

UC Irvine

UC Irvine Previously Published Works

Title

Interference of a pair of symmetric partially coherent beams

Permalink

<https://escholarship.org/uc/item/9ds1g9k0>

Journal

Optics Express, 18(5)

ISSN

1094-4087

Authors

Garcia-Guerrero, E. E.
Mendez, E. R.
Gu, Zu-Han
[et al.](#)

Publication Date

2010-02-23

DOI

10.1364/OE.18.004816

Copyright Information

This work is made available under the terms of a Creative Commons Attribution License, available at <https://creativecommons.org/licenses/by/4.0/>

Peer reviewed

Interference of a pair of symmetric partially coherent beams

E. E. García-Guerrero^{1*}, E. R. Méndez², Zu-Han Gu³,
T. A. Leskova⁴, and A. A. Maradudin⁴

¹ Universidad Autónoma de Baja California, Facultad de Ingeniería Ensenada
Km 103 carretera Tijuana–Ensenada, Ensenada, B. C. 22870, México

² División de Física Aplicada,

Centro de Investigación Científica y de Educación Superior de Ensenada,
Carretera Ensenada-Tijuana No. 3918, Ensenada, B. C., 22860, México

³ Surface Optics Corporation

11555 Rancho Bernardo Road, San Diego, CA 92127, USA

⁴ Department of Physics and Astronomy and Institute for Surface and Interface Science,
University of California, Irvine, CA 92697, USA

*eegarcia@uabc.mx

Abstract: We study theoretically and experimentally the interference of light produced by a pair of mutually correlated Schell-model sources. The spatial distributions of the fields produced by the two sources are inverted with respect to each other through their common center in the source plane. When the beams are in phase, a bright spot appears in the center of the spatial distribution of the beam intensity. When the beams have a phase shift $\phi = \pi$, a dark spot appears in the center of the spatial distribution of the beam intensity. Experimental results that illustrate these results are included. Both bright and dark spots diverge more slowly with the increasing distance from the sources than the beam itself.

© 2010 Optical Society of America

OCIS codes: (030.1670) Coherent optical effects; (140.3300) Laser beam shaping; (350.5500) Propagation.

References and links

1. Zu-Han Gu, E. R. Méndez, M. Ciftan, T. A. Leskova, and A. A. Maradudin, "Interference of a pair of symmetric Collett-Wolf beams," *Opt. Lett.* **30**, 1605–1607 (2005).
2. E. Collett and E. Wolf, "Is complete spatial coherence necessary for the generation of highly directional light beams?," *Opt. Lett.* **2**, 27–29 (1978).
3. E. Wolf and E. Collett, "Partially coherent sources which produce the same far-field intensity distribution as a laser," *Opt. Commun.* **25**, 293–296 (1978).
4. P. DeSantis, F. Gori, G. Guattari, and C. Palma, "An example of a Collett-Wolf source," *Opt. Commun.* **29**, 256–260 (1979).
5. A. C. Schell, *The Multiple Plate Antenna*, Ph. D. Dissertation, Massachusetts Institute of Technology, 1961, Section 7.5.
6. L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, New York, 1995), Section 4.3.2.
7. J. J. Foley and M. S. Zubairy, "The directionality of Gaussian Schell-model beams," *Opt. Commun.* **26**, 297–300 (1978).
8. Ref. 6, Section 5.2.1.
9. R. M. Fitzgerald, T. A. Leskova, and A. A. Maradudin, "Control of coherence of light scattered from a one-dimensional randomly rough surface that acts as a Schell-model source," *J. Lumin.* **125**, 147–155 (2007).

1. Introduction

In a recent paper [1] the interference of light produced by a pair of mutually correlated Gaussian Schell-model sources [2,3] was investigated. The spatial distributions of the fields produced by these sources were assumed to be symmetric with respect to a plane through their common center, and to differ by a phase factor $\exp(i\phi)$. One of the results of this investigation was that when $\phi = 0$ the resulting radiation is a beam with an intensity distribution that displays a narrow bright line at its center. When the parameters characterizing the Gaussian Schell-model source are chosen in such a way that it becomes a Collett-Wolf source [4], the resulting bright line diverges much more slowly than the radiated beam itself. These theoretical predictions were confirmed experimentally, and suggested that the interference of a pair of correlated Collett-Wolf beams can be used to produce a pseudo-nondiffracting beam.

In this paper we extend the investigation in [1] in several ways. First of all we assume that the spatial distributions of the fields produced by the two sources are inverted with respect to each other through their common center in the source plane and differ by a phase factor $\exp(i\phi)$. Second we assume a more general expression for the cross-spectral density in the source plane of each of the two sources producing the interfering beams than was used in [1], namely a Schell-model source [5] instead of a Gaussian Schell-model source. On the basis of scalar diffraction theory, we obtain an expression for the mean intensity of the field produced by the interference of these two beams in terms of the spectral density and spectral degree of coherence of each source. This result is illustrated by applying it to situations in which the cross-spectral density function in the source plane has a non-Gaussian form.

It is found that the angular divergence of the bright spot in the intensity distribution of the field produced by the interference of the fields produced by two sources with these properties is determined by the width of the initial beams and, as a result, is considerably smaller than that of the initial beams, which is determined by the spectral degree of coherence. Since the divergence of the interference feature can be much smaller than that of a beam of the same size, this result supports the suggestion that the interference of two mutually correlated beams produced by Schell-model sources can be used to produce a pseudo-nondiffracting beam.

2. The Radiated Intensity Distribution

We denote a component of the radiated field in the source plane ($x_3 = 0$) at frequency ω by $U(\mathbf{x}_{\parallel}, 0|\omega)$, where $\mathbf{x}_{\parallel} = (x_1, x_2, 0)$ is an arbitrary point in this plane. The cross-spectral density of this field in the source plane is then defined by [6]

$$W^{(0)}(\mathbf{x}_{\parallel}, 0|\mathbf{x}'_{\parallel}, 0) = \langle U(\mathbf{x}_{\parallel}, 0|\omega)U^*(\mathbf{x}'_{\parallel}, 0|\omega) \rangle, \quad (1)$$

where the angle brackets denote an average over the ensemble of realizations of the functions $\{U(\mathbf{x}_{\parallel}, 0|\omega)\}$. Since everything in this paper occurs at frequency ω , we omit it in writing the cross-spectral density. The superscript (0) in Eq. (1) and in subsequent expressions emphasizes that the corresponding quantity refers to the source plane $x_3 = 0$.

The class of cross-spectral density functions $W^{(0)}(\mathbf{x}_{\parallel}, 0|\mathbf{x}'_{\parallel}, 0)$ that we consider in this paper has the Schell-model form [5]

$$W^{(0)}(\mathbf{x}_{\parallel}, 0|\mathbf{x}'_{\parallel}, 0) = [S^{(0)}(\mathbf{x}_{\parallel})]^{\frac{1}{2}} g^{(0)}(\mathbf{x}_{\parallel} - \mathbf{x}'_{\parallel}) [S^{(0)}(\mathbf{x}'_{\parallel})]^{\frac{1}{2}}. \quad (2)$$

In this expression $S^{(0)}(\mathbf{x}_{\parallel}) = \langle |U(\mathbf{x}_{\parallel}, 0|\omega)|^2 \rangle$ is the spectral density (intensity) of the light at a typical point in the source plane, and $g^{(0)}(\mathbf{x}_{\parallel} - \mathbf{x}'_{\parallel})$ is the spectral degree of coherence of the source in the source plane. It is Hermitian,

$$g^{(0)}(\mathbf{x}_{\parallel} - \mathbf{x}'_{\parallel}) = g^{(0)}(\mathbf{x}'_{\parallel} - \mathbf{x}_{\parallel})^*, \quad (3)$$

and has the additional properties [6]

$$0 \leq |g^{(0)}(\mathbf{x}_{\parallel} - \mathbf{x}'_{\parallel})| \leq 1 \quad (4)$$

and

$$g^{(0)}(\mathbf{0}) = 1. \quad (5)$$

In the Fresnel approximation, the field that propagates in the x_3 direction can be expressed in terms of the field in the source plane as [7]

$$U(\mathbf{x}_{\parallel}, x_3 | \omega) = \left(\frac{\omega}{2\pi i c x_3} \right) \exp \left[i \frac{\omega}{c} x_3 \right] \int d^2 x'_{\parallel} \exp \left[i \frac{\omega}{2c x_3} (\mathbf{x}_{\parallel} - \mathbf{x}'_{\parallel})^2 \right] U(\mathbf{x}'_{\parallel}, 0 | \omega). \quad (6)$$

Let us consider a quasi-monochromatic beam in free space that is a superposition of two beams produced by mutually correlated Schell-model sources characterized by identical cross-spectral density functions of the form given by Eqs. (1), (2), so that

$$U(\mathbf{x}_{\parallel}, x_3 | \omega) = U_1(\mathbf{x}_{\parallel}, x_3 | \omega) + U_2(\mathbf{x}_{\parallel}, x_3 | \omega) \exp(i\phi), \quad (7)$$

where ϕ is a phase independent of the coordinates. The spatial distributions of the fields are assumed to be symmetric with respect to inversion in the $x_1 x_2$ plane,

$$U_2(x_1, x_2, x_3 | \omega) = U_1(-x_1, -x_2, x_3 | \omega). \quad (8)$$

Since the spatial distributions of the fields of the two beams obey the symmetry conditions expressed by Eq. (8) at a distance x_3 , they also obey the same conditions at $x_3 = 0$, i.e. in the source plane.

The mean intensity at a distance x_3 from the source plane is given by

$$\begin{aligned} \langle I(\mathbf{x}_{\parallel}, x_3 | \omega) \rangle &= \langle |U(\mathbf{x}_{\parallel}, x_3 | \omega)|^2 \rangle \\ &= \left(\frac{\omega}{2\pi c x_3} \right)^2 \int d^2 x'_{\parallel} \int d^2 x''_{\parallel} \exp \left[i \frac{\omega}{2c x_3} (\mathbf{x}_{\parallel} - \mathbf{x}'_{\parallel})^2 \right] \\ &\quad \times \exp \left[-i \frac{\omega}{2c x_3} (\mathbf{x}_{\parallel} - \mathbf{x}''_{\parallel})^2 \right] \langle U(\mathbf{x}'_{\parallel}, 0 | \omega) U^*(\mathbf{x}''_{\parallel}, 0 | \omega) \rangle \\ &= \left(\frac{\omega}{2\pi c x_3} \right)^2 \int d^2 x'_{\parallel} \int d^2 x''_{\parallel} \exp \left[i \frac{\omega}{2c x_3} (\mathbf{x}_{\parallel} - \mathbf{x}'_{\parallel})^2 \right] \\ &\quad \times \exp \left[-i \frac{\omega}{2c x_3} (\mathbf{x}_{\parallel} - \mathbf{x}''_{\parallel})^2 \right] \left\{ 2W^{(0)}(\mathbf{x}'_{\parallel}, 0 | \mathbf{x}''_{\parallel}, 0) \right. \\ &\quad \left. + \langle U_1(\mathbf{x}'_{\parallel}, 0 | \omega) U_2^*(\mathbf{x}''_{\parallel}, 0 | \omega) \rangle \exp(-i\phi) + \langle U_1^*(\mathbf{x}''_{\parallel}, 0 | \omega) U_2(\mathbf{x}'_{\parallel}, 0 | \omega) \rangle \exp(i\phi) \right\}. \quad (9) \end{aligned}$$

In view of the spatial symmetry of the fields the ensemble averages $\langle U_1(\mathbf{x}'_{\parallel}, 0 | \omega) U_2^*(\mathbf{x}''_{\parallel}, 0 | \omega) \rangle$ and $\langle U_1^*(\mathbf{x}''_{\parallel}, 0 | \omega) U_2(\mathbf{x}'_{\parallel}, 0 | \omega) \rangle$ entering Eq. (9) can be written in the forms

$$\begin{aligned} \langle U_1(\mathbf{x}'_{\parallel}, 0 | \omega) U_2^*(\mathbf{x}''_{\parallel}, 0 | \omega) \rangle &= \langle U_1(x'_1, x'_2, 0 | \omega) U_1^*(-x''_1, -x''_2, 0 | \omega) \rangle \\ &= W^{(0)}(\mathbf{x}'_{\parallel}, 0 | -\mathbf{x}''_{\parallel}, 0) \end{aligned} \quad (10a)$$

and

$$\begin{aligned} \langle U_1^*(\mathbf{x}''_{\parallel}, 0 | \omega) U_2(\mathbf{x}'_{\parallel}, 0 | \omega) \rangle &= \langle U_2^*(-x''_1, -x''_2, 0 | \omega) U_2(x'_1, x'_2, 0 | \omega) \rangle \\ &= W^{(0)}(\mathbf{x}'_{\parallel}, 0 | -\mathbf{x}''_{\parallel}, 0). \end{aligned} \quad (10b)$$

With these results the mean intensity of the beam, Eq. (9), becomes

$$\langle I(\mathbf{x}_{\parallel}, x_3 | \omega) \rangle = \langle I_+(\mathbf{x}_{\parallel}, x_3 | \omega) \rangle + \cos \phi \langle I_-(\mathbf{x}_{\parallel}, x_3 | \omega) \rangle, \quad (11)$$

where

$$\begin{aligned} \langle I_{\pm}(\mathbf{x}_{\parallel}, x_3 | \omega) \rangle &= 2 \left(\frac{\omega}{2\pi c x_3} \right)^2 \int d^2 x'_{\parallel} \int d^2 x''_{\parallel} \exp \left[i \frac{\omega}{2c x_3} (\mathbf{x}_{\parallel} - \mathbf{x}'_{\parallel})^2 \right] \\ &\times \exp \left[-i \frac{\omega}{2c x_3} (\mathbf{x}_{\parallel} - \mathbf{x}''_{\parallel})^2 \right] W^{(0)}(\mathbf{x}'_{\parallel}, 0 | \pm \mathbf{x}''_{\parallel}, 0). \end{aligned} \quad (12)$$

We substitute into Eq. (12) the expression for $W^{(0)}(\mathbf{x}'_{\parallel}, 0 | \pm \mathbf{x}''_{\parallel}, 0)$ given by Eq. (2) and obtain

$$\begin{aligned} \langle I_{\pm}(\mathbf{x}_{\parallel}, x_3 | \omega) \rangle &= 2 \left(\frac{\omega}{2\pi c x_3} \right)^2 \int d^2 x'_{\parallel} \int d^2 x''_{\parallel} g^{(0)}(\mathbf{x}'_{\parallel} \mp \mathbf{x}''_{\parallel}) \left[S^{(0)}(\mathbf{x}'_{\parallel}) \right]^{\frac{1}{2}} \\ &\times \left[S^{(0)}(\pm \mathbf{x}''_{\parallel}) \right]^{\frac{1}{2}} \exp \left[i \frac{\omega}{2c x_3} (x_{\parallel}^{\prime 2} - x_{\parallel}^{\prime\prime 2}) \right] \exp \left[-i \frac{\omega}{c x_3} (\mathbf{x}'_{\parallel} - \mathbf{x}''_{\parallel}) \cdot \mathbf{x}_{\parallel} \right]. \end{aligned} \quad (13)$$

The changes of variables $\mathbf{x}'_{\parallel} = \pm \mathbf{x}''_{\parallel} + \mathbf{u}_{\parallel}$ followed by the change $\pm \mathbf{x}''_{\parallel}$ to \mathbf{x}''_{\parallel} transform Eq. (13) into

$$\begin{aligned} \langle I_+(\mathbf{x}_{\parallel}, x_3 | \omega) \rangle &= 2 \left(\frac{\omega}{2\pi c x_3} \right)^2 \int d^2 u_{\parallel} g^{(0)}(\mathbf{u}_{\parallel}) \exp \left[i \frac{\omega}{2c x_3} u_{\parallel}^2 \right] \exp \left[-i \frac{\omega}{c x_3} \mathbf{x}_{\parallel} \cdot \mathbf{u}_{\parallel} \right] \\ &\times \int d^2 x''_{\parallel} \exp \left\{ i \frac{\omega}{c x_3} [\mathbf{u}_{\parallel} \cdot \mathbf{x}''_{\parallel}] \right\} \left[S^{(0)}(|\mathbf{x}''_{\parallel} + \mathbf{u}_{\parallel}|) \right]^{\frac{1}{2}} \left[S^{(0)}(\mathbf{x}''_{\parallel}) \right]^{\frac{1}{2}} \end{aligned} \quad (14a)$$

$$\begin{aligned} \langle I_-(\mathbf{x}_{\parallel}, x_3 | \omega) \rangle &= 2 \left(\frac{\omega}{2\pi c x_3} \right)^2 \int d^2 u_{\parallel} g^{(0)}(\mathbf{u}_{\parallel}) \exp \left[i \frac{\omega}{2c x_3} u_{\parallel}^2 \right] \exp \left[-i \frac{\omega}{c x_3} \mathbf{x}_{\parallel} \cdot \mathbf{u}_{\parallel} \right] \\ &\times \int d^2 x''_{\parallel} \exp \left\{ i \frac{\omega}{c x_3} [(\mathbf{u}_{\parallel} - 2\mathbf{x}_{\parallel}) \cdot \mathbf{x}''_{\parallel}] \right\} \left[S^{(0)}(|\mathbf{x}''_{\parallel} + \mathbf{u}_{\parallel}|) \right]^{\frac{1}{2}} \left[S^{(0)}(\mathbf{x}''_{\parallel}) \right]^{\frac{1}{2}}. \end{aligned} \quad (14b)$$

Equations (11) and (14), are the main results of this section.

We now turn to several applications of these results.

3. A Gaussian Spectral Density in the Source Plane

We begin by assuming that the spectral density of the radiated field in the source plane has the Gaussian form

$$S^{(0)}(\mathbf{x}_{\parallel}) = \exp(-x_{\parallel}^2 / 2\sigma_s^2). \quad (15)$$

On carrying out the integration over \mathbf{x}''_{\parallel} in Eq. (14) with the spectral density given by Eq. (15) we obtain

$$\begin{aligned} \langle I_+(\mathbf{x}_{\parallel}, x_3 | \omega) \rangle &= \frac{1}{\pi} \left(\frac{\omega^2 \sigma_s^2}{c^2 x_3^2} \right) \int d^2 u_{\parallel} g^{(0)}(\mathbf{u}_{\parallel}) \exp \left[-i \frac{\omega}{c x_3} \mathbf{x}_{\parallel} \cdot \mathbf{u}_{\parallel} \right] \\ &\times \exp \left\{ -u_{\parallel}^2 \left[\frac{1}{8\sigma_s^2} + \frac{\omega^2 \sigma_s^2}{2c^2 x_3^2} \right] \right\}, \end{aligned} \quad (16a)$$

$$\begin{aligned} \langle I_-(\mathbf{x}_{\parallel}, x_3 | \omega) \rangle &= \frac{1}{\pi} \left(\frac{\omega^2 \sigma_s^2}{c^2 x_3^2} \right) \exp \left[-2 \frac{\omega^2 \sigma_s^2}{c^2 x_3^2} x_{\parallel}^2 \right] \int d^2 u_{\parallel} g^{(0)}(\mathbf{u}_{\parallel}) \exp \left[2 \frac{\omega^2 \sigma_s^2}{c^2 x_3^2} \mathbf{u}_{\parallel} \cdot \mathbf{x}_{\parallel} \right] \\ &\times \exp \left\{ -u_{\parallel}^2 \left[\frac{1}{8\sigma_s^2} + \frac{\omega^2 \sigma_s^2}{2c^2 x_3^2} \right] \right\}. \end{aligned} \quad (16b)$$

Equations (16) describe the evolution of the beam as it propagates away from the source plane. The angular divergence of the beam, however, is determined by the behavior of these integrals in the far-field zone where $x_3 \gg 2(\omega/c)\sigma_s^2$. In this far-field regime, and if we also assume that the width of the spectral coherence function is much smaller than that of the source, Eqs. (16) take the forms

$$\langle I_+(\mathbf{x}_{\parallel}, x_3 | \omega) \rangle = \frac{1}{\pi} \left(\frac{\omega^2 \sigma_s^2}{c^2 x_3^2} \right) \int d^2 u_{\parallel} g^{(0)}(\mathbf{u}_{\parallel}) \exp \left(-i \frac{\omega}{c x_3} \mathbf{x}_{\parallel} \cdot \mathbf{u}_{\parallel} \right), \quad (17a)$$

$$\langle I_-(\mathbf{x}_{\parallel}, x_3 | \omega) \rangle = \frac{1}{\pi} \left(\frac{\omega^2 \sigma_s^2}{c^2 x_3^2} \right) \exp \left\{ -2 \left(\frac{\omega \sigma_s}{c x_3} \right)^2 x_{\parallel}^2 \right\} \int d^2 u_{\parallel} g^{(0)}(\mathbf{u}_{\parallel}). \quad (17b)$$

Two important conclusions can be drawn from Eqs. (17). First of all, it follows from Eq. (17a) that the angle of divergence of the primary beam, associated with $\langle I_+(\mathbf{x}_{\parallel}, x_3 | \omega) \rangle$, depends on the spectral degree of coherence through what is essentially a Fourier transform operation.

In contrast, from Eq. (17b), we see that the interference feature associated with $\langle I_-(\mathbf{x}_{\parallel}, x_3 | \omega) \rangle$ is independent of the form of the spectral degree of coherence, and has a Gaussian shape $\langle I_-(\mathbf{x}_{\parallel}, x_3 | \omega) \rangle \sim \exp[-(x_{\parallel}/x_3)^2/\Delta_i^2]$, where Δ_i is the angular divergence and is given by

$$\Delta_i = \frac{1}{\sqrt{2}(\omega/c)\sigma_s}. \quad (18)$$

We note, that the superposition of the initial beam and its inverted version leads to an interference feature only within a coherence area around the center of inversion. Outside this area the beams add incoherently. Thus, it is not surprising that in the far field the divergence properties of this interference feature coincide with those of a speckle. The relative intensity of the interference peak is determined by the phase shift ϕ between the initial beam and its inverted version and can be changed from 2 ($\phi = 0$) to 0 ($\phi = \pi$) as the interference regime changes from constructive to destructive interference.

Before proceeding, it is of interest to consider two extreme limits of the secondary source. In the case where the radiated field is completely coherent, i.e. the spectral degree of coherence is $g^{(0)}(\mathbf{u}_{\parallel}) = 1$, the initial beam and its inverted version interfere over the entire beam area, so that the total intensity of the beam is

$$\langle I(\mathbf{x}_{\parallel}, x_3 | \omega) \rangle = \frac{2(1 + \cos \phi)}{1 + \left(\frac{c x_3}{2\omega \sigma_s^2} \right)^2} \exp \left\{ - \frac{x_{\parallel}^2}{2\sigma_s^2 \left[1 + \left(\frac{c x_3}{2\omega \sigma_s^2} \right)^2 \right]} \right\}. \quad (19)$$

In contrast, if the field is incoherent, $g^{(0)}(\mathbf{u}_{\parallel})$ is essentially a delta function and far field conditions occur very rapidly. As a result, the interference feature appears on a uniform background,

$$\langle I(\mathbf{x}_{\parallel}, x_3 | \omega) \rangle = \frac{A_c}{\pi} \left(\frac{\omega \sigma_s}{c x_3} \right)^2 \left\{ 1 + \cos \phi \exp \left[-2 \left(\frac{\omega \sigma_s}{c x_3} \right)^2 x_{\parallel}^2 \right] \right\}, \quad (20)$$

where

$$A_c = \int d^2 u_{\parallel} g^{(0)}(\mathbf{u}_{\parallel}) \quad (21)$$

is a measure of the area under the coherence function.

We now consider different forms of the spectral degree of coherence, illustrating the results through some examples.

4. Examples

In this section we apply the results of the preceding section to several choices for the spectral degree of coherence $g^{(0)}(u_{\parallel})$. To make comparisons among the results meaningful, we will normalize each expression for $g^{(0)}(\mathbf{u}_{\parallel})$ in such a way that as $u_{\parallel} \rightarrow 0$ it has the form

$$g^{(0)}(\mathbf{u}_{\parallel}) = 1 - \frac{u_{\parallel}^2}{2\sigma_g^2} + o(u_{\parallel}^2). \quad (22)$$

In all calculations carried out here the values of the parameters σ_s , σ_g , and the wavelength λ are $\sigma_s = 2.432$ mm, $\sigma_g = 179.6 \mu\text{m}$, and $\lambda = 632.8$ nm. They correspond to the values characterizing the Collett-Wolf source studied experimentally in Ref. [1].

4.1. *Gaussian spectral degree of coherence:* $g^{(0)}(u_{\parallel}) = \exp(-u_{\parallel}^2/2\sigma_g^2)$.

For this circularly symmetric form for $g^{(0)}(u_{\parallel})$ we use Eqs. (11) and (16) to evaluate $\langle I_{\pm}(\mathbf{x}_{\parallel}, x_3 | \omega) \rangle$ analytically, with the result

$$\langle I(\mathbf{x}_{\parallel}, x_3 | \omega) \rangle = \frac{4\sigma_s^2}{\sigma_{eff}^2(x_3)} \exp\left(-\frac{x_{\parallel}^2}{\sigma_{eff}^2(x_3)}\right) \left\{ 1 + \cos\phi \exp\left[-4\frac{\sigma_s^2}{\sigma_g^2} \frac{x_{\parallel}^2}{\sigma_{eff}^2(x_3)}\right] \right\}, \quad (23)$$

where

$$\sigma_{eff}^2(x_3) = 2\sigma_s^2 + \frac{c^2 x_3^2}{\omega^2} \left(\frac{1}{2\sigma_s^2} + \frac{2}{\sigma_g^2} \right). \quad (24)$$

In Fig. 1 we present plots of the normalized intensities $I(\mathbf{x}_{\parallel}, x_3) = \langle I(\mathbf{x}_{\parallel}, x_3 | \omega) \rangle / \langle I(\mathbf{0}, x_3 | \omega) \rangle$ of the beams that are inverted with respect to their common center at two values of the distance from the source planes $x_3 = 1$ m (Figs. 1(a) and (b)), and $x_3 = 100$ m (Figs. 1(c) and (d)).

In the far field where $x_3 \gg 2(\omega/c)\sigma_s^2$ and when $\sigma_s \gg \sigma_g$ as a Collett-Wolf source [1, 4], Eq. (24) becomes

$$\sigma_{eff}(x_3) = \frac{\sqrt{2}}{(\omega/c)\sigma_g} x_3 = \Delta_b x_3, \quad (25)$$

so that Eq. (23) takes the form

$$\langle I(\mathbf{x}_{\parallel}, x_3 | \omega) \rangle = \left(\sqrt{2} \frac{\omega}{c} \frac{\sigma_s \sigma_g}{x_3} \right)^2 \exp\left\{-\frac{(x_{\parallel}/x_3)^2}{\Delta_b^2}\right\} \left[1 + \cos\phi \exp\left\{-\frac{(x_{\parallel}/x_3)^2}{\Delta_i^2}\right\} \right], \quad (26)$$

where Δ_b is the angular divergences of the beam and the angular divergence of the interference feature Δ_i is given by Eq. (18). Therefore, in the case considered ($\sigma_s \gg \sigma_g$) the bright or dark feature diverges significantly more slowly than the beam itself. The evolution of the beams that results from the interference of circularly symmetric beams as they propagate along the x_3 axis is shown in Fig. 2. It is seen that in both cases, when $\phi = 0$ and $\phi = \pi$, the central interference peak or dip diverges considerably more slowly than the beams themselves.

4.2. *Lorentzian spectral degree of coherence:* $g^{(0)}(u_{\parallel}) = 2\sigma_g^2/(2\sigma_g^2 + u_{\parallel}^2)$

For this form for $g^{(0)}(u_{\parallel})$ we use Eqs. (11) and (16) to evaluate $\langle I_{\pm}(\mathbf{x}_{\parallel}, x_3 | \omega) \rangle$. To simplify the calculations we use the fact that the spectral degree of coherence $g^{(0)}(\mathbf{u}_{\parallel}) \rightarrow g^{(0)}(u_{\parallel})$ is

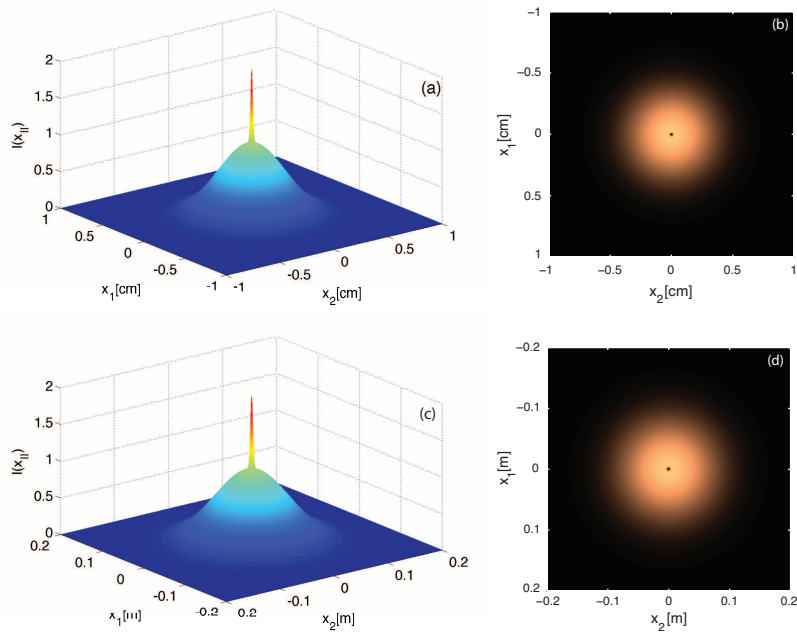


Fig. 1. The spatial distribution of the normalized intensity of the beams that are inverted with respect to their common center with (a) and (c) $\phi = 0$ and (b) and (d) $\phi = \pi$. The distance from the source plane is $x_3 = 1$ m (a) and (b), and $x_3 = 100$ m (c) and (d).

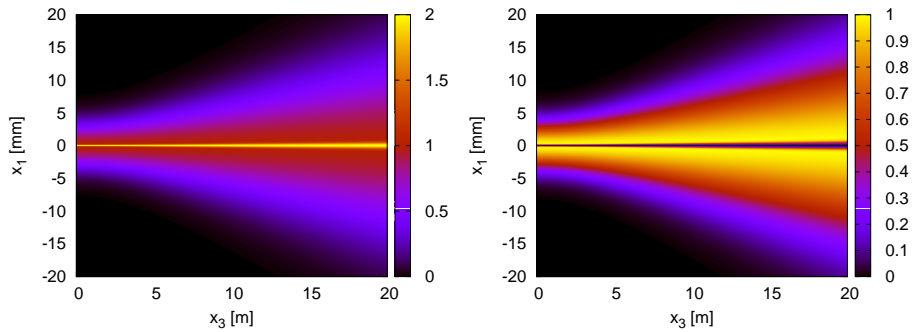


Fig. 2. Theoretical plot of the evolution along the x_3 axis of the cross section $x_2 = 0$ of the normalized intensity of the beams that are inverted with respect to their common center with $\phi = 0$ (a) and $\phi = \pi$ (b).

circularly symmetric. In this case the angular integrations in Eqs. (16) can be carried out analytically, leaving only one-dimensional integrals to evaluate numerically. Thus, if we denote the

azimuthal angles of the vectors \mathbf{x}_{\parallel} and \mathbf{u}_{\parallel} by ϕ and ϕ_u , respectively, we can rewrite Eq. (16a) as

$$\begin{aligned} \langle I_+(\mathbf{x}_{\parallel}, x_3 | \omega) \rangle &= \frac{1}{\pi} \left(\frac{\omega^2 \sigma_s^2}{c^2 x_3^2} \right) \int_0^{\infty} du_{\parallel} u_{\parallel} g^{(0)}(u_{\parallel}) \exp \left[- \left(\frac{\omega^2 \sigma_s^2}{2c^2 x_3^2} + \frac{1}{8\sigma_s^2} \right) u_{\parallel}^2 \right] \\ &\quad \times \int_{-\pi}^{\pi} d\phi_u \exp \left[-i \frac{\omega}{cx_3} x_{\parallel} u_{\parallel} \cos(\phi - \phi_u) \right] \\ &= 2 \left(\frac{\omega^2 \sigma_s^2}{c^2 x_3^2} \right) \int_0^{\infty} du_{\parallel} u_{\parallel} g^{(0)}(u_{\parallel}) J_0 \left(\frac{\omega}{cx_3} x_{\parallel} u_{\parallel} \right) \exp \left[- \left(\frac{\omega^2 \sigma_s^2}{2c^2 x_3^2} + \frac{1}{8\sigma_s^2} \right) u_{\parallel}^2 \right], \quad (27) \end{aligned}$$

where $J_0(x)$ is the Bessel function of the first kind and zero order.

The evaluation of the angular integral in the expression for $\langle I_-(\mathbf{x}_{\parallel}, x_3 | \omega) \rangle$ is only a bit more

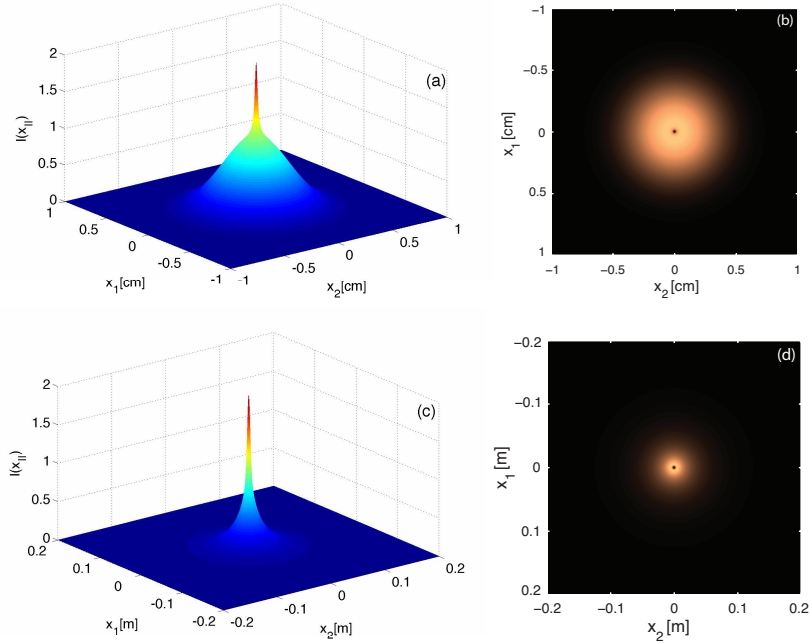


Fig. 3. The spatial distribution of the normalized intensity of the beams that have the spectral degree of coherence of the Lorentzian form and are inverted with respect to their common center with (a) and (c) $\phi = 0$ and (b) and (d) $\phi = \pi$. The distance from the source plane is $x_3 = 1$ m (a) and (b), and $x_3 = 100$ m (c) and (d).

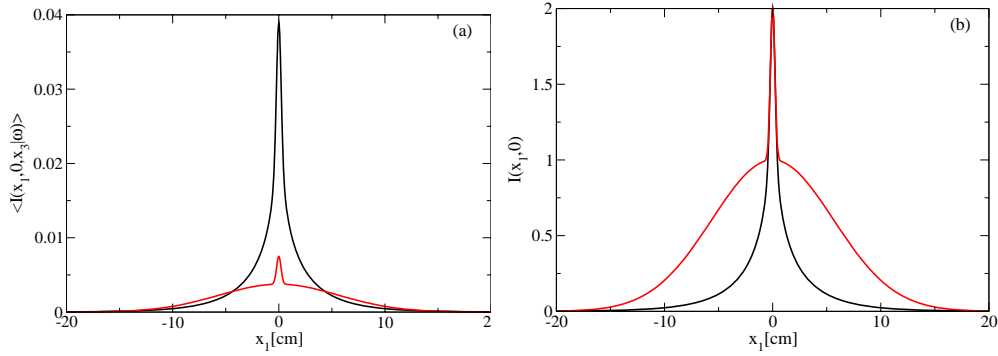


Fig. 4. The intensity (a) and the normalized intensity (b) of the beam as a function of x_1 at $x_2 = 0$ at a distance from the source plane $x_3 = 500$ m. The spectral degree of coherence of in the source plane has a Lorentzian form (black curve) or a Gaussian form (red curve).

subtle. Thus, the integral over \mathbf{u}_{\parallel} in Eq. (16b) can be written as

$$\begin{aligned}
 \langle I_-(\mathbf{x}_{\parallel}, x_3 | \omega) \rangle &= \frac{1}{\pi} \left(\frac{\omega^2 \sigma_s^2}{c^2 x_3^2} \right) \exp \left(-2 \frac{\omega^2 \sigma_s^2}{c^2 x_3^2} x_{\parallel}^2 \right) \int_0^{\infty} du_{\parallel} u_{\parallel} g^{(0)}(u_{\parallel}) \\
 &\times \exp \left[- \left(\frac{\omega^2 \sigma_s^2}{2c^2 x_3^2} + \frac{1}{8\sigma_s^2} \right) u_{\parallel}^2 \right] \int_{-\pi}^{\pi} d\phi_u \exp \left[2 \frac{\omega^2 \sigma_s^2}{c^2 x_3^2} x_{\parallel} u_{\parallel} \cos(\phi - \phi_u) \right] \\
 &= 2 \left(\frac{\omega^2 \sigma_s^2}{c^2 x_3^2} \right) \exp \left(-2 \frac{\omega^2 \sigma_s^2}{c^2 x_3^2} x_{\parallel}^2 \right) \int_0^{\infty} du_{\parallel} u_{\parallel} g^{(0)}(u_{\parallel}) I_0 \left(2 \left(\frac{\omega \sigma_s}{c x_3} \right)^2 x_{\parallel} u_{\parallel} \right) \\
 &\quad \times \exp \left[- \left(\frac{\omega^2 \sigma_s^2}{2c^2 x_3^2} + \frac{1}{8\sigma_s^2} \right) u_{\parallel}^2 \right], \tag{28}
 \end{aligned}$$

where $I_0(z)$ is the modified Bessel function of the first kind and zero order.

The results of a numerical evaluation of $\langle I_{\pm}(\mathbf{x}_{\parallel}, x_3 | \omega) \rangle$, Eqs. (27) and (28), are shown in Fig. 3. The striking result that can be easily seen when comparing Figs. 3(a) and 3(c) is that in the far field [Fig. 3(c)] the beam narrows down to the interference peak. This is easy to understand on the basis of Eq. (16a). Indeed, as $u_{\parallel} \rightarrow \infty$ the decay of the integrand is determined by the Gaussian function $\exp[-u_{\parallel}^2/(8\sigma_s^2)]$, rather than by the asymptotic behavior of the spectral degree of coherence $g^{(0)}(\mathbf{u}_{\parallel})$, as is the case for the Gauss-Schell model source. Therefore, the angle of divergence of the beam is determined by its initial width σ_s , $\Delta_b = c/(\sqrt{2}\omega\sigma_s)$, rather than by the correlation length σ_g . The analogous effect was pointed out in Ref. [9] for a one dimensional partially coherent beam whose spectral degree of coherence has a Lorentzian form.

To demonstrate this more clearly, in Fig. 4 we present the cross-sections $x_2 = 0$ of the far field intensity distribution of the beam produced by a pair of Schell-model sources whose spectral degree of coherence has a Lorentzian form, which produce fields inverted with respect to each other through their common center in the source plane and are in phase. The distance from the source plane is $x_3 = 500$ m. For comparison, in the same Figure we plot the cross-section of the far field intensity distribution of the beam produced by the interference of the two Collett-Wolf beams (red lines). For clarity, we present the absolute intensities in Fig. 4(a), and the normalized intensities in Fig. 4(b). We also point out that the integrated intensities of the two beams coincide, as they should.

This example shows that by choosing the spectral degree of coherence suitably, e.g. with a

Lorentzian form, the beam can be narrowed to the point where it is as narrow as the interference feature. We have not sought other functional forms for $g^{(0)}(\mathbf{u}_{\parallel})$ for which this is the case, but conjecture that they exist.

4.3. *Spectral degree of coherence* $g^{(0)}(\mathbf{u}_{\parallel}) = \text{sinc}(\sqrt{3}u_1/\sigma_g)\text{sinc}(\sqrt{3}u_2/\sigma_g)$

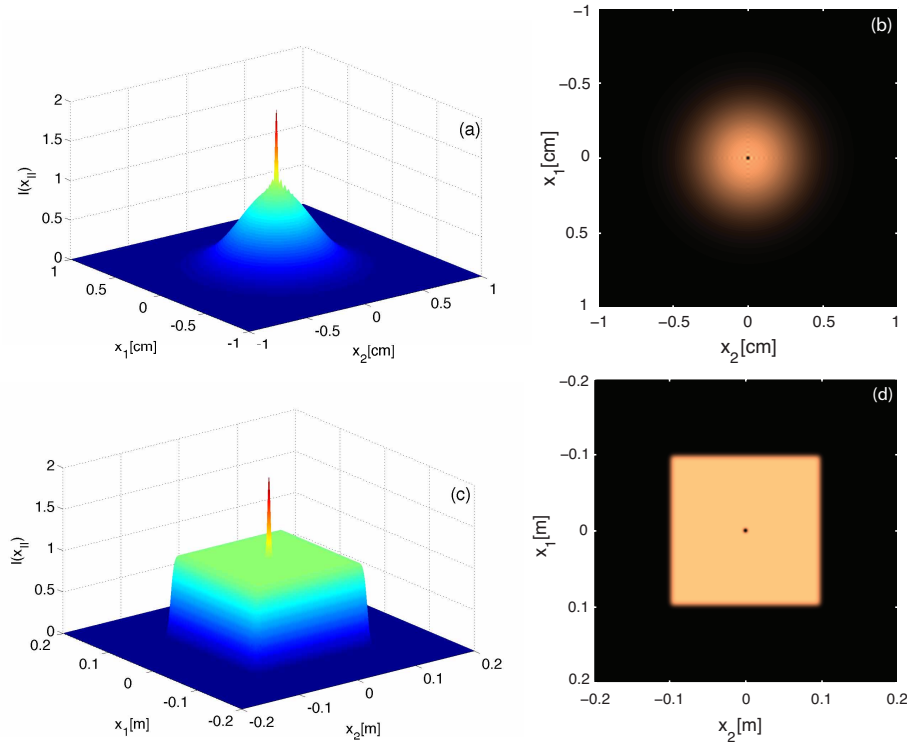


Fig. 5. Plots of the normalized intensity of the beams that have the spectral degree of coherence of the form $g^{(0)}(\mathbf{u}_{\parallel}) = \text{sinc}(\sqrt{3}u_1/\sigma_g)\text{sinc}(\sqrt{3}u_2/\sigma_g)$, and are inverted with respect to their common center with $\phi = 0$ (a) and (c) and $\phi = \pi$ (b) and (d). The distance from the source plane is $x_3 = 1$ m (a) and (b), and $x_3 = 100$ m (c) and (d).

For this non-circularly symmetric form for the spectral degree of coherence $g^{(0)}(u_{\parallel})$ we use Eqs. (11) and (16) to evaluate $\langle I_{\pm}(\mathbf{x}_{\parallel}, x_3 | \omega) \rangle$ numerically. The results are shown in Fig. 5.

The beam characterized by such a form of the spectral degree of coherence evolves on propagation into a flat top beam in a square domain. The angle of divergence of this beam is $\Delta_b = \sqrt{3}c/(\omega\sigma_g)$.

5. A General Circularly Symmetric Spectral Density in the Source Plane

Although the Gaussian form [Eq. (15)] is perhaps the most common form of the spectral density in the source plane, it is not the only one that has been used. As an example in this section we consider the interference of the beams produced by the sources whose spectral densities although circularly symmetric are not Gaussian, namely, they are constant in the circular domain (“flat top” beam). The beams are inverted with respect to their common center and can acquire an additional phase shift π .

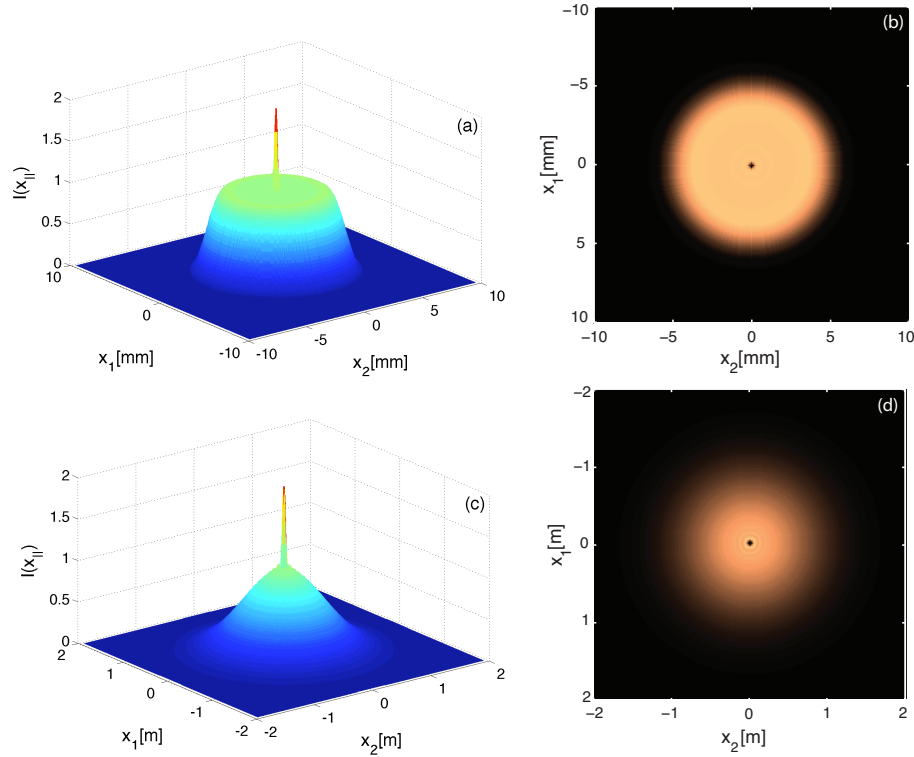


Fig. 6. Plots of the normalized intensity of the beams whose spectral density in the source plane is a constant within a circular domain and their spectral degree of coherence has a Gaussian form. The beams fields are inverted with respect to their common center with $\phi = 0$ (a) and (c) and $\phi = \pi$ (b) and (d). The distance from the source plane is $x_3 = 1$ m (a) and (b), and $x_3 = 1$ Km (c) and (d).

We assume that

$$S^{(0)}(\mathbf{x}_{\parallel}) = \begin{cases} 1, & x_{\parallel} < \sigma_g \\ 0, & x_{\parallel} > \sigma_g \end{cases}, \quad (29a)$$

$$g^{(0)}(\mathbf{x}_{\parallel}) = \exp(-x_{\parallel}^2/2\sigma_g^2). \quad (29b)$$

For this form for $S^{(0)}(\mathbf{x}_{\parallel})$ we use Eqs. (11) and (14) to evaluate $\langle I_{\pm}(\mathbf{x}_{\parallel}, x_3 | \omega) \rangle$ numerically. The results of this numerical evaluation are presented in Fig. 6.

6. Experimental Details and Results

The schematic diagram of the optical system employed is shown in Fig. 7. The illumination is provided by a HeNe laser beam ($\lambda = 633$ nm). After passing through a spatial filter, the diverging Gaussian beam passes through a rotating ground glass. The light emerging from the diffuser is allowed to propagate a small distance and illuminate a circular aperture, behind which we placed a collimating lens. The light emerging from the lens constitutes a secondary light source that can be regarded as a partially coherent source with a Gaussian spectral degree of coherence and a uniform intensity distribution within a circular domain. The radius of the beam is approximately 1 cm and the parameter $\sigma_g \approx 1$ mm. The value of σ_g is simply the size

of the speckle produced by the light transmitted through the rotating diffuser at the plane of the secondary source (plane of the lens), and was estimated from the size of the illumination beam on the diffuser and its distance to the lens.

To study the interference of the field produced by a pair of such partially coherent beams, the original beam is sent to a modified Michelson interferometer. The modification consists of the insertion of a spherical lens in one of the arms of the interferometer in such a way that the wave front is reversed about the axis of the lens. For this, the mirror of that arm of the interferometer must be placed on the focal plane of the lens. The output of the interferometer then consists of the superposition of a pair of partially coherent beams, where the first beam is symmetric with respect to the second about their common center.

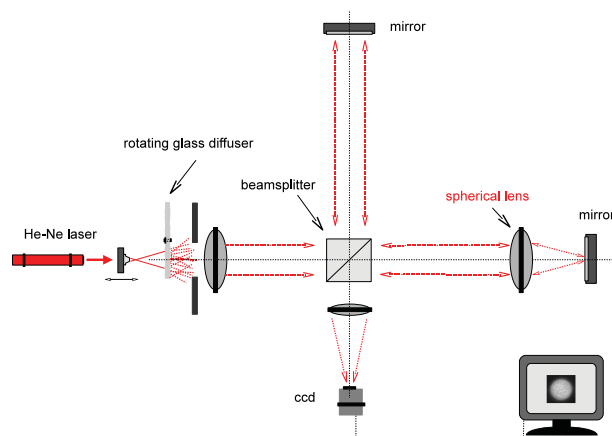


Fig. 7. Schematic diagram of the experimental arrangement used.

When the beams are in phase, a bright spot appears in the center of the beam spot. The experimental interference pattern showing constructive interference is shown in Fig. 8(a). When the mirrors of the modified Michelson interferometer are arranged to induce a path difference of $\lambda/2$, the output is a pair of Collet-Wolf beams that have a phase shift $\phi = \pi$. The resulting pattern displays a dark spot in the spatial distribution of the intensity, as in Fig. 8(b).

7. Summary and conclusion

We have shown theoretically and experimentally that the interference of the radiation produced by a pair of mutually correlated Schell-model sources produces a beam in the far field with an intensity distribution that displays a narrow bright or dark dot of small radius at its center, both of which diverge much more slowly with the increasing distance from the source plane than the beam itself. The bright dot arises when the fields produced by the two sources are inverted with respect to each other through their common center in the source plane, and are in phase. In the case where the inverted fields produced by the two sources are out of phase their interference produces the dark dot.

These results support the suggestion that the interference of the beams produced by two mutually correlated Schell-model sources can be used for the creation of a pseudo-nondiffracting beam.

We have also shown that when the spectral density of each source has a Gaussian form, while the spectral degree of coherence of each source has a Lorentzian form, the initial beams evolve upon propagation in such a way that in the far field the angular divergence of the initial beams

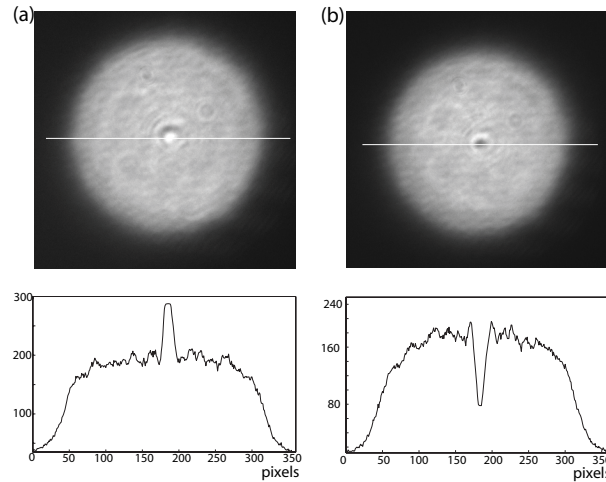


Fig. 8. Experimental gray-level images of the normalized intensity of the symmetric beams for (a) constructive interference and (b) destructive interference. The horizontal lines in the images show the positions where the intensity scans in the lower graphs were taken.

coincides with the angular divergence of the interference peak.

Acknowledgements

A. A. M. would like to thank Professors R. Righini and S. Califano for the hospitality of the European Laboratory for Nonlinear Spectroscopy (LENS) where his contribution to this work was carried out.

The research of E.E.G-G. was supported in part by projects PROMEP/103.5/07/2599 and the internal project 471, the research of E.R.M was supported by the Consejo Nacional de Ciencia y Tecnología (grant 47712-F), and the research of Z.-H.G., T.A.L., and A.A.M. was supported in part by AFRL contract FA9453-08-C-0230.