When does ignorance make us smart? Additional factors guiding heuristic inference.

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Abstract

"Fast & frugal" heuristics represent an appealing way of implementing bounded rationality and decision-making under pressure. The recognition heuristic is the simplest and most fundamental of these heuristics. Simulation and experimental studies have shown that this ignorance-driven heuristic inference can prove superior to knowledge based inference (Borges, Goldstein, Ortman & Gigerenzer, 1999; Goldstein & Gigerenzer, 2002) and have shown how the heuristic could develop from ACT-R's forgetting function (Schooler & Hertwig, 2005). Mathematical analyses also demonstrate that, under certain conditions, a "less-is-more effect" will always occur (Goldstein & Gigerenzer, 2002). The further analyses presented in this paper show, however, that these conditions may constitute a special case and that the less-is-more effect in decision-making is subject to the moderating influence of the number of options to be considered and the framing of the question.

The Less-Is-More Effect.

An interesting and counter-intuitive finding in decision-making research is the discovery that, under certain circumstances, individuals with less knowledge make more accurate judgments than those with greater (but still imperfect) knowledge. For example, 62% of American students tested could correctly state that San Diego has a higher population than San Antonio, but 100% of German students tested could do so (Goldstein & Gigerenzer, 1999). The argument here is that the American students were forced to consider multiple different, often unreliable, cues to size in making their judgment, whereas the German students simply needed to consider whether they had ever heard of the city: a more reliable size cue. In this case *Recognition Validity* (RV) exceeded *Knowledge Validity* (KV). A further study went on to examine the performance, over a period of time, of a recognition-based portfolio of shares with portfolios chosen on the basis of other knowledge. In this study, relative ignorance proved a better tool for playing the stock-market in a bull-market situation than did the other methods investigated (Borges, Goldstein, Ortmann & Gigerenzer, 1999, but see Boyd (2001) for a failure to replicate in a bear market). These examples show how an apparent use of recognition, via a recognition heuristic, was useful in practice. Other studies have queried the psychological status of the recognition heuristic (McCloy & Beaman, 2004; Newell & Shanks, 2004; Oppenheimer, 2003), here we are concerned with the in-principle usefulness of recognition as a decision-making criterion.

In Goldstein & Gigerenzer's (2002) study, they made use of the "cities task" as the basis for both their theoretical speculations and a test-bed for their empirical results. The cities task has been described as a "drosophila", or ideal research environment enabling the analysis of satisficing algorithms in a well-understood environment (Gigerenzer & Goldstein, 1996, p.651). Subsequently, Schooler & Hertwig (2005) demonstrated how the forgetting rate within the ACT-R cognitive architecture (Anderson & Lebiere, 1998) could give rise to the use of the recognition heuristic and, consequently, near-optimal performance on this task. A rational analysis of forgetting would therefore seem to encourage the formulation and use of a recognition heuristic. This conclusion is further strengthened by Goldstein and Gigerenzer's simulation of a "less-is-more" effect on the city task. The cities task requires participants to decide, given two alternatives, which city has the higher population. The recognition heuristic states:

If one of two objects is recognized and the other is not, then infer that the recognized object has the higher value (Goldstein & Gigerenzer, 1999, p. 41).

The 2-alternative forced choice (2AFC) cities task is used as a test-bed because it "is an elementary case to which many problems of greater complexity (multiple choice, for instance) are reducible" (Goldstein & Gigerenzer, 1999, p. 41). The cities task is thus seen as a representative of the set of decisions that involve selecting a subset of objects from a larger set. In this task, recognition works because the probability of recognition is influenced by a mediator variable which itself reflects the "real" but inaccessible criterion, as shown in Figure 1.

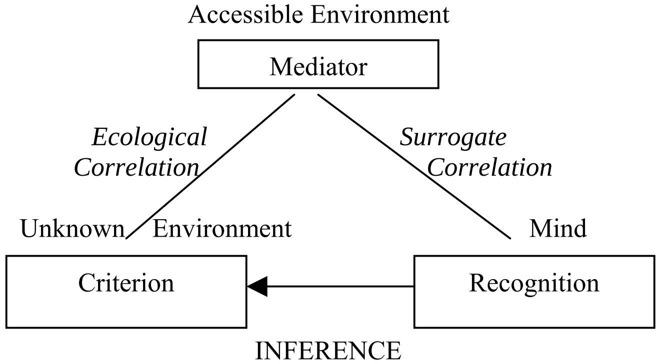

Figure 1: The relationship between criteria, mediating variable and recognition probability.

As a practical example, the mediating variable for many magnitude related choices, such as the cities task, might be the number of times the city has appeared in a newspaper report. This would correlate with city size (ecological correlation) and also influence the probability that the city name is recognized (surrogate correlation). Goldstein and Gigerenzer (2002) also showed how a degree of ignorance (recognizing only one of the two cities) could lead to superior performance on the cities task than knowledge of both cities. Mathematically, the demonstration goes as follows:

Let α = recognition validity, where recognition validity is the likelihood of correct choice made by the recognition heuristic.

Let β = knowledge validity, where knowledge validity is the likelihood of correct choice based on knowledge of the two options.

Let N = the set of objects from which pairs of objects are drawn.

Let n = the number of recognized objects.

There are therefore N-n unrecognized objects and n(N-n) pairs where only one object is recognized. There are also (N-n)(N-n-1)/2 pairs where neither item is recognized and n(n-1)/2 pairs where both objects are recognized. Finally, there are N(N-1)/2 possible pairs. Given this, the expected proportion of correct inferences, f(n) of an exhaustive pairing of objects is:

$$f(n)= [(N-n)(N-n-1]/[N(N-1)]\ .5 + [2n(N-n)]/[N(N-1)]\ \alpha + [n(n-1)]/[N(N-1)]\ \beta \quad (1)$$

The first term of this equation gives the proportion inferences correct where neither object is recognized and choice is random, the second term gives the proportion inferences correct where only one object is recognized and the recognition heuristic is employed and the third term gives the proportion inferences correct where both objects are recognized. When figures are substituted for the variables α and β (recognition and knowledge validities, respectively) the pattern shown in Figure 2 can be observed:

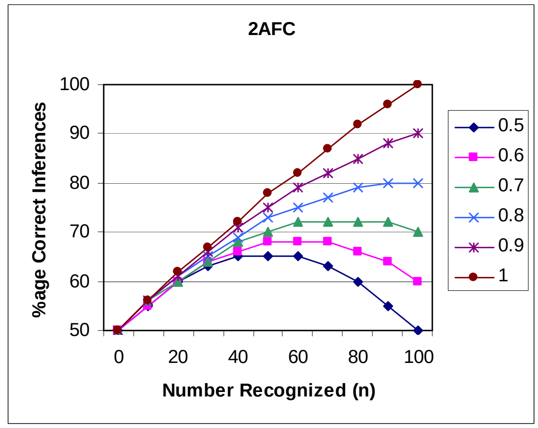

Figure 2: Inferences correct as a function of varying number recognized n (x-axis) and recognition validity α (given in the figure legend) where N = 100 and knowledge validity β is set to .8.

As observed by Goldstein and Gigerenzer (2002) a "less is more" effect, superior performance for conditions where only one object is recognized, appears at some point on the graph for all conditions where $\alpha > \beta$. This can best be seen by looking at the 100% recognition plots (right-most point) which represent the maximum performance levels on the basis of knowledge of all the objects. Any point on these lines that are graphed as higher along the y-axis (percentage inferences correct) are showing a less-is more effect. That is, 30% of the data points formally calculated for this figure. It is also possible to show mathematically that, where $\alpha > \beta$, a less-is-more effect *must* occur (Goldstein & Gigerenzer, 2002). Thus, the utility of recognition is demonstrated and its incorporation into an ACT-R architecture appears to be unquestionably consistent with the principle of rational analysis (Anderson, 1990). One question that has not been properly addressed, however, is how well, at a theoretical level, the 2AFC cities task represents other (for example multi-option) choices, whether the less-is-more effect holds up across the board or whether the 2AFC, far from being a case to which other tasks are reducible is, in fact, a special case.

Varying choice options and information required.

The first question to be addressed is how the less-is-more effect fares when the nature of the question is varied. For 2AFC, asking which of the two objects is smaller (*the lesser question*) should be equivalent to asking which of the two objects is larger (*the greater question*). So, for example, with the cities task the question "which has the smaller population, San Diego or San Antonio?" is equivalent to asking, "which has the larger population, San Diego or San Antonio?" This is because the use of recognition would entail, for the greater question, inferring that San Diego has the larger population (because it is recognized) and thus choosing it. For the lesser question the same inference is made, and on the same basis - so participants using the heuristic should choose the unrecognized city. Thus, the information required to make the choice is identical and the sort of pattern shown in Figure 2 will in principle follow from a comparison of recognition versus knowledge-based performance.[1]

Turning to multi-option problems, the task requirements are less obviously clear-cut. Consider the 3-alternative forced choice (3AFC) task. For this task, on a cities task given the greater question there are 4 possible states of affairs: Recognize 0, 1, 2 or 3 of the cities. Equation 1 can be expanded as follows:

$$\begin{aligned} f(n) = {} & [(N-n)(N-n-1)(N-n-2)] / [N(N-1)(N-2)]\ .333 + \\ & [3n(N-n)(N-n-1)] / [N(N-1)(N-2)]\ \alpha + \\ & [3n(n-1)(N-n)] / [N(N-1)(N-2)]\ \beta_1 + \\ & [n(n-1)(n-2)] / [N(N-1)(N-2)]\ \beta_2 \qquad (2) \end{aligned}$$

Where the 1st term is proportion correct inferences by chance, the 2nd term reflects the contribution of recognition validity (i.e., where only one item was recognised), the 3rd term reflects knowledge validity for choices out of 2 recognised (β_1) and the 4th term indicates the contribution of knowledge validity for choices out of 3 (β_2). If it is assumed that knowledge validity is the same regardless whether 2 or 3 options are recognized, i.e., $\beta_1 = \beta_2$ then the graph of performance shown in Figure 3 can be drawn:

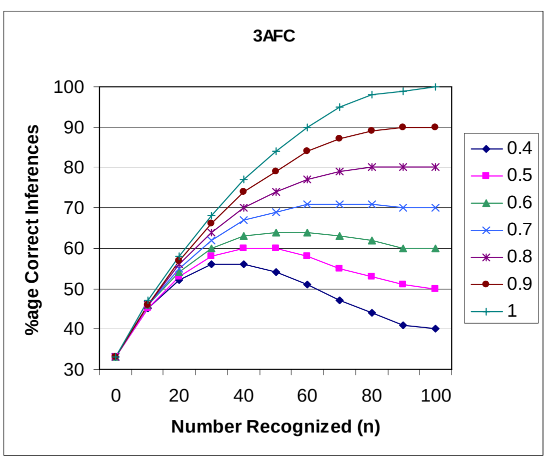

Figure 3. Inferences correct as a function of varying number recognized n (x-axis) and knowledge validity β (given in the figure legend) where N = 100 and recognition validity α is set to .8.

[1] Although psychological studies show that in practice choice varies as a function of the question wording (McCloy & Beaman, 2004).

Mathematically, the expansion of equation (1) to equation (2) has had little effect on the appearance of the less-is-more effect given the assumptions as stated above. The less-is-more effect still occurs at approximately the same rate (about 30% of the data-points plotted in this example) and always occurs whenever $\alpha > \beta$. However, it is not clear that the assumption that $\beta_1 = \beta_2$ is justified. Analysis of the cognitive operations that must take place for 3AFC tasks with the greater question reveals that although the cognitive operations for recognize 0, 1 or all of the alternatives can easily be mapped onto situations encountered in a 2AFC scenario, recognize 2 out of the 3 alternatives represents a possibility not previously encountered. Specifically, if 2/3 of the options are recognized, one can only apply knowledge after one has already used recognition to narrow down the choices to 2. The requirement to apply both recognition and knowledge is unavoidable and therefore when 2/3 of the options are recognized, validity of the judgment must be the product of recognition and knowledge validities. When equation 2 is amended to reflect this, the graph shown in Figure 4 is obtained.

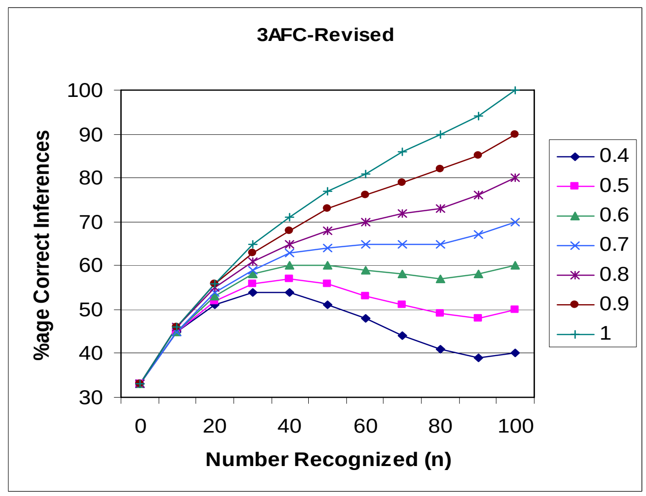

Figure 4. Inferences correct as a function of varying number recognized n (x-axis) and knowledge validity β (given in the figure legend) where N = 100, recognition validity α is set to .8, and validity of inference for recognize 2/3 is the product of α and β.

Figure 4 clearly shows the limitations of the "less-is-more effect" once one moves beyond the bounds of a 2AFC situation. As the figure demonstrates, although a less-is-more effect continues to occur, it does so only on 18% of the data-points plotted here and, significantly, does not always occur where $\alpha > \beta$, as previously.

It is also worth reconsidering the question wording at this point. For 2AFC the lesser and greater questions reduce to requesting the same information but this is not true of multi-option forced choice tasks.

For a 3AFC and the greater question, the following cognitive operations are required (broken down by situation):

Recognize 0: Guess
Recognize 1: Infer the recognized object has the higher value and choose that object accordingly.
Recognize 2: Infer the recognized objects have the higher value and then use knowledge to choose between them.
Recognize 3: Use knowledge.

In contrast, for a 3AFC task and the lesser questions the following cognitive operations are required:

Recognize 0: Guess
Recognize 1: Infer the recognized object has the higher value and then guess one of the remaining 2 objects.
Recognize 2: Infer the recognized objects have the higher value and choose the remaining object.
Recognize 3: Use knowledge.

As recognize 0 and recognize 3 situations do not differ in their knowledge requirements, the critical contrasts are between the recognize 1 and 2 conditions. With both questions, recognition can be used to guide choice unaided for only one of these two situations, and which situation can be resolved via recognition alone differs according to the question posed. Furthermore, whereas for the greater question choice in the recognize 2 situation must be made guided by recognition and knowledge validity, for the lesser question, there is no extra demand on knowledge validity. Instead, in the recognize 2 condition when set the lesser question, choice is made by a combination of recognition validity and guesswork as there is no more knowledge available to aid choice. Figure 5 gives the graph of a 3AFC situation if asked the lesser question.

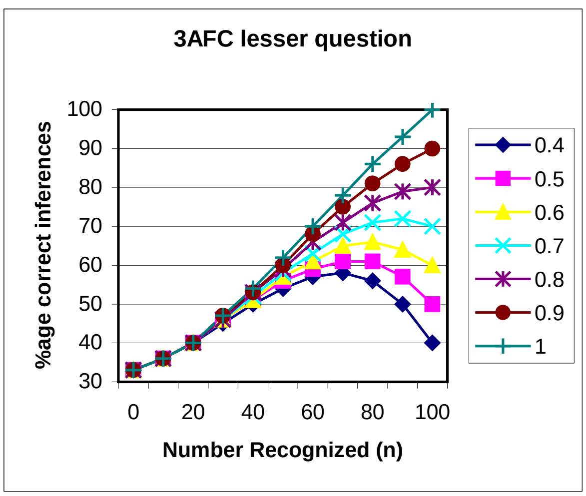

Figure 5. Inferences correct as a function of varying number recognized n (x-axis) and knowledge validity β (given in the figure legend) where N = 100 and recognition validity α is set to .8.

The pattern of performance shown in Figure 5 is closer to that obtained with a 2AFC question (Figure 2) than with the greater question on a 3AFC task (as shown in Figure 3). The less-is-more effect is reinstated and occurs once more on every occasion where recognition validity exceeds knowledge validity. On average, however, performance is lower with approximately 4% fewer correct inferences overall than in a 3AFC when asked the greater question (Figure 3). Thus, relative to knowledge, the utility of recognition is reinstated under these circumstances but expected performance overall drops slightly because this task is more open to the operation of random factors than is the same task when the greater question is posed.

n-AFC: The General Case.

To summarize the analysis so far: The usefulness of recognition as a cue to inference is supported but its limitations are also demonstrated. We have shown how the recognition heuristic might generalize beyond the 2AFC on which its empirical support depends (Goldstein & Gigerenzer, 1999, 2002). In doing so, we have also shown that the exact nature of the questions asked and the options given modifies the expected utility of the heuristic in an unanticipated and counter-intuitive manner. Specifically, the 2AFC task is revealed to be a special case where the nature of the question asked does not influence the information required. If the recognition heuristic had been directly applied to a 3AFC task using the greater question, which has so far been employed in all but one of the empirical psychological investigations of recognition-driven inference (Gigerenzer & Goldstein, 1996; Goldstein & Gigerenzer, 1999, 2002, McCloy & Beaman, 2004, Oppenheimer, 2003), the support derived from the counter-intuitive "less-is-more" effect would have been much weaker. In the situations simulated here it is only when the nature of the question posed is altered that the effect re-emerges to any great degree.

One potential concern is that the 3AFC situation that is the subject of the current set of mathematical models might be the "special case" rather than the 2AFC situation from which it is derived. This concern can be allayed, however, by expanding the situation out to *n*-AFC situations and examining the general cognitive demands posed. For reasons of space and of readability, this can easily be carried out in a non-mathematical way simply showing how the requirements to consult knowledge differ for *n*-AFC tasks as a function of the question posed. For *n*-AFC situations posing the lesser question, the operations required and their associated validity are shown in Table 1. As can be seen from this table, recognition can be used to eliminate a number of possible alternatives from the consideration set but having done so there is little role for knowledge to play. Under such circumstances, it is hardly surprising that a less-is-more effect frequently holds as recognition is simply applicable in more circumstances than knowledge. Equally, given that once recognition has been deployed choice amongst the remaining alternatives is dependent upon random factors, it is not surprising that overall performance is lower with the lesser question than with the greater question.

Table 1: Operations and operation validity for each of the possible situations encountered in an *n*-AFC task when asked the lesser question.

Recognise	Operation & Operation Validity
0	Chance (1/n)
1	Eliminate recognized (α) & chance (1/n-1)
2	Eliminate recognized (α) & chance (1/n-2)
….	
n-1	Eliminate recognized (α)
n	Choose amongst recognized (β)

An equivalent table for n-AFC tasks when the greater question is posed in shown in Table 2.

Table 2: Operations and operation validity for each of the possible situations encountered in an *n*-AFC task when asked the greater question.

Recognise	Operation & Operation Validity
0	Chance (1/n)
1	Choose recognized (α)
2	Choose recognized items (α) & select between recognized items (β)
….	
n-1	Choose recognized items (α) & select between recognized items (β)
N	Choose amongst recognized (β)

Table 2 shows how, when asked the greater question, the utility of recognition is bound, in all but one case, to the validity of knowledge. The one case where recognition utility is not bound to knowledge is where only one item is recognized. Thus, recognition is a powerful cue in 2AFC choices. In all other cases recognition may aid heuristic inference but performance ultimately is dependent upon the joint effects of recognition and knowledge validity. Thus, it appears that the role of recognition per se is overstated.

Conclusions.

The results of this set of simulations demonstrate that the rational application of a recognition heuristic is, as stated previously (e.g., McCloy & Beaman, 2004) not as straightforward as it might at first appear. The relative simplicity of a blocks-world style "drosophila" environment such as the cities task (Gigerenzer & Goldstein, 1996) fails to scale up to an *n*-AFC. Beyond 2AFC the utility of recognition alone is limited by the need to also consult the knowledge base for 50% of the decisions that may be made using the recognition heuristic within this environment (the "greater" questions). For the remaining 50% of choices (the "lesser" questions) the counter-intuitive less-is-more effect remains in force but overall the proportion of accurate inferences are reduced because for the majority of choices the recognition validity is used in conjunction with other factors that have only chance level validity (e.g., guesswork).

Thus, a recognition-based choice heuristic developed within a 2AFC environment fails to show that multi-option choices are reducible to binary choices in the way predicted by Goldstein and Gigerenzer (1999). 2AFC is demonstrably a special case by virtue of its relative immunity to knowledge and chance based influences in more complex tasks, and its symmetry between greater and lesser choices. Neither of these features are replicated in more complex, multi-option decisions. Whilst not denying the utility of the recognition heuristic under 2AFC situations, the results of this study show that the extent to which the heuristic can be said to be "ecologically rational" in tasks expanded beyond the basic blocks-world situation in which it was developed is limited.

Acknowledgments

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References

Anderson, J. R. (1990). *The adaptive character of thought*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Anderson, J. R. & Lebiere, C. (1998). *The atomic components of thought*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Borges, B, Goldstein, D. G., Ortmann, A., & Gigerenzer, G. (1999). Can ignorance beat the stock market? In G. Gigerenzer, P. M. Todd and the ABC Research Group. *Simple heuristics that make us smart.* Oxford: Oxford University Press.

Boyd, M. (2001). On ignorance, intuition and investing: A bear market test of the recognition heuristic. *Journal of Psychology and Financial Markets, 2,* 150-156.

Gigerenzer, G. & Goldstein, D. G. (1996). Reasoning the fast and frugal way: Models of bounded rationality. *Psychological Review, 103,* 650-669.

Goldstein, D. G. & Gigerenzer, G. (1999). The recognition heuristic: How ignorance makes us smart. In G. Gigerenzer, P. M. Todd and the ABC Research Group. *Simple heuristics that make us smart.* Oxford: Oxford University Press.

Goldstein, D. G. & Gigerenzer, G. (2002). Models of ecological rationality: The recognition heuristic. *Psychological Review, 109,* 75-90.

McCloy, R. & Beaman, C. P. (2004). The recognition heuristic: Fast and frugal but not as simple as it seems. In: *Proceedings of the Twenty-Sixth Annual Conference of the Cognitive Science Society* (pp 933-937). Mahwah, NJ: Lawrence Erlbaum Associates.

Newell, B. R. & Schanks, D. R. (2004). On the role of recognition in decision making. *Journal of Experimental Psychology: Learning, Memory & Cognition, 30,* 923-935.

Oppenheimer, D. M. (2003). Not so fast! (and not so frugal!): Rethinking the recognition heuristic. *Cognition, 90,* B1-B9.

Schooler, L. J. & Hertwig, R. (2005). How forgetting aids heuristic inference. *Psychological Review, 112,* 610-628.