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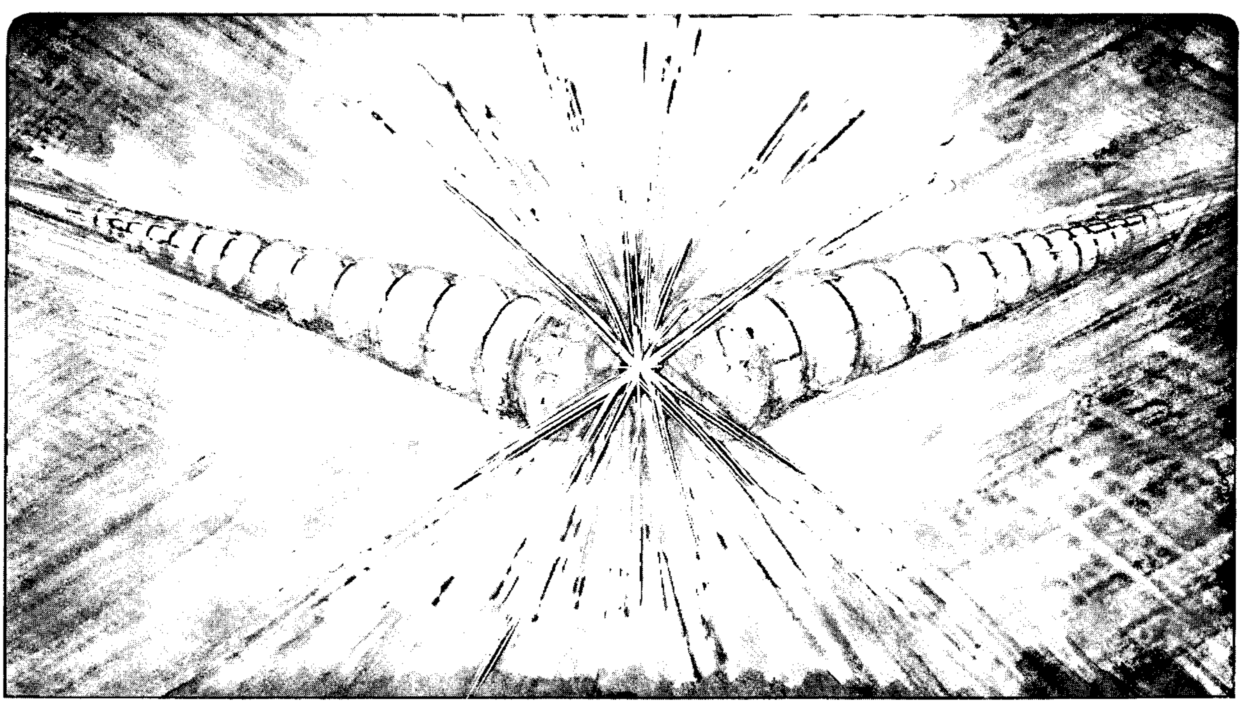

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Finite Pulse Effects in Self-Amplified-Spontaneous-Emission*

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Finite Pulse Effects in Self-Amplified-Spontaneous-Emission *

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Abstract

We study the effects of the electron density profile on self-amplified-spontaneous-emission(SASE). A general formalism in the linear regime is developed by deriving the coupled Maxwell-Klimontovich equations for an arbitrary density profile and including the effects of the energy spread, diffraction, and the betatron oscillation. An explicit solution is obtained for the one-dimensional(1-D) case. The temporal and the spectral intensity profiles of SASE depend linearly on the initial electron correlation function. The correlation function consists of two terms, a term giving rise to the usual spontaneous radiation and its amplification to SASE, and a term representing the coherent bunched beam effect. The latter term has been neglected so far in the treatments of SASE, but it could be significant when there is a variation in the electron density at a length scale comparable to the wavelength. The theory reproduces the well-known results when the electron density is uniform. It also reprodues a recent theory for a finite top-hat density profile and a vanishing energy spread.

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I. INTRODUCTION

The theory of SASE in the linear regime has been treated before for the case of an infinitely long, uniform electron beam [1,2]. Recently, Bonifacio et al. [3] have presented a theory of SASE for a finite, top-hat density profile with a vanishing electron's energy spread. In this paper, we extend the theory to a general density profile including the effects of electron's energy spread, diffraction and the betatron oscillation.

The Maxwell-Klimontovich equations describing the linear evolution of the radiation amplitude and the electron distribution function is derived in section II. The crucial step in this derivation is the specification of the electron coordinate relative to the bunch center with two variables, the macroscopic variable ζ and the microscopic phase variable θ_j. A ζ-dependent Klimontovich distribution function for the electron phase is introduced. The distribution function enters as a source term in Maxwell's equation, and satisfies an equation describing the evolution of the microscopic variables at a fixed ζ.

The equations are solved explicitely for the special 1-D case in section III. The solution reproduces the known results in the appropriate limits. The temporal and the spectral intensity profiles of SASE depend linearly on the initial electron correlation function. The correlation function consists of two terms, a term giving rise to the usual spontaneous radiation and a term representing the coherent bunched beam effect. The latter could be significant when there is a variation in the electron density at a length scale comparable to the radiation wavelength.

Section IV contains the concluding remarks.

II. INTERACTION OF RADIATION FIELD WITH FINITE PULSE ELECTRON BEAM

The slowly varying part $\mathcal{E}$ of the electric field $\mathbf{E}$ is defined by

$$\mathbf{E}(\mathbf{x},t,z) \;=\; \mathcal{E}(\mathbf{x},t,z)e^{ik_s(ct-z)} \;+\; c.c. \tag{1}$$

Here $\mathbf{x}, t, z$ are respectively the two dimensional transverse, time, and longitudinal coordinates, $k_s = 2\pi/\lambda_s$ is the radiation spatial frequency, λ_s is the wavelength, c is the speed of light, and *c.c.* implies complex conjugate.

To describe the source term in the Maxwell equation, the electron bunch is divided into bins of width Δ. The bin size Δ is normally taken to be the order of λ_s to be consistent with the slowly varying phase and amplitude approximation. However Δ is really the resolution of the macroscopic electron distribution, which could be much smaller than λ_s. The coordinate of the electron is written as

$$z = v_{\|}\bar{t}_j(z) \;+\; \zeta \;+\; \delta\zeta_j. \tag{2}$$

Here we choose z as the independent variable, $v_{\|}$ is the average longitudinal velocity, $\bar{t}_j(z)$ is the time the electron arrives at z (averaged over the velocity oscillation in the case of a planar undulator). In the above, the variable ζ is the macroscopic variable specifying the bin, while $\delta\zeta_j$ is the microscopic variable specifying the electron position within the bin. The phase θ_j is introduced via

$$\theta_j = (k_s + k_u)(\zeta + \delta\zeta_j) \simeq k_s(\zeta + \delta\zeta_j). \tag{3}$$

where $k_u = 2\pi/\lambda_u$ and λ_u is the undulator period. Figure 1 illustrates the coordinates ζ and $\delta\zeta_j$. Following the standard derivation [4] and changing the variable from (t,z) to (ζ,z), we obtain (MKS units are used throughout this paper)

$$\Big(\frac{\partial}{\partial z} + (1-\beta_{\|})\frac{\partial}{\partial \zeta} + i\frac{\nabla_\perp^2}{2k_s}\Big)\mathcal{E}(\mathbf{x},\zeta;z) = -\frac{eK_1}{4\epsilon_0\gamma_0}\frac{1}{\Delta}\sum_{j\in[\Delta,\zeta]} e^{i\theta_j}\delta^{(2)}(\mathbf{x}-\mathbf{x}_j). \tag{4}$$

Here e is the electron charge, $\beta_{\|} = v_{\|}/c$, $K_1 = K(J_0(\xi) - J_1(\xi))$, $K = eB_0/mck_u$, B_0 is the peak undulator magnetic field, m is the electron mass, $\xi = K^2/4(1+K^2/2)$, J's are Bessel functions, γ_0 is the energy of the reference electron in units of mc^2, and ϵ_0 is the vacuum dielectric constant. The summation in Eq. (4) is over the electrons belonging to the bin specified by ζ.

We introduce an electron distribution function as follows:

$$\hat{f}(\theta, \eta, \mathbf{x}, \dot{\mathbf{x}}, \zeta; z) = \frac{1}{N_\Delta} \sum_{j \in [\Delta, \zeta]} \delta(\theta - \theta_j)\delta(\eta - \eta_j)\delta^{(2)}(\mathbf{x} - \mathbf{x}_j)\delta^{(2)}(\dot{\mathbf{x}} - \dot{\mathbf{x}}_j). \tag{5}$$

where $\eta_j = (\gamma_j - \gamma_0)/\gamma_0$, γ_j is the energy of j-th electron in units of mc^2, $\dot{\mathbf{x}} = d\mathbf{x}/dz$ is the transverse angle, and N_Δ is the total number of electrons inside the bin of length Δ at the location of maximum density (which is taken to be at $\zeta = 0$). The source term in Maxwell's equation is proportional to the quantity

$$f_1(\mathbf{x}, \dot{\mathbf{x}}, \eta, \zeta; z) = \int d\theta e^{i\theta} \hat{f}. \tag{6}$$

f_1 is essentially the ζ-dependent bunching factor.

The equation for f_1 can be found from the Vlasov equation for $\hat{f}$, obtained by considering the change of $\hat{f}$ for a fixed ζ. The result to the lowest order in $\mathcal{E}$ is given by

$$\left(\frac{\partial}{\partial z} - i2\eta k_u - \frac{ik_u}{2}(\dot{\mathbf{x}}^2 + k_\beta^2 \mathbf{x}^2) + \dot{\mathbf{x}}\frac{\partial}{\partial \mathbf{x}} + \ddot{\mathbf{x}}\frac{\partial}{\partial \dot{\mathbf{x}}}\right) f_1 = -\frac{eK_1}{2mc^2\gamma_0^2}\mathcal{E}(\mathbf{x}, \zeta; z)\frac{\partial}{\partial \eta}\bar{f}(\mathbf{x}, \dot{\mathbf{x}}, \eta, \zeta). \tag{7}$$

Here we have assumed a constant betatron focusing with

$$\ddot{\mathbf{x}} \equiv \frac{d\dot{\mathbf{x}}}{dz} = -k_\beta \mathbf{x}. \tag{8}$$

where k_β is the betatron wave number. In Eq. (7), $\bar{f}$ is a smooth background distribution obtained by taking an ensemble average of $\int d\theta \hat{f}$ at $z = 0$ with the normalization

$$\int \bar{f}(\mathbf{x}, \dot{\mathbf{x}}, \eta, \zeta = 0) d^2\mathbf{x} d^2\dot{\mathbf{x}} d\eta = 1. \tag{9}$$

In the above equation, the point $\zeta = 0$ may be conventionally chosen to be the bunch center. Equations (4) and (7) completely specify the free-electron laser interaction for the case of a finite pulse electron beam.

III. SOLUTION FOR 1-D CASE

Let us now consider the 1-D equation obtained by neglecting the transverse dependence:

$$\left(\frac{\partial}{\partial z} + (1 - \beta_{\|})\frac{\partial}{\partial \zeta}\right)\mathcal{E}(\zeta; z) = -\kappa_1 \int f_1(\eta, \zeta; z) d\eta, \tag{10}$$

$$\left(\frac{\partial}{\partial z} - i2\eta k_u\right) f_1(\eta, \zeta; z) = -\kappa_2 \mathcal{E}(\zeta; z)\chi(\zeta)\frac{d}{d\eta}V(\eta). \tag{11}$$

where $\kappa_1 = eK_1 n_0/4\epsilon_0\gamma_0\Sigma$, $\kappa_2 = eK_1/2mc^2\gamma_0^2$, $n_0 = N_\Delta/\Delta$ is the peak linear density, and Σ is the beam cross section. Here we have assumed that the function $\bar{f}(\eta, \zeta)$ is factorized in the form

$$\bar{f}(\eta, \zeta) = V(\eta)\chi(\zeta), \tag{12}$$

with the normalization

$$\int V(\eta) d\eta = 1, \qquad \chi(0) = 1. \tag{13}$$

The coupled equations (10) and (11) can be solved by the Laplace transform technique in the variable z. The result consists of two terms, the coherent amplification term proportional to the initial coherent field $\mathcal{E}(\zeta; 0)$ and the self-amplified spontaneous emission (SASE) term proportional to the initial electron beam noise $f_1(\eta, \zeta; z = 0)$. The SASE term is given by

$$\mathcal{E}(\zeta;z) = i\frac{k_s\kappa_1}{2k_u^2}\int d\eta \int_{-\infty}^{\zeta} d\zeta' G(\zeta,\zeta',\eta;z) f_1(\eta,\zeta';0), \tag{14}$$

where

$$G(\zeta,\zeta',\eta;z) = \frac{1}{2\pi}\oint d\lambda \frac{e^{i[\lambda z - \psi_\lambda(\zeta) + \psi_\lambda(\zeta')]}}{\lambda/2k_u - \eta}. \tag{15}$$

Here the λ-integration is along a straight path parallel to the real axis, above all singularities in the integrand. The function $\psi_\lambda(\zeta)$ in Eq. (15) is

$$\psi_\lambda(\zeta) = \frac{k_s}{k_u}\lambda\zeta + 2k_s\rho^3 \int d\eta \frac{dV/d\eta}{\lambda/2k_u - \eta}\int_0^{\zeta} \chi(\zeta')d\zeta', \tag{16}$$

where $\rho = (\kappa_1\kappa_2/4k_u^2)^{1/3}$ is the fundamental scaling parameter.

Equation (14) reduces to the result in references [1] and [2] for the case of $\chi(\zeta) = 1$ (uniform pulse profile), and to the result in reference [3] for the case of a top-hat pulse profile with a vanishing energy spread. The case of a general pulse profile with a vanishing energy spread was also treated by Moore [5].

The temporal intensity profile of the SASE radiation is proportional to

$$\begin{aligned}\langle|\mathcal{E}(\zeta;z)|^2\rangle = &\left(\frac{k_s\kappa_1}{2k_u^2}\right)^2 \int d\eta \int d\eta' \int_{-\infty}^{\zeta} d\zeta' \int_{-\infty}^{\zeta} d\zeta'' \\ &G(\zeta,\zeta',\eta;z)G^*(\zeta,\zeta'',\eta';z)\langle f_1(\eta,\zeta';0)f_1^*(\eta',\zeta'';0)\rangle,\end{aligned} \tag{17}$$

where the brackets $\langle\ \rangle$ imply the ensemble average. The electron correlation function appearing in the integrand is

$$\begin{aligned}\langle f_1(\eta,\zeta;0)f_1^*(\eta',\zeta';0)\rangle = &\frac{1}{n_0}\delta(\zeta-\zeta')\delta(\eta-\eta')V(\eta)\chi(\zeta) \\ &+\chi(\zeta)\chi(\zeta')V(\eta)V(\eta')|\langle e^{i\theta}\rangle|^2,\end{aligned} \tag{18}$$

where

$$\langle e^{i\theta}\rangle \equiv \frac{\int \chi(\zeta)e^{ik_s\zeta}d\zeta}{\int \chi(\zeta)d\zeta}. \tag{19}$$

The first term in Eq. (18) represents the usual shot noise with a vanishing correlation length, while the second term represents the coherent bunched beam effect. Normally, the second term is negligible, and the radiation field evolves from the stochastic undulator radiation with the correlation length increasing as $\sqrt{\lambda_s z/\rho}$ [1,2]. The second term could become large when the density varies significantly in a length of the order of λ_s or shorter. This term gives rise to the so called coherent synchrotron radiation(see for example Ref. [6]), and may explain the large effective noise term observed in the LLNL experiment [7].

IV. CONCLUSION

We have presented a general formulation of the start-up from noise for a general electron beam density profile including the effects due to diffaction, electron beam focusing, and energy spread. Our treatment for the finite pulse profile takes into account the coherent bunched beam effect for $L_e \lesssim \lambda_s$. Based on the formalism presented here, we will carry out a detailed calculation of the radiation characteristics for the proposed short wavelength FEL scheme based on SASE [8].

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FIGURES

FIG. 1. Illustration of the coordinates ζ and $\delta\zeta_j$.

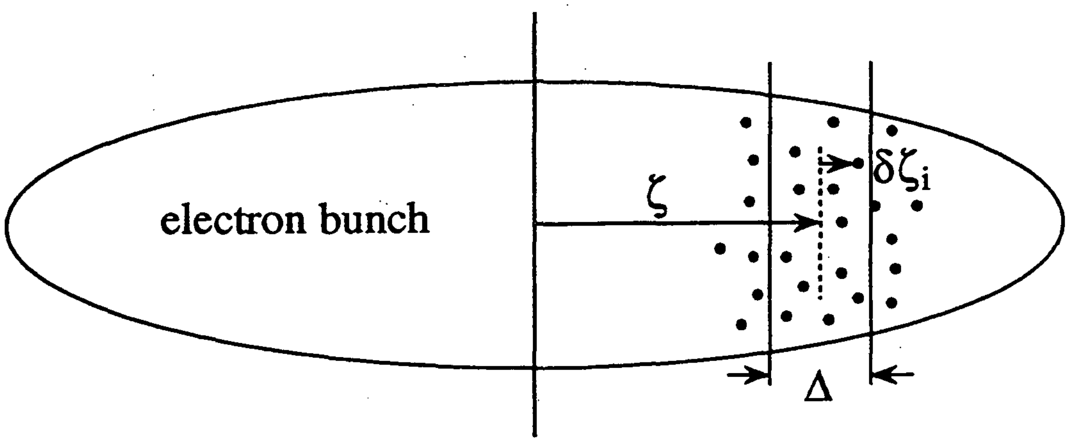

Fig. 1

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