Title
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SEMINAR IN HIGH ENERGY PHYSICS

ISOTOPIC SPIN, ISOTOPIC PARITY, AND CHARGE DEPENDENCE OF NUCLEAR FORCES

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Berkeley, California

SEMINAR IN HIGH ENERGY PHYSICS

Isotopic Spin, Isotopic Parity, and Charge Dependence of Nuclear Forces

Edward Vaughan

Introduction

The problem of distinguishing consequences of charge independence of nuclear forces from consequences of the weaker condition of charge symmetry has arisen recently in this laboratory in connection with the study of the related reactions

$$D + D \rightarrow p + t$$
$$D + D \rightarrow n + He^3$$
$$D + D \rightarrow D + D \quad .$$

This problem has been considered in a paper by Kroll and Foldy[1].

[1] Phys. Rev. 88, 1177 (1952).

These authors point out that certain selection rules obtained by Adair[2] from

[2] Phys. Rev. 87, 1041 (1952).

the hypothesis of charge independence also follow from the hypothesis of charge symmetry. Their method is to introduce an "isotopic parity" operator, R_γ, which replaces protons by neutrons and neutrons by protons, and so must commute with the nuclear Hamiltonian if the latter is charge symmetric.

Review of Isotopic Spin

Let us begin by reviewing the theory of isotopic multiplets based on the charge independence hypothesis.

The isotopic spin operators for a single nucleon are τ_1, τ_2, τ_3. They obey the commutation rule $[\tau_1, \tau_2] = i\,\tau_3$ and its cyclic permutations. Also, $\tau_1^2 = \tau_2^2 = \tau_3^2 = \frac{1}{4}$, $\tau^2 = \tau_1^2 + \tau_2^2 + \tau_3^2 = 3/4$, and $[\tau_1, \tau_2]_+ = 0$, etc. The Pauli matrices provide a representation of these operators, provided they are multiplied by one-half. The convention will be adopted that τ_3 is $+\frac{1}{2}$ for a proton, $-\frac{1}{2}$ for a neutron, as this is more convenient than the opposite convention when nucleon-meson couplings are involved.

For a nucleus,

$$T_i = \sum_{\ell=1}^{A} \tau_i^{(\ell)}$$

$$[T_1, T_2] = i\,T_3 \quad \text{and cyclic permutations}$$

$$[T_i, T^2] = 0$$

$$T^2 = T_1^2 + T_2^2 + T_3^2 \ .$$

Eigenvalues of T_3 will be called t_3; of T^2, $t(t+1)$. In analogy with angular momentum or ordinary spin, the levels will occur in sets such that all members of a set have the same value of t, while the values of t_3 differ among themselves in integer steps, and lie between $-t$ and t. Thus, there are $2t+1$ members of such a charge multiplet. Since t_3 varies, they all belong to different nuclei. However, they are degenerate except for charge-dependent perturbations, such as the n-p mass difference and the Coulomb effect.

These perturbations are, in the main, linear in t_3, so that the lowest level will have $t_3 = t$ or $t_3 = -t$. Now the possibility of β-processes ensures that the ground state of a stable nucleus must be the lowest member of its charge multiplet, from which it follows that all such states have $t = |t_3|$. This is the primary method of assigning values of t. An additional condition, holding for all levels, stable or not, is $t \geq |t_3|$.

Present ideas of nuclear structure would strongly suggest that $t = |t_3|$ for the ground state of any nucleus, whether β-stable or not.

Adair's idea was to note that the ground state of any stable nucleus having $t_3 = 0$ (equal numbers of neutrons and protons) must be a charge singlet (have $t = 0$). Moreover, the deuteron and α-particle are thought to be dynamically unstable except in their ground states, so that three of the four particles involved in (d, d'), (α, α'), (d, α), and (α, d) reactions must be in their ground states--since the target nucleus clearly can't be in an excited state--and must therefore be in $t = 0$ states. From this it follows that the fourth particle--namely, the residual nucleus, which may be in an excited state--must also be in a $t = 0$ state. The same holds for any resonant state of the compound nucleus that may be observed in the reaction.

Properties of Isotopic Parity

The consequences of adopting the hypothesis of charge symmetry, instead of the stronger one of charge independence, may be investigated by considering the operation of replacing all neutrons by protons and all protons by neutrons. This process is an operator on isotopic spin, and will be denoted R_γ. The charge symmetry hypothesis asserts that the operator R_γ commutes with the nuclear Hamiltonian, provided Coulomb forces and neutron-proton mass difference are ignored.

The effect of this operation is to replace a nucleus by its "mirror image". Because it commutes with the Hamiltonian, it cannot change the energy, so that for every state in a nucleus there is a corresponding state in its mirror nucleus, having the same energy except for neutron-proton mass difference and Coulomb energy. In this way, the present method of discussion leads to the well-known properties of mirror nuclei.

Although R_γ, since it commutes with the Hamiltonian, is a constant of the motion, it is nevertheless not a good quantum number in general, because it transforms a nucleus into a different nucleus. This ceases to be the case, however, if the nucleus is its own mirror image--that is, if it has the same number of neutrons as protons. The states of such nuclei must be eigenstates of R_γ, and the eigenvalues of R_γ yield an additional means of classifying these states, provided R_γ commutes with all other operators whose eigenvalues are being so used. Parity and angular momentum are such operators, and certainly commute with R_γ, since they do not operate on isotopic spin at all.

Double application of R_γ is equivalent to the identity operation, so that $R_\gamma^2 = 1$, and the possible eigenvalues of R_γ are ± 1. The eigenstates are said to have even or odd isotopic parity, or to be charge even or charge odd.

Isotopic parity, like isotopic spin, implies selection rules of the Adair type. These are weaker than the corresponding selection rules for isotopic spin, for two reasons. In the first place, they apply only to nuclei having zero neutron excess. In the second place, the isotopic parity separates nuclear levels into only two groups, whereas isotopic spin separates them into an infinite number of groups.

These differences have not yet been of practical importance. The reason is that applications have been made so far only to light nuclei of neutron excess zero. It may be possible to look experimentally for selection rules in nuclei having an odd neutron, and thus to distinguish $t = 3/2$ levels from $t = 1/2$ levels. Such selection rules would follow only from charge independence.

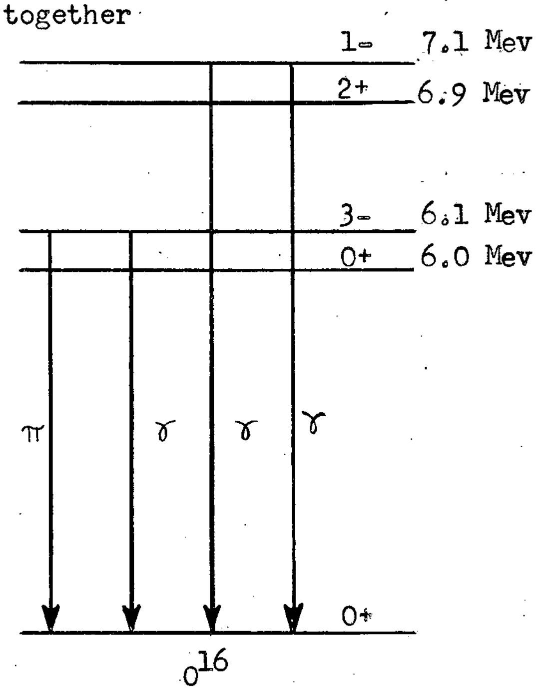
On the other hand, the restriction to light nuclei is essential because of the large Coulomb effects in heavy nuclei. Now it is shown in the formal appendix that, if T^2 is a good quantum number, then the $t_3 = 0$ member of a charge multiplet of multiplicity $2t + 1$ has $R_\gamma = (-1)^t$. That is, charge singlets are charge even, triplets odd, etc. But since there seem to be no stable states having $t \geq 2$ among the light nuclei (say $Z < 25$), it is clear that there is an exact identity between isotopic spin selection rules and isotopic parity selection rules, when applied to self-mirror nuclei.

Since pairing of levels in mirror nuclei, and selection rules of Adair type in self-mirror nuclei, are consequences of charge symmetry as well as of charge independence, experiments on these facts are unable to distinguish between the hypotheses. To do so, it is necessary either to demonstrate the operation of selection rules in nuclei having non-zero neutron excess, or else to show that correspondence of levels is not confined to mirror nuclei, but includes other members of isotopic multiplets. Evidence of the former sort is not available, but evidence of the latter sort will be presented below.

The isotopic parity also has a selection rule in γ-processes, provided they are electric dipole transitions. The reason is that the mass centers of the neutrons and protons lie at the same distance from the nuclear mass center, but in opposite directions, if the neutron and proton numbers are equal. The operator R_γ will thus reverse the direction of the electric dipole moment, without changing its magnitude. Now, this is exactly what happens in a space reflection, so that isotopic parity has the same selection rule in electric dipole transitions as does ordinary parity--that is, it must change. In case of charge independence, this implies the selection rule $\Delta t \neq 0$, which, together with the usual $\Delta t = 0, \pm 1$, leads to $\Delta t = \pm 1$. This rule applies only if $t_3 = 0$, of course.

Application of the Electric Dipole Selection Rule

The lowest five levels of O^{16} are shown, together with their angular momenta and parities as determined by studies of angular correlations of γ-rays, and other means. Our interest is in the state at 7.1 Mev. The selection rule for 2^{ℓ}-pole radiation is that $\Delta J \leq \ell \leq \Sigma J$, and requires $\ell = 1$ for the γ-ray emitted by this state. The parity change makes it electric dipole. Now, the ground state of N^{16} on this figure is at 10.3 Mev, and the $t_3 = 0$ member of the lowest charge triplet ought to lie still higher because of the Coulomb effect. This case therefore appears to contradict charge independence, and for this reason attracted considerable attention at the recent Rochester conference. The explanation appears to be that the charges on protons are



not the only source of electric dipole moment. In fact, in addition to the $\vec{A}\cdot\vec{j}$ term coupling the nucleons to the electromagnetic field, there is also the magnetic moment coupling term $\vec{\mu}\cdot\vec{H}$, and the multipole expansion of this term contains an electric dipole contribution. This contribution satisfies the same selection rule as to angular momentum and parity that the usual electric dipole does, and so can't usually be detected in its presence, but it does not satisfy the isotopic parity selection rule, and so can contribute when the larger term can't.

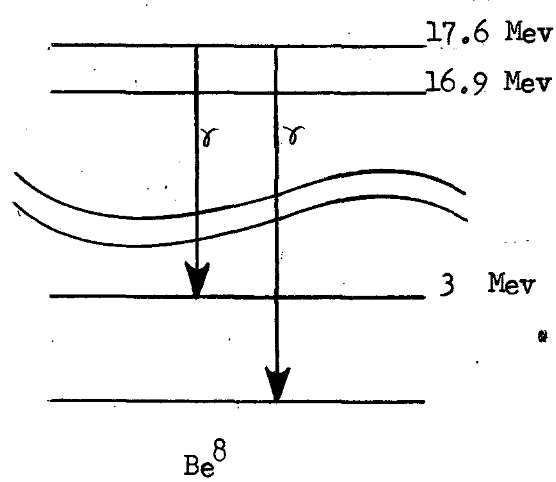
Thus, the selection rule requiring change of isotopic parity in electric dipole γ-processes is not rigorous, but reduces to a question of width. That is, it merely predicts that γ-widths of levels decaying without change of R_γ cannot be as great as expected for electric dipole transitions.

The R_γ selection rule for γ's can be applied to Be^8, some of whose levels are indicated. The application was made by Gell-Mann and Telegdi[3]. The experimental basis is the study of the photodisintegration of

[3] Phys. Rev. 91, 169 (1953).

C^{12} by Wilkins and Goward[4]. The reaction involved is $C^{12}(\gamma,\alpha)\,Be^8 \rightarrow 2\alpha$.

[4] Proc. Phys. Soc. (London) 64A, 1056 (1951).



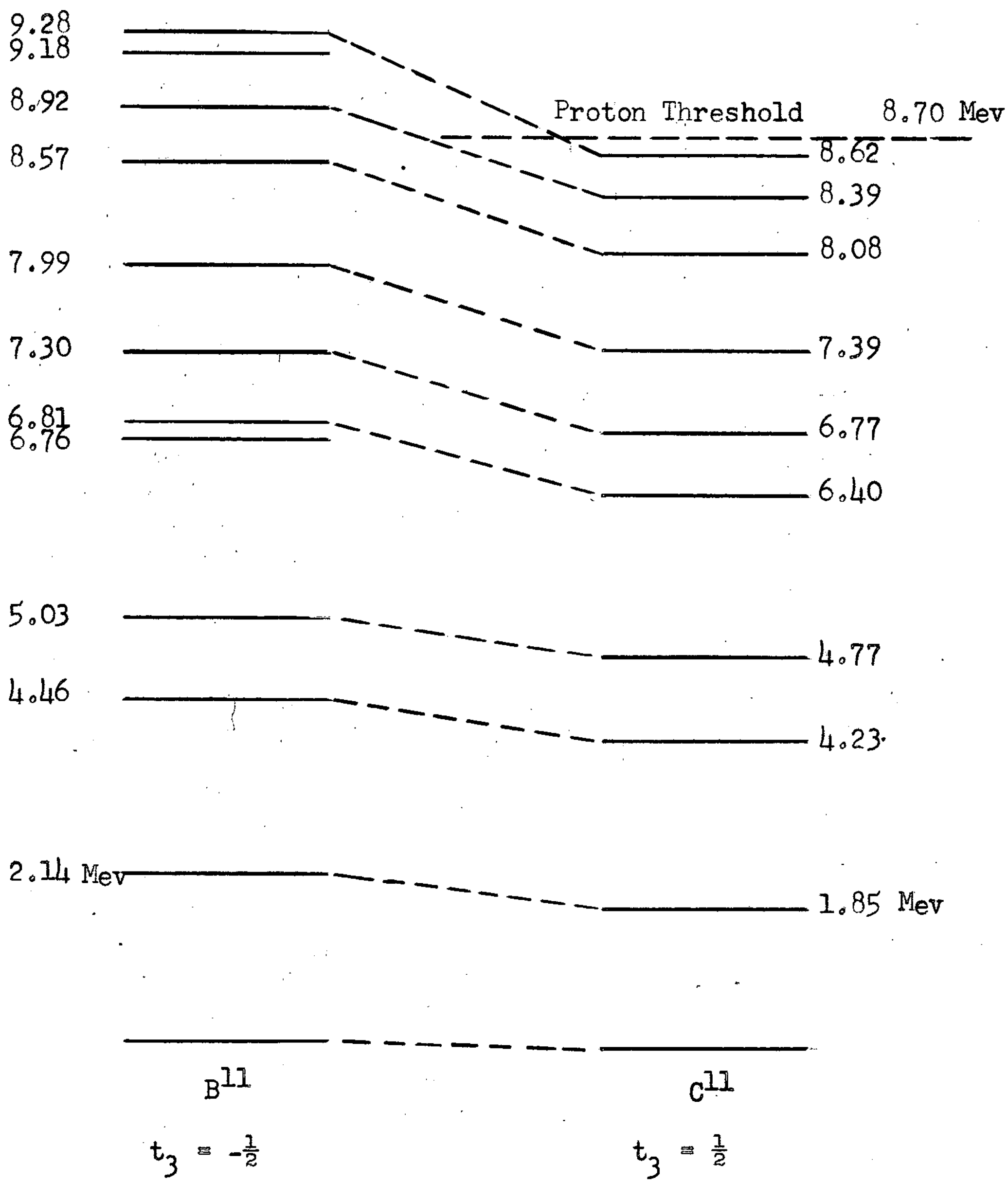
Their method was to observe, in emulsion exposed to 70 Mev bremsstrahlung, stars consisting of three α-particles. It was found that (1) the number of stars, as function of the total energy in the star, increases above 26 Mev, and (2) nearly every star having energy greater than 26 Mev contains a pair of α's, the sum of whose energies is 16.9 Mev. This is the evidence for the level shown at 16.9 Mev in the diagram. The threshold for exciting this level is 16.9 + 7.2 = 24.1 Mev, and with the Coulomb barrier of 2 or 3 Mev, it seems likely that the availability of this level is the explanation for the increase in cross-section. But there are other levels in Be^8 nearby, notably that at 17.6 Mev which emits the famous hard γ-rays. The most obvious explanation for the favoring of the 16.9 Mev level is that it can be reached by an electric dipole transition. This would require it to have odd R_γ, since R_γ of C^{12} ground state is even, by previous arguments.

Estimating the $t_3 = 0$ member of the triplet, whose other two members are the ground states of Li^8 and B^8, puts this level at 16.7 Mev, which is pretty good agreement. This is our first piece of evidence for charge independence.

It will be noted that the level decays into 2 α's. Each α has R_γ even, so the fact that the level of odd R_γ can so decay must be explained by the Coulombic perturbation. Moreover, there must be some selection rule inhibiting γ-emission. The experiment of Wilkins and Goward was not designed to detect γ's, so it is unknown whether this level can emit a γ, but in any case, the γ-emission must take long enough for the α-decay to compete, despite being forbidden. Now in fact, it is probable that the parity of this level is even, preventing electric dipole transitions to lower states (whose parity is also probably even). If, in addition, the angular momentum of the level should be an even multiple of $\hbar$, then, since lower levels are like 2 α's in having only even angular momenta, it would follow that $\Delta J = 1$ would be impossible, and the lowest order radiation possible would be electric quadrupole. Even if the angular momentum were odd, the parity would require the lowest order possible to be magnetic dipole.

Application to Charge Doublets of Odd-Mass Nuclei

The diagram indicates all the levels of C^{11} below the proton threshold at 8.70 Mev. There are ten of them, found largely as neutron groups in the stripping reaction $B^{10}(d, n)C^{11}$. As the neutron threshold of B^{11} is much higher, at 11.46 Mev, not all its levels are shown, but only the lowest twelve. The levels of B^{11} near 6.8 and 9.2 Mev are resolved into doublets, while the corresponding levels of C^{11} are not. This is no doubt because better energy resolution is possible for charged particles than for neutral ones.


9.28
9.18
8.92
8.57
7.99
7.30
6.81
6.76
5.03
4.46
2.14 Mev
Proton Threshold
8.70 Mev
8.62
8.39
8.08
7.39
6.77
6.40
4.77
4.23
1.85 Mev
B^{11}
$t_3 = -\frac{1}{2}$
C^{11}
$t_3 = \frac{1}{2}$

The energy scales have been adjusted arbitrarily to make the ground states degenerate. Then it is seen that corresponding low levels fail to coincide by 0.2 or 0.3 Mev, while corresponding levels near the C^{11} proton threshold fail to coincide by 0.5 to 0.7 Mev. Moreover, the levels are more closely spaced in the nucleus (C^{11}) having greater charge. The reason for this is presumably that the greater Coulomb forces tend to make the nuclear volume larger, and thus, by the uncertainty principle, to make the momentum uncertainty smaller, thus allowing the levels to crowd closer together.

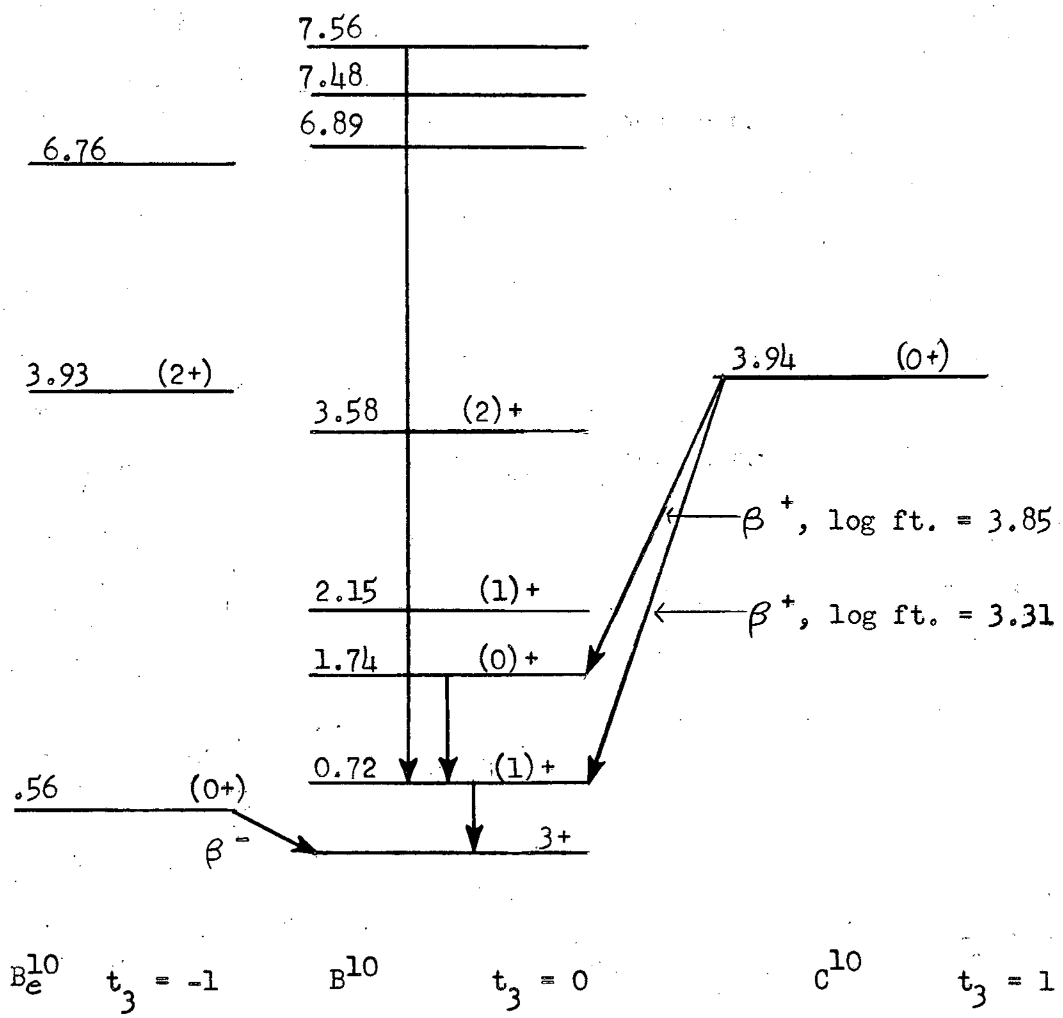
The discrepency of 0.6 or 0.7 Mev at the top of the diagram is not caused by the high excitation of about 9 Mev, but by the proton threshold in C^{11}. This is shown by the case of C^{13} - N^{13}, in which the proton threshold of the latter is at 1.95 Mev. The first excited state of N^{13} is at 2.37 Mev, while that of C^{13} is at 3.08 Mev, and the discrepency is again seen to be about 0.7 Mev, although the excitation energy is so much less.

It is true in general that the particle thresholds of $A = 4n + 1$ nuclei (He^5 - Li^5 unstable; Be^9 barely stable, B^9 unstable; C^{13} and N^{13} as seen) tend to be much lower than in the $A = 4n - 1$ nuclei ($A = 7, 11, 15$, etc.). The latter accordingly show the mirror structure more convincingly.

Nuclei Having $A = 4n + 2$

These nuclei are convenient because they have low-lying charge triplets which are readily accessible to experiment.

We begin with the case $A = 10$, part of whose level diagram is shown. The lower levels of B^{10} have been excited by inelastic scattering of protons and deuterons, with the results shown in the table. It appears that the 1.74 Mev level cannot be excited by the deuterons. The natural explanation is that the deuteron has even isotopic parity, and so is unable to change the isotopic parity of the target nucleus. This would not be true of the proton, which has $t_3 = \frac{1}{2} \neq 0$, and so can have no definite isotopic parity. Thus, the 1.74 Mev level must be charge even.



Comparison of (p, p') and (d, d') reactions in B^{10}		
B^{10} level Mev	Relative Yields	
	(p, p')	(d, d')
0	100	100
0.72	6.5	10
1.74	1.0	< 0.2
2.15	5	5
3.58	5	5

Linear interpolation between the ground states of Be^{10} and C^{10} gives $\frac{1}{2}(.56 + 3.94) = 2.25$ Mev. This requires correction to about 2 Mev (because the interpolation shouldn't be linear) as the predicted position of the $t_3 = 0$ member of the lowest charge triplet for $A = 10$. This agrees well enough with both the 1.74 Mev and the 2.15 Mev levels, but the selection rule evidence shows that the 1.74 Mev level is correct. Moreover, the agreement is good evidence for charge independence, while the mere existence of a level with a selection rule is evidence only for charge symmetry.

The parities of the low levels of B^{10} have been determined by application of Butler's method to the results of the stripping reaction $Be^9(d, n)B^{10}$. The angular momentum of the ground state of B^{10} has been measured. Other angular momenta and parities are not certain. However, the fact that the ground states of Be^{10} and C^{10} probably belong to the same charge multiplet as the 1.74 Mev state of B^{10} suggests that these ground states have even parity. Moreover, the rule that ground states of even-even nuclei have zero angular momentum would lead to the assignment $J = 0$ for the charge multiplet. Finally, we may take this multiplet as a triplet, because the β-decay of C^{10} shows that allowed β-transitions can occur between levels of the multiplet, and so could occur to a $t_3 = -2$ level if there were one; moreover, the β-decay of Be^{10} as observed is highly forbidden, and would be unable to compete with such a process.

There is some evidence for the $t_3 = -1$ and $t_3 = 0$ members of an excited charge multiplet. The diagram shows three excited states of B^{10} which have been observed as resonances in bombardment of Be^9 with protons. The levels at 6.89 and 7.48 Mev are seen as (p, p'), (p, d), and (p, α) resonances, while that at 7.56 Mev is seen as (p, p') and (p, γ) resonances. The fact that the γ-transition leads to the 0.72 level, rather than to the $J = 3$ ground state, suggests low angular momentum, so that the failure of the 7.56 level to emit deuterons or alphas cannot well be ascribed to a centrifugal barrier. The Coulomb barrier is eliminated by the lower-lying levels which are able to emit alphas and deuterons. While it is impossible to eliminate all alternatives, the easiest explanation of the anomaly is that the 7.56 Mev state has odd isotopic parity, and so would have to leave the residual Be^8 or Li^6 in a charge-odd state in emitting d or α. Such states are excited, however, so less energy is available for the process, and consequently, the Coulomb barrier is effective.

The corresponding level in Be^{10} would be at $7.56 - 1.74 + .56 = 6.38$ Mev. A doubtful proton group in $Be^9(d, p)Be^{10}$ indicates a level at 6.76 Mev, which would be just right if it could be confirmed, since it is too high, as expected for the nucleus of smaller charge.

After this exhaustive discussion of $A = 10$, we can be satisfied with a briefer treatment of other cases. In the case of Li^6, the reaction $Li^7(p, d)Li^6$ yields deuteron groups corresponding to levels at 2.19 and

3.58 Mev, as well as the ground state. The reaction $Be^9(p, \alpha)Li^6$ is similar, except that the 3.58 Mev level has been detected by its emitting a γ-ray, rather than by an alpha group. The scattering of deuterons from helium, or of alphas from deuterium, show some resonance for the 2.19 Mev level of Li^6, but none for the 3.58 Mev level.

The failure of the 3.58 Mev level to influence the α-d scattering can be explained if its isotopic parity is odd, since both d and α have even isotopic parity. However, if the level has even parity and $J = 0$, the experiment would be explained just as well. The reason is that an even parity state of $(\alpha + d)$ would have $\ell = 0, 2, 4, \ldots$, and $\vec{J} = \vec{\ell} + \vec{S}$, with $S = 1$. Thus, if $\ell = 0$, $J = 1$, while if $\ell \geq 2$, $J \geq 1$, so that $J = 0$ is never possible. Since the ground state of He^6 probably has even parity and $J = 0$, it is most probable that both types of selection rules are active in the Li^6 3.58 Mev level. The γ-ray from the 3.58 Mev level is further evidence of a selection rule against its decay into $d + \alpha$.

In the case of N^{14}, levels have been observed at 0, 2.3, 3.96, 4.85, 4.97, and 5.78 Mev, especially as neutron groups in the reaction $C^{13}(d, n)N^{14}$. In the reaction $O^{16}(d, \alpha)N^{14}$, the α-groups observed correspond to levels at 0, 3.98 $\pm$ .04, 5.06 $\pm$ .05, and 5.7 Mev. Since the two levels near 5 Mev may not be resolved, the main feature is the absence of the 2.3 Mev level from the (d, α) reaction. Preliminary results show that the inelastic scattering of deuterons is also unable to excite this level, thought it excites the level at 3.9 Mev. The conclusion is that the 2.3 Mev level has opposite isotopic parity from the other low levels.

Calculation from the ground states of C^{14} and O^{14} puts the $t_3 = 0$ member of their charge multiplet at 2.3 Mev, so the confirmation of charge independence is excellent. Forbidden character of the C^{14} β-decay shows that the multiplet has no $t_3 = -2$ member, for the β-transition to it would be allowed, as shown by the value log ft = 3.52 for the β-transition of O^{14} to the 2.3 Mev level of N^{14}. Thus, the levels form a charge triplet.

This charge triplet is even better known than the one at A = 10, because the parity and angular momentum of C^{14} have been measured, and found to be J = 0, even. These assignments should apply to the entire triplet.

Higher members of the series A = 4n + 2 have not been studied enough to produce any states of odd isotopic parity. For example, the principle reactions used to investigate F^{18} have been $Ne^{20}(d, \alpha)F^{18}$, $N^{14} + \alpha$ resonances, and $O^{16} + d$ resonances. The only other reaction which has, so far, been applied to any state other than the ground state is $F^{19}(d, t)F^{18}$, which gives a triton group corresponding to a lvel at 1.05 Mev. Since this level has also been observed in $Ne^{20}(d, \alpha)F^{18}$, it cannot be a state of odd isotopic parity.

Nuclei Having A = 4n

Be^8 has already been discussed. In C^{12}, O^{16}, and Ne^{20}, evidence of a somewhat different type exists. Each of these nuclei has a rather high-lying level which decays by both γ-emission and α-emission. It appears likely that some special suppression of α-emission or favoring of γ-emission is necessary to enable these processes to compete, as the levels all lie above the barrier against α-emission. Since α-decay can occur, a rigorous selection rule based on the rotation group cannot be invoked. Thus, the most likely explanations are that the emission of alphas is suppressed by conservation of isotopic parity, or that the γ-process is electric dipole. Either explanation would result in odd isotopic parity for these levels; for the α-emission leads to low-lying levels of Be^8,

C^{12}, or O^{16}, all having (presumably) even R_γ, while the γ-emission leads to low-lying levels of C^{12}, O^{16}, or Ne^{20}, which also have even R_γ.

The argument is not perfect, because there may be other approximate selection rules based on the α-particle model of these nuclei, and also because, at least for Ne^{20}, the level is just barely above the barrier against α-emission. Comparison with levels of neighbor isobars is somewhat unsatisfactory. In the case of B^{12}, the levels are dense and so unable to select a level in C^{12}. No suitable level is found in N^{16}. In the case of $A = 20$, the ground states of F^{20} and Na^{20} suggest that the lowest charge-odd level of Ne^{20} should be at 10.7 Mev or a bit lower. The relevant level in Ne^{20} is at 11.85 Mev, which is not good agreement.

Charge Quartets in Odd-Mass Nuclei

After doublets and triplets, the next subject is naturally quartets. Since these have no $t_3 = 0$ members, selection rules otherwise unaccounted for must be selection rules on t, rather than on R_γ. It is perfectly possible to conceive reactions which would distinguish between doublets and quartets. Thus, assuming the target nucleus to be in its (doublet) ground state, such reactions as (d, α), (α, d), or inelastic scattering of d or α would all lead to resonances and particle groups corresponding to charge doublet states. On the other hand, such reactions as inelastic proton scattering, (t, n) or (n, t), (p, He^3) or (He^3, p) could excite charge quartets as well. Also, such reactions as (d, p), (p, d), (d, n), or (n, d), applied to target nuclei in charge triplet states, such as Be^{10}, C^{14}, O^{18}, or Ne^{22}, would lead to charge doublet and quartet states of the residual nuclei.

The existence of the dynamically stable nuclei Li^9, C^{15}, N^{17}, and O^{19}, having neutron excess 3, suggests the existence of stable charge quartet states in Be^9, N^{15}, O^{17}, and F^{19}, which could be investigated by the above methods. However, very little work of this kind has been done yet, so that definite evidence for charge quartets from isotopic spin selection rules is not available.

Application to β -Decay

One of the most important problems in the theory of β -decay has been whether Fermi or Gamow-Teller selection rules are followed by allowed β -transitions. The principal difficulty has been that the initial state of an allowed transition doesn't usually live long enough to permit a measurement of its angular momentum. The exceptions, such as the neutron and triton, are transitions which do not permit a decision between the selection rules, being allowed by both of them. As a result, the decision depends on angular momenta assigned by extrapolation to unmeasured cases of rules found to apply in measured cases. That this process is dangerous is seen in the cases of Be^{10} and Na^{22}, whose large ft values were considered anomalous until measurement showed that the angular momenta of B^{10} and Na^{22} are 3, rather than 1 as had been guessed by extrapolation.

In this situation, the additional information obtained from the use of selection rules on isotopic spin has been very valuable. At the same time, knowledge of these rules will provide means for assigning the isotopic spins of ground states of many β -active nuclei, in addition to the stable nuclei for which rules have already been found.

The most obvious selection rule is $\Delta t_3 = \pm 1$. This rule forbids $t = 0 \rightarrow t = 0$ transitions. Since R_γ can be diagonal only if $t_3 = 0$, and t_3 can't be zero for both initial and final states, it also follows that there is no useful selection rule for R_γ. Nevertheless, consideration of isotopic parity will be of value in certain later arguments.

The terms in the Hamiltonian which bring about β -transitions must change neutrons into protons or protons into neutrons, and so must have the form

$$\sum_{\ell=1}^{A}\left(\tau_+^{(\ell)}\ \Omega_+^{(\ell)} + \tau_-^{(\ell)}\ \Omega_-^{(\ell)}\right)$$

where $\Omega_+^{(\ell)}$ and $\Omega_-^{(\ell)}$ are operators on the space and spin coordinates of the ℓ'th particle, and are Hermitian conjugates in order to make the term Hermitian. This expression is a vector in isotopic space. Just as electromagnetic dipole transitions obey the selection rule $\Delta J = 0, \pm 1$, because the dipole is a vector, so this isotopic vector results in the selection rule $\Delta t = 0, \pm 1$ for β-transitions.

The most stringent selection rule is the Fermi selection rule for allowed transitions. In this case, the coupling is independent of spin and space variables, so that the Ω's are constants, and the coupling operator reduces to

$$\Omega_+ \sum_{\ell=1}^{A} \tau_+^{(\ell)} + \Omega_- \sum_{\ell=1}^{A} \tau_-^{(\ell)} = \Omega_+ T_+ + \Omega_- T_-$$

This operator commutes with T^2. It follows that the selection rule is $\Delta t = 0$. In fact, T_+ and T_- are able to produce transition only between states which all belong to the same isotopic multiplet.

This selection rule probably explains why unambiguous evidence for Fermi-type coupling has been so difficult to obtain. In the first place, because of the Coulomb energy, only positron emission and K-capture can be allowed, with the exception of the neutron and triton β -decays. In the second place, because nuclear ground states generally have $t = |t_3|$, and β-decay generally takes place from the ground state, it is usually impossible for a nucleus with a neutron excess to make a β^+-transition without increasing t, so that positron emitters with proton excess are required. Finally, there is the well-known fact that, except for $J = 0 \longrightarrow J = 0$ transitions, Fermi-allowed decays are also allowed by Gamow-Teller selection rules. The result is that only nuclei having $A = 4n + 2$, $t_3 = 1$ are probable candidates. Nuclei having $A = 4n$, $t_3 = 1$, such as B^8 and Al^{24}, are useless because the $t = 1 \longrightarrow t = 1$ transition cannot compete with transitions to much lower-lying levels having $t = 0$. It follows that C^{10} and O^{14} are almost the only known nuclei capable of deciding whether a Fermi coupling exists. The best chance for an addition to this short list would be for the lowest $t = 1$ state of a $t_3 = 0$ nucleus to be an isomer. The shell model suggests that this is possible in the case of Al^{26}, but no experimental test has been made.

In the case of C^{10}, the transition to the $t = 1$ state of B^{10} at 1.74 Mev has log ft = 3.85, and thus is allowed. This establishes the existence of a Fermi coupling, provided the assignment $J = 0$ is accepted. This assignment not only follows from the rule for even-even nuclei, but is confirmed by the shape and ft value of the Be^{10} β-decay. These imply $\Delta J = 3$, and since B^{10} has $J = 3$, Be^{10} must have $J = 0$ or 6, of which possibilities the latter is somewhat fantastic.

In the case of O^{14}, the transition to the $t = 1$ state of N^{14} at 2.3 Mev has log ft = 3.52, which is allowed. In this case, the angular momentum of C^{14} has been measured and found to vanish, thus establishing $J = 0$ for the whole charge triplet. It was Adair who pointed out that the failure of the 2.3 Mev level to appear in the $O^{16}(d, \alpha)N^{14}$ reaction established its J as 0 by certifying its membership in the charge triplet with the ground states of C^{14} and O^{14}.

The Fermi selection rule on t can also be used to show the existence of Gamow-Teller coupling, by finding cases in which it is violated. This procedure has the advantage of being independent of the $J = 0$ rule for even-even nuclei. In He^6, for example, log ft = 2.91, which is allowed. $J = 1$ for Li^6, but cannot be measured for He^6. However, $t = t_3 = 0$ for Li^6 because it is stable, whereas $t \geq |t_3| = 1$ for He^6, so that the Fermi selection rule $\Delta t = 0$ is violated. This argument also applies to the decay of C^{10} to the 0.72 Mev level of B^{10}. Since this level can be produced by inelastic deuteron scattering, it has $t = 0$, while the states of C^{10} must have $t \geqslant 1$. The value log ft = 3.31 establishes the Gamow-Teller coupling, without the necessity of knowing the value of J for the B^{10} level.

These arguments are based on charge independence, but can be made to follow nearly as conclusively from charge symmetry. The part of the Hamiltonian which is charge-dependent can be considered as mixing the charge multiplets, but the necessity for preserving charge parity keeps the multiplets of even t separate from those of odd t, when $t_3 = 0$. Thus, only the $t = 2, 4, \ldots$ impurity in the ground states $t = 1$ of He^6 and C^{10}, and in the $t = 0$ ground states of Li^6 and B^{10}, can bring about transitions. It would be unlikely that such impurity would be great enough to account for the small ft values observed. Similarly, it would be unlikely that such mixing would change the value of the angular momentum of the lowest charge-odd state of B^{10} or N^{14}; thus, the $J = 0$ assignments would continue to hold. The $J = 0$ assignment for C^{10} and O^{14} would follow from those for Be^{10} and C^{14}, since they are respective mirror images. Accordingly, the transitions must be $J = 0 \rightarrow J = 0$ transitions, even if $\Delta t = 0$ loses its meaning.

Isotopic Spins of β -Unstable Levels

So far, our isotopic spin assignments have been based on the rule that $t = |t_3|$ for ground states of stable nuclei. The selection rules have merely provided a means of comparing excited states to the ground states. The selection rules for β -transitions, however, permit additional assignments by comparing states of isobars.

The basic general relation is $-t \leq t_3 \leq t$, which can be written $t \geq |t_3|$. The selection rule for β -processes is $\Delta t = 0, \pm 1$. First, consider β^--decay, in which $|t_3|$ always <u>decreases</u> (except in the cases of neutron and triton, which can be handled as mirror images of stable nuclei). If the final state has $t_f = |t_{3f}|$, then it is easy to show that the initial state must have $t_i = |t_{3i}|$. The reason is that

$$t_i \geq |t_{3i}| = |t_{3f}| + 1 = t_f + 1,$$

but t_i cannot be greater than $t_f + 1$ because of the selection rule, so that $t_i = t_f + 1 = |t_{3i}|$. Now, the final state of a β^--process must have $t_f = |t_{3f}|$, provided the initial state is the ground state of its nucleus. For if $t_f > |t_{3f}|$, then $|t_{3i}| = |t_{3f}| + 1 \leq t_f$, so that there is a state in the initial nucleus belonging to the same charge multiplet as the final state, and lying lower because of the Coulomb energy, and therefore, contrary to hypothesis, lying lower than the initial state.

Thus, $t = |t_3|$ not only for stable states, but for all initial and final states of β^--processes whose initial states are the ground states of their nuclei.

In the case of β^+-activity or K-capture, a different approach is necessary. These transitions can always take place between states belonging to the same charge multiplet, and will do so if no lower-lying state is available in either nucleus. When a new lower charge multiplet becomes available (or ceases to be available), the state satisfies $t = |t_3|$. Thus, cases in which $t > |t_3|$ are associated with transitions within a single multiplet. This often makes it possible to show that $t = |t_3|$ by showing that the energy of the process is quite different from the charge multiplet splitting, so that it cannot be a transition between states of a single charge multiplet.

Examples of the first rule are He^6, Be^{10}, C^{14}, etc., which have $|t_3| = 1$, and so $t \geq 1$. In order to decay into Li^6, B^{10}, and N^{14}, which are stable and so must have $t = 0$, these nuclei must all have $t = 1$ in their ground states.

An example of positron emission is F^{18}. It emits a 0.64 Mev β^+, so that the energy difference between the levels of F^{18} and O^{18} is 0.64 Mev $+ 2\ mc^2 = 1.66$ Mev. An estimate of the multiplet splitting is given by the β^+ from F^{17}, of 1.72 Mev energy, giving 1.72 Mev $+ 2\ mc^2 = 2.74$ Mev. The correction for the volume difference between $A = 17$ and $A = 18$ is very small, so that the large difference between 1.66 Mev and 2.74 Mev shows at once that the ground states of O^{18} and F^{18} cannot belong to the same charge multiplet. This means that a level is available to F^{18} which is not available to O^{18}, so that the ground state of F^{18} must have $t = |t_3| = 0$. Then the selection rule $\Delta t = 0, \pm 1$ forces the ground state of O^{18} to have $t = 1$. This transition is allowed (log ft = 3.62), so that the change in t is another proof of Gamow-Teller selection rules.

Another means of dealing with positron emission is to use the fact that transitions within a charge multiplet are allowed. Then if a β^+-process is forbidden between ground states of neighbor isobars, they must both have

$t = |t_3|$. An example is Na^{22}, which decays primarily to an excited state of Ne^{22}. The conclusion $t = |t_3|$ can be confirmed by the energy method used for F^{18}.

These methods permit the assignment of t-values to a great many nuclear ground states, and in all such cases, it appears that $t = |t_3|$. This accords with our conceptions of nuclear structure, which require ground states to be as symmetrical as possible in the spatial coordinates and spins of the nuclei, and therefore as anti-symmetrical as possible in the charges. Just as anti-symmetrizing spins leads to low values of S, so anti-symmetrizing charges leads to low values of t. Subject to $t \geq |t_3|$, the lowest value of t is $t = |t_3|$.

Applications to High-Energy Experiments

The preceding developments provide the means for discussing the reactions mentioned in the introduction. These are:

$$
\begin{aligned}
D + D &\rightarrow D + D \\
&\rightarrow n + He^3 \\
&\rightarrow p + t.
\end{aligned}
$$

It is apparent at once that $n + He^3$ and $p + t$ are mirror systems, and should be produced with equal cross-sections provided charge symmetry holds. It was suggested that the fact that $D + D$ has $t = 0$ would prevent $t = 1$ states of $n + He^3$ or $p + t$ from appearing, and that the consequent reduction of their production cross-section, relative to elastic scattering, might give some information on charge independence. Aside from other difficulties with this program, it is clear that the suppression of charge-odd states, arising from the fact that $D + D$ is charge even, would accomplish all that suppression of $t = 1$ states could. In fact, one can construct the two linear combinations (n, He^3) $\pm$ (p, t), and find that they have opposite charge parities, so that one of them is suppressed. Since n, p, t, and He^3 all have $t = \frac{1}{2}$, it follows that only states having $t = 0$ and $t = 1$ are possible, and since these two states have opposite charge parities, they must correspond to the same two linear combinations already constructed. Thus, no stronger results follow from charge independence than from charge symmetry.

The reason the states given have opposite charge parity is that (n, He^3) and (p, t) are interchanged by R_γ. Which of them is charge-even, and which charge-odd, is not material.

A more informative experiment is that of Hildebrand on the reaction

$$p + n \longrightarrow d + \pi^0 .$$

He finds the angular distribution of deuterons to be the same as in the reaction

$$p + p \longrightarrow d + \pi^+ .$$

This is taken as strong evidence for charge independence. In order to appreciate the strength of this evidence, let us consider the experiment from the standpoint of charge symmetry. To do so, it is simplest to consider the reverse reaction. In the center of gravity, the angular distribution of direct and reverse reactions is the same, since there is really only one angle--namely, that between the relative momentum of the neutron and proton and the relative momentum of the meson and deuteron.

The consequence of charge symmetry is that the neutrons and protons have the same angular distribution. A proton detector would give the same reading as a neutron detector in the same place. However, a proton detector at angle θ would read the same as a neutron detector at $180^\circ - \theta$, because every proton is accompanied by a neutron in the opposite direction. It follows that a proton detector at θ and a proton detector at $180^\circ - \theta$, would read the same. It is evident, therefore, that charge symmetry requires the angular distribution to be symmetric about 90° in the center of gravity system.

Now, the identity of the two protons requires the same fore-and-aft symmetry in the comparison reaction. This common feature of the two reactions, predicted by charge symmetry, is clearly much weaker than the <u>identity</u> actually observed. Moreover, no stronger prediction can be made from charge symmetry, because the operator R_{γ} is unable to transform any of the states of one reaction into a state of the other.

The experiment, therefore, really constitutes evidence for charge independence. As the result of a single experiment might conceivably be accidental, it would be desirable to confirm the charge independence by studying other high-energy reactions.

Appendix--Formalism of Isotopic Parity

The process of replacing all protons by neutrons and all neutrons by protons changes the sign of T_3. In order to preserve the commutation rules (such as $\left[T_1, T_2 \right] = i\, T_3$) it is necessary to change the sign of either T_1 or T_2, also. If the signs of T_2 and T_3 are both changed, then the operation is a 180°-rotation about the T_1 axis.

Thus,

$$R_\tau = e^{i \pi T_1} = e^{i \pi \sum_{\ell=1}^{A} \tau_1^{(\ell)}} = \prod_{\ell=1}^{A} e^{i \pi \tau_1^{(\ell)}}$$

$$= \prod_{\ell=1}^{A} \left[\cos (\pi \tau_1^{(\ell)}) + i \sin (\pi \tau_1^{(\ell)}) \right]$$

$$= \prod_{\ell=1}^{A} \left[\cos (\pi \tau_1^{(\ell)}) + i\, \tau_1^{(\ell)} \frac{\sin (\pi \tau_1^{(\ell)})}{\tau_1^{(\ell)}} \right]$$

Now $\cos (\pi \tau_1)$ and $\dfrac{\sin (\pi \tau_1)}{\tau_1}$ are functions of $\tau_1^2 = \frac{1}{4}$, and so can be evaluated by replacing τ_1 by 1/2. The result is:

$$R_\tau = e^{i \pi T_1} = \prod_{\ell=1}^{A} 2\, i\, \tau_1^{(\ell)} .$$

Of course, $R_\tau^2 = \prod_{\ell=1}^{A} (2i\, \tau_1^{(\ell)})^2 = (-1)^A$, so that, for even A, eigenvalues of R_τ must be ± 1. Now it is easy to show that R_τ anti-commutes with $\tau_3^{(\ell)}$.

$$\left[R_{\gamma}, \tau_3^{(\ell)} \right]_+ = \left\{ \prod_{\substack{\ell'=1 \\ \ell' \neq \ell}}^{A} 2i\,\tau_1^{(\ell')} \right\} \cdot 2i \left[\tau_1^{(\ell)}, \tau_3^{(\ell)} \right]_+ = 0$$

Since $T_3 = \sum_{\ell=1}^{A} \tau_3^{(\ell)}$, it follows at once that $\left[R_{\gamma}, T_3 \right]_+ = 0$.

Then $\left[R_{\gamma}, T_3^2 \right] = \left[R_{\gamma}, T_3 \right]_+ T_3 - T_3 \left[R_{\gamma}, T_3 \right]_+ = 0$.

Similarly, $\left[R_{\gamma}, T_2 \right]_+ = 0$ and $\left[R_{\gamma}, T_2^2 \right] = 0$.

Clearly $\left[R_{\gamma}, T_1 \right] = 0$, so $\left[R_{\gamma}, T_1^2 \right] = 0$. Now it follows that $\left[R_{\gamma}, T^2 \right] = 0$.

The significance of these relations is that R_{γ} cannot in general be diagonal if T_3 is (that is, if a definite nucleus is considered), but that it can be diagonal if T_3^2 is, or if T^2 is.

It is now possible to display the structure of the matrix representing R_{γ} , in a representation in which T^2 is diagonal with eigenvalue $t(t+1)$, and T_3 is diagonal with eigenvalues t_3. Consider a state $\chi_t^{t_3}$. Operate with R_{γ} to form the state $R_{\gamma} \chi_t^{t_3}$. Find the value of t_3 for the new state:

$$T_3 \left(R_{\gamma} \chi_t^{t_3} \right) = - R_{\gamma} T_3 \chi_t^{t_3} = - t_3 \left(R_{\gamma} \chi_t^{t_3} \right) .$$

Thus, $\left(R_{\gamma} \chi_t^{t_3} \right)$ is some multiple of $\chi_t^{-t_3}$, since the charge multiplet has only one state for which T_3 has the eigenvalue $-t_3$. Because $\left[R_{\gamma}, T_3^2 \right] = 0$, it follows in a well-known manner that there are no matrix elements of R_γ connecting states having differing values of t_3^2. The only matrix elements then remaining are those on the "back diagonal" of the $(2t+1) \times (2t+1)$ matrix.

$$R_\gamma = \begin{pmatrix} 0 & & & x \\ & & x & \\ & x & & \\ x & & & 0 \end{pmatrix} .$$

It is necessary to distinguish between even A and odd A. If A is odd, then t is half an odd integer and $2t+1$ is even. In this case, the back diagonal and the main diagonal have no common element. If A is even, then t is an integer, and $2t+1$ is odd. In this case, the diagonal element for $t_3 = 0$ also lies on the back diagonal. Thus, in the state $(t, t_3 = 0)$, R_γ is diagonal. The reason is that $\left[R_\gamma, T_3\right] = 2R_\gamma T_3 \rightarrow 0$ if $t_3 = 0$. The situation is like that in S-states, in which L_x and L_y can be diagonal as well as L_z, though this is generally impossible.

It is important to know the value of the diagonal matrix element of R_γ. The way to find it is to take the spur of the matrix, since there is only one element. The spur is invariant under unitary transformations, so instead of diagonalizing T_3, it is possible to diagonalize T_1 instead. The eigenvalues of T_1 are the same as those of T_3. So get:

$$Sp(R_\gamma) = Sp(e^{i\pi T_1}) = Sp(e^{i\pi T_3}) = Sp\left[(-1)^{T_3}\right]$$

$$= \sum_{t_3=-t}^{t} (-1)^{t_3} = 1 + 2\sum_{t_3=1}^{t} (-1)^{t_3} = 1 + 2\left[-1 + 1 - 1 \ldots\right]$$

$$= 1 + 2 \begin{Bmatrix} 0 & \text{if } t \text{ is even} \\ -1 & \text{if } t \text{ is odd} \end{Bmatrix} = (-1)^t .$$

This relation tells us that the $t_3 = 0$ member of a charge multiplet of multiplicity $2t + 1$ is "charge even" if t is even, "charge odd" if t is odd. A simple application is to ground states of stable nuclei having $t_3 = 0$. By earlier arguments, such states have $t = 0$, and it now follows that they must have $R_\gamma = 1$. Similarly, the $t_3 = 0$ member of the lowest isotopic triplet must have $R_\gamma = -1$.

Since $\left[R_\gamma, T_3^2\right] = 0$, it is possible for R_γ and T_3^2 to be diagonal simultaneously. But neither state $t_3 = \pm\sqrt{t_3^2}$ can diagonalize R_γ, so some linear combination of them must do so. Accordingly, these two states must have the same energy. This result is the degeneracy of mirror nuclei, which is already a well-known consequence of charge symmetry.

There is one more formal property of R_γ which has applications. This relates to electric dipole γ-processes. The charge operator for a nucleon is:

$$e\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix} = e\begin{pmatrix}\frac{1}{2}+\frac{1}{2} & 0\\ 0 & \frac{1}{2}-\frac{1}{2}\end{pmatrix} = e\left[\begin{pmatrix}\frac{1}{2} & 0\\ 0 & \frac{1}{2}\end{pmatrix} + \begin{pmatrix}\frac{1}{2} & 0\\ 0 & -\frac{1}{2}\end{pmatrix}\right]$$

$$= \frac{1 + \tau_3}{2}\, e \; .$$

Then the electric dipole operator is

$$\vec{P} = \frac{e}{2} \sum_{\ell=1}^{A} \left[1 + \tau_3^{(\ell)}\right] \vec{x}_\ell \; .$$

If the origin is taken at the center of mass of the nucleus--as is allowable, since the nuclear recoil is very small in γ-processes--then $\sum_{\ell=1}^{A} \vec{x}_\ell = 0$, and it follows that $\vec{P} = \frac{e}{2} \sum_{\ell=1}^{A} \tau_3^{(\ell)} \vec{x}_\ell$. From our earlier result, that R_γ anticommutes with the $\tau_3^{(\ell)}$, it now follows that R_γ anticommutes with $\vec{P}$. We know that the anti-commutation of $\vec{P}$ with the space reflection implies change of parity in electric dipole radiation. In the same way, anti-commutation with R_γ implies change of isotopic parity.

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