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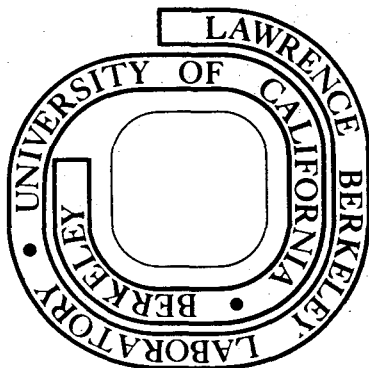
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A STATISTICAL-WEIGHT INTERPRETATION FOR THE $1/N^2$ CONVERGENCE FACTORS

OF THE TOPOLOGICAL EXPANSION[*]

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ABSTRACT

In a world with exact SU_N symmetry Veneziano has proposed a
topological expansion in terms of the small dimensionless parameter
$1/N^2$. We relate this convergence factor to the density of hadronic
states. The weakness of higher order corrections (i.e. $1/N^2$)
is interpreted as reflecting the small proportion of states that can
flow through a "cylinder." States that communicate with the cylinder
may be strongly affected thereby, but the number of such states is
relatively small.

I. INTRODUCTION

With SU_N internal symmetry, successive components of
Veneziano's topological expansion in orientable two-dimensional
surfaces have been shown to carry factors $(1/N^2)^h$, where h is the
number of handles.[1] A mnemonic for this rule is to visualize a handle
as a cylinder whose two ends are internally attached to the two-
dimensional surface and to characterize the coupling of each cylinder
end as carrying a factor $1/N$. In the first part of this paper we
point out that this $1/N^2$ cylinder coupling reflects nothing more
than the statement that SU_N-singlets are the only representations
allowed to "flow" along the cylinder. That is, the $1/N^2$ convergence
of the topological expansion is a consequence of the low relative
probability of states that communicate with the cylinder (or with
handles). Such a line of thought is helpful when contemplating the
real hadronic world where SU_N symmetry is badly broken for $N > 2$
but where the topological expansion turns out to possess great utility.[2]
Such an interpretation also eliminates any need for field-theoretic
models to motivate $1/N^2$ convergence of the S-matrix expansion.

In Section II we show how the weakness of the cylinder coupling
$(g/\sqrt{N})$ arises from the SU_N singlet nature of the cylinder. As
illustrations of the subtleties which arise from the singlet character
of the cylinder we cite in Sections III and IV examples where the
cylinder effect is large and prominent. In Section III we talk about
the pomeron, while in Section IV we examine a puzzling physical
question relating to cylinder weakness and charge conjugation (or
signature). The topological distinction between simple planar sur-
faces and cylinders or handles rests on the possibility of "twisting"

the two-dimensional surface to reverse its orientation. Since passage
from one orientation to the other is related to charge conjugation and
signature, it may be wondered how twists under any circumstances can
be regarded as "weak"? We shall discuss this question with a concrete
example, hoping to clarify misunderstandings about the physical
significance of the topological expansion--misunderstandings that have
plagued our own thinking and that may have troubled others as well.

We are then led in Section V to conjecture that, independent
of internal symmetry, there will occur "$1/N^2$ convergence" of the
typological expansion to the extent that two conditions are satisfied:
(a) Within planar unitarity sums over intermediates states, there are
significant contributions from a variety of different combinations of
internal quantum numbers. (b) Several different planar states are
mutually coupled to a substantial degree by the cylinder. We argue
that such conditions are better and better satisfied as t increases
negatively.

In discussing the "$1/N^2$ weakness" of cylinders and handles
there is risk of confusion with the concept of "asymptotic planarity"[3]
--the N-independent weakening of the cylinder with increasing mass-
squared (t) flowing along the cylinder. The latter weakening results
from the phase factor and consequent sign oscillation associated with
surface twists.[4,5] The present paper deals only with that aspect of
cylinder weakness stemming from the largness of N at any value of t.
The cylinder weakness of concern here is present at $t = 0$, being,
for example, responsible for the relative weakness of $t = 0$ pomeron
couplings. There is no connection between $1/N^2$ cylinder weakness
and the Okubo-Zweig-Iizuka rule, which becomes useful at large t as

a consequence of asymptotic planarity.

The analysis presented in the body of this paper is essentially
perturbative in nature. We make a first step toward removing this
perturbative restriction on our reasoning by presenting in our
Appendix an exact unitarity equation for the cylinder. The planar
⊗ planar couplings which are the basis of our $1/N$ counting in
Sections II-V, appear in the unitarity product, so we expect the
unitarity equation to provide nonperturbative justification for our
arguments. Such justification is in fact achieved for the results
of Section IV and is included in our Appendix.

II. THE INTRINSIC WEAKNESS OF SINGLET COUPLINGS

It is well-known that with SU_N symmetry the cylinder
communicates along its axis only with singlet states. One may say,
correspondingly, that handles "carry" only singlets. We wish to point
out how this fact immediately suffices to explain the $1/N^2$ weak
coupling of cylinders and handles. Our reasoning will employ dual
diagrams whose boundaries carry quark indices.

Consider a planar vertex with strength g independent of quark
index, as indicated in Fig.1. What is the residue of a singlet pole
in a four-line elastic planar connected part such as that of Fig. 2?
On the right side of Fig. 2 we show the "stretched" form of the surface
that focuses attention on horizontal-channel poles. The complete
planar pole residue is evidently g^2. To project out the singlet
component we recognize that a singlet (in the horizontal channel)
corresponds to the "wave function" of Fig. 3, which must be "emitted"
at one vertex and "absorbed" at the other. Thus a horizontal-channel
pole of the planar amplitude of Fig. 2, when projected onto the

singlet, has a residue g^2/N . The full planar residue is g^2 because of the degenerate presence of higher dimensional representations in addition to the singlet. The factor $1/N$ reflects the low statistical weight of the singlet. According to planar bootstrap requirements, to be discussed further below, g^2N is of order unity,[6] so singlet couplings are already at the planar level of order $1/N^2$.

Let us look next at a two-particle discontinuity in the horizontal channel, where a cylinder contribution will arise. The planar part of the discontinuity is depicted in Fig. 4, where the symbols (+) and (-) indicate that the separate factors are to be evaluated on opposite sides of the cut. We shall henceforth drop (+, -) symbols as irrelevant to the N dependence. The important aspect of Fig. 4 for our purposes is the occurrence of several different but equivalent intermediate states. The contribution of all possible intermediaries when weighted by the appropriate SU_N Clebsch Gordon coefficients is to provide an extra factor of N. This is readily reproduced in the quark diagram mnemonic by the presence of N different quark lines giving rise to all the intermediary particles. Henceforth we shall count intermediaries by the different number of quark lines. If each member of the product is of order g^2 , the two-particle discontinuity is of order g^4N --the same order of magnitude as the amplitude itself if $g^2N \sim 1$. The simplest "derivation" of the latter condition, in fact, is to observe that for Regge behavior amplitudes and discontinuities must be of the same order.

The cylinder may be said to arise because the planar S matrix fails to satisfy unitarity. Planar discontinuity products such as that of Fig. 4 correspond to matching permutations for the two product

members (strong ordering) but since the full planar connected part is a sum over permutations of external lines, unitarity requires bilinear products with differing permutations for the two members (see Appendix). An example of a nonmatching product is shown in Fig. 5, which may be expressed with twist notation as Fig. 6. Here we have a typical cylinder configuration (with horizontal cylinder axis). Either in the form of Fig. 5 or that of Fig. 6, the essential feature of this cylinder product is the presence of only one intermediate state. For the other N-1 intermediate states, strong ordering is unavoidable. The cylinder part of the discontinuity is thus of order g^4 , smaller than the planar discontinuity by a factor $1/N$. But, if we project out of the planar discontinuity its singlet component we shall find a factor $1/N$ --just as in the pole-residue projection. In contrast such a projection of the cylinder discontinuity gives a factor 1 because the cylinder is purely singlet in character. Within the singlet component the cylinder discontinuity is thus of the same order of magnitude as the planar discontinuity, both being of the same order as the singlet amplitude itself.

The $1/N^2$ cylinder weakness is therefore to be understood as an intrinsic weakness of singlet couplings. Within a singlet amplitude the contribution of the cylinder is not $1/N^2$ weak; it is equally as important as the planar component in the sense of a $1/N^2$ expansion.

Before leaving our example of the two-particle discontinuity it may be helpful to see explicitly why the singlet projection of the cylinder does not lead to a factor $1/N$. The reason is that cylinder discontinuites arise not only for reactions of the type of Fig. 2 but also for reactions forbidden at the planar level. That is, with SU_N

symmetry the cylinder discontinuity depicted in Fig. 7 for any com-
bination of j and j' is equal to that of Fig. 6. In the singlet
projection one must sum over these additional amplitudes, leading to
a factor N that compensates the $1/N$ in the singlet wave function
normalization.

It may trouble readers that "full strength" of the cylinder
in planar amplitudes depends on reactions of the type of Fig. 7 with
j ≠ j' --forbidden by the Okubo-Zweig-Iizuka (OZI) rule. It is
necessary to appreciate that the phenomenological validity of the
latter rule is not a consequence of $1/N^2$ cylinder weakness but
rather of SU_N-symmetry breaking in combination with asymptotic
planarity--the latter term describing the suppression of nonplanar
effects by the alternation in sign of twisted-link pole residues.

III. POMERON PROPERTIES AS AN ILLUSTRATION

The best-known physical manifestation of the cylinder (and of
handles) is through the bare pomeron. The $1/N^2$ weakness of bare
pomeron couplings is of great practical importance, being reflected
in the small magnitude of total cross sections at high energy and the
relative weakness of pomeron cuts. The usefulness for high-energy
multiple production of the concept of short-range order in rapidity
correlations depends crucially on pomeron weakness. What we are
emphasizing here is that all these physical effects become understan-
dable as soon as the bare pomeron is identified as a singlet in a
spectrum where higher multiplets occur. This explanation of pomeron
weakness was proposed before development of the topological expansion
by Abarbanel et al.,[7] on the basis of a multiperipheral model.

In previous publications we have emphasized the usefulness of
recognizing the pomeron to be a planar f trajectory that has been
displaced upward and mixed with the f' by the effect of the cylinder.
It is automatic in this view that pomeron couplings should be of the
same order as planar f and f' couplings. All are of order $1/N^2$.
The trajectory shift at t = 0 on the other hand is of order unity
since this is an effect within the singlet amplitude. Also, the
degree of mixing of planar f and f' depends not on $1/N^2$ but on
the ratio of pomeron shift to f-f' splitting. The asymptotic
planarity mechanism, unrelated to $1/N^2$, reduces the shift and the
mixing as t increases.

IV. THE ρ-ω WIDTH PARADOX: A LARGE ROLE FOR THE CYLINDER

Let us turn now to an aspect of the cylinder that involves
charge conjugation and signature. At the planar level, not including
cylinder effects, trajectories are grouped into degenerate families.
With SU_2 symmetry (equivalent p and n quark indices) there are
four leading degenerate vacuum-communicating trajectories-- p$\bar{p}$ or
n$\bar{n}$ boundaries with either even or odd charge conjugation symmetry.
Although by taking suitable linear combinations of p$\bar{p}$ and n$\bar{n}$ one
may form I = 0 and I = 1 trajectories, at the planar level the
isosinglet is degenerate with the isotriplet.

Consider the two odd-charge-conjugation planar trajectories
that we expect to associate with ω (I = 0) and ρ (I = 1).
Degeneracy means not only that the real parts of ω and ρ masses
should be equal but also the imaginary parts or widths. Since the
cylinder does not communicate with I = 1, the physical ρ width of
140 MeV, dominated by ππ channels, should be close to the simple

planar approximation. It follows that the planar $I = 0$ ω width is also predominently two-pion (and ~ 140 MeV). But the ω, with odd G-parity does not communicate with the $\pi\pi$ channel! At the planar level $\pi\eta$ and $\eta\eta$ channels are degenerate with $\pi\pi$, but $\pi\eta$ has the wrong isospin and $\eta\eta$ the wrong G-parity to communicate with ω. Lacking the $\pi\pi$ mode of decay the actual ω width is only 10 MeV.

Resolution of this dilemma is achieved through the cylinder terms in the $I = 0$ odd charge-conjugation discontinuity, which precisely cancel the $\pi\pi$ planar contribution. In other words, G parity is not consistent with two-particle discontinuities at the planar level but only with the cylinder-planar combination.

The planar amplitude itself forbids $\omega \to 2\pi$ physical transitions inasmuch as the two planar terms shown in Fig. 8, when superposed in a manner appropriate to $I = 0$ and odd charge conjugation, cancel each other. But among the bilinear terms associated with 2π intermediate states in a unitarity product there appear not only planar forms such as shown in Fig. 9, but an equal number of cylinder forms such as that in Fig. 10. For odd G parity it is the sum of simply-planar and cylinder discontinuity products that vanishes. The charged pion, in other words, is the superposition indicated in Fig.11. If only simply-planar products are included, odd and even G parity have the same 2π discontinuity.

The foregoing argument is incomplete in that we have used a simply-planar representation for the ω. This defect is remedied in the Appendix where t-channel unitarity equations for the cylinder are discussed. The ρ-ω width difference illustrates that within individual singlet states the cylinder may not be regarded as weak.

Why is the real part of the ρ-ω mass difference so small? Firstly, it is probably accidental that the real-part difference is much smaller than the width difference. Secondly, a mechanism to explain a ρ-ω mass difference of the order of 40 MeV has been presented in Ref. (4) within a general discussion of asymptotic planarity. This mechanism is unrelated to $1/N$ factors.

V. WHAT IS N^2 IN THE ABSENCE OF SU_N SYMMETRY?

How are such parameters as pomeron couplings determined in the real world where, although SU_2 symmetry is accurate, already SU_3 is badly broken? This question must be answerable within the topological expansion, which does not require internal symmetry for its formulation. Although clean understanding is so far lacking, the considerations of Section II motivate the speculations that follow.

Corresponding to an average planar coupling g^2 we conjecture that the cylinder coupling will be g^2/N_{eff}^{cyl}, where N_{eff}^{cyl} is in general the average number of planar trajectories strongly mixed by the cylinder. It is plausible that the trajectories to be counted are those within the J interval "spanned" by cylinder mixing--this span being of the order of magnitude of the (N-independent) trajectory shift induced by the cylinder. Trajectories whose separations are much larger than the cylinder span will not be appreciably mixed.

At $t = 0$ one expects that N_{eff}^{cyl} is greater than 2 but substantially below 3, because the cylinder shift of about 0.4 in J is large compared to the gap between planar $n\bar{n}$ and $p\bar{p}$ trajectories but of the same order as the spacing between these two and the $\lambda\bar{\lambda}$ trajectory. Trajectories like ϕ are thus only

partially mixed by the cylinder with trajectories like ω .

These considerations can be made concrete by referring to the simple model of Ref. (3) and the detailed fits for the parameters of this model in Ref. (8). It was found[8] that the pomeron was shifted upward from the planar value of 0.58 by 0.38. This is to be compared to a $\lambda\bar{\lambda}$ gap of 0.35 from the $p\bar{p}$ and $n\bar{n}$ trajectories. The trajectory shifts determine mixing parameters, and it was found that at $t = 0$ the ϕ, ω mixing angle is $\sim -34^{\circ}$, where full mixing would correspond to -55°.

We can calculate an N_{eff}^{cyl} in this model by considering the triple cylinder coupling which will be $g/\sqrt{N_{eff}^{cyl}}$ compared to a triple planar coupling of g . In terms of θ , the f, f' mixing angle, the triple cylinder coupling is

$$g/\sqrt{N_{eff}^{cyl}} = g\left(\frac{\cos^3 \theta}{\sqrt{2}} + \sin^3 \theta\right)$$

implying on $N_{eff}^{cyl} = 2.56$ for $\theta = 20.3^{\circ}$.

Such an effective N depends on t to the extent that the cylinder span varies with t . According to asymptotic planarity, the cylinder shift is monotonic decreasing; so if planar trajectory separations are roughly constant, we expect N_{eff}^{cyl} to diminish as t grows. The cylinder "quenching interval," $t_c \approx 0.5$ GeV^2, estimated in Ref. (4), implies $N_{eff}^{cyl} \approx 2$ for $t \gtrsim 0.5$ GeV^2 because here the cylinder span is much less than the separation between $\lambda\bar{\lambda}$ and $(p\bar{p}, n\bar{n})$. On the other hand, N_{eff}^{cyl} should increase to values near 3 as t decreases to values below -0.5 GeV^2, because here the cylinder span has become large in comparison to SU_3 symmetry breaking. It is plausible that N_{eff}^{cyl} will grow further as t

becomes more negative and the cylinder span expands to include additional degrees of freedom; at some stage the ψ and even lower trajectories may begin to be significant. If there exist Rosner-type trajectories[9]--associated with baryon number--these may cause N_{eff}^{cyl} to increase beyond 3 already at modestly negative values of t .

Should the foregoing conjecture prove correct, pomeron coupling will progressively weaken as t grows in the negative direction and the pomeron "wave function" absorbs more and more hadronic degrees of freedom. Such behavior is a curious counterpart to asymptotic planarity--which corresponds to a weakening of cylinder influence as t grows positively. The distinction between these two different trends toward simplicity--one stemming from an increasing number of active internal hadronic degrees of freedom and the other from sign alternation of pole residues in twisted links--requires a distinction between pomeron and cylinder. Cylinder weakening at large positive t does not imply pomeron weakening; the pomeron asymptotically becomes more and more nearly a planar trajectory (the f) and its coupling does not diminish. At large negative t , in contrast, the cylinder influence might be said to grow in that it mixes together more and more planar trajectories; the result is a weakening of pomeron coupling.

Although in both limits--large positive t and large negative t --similarities may be recognized to the field-theoretical notion of asymptotic freedom (associating cylinders somehow with gluons), the statements are different. A profound distinction, not to be forgotten, is that the planar limit of the topological expansion does not correspond to weak coupling.

We have proposed the replacement of $1/N^2$ by g^2/N_{eff}^{cyl} , but there remains the question of the planar coupling g^2. What magnitude do we expect for g^2 in the absence of SU_N symmetry? Once again it is useful to think of an effective N , but this time we want to count the number of important intermediate states that contribute to planar discontinuities. The bootstrap condition that a discontinuity is of the same order of magnitude as the amplitude then leads to the requirement that $g^2 N_{eff}^{plan}$ be of the order unity. Combining the two effective N's , $1/N^2$ becomes $1/N_{eff}^{cyl} N_{eff}^{plan}$.

In Ref. (10) a multiperipheral model was used to estimate that $N_{eff}^{plan} \approx 2.3$ at $t = 0$, roughly the magnitude we expect there for N_{eff}^{cyl} . Given the positive definite structure of planar discontinuity formulas (no alternation of sign) it is unlikely that N_{eff}^{plan} should ever be less than 2 . Although planar bootstrap models are as yet insufficiently developed to provide guidance as to the t dependence of N_{eff}^{plan} , one notes the experimental fact that at increasing transverse momenta an increasing variety of comparably-probable particles is observed. It seems plausible that as t grows negatively there will be a corresponding increase in N_{eff}^{plan} . It might even turn out--from a mechanism as yet unrecognized--that at all t , $N_{eff}^{plan} \approx N_{eff}^{cyl}$.

SUMMARY

We have pointed out how the $1/N^2$ convergence factors of the topological expansion reflect the statistical weight of cylinder communicating states. The $1/N^2$ weakness of cylinder couplings corresponds to the small proportion of states allowed to "flow" through

the cylinder. For $t \lesssim 0$ the cylinder is not weak with respect to states with which it communicates and indeed produces large shifts and mixings. In the absence of exact SU_N symmetry the number of different planar states substantially mixed by the cylinder determines an effective N which, according to asymptotic planarity, is a monotonically decreasing function of t . The cylinder or handle coupling is then of order g^2/N_{eff}^{cyl} with g^2 a typical planar coupling. With N_{eff}^{plan} determined by the average number of intermediate states in planar discontinuity products such that $g^2 N_{eff}^{plan} \sim 1$ the convergence factor for the typological expansion is of order $1/N_{eff}^{cyl} N_{eff}^{plan}$. While this is not an especially small number (e.g. $\sim 1/6$) it is sufficiently small to justify treating the topological expansion as an expansion in powers of $1/N_{eff}^{cyl} N_{eff}^{plan}$.[11]

APPENDIX: UNITARITY FOR THE CYLINDER

It has recently been shown by Veneziano[12] that for two-particle channels a particular combination of cylinder and planar amplitudes satisfies a simple t-channel unitarity equation. We generalize his results to multiparticle unitarity and discuss some implications of these equations.

If we denote by $C_{a_1\cdots a_k; b_1\cdots b_j}$ the cylinder amplitude for $a_1\cdots a_k \to b_1\cdots b_j$ shown in Fig. 12 we can define the "full" cylinder amplitude $\bar{C}$ as

$$\bar{C}_{a_1\cdots a_k; b_1\cdots b_j} \equiv C_{a_1\cdots a_k; b_1\cdots b_j}$$

$$+ \sum_{P[a_1\cdots a_k]} \sum_{P[b_1\cdots b_j]} P_{a_1\cdots a_k; b_1\cdots b_j} .$$

(A.1)

$P_{a_1\cdots a_k; b_1\cdots b_j}$ is the planar amplitude for $a_1\cdots a_k \to b_1\cdots b_j$ (see Fig. 13) and $P[\cdots]$ implies that we should sum over all possible cyclic permutations of the objects in [ ]. The unitarity equation for $\bar{C}$ then reads

$$\text{Disc}_t \ \bar{C}_{a_1\cdots a_k; b_1\cdots b_j} = \sum_{n=2}^{\infty} \bar{C}_{a_1\cdots a_k; 1\cdots n} \otimes \bar{C}^*_{1\cdots n; b_1\cdots b_j} .$$

(A.2)

A typical contribution from $P \otimes P^*$ to the unitarity sum for $\bar{C}_{ab;cd}$ is displayed in Fig. 14.

The unitarity equation (A.2) makes the discussion of the $\pi\pi$ contribution to the discontinuity of the ω pole particularly transparent. If we project (A.2) onto the ω pole and restrict ourselves to the two-particle intermediate state, the cyclic permutations of the intermediate particles will include the anticyclic permutation for $\omega \to ab$ (i.e. $\omega \to ba$). In general the restoration of G parity depends on summing over cyclic plus anticyclic orderings of the planar diagrams.[13] Thus for all two-particle (or quasi two-particle) intermediate states the full cylinder, since it includes all permutations required by unitarity, will respect G parity. For multiparticle intermediate states, permutations other than cyclic will be necessary to restore the G-parity constraints. This will lead to more complex topologies. In general, to respect G parity at the n-particle level, we have to consider topologies with n-2 handles.

The reader may also verify from the prescription (A.2) that the simple counting exercises undertaken in Section II remain valid for multiparticle contributions to the cylinder.
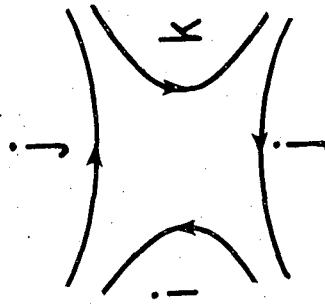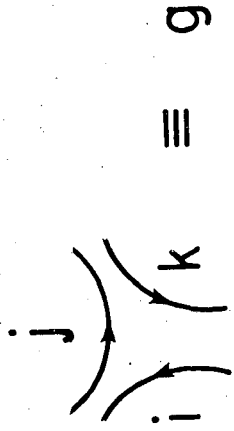
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FIGURE CAPTIONS

1. A planar vertex.

2. A planar elastic amplitude.

3. The singlet "wave function" corresponding to a planar pole.

4. A two-particle planar discontinuity product.

5. A cylinder term in a two-particle discontinuity product.

6. The cylinder term of Fig. 5 expressed in twist notation.

7. The off-diagonal cylinder discontinuity corresponding to Fig. 6.

8. Planar vertices involved in the transitions $\omega \to 2\pi$ and $\rho \to 2\pi$.

9. A planar contribution to the width of the ρ and the ω.

10. A cylinder contribution to the width of the ρ and the ω.

11. The superposition of planar poles corresponding to the charged pion.

12. A general cylinder amplitude.

13. A general planar amplitude.

14. A planar $\otimes$ planar contribution to the discontinuity of the cylinder.

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Figs. 1 and 2

$$\frac{1}{\sqrt{N}} \left\{ \underset{1}{\overset{1}{\longrightarrow}} + \underset{2}{\overset{2}{\longrightarrow}} + \underset{3}{\overset{3}{\longrightarrow}} + \cdots \underset{N}{\overset{N}{\longrightarrow}} \right\}$$

$$\sum_{\ell} \quad i \,\big)\, (+) \,\big(\, \ell \qquad \ell \,\big)\, (-) \,\big(\, k$$

Figs. 3 and 4

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Figs. 5 and 6

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Figs. 7 and 8

Figs. 9 and 10.

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Figs. 11 and 12

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Figs. 13 and 14

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