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PREDICTIONS OF THE PARTON DUAL RESONANCE MODEL FOR THE TOTAL CROSS SECTION OF ELECTRON-POSITRON ANNIHILATING INTO HADRONS




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# PREDICTIONS OF THE PARTON DUAL RESONANCE MODEL FOR THE TOTAL CROSS SECTION OF ELECTRON-POSITRON ANNIHILATING INTO HADRONS

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March 29, 1971

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# ERRATA

TO: All recipients of UCRL-20646

FROM: Technical Information Division

SUBJECT: UCRL-20646: Predictions of the Parton Dual Resonance Model for the Total Cross Section of Electron-Positron Annihilating into Hadrons, Loh-ping Yu, March 29, 1971

Please make the following corrections on subject report:

Page 1, Abstract, line 5: $C(|q^2|\ell n|q^2|)^{-1}$ should read $Cq^{-4}(|q^2|\ell n|q^2|)^{-1}$

Page 1, Abstract, line 6: $C'(\ell n|q^2|)^{-1}$ should read $C'q^{-4}(\ell n|q^2|)^{-1}$

Page 1, text, line 3: $e^- + N \to N +$ Hadrons should read $e^- + n \to e^- +$ Hadrons

Page 2, last line: $\omega^3$ should read $\omega \ell n^{-3}\omega$

Page 3, in Eq. (1): $4\pi^2\alpha$ should read $\frac{4\pi^2\alpha}{q^4}$

Page 4, in Eq. (6): $x^{-\alpha_{12}^{-1}}$ should read $x^{-\alpha_{12}}$

Page 7, in Eq. (16): $\frac{1}{\ell n|q^2|}$ should read $\frac{1}{q^4\ell n|q^2|}$

Page 7, first paragraph, next to last line: Delete sentence "This prediction is in consistency..." and replace with: "This prediction indicates that the total cross section for the process $e^+ + e^- \to$ hadrons is of the same order of magnitude as the point-like interaction."

Page 9, lines 1, 2, 3, 4, 5: delete entirely.

# PREDICTIONS OF THE PARTON DUAL RESONANCE MODEL FOR THE TOTAL CROSS SECTION OF ELECTRON-POSITRON ANNIHILATING INTO HADRONS*

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March 29, 1971

ABSTRACT

Based on the idea that a heavy virtual photon behaves like a parton-antiparton pair in participating the strong interaction, we predict that the total cross section for $e^+e^- \to$ hadrons, as $q^2 \to \infty$, behaves like $C(|q^2| \ln|q^2|)^{-1}$ for spin-0 partons, and like $C'(\ln|q^2|)^{-1}$ for spin-$\frac{1}{2}$ partons, where $q^2$ is the virtual photon's mass square. This provides a simple, yet important test of the parton dual resonance model for the deep inelastic lepton-hadronic scattering, proposed in a previous paper.

Because the final-state interaction among partons has not been taken into account, all previous parton models for the scatterings $e^- + N \to N +$ Hadrons and $e^+ + e^- \to N +$ Hadrons, consider a parton successively coupled (point-like) to a pair of heavy virtual photons; and to obtain the structure functions $W_1$, $\nu W_2$, an imaginary part cut across this parton is necessary. On doing this, one immediately faces the puzzle: What is a parton? Why the parton is not observed



experimentally? Motivated by this, we have proposed[1] a six-point function dual resonance model for the virtual forward Compton scattering amplitude, in order to take the final-state interaction into account and hence to resolve the puzzle. In this model,[1] the two virtual photons are point-like coupled to two pairs off-shelled partons of field theoretical type. One then integrates two loop momenta over the standard six-point generalized Veneziano formula (with four legs off the mass shells). We briefly summarize the outcome[1] of this "parton dual resonance model." The model furnishes explicit formulas for $W_1$ and $\nu W_2$ over the whole range of the scaling variable $\omega$ between $0$ and $\infty$; and, through the duality property of the six-point function, it correctly reproduces, apart from a nonscaling factor of $(a + b \ln|q^2|)^{-1}$, the Bjorken scaling law, the regge limit $\omega \to \infty$, the "fixed-angle" limit $\omega \to 1 \pm \epsilon$, and the threshold behavior $\omega \to 1^{\pm}$ of Bloom and Gilman. For $e^+ + e^- \to N +$ Hadrons process, it further predicts the pionization (nucleonization) limit $\omega \to 0$ to vanish like $\omega^3$.

It is also suggested[1] that the final-state interaction among the partons is of diffractive type (Pomeron exchange in the parton-parton channel), and it breaks the scaling law by a factor of $(a + b\ \ell n|q^2|)^{-1}$. Two crucial assumptions[1] in constructing that model are: that the parton is point-like coupled to the heavy virtual photon, and that the final-state interaction effect does not allow the particular parton, which absorbs the virtual photon, to be observed experimentally. Because of these two assumptions, a heavy virtual photon is naturally pictured as a parton-antiparton pair in participating the strong interaction. In this note, we suggest a simple experimental test to this interesting idea.

We consider the measurement of the total cross section for the process $e^+e^- \to$ hadrons. Within the framework of the parton dual resonance model discussed in Ref. 1, we therefore consider a four-parton-leg Veneziano formula (with two loop integrations) for the total cross section $\sigma_{e\bar{e}}$, shown in Fig. 1; in which the dotted line indicates the imaginary part in the virtual photon mass variable $q^2$. We formulate the model for the total cross section as

$$\sigma^{(i)}_{e\bar{e}}(q^2) = 4\pi^2\alpha \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)^{-1} \left[\mathrm{Im}_{q^2}\, T^{(i)}_{\mu\nu}\right], \tag{1}$$

with

$$T^{(i)}_{\mu\nu} = \int d^4k_2\, d^4k_3 \times \frac{K^{(i)}_{\mu\nu}\, \bar{B}_4(q - k_2, k_2, k_3, -q - k_3)}{[(k_2 - q)^2 - m^2](k_2^2 - m^2)(k_3^2 - m^2)[(k_3 + q)^2 - m^2]}, \tag{2}$$

where $m$ is the common mass of the four partons,



$$K^{(i)}_{\mu\nu} = \begin{cases} -(2k_2 - q)_\mu (2k_3 + q)_\nu, & \text{for spin-0 partons, } (i = 1), \\ -(2k_2 - q)_\mu (2k_3 + q)_\nu + q^2\left(g_{\mu\nu} - \dfrac{q_\mu q_\nu}{q^2}\right), & \text{for spin-}\tfrac{1}{2}\text{ partons, } (i = 2), \end{cases} \tag{3}$$

and

$$\bar{B}_4 = \int_0^1 dx\, x^{-\alpha_{12}(q)-1} (1 - x)^{-\alpha_{23}(k_2+k_3)-1}. \tag{4}$$

As in Ref. 1, and again in Eq. (4), the $q^2$ variable belonging to the legs 1,2, must be analytically continued in opposite direction to that belonging to the legs 3,4, in crossing the $q^2$-plane cut. We indicate this by a "bar" over the ordinary four-point Veneziano formula.

Substitute Eqs. (3), (4) in Eq. (2), and use the identity

$$\frac{1}{(k_j^2 - m^2 + i\epsilon)} = -\int_0^\infty da_j\, \exp[a_j(k_j^2 - m^2) + i\epsilon], \quad j = 1,2,3,4, \tag{5}$$

we can perform the double-loop integrations over $k_2$ and $k_3$, and obtain[2]

$$T^{(i)}_{\mu\nu} = \int_0^\infty da_1\, da_2\, da_3\, da_4 \int_0^\infty d(\ell n\, \tfrac{1}{x})\, x^{-\alpha_{12}-1}(1 - x)^{-\alpha_{23}-1}\, K^{(i)}_{\mu\nu} \times \frac{\pi^4}{|C|^2} \exp[q^2(\ell n\, \tfrac{1}{x} + \tfrac{D}{C})]\, \exp(-J), \tag{6}$$

where

$$C = (a_1 + a_2 + a_3 + a_4)\, \ell n(1 - x)^{-1} + (a_1 + a_2)(a_3 + a_4),$$

$$D = (a_1 + a_4)C - \{2a_1a_2\, \ell n(1 - x)^{-1} + a_4^2[a_1 + a_2 + \ell n(1 - x)^{-1}] + a_1^2[a_3 + a_4 + \ell n(1 - x)^{-1}]\}, \tag{7}$$

and $J = m^2(a_1 + a_2 + a_3 + a_4) + i\epsilon$.

We now take the limit $q^2 \to -\infty$, where the Veneziano formula convergents nicely. From Eqs. (6), (7), we see that the important region is when $\ln\frac{1}{x}$, $a_1$, $a_4$ are small. We thus make the scale transformation[3]

$$\begin{aligned} a_1 &= \rho\beta_1, \\ a_4 &= \rho\beta_2, \\ \ln\frac{1}{x} &= \rho(1 - \beta_1 - \beta_2). \end{aligned} \tag{8}$$

Expand $(1 - x) \approx \rho(1 - \beta_1 - \beta_2)$, $\ln(1 - x)^{-1} \approx \ln q^2$, and put $\rho \approx 0$ everywhere else except the coefficient of $q^2$ in the exponent of Eq. (6). We then do the $\rho$ integral, and set[4] $\alpha_{23} \equiv 1$, as suggested from Ref. 1. We find

$$T^{(i)}_{\mu\nu} \underset{\substack{q^2\to -\infty \\ \alpha_{23}\equiv 1}}{\widetilde{\quad}} \pi^4 \int_0^\infty da_2\, da_3\, \tilde{K}^{(i)}_{\mu\nu} \frac{\exp[-m^2(a_2 + a_3)]}{|a_2a_3 + (a_2 + a_3)\ln q^2|^2}$$

$$\times \int_0^1 d\beta_1 \int_0^{1-\beta_1} d\beta_2 \frac{1}{[q^2(1 - \beta_1 - \beta_2)^2 + i\epsilon]}, \tag{9}$$

where

$$\tilde{K}^{(i)}_{\mu\nu} \approx \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\begin{cases} 1 - \dfrac{2}{a_2 + a_3 + i\epsilon}, & i = 1, \\ q^2 & i = 2. \end{cases} \tag{10}$$

The $i\epsilon$ prescription in Eqs. (9), (10) is from the Feynman's field propagators prescription for the four parton legs. It is crucial to have it here, otherwise the imaginary part in $q^2$ vanishes. Because the $\beta_1$, $\beta_2$ integrals divergent, a further enhancement in the $q^2$ asymptotic behavior comes from the region $\beta_1 \approx 0$, $\beta_2 \approx 1$. We make the second scale transformation



$$\begin{aligned} \beta_1 &= \rho'\alpha_1, \\ 1 - \beta_2 &= \rho'(1 - \alpha_1). \end{aligned} \tag{11}$$

We then integrate $\rho'$ over a small region, i.e.,

$$\begin{aligned} &\int_0^1 d\beta_1 \int_0^{1-\beta_1} d\beta_2 \frac{1}{[q^2(1 - \beta_1 - \beta_2)^2 + i\epsilon]} \\ &\approx \int_0^1 d\alpha_1 \int_0^{\epsilon'} d\rho' \frac{1}{(q^2\rho' + i\epsilon)} \\ &= \ln\left(\frac{\epsilon' q^2 + i\epsilon}{i\epsilon}\right) \approx \ln q^2[1 + O(\ln^{-1} q^2)]. \end{aligned} \tag{12}$$

The double integrals over $a_2$ and $a_3$, in Eq. (9), can also be reduced to a single integral over $x \equiv a_2 + a_3$, as has been shown in the Appendix C of Ref. 1. We thus find,[5] from Eq. (9),

$$T^{(i)}_{\mu\nu} \underset{\substack{q^2\to -\infty \\ \alpha_{23}\equiv 1}}{\widetilde{\quad}} \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) \pi^4 \int_0^\infty dx \frac{\exp(-m^2 x)\, K^{(i)}}{|\frac{x}{4} + \ln q^2|} \left(\frac{\ln q^2}{q^2}\right), \tag{13}$$

with

$$K^{(i)} = \begin{cases} 1 - \dfrac{2}{x + i\epsilon}, & i = 1, \\ q^2, & i = 2. \end{cases} \tag{14}$$

Now we analytically continue[6] Eq. (13) to $q^2 \to +\infty \pm i\epsilon$ and take its imaginary part in $q^2$. Remember that, the $q^2$ variable belonging to the legs 1,2, must be analytically continued opposite to the $q^2$ variable belonging to the legs 3,4. This is reflected in the fact that the $\ln q^2$ factor is changed into $\ln|q^2| \pm i\pi$. On the other hand, the denominator factor in Eq. (13), being contributed only to the residues of the resonance poles in $q^2$, must remain real and positive definite when it is analytically continued to $q^2 \to +\infty \pm i\epsilon$

region. This is due to the optic theorem for cross section. Therefore the denominator factor is changed into $[\frac{x}{4} + (\ell n^2|q^2| + \pi^2)^{\frac{1}{2}}]$. We thus take the imaginary part in $q^2$ from Eq. (13), resulting in

$$\mathrm{Im}\, T^{(i)}_{\mu\nu} \underset{q^2\to+\infty}{\sim} \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) \pi^5 \int_0^\infty dx \, \frac{\exp(-m^2x)}{[\frac{x}{4} + (\ell n^2|q^2| + \pi^2)^{\frac{1}{2}}]} \times \begin{cases} \dfrac{C}{|q^2|}, & i = 1, \\ 1, & i = 2. \end{cases} \tag{15}$$

We thus predict, from Eqs. (15), (1), that

$$\sigma^{(i)}_{e\bar{e}}(q^2) \underset{\substack{q^2\to+\infty \\ \ell n|q^2|\to\infty}}{\sim} \frac{1}{\ell n|q^2|} \begin{cases} \dfrac{C}{|q^2|}, & \text{for spin-0 partons}, \quad (i = 1), \\ C', & \text{for spin-}\tfrac{1}{2}\text{ partons}, \quad (i = 2). \end{cases} \tag{16}$$

The C, C' are two constants. This prediction is in consistency with the models of compound field algebra or scale invariance.[7,8]

ACKNOWLEDGMENTS

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FOOTNOTES AND REFERENCES

* This work was done under the auspices of the U.S. Atomic Energy Commission.

1. L. P. Yu, A Simple Parton Dual Resonance Model for the Electroproduction and the Lepton Pair Annihilation Processes--Incorporates the Final State Interaction, Lawrence Radiation Laboratory Report UCRL-20610.

2. The determinant factor C must be real and positive definite, since we are making a model for the cross section.

3. Contrary to Ref. 1, here we don't need to consider the extra pieces of imaginary part cutting across the two partons. Because taking the imaginary part along the line cutting through the two partons, is equivalent to putting the two partons on the mass shells, which then vanishes by the energy-momentum conservation at the $\gamma p\bar{p}$ vertex.

4. This crucial step was used in Ref. 1 to obtain a result consistent with the scaling law. We have interpreted this by saying that the final-state interaction among partons is diffractive in nature, see Ref. 1.

5. The exact $\ell n|q^2|$ dependence, as shown in the Appendix C of Ref. 1, is

$$\frac{1}{\left|\frac{x^2}{4} + x\,\ell n|q^2|\right|^{\frac{1}{2}}} \ell n\left[\frac{\frac{x}{2} + (\frac{x^2}{4} + x\,\ell n|q^2|)^{\frac{1}{2}}}{\frac{x}{2} - (\frac{x^2}{4} + x\,\ell n|q^2|)^{\frac{1}{2}}}\right].$$

We have approximated the complicated $\ell n[\ ]$ factor by assuming $\ell n|q^2|$ large.

6. We take the usual smooth average assumption in the $+\infty$ direction of the Veneziano amplitude.

7. P. Langacker and M. Suzuki, The Total Cross Section for Electron-Positron Annihilation and the Hadronic Contribution to the Muon Magnetic Moment, Lawrence Radiation Laboratory Report UCRL-20608.

8. I thank Professor M. Suzuki for informing me of the results of their work (UCRL-20608) before publication.



FIGURE CAPTIONS

Fig. 1. The four legs off-shelled parton dual resonance model for the total cross section $\sigma(e^+e^- \to \text{hadrons})$. The dotted line indicates the imaginary part in $q^2$.

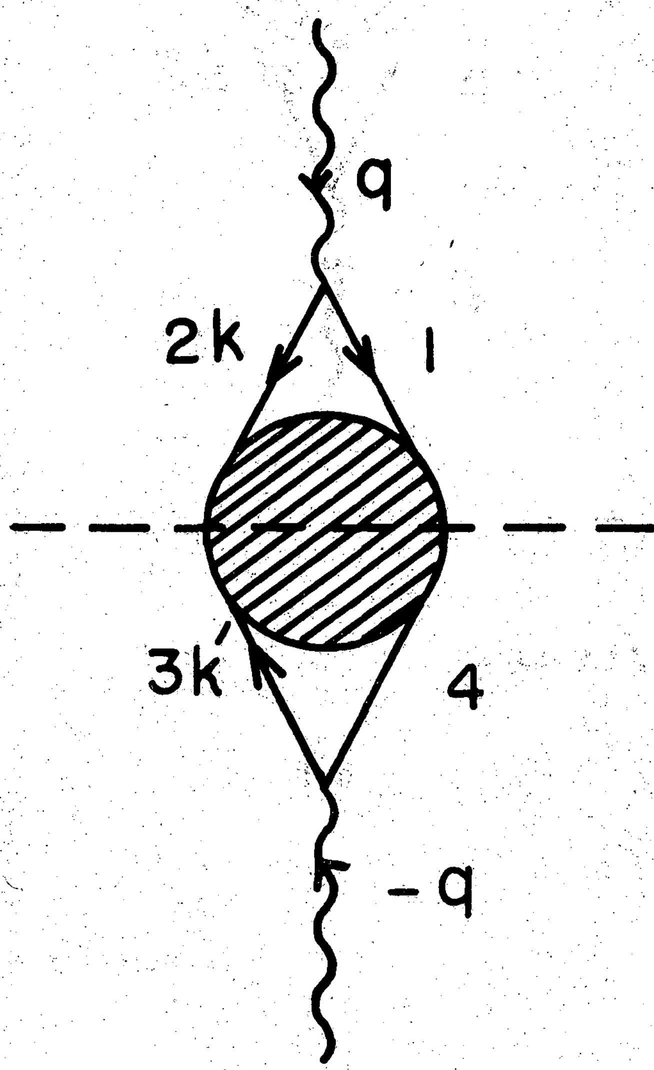


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Fig. 1

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