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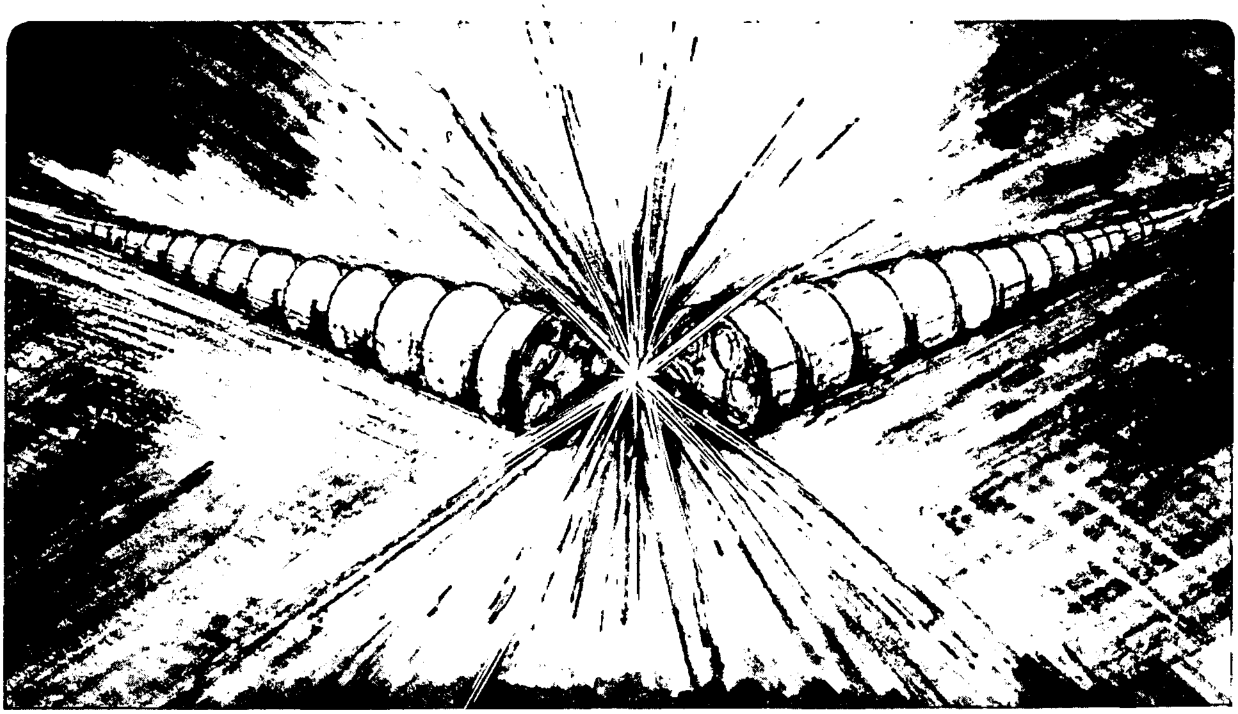

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Technical Report of the Betatron Design Study

SPACE-CHARGE AND CURRENT FORCE CANCELLATION IN TOROIDAL GEOMETRY*

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25 September 1981

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)
The cancellation in linear geometry of electrostatic and magnetic space-charge forces including image effects from conducting boundaries occurs to the degree $(1 - \beta^2) = \gamma^{-2}$. In toroidal geometry the cancellation is less. For a cylindral sheet beam at R_o, the degree of cancellation is $(R_o/R - \beta^2)$, or approximately $\Delta R/R_o$ for a relativistic beam. Several consequences are suggested.

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SPACE-CHARGE AND CURRENT FORCE CANCELLATION IN TOROIDAL GEOMETERY*

Donald W. Kerst

Electrostatic fields are derived from $\epsilon = -\frac{\partial V}{\partial n}\hat{n}$, with V = constant on conducting boundaries, while magnetic fields are derived from $\vec{B} = \frac{1}{2\pi R}\frac{\partial \Psi}{\partial n}[\hat{n} \times \hat{\theta}]$ also with Ψ = constant on the same conducting boundaries. ($\Psi/2\pi R = A_\theta$ is a flux or stream function). Furthermore, Ψ and V satisfy different differential equations: ($\partial/\partial\theta = 0$)

$$\left.\begin{aligned} 0 = \nabla^2 V &= \frac{\partial^2 V}{\partial R^2} + \frac{1}{R}\frac{\partial V}{\partial R} + \frac{\partial^2 V}{\partial Z^2} && \text{electrostatic} \\ \text{while } 0 &= \frac{\partial^2}{\partial R^2} - \frac{1}{R}\frac{\partial \Psi}{\partial R} + \frac{\partial^2 \Psi}{\partial Z^2} && \text{magnetostatic} \end{aligned}\right\} \quad (1)$$

The first derivative sign is reversed.

It cannot then be expected that in toroidal geometry $\beta|\epsilon| = |B|$, which is necessary for approximate cancellation of space-charge and space-current forces on incoherent motion with $\beta \sim 1$. While we speak of "images" of current or charge in the poles or wall, the image model is possible only in two dimensions. In 3D charges at infinity are needed.

A geometric way to see this for cylindrical geometry is shown in Fig. 1. We see that $\beta|\epsilon| = |B|$ holds only at $R = R_0$, and that at other values of R, $\beta|\epsilon| = |B|\frac{R_0}{R}$ and thus force cancellation is

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$$\Delta F = 4\pi\sigma_0 e\left[\frac{R_0}{R} - \beta^2\right] \qquad \text{cylindrical beam}$$

$$\text{instead of } \Delta F = 4\pi\sigma_0 e\,[1 - \beta^2] \qquad \text{linear beam} \tag{2}$$

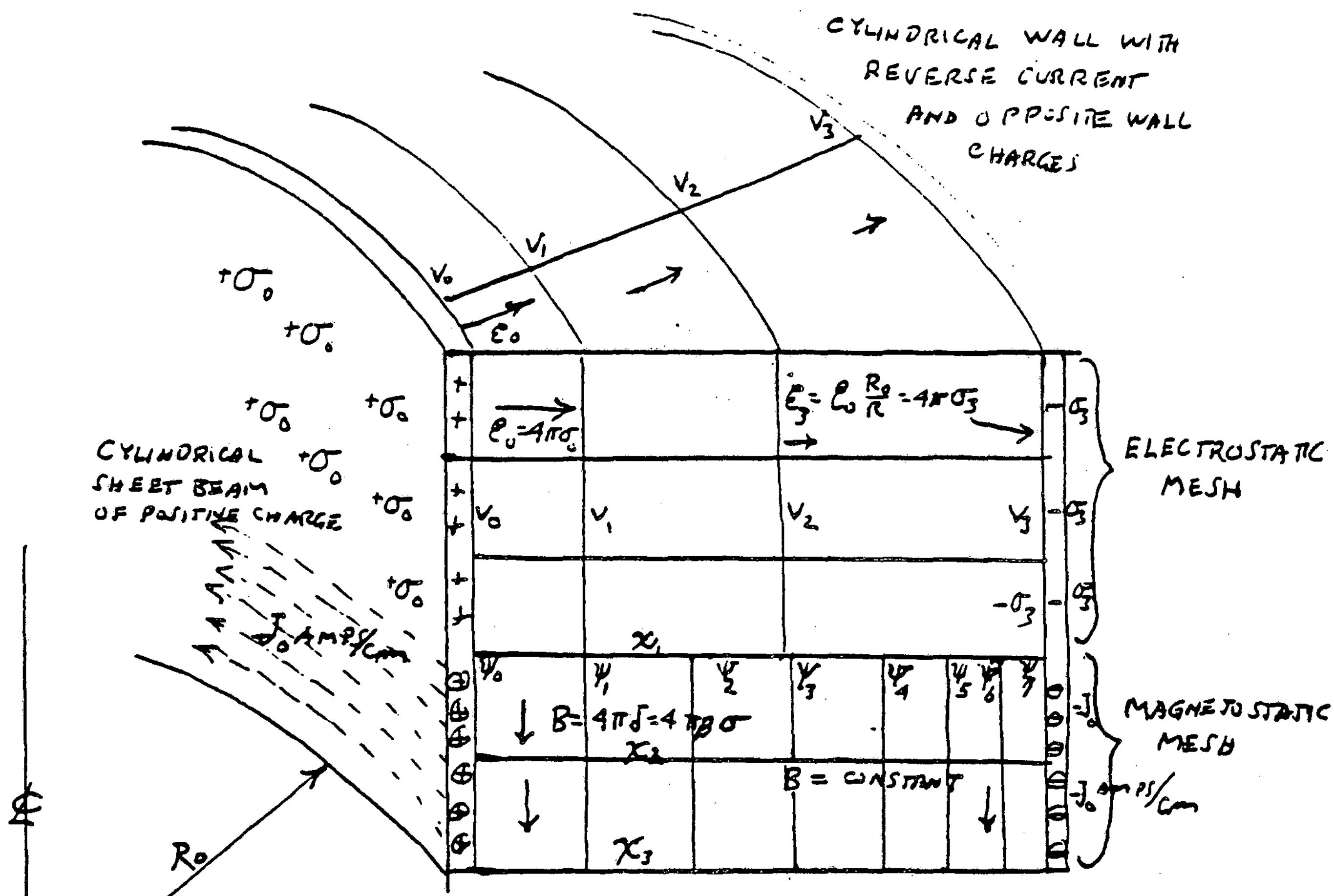
Stated the usual way, for a linear beam cancellation occurs to within γ^{-2} of the predominant electric force, but in cylindrical fields cancellation is within

$$\gamma^{-2} + (R_0/R - 1) \approx -\frac{\Delta R}{R_0} \tag{3}$$

of the electric force. So electric force predominates for $R < R_0$, and magnetic force predominates for $R > R_0$ for γ^{-2} small. For 50 MEV electrons $\gamma = 100$, and in an orbit of radius 100 cm the toroidal effect governs for particles a distance of 0.1 mm or more away from the sheet beam. At high currents electric fields of 200 KV/cm exist a few centimeters from beam centers, giving 200 KV/cm x 10^{-2} or ± 10 KV/cm or its equivalent magnetic field, $\Delta B = 30$ gauss, of unbalanced force field a few centimeters from the orbital radius which can be several times the focussing force of the guide field in our cases.

The magnitude of these fields suggests (1) that changes in the radius of curvature of a beam pipe or chamber or bumps on the wall could make force bumps at fixed points along the orbit of intense beams, and (2) that a test particle oscillating radially fully and symetrically across the beam would have effective cancellation of the force on its oscillation frequency because the unbalance force is always radially inward and does not reverse on opposite sides of the beam center. (3) Off momentum particles would have a negative increment in momentum compaction at $R > R_0$ and thus ride closer than normal to R_0, and at $R < R_0$ they would ride farther than normal from R_0. (4) There must be an effect, perhaps large, on the space charge limit on radial forces. (5) With electrostatic "images" in their automatically induced locations, perhaps the current "images" can be distributed so the resulting forces cancel. That is, the scalar magnetic potential problem obeys $\nabla^2 \chi = 0$ in toroidal geometry, but it takes its boundary condition from the distribution of Gilberts or amperes on the walls, -- this we can adjust in 3D.

FIG. 1



V = ELECTROSTATIC POTENTIAL (VOLTS)

$\vec{\mathcal{E}} = -\vec{\nabla} V$ ELECTROSTATIC FIELD $= -\frac{\partial V}{\partial n}\hat{n}$ (VOLTS/METER)

χ = MAGNETOSTATIC SCALAR POTENTIAL (GILBERTS $= 4\pi I$)

$\vec{H} = \vec{\nabla}\chi$ MAGNETIC FIELD (OERSTED=GILBERTS/cm)

Ψ = MAGNETOSTATIC FLUX FUNCTION (WEBERS)

$\vec{B} = \frac{1}{2\pi R}\frac{\partial \Psi}{\partial n}\left[\hat{n}\times\hat{\theta}\right]$ (WEBERS/METER^2 = TESLAS)

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