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EFFECTIVE MOMENT OPERATOR FOR MAGNETIC MOMENTS AND M1 TRANSITIONS IN THE Pb REGION*

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Abstract

The effect of core polarization on magnetic moments in the Pb region is investigated using first order perturbation theory with a central zero-range coupling interaction. The results are expressed in the form of a state dependent effective moment operator which includes an anomalous orbital g-factor introduced previously. This operator gives a better account of the experimental data than a state independent operator proposed earlier by Maier et al., particularly in the case of the known M1 transition rates which are quite sensitive to the magnitude of the polarization. The force required to fit the data is somewhat larger than realistic interactions currently in use.

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1. Introduction

The deviation of experimental magnetic moments from the Schmidt values and the retardation of M1 transition rates have been topics of interest ever since the inception of the shell model. The two are intimately related and the effects which bring about the deviations are, by now, fairly well known[1]. Most important is the effect of configuration admixtures resulting from the interaction of the shell model valence nucleons with the core, i.e. core polarization. This was first investigated by Horie and Arima[2] and Blin-Stoyle and Perks[3] in 1954. Of lesser importance, and not so well understood, are interaction and mesonic effects which actually result in a modification of the form of the "bare" magnetic operator[4]. In addition to these Bertsch[5] has also suggested that Brueckner correlations can affect the magnetic moments.

At present there is a substantial amount of experimental data available on magnetic moments and M1 transition rates for nuclei in the Pb region. The effect of core polarization in this region has been investigated in several theoretical calculations[6-10]. All of these succeed in giving a good qualitative explanation of the experimental data, but fail to account completely for the observed deviations. In view of the, as yet unexplained, discrepancy between theory and experiment Maier _et al._[11] have attempted to parameterize the experimental data in terms of an effective moment operator. Their effort was only partially successful.

The "bare" magnetic moment operator is given by

$$\overline{\mu}(q) = g_\ell(q)\overline{\ell} + g_s(q)\overline{s} = g_\ell(q)\overline{j} + G(q)\overline{s} \qquad (1)$$

where $G(q) = g_s(q) - g_\ell(q)$ and q is a charge index. For protons g_ℓ=1 and g_s = 5.58 while for neutrons g_ℓ = 0 and g_s = 3.82 all in nuclear magnetons. The effective magnetic moment operator of ref.[11] is

$$\bar{\mu}_{eff}(q) = \bar{\mu}^*(q) + \delta\bar{\mu}(q) \tag{2a}$$

$$\delta\bar{\mu}(q) = G_0(q)\bar{s} + G_2(q)\bar{p} \tag{2b}$$

where $\bar{p}$ is a vector with z-component

$$p_z = i^2[Y_2 \times \bar{s}]^1_z \tag{2c}$$

and $\mu^*(q)$ is the "bare" magnetic moment operator modified by the inclusion of an anomalous orbital g-factor, i.e. $g_\ell(q) \longrightarrow g_\ell(q) + \delta g_\ell(q)$. The term $\delta\bar{\mu}(q)$ is associated with core polarization effects while the anomalous orbital g-factor was first proposed by Yamazaki et al.[12] in order to explain the experimental magnetic moment of the $|1h_{9/2}\ 1i_{13/2};\ 11^-\rangle$ state in ^{210}Po. It presumably can be associated with mesonic effects[13]. Maier et al. assumed that (1) $G_i(n) = -G_i(p)$ (iso-vector assumption) and (2) that G_i did not depend on the orbital of the valence nucleon. With $G_0(n) = 3.43$, $G_2(n) = 4.55$, $\delta g_\ell(p) = 0.09$, and $\delta g_\ell\ (n) = -0.06$ they obtained a reasonable fit to the known magnetic moments, but overestimated the retardation of the $p_{3/2} \longrightarrow p_{1/2}$ and $f_{7/2} \longrightarrow f_{5/2}$ M1 transition rates in ^{207}Pb by 2-3 orders of magnitude.

The present study was undertaken in an effort to understand the above difficulty. We have investigated the effect of core polarization on the magnetic moments and M1 transition rates using first order perturbation theory with the assumption that the interaction between the valence nucleons and the core can be represented by a zero-range force. The results of these calculations can be expressed, exactly, as a contribution to the magnetic moment operator of the form given in eq. (2b). In this approach the $G_i(q)$ are proportional to integrals which express the overlap between the radial wave functions of the valence nucleons and the core admixtures, therefore, the resulting effective magnetic moment operator is state dependent. We find this state dependence to be quite important.

In the following sections of this paper we review the theory of effective operators and discuss the results of our perturbative calculations.

2. Theory of Effective Operators

The theory of effective operators is well known and is reviewed here for the purpose of deriving certain relations pertinent to this paper. In first order perturbation theory the admixtures of core excited states in the single particle state (jm) are computed according to[14])

$$|\tilde{jm}\rangle = |jm\rangle + (E_j - H_o)^{-1} PV|jm\rangle \quad (3)$$

where $V = \sum_{i<j} v_{ij}$ is the interaction between the single particle and the core, P is a projection operator defining the core excited stated to be included in the calculations, and H_0 is the Hamiltonian which defines the energy of these intermediate states. The reduced matrix element[15]) of a one body operator of rank J, $T^J = \sum_i t_i^J$, between two such states is given by

$$M = M_0 + \delta M$$

$$M_0 = \langle j_f \| t^J(q) \| j_i \rangle \quad (4)$$

$$\delta M = \langle j_f \| T^J (E_{j_i} - H_o)^{-1} PV + VP(E_{j_f} - H_o)^{-1} T^J \| j_i \rangle$$

where M_0 is the single particle matrix element and δM is the modification due to core polarization. As before q is being used as a charge index.

In order to obtain explicit expressions for δM the nature of the core excited states must be specified. If we ignore interactions between the core particles and take P to be the projection operator for uncorrelated 2p-1h states

$$|j(j_p\bar{j}_h)J;\, j'm'\rangle =$$

$$\sum_{\substack{m_p m_h \\ mM}} \langle jJmM|j'm'\rangle\langle j_p j_h m_p - m_h|JM\rangle\ (-1)^{j_h-m_h}\ a^+_{jm}(q)\ a^+_{j_p m_p}(q')\ a_{j_h m_h}(q')|C\rangle \tag{5}$$

we obtain

$$\delta M = \sum_{\substack{j_p j_h \\ q'}} \left\{\frac{1}{E_{if}(ph)} + \frac{1}{E_{fi}(ph)}\quad P_{ph}\right\}\ (-1)^{j_p-j_h}\ \frac{\hat{j}_h\hat{j}_p}{\hat{J}}\langle j_h\|t^J(q')\|j_p\rangle\,\alpha^J_{qq'}(j_p j_h, j_f j_i) \tag{6}$$

where P_{ph} acts to the right and means to interchange j_p and j_h, $E_{if}(ph) = E_{j_i} - E_{j_f} - E_{j_p} + E_{h}$, and $\alpha^J_{qq'}$ is the Jth multipole coefficient of the interaction defined by

$$\alpha^J_{qq'}(j_p j_h,\ j_f j_i) = \sum_{J'} (-1)^{j_f+j_h-J'}\frac{\hat{J}\ \hat{J}'^2}{\hat{j}_p\hat{j}_f}\begin{Bmatrix} j_p j_f J' \\ j_i j_h J\end{Bmatrix}\ \langle j_p j_f J'|v_{qq'}|j_h j_i J'\rangle\quad . \tag{7}$$

The two body matrix element in this equation is antisymmetrized but not normalized and the subscript qq' distinguishes between the p-p(n-n) and p-n components of the two body force. If we treat interactions between the core particles in the T.D.A. approximation[8,16] then P projects onto correlated 2p-1h states

$$|j(\nu)J;\, j'm'\rangle = \sum_{\substack{j_p j_h \\ q'}} X^J_\nu(j_p\ j_h\ q')|j(j_p\ \bar{j}_h)J;\ j'm'\rangle \tag{8}$$

and

$$\delta M =$$

$$\sum_{\substack{j_p j_h \\ j_p' j_h' \\ q' q'' \\ \nu}} X_\nu^J(j_p\ j_h\ q') X_\nu^J(j_p'\ j_h'\ q'') \left\{ \frac{1}{E_{if}(\nu)} (-1)^{j_p - j_h} \frac{\hat{j}_h\ \hat{j}_{p'}}{\hat{J}} \langle j_h \| t^J(q') \| j_p \rangle \alpha^J_{qq''}(j_p' j_h', j_f j_i) \right.$$

$$\left. + \frac{1}{E_{fi}(\nu)} (-1)^{j_p' - j_h} \frac{\hat{j}_h'\ \hat{j}_p}{\hat{J}} \langle j_p \| t^J(q') \| j_h \rangle\ \alpha^J_{qq''}(j_h' j_p', j_f j_i) \right\} \tag{9}$$

where $E_{if}(\nu) = E_{j_i} - E_{j_f} - E_\nu$. The energies E_ν and the amplitudes X_ν^J are obtained by performing a diagonalization in the space of particle-hole core excitations.

For a central zero-range force

$$v_{qq'}(1,2) = (A^0_{qq'} + A^1_{qq'}\ \bar{\sigma}_1 \cdot \bar{\sigma}_2)\delta(\bar{r}_1 - \bar{r}_2) \tag{10}$$

the exchange component of a two body matrix element is equal to the direct component and

$$\alpha^J_{qq'}(j_p j_h, j_f j_i) = \frac{2}{\hat{J}} \sum_{LS} A^S_{qq'}\ (-1)^{S+J}\ \langle\ j_p \| T^{LSJ} \| j_h\ \rangle \langle\ j_f \| T^{LSJ} \| j_i\ \rangle\ \ I(\ell_p \ell_f \ell_h \ell_i) \tag{11}$$

where the reduced matrix elements contain integrations only over spin and angular coordinates, T^{LSJ} is the spin-angle tensor

$$T^{LSJ}_{M_J} = \sum_{M\lambda} \langle LSM\lambda | JM_J \rangle\ i^L\ Y_{LM}\ \sigma^S_\lambda\ , \tag{12}$$

and I is the radial overlap integral

$$I(\ell_p \ell_f \ell_h \ell_i) = \int u_{n_p \ell_p}(r)\, u_{n_f \ell_f}(r)\, u_{n_h \ell_h}(r)\, u_{n_i \ell_i}(r)\, r^2 dr \quad . \tag{13}$$

The separation of the core and valence coordinates achieved in eq. (11) allows δM to be written in the following form

$$\begin{aligned} \delta M &= \langle j_f \| \delta t^J(q) \| j_i \rangle \\ \delta t^J_{M_j}(q) &= \sum_{LS} t_q^{LSJ}\, T^{LSJ}_{M_J} \quad . \end{aligned} \tag{14}$$

The reduced matrix element of the effective operator $\delta t^J(q)$ can be interpreted as an integral over spin and angular variables only, in which case t_q^{LSJ} will depend on the states j_i and j_f through the radial integrals I, or it can be interpreted as an integral over all variables, in which case t_q^{LSJ} will be an explicit function of r. If the first interpretation is adopted, the expressions for t_q^{LSJ} corresponding to eq. (6) and eq. (9) are

$$t_q^{LSJ} = \sum_{\substack{j_p j_h \\ q'}} \frac{E(ph)}{Q^2 - E^2(ph)}\, \frac{4\,\hat{j}_p^2}{\hat{J}^2}\, A^S_{qq'}\, \langle j_p \| t^J(q') \| j_h \rangle \langle j_p \| T^{LSJ} \| j_h \rangle\, I(\ell_p \ell_f \ell_h \ell_i) \tag{15}$$

where $Q = E_{j_i} - E_{j_f}$ and $E(ph) = E_{j_p} - E_{j_h}$, and

$$t_q^{LSJ} = \sum_{\substack{j_p j_h \\ j_p' j_h' \\ q' q'' \\ \nu}} \frac{E(\nu)}{Q^2 - E^2(\nu)} X_\nu^J(j_p j_h q')\, X_\nu^J(j_p'\, j_h'\, q'')$$

$$\times \frac{4\,\hat{j}_p\,\hat{j}_p'}{\hat{J}^2} A^S_{qq''} \langle j_p \| t^J(q') \| j_h \rangle \langle j_p' \| T^{LSJ} \| j_h' \rangle\; I(\ell_p' \ell_f \ell_h' \ell_i) \quad , \qquad (16)$$

respectively. In deriving these equations it has been assumed that t^J and T^{LSJ} have the same conjugation properties. The second interpretation leads to identical expressions, except that $u_{n_p \ell_p}(r) u_{n_h \ell_h}(r)$ appears in place of $I(\ell_p \ell_f \ell_h \ell_i)$.

In the case of magnetic moments and M1 transitions the one body operator of interest is the "bare" magnetic moment operator given in eq. (1). Specializing the above development to this case immediately leads to the effective moment operator defined in eq. (2). Observe that the selection rules for $\overline{\mu}$ (and $\overline{\mu}^*$) are $\Delta\pi = 1$, $\Delta L = 0$, and $\Delta S = \Delta J = 0,1$ while $\overline{\mu}_{eff}$ allows $\Delta L = 2$ as well. As a result of the selection rules on $\overline{\mu}$, the only particle-hole admixtures which can contribute directly to $\delta\overline{\mu}$ in lowest order are $J = 1^+$ states formed from spin-orbit partners. In the Pb region these are the $1\, h_{9/2} - 1\, h_{11/2}^{-1}$ proton and $1\, i_{11/2} - 1\, i_{13/2}^{-1}$ neutron particle-hole pairs. Other particle-hole pairs do contribute indirectly to $\delta\overline{\mu}$ when core interactions are taken into account.

The following expressions for $G_0(q)$ and $G_2(q)$ are obtained from eq. (15).

$$G_0(q) = \frac{8}{3\pi} K(q) \qquad\qquad G_2(q) = \frac{8}{3} (2\pi)^{-1/2} K(q)$$

$$K(q) = \sum_{\substack{j_p j_h \\ q'}} \delta_{\ell_p \ell_h} \frac{E(ph)}{Q^2 - E^2(ph)} \frac{\ell_p(\ell_p + 1)}{2\ell_p + 1} G(q') A^1_{qq'}\, I(\ell_p \ell_f \ell_h \ell_i) \qquad (17)$$

Note that the above sum contains only two terms and that the sign of $K(q)$ is negative for protons and positive for neutrons as $AG > 0$ and < in these cases for most reasonable forces while $Q^2 - E^2(ph) < 0$. It is also interesting that G_0 and G_2 are simply related in this approximation. From eq. (16)

$$G_0(q) = \frac{8}{3\pi} \sum_{\substack{j_p j_h \\ j_p' j_h' \\ q' q'' \\ \nu}} A(phq', p'h'q'', \nu)\, \delta_{\ell_p \ell_h}\, \delta_{\ell_p' \ell_h'} \left[\frac{\ell_p(\ell_p + 1)}{2\ell_p + 1}\right]^{1/2} \left[\frac{\ell_p'(\ell_p' + 1)}{2\ell_p' + 1}\right]^{1/2}$$

$$G_2(q) = \frac{8\sqrt{2}}{3} \sum_{\substack{j_p j_h \\ j_p' j_h' \\ q' q'' \\ \nu}} A(phq', p'h'q'', \nu)\, \delta_{\ell_p \ell_h} \left[\frac{\ell_p(\ell_p + 1)}{2\ell_p + 1}\right]^{1/2} \hat{j}_p' \langle j_p' \| T^{211} \| j_h' \rangle \tag{18}$$

where

$$A(phq', p'h'q'', \nu) = \frac{E(\nu)}{Q^2 - E^2(\nu)} X_\nu^J(j_p\ j_h\ q') X_\nu^J(j_p'\ j_h'\ q'') G(q') A^1_{qq''}\ I(\ell_p' \ell_f \ell_h' \ell_i) \quad . \tag{19}$$

Here contributions from $J = 1^+$ particle-hole states other than those formed from spin-orbit partners appear implicitly in the $E(\nu)$ and explicitly in the unrestricted sum over j_p' and j_h' in the second of eq. (18). There is no simple relationship between G_0 and G_2 in this case, although deviation from the relationship of eq. (17) will be small in the event that the $1\ h_{9/2} - 1\ h^{-1}_{11/2}$ and $1\ i_{11/2} - 1\ i^{-1}_{13/2}$ p-h pairs are not strongly mixed with other p-h pairs by the core interactions.

3. Results and Discussion

There is experimental information on eight single particle (single hole) magnetic moments in the Pb region and the B(M1) for the two single hole M1 transitions in ^{207}Pb are known. In these cases we have calculated the Schmidt values and experimental deviations assuming (1) that $\delta g_\ell(p) = \delta g_\ell(n) = 0$ and (2) $\delta g_\ell(p) = 0.09$ and $\delta g_\ell(n) = -0.06$ from ref.[11]). Estimates of the values of $G_0(q)$ needed to fit the data have been obtained directly from the experimental deviations by assuming that $G_2(q) = (\pi/2)^{1/2} G_0(q)$ as suggested by eq. (17). The relations between $G_0(q)$ and the experimental diviations are easily obtained from eq. (2). For the magnetic moments

$$G_0(q) = \delta\mu_{exp} \left\{ \pm \frac{j}{2\ell + 1} + \frac{1 \mp (j+1/2)}{8\,(j+1)} \right\}^{-1} \qquad (j = \ell \pm 1/2) \qquad (20)$$

and for the M1 transitions

$$G_0(q) = \frac{8}{9}\, G(q) \left\{ B(M1)^{1/2}_{exp} - 1 - \frac{\delta g_\ell(q)}{G(q)} \right\} \qquad (21)$$

where $B(M1)_{exp}$ is expressed as a ratio to the Schmidt value. These results are summarized in Table 1.

The purpose here is to illustrate the need for the anomalous orbital g-factor and state dependence in the effective moment operator. The importance of the anomalous orbital g-factor is quite evident if one compares the values of $G_0(1)$ and $G_0(2)$ in the table, particularly those for the 1 $h_{9/2}$ and 1 $i_{13/2}$ states. Without this factor one is led to the conclusion that core polarization is considerably stronger for the 1 $h_{9/2}$ state than for the 1 $i_{13/2}$ states -- a fact which cannot be explained with the model being considered in this work. The values of $G_0(2)$ show a definite decrease as the number of nodes in the radial wave function of the valence nucleons increase. The $2f_{7/2}$ state in ^{209}Bi is an exception to this, but there is a large experimental uncertainty in the magnetic moment for this state and there is some question concerning the purity of the configuration assumed in extracting this value[11]). This state dependence is consistent with theoretical expectations, as can be seen from the overlap integrals given in Table 2.

The values of $G_0(2)$ should also be compared with the results of Maier et al. who found that $G_0(n) = -G_0(p) = 3.43$ and $G_2(n) = -G_2(p) = 4.55$ which give $G_2/G_0 = 1.33$ which is quite close to $(\pi/2)^{1/2}$. This comparison suggests that their searches were biased to the magnetic moments of the high spin states. They were able to fit the moments for the states with lower spin because the $L = 0$ and $L = 2$ terms in $\delta\bar{\mu}$ cancel one another in the moment calculations. This can be seen from eq.(20). In the case of the M1 transition rates the two terms are additive which explains the large retardations they obtained.

Observe finally that the $G_0(2)$ for the f and p states deduced from the transition rates are about 30% smaller than the corresponding values obtained from the magnetic moments. This means that even with the state dependence included there will be a tendency to overestimate the M1 transition rates, but not by orders of magnitude. Deviations from the assumed relationship between G_0 and G_2 will have additional bearing on this point.

Two perturbative calculations have been made with a zero-range interaction as described in the preceding section of this paper. In the first calculation we neglected the effect of interactions between the core particles and assumed that $E(1\ h_{9/2} - 1\ h^{-1}_{11/2}) = 5.60$ MeV and $E(1\ i_{11/2} - 1\ i^{-1}_{13/2}) = 5.86$ MeV as observed experimentally. In the second calculation core interactions have been treated by projecting onto correlated 2p-1h states constructed from the T.D.A. wave functions for the magnetic dipole states of ^{208}Pb given in ref.[10]). These wave functions have been selected because they give a reasonable description of the known properties of the core states. In a more consistent calculation the core states would be calculated with the same force that is used in estimating the effect of core polarization. Harmonic oscillator wave functions have been used throughout with b = 2.33fm[17]) and in both calculations the force strengths A^1_{pp} and A^1_{pn} as well as the anomalous orbital g-factors have been varied to give the best fit to the data.

The results of these two calculations are summarized in Table 3. Both calculations give quite similar results and they compare more favorably with experiment that the results of Maier et al. particularly in the case of the ^{207}Tl($3s_{1/2}$) and ^{207}Pb($3p_{3/2}$) magnetic moments and the M1 transitions in ^{207}Pb. The magnetic moment for the $2f_{7/2}$ state in ^{209}Bi remains an exception. The state dependence in the theoretical effective operator is somewhat more severe than that indicated by the experimental data. This can be seen by comparing the values of G_0 in Table 3 with the $G_0(2)$ of Table 1. If finite well wave functions[18]) had been used instead of harmonic oscillator wave functions the differences would be larger still. In view of the approximations involved in these calculations these differences are not considered serious. It is interesting that the calculations give a qualitative reproduction of the state dependence in the M1 transition rates. The retardation of these rates is still overestimated, however, the discrepancy in the M1 matrix elements is less than a factor of 1.25 as compared to the factor of about 17 obtained in ref.[11]).

We find $\delta g_\ell(p) = -0.08$ and $\delta g_\ell(n) = -0.06$ which is in reasonable agreement with the previous estimates[11,12]). The major effect of the core interactions is to push the isovector component of the magnetic dipole core strength up in energy[10]. This is reflected in the force strengths which have been obtained. The isovector component of the force obtained in Approximation 2 is about 29% larger than that obtained in Approximation 1. The agreement between the theoretical and experimental M1 transition rates is a little better in Approximation 2 than in Approximation 1. This is partially due to the fact that $G_2 \approx 1.11G_0$ in the former calculation as compared to $G_2 = 1.25G_0$ in the latter. In addition the importance of Q in the energy denominators [see eqs. (17) and (19)] is diminished in Approximation 2 as the core strength lies at a higher energy in this case.

In order to gain some estimate as to the uncertainty in the parameters which have been obtained additional fits were attempted with various pieces of the data omitted. This check is important particularly when there is a question concerning the purity of the configuration assumed in extracting a piece of data. In carrying out this test the values for the anomalous orbital g-factors and the isovector component of the interaction remained constant within 15%, but the isovector component of the interaction exhibited large fluctuations. It is concluded that both of the anomalous orbital g-factors but only one of the force strength parameters are well determined. This is expected as the isovector component of G(q) is more than 10 times greater than the isoscalar component.

Magnetic moments for other single particle states in the Pb region have been calculated using the parameters given in Table 3. The results are given in Table 4. As the results in Approximation 1 and 2 do not differ greatly, only the latter are shown in the table.

Table 5 contains a summary of the results obtained in calculations using more realistic interactions. The BGT and HJ results are from ref.[6,10], respectively.

The effect of core interactions have been included in these calculations and Blomqvist <u>et al.</u>[6]) have also included a small correction for interaction effects. The KK results have been obtained in this work using the "semi-realistic" Kallio-Kolltveit force[19]). Approximation 1 has been used and the results have been reduced by 30% to account for core interactions. The BGT and HJ interactions are similar and give larger deviations which are in better agreement with experiment than those obtained with the KK force. The former interactions contain repulsive odd central components and non-central components (the most important of which is the tensor force) while the latter is an s-wave central interaction. As a result of these additional components the BGT and HJ interactions give larger isovector coupling than the KK force.

The KK matrix elements can be reproduced quite well with a zero-range force with $A^{1}_{pp} = 124$ MeV·fm^3 and $A^{1}_{pn} = -24$ MeV·fm^3. The average of the BGT and HJ matric elements can be reproduced reasonably well with a zero-range force with $A^{1}_{pp} = 187$ MeV·fm^3 and $A^{1}_{pn} = -59$ MeV·fm^3. Results obtained with these forces have also been included in Table 5 for purpose of comparison. The isovector components of these forces are 74 and 123 MeV·fm^3, respectively, as compared to the value of 175 MeV·fm^3 for the interaction required to fit the data in Approximation 2 which has been given in Table 3.

Mavromatis and Zamick[8]) have given detailed expressions for calculating those corrections to the magnetic moments which arise in 2nd order perturbation theory. They have also indicated how certain terms which are first order in the coupling to the valence nucleon may be summed to all orders. Application was made to the 1 $h_{9/2}$ single proton state in ^{209}Bi where they found that by far the most important correction came from the term which corresponds to our use of correlated intermediate states (Approximation 2). The 1 $h_{9/2}$ level in ^{209}Bi is thought to be quite pure, while other single particle states in the Pb region might be mixed appreciably with low lying particle vibration coupled states[20]. This mixing might have important consequences.

As an example consider the mixing between the 1 $i_{13/2}$ single proton state and $|\ 1\ h_{9/2} \times 3^-; 13/2\ \rangle$ particle-vibration state in ^{209}Bi. A recent analysis[21] of the ^{209}Bi(p,p')^{209}Bi* reaction at 61.2 suggests that these states are admixed about 8%. The magnetic moment of the 3^-(2.62 MeV) vibrational state in ^{208}Pb is known to be 1.7 - 2.2 nm[22,23]. The correction to the magnetic moment of the 1 $i_{13/2}$ state calculated according to

$$\mu(1\ i_{13/2}) = .92\mu_{sp}\ (1\ i_{13/2}) + .96\ \delta\mu + .08\mu\ (1\ h_{9/2} \times 3^-;\ 13/2) \quad (22)$$

with $\delta g_\ell(p) = .08$ nm turns out to -.34 nm which is substantial. It is not known if this correction might be cancelled by other contributions, but it suffices to demonstrate that higher order terms besides the core interaction terms may be important in some instances. Experimental magnetic moments for other low lying vibrational states in ^{208}Pb might provide useful information concerning this question.

Following the phenomenological point of view taken in this work, we point out that the above admixture might be compensated for by increasing $\delta g_\ell(p)$ to 0.11 nm. This then gives $\mu(1\ h_{9/2}) = 4.19$ nm and $\mu(1\ i_{13/2}) = 7.72$ which is still in reasonable agreement with experiment.

4. Summary

It has been shown that the single particle magnetic moments and M1 transition rates in the Pb region can be understood by introducing an anomalous orbital g-factor in conjunction with a first order treatment of core polarization which assumes a zero-range coupling interaction. The results have been expressed as a state dependent effective moment operator of the form given in eq. (2). The properties of this operator may be summarized by the relations

$$G_i(q) = g_i(q)\ I_{ave}$$

$$g_0(q) = \alpha\ g_2(q)$$

$$g_i(n) = \beta\ g_i(p)$$

where I_{ave} is the average radial overlap integral, $\alpha = 1.11 - 1.25$, $\beta = .91$, and $g_0(n) = 473$ nm. This operator gives a better reproduction of the experimental data than the earlier state independent operator of Maier *et al*[11]).

It has also been found that the coupling interaction required to fit the data is from 1.4 - 2.3 times stronger than current realistic interactions. In addition the values for the anomalous orbital g-factors, $\delta g_\ell(p) = 0.08$ nm and $\delta g_\ell(n) = -0.06$ nm, are larger than previous theoretical estimates of corrections for mesonic and interaction effects[4,6]). Although a good, and hopefully useful fit to the data has been achieved with this simple model, theoretical puzzles still remain.

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Table 1. Summary of experimental information on single particle (single hole) magnetic moments and M1 transitions in the Pb region. Experimental $G_0(q)$ are also shown.

Magnetic Moments[a]

State	μ_{exp}[b]	$\mu_{sp}(1)$	$\delta\mu_{exp}(1)$	$G_0(1)$	$\mu_{sp}(2)$	$\delta\mu_{exp}(2)$	$G_0(2)$
$^{209}Bi(1h_{9/2})$	4.08	2.63	1.45	-5.33	3.08	1.00	-3.66
$^{209}Bi(2f_{7/2})$	4.41(65)	5.79	-1.38(65)	-3.31	6.06	-1.65(65)	-3.96
$^{209}Bi(1i_{13/2})$	7.9	8.79	-0.89	-2.23	9.34	-1.44	-3.60
$^{207}Tl(3s_{1/2})$	1.63	2.79	-1.16	-2.32	2.79	-1.16	-2.32
$^{207}Pb(3p_{1/2})$	0.59	0.64	-0.05	—	0.60	-0.01	—
$^{207}Pb(2f_{5/2})$	0.65(5)	1.36	-0.71(5)	3.31	1.19	-0.54(5)	2.52
$^{207}Pb(3p_{3/2})$	-1.09	-1.91	0.82	1.82	-1.97	0.88	1.96
$^{207}Pb(1i_{13/2})$	-0.98	-1.91	0.93	2.33	-2.27	1.29	3.23

M1 Transitions[c]

Transition	$B(M1)_{exp}$	$G_0(1)$	$G_0(2)$
$^{207}Pb(p_{3/2} \rightarrow p_{1/2})$	.32 ± .08	1.47	1.42
$^{207}Pb(f_{7/2} \rightarrow f_{5/2})$	.25 ± .06	1.70	1.64

[a] μ, $\delta\mu$, and G_0 are all given in nm units. The arguments 1 and 2 differentiate between the results with and without the anomalous g-factor.

[b] Not all of the single particle moments shown are the result of direct experimental measurement. Some have been extrapolated from measurements on states involving more complicated configurations. See ref.[6-11]), particularly ref.[11]) for origin of data.

[c] $B(M1)_{exp}$ is the dimensionless quantity $B(M1)/B(M1)_{sp}$ and G_0 is given in nm.

Table 2. Overlap integrals in fm^{-3} computed with harmonic oscillator wave functions with the range parameter b = 2.33 fm.

$n\ell$	$I(1h, n\ell, 1h, n\ell)$	$I(1i, n\ell, 1i, n\ell)$
3s	0.00429	0.00418
3p	0.00357	0.00338
2f	0.00421	0.00402
1h	0.00783	0.00692
1i	0.00692	0.00666

Table 3. Summary of magnetic moments and M1 transition rates obtained in calculations with zero-range interaction. Approximation 1 and 2 refer to calculations made with uncorrelated and correlated intermediate states, respectively.

Magnetic Moments

		Approximation 1[a]		Approximation 2[b]			Maier *et al.*[11]
State	μ_{exp}	G_0	μ	G_0	G_2	μ	μ
$^{209}Bi(1h_{9/2})$	4.08	-3.82	4.06	-3.78	-4.26	4.10	3.98
$^{209}Bi(2f_{7/2})$	4.41(65)	-2.08	5.16	-2.07	-2.33	5.15	4.65
$^{209}Bi(1i_{13/2})$	7.9	-3.42	7.90	-3.41	-3.84	7.87	7.98
$^{207}T\ell(3s_{1/2})$	1.63	-2.12	1.73	-2.12	-2.38	1.73	1.08
$^{207}Pb(3p_{1/2})$	0.59	1.64	0.60	1.61	1.74	0.56	0.63
$^{207}Pb(2f_{5/2})$	0.65(5)	1.95	0.78	1.91	2.06	0.74	0.49
$^{207}Pb(3p_{3/2})$	-1.09	1.64	-1.23	1.61	1.74	-1.23	-0.44
$^{207}Pb(1i_{13/2})$	-0.98	3.22	-0.98	3.16	3.40	-0.96	-0.91

M1 Transitions

Transition	B(M1)$_{exp}$	G_0	B(M1)	G_0	G_2	B(M1)	B(M1)
$^{207}Pb(p_{3/2} \rightarrow p_{1/2})$	.32 ± .08	1.68	0.24	1.63	1.75	0.26	0.001
$^{207}Pb(f_{7/2} \rightarrow f_{5/2})$	.25 ± .06	2.15	0.12	2.01	2.15	0.16	0.001

[a]These results are obtained with A^1_{pp} = 221 MeV·fm^3, A^1_{pn} = -49 MeV·fm^3, $\delta g_\ell(p)$ = 0.08 nm, and δg_ℓ (n) = -0.06 nm.

[b]These results are obtained with A^1_{pp} = 255 MeV·fm^3, A^1_{pn} = -94 MeV fm^3, $\delta g_\ell(p)$ = 0.08 nm, and $\delta g_\ell(n)$ = 0.06 nm.

Table 4. Predicted magnetic moments for single particle (single hole) states in the Pb region for which there is not experimental data at present. Magnetic moments obtained with effective moment operator of ref.[11]) are also shown.

Magnetic Moments

State	μ_{sp}^{a}	G_0	G_2	μ	Maier et al. μ
$^{209}Bi(3p_{1/2})$	- .263	-1.75	-1.97	- .181	- .237
$^{209}Bi(2f_{5/2})$	.864	-2.06	-2.33	1.57	1.83
$^{209}Bi(3p_{3/2})$	3.79	-1.75	-1.97	3.07	2.35
$^{207}T\ell(2d_{3/2})$	.126	-2.41	-2.72	.669	.772
$^{207}T\ell(1h_{11/2})$	7.79	-3.78	-4.26	6.62	6.87
$^{207}T\ell(2d_{5/2})$	4.79	-2.41	-2.72	3.90	3.52
$^{207}T\ell(1g_{7/2})$	1.72	-3.88	-4.38	3.05	2.90
$^{209}Pb(3d_{3/2})$	1.15	1.42	1.53	.796	.554
$^{209}Pb(2g_{7/2})$	1.49	1.72	1.86	.789	.442
$^{209}Pb(4s_{1/2})$	-1.91	1.33	1.43	-1.25	- .195
$^{209}Pb(1j_{15/2})$	-1.91	2.78	2.99	-1.19	- .989
$^{209}Pb(3d_{5/2})$	-1.91	1.42	1.53	-1.41	- .574
$^{209}Pb(1i_{11/2})$	1.62	3.16	3.40	.291	.298
$^{209}Pb(2g_{9/2})$	-1.91	1.72	1.86	-1.42	- .765
$^{207}Pb(2f_{7/2})$	-1.91	1.91	2.06	-1.27	- .677
$^{207}Pb(1h_{9/2})$	1.56	3.37	3.64	.286	.360

[a]In calculating μ_{sp} it has been assumed that $\delta g_\ell(p) = \delta g_\ell(n) = 0$.

Table 5. Summary of theoretical magnetic moments and M1 rates obtained with realistic interactions. "Equivalent" zero-range results are also shown.

Magnetic Moments

State	$\delta\mu_{exp}^{a}$	$\delta\mu_{BGT}$	$\delta\mu_{HJ}^{b}$	$\delta\mu_{KK}$	$\delta\mu_{ZR}^{d}$	$\delta\mu_{ZR}^{e}$
$^{209}Bi(1h_{9/2})$	1.05	0.72	0.70[c]	0.43	0.78	0.41
$^{209}Bi(2f_{7/2})$	-1.60(65)			-0.37	-0.63	-0.33
$^{209}Bi(1i_{13/2})$	-1.39			-0.49	-1.01	-0.53
$^{207}Tl(3s_{1/2})$	-1.16		-0.91	-0.47	-0.76	-0.41
$^{207}Pb(3p_{1/2})$	-0.01	-0.19	-0.12	-0.01	-0.03	0
$^{207}Pb(2f_{5/2})$	-0.54(5)	-0.46	-0.40	-0.19	-0.32	-0.17
$^{207}Pb(3p_{3/2})$	0.88		0.52	0.31	0.52	0.28
$^{207}Pb(1i_{13/2})$	1.29			0.51	0.94	0.50

M1 Transitions

Transition	$B(M1)_{exp}$	$B(M1)_{HJ}$	$B(M1)_{KK}$	$B(M1)_{ZR}^{c}$	$B(M1)_{ZR}^{d}$
$^{207}Pb(p_{3/2} \rightarrow p_{1/2})$	0.32 ± 0.08	0.44	0.66	0.44	0.64
$^{207}Pb(f_{7/2} \rightarrow f_{5/2})$	0.25 ± 0.06		0.57	0.34	0.56

[a]The values of $\delta g_{\ell}(q)$ given in Table 3 have been used in extracting $\delta\mu_{exp}$.

[b]Deviations have been multiplied by 1.19 to correct for differences in harmonic oscillator wave functions.

[c]This result has been taken from the work of Mavromatis _et al._[8]).

[d]Results obtained with zero-range force matched to BGT and HJ matrix elements.

[e]Results obtained with zero-range force matched to KK matrix elements.

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