Title
E2/M1 Y RAY MULTIPOLE MIXING RATIOS IN EVEN-EVEN DEFORMED NUCLEI

Author
Krane, K.S.

Publication Date
1973

Submitted to Physical
Review Letters

LBL-1625
Preprint

E2/M1 γ RAY MULTIPOLE MIXING RATIOS IN EVEN-EVEN DEFORMED NUCLEI

K. S. Krane

January 1973

Prepared for the U. S. Atomic Energy Commission
under Contract W-7405-ENG-48

DISCLAIMER

E2/M1 γ RAY MULTIPOLE MIXING RATIOS IN EVEN-EVEN DEFORMED NUCLEI*

K. S. Krane

Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

January 1973

Abstract:

A summary is presented of the magnitudes and phases of previously measured E2/M1 multipole mixing ratios of γ-transitions de-exciting levels of the β- and γ-vibrational bands to the ground-state band in even-even deformed nuclei. It is shown that none of the previously proposed theoretical interpretations is sufficient to explain both the magnitudes and relative phases of these mixing ratios.

— — —

In the model of adiabatic vibrations of an ellipsoidally-deformed nucleus, magnetic dipole (M1) transitions are forbidden to exist in γ-transitions connecting rotational levels built on the vibrational excitations with those of the ground-state band; such transitions are expected to be pure electric quadrupole radiation (E2). However, non-vanishing M1-admixtures are found in such transitions in even-even nuclei throughout the mass region $150 < A < 190$; the M1-intensity generally comprises 0.5% - 2% of the transition intensity.

The measurement of γ-ray angular distributions or correlations is sensitive to interference effects between the M1 and E2 amplitudes, and thus

depends on the relative phase of the M1 and E2 matrix elements. Some confusion has existed in the literature regarding the relationship of this phase to the observed angular distribution. This situation results from the various formalisms which have been proposed for interpreting angular correlation data. In the present work, the phase convention of Krane and Steffen[1] is used; this choice permits a direct comparison of the deduced mixing ratio with theoretical predictions, and can be written in terms of the Bohr-Mottelson[2] multipole operators as

$$\frac{\delta}{E_\gamma(\mathrm{MeV})} = 0.835 \frac{\langle I_f \| \mathcal{M}(E2) \| I_i \rangle}{\langle I_f \| \mathcal{M}(M1) \| I_i \rangle} \quad , \tag{1}$$

with the E2 matrix element in units of electron-barns (eb) and the M1 matrix element in units of nuclear magnetons (nm); E_γ is the energy of the transition in MeV.

A comprehensive discussion of the electromagnetic transition operators is given in the work of Alder and Steffen.[3]

Table I presents a summary of the results obtained from an analysis of the angular correlation literature in terms of the present phase convention. The tabulated value is the "reduced" mixing ratio δ/E_γ given by Eq. (1). The quoted uncertainties are those arising from one standard deviation of the measured angular distribution coefficients. Transitions depopulating states of the β and γ bands of $I \leq 4$ have been analyzed; the identification of the γ-band is usually obvious, and the β-band has generally been assigned as a $K = 0^+$ excitation showing, for example, a large E2 excitation probability in a Coulomb excitation measurement.

The systematic behavior of the phase of the mixing ratio is apparent from an inspection of the table. With minor exceptions, transitions from the γ-band have negative phase, while transitions from the β-band seem to show the opposite phase.

The magnitudes and phases of the mixing ratios may be predicted from a variety of different models.

1. ΔK = 2 Band Mixing. This type of analysis takes into account mutual mixing of the ground-state, β, and γ bands, and has been widely used with reasonable success to interpret deviations of the relative reduced transition probabilities of transitions from the γ-band from the predictions of the adiabatic rotational model. The interpretation of transitions from the β-band has met with considerably less success. The present notation for the band mixing parameters is that of Marshalek[5] and of the Oak Ridge-Vanderbilt group.[6] A similar analysis has been done by Rud and Bonde Nielsen.[7] The M1 matrix elements are now given in terms of the static magnetic moments of the admixed intrinsic states, and the mixing ratios are given by

$$I_\gamma - I_g \qquad \left(\frac{\delta}{E}\right) = \frac{-AQ_o}{Z_\gamma(g_K - g_R) + B\, g_R\, Z_\beta\, Z_{\beta\gamma}} \quad , \tag{2}$$

$$I_\beta - I_g \qquad \left(\frac{\delta}{E}\right) = \frac{-AQ_o}{g_R\, Z_\gamma\, Z_{\beta\gamma}} \quad , \tag{3}$$

where A and B have the following values:

	$2_\gamma - 2_g$	$3_\gamma - 2_g$	$3_\gamma - 4_g$	$4_\gamma - 4_g$	$2_\beta - 2_g$	$4_\beta - 4_g$
A =	0.176	0.330	0.048	0.092	0.029	0.001
B =	18	0	0	200		

This calculation assumes that the intrinsic quadrupole moment Q_o and rotational g-factor g_R are constant for the three bands; g_K is the intrinsic g-factor evaluated for the γ-band. The band mixing parameters are in the notation of Ref. 6. With $Z_\gamma \simeq 5 \times 10^{-2}$, this model gives δ-values for γ-band transitions too large by at least an order of magnitude. Independent of Z_γ, the relative magnitudes of δ for the four γ-band de-excitations are not in agreement with experiment. The phases of the γ-band transitions are not easily calculable, depending on the values of $(g_K - g_R)$ and $Z_{\beta\gamma}$, which numbers are not widely available for all the nuclei considered.

2. <u>ΔK = 1 Band Mixing</u>. The first-order Coriolis interaction can mix K = 1 states into the K = 0 and K = 2 bands. The M1 matrix element resulting from such mixing is given by[8]

$$\langle I_1\ K = 0 \| \mathcal{M}(M1) \| I_2\ K = 2 \rangle = (-1)^{I_1 + I_2 + 1} \sqrt{I_1(I_1 + 1)(2I_1 + 1)} \times \langle I_1\ 1\ 1\ 1 | I_2\ 2 \rangle\ M_1 \tag{4}$$

where

$$M_1 = \sqrt{2}\, \langle K = 2 | [\epsilon_{+1}, \mathcal{M}'(M1, \nu = 1)] | K = 0 \rangle$$

$$= - \left\{ \frac{\langle K = 2 | h_{+1} | K = 1 \rangle}{E_{K=1} - E_{K=2}} \langle 00 \| \mathcal{M}(M1) \| 11 \rangle \right.$$

$$\left. + \sqrt{\frac{2}{3}}\, \frac{\langle K = 1 | h_{+1} | K = 0 \rangle}{E_{K=1} - E_{K=0}} \langle 22 \| \mathcal{M}(M1) \| 11 \rangle \right\}, \tag{5}$$

where $\mathcal{M}'$ refers to the intrinsic system and h_{+1} is the operator associated with ΔK = 1 Coriolis mixing.

At present there exists insufficient knowledge of K = 1 excitations to predict either the coupling or M1 matrix elements of Eq. (5). However, conclusions are possible regarding the relative phases and magnitudes of the mixing ratios. The relative magnitudes are as follows:

$$\frac{\delta}{E}(2_\gamma \to 2_g) : \frac{\delta}{E}(3_\gamma \to 2_g) : \frac{\delta}{E}(3_\gamma \to 4_g) : \frac{\delta}{E}(4_\gamma \to 4_g) = 1 : 0.94 : 0.68 : 0.44$$

These relationships are in better agreement with the observed values than are the relationships deduced above for $\Delta K = 2$ mixing. The relative phases of the mixing ratios are predicted to be the same, which is likewise in agreement with experiment.

An estimate of the magnitude of the required coupling strength indicates that the observed magnitudes of the mixing ratios require, for $\langle 00 \| \mathcal{M}(M1) \| 11 \rangle$ $\sim$ one single particle unit, a coupling matrix element $\langle K+1 | h_{+1} | K \rangle \simeq 10$ keV, which is not an unreasonably large value.

3. Microscopic Theory of the γ-Band. Bès et al.[9] have considered the microscopic structure of the γ-vibrational state, in which the intrinsic state is treated as a superposition of quasiparticle pairs. The M1 amplitudes are obtained through Coriolis band-mixing of the γ-band and ground-state band. The predictions of Bès et al. for the magnitudes of the E2/M1 mixing ratios are given in Table I. The phase of the mixing ratio is not uniquely determined in this model, but rather depends on the competition between the rotational motion and the orbital motion of the protons. If, as concluded by Bès et al.,[9] the contribution from the rotational motion dominates, this model predicts $\delta > 0$, in disagreement with experiment, although the predicted magnitudes seem to be in good agreement with experimental values.

A similar microscopic calculation was done by Tamura and Yoshida,[10] who considered the magnitude and phase of the M1 matrix element in terms of the lowest-lying K = 2 two-quasiparticle states which can mix with both the γ and ground-state bands. They estimated $|\delta| \sim 10$, in reasonable agreement with observed values, and also $\delta > 0$. However, their δ was defined in terms of absorption matrix elements, and the transformation to the presently-employed emission matrix elements requires a knowledge of the spatial and temporal symmetry properties of the nuclear wave-functions and multipole operators used. (A complete discussion of this problem is given by Alder and Steffen.[3]) If we assume the convention of Biedenharn-Rose[1] was used, then in terms of the present convention, $\delta < 0$, in agreement with experiment.

4. g-Factor Variation. In the ΔK = 2 band-mixing analysis given above, it was assumed that the g_R-factors were identical for the β-, γ-, and ground-state bands. Relaxing this requirement gives rise to M1 transitions which depend on the variation of g_R; however, this additional contribution to the M1 matrix element occurs only for ΔI = 0 transitions. The $2_\gamma - 2_g$ and $2_\beta - 2_g$ mixing ratios both require that the g_R-factor difference between the ground and vibrational bands be

$$\Delta g = g_R(g) - g_R(\beta,\gamma) \approx -0.5 \quad ,$$

which implies an increase in the g_R-factor by 2-1/2 times in the excited bands. Such an increase seems highly unlikely.

Greiner[11] has discussed the lowering of g_R-factors from the value Z/A in terms of a model in which the proton distribution is characterized by a somewhat smaller deformation than the neutrons. The M1 transition operator then obtains a tensor character dependent on the collective variables, and thus has non-vanishing

matrix elements between the collective bands. The predictions of this model for the magnitudes of the E2/M1 mixing ratios are given in Table I. This model is characterized by a smooth variation of δ from nucleus to nucleus, and thus is unable to account for the sudden changes in δ in the Er and Yb nuclei. The phase of the mixing ratio appears in this model to be positive for transitions from both the β and γ bands; however, as discussed above, the absorption matrix elements experience a charge of phase when converted to emission matrix elements. Consequently, although the predicted phase of the γ-band mixing ratio agrees with experiment, the identical phase predicted for the β-band does not agree.

5. Pairing-Plus-Quadrupole Model. The apparent increase in δ/E for the osmium nuclei comes about through a decrease in the energy of the $K = 2^+$ level associated with the γ-vibration, rather than through an increase in δ. For these nuclei, which are in a region of transition from deformed to spherical equilibrium shapes, Kumar and Baranger[12] have employed the pairing-plus-quadrupole model to predict energy levels and electromagnetic multipole moments. The E2 and M1 moments were calculated by Kumar,[13] and were found to be in good agreement with experimental E2/M1 mixing ratios (magnitude as well as phase) for Os nuclei, although the agreement is somewhat poorer for the (more deformed) W nuclei (see Refs. m-p of Table I).

It can be concluded from this investigation that at present there is no satisfactory interpretation of both the magnitudes and phases of M1 admixtures in collective transitions in even-even deformed nuclei, although the $\Delta K = 1$ coupling through $K = 1^+$ excitations seems to hold the most promise for a successful theory. Further insight into this problem must await studies of $K = 1^+$ excitations, in order that the matrix elements entering into Eq. (5) may be evaluated. Additionally, the agreement between the various theories and

experiment seems to be poorest for the Er, Yb, and Hf nuclei, and it would thus be interesting to reduce the experimental uncertainty for the Er results and to obtain additional results for Yb and Hf nuclei.

Finally, we note that, while most reasonable theories predict a unique phase for all mixing ratios of transitions depopulating the γ-vibrational band, $\delta(3_\gamma - 2_g)$ in ^{168}Er and $\delta(2_\gamma - 2_g)$ in ^{182}W are at variance with the remainder of the cases studied. While no explanation for the former case is apparent, ^{182}W also shows an anomalous phase and magnitude of $\delta(2_\beta - 2_g)$. (While Refs. m and n of Table I chose the larger root for δ, the directional correlation data of Herzog et al.[14] are more consistent with the smaller root.) Although the K = 0 excitation of ^{182}W is not a good β-vibration, it is coupled rather strongly to the γ-vibration, owing primarily to the small energy spacing.[15] In the ΔK = 2 formulism, the anomalous $2_\gamma - 2_g$ value could arise from a contribution from the 2nd term of the denominator of Eq. (2), and the $2_\beta - 2_g$ phase (compared with ^{184}W) is consistent with the sign change of the $Z_{\beta\gamma}$ matrix element[15] between ^{182}W and ^{184}W. A measurement of $\delta(4_\gamma - 4_g)$ in ^{182}W would shed considerable light on this problem.

FOOTNOTES AND REFERENCES

*Work performed under the auspices of the U. S. Atomic Energy Commission.

1. K. S. Krane and R. M. Steffen, Phys. Rev. C 2, 724 (1970). In this convention, the mixing ratio is defined in terms of emission matrix elements, and the phase of the mixing ratio is thus independent of the position of the γ-ray in the cascade. The relationship to the Biedenharn-Rose convention [Rev. Mod. Phys. 25, 729 (1953)] for the cascade $\gamma_1 - \gamma_2$ is $\delta(\gamma_1)_{KS} = -\delta(\gamma_1)_{BR}$ and $\delta(\gamma_2)_{KS} = \delta(\gamma_2)_{BR}$.

2. A. Bohr and B. R. Mottelson, Nuclear Structure (W. A. Benjamin, New York, 1969), Vol. I, p. 381

3. K. Alder and R. M. Steffen, in The Electromagnetic Interaction in Nuclear Physics, ed. by W. D. Hamilton (North-Holland, Amsterdam, 1972).

4. J. H. Hamilton, in Angular Correlation in Nuclear Disintegrations, ed. by H. van Krugten and B. van Nooijen (Rotterdam University Press, Wolters-Nordhoff Publishing, Groningen, The Netherlands, 1971), p. 181.

5. E. R. Marshalek, Phys. Rev. 158, 993 (1967).

6. L. L. Riedlinger, N. R. Johnson, and J. H. Hamilton, Phys. Rev. 179, 1214 (1969); J. H. Hamilton et al., Phys. Rev. C 5, 899 (1972); P. Little et al., Phys. Rev. C 5, 252 (1972).

7. N. Rud and K. Bonde Nielsen, Nucl. Phys. A158, 546 (1970).

8. A. Bohr and B. R. Mottelson, Nuclear Structure, Vol. II (to be published).

9. D. R. Bès, P. Federman, E. Maqueda, and A. Zuker, Nucl. Phys. 65, 1 (1965).

10. T. Tamura and H. Yoshida, Nucl. Phys. 30, 579 (1962).

11. W. Greiner, Nucl. Phys. 80, 417 (1966).

12. K. Kumar and M. Baranger, Nucl. Phys. A122, 273 (1968).

13. K. Kumar, Phys. Letters 29B, 25 (1969).

14. P. Herzog, M. J. Canty, and K. D. Killig, Nucl. Phys. A187, 49 (1972).

15. C. Günther, P. Kleinheinz, R. F. Casten, and B. Elbek, Nucl. Phys. A172 273 (1971).

Table I. Reduced E2/M1 mixing ratios δ/E_γ(MeV) of transitions from levels in β- and γ-bands to levels in ground-state bands[a]

Nucleus	$2_\gamma - 2_g$	$3_\gamma - 2_g$	$3_\gamma - 4_g$	$4_\gamma - 4_g$	$2_\beta - 2_g$	$4_\beta - 4_g$
^{152}Sm	$-9.5(2)^b$	$-8.0(9)^c$	$-7.0(3)^b$	$-2.8(3)^b$	$+(25^{+7}_{-4})^b$	$+4.7(21)^b$
	(8.5; 12.5)	(7.4; 11.7)	(6.5; 8.5)	(5.1; 6.5)	(6.6)	(3.4)
^{154}Gd	$-11.6(11)^c$	$-6.6(7)^c$	$-7.5(2)^d$	$-4.9(6)^d$	$+16(4)^d$	$+9(3)^d$
	(7.3; 12.4)	(6.4; 11.6)	(5.6; 8.4)	(4.4; 6.4)	(6.4)	(3.4)
^{156}Gd	$-17(3)^e$				$-(5.7^{+2.7}_{-1.4})^e$	
	(9.5; 14.1)				(7.3)	
^{160}Dy	$-12.5(19)^f$	$-9.4(25)^g$	$-(6^{+6}_{-2})^g$			
	(8.5; 13.6)	(7.4; 12.7)	(6.5; 9.3)			
^{162}Dy	$-(9^{+\infty}_{-7})^h$			$-(3^{+6}_{-1})^i$		
	(10.0; 14.9)			(6.0; 7.8)		
^{164}Dy	$-(12^{+\infty}_{-9})^h$					
	(12.0; 15.6)					
^{166}Er	$-(27^{+54}_{-13})^i$			$-(5^{+4}_{-2})^i$		
	(9.0; 15.7)			(5.4; 8.1)		

(continued)

Table I. (continued)

Nucleus	$2_\gamma - 2_g$	$3_\gamma - 2_g$	$3_\gamma - 4_g$	$4_\gamma - 4_g$	$2_\beta - 2_g$	$4_\beta - 4_g$
^{168}Er	$-(39^{+30}_{-12})^{j}$	$+20(3)^{j}$	$-7.7(5)^{j}$	$-(8^{+8}_{-5})^{i}$		
	(10.8; 15.9)	(9.4; 14.8)	(8.3; 10.8)	(6.5; 8.3)		
^{170}Er	$-(67^{+\infty}_{-48})^{i}$			$-(45^{+\infty}_{-26})^{i}$		
	(11.2; 16.1)			(6.7; 8.5)		
^{172}Yb	$-(7^{+4}_{-2})^{g}$	$-(4^{+2}_{-1})^{g}$				
	(28; 16.5)	(24; 15.4)				
^{174}Hf					$< -4^{k}$	$-3(1)^{k}$
					(8.2)	(4.2)
^{178}Hf	$-(30^{+\infty}_{-19})^{\ell}$					
	(3.0; 14.3)					
^{182}W	$+(19^{+17}_{-5})^{m}$	$-(49^{+81}_{-16})^{m}$	$-9(2)^{m}$		$-0.51(5)^{n}$	
	(4.2; 13.4)	(3.6; 12.5)	(3.2; 9.1)		(6.7)	
^{184}W	$-20(1)^{o}$	$-14.7(10)^{o}$	$-13.2(12)^{o}$	$-(8^{+4}_{-3})^{o}$	$+2.3(6)^{o}$	
	(5.2; 13.0)	(4.5; 12.1)	(4.0; 8.8)	(3.1; 6.8)	(6.5)	

(continued)

Table I. (continued)

Nucleus	$2_\gamma - 2_g$	$3_\gamma - 2_g$	$3_\gamma - 4_g$	$4_\gamma - 4_g$	$2_\beta - 2_g$	$4_\beta - 4_g$
^{186}W	$-(18^{+6}_{-5})^n$				$+(15^{+80}_{-7})^n$	
	(5.2; 13.2)				(6.6)	
^{186}Os	$-(16^{+24}_{-6})^p$	$-(17^{+12}_{-8})^p$				
	(5.2; 14.0)	(4.5; 13.1)				
^{188}Os	$-26(6)^p$	$-11(5)^p$				
	(; 13.5)	(; 12.6)				
^{190}Os	$-23(3)^p$	$-16(3)^p$				
	(; 13.7)	(; 12.8)				

[a]Explanation of Table: The subscripts γ, β, and g refer to states of the γ-, β-, and ground-state bands, respectively. The experimental uncertainty of the last place is given in parentheses following each entry. The two numbers under each entry give respectively the prediction for the magnitude of δ as calculated according to the methods of Bès et al. (Ref. 9) and Greiner (Ref. 11), except for transitions from the β-band, where only the predictions of Greiner are given.

[b]A. V. Ramayya et al., Angular Correlations in Nuclear Disintegration, ed. by H. van Krugten and B. van Nooijen (Groningen, The Netherlands, Rotterdam University Press, 1971), p. 247.

[c]J. Lange et al., Nucl. Phys. A171, 92 (1971).

[d]L. C. Whitlock et al., Phys. Rev. C 3, 313 (1971).

(continued)

Table I. (continued)

[e] J. H. Hamilton et al., Phys. Rev. C 5, 899 (1972).

[f] K. S. Krane and R. M. Steffen, Nucl. Phys. A164, 439 (1971).

[g] J. Lange, Angular Correlations in Nuclear Disintegration, p. 242.

[h] G. Engler and S. Lane, Bull. Am. Phys. Soc. 15, 100 (1970).

[i] J. M. Domingos et al., Nucl. Phys. A180, 600 (1972).

[j] L. M. Quinones et al., Bull. Am. Phys. Soc. 18, 37 (1973) and private communication from M. Behar (January 1973).

[k] H. Ejiri and G. B. Hagemann, Nucl. Phys. A161, 449 (1970).

[l] L. Varnell et al., Phys. Rev. C 3, 1265 (1971).

[m] K. S. Krane et al., Phys. Rev. C 5, 1104 (1972).

[n] W. T. Milner et al., Nucl. Phys. A177, 1 (1971).

[o] K. S. Krane et al., Phys. Rev. C 7 (to be published).

[p] K. S. Krane and R. M. Steffen, Phys. Rev. C 3, 240 (1971).

TECHNICAL INFORMATION DIVISION
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720