accuracy. We shall take those constraints as fixed and not as varying parameters. They are the value of $f_+(0)$ and that of d(t) at $t = m_{K^0}^2$. The form factor $f_+(0)$ has been proved to deviate from its SU(3) value ($f_+(0) = 1$) only at second order in the SU(3) symmetry breaking parameter. $(O\varepsilon_8^2/\varepsilon_0)$ [10, 11a]. We shall use the most recent evaluation of the breaking given in Ref. [11b]:

$$f_+(0) - 1 = -\frac{3.57\,(M_{K^0}^2 - M_{\pi^-}^2)}{384\,\pi^2\, F_\pi^2\, M_{K^0}^2}, \tag{3.1}$$

which leads to

$$f_+(0) = .977 \quad . \tag{3.2}$$

This value is in good agreement with the experimental result of Ref. [12]. We shall take the Callan Treiman relation [13a] including m_π^2/m_K^2 corrections as given in Ref. [13b]:

$$d\,(M_{K^0}^2) = M_{K^0}^2\, \frac{F_K}{F_\pi} \left(1 - \frac{m_u + m_d}{m_d + m_s}\right) \tag{3.3}$$

In the SU(3) limit <uu> = <dd> = <ss>, the ratio $m_u + m_d/m_d + m_s$ may be extracted from current algebra Ward identities, which yield:

$$\frac{m_u + m_d}{m_d + m_s} = \frac{F_\pi^2\, M_{\pi^-}^2}{F_K^2\, M_{K^0}^2} \quad . \tag{3.4}$$

We estimate the ratio f_π/f_K via the experimental data on the decays $\pi \to \mu\nu$ and $K \to \mu\nu$. We obtain

$$\frac{F_K}{F_\pi} = 1.175 \quad , \tag{3.5}$$

and

$$d(M_{K^0}^2) = 1.11\; M_{K^0}^2 . \tag{3.6}$$

2. Values of the slope at $t = 0$:

The method employed only enables us to constrain the derivative of the product $d \times \phi(z)$, which means that we need the knowledge of both d and its derivative at the choosen point. $t = 0$ is particularly suitable because, up to now experimental data do not give any evidence for d(t) deviating from a linear behavior in the physical domain of t.

Following the usual parametrization:

$$d(t) = d(0)\left(1 + \frac{\lambda_0 t}{m_\pi^2}\right) \tag{3.7}$$

we shall consequently identify $d'(0)$ with $d(0)\,\lambda_0/m_\pi^2$ that is:

$$d'(0) = \left(M_{K^0}^2 - M_{\pi^-}^2\right) f_+(0)\, \lambda_0\, \frac{1}{M_{\pi^-}^2} \tag{3.8}$$

We may compare this assumption with the results given by the phase shift analysis of Ref. [3,4] (d(0) is accordingly expected to be a convex function). It gives $d'(0)/d(0) = .96\ \mathrm{GeV}^{-2}$, corresponding to $\lambda_0 \approx .022$. So, in this approach, the deviation from linearity in the physical region is not expected to exceed 15%, which is smaller than the smallest experimental uncertainty on λ_0.

Let us summarize the experimental situation: the higher statistics has been obtained in Ref. [15] which gives $\lambda_0 = .019 \pm 0.006$. All the posterior data give bigger values, which may reach $\lambda_0 = .046$ [16]. (However the statistics is often rather poor.) The most recent data are those of Ref. [17], with $\lambda_0 = .034 \pm .007$. Several older experiments had given a negative slope, which restore the present world average to

$$\lambda_0 = .028 \pm .006 \quad {}^{14},$$

and the fitted value to $\lambda_0 = .025 \pm .006$.[14] Though a negative slope at the origin seems now out of the question and is in contradiction with the analyis of Refs. [3, 4], we have no reason *a priori* to disregard those results. We shall subsequenty give our bounds for the four choices: λ_0 best statistics, λ_0 fitted, λ_0 average and λ_0 most recent.

3. The choice of the QCD scale Q^2

All QCD information lies in the function F given in Eq. (2.19). The bound on $\hat{m}_s - \hat{m}_u$ behaves like $F^{1/2}$. Let us study separately the strict perturbative series:

$$G(Q^2, m) = 1 - 2\,\frac{\bar{m}_u^2 + \bar{m}_s^2 + \bar{m}_u \bar{m}_s}{Q^2} + \frac{11}{3}\frac{\bar{\alpha}_s}{\pi} \qquad (3.9)$$
$$+ \frac{16\pi^2}{3}\,\frac{(m_u + \tfrac{1}{2} m_s)\langle \bar{s}s\rangle + (m_s + \tfrac{1}{2} m_u)\langle \bar{u}u\rangle}{Q^4} + \frac{2\pi}{3}\,\frac{\langle \alpha_s FF\rangle}{Q^4},$$

and slightly anticipate our results to give an order of magnitude of the corrections: taking $\hat{m}_s \sim .25$ GeV, $\hat{m}_u \sim .007$ GeV and using the Eqs. (2, 9-13) we obtain the parametrization:

$$G(Q^2) = 1 - \frac{.129}{\left(\tfrac{1}{2}\ln Q^2/\Lambda^2\right)^{-2\gamma_1/\beta_1} Q^2} + \frac{11}{3}\,\frac{2}{-\beta_1 \ln Q^2/\Lambda^2} - 8\pi^2\,\frac{f_K^2 M_K^2}{Q^4} + \frac{2\pi}{3}\,\frac{.1}{Q^4}. \qquad (3.10)$$

Taking $\Lambda = 150$ MeV, we obtain the set of corrections displayed in Table 1. The series in α_s is evidently poorly convergent at this order, and the relatively low (~ 30%) global correction in G at low Q's is due to some fortunate cancellations that we reasonably cannot advocate to trust the final result. The non perturbative contributions are rapidly damped as expected, and take reasonable values at $Q > 1.4$ GeV. At this value the mass corrections are also small and the series is mainly driven by its α_s expansion. We shall take the conservative attitude not to trust the results below $Q^2 \approx 2\ \mathrm{GeV}^2$, at which scale the QCD corrections to $\hat{m}_s - \hat{m}_u$ amount to $\approx 13\%$, which is within an acceptable range.

IV. RESULTS AND DISCUSSION

The bounds on $\hat{m}_s - \hat{m}_u$ as functions of Q^2, for $\Lambda = 100$, 150 and 200 MeV and for the 4 choices of λ_0 exposed in the last section are shown on Fig. 1. Taking $Q \approx 1.4$ GeV, the present experimental data on λ_0 allow a lower bound ranging from 160 to 370 MeV. The present experimental trend favors the bigger results, while the data with the best statistics (and the phase shift analysis of Ref. [3.4]) favors the lower ones.

If the present trend persists, we may have two attitudes: either take for granted a big mass difference or question our theoretical inputs. In our opinion the value of $f_+(0)$ seems reliable; It is the Callan-Treiman relation that could be the most subject to caution. Indeed, the analysis of Ref. [3.4], (though not itself exempt of uncertainly) gives, through an Omnes relation and for $f_+(0)$ given by Eq. (3.2), $d(M_{K_0}{}^2) = 1.20\ M_{K_0}{}^2$ instead of our input of $1.11\ M_{K_0}{}^2$. This small (= 8%) variation has the effect of decreasing our bounds by around 50 MeV (for $Q = 1.4$ GeV and $\Lambda = 150$ MeV). This rather high sensitivity together with present experimental uncertainty keeps us from giving very precise results at the moment. As the Callan Treiman point lies outside the physical region and consequently outside experimental range, it is through more precise measurements of the slope of the form factor that we could strengthen the constraint on $\hat{m}_s - \hat{m}_u$ quark mass difference. The sensitivity of this quantity to experimental data makes such measurements very desired.

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Table 1.

Q GeV	mass	α_s	$m\langle\psi\psi\rangle$	$\alpha_s\langle FF\rangle$	G	$1/\sqrt{G}$	$\alpha(m_s - m_u)$ inf
.8	-.1275	.4868	-.57	.5113	1.3	1	-.1229
1.0	$-7.30 \, 10^{-2}$	.4295	-.2338	.2094	1.332	1	-.1335
1.2	$-4.67 \, 10^{-2}$	.3918	-.1128	.1010	1.333	1	-.1339
1.4	$-3.22 \, 10^{-2}$	.3648	$-6.09 \, 10^{-2}$	$5.45 \, 10^{-2}$	1.326	1	-.132
1.6	$-2.34 \, 10^{-2}$	.3442	$-3.57 \, 10^{-2}$	$3.20 \, 10^{-2}$	1.317	1	-.1286
1.8	$-1.77 \, 10^{-2}$	.3279	$-2.23 \, 10^{-2}$	$2. \, 10^{-2}$	1.308	1	-.1256
2.0	$-1.38 \, 10^{-2}$	.3146	$-1.46 \, 10^{-2}$	$1.30 \, 10^{-2}$	1.30	1	-.1229

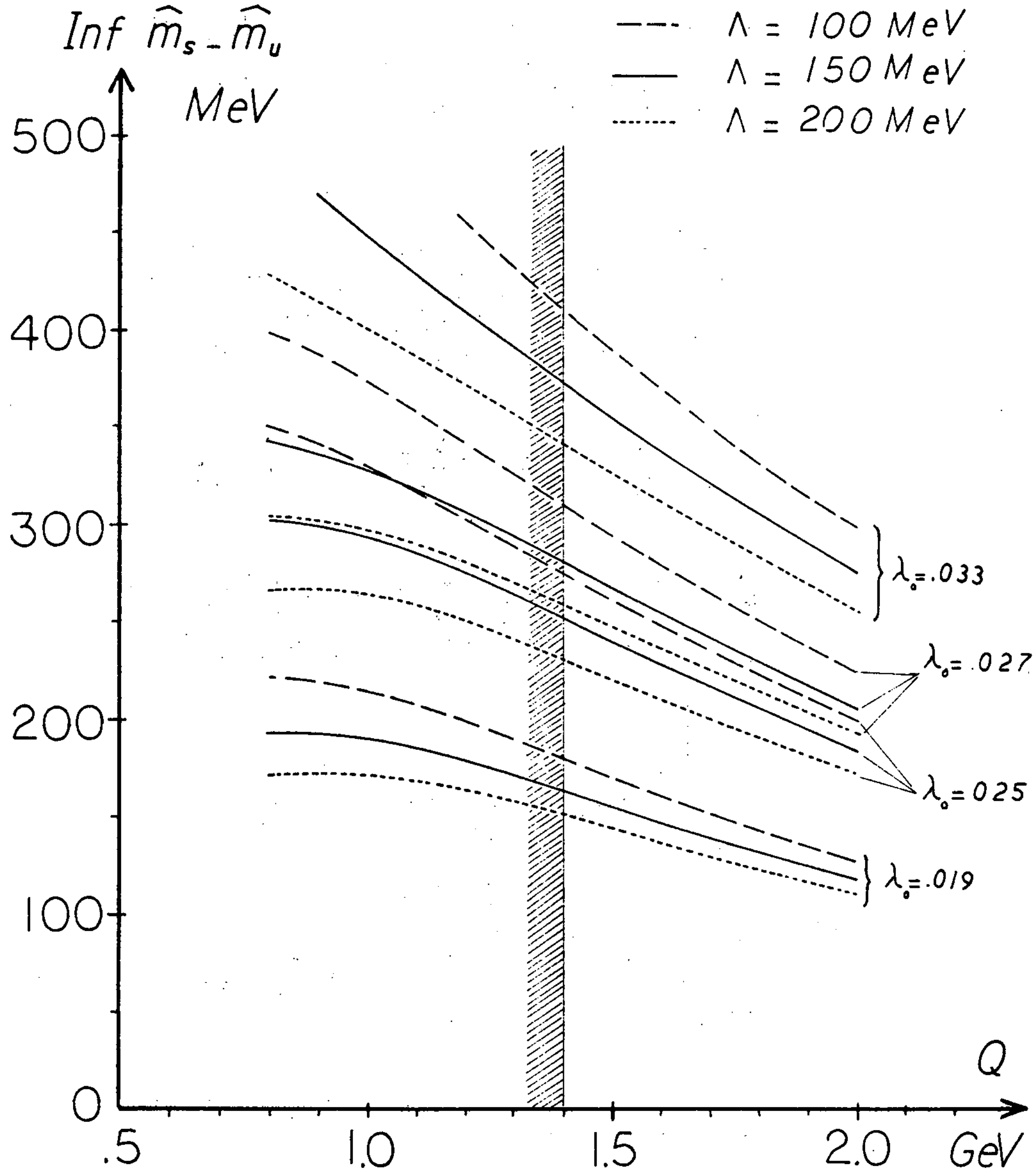

Fig 1: lower bound for $\widehat{m}_s - \widehat{m}_u$ as a function of Q^2

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