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AN AMPLITUDE ANALYSIS FOR THE REACTION $\pi^+ p \to \omega\Delta^{++}$ AT 7.0 GeV/c*

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ABSTRACT: An amplitude analysis of the reaction $\pi^+ p \to \pi^+\pi^-\pi^0\Delta^{++}$ at 7 GeV/c has been performed in the ω mass region ($0.740 < M(3\pi)^0 < 0.820$ GeV). Using the isobar model for the $(3\pi)^0$ system together with the Δ^{++} decay moments, we have determined the background under the omega to be an $(\epsilon\pi)$ S-wave. The production mechanism of this reaction has also been studied and the results have been compared with the predictions of an absorptive Regge-exchange model and of the quark model.

Results on the quasi-2-body reaction

$$\pi^+ p \to \omega\Delta^{++} \tag{1}$$

have been previously reported at different energies,[1] using the information from the joint decay angular distribution of the ω and the Δ^{++}. The dominant exchange processes could partially be deduced; however, results in this form cannot be unfolded to give production amplitudes because of large errors on individual correlated moments. On the other hand, if only single

ω or Δ^{++} moments are used, correlation information is lost; this type of analysis is further complicated by the presence of background which cannot be extracted beneath the resonances. Consequently, we have chosen to apply an amplitude analysis to the reaction

$$\pi^+ p \rightarrow \pi^+ \pi^- \pi^0 \Delta^{++}. \tag{2}$$

The technique used was originally developed[2] for an isobar model analysis of the formation reaction $\pi N \rightarrow \pi\ \pi N$ at low energy, and modified[3] to study the production reaction $\pi^+ p \rightarrow (3\pi)^+ p$. The latter analysis produced results consistent with those obtained by a density-matrix analysis using an independent fitting program.[4] We will first describe how the data were selected, then the method we used and our notations, and then the results we obtained. We will end with some comparisons with theoretical predictions.

The data that we used have been previously studied[1b] with a density matrix analysis of the ω. Experimental details together with cross sections were then published. In this analysis, the events of reaction (1) have been selected from approximately 86000 events (σ = 2.16 ± 0.09 mb) fitting the reaction:

$$\pi^+ p \rightarrow \pi^+ \pi^- \pi^0 p \pi^+ \tag{3}$$

by imposing the mass selections: $0.740 < M(3\pi)^0 < 0.820$ GeV and $1.16 < M(p\pi) < 1.28$ GeV. Less than 2% of those events had 2 ambiguous Δ^{++} sections. These were weighted by the Δ Breit-Wigner formula. Then we selected events with momentum transfer $|t_{p\Delta}| < 1.0$ GeV^2; the data were binned in $|t|$ so that the minimum number of events was $\sim$ 400 per bin, the total number of events so selected being 3075. As in the method described in Ref. 2 and 3, we used the isobar model, i.e. we approximated the $(3\pi)^o$ state with spin -parity J^P, helicity M and isospin I by a resonance (isobar) decaying into 2π (ε,ρ,f) of spin ℓ, and a pion with relative angular momentum L.

If we abbreviate by K all $(3\pi)^o$ quantum numbers except M, (i.e. K represents $IJ^PL\ \varepsilon\pi/\rho\pi/f\pi$), the most general amplitude for reaction (2) can be written as:

$$T_{\lambda_p,\lambda'_p} = \sum_{K,M}\sum_{\lambda_\Delta} G^{K,M}(\alpha,\beta,\gamma,\Theta_h)\cdot T^{M}_{\lambda_p\lambda_\Delta}(K,M_{3\pi},t)\cdot D^{*3/2}_{\lambda_\Delta\lambda'_p}(\Omega_\Delta)$$

where λ_p,λ_Δ, and λ'_p are the helicities of the incoming proton, delta, and outgoing proton respectively; $D^{*3/2}_{\lambda_\Delta\lambda'_p}(\Omega_\Delta)$ and $G(\alpha,\beta,\gamma,\theta_h)$ describe the decay of the Δ and the $(3\pi)^o$ system respectively, θ_h is the helicity decay angle for the isobar, α,β and γ are the Euler angles defining a rotation from the production coordinate system to a system with Z axis in the meson rest frame parallel to the vector

$$\vec{C} = \vec{q}_\pi + \frac{t-m_\pi^2}{2s}(\vec{q}_p + \vec{q}_\Delta)$$

In this expression $\vec{q}_\pi,\vec{q}_p$ and $\vec{q}_\Delta$ are the momenta of the incoming π, proton and outgoing Δ respectively, s is the square of the center of mass energy, and m_π is the mass of the meson π. This vector $\vec{C}$ first introduced by Cho and Sakurai,[5] was motivated by the coupling of vector mesons to a conserved current; it has the property that it leads to helicity conservation in the meson system[6] and was consequently used in an amplitude analysis of higher mass $(3\pi)^o$ systems.[7] Due to the proximity of the $\vec{C}$ coordinate system to the t-channel system where no waves with helicity flip of 2 units are observed,[3] there should not exist amplitudes with an helicity flip M=2 at the meson vertex. In a similar way, the Z axis in the Δ rest frame is chosen to be the vector:

$$\vec{C}' = \vec{q}_p + \frac{t-m_p^2}{2\,s}\,(\vec{q}_\pi + \vec{q}_{3\pi})$$

and by analogy with what has been said above, we propose to neglect the waves with two units of helicity flip (μ =2) at the proton- vertex.

Parity conservation in the production process restricts the number of independent helicity amplitudes. In particular by reflecting in the production plane, the helicity amplitudes are such that:[8]

$$T^M_{\lambda_p\lambda_\Delta} = \mathscr{P}(-1)^{J-M-\lambda_p-\lambda_\Delta} \cdot T^{-M}_{-\lambda_p-\lambda_\Delta}$$

where $\mathscr{P}$ is the product of the intrinsic parities, equaling $(-1)^{L+\ell}$ for the Δ^{++}(and $(-1)^{L+\ell+1}$ for the $(p\pi)$S wave ($J^P = \frac{1}{2}^-$) that we include to describe the background under the Δ^{++}). Performing the Kronecker-product decomposition of the baryon spin representation, we can also write for the Δ:

$$T^M_{\lambda_p\lambda_\Delta} = \sum_{S,\mu} \left\langle \frac{1}{2}\ \lambda_p S\ \mu \,\middle|\, \frac{3}{2}\ \lambda_\Delta \right\rangle T^M_{S\mu}$$

where S is the exchanged spin between the incoming proton and outgoing Δ, and μ is the helicity flip (its Z component). A similar expression can be worked out for the $(p\pi)$ S wave with the necessary modifications. The amplitudes $T^M_{S\mu}$ have the following parity property:

$$T^M_{S\mu} = (-1)^{L+\ell+J-M+S-\mu}\ T^{-M}_{S-\mu}.$$

These restrictions are built into our formalism as follows. If we define $Y_m = (-1)^{L+\ell+J-|M|}$ for the meson vertex, while $Y_B = (-1)^{S-|\mu|}$ for the Δ^{++}

(S=1 or 2), or $Y_B = (-1)^{S-|\mu|+1}$ for the $(p\pi)$ S wave (S=0), then for a given J and S, the amplitude can be expressed in a manner which manifests parity conservation for the whole production reaction:

$$\left[T^{M}_{S\mu} + \eta_m Y_m T^{-M}_{S\mu}\right] + \eta_B Y_B \left[T^{M}_{S-\mu} + \eta_m Y_m T^{-M}_{S-\mu}\right],$$

where the quantities η_m and η_B can have the values ± 1. One observes that unless $\eta_m = \eta_B = \eta$, this expression vanishes. To the order (1/s), $\eta = +1$ (-1) corresponds to natural (unnatural) parity exchange.[9] As a matter of fact, in addition to the simple expression of parity conservation, there exists another justification for our use of this particular decomposition: it is suitable for testing some quark-model predictions as we will see below.

Under these conditions, the unpolarized differential cross-section W including a symmetric matrix A which depends on the Δ^{++} decay distribution, is given by:

$$W = \sum_{S,\bar{S}|\mu||\bar{\mu}|} \sum_{K\bar{K}\eta\bar{\eta}|M||\bar{M}|} A^{\eta\bar{\eta}}_{S|\mu|\bar{S}|\bar{\mu}|} \cdot G^{K\eta|M|} \cdot G^{*\bar{K}\bar{\eta}|\bar{M}|} \cdot T^{\eta|M|}_{S\mu} \cdot T^{*\bar{\eta}|\bar{M}|}_{\bar{S}|\bar{\mu}|} .$$

The amplitudes $T^{\eta|M|}_{S|\mu|}$ are parameters in a maximum likehood fit.[10] More details can be found in Ref. 11.

Table I gives the quantum numbers of the meson states considered in this analysis and the allowed exchanged quantum numbers at the proton-Δ vertex (colume 1). Columns 2 and 3 list combinations, taking into account the parity restrictions exposed above, with abbreviated names for the waves that we will refer to hereafter. For waves that are significantly different from zero, the results are displayed on Fig. 1, which shows the proportion of the different so-called P waves (S=1), as well as the $(\varepsilon\pi)$S wave (S-2), called Z, as a function of $|t_{p\Delta}|$ (Fig. 1a and 1b); one sees that only 3 waves are significantly bigger than 10%, corresponding to both natural exchange (P_{++}) and to unnatural exchange (P_{00}) for the production of the omega as well as the background determined to be an $(\varepsilon\pi)$S-state; the $(p\pi)$ S-wave has been found to be negligible under the Δ^{++} (Table I). We observe that all these waves are important at low $|t|$. The last fact is in contradiction with the results of Ref. 1a. The average ratio of natural to unnatural exchange is 0.8 over the whole $|t|$ range. The complete variation of this ratio as a function of $|t|$ is consistent with the ratio determined in Ref. 1b.

Regarding the relative phases between these waves, we have found that the 3 smallest unnatural exchange waves (P_{0-}, P_{-0}, P_{--}) were in phase; Fig. 1c shows these phases (relative to P_{00}). We have subsequently considered them as real. Finally we have found only one ambiguous solution compatible with a change of phase of 180°, so we consider our result as coming from a unique solution.

In order to interpret these results, we have used an absorptive Regge exchange model, and by analogy with what has been done in Ref. 12 we have parametrized the amplitudes in the s-channel by:

$$P^{(s)}_{++} = C_A - 4tA_N, \qquad P^{(s)}_{--} = - C_A + 4tA_U, \qquad P^{(s)}_{00} = M_\omega M'_\Delta A_U,$$

$$P^{(s)}_{0-} = - 2 M'_\Delta \sqrt{-t}\, A_U \quad \text{and} \quad P^{(s)}_{-0} = -2 M_\omega \sqrt{-t}\, A_U \ ;$$

C_A is a cut effect (parametrized by $\gamma e^{\alpha|t|}$)required by the fact that neither P_{++} nor P_{--} vanishes in the forward direction; A_N and A_U

(whose t dependence is parametrized by $\gamma_j e^{\alpha_j |t|} \cdot e^{-\frac{i\pi}{2}(\beta_j + \delta_j t) - \frac{i\pi}{2}}$, where j = N or U)represent respectively pure natural exchange (ρ) or pure unnatural exchange (B). The angles of rotation χ and χ' which relate the $\vec{C}$ and $\vec{C}'$ coordinate systems to the s channel are such that:

$$\mathrm{tg}\,\chi = -2\sqrt{-t}/M_\omega \quad \text{and} \quad \mathrm{tg}\,\chi' = -2\sqrt{-t}/M'_\Delta .$$

The amplitudes that we have obtained are then parametrized by:

$$P_{++} = C_A - 4t\,A_N, \quad P_{--} = -C_A \cos\chi \cdot \cos\chi'$$

and

$$P_{00} = -C_A \sin\chi \sin\chi' + \sqrt{M_\omega^2 - 4t}\sqrt{M_\Delta'^2 - 4t}\,A_U ,$$

the amplitudes P_{-0} and P_{0-} being such that:

$$P_{-0} = \mathrm{tg}\,\chi \cdot P_{--} \quad \text{and} \quad P_{0-} = \mathrm{tg}\,\chi' \cdot P_{--} .$$

The results of this fit (χ^2/ND = 26.3/15) are displayed in Fig.1d as well as the curves fitting the intensities of the waves. The cut is found to have a t slope compatible with zero, while the difference of the intercept of the ρ and the B trajectories is found to be:

$$\alpha_\rho - \alpha_B = (0.95 \pm 0.22) .$$

This value is in agreement with the prediction coming from the masses of the ρ and B mesons, assuming a slope $\simeq 1$ in the Chew-Frautschi plot. We conclude that this type of model and parametrization describes our data well.

The allowed values for the exchanged spin S at the proton-delta vertex are 1 or 2; the quark model[13] on the other hand requires S=2 to vanish. Consequently, we have considered the waves with S = 2, associated either with the $(\rho\pi)$ or $(\epsilon\pi)$ S and P waves. We have already observed that the Z waves (S = 2 and $(\epsilon\pi)$ S-wave) is comparable in magnitude to the waves producing the ω and constitutes the essence of the background (Fig.1a). Figure 2 shows the R waves (S = 2 and ω waves); all are found compatible with zero. The waves with S = 2, $J^P = 0^-$ ($\rho\pi$ P-wave), $J^P = 1^+$ ($\rho\pi$ S-wave and $\epsilon\pi$ P-wave) were all found compatible with zero (Table I). Consequently, the predictions of the quark model are verified for the resonance production of $\omega^o\Delta^{++}$ and also for the background waves of the $(3\pi)^o$ system associated with the Δ^{++}, except for the $(\epsilon\pi)$ S-wave (the Z wave in our notations of Table I). These results are in agreement with the corresponding results of a similar analysis performed for reaction (2) in the higher mass range of the $(3\pi)^o$ system.[7]

In conclusion, we have been able with our amplitude analysis to determine the nature of the background wave under the omega produced in the reaction $\pi^+p \to (3\pi)^o\Delta^{++}$. We have also been able to determine that the omega is produced by both natural and unnatural exchange, their ratio being 0.8 as an average over $|t|$. Finally, the absorptive Regge exchange model provides a good parametrization for the results of our analysis. On the other hand, in the production of the ω, the waves corresponding to an exchanged spin of 2 units are found compatible with zero in agreement with the quark model. However, the predictions of this model fail for the background $(\epsilon\pi)$ S-wave.

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Table I

Characteristics of the waves considered in the amplitude analysis of $\pi^+ p \to (3\pi)^0 \Delta^{++}$ at 7.0 GeV/c/

Individual vertices waves

	J^P	$M(\eta)$	ℓ, L	name
Meson vertex	1^-	0(-) 1(-) 1(+)	1,1	ω
	0^-	0(+) 0(+)	0,0 1,1	$(\varepsilon\pi)$S wave $(\rho\pi)$P wave
	1^+	0(+) 1(+) 1(-)	0,1	$(\varepsilon\pi)$P wave
		0(+) 1(+) 1(-)	1,0	$(\rho\pi)$S wave

	S	$\mu(\eta)$	name
Baryon vertex	1	0(-) 1(-) 1(+)	Δ^{++}(S=1)
	2	0(+) 1(+) 1(-)	Δ^{++}(S=2)
	0	0(-)	$(p\pi)$S wave

Waves considered in this analysis

Waves associated with ω^0 production

J^P	M	L	S	μ	η	name	Results
1^-	0	1	1	0	-	P_{00}	Fig.1
	0			1	-	P_{-0}	
	1			0	-	P_{0-}	
	1			1	-	P_{--}	
	1			1	+	P_{++}	
	1		2	0	+	R_{0+}	Fig.2
	1			1	+	R_{++}	
	0			1	-	R_{-0}	
	1			1	-	R_{--}	
	0		0	0	-	S_{00}	$\simeq 0$
	1			0	-	S_{0-}	

Waves associated with $(3\pi)^0$ non-resonant background

J^P	M	L	S	μ	η	name (results)
0^-	0	0 1	1	1	+	$\simeq 0$
1^+	0	1		0 1	+	$\simeq 0$
	1			1	+	
	1			0 1	-	
	0	0		1	+	
	1			1	+	
	1			0 1	-	
0^-	0	0	2	0 1	+	Z (Fig.1)
	0	1		0 1	+	$\simeq 0$
1^+	0	1		1	+	$\simeq 0$
	1			0 1	+	
	1			1	-	
	0	0		0 1	+	
	1			0 1	+	
	1			1	-	

Figure Captions

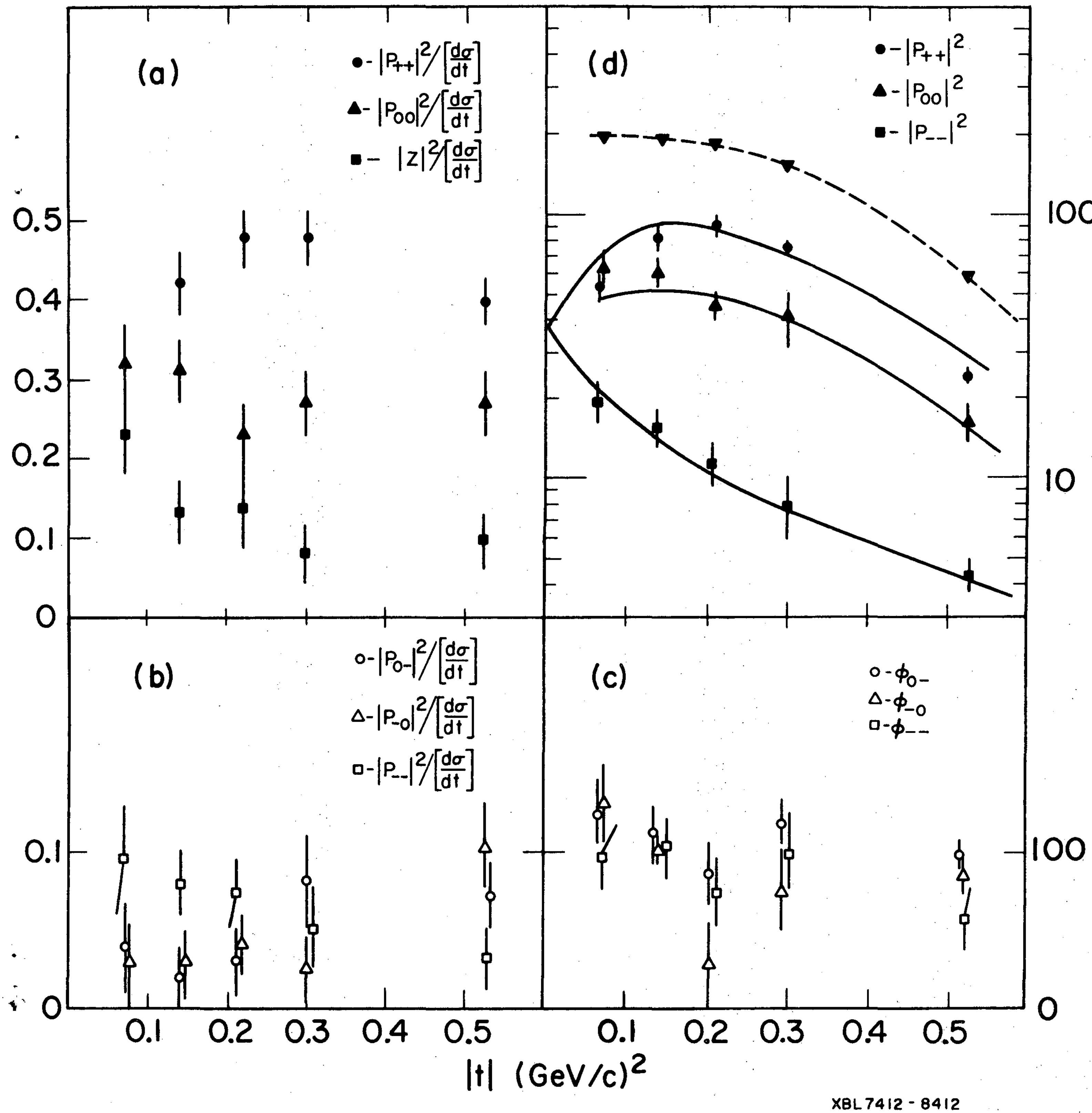
Fig. 1a) The ratio of the intensity of the waves P_{++} (natural exchange), P_{00} (unnatural exchange), Z(background found to be an ($\varepsilon\pi$) S-wave) to the total differential cross-section, as a function of the momentum transfer $|t_{p\Delta}|$.

1b) The ratio of the intensity of the waves P_{0-}, P_{-0} and P_{--} (all unnatural exchange) to the total differential cross section, as a function of the momentum transfer $|t_{p\Delta}|$.

1c) The phases of P_{0-}, P_{-0}, and P_{--} relatives to the phase of P_{00}, as a function of $|t_{p\Delta}|$.

1d) Results of the absorptive Regge fit (curve ———) to the P_{++}, P_{00} and P_{--} wave intensities, as well the total differential cross section (-- ▼ --), as a function of $|t_{p\Delta}|$.

Fig. 2. The S = 2 waves associated with the ω, as a function of $|t_{p\Delta}|$.



(a)
● - $|P_{++}|^2/\left[\frac{d\sigma}{dt}\right]$
▲ - $|P_{00}|^2/\left[\frac{d\sigma}{dt}\right]$
■ - $|Z|^2/\left[\frac{d\sigma}{dt}\right]$
(d)
● - $|P_{++}|^2$
▲ - $|P_{00}|^2$
■ - $|P_{--}|^2$
(b)
○ - $|P_{0-}|^2/\left[\frac{d\sigma}{dt}\right]$
△ - $|P_{-0}|^2/\left[\frac{d\sigma}{dt}\right]$
□ - $|P_{--}|^2/\left[\frac{d\sigma}{dt}\right]$
(c)
○ - ϕ_{0-}
△ - ϕ_{-0}
□ - ϕ_{--}
0.5
0.4
0.3
0.2
0.1
0
100
10
0.1
0
100
0
0.1
0.2
0.3
0.4
0.5
$|t|$ (GeV/c)^2
XBL 7412 - 8412

Fig. 1

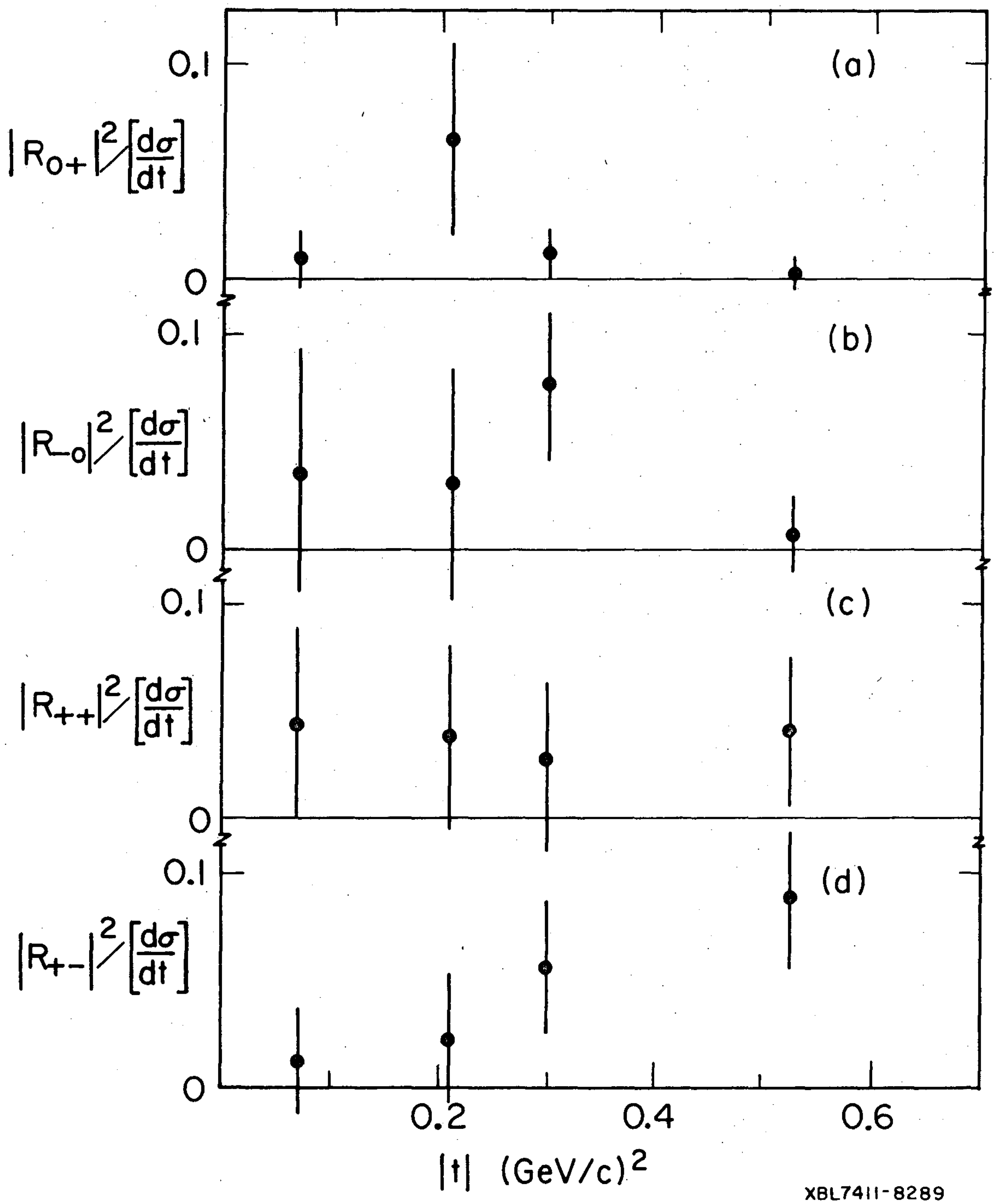

0.1
(a)
$|R_{0+}|^2 / \left[\frac{d\sigma}{dt}\right]$
0
0.1
(b)
$|R_{-0}|^2 / \left[\frac{d\sigma}{dt}\right]$
0
0.1
(c)
$|R_{++}|^2 / \left[\frac{d\sigma}{dt}\right]$
0
0.1
(d)
$|R_{+-}|^2 / \left[\frac{d\sigma}{dt}\right]$
0
0.2
0.4
0.6
$|t|$ $(GeV/c)^2$
XBL7411-8289

Fig. 2

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