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ABSTRACT

For the past twenty-five years, Dusansky and his associated co-authors have published a long series of papers which are based on the same price-dependent utility function. The alleged price dependence, however, is fictitious in the sense that the level of exogenous money income can replace the commodity prices. The consequence is that the demand functions derived from Dusansky's utility function are identical and observationally equivalent to the demand functions obtained from a prototypical utility function. Since all the market and environmental effects are revealed only through the demand functions, the specification and use of a utility function such as that used by Dusansky is irrelevant and uninformative for the analysis of any economic problem where prices enter the consumer utility function and whose goal is the detection of the effects of price-dependent preferences on the demand for real goods.

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A Recurrent Utility Function of Fictitious Generality

For the past twenty five years, Dusansky and associated co-authors have published a series of at least eight papers (see Dusansky and Wilson, Dusansky 1989 and 1980, Dusansky and Kalman 1976, 1974, 1972 together with the "Erratum" of 1973, Kalman, Dusansky and Wickström, and Kalman and Intriligator) which are essentially based on the same price-dependent utility function. All these papers use a peculiar utility function that, in spite of its dependence upon all commodity prices, generates demand functions that satisfy the traditional properties, including the symmetry and negative semidefiniteness of the Slutsky matrix.

We will demonstrate that the recurrent utility function of Dusansky and associated co-authors is operationally indistinguishable from an archetype utility function dependent only on real goods, i.e., it is simply an affine transformation of a prototype utility function dependent only on real goods. Hence, it is well known that two utility functions related in such a manner are equivalent in the sense that they generate identical demand functions with identical empirical properties, and thus are observationally indistinguishable. Given that this fundamental result was established at least a hundred years ago, it is natural to ask the following two questions:

1. Why is it of interest to seek out the traditional Slutsky matrix and its properties as a final target of any consumer's utility specification, including those that are price-dependent?

2. Why is it not more interesting to expect that the price dependency of the utility function will likely require a modification of the traditional Slutsky matrix and, therefore, provide results with an eye towards generalization?

There are at least two possible answers to the first question. One is to prove observational equivalence, while another is the desire to derive necessary and sufficient conditions on the general model that yield the prototype testable implications even though they could be

delivered by the prototype model. Having derived a form of the utility function necessary to attain this objective (see "Erratum" in JET, 1973, p. 107), five of the aforementioned papers essentially prove that such a utility function is necessary. In fact, the proofs that the recurrent price dependent utility function is necessary to recover the prototype Slutsky properties are identical in Dusansky and Wilson (1993), Dusansky (1989) and Dusansky and Kalman (1976), and are only slightly more general in Kalman, Dusansky and Wickström (1974) and Kalman and Intriligator (1973) due to the more general model considered.

The answer to the second question has been given over the years by several authors beginning with Lloyd, Berglas and Razin, and Samuelson and Sato. All these authors have pointed out the necessity of generalizing the archetype Slutsky matrix when preferences are price dependent. Furthermore, Clower and Riley, followed by Howitt and Patinkin, have been critical of Dusansky and co-authors regarding related money illusion claims and have pointed out the "*spurious generality* " of their results. Their conclusions, however, went unheeded by Dusansky who, after the debate, derived the same price-dependent utility function in two other papers (Dusansky, 1989 and Dusansky and Wilson 1993).

The recurrent price-dependent utility function of Dusansky and his co-authors has the following structure:

$$(1) \qquad\qquad f(q,p) = \pi(q) + \sigma(p) + K \sum_j p_j q_j,$$

where p and q are vectors of commodity price and quantities, and K is an arbitrary constant. We note that in Dusansky and Wilson, $f(q,p)$ is an expected utility function. In the five papers referred to above, Dusansky and his co-authors have shown that one can integrate back to this utility function given demand functions that satisfy the traditional Slutsky properties.

This statement must be viewed in the context of consumer theory, where all the measurable relations are expressed by the consumer demand functions and not by the utility function. Within this context— which is the only admissible one—the above price-dependent

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utility function is equivalent to the traditional utility function, say $\pi(q)$, that depends only on the quantities of consumed commodities, in the sense that $f(q,p)$ and $\pi(q)$ generate identical demand functions. Since only demand functions are observable, it is impossible to distinguish whether they have been generated by one or the other utility function and, therefore, claims that $f(q,p)$ is a more general utility function than $\pi(q)$ are specious.

This conclusion is so obvious that its restatement may appear to be a waste of time and effort. Unfortunately, as long as Dusansky and his co-authors keep publishing papers which claim to offer an insight into the issue pertaining to price-dependent preferences using the utility function presented in Eq. (1), it will be necessary to keep reminding the audience of its fictitious generality and empirical emptiness for modeling price-dependent preferences, no matter how sophisticated the mathematical presentation may appear.

We demonstrate the fictitious generality of the utility function in Eq. (1) by discussing the PROPOSITION in Dusansky and Wilson's paper published in *JET* in 1993. We follow the notation in their Eq. (19). Note that we could have equivalently adopted the notation in Dusansky (1989, Eq. (A11)), or Dusansky and Kalman (1976, Eq. (12)), as the proposition that all three papers prove is identical, as noted above. Then, the main result of Dusansky and Wilson (see p. 129) is contained in their

"PROPOSITION. *Given symmetry of the Slutsky unobservables, $S_{ij} = S_{ji}$, a general form of $f(q,p)$ is*

(1)
$$f(q,p) = \pi(q) + \sigma(p) + K \sum_j p_j q_j,$$

where $\pi(\cdot)$ is any real valued twice continuously differentiable function, strictly quasiconcave and increasing in its arguments, $\sigma(\cdot)$ is any real valued twice continuously differentiable function, and K is any non-negative constant."

We note that this PROPOSITION gives necessary conditions for a price-dependent utility function to yield the prototype Slutsky matrix and its properties.

I. Discussion

We state and prove the following sufficiency counterpart to the above PROPOSITION of Dusansky and Wilson.

LEMMA. If the utility function $f(q,p)$ is specified as in Eq. (1), then the demand functions obtained from it have the prototype Slutsky properties.

Proof: We begin the proof by demonstrating the equivalence of three related utility specifications, whose corresponding indirect utility functions are defined as

$$(2) \qquad \Phi_1(p,y) := \max_q \{\pi(q) + \sigma(p) + Kq'p \mid q'p = y\}$$

$$(3) \qquad \Phi_2(p,y) := \max_q \{\pi(q) + \sigma(p) + Ky \mid q'p = y\}$$

$$(4) \qquad \Phi_3(p,y) := \max_q \{\pi(q) \mid q'p = y\}.$$

By definition of $\Phi_1(p,y)$,

$$\Phi_1(p,y) = \max_q \{\pi(q) + \sigma(p) + Kq'p \mid q'p = y\}$$

$$= \max_q \{\pi(q) + \sigma(p) + Ky \mid q'p = y\} \qquad \text{for all feasible } q$$

$$= \Phi_2(p,y) \qquad \text{by definition of } \Phi_2(p,y)$$

$$= \sigma(p) + Ky + \max_q \{\pi(q) \mid q'p = y\}$$

$$= \sigma(p) + Ky + \Phi_3(p,y) \qquad \text{by definition of } \Phi_3(p,y).$$

Thus,

$$(5) \qquad \Phi_1(p,y) \equiv \Phi_2(p,y) \equiv \Phi_3(p,y) + \sigma(p) + Ky.$$

By differentiation of the identities in Eq. (5) with respect to p and y it follows that

$$(6) \qquad \frac{\partial \Phi_1(p,y)}{\partial p} \equiv \frac{\partial \Phi_2(p,y)}{\partial p} \equiv \frac{\partial \Phi_3(p,y)}{\partial p} + \frac{\partial \sigma(p)}{\partial p}$$

$$(7) \qquad \frac{\partial \Phi_1(p,y)}{\partial y} \equiv \frac{\partial \Phi_2(p,y)}{\partial y} \equiv \frac{\partial \Phi_3(p,y)}{\partial y} + K.$$

Let $q^1(p,y), q^2(p,y)$ and $q^3(p,y)$ be the vectors of demand functions and let $\lambda^1(p,y), \lambda^2(p,y)$ and $\lambda^2(p,y)$ be the associated Lagrange multipliers resulting from the solutions of the three

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problems defined by Eq. (2), Eq. (3) and Eq. (4). By using the envelope theorem on $\Phi_1(p, y)$:

(8)
$$\frac{\partial \Phi_1(p, y)}{\partial p} \equiv \frac{\partial \sigma(p)}{\partial p} - [\lambda^1(p, y) - K]q^1(p, y)$$

(9)
$$\frac{\partial \Phi_1(p, y)}{\partial y} \equiv \lambda^1(p, y)$$

where $[\lambda^1(p, y) - K] > 0$ by the non-satiation hypothesis. Hence, Eq. (8) and Eq. (9) generate the Roy's identity

(10)
$$q^1(p, y) \equiv \frac{\dfrac{\partial \sigma(p)}{\partial p} - \dfrac{\partial \Phi_1(p, y)}{\partial p}}{\dfrac{\partial \Phi_1(p, y)}{\partial y} - K} \geq 0.$$

Similarly, using the envelope theorem on $\Phi_2(p, y)$ yields

(11)
$$\frac{\partial \Phi_2(p, y)}{\partial p} \equiv \frac{\partial \sigma(p)}{\partial p} - \lambda^2(p, y)q^2(p, y)$$

(12)
$$\frac{\partial \Phi_2(p, y)}{\partial y} \equiv \lambda^2(p, y) + K$$

where $\lambda^2(p, y) > 0$ by non-satiation. Hence, Eq. (11) and Eq. (12) generate the Roy's identity

(13)
$$q^2(p, y) \equiv \frac{\dfrac{\partial \sigma(p)}{\partial p} - \dfrac{\partial \Phi_2(p, y)}{\partial p}}{\dfrac{\partial \Phi_2(p, y)}{\partial y} - K} \geq 0.$$

Finally, using the envelope theorem on $\Phi_3(p, y)$ yields

(14)
$$\frac{\partial \Phi_3(p, y)}{\partial p} \equiv -\lambda^3(p, y)q^3(p, y)$$

(15)
$$\frac{\partial \Phi_3(p, y)}{\partial y} \equiv \lambda^3(p, y)$$

where $\lambda^3(p, y) > 0$ by non-satiation. Hence, Eq. (14) and Eq. (15) generate the Roy's identity

(16)
$$q^3(p, y) \equiv \frac{-\dfrac{\partial \Phi_3(p, y)}{\partial p}}{\dfrac{\partial \Phi_3(p, y)}{\partial y}} \geq 0.$$

Equations (10), (13), and (16) are identical by Eq. (6) and Eq. (7). Since all three demand functions are identical, they are indistinguishable from the neoclassical prototype $q^3(p, y)$ and, therefore, all have the identical and traditional Slutsky properties. *Q.E.D.*

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The Proposition and Lemma, therefore, show that the utility function $f(q,p)$ as specified in Eq. (1), is equivalent to and indistinguishable from a traditional utility function that depends only on real goods, say $\pi(q)$, in the sense that the two sets of demand functions obtained from the utility functions $f(q,p)$ and $\pi(q)$ are identical and thus have indistinguishable properties. This means that adopting the utility function $f(q,p)$ as a basis for investigating the effects of price-dependent preferences on commodity demands is of no value since it cannot yield any insights into such matters.

Another troubling point about this literature is the apparently incomplete understanding of the model by its authors. Take footnote 7 in Dusansky and Wilson (1993, p.131), for example:

" It can be shown that the negative semi-definiteness property holds if $(\lambda - K) > 0$. We know that $\lambda > 0$, by the construction of the Lagrangean, and that K can be arbitrarily small. Hence, this technical condition is satisfied."

In general, by the construction of the Lagrangean alone, the Lagrange multiplier $\lambda^1(p,y)$ can be either positive or negative and may also be equal to zero, for the budget constraint is an equality. From Eqs. (7), (9) and Eq. (15), $(\lambda^1(p,y) - K) = \lambda^3(p,y) > 0$ by the non-satiation hypothesis, regardless of the magnitude *and* sign of the constant K and the sign of $\lambda^1(p,y)$. This means that K can be *arbitrarily large* in absolute value without ever violating the technical condition that $(\lambda^1(p,y) - K) > 0$. Requiring that K be arbitrarily small would render the effect of $Kp'q$ insignificant, contrary to the objective of the authors.

Therefore, the sufficient condition $(\lambda - K) > 0$ of Dusansky and Wilson is always satisfied under the non-satiation hypothesis. Hence, Proposition 2 and footnote 5 in Dusansky (1989, p. 898), footnote 5 in Dusansky and Kalman (1976, p. 195), and footnote 7 in Dusansky and Wilson (1993, p. 131) are irrelevant for the negative semi-definiteness of the Slutsky matrix.

We note finally that the utility function $f(q,p)$ of Eq. (1) is a positive affine transformation

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of the function $\pi(q)$. This is seen by recalling that, by Lemma, it is sufficient to choose $g(p, y) := \sigma(p) + Ky$, where K is an arbitrary constant. Then, $f(q, p) = \pi(q) + g(p, y)$, and the Slutsky conditions associated with $f(q, p)$ are identical to those associated with $\pi(q)$.

II. Conclusion

We have shown that the utility function $f(q, p)$ in Eq. (1) is operationally identical to and, therefore, indistinguishable from a conventional utility function which depends only on real goods. The eight papers by Dusansky and his co-authors, therefore, contain a meta-contradiction, that is, a contradiction involving two conceptual frameworks of analysis. They intended to model the demand for real goods by formulating a utility function that included the prices of real goods. They combined this objective with the further requirement that the resulting model reproduce the Slutsky matrix and its properties of the standard consumer theory. The use of a price-dependent utility function that is necessary and sufficient for re-covering the prototype Slutsky matrix and its properties, however, eliminated any possibility of detecting the effects of price-dependent preferences on the demand for real goods. The meta-contradiction, therefore, consists of wishing to formulate a model that expresses the effects of price-dependent preferences on demand and, at the same time, requiring that the Slutsky conditions of the standard consumer model hold. In other words, since all the market and environmental effects are revealed only through the demand functions, the specification and use of a utility function such as $f(q, p)$ of Eq. (1) is irrelevant and uninformative for the analysis of any economic problem where prices enter the consumer utility function and whose goal is the detection of the effects of price-dependent preferences on the demand for real goods.

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