UNIVERSITY OF CALIFORNIA, SAN DIEGO

Adventures in model-building beyond the Standard Model and esoterica in six dimensions

A dissertation submitted in partial satisfaction of the requirements for the degree
Doctor of Philosophy

in

Physics

by

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The Dissertation of David C. Stone is approved, and it is acceptable in quality and form for publication on micro lm and electronically:

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Chair

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Chapter 2 and Appendix A are a reprint of the material as it appears in \Explaining the $t\bar{t}$ Forward-Backward Asymmetry from a GUT-Inspired Model," JHEP 1201, 096 (2012) [arXiv:1111.2050 [hep-ph]].

Chapter 3 is a reprint of the material as it appears in \Spontaneous R-symmetry breaking from the renormalization group ow," JHEP 1301, 092 (2013) [arXiv:1210.3028 [hep-th]].

Chapter 4 and Appendix B are a reprint of the material as it appears in \B decays to two pseudoscalars and a generalized $I = \frac{1}{2}$ rule," Phys.Rev. D89 (2014) 114014 [arXiv:1402.1164 [hep-ph]].

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D. C. Stone and P. Uttayarat, \Explaining the $t\bar{t}$ Forward-Backward Asymmetry from a GUT-Inspired Model," JHEP 1201, 096 (2012) [arXiv:1111.2050 [hep-ph]].

A. Amariti and D. C. Stone, \Spontaneous R-symmetry breaking from the renormalization group ow," JHEP 1301, 092 (2013) [arXiv:1210.3028 [hep-th]].

B. Grinstein, A. Stergiou, and D. C. Stone, \Consequences of Weyl Consistency Conditions," JHEP 1311, 195 (2013) [arXiv:1308.1096 [hep-th]].

B. Grinstein, D. Pirtskhalava, D. C. Stone, and P. Uttayarat, \B decays to two pseudoscalars and a generalized $I = \frac{1}{2}$ rule," Phys.Rev. D89 (2014) 114014 [arXiv:1402.1164 [hep-ph]].

B. Grinstein, A. Stergiou, D. C. Stone, and M. Zhong, \A challenge to the a-theorem in six dimensions," [arXiv:1406.3626 [hep-th]]. In submission to PRL.

ABSTRACT OF THE DISSERTATION

Adventures in model-building beyond the Standard Model and esoterica in six dimensions

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This dissertation is most easily understood as two distinct periods of research. The first three chapters are dedicated to phenomenological interests in physics. An anomalous measurement of the top quark forward-backward asymmetry in both detectors at the Tevatron collider is explained by particle content from beyond the Standard Model. The extra field content is assumed to have originated from a grand unified group SU(5), and so only specific content may be added. Methods for spontaneously breaking the R-symmetry of supersymmetric theories, of phenomenological interest for any realistic supersymmetric model, are studied in the context of two-loop Coleman-Weinberg potentials. For a superpotential with a certain structure, which must include two different couplings, a robust method of spontaneously breaking the R-symmetry is established. The phenomenological studies conclude with an isospin analysis of B decays to kaons and pions. When the parameters of the analysis are fit to data, it is seen that an enhancement of matrix

elements in certain representations of isospin emerge. This is highly reminiscent of the infamous and unexplained enhancements seen in the $K \to \pi\pi$ system. We conjecture that this enhancement may be a universal feature of the flavor group, isospin in this case, rather than of just the $K \to \pi\pi$ system.

The final two chapters approach the problem of counting degrees of freedom in quantum field theories. We examine the form of the Weyl anomaly in six dimensions with the Weyl consistency conditions. These consistency conditions impose constraints that lead to a candidate for the a-theorem in six dimensions. This candidate has all the properties that the equivalent theorems in two and four dimensions did, and, in fact, we show that in an even number of dimensions the form of the Euler density, the generalized Einstein tensor, and the Weyl transformations guarantee such a candidate exists. We go on to show that, unlike in two and four dimensions, the a-theorem is six dimensions has the opposite sign of its counterparts in lower dimensions, at least in perturbation theory. This would imply, if the result could be extended without the use of perturbation theory, that the number of degrees of freedom accessible at a certain energy scale would **increase** as that energy scale is decreased. This is contrary to the intuition from two and four dimensions. We comment on what renormalization group flows, if any, we might find to exhibit this behavior.

Chapter 1

Introduction

I attempt to motivate the lines of inquiry I have pursued in my research. Section 1.2 establishes the Standard Model and a few of its most famous extensions. This section is mostly phenomenologically motivated. Section 1.3 is more theoretical in nature and discusses the question \What is a quantum eld theory?" and attempts to elucidate an answer to the question by appealing to the notion of degrees of freedom.

1.1 Preface

Whilst studying the quantization of the electromagnetic eld in a quantum mechanics course during my undergraduate studies I was told of the \jewel" of physics{ quantum electrodynamics, or QED (as made famous by the inimitable Richard P. Feynman). In QED one can compute the magnetic moment of the electron in perturbation theory in ~, the fundamental constant that is the harbinger of quantum mechanical e ects. The prediction from QED, I was told, matches the experimental measurement of this quantity to one part in 100 trillion! It would seem that the only limit of the success of QED was the computational complexity that is endowed in the higher order corrections in perturbation theory, and experimental limitations. With this view of quantum field theory in mind, I came to graduate school looking to become a scholar of quantum field theory— the master theory of physical phenomena at the fundamental scale.

Unfortunately, one quickly finds out that the spectacular success of QED does not carry over to many of the other incarnations of quantum field theory in nature. QED is special, one finds, because it has a gauge group that is Abelian whose interactions with matter decrease in importance as the number of simultaneous interactions is increased. In nature, where QED is the descriptor of electromagnetism, there are many other manifestations of quantum field theories, as we shall see. These include the weak interaction of subatomic particles, the strong interaction that describes subatomic nuclear physics, and numerous descriptions of condensed matter systems in terms of quantum field theories (including the quantum Hall effect, topological insulators, anti-/ferromagnets, *et cetera*).

My graduate studies progressed from work closer in nature to the perturbative quantum field theory calculations of yore to much broader considerations of the general features of quantum field theories. Because of the academic liberties afforded to me by my advisor, it is difficult to establish a unifying assertion that classifies my research, other than the aforementioned direction. I shall strive in this introduction to provide basic details that can put the work I completed in context.

1.2 Quantum field theory as the theory of fundamental particles

Being a student of particle physics, we focus on quantum field theories as the mathematical structures that describe subatomic physics. Quantum field theory was necessitated, from the theoretical side, by the union of special relativity, produced in 1905 by Einstein, and quantum mechanics, which rapidly developed in its second incarnation in the late 1920s. On the experimental side, the decay of massive particles into many other massive particles did not seem to be computable within the scope of quantum mechanics.

The interpretation of particles as the excitations, or quanta, of elds freed physicists from the unitary interactions in quantum mechanics that preserved particle number while still maintaining unitarity (which is to say, probability conservation) of the eld theory. On the other hand, it permitted one to take the Lagrangian and Hamiltonian operators from quantum mechanics and recycle them in the eld interpretation, where they become Lagrangian and Hamiltonian densities. This permitted a straightforward way to compute observable quantities in quantum eld theory that led to many of its early successes.

So in the early years (up until the mid 1950s), it was thought that the Lagrangian descriptions of eld theory were the de ning aspects of quantum eld theory, and a de nition that would forever encompass the capabilities of describing subatomic phenomena within a mathematical framework. Although this changed when the elements of the strong force began to emerge theoretically in the late 1950s, and when S-matrix theory rose as a competitor to this \orthodox" view of quantum eld theory, we will will focus on this orthodox notion of the de ning features of a quantum eld theory, as it characterized the early aspects of my research.

1.2.1 The Standard Model

The constituents of subatomic matter (and force carriers) are well-described by a quantum eld theory with a particular speci cation of its Lagrangian. This form of this Lagrangian is constrained by the symmetries in Nature that we believe to exist. This is a very powerful statement that makes the construction of a quantum eld theory of subatomic interactions tractable. This Lagrangian is the main content of the Standard Model of particle physics. Let us brie y establish its main features and symmetries.

First, there are the spacetime symmetries. We require the physics of our subatomic particles to be the same under Lorentz boosts, rotations, and translations, which together form the Poincaré group. The Standard Model Lagrangian is Poincaré invariant. This classifies the content of the Standard Model into irreducible representations of the Poincaré group, labeled by their momenta and spin.

The second important symmetry is that of the gauge "symmetries".[1] These symmetries classify the content of the Standard Model by their "charge." In the Standard Model, there are three types of charges for the particle content. These are the electric charge of electromagnetism, the weak charge, and the color charge.

Electric charge is the most familiar and is a scalar quantity— simply a number. In fact, it is manifest by the fact that charged particles, like the electron, couple to the photon, and the strength of their coupling is given by their electric charge. That the electric charge is a scalar quantity, and is also a conserved quantity, is a manifestation of the fact that the Lagrangian of the Standard Model is invariant under a local $U(1)$ symmetry if one assumes that the photon transforms under the adjoint representation of this local symmetry group.[2] Historically this transformation of the photon under this symmetry was called a "gauge" transformation and so we can state this symmetry feature of the Standard Model with the language "the Standard Model has a $U(1)$ gauge symmetry that represents the effects of electromagnetism with the photon, transforming in the adjoint of this gauge group, being the force carrier associated with this symmetry." A consequence of this statement is that the photon is a spin 1 boson, as is familiar from electrodynamics.

The other two types of (implicitly) observed charges, that of the weak and color charge, are a bit more complicated. To use the language of the previous paragraph, the weak charge comes from a gauge symmetry with gauge group $SU(2)$, with its charge communicators/force carriers transforming in the adjoint of $SU(2)$. This means there are 3 of these force carriers, the W^+, W^-, and Z^0 bosons, where the subscripts indicate their electric charge. That these force carriers are also

[1] Really, gauge redundancies, as emphasized to me so often by John McGreevy.

[2] For $U(1)$, an Abelian (Lie) group, the adjoint representation is not distinct from the fundamental (or any other) representation, but this distinction will be useful for later examples.

charged under the electromagnetic force will be discussed below. These bosons also carry weak charge— the same charge that they communicate to other particles. The color charge is described by a gauge symmetry with gauge group $SU(3)$, implying that there are 8 force carriers, called gluons. The gluons carry no charge other than the color charge itself. What is curious about these two types of charges is that their gauge group is non-Abelian, unlike the $U(1)$ gauge group of electromagnetism. Among other things, this implies that the force carriers carry the same charge that they communicate, as mentioned.

So far we have stated that the Standard Model has 1 photon, 3 W/Z bosons, and 8 gluons. These are only the force communicators- what matter do they communicate to? The actual matter content comes from direct or indirect evidence in experiments. From the discovery of Thompson in 1897 of the most well-known elementary particle, the electron, to the discovery of the top quark at Fermilab in 1995 and the Higgs boson at CERN in 2012, all observed matter falls into a remarkably simple organization under the aforementioned symmetries (or redundancies) of the Standard Model. Until 2012, all matter was observed to have spin $\frac{1}{2}$. These fermions were classified by whether they carried color charge— in which case they are deemed "quarks"— or not— in which case they are deemed "leptons." Both quarks and leptons fit neatly into three sets of what would be identical replicas of each other if their masses and, in the case of the quarks, their interactions with the charged W bosons were identical. For example, the first "generation" of the leptons has the charged electron with both of its left- and right-handed states (i.e. for a massless electron, its helicity states) plus an electrically-neutral neutrino, called the electron neutrino. There are two more copies of this generation which are, in order of increasing mass of the charged lepton, the charged muon plus neutral muon neutrino and the charged tau plus neutral tau neutrino. In the quark sector, the lightest generation includes what are called the up quark and down quark, in both of their left- and right-handed states. Both these quarks are charged, the up quark with electric charge $+\frac{2}{3}$ and the down quark with $\frac{1}{3}$. The two heavier copies of this generation include the paired charm and strange quarks and the paired top and bottom quarks, respectively.

Now, there is an intensely curious aspect of this whole method of classifying the content of the Standard Model, which are the masses of said content. Our best experimental limits show that the photon and gluon are massless, in accordance with what we expect from this theoretical framework[3]. However, the W and Z bosons are massive, as are all the quarks and leptons. However, invariance under the $SU(2)$ gauge symmetry mentioned above, whose exact nature I have not specified, would prohibit all such masses. What permits this description of the gauge symmetries of the Standard Model to remain acceptable is the existence of another matter particle, the Higgs scalar boson. The Higgs boson is a curious particle in many ways; it is the only scalar in the Standard Model and, as a consequence, it is the only particle in the Standard Model that does not have a symmetry that would lead us to believe it was naturally massless. I will not discuss how the Higgs provides a mechanism to allow gauge theory descriptions of the Standard Model to remain viable while allowing masses for some of the gauge bosons, which would naïvely destroy the gauge symmetry, because my research did not focus on Higgs phenomenology. I will briefly state that the Higgs allows a further clarification of the structure of the Standard Model in the following sense: The reader has perhaps noticed that I have only mentioned the electric and color charges of the matter content and not the weak charges. This is because the Higgs boson plays a major role in how the weak force is realized in experiments. Without further digression, I will say that the actual gauge symmetries of the Standard Model have a $SU(2)$ group under which only the left-handed particles are charged and a $U(1)$ group that we call "hypercharge" that is distinct from the $U(1)$ of electromagnetism. The distinction occurs because of the Higgs mechanism, which can be found in any quantum field

[3] In order for gauge symmetries, along with unitarity of the theory at hand, to be preserved, the force carriers must be massless. The caveat to this statement lies in the Higgs mechanism, to be described now.

theory textbook (e.g. see [2]). The Higgs mechanism provides masses to the $SU(2)$ W and Z gauge bosons and also the quarks and leptons. In doing so, the $SU(2)$ gauge symmetry is hidden by the masses of the gauge bosons and the combination of the $SU(2)$ weak and $U(1)$ hypercharge groups are \broken" to a remaining $U(1)$ gauge group that is our beloved electromagnetism gauge theory.

To summarize, the content of the Standard Model is stated in table 1.1. The gauge group of the Standard Model can be written as the direct product $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ for the color, weak, and hypercharge gauge groups, respectively.

1.2.2 Beyond the Standard Model

The description above is simple in hindsight, but rather remarkable by way of its record of experimental successes. Where it is possible to compute a quantity within the framework of the Standard Model, the predictions are found to match experiment in all but a few measurements. These measurements, anomalous from the Standard Model, drive an entire industry in particle physics known as \model-building" and serve as an impetus to consider that the Standard Model, while almost entirely satisfactory to our present understanding of subatomic physics, is an incomplete description of Nature as a whole.

The most famous exploration beyond the Standard Model came in the form of the unification of the Standard Model gauge groups into a single gauge group, $SU(5)$. In 1974 it was conjectured [3] that, since $SU(3) \otimes SU(2) \otimes U(1) \subset SU(5)$, the Standard Model could come from a specific pattern of Higgs mechanisms that broke the $SU(5)$ gauge invariance but preserved the specific combination of the Standard Model. The matter content of the Standard Model fit perfectly, in my opinion, into representations of $SU(5)$ that are both chiral and anomaly-free. The

Table 1.1: A list of the particle content of the Standard Model, as organized by their quantum numbers that show up in the Standard Model Lagrangian (i.e. before spontaneous symmetry breaking via the Higgs mechanism). The indicates that the field transforms in the fundamental representation of a given group (for $SU(N)$ this is sometimes written as N, for the dimensionality of the fundamental representational of $SU(N)$), Ad. in the adjoint, and 1 that the field is a singlet (does not transform). In the $U(1)_Y$ hypercharge column, the number indicates its charge. The i index runs from $i = 1; 2; 3$ and represents the three generations of the quark and lepton matter fields.

Field	Name	Type	Transformation under gauge group		
			$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
G^a	Gluon	Gauge boson	Ad.	1	0
W^a	W/Z boson	Gauge boson	1	Ad.	0
B	Hypercharge boson	Gauge boson	1	1	0
$\begin{pmatrix} u_L \\ d_L \end{pmatrix}^i = Q_L$	Left-handed quark	Spin $\frac{1}{2}$ matter			$\frac{1}{6}$
u^i_R	Right-handed up-type quark	Spin $\frac{1}{2}$ matter		1	$\frac{2}{3}$
d^i_R	Right-handed down-type quark	Spin $\frac{1}{2}$ matter		1	$\frac{1}{3}$
$\begin{pmatrix} {}_L \\ e_L \end{pmatrix}^i = Q_L$	Left-handed lepton elds	Spin $\frac{1}{2}$ matter			$\frac{1}{6}$
e^i_R	Right-handed charged lepton	Spin $\frac{1}{2}$ matter	1	1	1
H	Higgs boson	Spin 0 matter	1		$\frac{1}{2}$

charge-conjugated right-handed down-type quarks[4] have transformation properties under $(SU(3)_c; SU(2)_L)_{U(1)_Y}$ being $(\bar{\ }; 1)_{1=3}$ or, equivalently, $(3; 1)_{1=3}$; the left-handed lepton doublet has transformation properties $(1\ )_{1=2}$ or, equivalently, $(1; 2)_{1=2}$. These two t into the anti-fundamental representation of $SU(5)$, the 5. We also have that the charge-conjugated right-handed up-type quarks transform as $(\bar{\ }; 1;)_{2=3}$ or $(3; 1)_{2=3}$; the charge-conjugated right-handed charged leptons as $(1; 1)_1$; the left-handed quark doublet as $(;\ )_{1=6}$ or $(3; 2)_{1=6}$. These three t into the 10-dimensional anti-symmetric representation of $SU(5)$, called the 10.

This model was not conjectured to explain any known anomalous experimental measurements, but suggested out of the pure aesthetic beauty of the model.[5] Although predictions of the simplest version of this model failed experimentally (most importantly with its prediction of the proton lifetime), it inspired Chapter 2, which makes an attempt to explain an anomalous measurement at the Tevatron collider in the context of a grand uni ed model{ in fact, in a $SU(5)$ model with additional matter content beyond that proposed in the original model.

The other famous theoretical expedition beyond the Standard Model relevant to this thesis is its supersymmetric extension. Unlike the $SU(5)$ example, where the gauge and matter content of the Standard Model were subsumed into a larger, cohesive gauge symmetry, supersymmetry subsumes the Standard Model in a larger spacetime symmetry with fermionic dimensions added to the usual space and time. By Z_2-grading the Poincaré algebra[6] that classi es particles by their spacetime properties, one arrives at its most \natural" extension of the spacetime symmetries that does not give trivial or non-analytic scattering processes [4, 5], the super-Poincaré algebra. The e ect of this gradation is to double the particle spectrum

[4] In the following classi cation of particles under $SU(5)$ representations, we write all particles or anti-particles so that they transform under Lorentz transformations as left-handed particles. A charge conjugated right-handed fermion (i.e. a right-handed anti-fermion) transforms under Lorentz transformations as a left-handed fermion. This simpli es further analysis.

[5] The paper cited above [3], titled \Unity of All Elementary Particle Forces," is quite hubristic in its tone.

[6] We only consider the $N = 1$ supersymmetric algebra, and do not consider extended supersymmetries.

of the Standard Model by pairing each particle with a new particle that has the opposite spin statistics as the Standard Model one. The mantra for supersymmetry is that there is a boson-fermion symmetry, such that every boson has a fermion partner and vice versa (much like every left-handed particle has a right-handed partner because of the Poincaré symmetry).

The consequences of imposing supersymmetry are much larger in scope than simply doubling the particle spectrum. In particular, only certain interactions are allowed such that the potential of the supersymmetric theory, aptly named the superpotential, must be holomorphic in the superfields that pair the bosonic and fermionic fields of the theory (roughly, they can only involve fermions of a single chirality). This constraint on the interactions dictates what models can be built that use supersymmetry as the basis for beyond the Standard Model phenomenology[7].

One of the interesting consequences of supersymmetry is a specific portion of the algebra. While the Poincaré group is the semi-direct product $\mathbb{R}^{3,1} \rtimes SO(3,1)$ in 3+1-dimensional Minkowski space, the supersymmetry algebra gives an extra $U(1)_R$ symmetry called the R-symmetry, which, in a sense, counts the supersymmetric "charge" of a particle– particles in the Standard Model have charge $R = 0$, while their supersymmetric partners have non-zero R charge. One particular feature of this R-symmetry is that is prevents a certain supersymmetric particle from gaining a mass. Since no such particles are observed experimentally, something must break this R-symmetry. Doing so, within the constraints of having a supersymmetric theory at high energies, is not as trivial as it sounds; chapter 3 concerns a particular method to solve this problem.

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[7]We will not discuss supersymmetry breaking, which must be true if supersymmetry is a symmetry of Nature, as supersymmetry is clearly not seen experimentally at the energy scales that we have thus far explored.

1.3 Quantum field theory as a generic mathematical structure of the universe

The previous section devoted itself to the description of the content of an established quantum field theory – the Standard Model. In this section we turn to more formal discussions on the structure of quantum field theory. Although the description in the previous section seems straightforward, many aspects of quantum field theory are still not understood.

1.3.1 What defines a quantum field theory?

I think that the best way to approach the notion of what defines a quantum field theory is to quote the preface in one of the chapters of Anthony Zee's *Quantum Field Theory in a Nutshell* [6]:

> . . . I like to tell the students that by the mid-1970s field theorists were breaking the shackles of Feynman diagrams. A bit melodramatic, yes, but by that time Feynman diagrams, because of their spectacular successes in quantum electrodynamics, were dominating the thinking of many field theorists, perhaps to excess. As a student I was even told that Feynman diagrams define quantum field theory, that quantum fields were merely "slices of venison" used to derive the Feynman rules, and should be discarded once the rules were obtained. The prevailing view was that it barely made sense to write down $\varphi(x)$.

Zee mentions in a footnote that this "slices of venison" comment originates from Gell-Mann, who "used to speak about how pheasant meat is cooked in France between two slices of venison which are then discarded. He forcefully advocated a program to extract and study the algebraic structure of quantum field theories which are then discarded." This is much the way in which I was taught quantum field theory from standard particle physics textbooks – in a more subtle but equivalent way, to compute Green's functions and use those in the computation of any quantum mechanical process.

However, the reader will notice that the eld content of a quantum eld theory is very much an intuitive and important notion in how we distinguish di erent quantum eld theories{ for example, in table 1.1 we establish the Standard Model not by some predictions of speci c scattering cross sections or magnetic dipole moments, but by the elementary quantum elds that specify its \content." There is some issue to be taken with this classi cation. One is taught that, in the leap from the harmonic oscillator in quantum mechanics to a free quantum eld theory, we proceed from a single harmonic oscillator and its associated degrees of freedom to an in nite collection of these harmonic oscillators, with a harmonic oscillator and its associated degrees of freedom at every point in space in the eld description.

But there clearly is some value in classifying a quantum eld theory as we did for the Standard Model{ by counting (real) scalar elds as a single degree of freedom, massless Weyl fermions as having two degrees of freedom (one for each spin state), massless gauge bosons having two degrees of freedom (one for each polarization), and so on{ one can also include the dimensionality of the representation of some gauge or global symmetry that these elds transform under in the counting of the degrees of freedom. This is not captured by the in nite collection of harmonic oscillators mentioned in the usual introduction to quantum eld theory.

There are quantities that seem to exist in every quantum eld theory that do this naïve counting of degrees of freedom and provide a great deal of further information about the eld theory in question. This quantity, often called the central charge of the theory, is best understood in conformal eld theories, to which we now turn for the sake of motivating my research in this area.

1.3.2 A brief digression into conformal eld theories

To make these statements more poignant, let us make a brief digression to discuss one of the more salient features of a special type of quantum eld theory{

conformal field theories in two dimensions. Many exact results can be found in this context because of the high degree of symmetry latent in the theory and the low-dimensionality of spacetime. It was also because of my forays into this topic [7] that I came to be interested in the latter topics of my thesis work.

The conformal symmetry is a spacetime symmetry that scales the metric tensor by an overall factor:

$$g'_{\mu\nu}(x') = \Omega(x) g_{\mu\nu}(x); \tag{1.1}$$

where $\Omega(x)$ is real-valued. It can be shown that this includes, for finite conformal transformations on the spacetime coordinates, dilatations,

$$x'^{\mu} = \lambda x^{\mu} \tag{1.2}$$

which rescales all lengths by a common scalar factor λ, and special conformal transformations,

$$x'^{\mu} = \frac{x^{\mu} - b^{\mu} x^2}{1 - 2b \cdot x + b^2 x^2} \tag{1.3}$$

which amount to an inversion of the coordinate x^{μ}, followed by a translation by the vector b^{μ}, followed by another inversion of the resulting vector. Quantum fields transform under conformal transformations in a way that depends on their spin and scaling dimension. For example, in the simplest case of scalar fields, they transform under conformal transformations that send $x \to x'$ as

$$\phi'(x') = \left|\frac{\partial x'}{\partial x}\right|^{-\Delta/d} \phi(x) \tag{1.4}$$

where d is the dimension of spacetime and Δ is called the scaling dimension of the quantum field $\phi(x)$.

A quantum field theory that is invariant under local conformal transformations on all the content of the theory is called a conformal field theory. When that theory lives in two dimensions, a remarkable number of results can be computed exactly (rather than in perturbation theory). This is primarily because the conformal group in two dimensions consists of the set of all analytic maps on the complex plane, which is an infinite-dimensional symmetry[8]. More precisely, in two dimensions it is convenient to switch to "left-" and "right-moving" complex coordinates, $z = x^0 + ix^1$ and $\bar{z} = x^0 - ix^1$, respectively. The z and $\bar{z}$ coordinates are also called holomorphic and anti-holomorphic and are taken to be independent of each other for many calculations in field theory because the two variables often decouple in calculations[9].

Using these anti-/holomorphic coordinates we can elucidate the main points I would like to make about two-dimensional conformal field theories. For example, the transformation of a scalar in eq. (1.4) may be written in the $z, \bar{z}$ coordinates in two dimensions as

$$\phi'(z', \bar{z}') = \left(\frac{dz'}{dz}\right)^{-h} \left(\frac{d\bar{z}'}{d\bar{z}}\right)^{-\bar{h}} \phi(z, \bar{z}) \tag{1.5}$$

where h and $\bar{h}$ are equal to the scaling dimension $\Delta/2$ up to a piece from any planar spin we might associate with more general fields. Now, the most useful piece of a two-dimensional conformal field theory is undoubtedly the stress-energy tensor of the theory. Given a Lagrangian for the theory, one may compute the stress-energy tensor using textbook methods[10]. One can show that the stress energy tensor in a conformal field theory is traceless[11]. More important to this discussion is the

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[8] The group multiplication law would be defined by the composition of these analytic functions.

[9] At the end of the day, of course, we must impose that $\bar{z} = z^*$.

[10] The easiest of which is to introduce a background metric $g_{\mu\nu}(x)$ into the action of the theory, construct the diffeomorphism-invariant measure, and functionally differentiate with respect to the metric to obtain the stress-energy tensor:

$$T_{\mu\nu}(x) = \frac{\delta}{\delta g^{\mu\nu}(x)} \int d^2x \sqrt{-g}\, L[g_{\mu\nu}(x)] \tag{1.6}$$

ensuring a symmetric stress-energy tensor. Of course, afterwards we normal order this operator to ensure that it has all the nice properties, like a vanishing one-point vacuum expectation value.

[11] This fact has a very interesting story behind it; for some of the most recent advances, see the thesis of my collaborator, Andy Stergiou [8].

effect of taking the operator product expansion (OPE) of the anti-/holomorphic components of the stress energy tensor. If one works in the $z, \bar{z}$ basis and defines the holomorphic part of the stress energy tensor

$$T(z) = -2\pi T_{zz} \tag{1.7}$$

and equivalently for the anti-holomorphic part, then one can show for a general field $\varphi(z, \bar{z})$ the OPE with $T(z)$ is of the form

$$T(z)\varphi(z', \bar{z}') \sim \frac{h}{(z-z')^2}\varphi(z, \bar{z}') + \frac{1}{z-z'}\frac{\partial}{\partial z'}\varphi(z', \bar{z}') \tag{1.8}$$

where the $\sim$ indicates that we are only extracting the singular short-distance behavior of the product of the two operators; there exist other terms that are finite as $z \to z'$. The second term in the OPE is the usual transformation of any field under spacetime symmetry (essentially a translation from z' to z); the remarkable property is the first piece, where the coefficient of $(z - z')^{-2}$ gives the scaling dimension of the field in question. For example, in a (Euclidean) field theory of a massless scalar with action

$$S = \frac{g}{2}\int d^2x\, \partial^\mu\varphi\, \partial_\mu\varphi = g\int dz\, d\bar{z}\, \partial_z\varphi\, \partial_{\bar{z}}\varphi \tag{1.9}$$

one can compute[12]

$$T(z)\partial_{z'}\varphi(z') \sim \frac{\partial_{z'}\varphi(z')}{(z-z')^2} + \frac{\partial^2_{z'}\varphi(z')}{(z-z')} \tag{1.10}$$

so that evidently the derivative on the free scalar field has scaling dimension $h = 1$, as we would expect in two dimensions where the scalar field has mass-dimension zero (and so the derivative picks up all the mass dimensions). For a theory of free

[12] The scalar field φ is a peculiar field with mass dimension 0 in two dimensions, so we study $\partial_z\varphi$ instead, which has the nice property of transforming as a "primary field." We also drop anti-holomorphic dependency here, as focusing on the holomorphic fields is sufficient.

fermions with a two component spinor $\Psi(z,\bar{z}) = (\psi(z), \bar{\psi}(\bar{z}))$ and

$$S = g \int dz\, d\bar{z}\, \left(\psi \partial_{\bar{z}} \psi + \bar{\psi} \partial_z \bar{\psi}\right) \tag{1.11}$$

one finds from the OPE $T(z)\psi(z')$ that $\psi(z)$ has scaling dimension $h = \frac{1}{2}$, as we would expect for a free fermion in two dimensions.

Now here is the punchline: if we take the OPE of $T(z)$ with itself in the above theories we find an anomalous term that we would not expect from the general form

$$T(z)T(z') \sim \frac{c/2}{(z-z')^4} + \frac{2T(z')}{(z-z')^2} + \frac{\partial_{z'} T(z')}{(z-z')}. \tag{1.12}$$

The OPE of the holomorphic part of the stress energy tensor with itself should yield the scaling dimension of the stress energy tensor – this is the coefficient 2 of $(z-z')^{-2}$ in the second term. We do not expect the term that goes as $(z-z')^{-4}$. Its coefficient, the *central charge* c, is an incredibly important and universal quantity in two-dimensional conformal field theories. It is the same quantity c that shows up in the quantum Virasoro algebra that classifies the spectrum of a conformal field theory (which is why it is called the "central charge"). For the free massless scalar, we get $c = 1$; for the free massless fermion we get $c = \frac{1}{2}$; for N free, non-interacting massless scalars with an $O(N)$ symmetry in eq. (1.10) we would get $c = N$. Apparently c is somehow measuring the degrees of freedom in the naïve way that I did for the Standard Model. This is true in a much broader sense, and has been shown to capture this notion of "degrees of freedom" in many other systems[13].

In fact, c can be shown to be related to a slew of other important quantities [9]. It shows up [10]

- in the Casimir energy for a theory defined on a cylinder with periodic boundary conditions,
- in the coordinate transformation law for the stress energy tensor,

[13] However, c is not necessarily integer, though it is positive in unitary theories.

- in the free energy per unit length for a system with finite size,
- as the coefficient of the trace anomaly for a theory on a curved background,
- as the value of Zamolodchikov's $\tilde{c}$-function at critical points of a theory.

The last two points are, perhaps, the two most important points. They have led to a slew of work in four dimensions (and, in the case of my research, six dimensions) to see if something like c exists in higher dimensions. We discuss this work in the next section.

1.3.3 Degrees of freedom in quantum field theories

In the mid 1980s Zamolodchikov showed [11] that there existed a function of the couplings (g^i) in a quantum field theory, $\tilde{c}(g)$, which decreased monotonically[14] from a UV fixed point to an IR fixed point. At each fixed point the field theory becomes critical, becomes conformally invariant, and has vanishing beta functions. The beta functions β^i describe how the couplings flow as we change the energy scale with which we probe the interactions of the theory; this is to say,

$$\beta^i(g) \equiv \mu \frac{dg^i}{d\mu}, \tag{1.13}$$

where μ, the renormalization scale, is effectively the energy at which we probe our theory. At these fixed points, which occur at some value of the couplings $g^i = g^i_*$ with $\beta^i(g_*) = 0$, Zamolodchikov showed that this same function was stationary, $\partial\tilde{c}/\partial g^i|_{g_*} = 0$, and equal to the central charge of the conformal field theory at that fixed point, $\tilde{c}(g_*) = c(g_*)$. This result was called the c-theorem.

The interpretation of this result, from our discussion of the central charge in the previous section, is that the number of degrees of freedom we see in a quantum field theory decreases as we probe it at lower energies. This fits quite nicely with the effective field theory notion of "integrating out" heavy (*i.e.* high energy) degrees of

[14] Actually, he showed an even stronger condition, that the flow of $\tilde{c}(g)$ is a gradient flow.

freedom, which we lose information about as we look at the low energy description of a theory.

The immediate question was whether such a powerful theorem existed in four dimensions (where we humans live), and, if so, what quantity would be the analog of $c(g)$. The answer lies in the penultimate point of the list in section 1.3.2 above. The trace anomaly, where the expectation value of the trace of the stress energy tensor does not vanish in a curved background, exists in any even number of dimensions with a well-defined structure [12, 13]. In particular, in two dimensions in conformal field theories, the trace anomaly has the form, in the presence of a background metric $g_{\mu\nu}(x)$,

$$\langle T^{\mu}_{\mu}(x)\rangle_{g_{\mu\nu}(x)} = -\frac{c}{24}R(x) \tag{1.14}$$

where c is the same central charge from before and $R(x)$ is the Ricci scalar curvature. It turns out that in two dimensions the Ricci scalar is also, when integrated, a constant, and hence a topological quantity – it is the Euler density of two dimensions, and its integral gives the Euler character of the space in question. This clued Cardy in to conjecture [14] that in four dimensions the quantity that "counts degrees of freedom" is also related to the topology of the spacetime and can be computed by extracting the coefficient of the Euler density in four-dimensional curved backgrounds.

The coefficient of the Euler density in four dimensions, called a, was subsequently shown to have all the nice properties of the central charge when the theory was conformal. A quantity $\tilde{a}$, analogous to c, was shown to decrease monotonically in perturbation theory [15], and a weak version of the c-theorem in four dimensions, called the a-theorem, was proven rather recently [16][15]. No proof of the strongest

[15] In the weak version of the a-theorem it is shown that the change in a between the fixed points is greater than zero: $\Delta a = a_{UV} - a_{IR} > 0$. This is a necessary consequence of the strongest version of the a-theorem, where $\tilde{a}$ has a monotonically decreasing flow, but the converse is obviously not true.

version of the a-theorem has been given in four dimensions.

And so we arrive at the pursuit of this line of work in six dimensions, first given impetus by a paper that pursued the weak version of the a-theorem in six dimensions [17]. The main motivation was to ascertain whether or not patterns emerge in studying the a-theorem in various dimensions, and determine if a general statement about degrees of freedom in quantum field theories could be made. Chapters 5 and 6 tell the rest of this story, up to the current state of the field.

1.4 An Outline of this Dissertation

The topics of this dissertation do not lend themselves to smooth (perhaps in the mathematical sense) narrative. Because of the gracious liberties my advisor, Ben, allowed me, the topics that I pursued during my graduate career often came about via serendipitous interactions with other graduate students and postdocs. While this type of academic freedom is delightful, it also indicates a lack of academic discipline that I only developed in the later years of my graduate career. This final year and a half has narrowed my focus to a more consistent topic, which I will reiterate below.

And so, to recapitulate, my studies proceeded thusly:

In Chapter 2 we pursued an explanation of a measurement at the Tevatron collider at Fermilab that was anomalous from Standard Model predictions. We did so in the context of a grand unified theory approach, adding certain field content to produce a shift in prediction for the measured value while maintaining desirable theoretical features like unification of gauge couplings and the origin of additional particles from $SU(5)$ multiplets.

In Chapter 3 we engage in more model building, this time with the goal of producing spontaneous R-symmetry breaking by introducing a superpotential with fields having only R-charge $R = 0$ or $R = 2$ (a common restriction when supersymmetry is broken in a certain manner) and two distinct couplings. The couplings are constructed to have different beta functions such that they can compete to produce a non-trivial two-loop Coleman-Weinberg effective potential that gives a vacuum expecation value to a field with $R = 2$ away from the origin. This spontaneously breaks the R-symmetry and avoids excessive fine-tuning.

In Chapter 4 we finally turn away from model building and examine the decays of B mesons to light pseudoscalar particles; namely, pions and kaons. What started off as a project to classify these decays in terms of group invariants of the approximate $SU(3)$ flavor symmetry of the u, d, s quarks took a surprising turn when we found that, under the (more exact) $SU(2)$ isospin symmetry of the u and d, a significant enhancement of matrix elements in certain representations of $SU(2)$ emerges from analysis of the data. We recognized that this enhancement was very reminiscent of the enhancements seen in the decay of kaons to two pions, analysed over 40 years ago and still largely unexplained by perturbative QCD computations. We postulate that perhaps this enhancement is not specific to kaon system, but perhaps to any system whose decays can be accurately classified by isospin. This suggests that the strong dynamics of QCD know a lot more about flavor symmetries than previously thought.

In Chapter 5 we diverge from the line of research in previous projects and study six-dimensional quantum field theories with an eye of producing a candidate for the a-thereom in six dimensions. To do so we use the Weyl consistency conditions to derive integrability conditions on the trace anomaly in six dimensions. We do indeed find an integrability condition that presents itself in the exact manner as is done in two and four dimensions and identify a candidate for a quantity that decreases monotonically along the renormalization group flow. The monotonicity of this flow hinges on the positivity of a tensor in the space of couplings that can be thought of as a "metric" in the space of renormalization group flows. Proving the positivity of this metric (or showing the contrary!) would establish the strongest version of the a-theorem, akin to two-dimensional results. However, we are no closer in finding traction on solving this problem in general then for the equivalent case in four dimensions. We note, however, that there is a beautiful regularity in the production of a candidate for the a-theorem in any even number of dimensions that relates the variation of the $2n$-dimensional Euler density to the generalization of the Einstein tensor in $2n$ dimensions.

In Chapter 6 we build off the tedious work done in Chapter 5 and actually compute the aforementioned metric on the space of couplings. Surprisingly (or perhaps not, in hindsight — though we do not have the benefit of hindsight yet) we find that this metric is *negative definite* in perturbation theory, so that the quantity $\tilde{a}$ mentioned in section 1.3.3 *increases* away from the (perturbative) UV fixed point that we have at our disposal. This implies that, contrary to the intuition developed in the latter parts of the introduction, the number of degrees of freedom can increase as we probe our theory at lower and lower energies, at least somewhere along the renormalization group flow, and at least in six dimensions. We also compute the beta functions and anomalous dimensions for the theory with multiple couplings and interaction $S_{int} = \int d^6x \, \frac{1}{3!} \lambda_{ijk} \phi^i \phi^j \phi^k$, which have not been computed in the literature. We conclude with some speculation about the nature of renormalization group flows in quantum field theories in six dimensions, long a topic of little revelation, and apparently just as much so now.

Chapter 2

Early curiosities in model building: A grand uni ed model to explain the $t\bar{t}$ forward-backward asymmetry

We consider a model that includes light colored scalars from the 45 and 50 representations of $SU(5)$ in order to explain the CDF- and DO-reported $t\bar{t}$ forward-backward asymmetry, $A_{FB}^{t\bar{t}}$. These light scalars are, labeled by their charges under the Standard Model gauge groups, the $(6,1)_{4/3}$ and $(6,3)_{1/3}$ from the 50 and the $(8,2)_{1/2}$ from the 45. When the Yukawa coupling of the 50 is reasonably chosen and that of the 45 kept negligible at the scale of M_Z, the model yields phenomenologically viable results in agreement with the total $A_{FB}^{t\bar{t}}$ reported by CDF at the 0.5 level and with $A_{FB}^{t\bar{t}}(M_{t\bar{t}}$ 450 GeV) at the 2 level.

2.1 Introduction

The Standard Model (SM) has been a very successful model when confronted with experimental observations. However, there are motivations to study New Physics (NP) that supersede or extend the SM. The reasons are two-fold. On the one hand, there are anomalies reported from various experiments that cannot be explained by the SM. If these anomalies are verified, they necessary imply NP. On the other hand, the study of NP has been fueled by theoretical curiosities. One of the often-studied scenarios is the possibility of the unification of fundamental forces. In this paper, we will explore a NP model that could explain reported anomalies while at the same time allowing for the unification of fundamental forces.

From the theoretical point of view, the SM suggests the three gauge forces of $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ unify at at high scale ($\sim 10^{15}$ GeV). This observation leads to the formulation of a grand unified theory (GUT) in which all three gauge forces originate from just one fundamental gauge group. The simplest such model is the minimal $SU(5)$ model of Georgi and Glashow [3]. However, this minimal model predicts an incorrect fermion mass ratio. To make the model phenomenologically viable, scalar fields transforming in the 45-dimensional representation of $SU(5)$ are introduced [18]. This raises the possibility that there could also be more scalar fields transforming in some other representations of $SU(5)$. If some components of these scalar fields are light, they could be relevant for low energy physics.

On experimental side, the CDF and DØ collaboration have recently reported a measurement of the $t\bar{t}$ forward-backward asymmetry ($A_{FB}^{t\bar{t}}$) [19, 20, 21] which deviates from the SM prediction [22] at more than the 2σ level. Moreover, CDF also reports that the asymmetry grows with the invariant mass of the $t\bar{t}$ system. In particular, the CDF measurement of $A_{FB}^{t\bar{t}}$ for $M_{t\bar{t}} \geq 450$ GeV [20] is more than 3σ away from the SM predictions [22]. These discrepancies invite NP explanations. There are many models proposed in the literature to explain the $A_{FB}^{t\bar{t}}$ anomaly. Most of them involve the introduction of a new particle near the electroweak scale, see refs. [23, 24, 25, 26, 27, 28, 29, 30, 31] for a partial list of references.

In this work we focus our attention on models involving new colored scalar fields. This class of models is interesting in our opinion since the scalar fields could arise from GUT scalar multiplets. The model with an extra scalar field in various representations has been previously studied in ref. [23]. However, to generate a large $A_{FB}^{t\bar{t}}$ consistent with CDF and D0 measurements, the scalar Yukawa couplings are generally taken to be large. Such a large Yukawa coupling would become non-perturbative at a scale not far above the weak scale. This difficulty can be overcome by having multiple light scalars contribute to the $A_{FB}^{t\bar{t}}$. As an added benefit, multiple light scalar fields can conspire to give gauge coupling unification. This idea has been previously explored by Dorsner et. al. [24, 26]. However, the scalar field studied by Dorsner et. al. can couple quarks to leptons and mediate proton decay via a dimension-9 operator. Bounds on proton decay lead to a lower bound on the mass of their scalar of 10^{10} GeV, far too high to be of relevance to $t\bar{t}$ phenomenology. Thus we seek different scalar representations which could unify gauge couplings, explain the $A_{FB}^{t\bar{t}}$ and not lead to proton decay. Previous papers have had some success with this approach; in particular, a model with multiple light colored scalars from a $SO(10)$ GUT has had some success in these regards [32]. However, those results introduced scalars with masses at both the electroweak scale and at an intermediate scale. In our model, it is not necessary to introduce any scales other than the electroweak and GUT scale; we consider this a distinct advantage.

One might object that adding light scalars leads to a hierarchy problem. In fact, even in the minimal $SU(5)$ model the mass of the Higgs doublet has to be fine tuned to be at the electroweak scale. We will assume here that the hierarchy problem can be solved in a similar fashion, and that a similar mechanism is responsible for masses of the new scalar fields at the electroweak scale.

The structure of this paper is as follows: In section 2.2 we discuss the effects these colored scalars have on gauge coupling unification. In section 2.3 we discuss the effects these particles have on general $t\bar{t}$ phenomenology, including the total cross-section and $A_{FB}^{t\bar{t}}$. Finally, in section 2.4 we draw general conclusions of the merits of this model, its weaknesses, and look towards the LHC phenomenology of the model.

2.2 Gauge Coupling Unification

Upon closer inspection, the three gauge couplings of the SM do not quite unify. However, if one allows for more fields at low energy, the unification of gauge coupling could be achieved. To have a consistent GUT, these light particles must be part of some incomplete representation of the GUT gauge group. For definiteness, we will consider GUT model based on the $SU(5)$ gauge group. We will first give a brief review of the $SU(5)$ GUT model.

In the minimal $SU(5)$ model, each family of the SM fermion contents are embedded in the $\bar{5}$ and the 10 representation of $SU(5)$ as follows

$$ {}_R = \begin{pmatrix} d_{R_1} \\ d_{R_2} \\ d_{R_3} \\ \overline{e_L} \\ \overline{\ {}_L} \end{pmatrix}; \qquad {}_L = \begin{pmatrix} 0 & \overline{u_R}_3 & \overline{u_R}_2 & u_{L_1} & d_{L_1} \\ \overline{u_R}_3 & 0 & \overline{u_R}_1 & u_{L_2} & d_{L_2} \\ \overline{u_R}_2 & \overline{u_R}_1 & 0 & u_{L_3} & d_{L_3} \\ u_{L_1} & u_{L_2} & u_{L_3} & 0 & \overline{e_R} \\ d_{L_1} & d_{L_2} & d_{L_3} & \overline{e_R} & 0 \end{pmatrix}; \tag{2.1}$$

where we use the convention that the $\bar{5}$ of $SU(5)$ decomposes to $(\bar{3},1)_{1/3}$ $(1,2)_{1/2}$ under the SM gauge group. In the minimal setup, there are two scalar representations, the 5 and the 24, denoted by H_5 and H_{24} respectively. The scalars in the 24 spontaneously break $SU(5) \to SU(3)_C \times SU(2)_L \times U(1)_Y$ while the 5 contains a Higgs doublet responsible for electroweak symmetry breaking.[1] However,

[1]Note that to accomodate neutriono masses, the matter content of the theory must be extended

such a minimal setup predicts the light fermion mass ratios to be m_e/m m_d/m_s which is in contradiction with experimental measurement. The solution to this fermion mass ratio problem is to introduce another scalar multiplet, the 45 [18]. With this extra multiplet, the correct fermion mass ratio can be achieved.

We take the introduction of scalars in the 45, denoted by H_{45}, as evidence that there could be addition scalars transforming in some representation of $SU(5)$, denoted by . Since we want some light components of to contribute to A^{tt}_{FB}, must have a Yukawa coupling to the product of $_L$ and $_L$. Thus could either be in the 45 or the 50 representation[2].

2.2.1 Possible Light Scalar Representations

In this subsection we will explore possible light components of that could unify the SM gauge couplings. Recall that the decomposition of the 45 and 50 under the SM gauge group are

$$
\begin{aligned}
45 &= (8;2)_{1/2} \quad (6;1)_{1/3} \quad (3;3)_{1/3} \quad (3;2)_{7/6} \quad (3;1)_{4/3} \quad (3;1)_{1/3} \quad (1;2)_{1/2}; \\
50 &= (8;2)_{1/2} \quad (6;3)_{1/3} \quad (6;1)_{4/3} \quad (3;2)_{7/6} \quad (3;1)_{1/3} \quad (1;1)_{2}:
\end{aligned}
\tag{2.2}
$$

To avoid problems with light scalars mediating proton decay, we consider only the light scalars that couple to quarks but not leptons. For the 45, the qualified components are the $(8;2)_{1/2}$ and $(6;1)_{1/3}$, while for the 50 the qualified components are the $(8;2)_{1/2}$, $(6;3)_{1/3}$ and $(6;1)_{4/3}$. Now we are ready to address the issue of gauge coupling unification in the presence of these light scalar fields.

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to include an $SU(5)$ singlet, i.e. the right-handed neutrinos.

[2] We ignore the possibility that is in the 5 representation.

2.2.2 Gauge Coupling Evolution

The evolution of gauge couplings is governed by the β-functions. At 1-loop level the running of the couplings in the presence of additional scalar particles is given by

$$\alpha_i^{-1}(t) = \alpha_i^{-1}(M_Z) + \frac{b_i}{2\pi} t + \sum_{t_i} \Theta(t - t_i) \frac{\Delta b_i}{2\pi}(t - t_i); \qquad (2.3)$$

where $t = \ln(\mu/M_Z)$, Θ is the Heaviside function, t_i is the scale that new scalars start to contribute and Δb_i is the contribution due to these new scalars. The coefficients of the β functions from the SM fields are (with 3 generations of fermion and SU(5) normalization for $U(1)_Y$)

$$(b_3; b_2; b_1) = \left(7; \frac{19}{6}; \frac{41}{10}\right); \qquad (2.4)$$

while the contributions from additional scalar fields are

$$\begin{aligned} (\Delta b_3;\ \Delta b_2;\ \Delta b_1)_{(8;2)_{1/2}} &= \left(2; \frac{4}{3}; \frac{4}{5}\right); \\ (\Delta b_3;\ \Delta b_2;\ \Delta b_1)_{(6;1)_{1/3}} &= \left(\frac{5}{6}; 0; \frac{2}{15}\right); \\ (\Delta b_3;\ \Delta b_2;\ \Delta b_1)_{(6;3)_{1/3}} &= \left(\frac{5}{2}; 4; \frac{2}{5}\right); \\ (\Delta b_3;\ \Delta b_2;\ \Delta b_1)_{(6;1)_{4/3}} &= \left(\frac{5}{6}; 0; \frac{32}{15}\right): \end{aligned} \qquad (2.5)$$

We found that in the case where Φ is in the 45, the SM with additional light $(8;2)_{1/2}$ and $(6;1)_{1/3}$ scalar fields do not lead to gauge coupling unification without having an additional particle at an intermediate scale.[3] However, this is not the case for Φ in the 50. The SM fields with an additional light $(8;2)_{1/2}$, $(6;3)_{1/3}$ and $(6;1)_{4/3}$ lead to gauge coupling unification at the scale 10^{17} GeV when the mass of these extra scalar fields are taken to be around 500 GeV, see FIG. 2.1. Note that

[3]With an addition of $(3;3)_{1/3}$ at low scale, one could achieve gauge coupling unifications. However, the $(3;3)_{1/3}$ could mediate proton decay unless Φ is constrained to have Yukawa coupling with ν_L but not ν_R. We will not pursue this possibility in this paper.

(a) SM (b) Model with in the 50

Figure 2.1: Gauge couplings running. $_i = g_i^2 = 4$

this is actually an improvement over typical minimal supersymmetric SM (MSSM) uni cation scales of 2 10^{16} GeV [33, 34]. Thus we will ignore the case where is in the 45 and focus only on the case where is in the 50.

Note that actually all we need to achieve gauge coupling uni cation is to have $(8;2)_{1=2}$, $(6;3)_{1=3}$ and $(6;1)_{4=3}$ at a low scale. However, they all don't have to come from the same multiplet. For example, it is equally valid to have the $(8;2)_{1=2}$ in the same multiplet as the H_{45} while the $(6;3)_{1=3}$ and $(6;1)_{4=3}$ are part of which is in the 50 representation. Since we are interested in having these light scalar mediating positive A^{tt}_{FB}, and it is well known in the literature that $(8;2)_{1=2}$ leads to negative A^{tt}_{FB} [23], we will focus only in the case where $(8;2)_{1=2}$ is part of the H_{45} while $(6;3)_{1=3}$ and $(6;1)_{4=3}$ are part of the . Then we can take the Yukawa coupling of H_{45} to be negligible with impunity. After all, such a coupling must be small in order to e ect only light fermion mass ratios and not the heavier fermion masses.

Finally we note that the mass of the $(6;3)_{1=3}$ and $(6;1)_{4=3}$ can be arranged to be close to the weak scale while the other components remain at GUT scale, see Appendix A for more detail.

2.2.3 Yukawa Couplings of Light Scalars

To have a consistent GUT model, the Yukawa couplings of light scalars at low scale cannot be arbitrary. In particular, they must remain perturbative and unify at the GUT scale. Put another way, the Yukawa couplings at a low scale are determined from the GUT scale Yukawa coupling by the renormalization group (RG) running. In this subsection we compute the Yukawa couplings of these light scalars at a low scale via RG running down from GUT scale. The Yukawa coupling at the GUT scale of the 50 ( ) and two $_L$s is

$$\mathcal{L} = \frac{Y_G^{ab}}{2} \; {}_{aAB} \; {}_{bCD} \; {}^{AB;CD} \; ; \qquad (2.6)$$

where ${}^{AB;CD} = {}^{CD;AB} = {}^{BA;CD} = {}^{AB;DC}$. Here $A;B$ denote $SU(5)$ fundamental indices while $a;b$ are avor indices. We denote the light components of by $_1 = (6;3)_{1=3}$, $_2 = (6;1)_{4=3}$ and $_3 = (8;2)_{1=2}$. Projecting the Lagrangian onto the basis of light elds yields

$$L = \frac{Y_6^{ab}}{2} q_{La}^T \, C q_{Lb} \; {}_1 + \frac{Y_6^{ab}}{2} \overline{u_R}{}^i_a C \overline{u_R}{}^j_b \; {}_{2ij} + h.c.; \qquad (2.7)$$

where $C = i \; {}^0 \; {}^2$ is the charge conjugation matrix, $i;j$ are $SU(3)_C$ indices and $;$ are $SU(2)_L$ indices. Here the q_L are the left-handed $SU(2)_L$ quark doublets, $q_L = \begin{pmatrix} u^i \\ d^i \end{pmatrix}$, where the u^i (d^i) are the i^{th} generation of the up-type (down-type) quarks. Similarly, the u_R are the right-handed $SU(2)_L$ singlets.

In general, the Yukawa coupling can be any 3 3 symmetric matrices in avor space. However, to avoid problems with avor changing neutral currents in

Table 2.1: Running of SU(5) Yukawa couplings at GUT scale (Y_G) to M_z scale.

Y_G	0.05	0.1	0.5	1	1.25	1.5
y_6	0.0049	0.0181	0.1173	0.1433	0.1474	0.1497
y_6	0.0119	0.0392	0.1455	0.1590	0.1608	0.1618

the light quark sector, we take the Yukawa matrix at GUT scale to be

$$Y_6^{ab} = Y_6 \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \text{and} \quad Y_6^{ab} = Y_6 \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \tag{2.8}$$

This structure is preserved by renormalization. We computed the one-loop running of the Yukawa couplings, assuming that only the top Yukawa coupling, Y_t, and Yukawa coupling, Y_i's, are sizable. The relevant -functions are

$$\begin{aligned}
(2\ )\frac{d\ _{y_t}}{dt} &= \frac{9}{2}\ _{y_t} + \ _{y_6} + \frac{3}{2}\ _{y_6} \quad 8\ _3 + \frac{9}{4}\ _2 + \frac{17}{20}\ _1 \quad _{y_t}; \\
(2\ )\frac{d\ _{y_6}}{dt} &= 4\ _{y_6} + 2\ _{y_t} \quad 8\ _3 + \frac{8}{5}\ _1 \quad _{y_6}; \\
(2\ )\frac{d\ _{y_6}}{dt} &= 5\ _{y_6} + \ _{y_t} \quad 8\ _3 + \frac{9}{2}\ _2 + \frac{1}{10}\ _1 \quad _{y_6};
\end{aligned} \tag{2.9}$$

where we take $_{y_t} = \frac{Y_t^2}{4}$, $_{y_6} = \frac{Y_6^2}{4}$ and $_{y_6} = \frac{Y_6^2}{4}$.

Typical values of the new Yukawas at M_Z are computed for various perturbative GUT Yukawas in Table 2.1. They indicate that reasonable Yukawas at M_Z can yield both a large $t\bar{t}$ A_{FB}^{tt}, as we will see in section 2.3.

2.3 $t\bar{t}$ Phenomenology at the Tevatron

2.3.1 General Considerations of $A_{FB}^{t\bar{t}}$

The forward-backward asymmetry is defined to be

$$A_{FB}^{t\bar{t}} = \frac{\sigma_F^{t\bar{t}} - \sigma_B^{t\bar{t}}}{\sigma_{tot}^{t\bar{t}}}; \tag{2.10}$$

where forward and backward are defined with respect to the direction of the proton. In the presence of NP, it is convenient to characterize the asymmetry in terms of the SM and NP contributions. We follow [30] to define $A_{FB}^{t\bar{t}}$ as

$$A_{FB}^{NP+SM} = \frac{\sigma_F^{NP} - \sigma_B^{NP}}{(\sigma^{NP+SM})_{LO}} + A_{FB}^{SM}\left(\frac{\sigma^{SM}}{\sigma^{SM} + \sigma^{NP}}\right): \tag{2.11}$$

Note that the first term comes from the leading effect of NP while the second term is the dilution of A_{FB}^{SM} due to NP. The observed $A_{FB}^{t\bar{t}}$ reported by the CDF collaboration is $A_{FB}^{CDF} = 0.201 \pm 0.065_{stat} \pm 0.018_{sys} = 0.201 \pm 0.067$ [21], where we have combined the uncertainties in quadrature. DØ reports a value of $A_{FB}^{DØ} = 0.196 \pm 0.065$ [19]. The asymmetry as predicted by SM is estimated to be 0.073 [22] which is about 2σ away from either observed value. However, CDF observed that the asymmetry increased with energy, with $A_{FB}^{CDF} = 0.475 \pm 0.114$ for $M_{t\bar{t}} \geq 450$ GeV [20]. The corresponding SM prediction is 0.111 [22], a 3.5σ deviation. We take this discrepancy as a hint for NP.

It is worth mentioning that any NP models that wish to explain the $A_{FB}^{t\bar{t}}$ must not violate the measured $t\bar{t}$ production cross-section, $\sigma^{t\bar{t}}$. The latest measurement reported by the CDF is $\sigma_{t\bar{t}} = 8.5 \pm 0.6_{stat} \pm 0.7_{sys} = 8.5 \pm 0.9$ pb [35]. This is to be compared with the SM prediction of $\sigma^{SM} = 6.63$ pb [22]. For further reference, we compile the CDF measurements as well as the SM prediction [22] in Table 2.2.

Table 2.2: Measurements and SM predictions of $t\bar{t}$ observables at the Tevatron.

Observable	Measured Value	SM Prediction [22]
$A_{FB}^{t\bar{t}}$	0.196 ± 0.065 [19] $0.201 \pm 0.065_{stat} \pm 0.018_{sys}$ [21]	0.073
$A_{FB}^{t\bar{t}}(M_{t\bar{t}} < 450\ \mathrm{GeV})$	-0.116 ± 0.153 [20]	0.052
$A_{FB}^{t\bar{t}}(M_{t\bar{t}} \geq 450\ \mathrm{GeV})$	0.475 ± 0.114 [20]	0.111
$\sigma_{t\bar{t}}$	$8.5 \pm 0.6_{stat} \pm 0.7_{sys}$ pb [35]	6.63

2.3.2 Differential Cross-section for $t\bar{t}$ Production

To study $t\bar{t}$ phenomenology, it is convenient to expand the $SU(2)$ indices in the above Lagrangian, Eq. (2.7), and keep terms relevant for tree-level $t\bar{t}$ production cross-section:

$$\mathcal{L} = \frac{Y_6^{ut} + Y_6^{tu}}{2}\left(u^T C P_L t\, \Phi_1^1 + \frac{1}{\sqrt{2}} d^T C P_L t\, \Phi_1^2\right) + \frac{Y_6^{ut} + Y_6^{tu}}{2} u C P_L t^T \Phi_2 + \mathrm{h.c.}; \tag{2.12}$$

where in the above expression $SU(3)_C$ indices have been suppressed. We take the $SU(2)$ two-index symmetric tensor to be $\Phi_1 = \begin{pmatrix} \Phi_1^1 & \Phi_1^2/\sqrt{2} \\ \Phi_1^2/\sqrt{2} & \Phi_1^3 \end{pmatrix}$. Note that we allow for the possibility that the Yukawa coupling matrices can become non-symmetric due to radiative corrections. At the Tevatron, $t\bar{t}$ are dominantly produced from the u quarks or the d quarks, while other quarks or gluon initial states are PDF suppressed. The differential cross-section for $t\bar{t}$ production initiated from the u quarks is

$$\frac{d\sigma^{(NP)}_{(6;3)}}{d\hat{t}}(u\bar{u} \to t\bar{t}) = \frac{1}{16\pi\hat{s}^2}\frac{1}{4}\frac{1}{9}\left(\frac{8 g_s^2 (4\pi\alpha_{y_6})}{\hat{s}(\hat{u} - m_{\Phi_1}^2)}\left((\hat{u} - m_t^2)^2 + \hat{s} m_t^2\right)\right.$$
$$\left. + 4(4\pi\alpha_{y_6})^2 \frac{3}{2}\frac{(\hat{u} - m_t^2)^2}{(\hat{u} - m_{\Phi_1}^2)^2}\right); \tag{2.13}$$

$$\frac{d\sigma^{(NP)}_{(6;1)}}{d\hat{t}}(u\bar{u} \to t\bar{t}) = \frac{1}{16\pi\hat{s}^2}\frac{1}{4}\frac{1}{9}\left(\frac{8g_s^2(4\pi\alpha_{y_6})}{\hat{s}(\hat{u}-m^2_{\Phi_2})}\left((\hat{u}-m_t^2)^2+\hat{s}m_t^2\right)\right.$$
$$\left.+4(4\pi\alpha_{y_6})^2\frac{3}{2}\frac{(\hat{u}-m_t^2)^2}{(\hat{u}-m^2_{\Phi_2})^2}\right); \qquad (2.14)$$

where we have defined $\alpha_{y_6} = \left(\frac{|Y_6^{ut}+Y_6^{tu}|}{2}\right)^2/4\pi$, and α'_{y_6} is also defined analogously. Note that we have included interference with the SM in our NP cross-section. Similarly, the differential cross-section initiated from the d quarks is

$$\frac{d\sigma^{(NP)}_{(6;3)}}{d\hat{t}}(d\bar{d} \to t\bar{t}) = \frac{1}{16\pi\hat{s}^2}\frac{1}{4}\frac{1}{9}\left(\frac{4g_s^2(4\pi\alpha'_{y_6})}{\hat{s}(\hat{u}-m^2_{\Phi_1})}\left((\hat{u}-m_t^2)^2+\hat{s}m_t^2\right)\right.$$
$$\left.+(4\pi\alpha'_{y_6})^2\frac{3}{2}\frac{(\hat{u}-m_t^2)^2}{(\hat{u}-m^2_{\Phi_1})^2}\right). \qquad (2.15)$$

2.3.3 Results from the Colored Scalar Model

The above differential cross-sections must be convoluted with parton distribution functions (PDFs) of the proton and anti-proton to give $A^{t\bar{t}}_{FB}$ comparable with accelerator measurements. We compute the total cross-sections and asymmetries using the NLO MSTW 2008 PDFs [36]. We find that for a suitable set of parameters, our model can accommodate the large $A^{t\bar{t}}_{FB}$ and be consistent with the $t\bar{t}$ production cross-section constraint as can be seen in FIG. 2.2 and 2.3.

The phenomenological aspects of the model provide nice improvements over SM predictions. The $A^{t\bar{t}}_{FB}$ from our model agrees with CDF data within 2.2σ for the high-mass bin, 1.4σ for the low-mass bin, and 0.8σ for the total asymmetry. Additionally, our model agrees with the CDF total $t\bar{t}$ production within 0.9σ. While the high-mass bin result seems most problematic with the model, it is actually the most significant improvement over SM results. The CDF low-mass bin and total asymmetry measurements are discrepant from the SM predictions (NLO+LL) at 2σ or less and are not statistically significant [22]. However, the high-mass bin is discrepant at 3.5σ, and so our model reduces this deviation to a less stastistically

(a) High invariant mass bin asymmetry

(b) Low invariant mass bin asymmetry

(c) Total asymmetry

(d) Total (NP+SM) $t\bar{t}$ production

Figure 2.2: Computational results for $t\bar{t}$ phenomenology at a GUT Yukawa coupling of $Y_G = 0.5$. The contours in each plot, from bottom to top, are decreasing in $m_{(6;3)}$ from 550 to 400 GeV. The gray regions are, in plot (a), CDF 2σ allowed regions, in plot (b), CDF 1.5σ allowed regions, and, in plots (c) and (d), CDF 1σ allowed regions. Plot (d) also shows the central value for the CDF $t\bar{t}$ cross-section.

significant result. The agreement within less than 1.4σ of the rest of the results serves as a check on the merits of the model.

2.4 Conclusion and Discussion

Light colored scalars extension of SM can account for the observed $A^{t\bar{t}}_{FB}$ reported by CDF and D0. At the same times they also allow for unification of fundamental forces at sufficiently high scale consistent with bound from proton

(a) High invariant mass bin asymmetry

(b) Low invariant mass bin asymmetry

(c) Total asymmetry

(d) Total (NP+SM) $t\bar{t}$ production

Figure 2.3: Same as FIG. 2.2, except with $Y_G = 1.0$. This coupling provides better agreement with measurements.

decay. In our explicit model with SU(5) GUT, we extend SM by introducing 3 multiplets of light scalars: $(8,2)_{1/2} \in 45$, $(6,3)_{1/3}$ and $(6,1)_{4/3} \in 50$. These new scalars lead to gauge coupling unification at scale 10^{17} GeV which is considerably higher compared to typical unification scale suggested by MSSM, 2×10^{16} GeV. Notice that the quantum number of these scalars forbids Yukawa coupling to leptons. Hence there is no light scalar leptoquark mediated proton decay. In this work we do not attempt to solve the hierarchy problem associated with these light scalars.

For suitable value of Yukawa coupling and masses these colored scalars that contribute to $A_{FB}^{t\bar{t}}$ yield values of 9.5-11.5% (CDF: -11.6±15.3%) for $M_{t\bar{t}} \leq 450$ GeV, 21-22.5% (CDF: 47.5±11.4%) for $M_{t\bar{t}} > 450$, and 14-16% (CDF: 20.7±6.7%, DO: 19.6 ± 6.5%) for the total forward-backward asymmetry. The corresponding total $t\bar{t}$ production cross-section, including these scalars contribution, is around $8.4 - 9.6$ pb (CDF: 8.5 ± 0.9 pb). These computations have been checked under variations of the SM input parameters (m_t, α_s, and PDF parameters) within their reported 1σ limits; our results show sub-percent variations and thus the ranges of the values reported above can be trusted.

It is interesting to explore LHC phenomenology associated with this model. In principle, a single color sextet scalar $\Phi \in \{(6,3)_{1/3}$ and $(6,1)_{4/3}\}$ can be singly produced from a pair of quark initial states $qq \to \Phi$. The single production channel does depends on the form of Yukawa coupling matrix. In the case where the Yukawa coupling is diagonal and of $O(1)$, the production cross-section from the uu initial state, $gg \to \Phi$, is ~ 10 nb [37]. However in our model, due to the particular from of the Yukawa coupling, the possible initial states are ut or db. Thus single production will be suppressed by the $t(b)$ PDF. The Φ would decay into a pair of $ut(db)$ which lead to 2 jets with or without lepton.

Nevertheless, the LHC is known as a gluon factory, thus more promising production mechanism is a pair production from gluon fusion $gg \to \Phi\Phi^*$. The production cross-section in this channel taken the mass of Φ 500 GeV is at the order of a few pb [38]. The Φ would decay into a pair of $ut(db)$ hence would lead to 4 jets, 2 of which are b jets, with or without lepton. Thus in this case it is possible to observed two widely separated b jets and the invariant mass of these 4 jets displays a resonant structure at $2m$.

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Chapter 3

Model building with theoretical motivations: Spontaneous R-symmetry breaking from the renormalization group flow

We propose a mechanism of R-symmetry breaking in four-dimensional DSB models based on the RG properties of the coupling constants. By constraining the UV sector, we generate new hierarchies amongst the couplings that allow a spontaneously broken R-symmetry in models with pure chiral fields of R-charges $R = 0$ and $R = 2$ only. The result is obtained by a combination of one- and two-loop effects, both at the origin of field space and in the region dominated by leading log potentials.

3.1 Introduction

In the last decade many different mechanisms of supersymmetry breaking have been investigated. Dynamical supersymmetry breaking (DSB) is an attractive possibility because it can evade constraints imposed by the supertrace formula $\mathrm{STr}(M^2)$. Unfortunately, DSB models often lead to non-calculable strongly coupled sectors, in which the knowledge of the spectrum requires the use of non-perturbative techniques that are not always available. A new scenario for DSB was proposed in [39]. There a weakly coupled IR supersymmetry breaking sector was obtained from supersymmetric duality. A mass deformed $\mathcal{N} = 1$ asymptotically free supersymmetric field theory flows in the IR to a weakly coupled dual theory with parametrically long-lived metastable minima that break the supersymmetry. At the lowest orders in the perturbative expansion the dynamics are dominated by a model of pure chiral fields, like the O'Raifeartaigh model. It is therefore important to know the exact and general properties of O'Raifeartaigh-like models for the study of DSB. To provide a phenomenologically viable scenario, we must also break the R-symmetry that generically accompanies these models to give the gaugino a non-zero Majorana mass.

In this work we focus on R-symmetric O'Raifeartaigh-like models whose field content has R-charge $R = 0$ or $R = 2$ only. This property is typical of generalizations of the ISS mechanism but these models suffer from broad constraints that limit the possibility of spontaneous R-symmetry breaking, which is what we seek to achieve in this work. In particular, the generic (pseudo)moduli fields that accompany the supersymmetry breaking superpotential that have $R = 2$ receive positive corrections from the one-loop Coleman-Weinberg potential, eliminating the possibility of R-symmetry breaking via a non-zero modulus field vev, or they remain flat. There is no general proof for the behavior of the pseudomoduli at higher loops, leaving open the possibility of spontaneous R-symmetry breaking via higher loop corrections to pseudomoduli that are one-loop flat. In fact, most examples that have one-loop flat directions receive negative two-loop corrections that destabilize the origin [40, 41]. Unfortunately, in all of these examples the tachyonic behavior near the origin is never stabilized at a non-zero pseudomodulus vev, and the potentials run away to a supersymmetric vacuum or infinite field value.

In principle, a model could be constructed that stabilizes these potentials with tachyonic behavior at the origin by going far out in field space and using the quantum effective potential methods developed by [42]. It then becomes necessary to introduce one-loop corrections to the pseudomodulus; however, these corrections, at least at the origin, must be subdominant to the tachyonic two-loop effect. We give a rough sketch that shows that having both a tachyonic origin and a stabilizing (i.e. positive) slope in the far field potential cannot be accomplished with a single superpotential coupling if one-loop effects are subdominant at the origin, in the perturbative regime, they will continue to be subdominant to higher-loop order effects far in field space.

This suggests that the myriad obstructions already evident might be evaded by using more than one superpotential coupling. A mechanism could be introduced to invert the behavior of the couplings in the two regions of field space and induce the desired behavior of the effective potential. More concretely, we invert the natural hierarchy of the perturbative expansion so that at the origin of field space two-loop effects are dominant, but, far in field space, the one-loop effects become more important. We achieve this through a new coupling associated with massive degrees of freedom that are integrated out at small field values but that contribute far from the origin. This is reminiscent of the interplay between the gauge and interaction couplings in [43], where the coupling hierarchy is inverted in the field space because of asymptotic freedom.

In section 3.2 we elaborate on the obstructions to spontaneous R symmetry breaking at one and two loops in models with charges $R = 0$ and $R = 2$ only. Then in section 3.3 we propose our mechanism, explicitly check its validity in a toy model, and provide a UV completion. In section 3.4 we conclude and discuss some open questions.

3.2 R-symmetry breaking with R = 0 and R = 2: Obstructions

The one-loop correction to the mass of the O'Raifeartaigh field is non-negative in models of pure chiral fields with charges R = 0 and R = 2 [44]. This result holds when more than one pseudomodulus is present [45]; however, the fate of these pseudomoduli at higher-loop order is generically unconstrained and R-symmetry breaking is left as a possibility. Unlike the one-loop Coleman-Weinberg effective potential, which can be calculated in terms of the mass matrices only, at two-loop order the effective potential must be explicitly calculated by including the Yukawa and quartic couplings[1].

Explicit examples show that at two-loop order there are no non-negativity constraints on the pseudomoduli masses as in the one-loop case. For example, in the model studied in [40] the superpotential is[2]

$$W = fX + hX\phi_1^2 + h\mu\phi_1\phi_2 + hY\phi_1\phi_4 + hZ\phi_4^2 + hm\phi_4\phi_5 \tag{3.1}$$

where $\sqrt{f}$, μ, and m are mass scales in the theory, h is the superpotential coupling, the ϕ_i fields are tree-level stable at the origin, the X and Y are pseudomoduli stabilized at one-loop, and the Z field is still a pseudomodulus at one loop that acquires a negative mass at two loops.

A different possibility has been studied in [41], by starting from the superpotential

$$W = fX + hX\phi_1^2 + h\mu\phi_1\phi_2 + hY\phi_1\phi_5 + hZ\phi_4^2 + hm\phi_4\phi_5. \tag{3.2}$$

In this case the one-loop pseudo at direction Z has a positive two-loop mass that is stabilized around the R-symmetric vacuum $\langle Z\rangle = 0$.

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[1] If the supersymmetry breaking scale F is smaller than the messenger scale M, $F \ll M^2$, there are simpler results for the two-loop effective potential. [46]

[2] The model studied in [40] is slightly different, but the quantum corrections are computed in a similar manner and the final result is the same.

Clearly, while in (3.2) the R-symmetry is not spontaneously broken, the possibility to break the R-symmetry exists in (3.1). The vacuum structure for this model must be determined by calculating the behavior of the Z potential away from the origin. This can be explored by applying the analysis of [42]. There, one reconstructs the effective potential for a pseudo flat direction far from the origin but below the cutoff scale by computing the discontinuity in the anomalous dimension of the massive messengers in the theory. The pseudomodulus is treated as a background field with non-zero vev. By applying this idea the leading log potential is obtained order-by-order in perturbation theory- schematically, with loop order n, one has

$$V_{\rm eff}(\ ) \simeq \text{const.} + \sum_n (-1)^{(n+1)} \frac{2}{n!} |f|^2 \ \gamma_X^{(n)} \log^n \frac{|\ |}{m_0} \qquad (3.3)$$

and the sign of the coefficient $(-1)^{(n+1)} \ \gamma_X^{(n)}$ determines the sign of the potential of the pseudomodulus at large $|\ |$. The discontinuity in the anomalous dimension is captured in $\gamma_X^{(n)} = \frac{d^{n-1}\gamma_X}{dt^{n-1}}\Big|_{t^-}^{t^+}$. As explained in [42], each derivative of γ_X gives a loop factor. This formula is only valid in the region $\sqrt{F} \ll \langle\ \rangle \ll$, where F is the scale set by the supersymmetry-breaking F-terms of $\ $.

In the case of (3.1) the leading log potential for Z is negative and the potential flows towards a supersymmetric minimum (or a runaway)[3]. So there are no R-symmetry breaking vacua in (3.1), even though the potential is destabilized at the origin.

One can still try to break the R-symmetry with the addition of a tree-level term $W \supset f_2 Z$ to the superpotential. Indeed, this term generates a one-loop contribution to the mass of Z (which is automatically positive) and there is a tension between the one- and two-loop contributions, potentially giving a non-supersymmetric vacuum at $\langle Z \rangle \neq 0$. One can then distinguish the two cases $f_2 \simeq f$

[3] The supersymmetric vacuum structure is usually associated with the UV completion of the model.

and $f_2 \lesssim f$[4]. If $f_2 \simeq f$ the positive one-loop correction dominates at the origin and the negative two-loop effect dominates at large vev, so the potential has a local maximum at the origin. On the contrary, if $f_2 \ll f$ the negative two-loop potential dominates everywhere, since the one-loop effects at the origin are suppressed by $f_2/f \ll 1$. In both cases there are no R-symmetry breaking minima.

It would appear that this outcome is generic in the models presented. This is argued as follows: To achieve spontaneous R-symmetry breaking in these O'Raifeartaigh-like models, we require that the Z potential be (a) tachyonic at the origin and (b) increasing (*i.e.* with positive slope) somewhere further out in field space. To satisfy (a), we must have two-loop effects that are dominant at the origin, since one-loop effects will never afford this behavior. Since the two-loop effects are suppressed by a factor of h^2 compared to the one-loop effects (but aided by a factor of $F_X/F \gg 1$), this puts a lower bound on the value of h[5]. As we move farther out in field space to the regime where (3.3) is applicable we begin to lose perturbativity as higher loops become increasingly important. However, in a model of only chiral fields with one coupling, if the two-loop contribution dominates the one-loop contributions at the origin it will dominate the one-loop contribution everywhere in field space, since no new field content is introduced. To satisfy (b), one could argue that the three-loop behavior might accomplish what the one-loop contribution sought to do, but then our "leading" log arguments are foregone as we begin to consider all loop contributions. More quantitatively, the requirement from (b) that the (positive) one-loop leading log dominate the two-loop leading log out in field space puts an *upper* bound on the value of h, which will eliminate any parameter space in h left from the previous *lower* bound.

We now search for a loophole in this argument based upon the RG properties of the model in the perturbative large field region with multiple couplings. In

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[4]The case $f_2 \gg f$ is irrelevant because it reverses the role of Z and X in the $f_2 \ll f$ case

[5]Hereafter we assume h is a real coupling by absorbing the imaginary part in the phases of the fields.

the next section we provide a way to invert the hierarchy amongst the one- and two-loop effects when the potential is dominated by the leading log.

3.3 R-symmetry breaking from the renormalization group flow

We have seen that in O'Raifeartaigh-like models with only $R = 0$ and $R = 2$ fields R-symmetry breaking is quite constrained. One-loop quantum corrections will leave pseudomoduli flat or stabilize them at the origin, while two-loop corrections can be either positive or negative. At the quantum level, this means that there can exist tension between a positive one-loop and a negative two-loop correction[6]. In the models previously studied this leads to runaway behavior, but here we will attempt to circumvent their fate with a loophole based upon the RG properties of superpotentials and their moduli spaces.

3.3.1 Generalities

Consider a model with chiral fields, a canonical Kähler potential and a superpotential W with all fields assigned R-charges $R = 0$ or $R = 2$ such that the R-symmetry is preserved. Let the superpotential be of the form

$$W = W_1(X;\ _i;\ _i) + W_2(\ _i;'_i) \tag{3.4}$$

where we identify the tree-level flat direction with X and $_i$ and the other fields are the $_i$ and $'_i$. The W_1 sector has the usual O'Raifeartaigh field X in addition to other pseudomoduli $_i$ and massive messengers $_i$. The second sector, W_2, contains some of the (pseudo)moduli $_i$ with non-zero F-terms such that $F \quad F_X$ and some massive fields $'_i$. We assume the masses of the $'_i$ are much larger than those of the $_i$ from the first sector, $m_{'_i} \quad m_{_i}$.

In this limit the W_2 sector decouples around the origin of the (pseudo)moduli

[6] We ignore higher loop corrections.

space and the non-supersymmetric vacuum structure is encrypted in W_1[7]. We consider W_1 such that one of the $_i$ has a vanishing one-loop mass correction but a non-zero, negative two-loop correction. The e ective potential for this eld is negative around the origin and it remains negative in the region $|F_X| \; |h \; _i| \;$, where is the strong coupling scale determined by the UV completion of the model [42]. This model does not generically break the R-symmetry spontaneously, at least not without the W_2 sector. The contributions from W_2 become important at a scale $|h \; _i| \, ' \; m_{\cdot \; i}$, where the presence of a non-zero F-term for $_i$ gives a positive leading log correction to the e ective potential. The potential in this region is

$$V(\; _i) \; ' \; V^{(1)}(h_2; F \; _i) + V^{(2)}(h_1; F_X) \tag{3.5}$$

where h_i is the coupling in the W_i sector. The R-symmetry can be broken if $h_2 \quad h_1 \; (F_X; F \; _i)$ where $(F_X; F \; _i)$ is a model-dependent function. Figure 3.1 gives a schematic picture of the e ective potential for the eld .

3.3.2 A toy model

Here we propose a toy model that spontaneously breaks the R-symmetry in an O'Raifeartaigh-like model with elds that have R-charges 0 and 2 only. We follow the strategy explained above. The superpotential W_1 is

$$W_1 = f_X X + h_X X \; {}^2_1 + m_1 \; _1 \; _2 + Y \; _1 \; _4 + h_1 Z \; {}^2_4 + m_2 \; _4 \; _5 \tag{3.6}$$

while W_2 is

$$W_2 = f_Z Z + h_2 Z \; {}^2_4 + m_3 \; _4 \; _5 \tag{3.7}$$

---

[7] There is still a non-zero F-term associated to $_i$ in W_2 but it is subleading in the limit $F \quad F_X$.

Figure 3.1: A schematic picture of the e ective potential for the eld . Near the origin in h i there is a positive one-loop correction to the tree-level at potential for h i. This contribution is suppressed by $\frac{F}{F_X}$ in comparison to a negative two-loop correction that dominates the one-loop contribution. Both are computed perturbatively. As we move away from the origin and lose computational control, we approach the far- eld region, where h i m$_{\cdot\ i}$ but h i , where is the cuto scale. Here the potential is computed using the leading log expansion and the one-loop leading log h i dominates the two-loop leading $\log^2$ h i by a careful choice of parameters in the model. As h i , we lose all perturbative control over the behavior of the potential.

We impose a hierarchy amongst the scales

$$\sqrt{f_Z} \ll \sqrt{f_X} \ll m_1; m_2 \ll m_3 \quad : \tag{3.8}$$

Around the origin, ϕ_4 and ϕ_5 are integrated out at zero vev and the vacuum structure is well described by W_1. The fields ϕ_i acquire a tree-level mass at zero vev while the fields X and Y are tree-level flat directions, stabilized at the origin by one-loop corrections. The field Z is flat at tree level and its quantum mass is dominated by the two-loop effect if

$$\frac{f_Z^2}{f_X^2 h_X^2} \ll 1: \tag{3.9}$$

At larger $\langle Z \rangle$ the effects of m_3 are no longer suppressed. In the region

$$m_3 \ll \langle Z \rangle \tag{3.10}$$

the leading log potential is

$$V_{eff} = f_Z^2 (h_1^2 + h_2^2) \log Z - f_X^2 h_X^2 h_1^2 \log^2 Z: \tag{3.11}$$

There can still be an R-symmetry breaking minimum if the inequality

$$h_2 > h_1 \sqrt{\frac{2 \log Z}{\;} - 1} \tag{3.12}$$

is satisfied (note this is compatible with (3.8)). The R-symmetry is broken at the quantum level by the vev of Z, with $R(Z) = 2$. The presence of two couplings in this simple example follows the construction outlined in section 3.3.1 and accomplishes spontaneous R-symmetry breaking.

3.3.3 A UV completion

In this section we discuss a supersymmetric gauge theory with supersymmetry-breaking metastable vacua that also break its R-symmetry. In the IR the model reduces to the class introduced above, where W_1 and W_2 provide a generalization of the toy model.

In this model we tune the masses of the fields in the UV sector, while the tuning on the couplings is dynamical. This provides a more natural explanation of the necessary hierarchies amongst the couplings required by our construction. The field content is (see Figure 3.2 for a quiver representation of the model)

Field	$SU(N_{F_1})$	$SU(N_c)$	$SU(N_{F_2})$	$SU(M)$
$Q_1 \quad \tilde{Q}_1$	$N_{F_1} + \tilde{N}_{F_1}$	$\tilde{N}_c \quad N_c$	$1 \quad 1$	$1 \quad 1$
$Q_2 \quad Q_2$	$1 \quad 1$	$N_c \quad \tilde{N}_c$	$\tilde{N}_{F_2} + N_{F_2}$	$1 \quad 1$
$q_3^{(i)} \quad \tilde{q}_3^{(i)}$ with $i = 1;2$	$1 \quad 1$	$1 \quad 1$	$N_{F_2} + \tilde{N}_{F_2}$	$\overline{M} \quad M$

with superpotential

$$W = m_1 Q_1 \tilde{Q}_1 + m_2 Q_2 \tilde{Q}_2 + m_3 q_3^{(1)} \tilde{q}_3^{(2)} + m_3 q_3^{(2)} \tilde{q}_3^{(1)} + \frac{1}{\Lambda_0} Q_2 \tilde{Q}_2 q_3^{(1)} \tilde{q}_3^{(1)} \tag{3.13}$$

The groups $SU(N_{F_1})$ and $SU(N_{F_2})$ are flavor symmetries while $SU(N_c)$ is the gauge symmetry. At this level we do not specify the dynamics of $SU(M)$; Figure 3.2 indicates the possibilities for this $SU(M)$ in the context of a quiver diagram.

Figure 3.2: A quiver representing the electric theory. The green boxes are flavor nodes, the red the gauge node. We do not fix the nature of the blue node: it can be either a flavor symmetry or a weakly gauged global symmetry.

We consider this $SU(N_c)$ gauge symmetry in the free magnetic range,

$$N_c + 1 < N_{F_1} + N_{F_2} < \frac{3}{2} N_c \tag{3.14}$$

so that the model is described in the IR by the Seiberg dual with field content (see Figure 3.3 for the quiver representation)

Field	$SU(N_{F_1})$	$SU(\tilde N_c)$	$SU(N_{F_2})$	$SU(M)$
q_1 $\tilde q_1$	$N_{F_1} + \tilde N_{F_1}$	$\tilde N_c$ N_c	1 1	1 1
q_2 $\tilde q_2$	1 1	N_c $\tilde N_c$	$\tilde N_{F_2} + N_{F_2}$	1 1
M_{11}	N_{F_1} $\tilde N_{F_1}$	1	1	1
M_{12} M_{21}	$\tilde N_{F_1} + N_{F_1}$	1	$N_{F_2} + \tilde N_{F_2}$	1
M_{22}	1	1	N_{F_2} $\tilde N_{F_2}$	1
$q_3^{(i)}$ $\tilde q_3^{(i)}$ with $i = 1; 2$	1 1	1 1	$N_{F_2} + \tilde N_{F_2}$	$\tilde M$ M

where $\tilde N_c = N_{F_1} + N_{F_2} - N_c$ and the superpotential is

$$W = -h\,\mu_1^2 M_{11} + h\,\mu_2^2 M_{22} + h \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} \begin{pmatrix} \tilde q_1 & \tilde q_2 \end{pmatrix}$$

$$+ \frac{g}{\Lambda_0} M_{22}\, q_3^{(1)} \tilde q_3^{(1)} + m_3 \left(q_3^{(1)} \tilde q_3^{(2)} + q_3^{(2)} \tilde q_3^{(1)} \right) : \tag{3.15}$$

If we fix the hierarchy among the electric masses as

$$m_3 \ll m_1 \ll m_2 \tag{3.16}$$

there is a classical vacuum solution that breaks supersymmetry which can be

Figure 3.3: A quiver representing the magnetic theory. The green boxes are flavor nodes, the red one is the gauge node, while the blue one can be both.

written as

$$\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} {}_1 1_{\tilde N_c \times \tilde N_c} \\ 0_{(N_{F_1}-\tilde N_c)\times \tilde N_c} \\ \hline 0_{N_{F_2}\times \tilde N_c} \end{pmatrix};$$

$$\begin{pmatrix} \tilde q_1 & \tilde q_2 \end{pmatrix} = \left(\begin{array}{cc|c} {}_1 1_{\tilde N_c \times \tilde N_c} & 0_{\tilde N_c \times (N_{F_1}-\tilde N_c)} & 0_{\tilde N_c \times N_{F_2}} \end{array} \right) \tag{3.17}$$

with the rest of the fields at zero expectation value. We can expand about this vacuum and choose a convenient parametrization of the field fluctuations:

$$\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} {}_1 + {}_1 \\ {}_1 \\ \hline {}_5 \end{pmatrix}; \quad \begin{pmatrix} \tilde q_1 & \tilde q_2 \end{pmatrix} = \left(\begin{array}{cc|c} {}_1 + \tilde{}_1 & {}_2 & {}_4 \end{array} \right)$$

$$M_{11} = \begin{pmatrix} {}_{11} & {}_{6} \\ {}_{7} & X \end{pmatrix} ; \quad M_{12} = \begin{pmatrix} {}_{8} \\ \tilde{Y} \end{pmatrix} ; \quad M_{21} = \begin{pmatrix} {}_{9} & Y \end{pmatrix} ; \quad M_{22} = Z$$

$$\begin{pmatrix} q_3^{(1)} \\ q_3^{(2)} \end{pmatrix} = \begin{pmatrix} {}_{4} \\ {}_{6} \end{pmatrix} ; \quad \begin{pmatrix} \tilde{q}_3^{(1)} & \tilde{q}_3^{(2)} \end{pmatrix} = \begin{pmatrix} {}_{5} & {}_{7} \end{pmatrix} \qquad (3.18)$$

This yields the IR superpotential

$$\begin{aligned} W &= \mathrm{Tr}[\, h\ {}_1^2 X + hX\ {}_1\ {}_2 + h\ {}_1(\ {}_1\ {}_6 + \ {}_2\ {}_7 + \ {}_4\ {}_8 + \ {}_5\ {}_9) \\ &+ h\ {}_2^2 Z + hZ\ {}_4\ {}_5 + h_2 Z\ {}_4\ {}_5 + m_3(\ {}_4\ {}_6 + \ {}_5\ {}_7) \\ &+ h\ {}_1 Y\ {}_4 + h\ {}_2 \tilde{Y}\ {}_5] \end{aligned} \qquad (3.19)$$

plus terms that are supersymmetric at two loops, which is the order to which we study supersymmetry breaking e ects in this work. Here we have de ned $h_2 \quad g = \ _0$.

From (3.16) we have

$$m_3^2 \quad h\ {}_1^2 \quad h\ {}_2^2 \qquad (3.20)$$

and we can integrate out the $q_3^{(i)}$ and $\tilde{q}_3^{(i)}$ (with $\langle q_3^{(i)} \rangle = \langle \tilde{q}_3^{(i)} \rangle = 0$). Deep in the IR we have the usual W_1 model, with $m_Z^2 < 0$ from two-loop quantum e ects. There is also a one-loop contribution that is suppressed ($_2$). In particular, the one- and two-loop Z masses are

$$\begin{aligned} m_Z^{(1)\,2} &= 4\frac{h^2\ {}_1^2}{24\ {}^2} N_{F_2} \tilde{N}_c \quad h^2 + h_2^2 \ \frac{1}{m_3}\ ^2 + O(\ ^6) \\ m_Z^{(2)\,2} &= \frac{2h^4\ {}_1^2}{(16\ {}^2)^2} N_{F_2} \tilde{N}_c \ \Big\{ \log 4 \quad 1 \quad \frac{^2}{6} + \ ^4 g(\ ) \Big\} + O(\ ^6) \end{aligned} \qquad (3.21)$$

where $g(\ )$ is a complicated but well-behaved function that depends on[8] $\log^2$ and we have defined $_2 = {}_1$. The trace in (3.19) gives a factor of $N_{F_2} N_c$.

These masses indicate how the potential for Z behaves at the origin. Clearly, to have a R-symmetry breaking minimum, we must have $m_Z^2 = m_Z^{(1)\,2} + m_Z^{(2)\,2} < 0$. However, there is another constraint on the parameters in m_Z^2 that comes from the behavior of the potential for Z in the far field region. Here $_1 << \langle Z \rangle <<$; the one- and two-loop contributions to the potential in this region are in tension with one another, since they are introduced with opposite signs (*cf.* (3.3)), and we must include the effects of the m_3 mass terms. Then, to two-loop order,

$$
\begin{aligned}
V_e(Z) &= V_e^{(1)}(Z) + V_e^{(2)}(Z) \\
&= \frac{2\ {}_2^4}{16\ ^2}\ h^2 + h_2^2\ \log\frac{\langle Z\rangle}{_2} \\
&\quad \frac{1}{(16\ ^2)^2}\ 4\ {}_1^4 h^4 \log^2\frac{\langle Z\rangle}{_1} + 2\ {}_2^4\ h^2 + h_2^2\ {}^2 \log^2\frac{\langle Z\rangle}{_2}
\end{aligned}
\qquad (3.22)
$$

up to an unimportant constant. To have a stable R-symmetry breaking minimum in the pseudomodulus Z, we require that the slope of the potential far in field space be positive, so that an intermediate minimum is guaranteed (*cf.* Figure 3.1). This further constricts the allowed values of $,\ h$, and h_2; however, the allowed parameter space is substantial, depending on the ratio between h and h_2, which we define as $\frac{h^2}{h_2^2}$. Figure 3.4 illustrates the allowed values of and h_2 as a function of the ratio .

This model is a UV completion of the former toy model in the sense that it provides a gauge theory that underlies the model of the chiral fields. This completion has two sources of tuning, the first being the mass hierarchy that is necessary to enforce the decoupling of the W_2 messenger sector in (3.4). There is also tuning in the value of . According to Figure 3.4, values where $\ 1$[9] are

---

[8]The dependence on the cutoff in $m_Z^{(2)\,2}$ is introduced through the Z-self corrections, and vanishes in the limit that $_2 \to 0$.

[9]But not too small, as the two-loop effects in (3.6) would vanish completely as $\to 0$! Figure 3.4 shows that very small values of are disfavored.

preferred to maximize the available parameter space. For the lowest value depicted, $= 0{:}0001$, this corresponds to $h \;\; \frac{1}{100} h_2$, but there is still appreciable available parameter space for $h \;\; \frac{1}{10} h_2$ ($= 0{:}01$). We also know that this ratio is related to the scales in (3.13) and (3.15), $= h^2 \;\; \frac{0}{g}^{\;2}$. Indeed, dynamically it is more natural to have $\frac{1}{h} \frac{g}{0} = \frac{1}{p=} \;\; 1$ than the case preferred here, where $\frac{g}{0} \;\; h$ or $\;\; 1$. For example, if the quartic term in (3.13) arises from a massive field that is integrated out at $_0$, then $_0$ is roughly its mass and is generically larger than $_g$, the duality scale.

The tuning in can be accommodated by assuming that the h_2 sector is a generic strongly coupled sector. After integrating out the massive field associated to, $_0$ the RG flow reduces the effective $_0 = \;_g$ such that $\frac{g}{0(\;\; g)} \;\; h$ at the scale $_g$, where the flow changes. These ideas are illustrated in Figure 3.5.

3.4 Conclusions

We have shown that there exist R-symmetric O'Raifeartaigh-like models with fields having only R-charge 0 and 2 that spontaneously break their R-symmetry. The model we examined had two couplings in the superpotential that exhibited distinct behaviors under their renormalization group flow; in particular, one of the couplings (h_2 in (3.19)) had to be tuned to within 10% by the RG evolution to achieve a spontaneously broken R-symmetry. There are also two scales in the model that were arranged in a hierarchy, with tuning of order 10 20%. The R-symmetry is broken by the non-zero vev of a R-charged pseudomodulus in the model that has a potential dictated by one- and two-loop quantum corrections to its tree-level flat potential. The parameter space that allows a non-trivial minimum of the potential is substantial but prefers the tuning in the scales and couplings already mentioned.

Many extensions of our work are possible. One may look at a brane engineering of the UV model (or some generalizations), as done in [47, 48, 49] for the

ISS model. It would be interesting to check if the brane action can capture the physics of the non-supersymmetric state that we discovered in this field theory.

Because there is tuning in its marginal couplings, a better understanding of our UV completion is also necessary. A possible explanation of this tuning can come from the strong dynamics of the UV sector- for example, one can suppose that the UV dynamics are governed by an approximate CFT that generates a hierarchy amongst the couplings from their anomalous dimensions, as in [50].

One can ponder the possibility of a general result (like in [44]) for the sign of the two-loop masses, possibly associated to some (global) charge assignment. In the case of $R = 0$ and $R = 2$ there is no sign constraint on the mass at two loops, but extra conditions might provide such a constraint (at least at the origin).

We conclude by discussing the embedding of the model in a phenomenological scenario. One can imagine gauging some of the global symmetries and gauge mediating the supersymmetry breaking effects to a SSM sector. This requires the existence of an explicit R-symmetry breaking sector to prevent massless axions [51]. It would be important to generate the explicit R-symmetry breaking term in the UV theory and study the possible constraints of such a term on the other couplings.

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Figure 3.4: The parameter space that satis es the requirements (a) that the mass of the Z pseudomodulus is tachyonic at the origin and (b) that the slope of the far eld potential be positive. This space is parametrized by the ratio of scales $\frac{{}_2}{{}_1}$ and the m_3 sector coupling h_2 as a function of the ratio $= \frac{h^2}{h_2^2}$. Note that small values of are preferred, but not too small. The behavior of the allowed regions is smooth everywhere.

Figure 3.5: A complete arrangement of the scales introduced into (3.19) so as to accomplish spontaneous R-symmetry breaking. The ratio $_0(\ _g) = \ _g$ is arranged to be smaller than one via running in a strongly coupled sector between the scales $_0$ down to $_g$. The tuning in is actually quite mild- for appreciable parameter space that allows a spontaneously broken R-symmetry, can be as large as $\frac{1}{100}$ (corresponding to $h \ \frac{1}{10} h_2$ - see Figure 3.4).

Chapter 4

Model independent studies of Standard Model phenomena: B decays to two pseudoscalars and a generalized $\Delta I = \frac{1}{2}$ rule

We perform an isospin analysis of B decays to two pseudoscalars. The analysis extracts appropriate CKM and short distance loop factors to allow for comparison of non-perturbative QCD effects in the reduced matrix elements of the amplitudes. In decays where penguin diagrams compete with tree-level diagrams we find that the reduced matrix elements of the penguin diagrams, which are singlets or doublets under isospin, are significantly enhanced compared with the triplet and fourplet contributions of the weak Hamiltonian. This similarity to the $\Delta I = \frac{1}{2}$ rule in $K \to \pi\pi$ decays suggests that, more generally, processes mediated by Hamiltonians in lower-dimensional isospin representations see enhancement over higher-dimensional ones in QCD.

4.1 Introduction

One of the longstanding puzzles in avor physics is the $I = 1{=}2$ rule. An isospin-$\frac{1}{2}$ neutral kaon may decay into two pions in either an isospin-0 or isospin-2 (s-wave) state with amplitude A_0 or A_2, respectively. Empirically,

$$\frac{\mathrm{Re} A_0}{\mathrm{Re} A_2} = 22{:}5\, : \tag{4.1}$$

The $I = 1{=}2$ rule is the statement that the amplitude A_0, mediated by the part of the weak Hamiltonian that transforms as an $I = 1{=}2$ tensor, is much larger than A_2, mediated by the larger $I = 3{=}2$ tensor.

There is no satisfactory understanding of this rule. In Refs [52, 53, 54] and, more recently, Ref. [55] the rule was investigated in chiral perturbation theory, in the large N_c limit. However, it was argued in Ref. [56] that for QCD, $N_c = 3$ is not large enough for this limit to be useful. More recent studies using Monte Carlo simulations of QCD in the lattice have addressed the $I = 1{=}2$ rule [57]; a very recent study on the lattice of the validity of the vacuum insertion approximation was done in [58]. The ratio in (4.1) is still twice as large as any values obtained on the lattice with unphysical quark masses, but it is expected that simulations at physical quark masses will reproduce the empirically observed ratio and shed light on the origin of the enhancement [59]. This begs the question{ does this enhancement occur in systems other than the $K \,!$ system?

There is evidence that answers this question in the a rmative. Identifying any patterns of enhancements will give new insights into the long distance dynamics of QCD. For example, the $SU(3)$ analysis of $D \,!\, KK;$ decays reveals a similar enhancement. In that system, the $D^0 \,!\, K^+K$ and $D^0 \,!\, {}^+$ amplitudes may be written as [60]

$$A(D^0 \,!\, K^+K\ ) = (2T + E \quad S) + \tfrac{1}{2}(3T + 2G + F \quad E)$$

$$A(D^0 ! \quad ^+ \quad) = \quad (2T + E \quad S) + \tfrac{1}{2}(3T + 2G + F \quad E)$$

where $\tfrac{1}{2}(V_{cs}V_{us} \quad V_{cd}V_{ud})$ and $\tfrac{1}{2}(V_{cs}V_{us} + V_{cd}V_{ud})$. S, E and F are the invariant matrix elements between a D meson and a meson pair in an octet of the 6, 15 and 3 components of the weak Hamiltonian, respectively, G of the 3 to a singlet pair and T of the 15 to a meson pair in the 27. Note that $= \sin_C$, while $j \quad j \quad ^5$, so that $j \quad j = \quad 10^{\ 3}$. Neglecting one would have $(D^0 ! \quad K^+K \quad) = (\ D^0 ! \quad ^+ \quad)$ in the SU(3) limit. Experimentally $(D^0 ! \quad K^+K \quad) = (\ D^0 ! \quad ^+ \quad) \quad 3$ requires both the and terms in the amplitude to contribute with similar strengths. Barring accidental cancellations this means that the matrix elements G and F are signi cantly enhanced. Since has a large phase, signi cant CP-violation in these decays was predicted [61] and recently con rmed by experiment [62, 63, 64].

If SU(3)-breaking e ects are included, the ratio $(D^0 ! \quad K^+K \quad) = (\ D^0 ! \quad ^+ \quad) \quad 3$ can be attained with only a \mild" enhancement of F and G relative to the other reduced matrix elements of about an order of magnitude [65, 66, 67, 68, 69, 70]. The enhancement in F and G is similar to that of the $I = 1{=}2$ rule in that it appears in matrix elements of the smallest SU(3)-representation of the Hamiltonian. In this case, the dominant contributions are from the 3 Hamiltonian (as opposed to the 6 and 15), whereas for the $I = 1{=}2$ rule the dominant piece is from the $I = 1{=}2$ Hamiltonian (as opposed to the $I = 3{=}2$ piece).

In this work we investigate the possibility of similar enhancements in B decays. We will show that an isospin analysis of $B ! \quad K$ decays and CP-asymmetries shows a marked enhancement of amplitudes mediated by the weak Hamiltonian in the lowest isospin representation. An analysis of $B !$ decays shows that, although there is little enhancement of doublet versus fourplet amplitudes, the matrix elements of penguin contributions (which are purely $I = 1{=}2$) are still enhanced to produce the observed data. Both these analyses support the general rule that amplitudes mediated by the piece of the weak Hamiltonian in the smallest representation of the symmetry group are enhanced.

It should go without saying that we have no dynamical explanation of the enhancement. This comes as no surprise, since the very $\Delta I = 1/2$ rule has resisted explanation for more than a half century. But we hope that insights provided by this new, generalized rule may eventually lead to a global understanding of these enhancements.

4.2 Isospin analysis

The strong interactions, to a good approximation, obey isospin symmetry. In hadronic spectra and decays isospin violating effects are no larger than a few per cent. We study the amplitudes for the decay of B-mesons to two light scalar mesons using isospin symmetry, under which kaons and B-mesons transform as doublets and pions as a triplet. The possible two-body final states are easily classified according to their transformation properties under isospin. We also need the transformation properties of the effective Hamiltonian responsible for the weak decay. The effective Hamiltonian is given in terms of four-quark operators, whose transformation properties are readily determined.

4.2.1 $B \to \pi K$

The effective Hamiltonian density for the $\Delta B = -1$, $\Delta S = -1$ decays, to leading order in the Fermi constant G_F, can be written as [71, 72]

$$H = \frac{G_F}{\sqrt{2}} \left[\lambda_u (C_1 Q_1 + C_2 Q_2) - \lambda_t \sum_{i=3}^{6} C_i Q_i \right] . \qquad (4.2)$$

Here $\lambda_q \equiv V_{qb} V^*_{qs}$ are CKM factors and C_i's are the Wilson coefficients. The "tree" ($Q_{1;2}$) and "penguin" (Q_{3-6}) operators are defined as

$$Q_1 = \left(\bar{b}_a u_b\right)_{V-A} \left(\bar{u}_b s_a\right)_{V-A} ;$$

$$
\begin{aligned}
Q_2 &= (\bar b u)_{V-A}(\bar u s)_{V-A} ;\\
Q_3 &= (\bar b s)_{V-A} \sum_{q=u,d} (\bar q q)_{V-A} ;\\
Q_4 &= (\bar b_a s_b)_{V-A} \sum_{q=u,d} (\bar q_b q_a)_{V-A} ;\\
Q_5 &= (\bar b s)_{V-A} \sum_{q=u,d} (\bar q q)_{V+A} ;\\
Q_6 &= (\bar b_a s_b)_{V-A} \sum_{q=u,d} (\bar q_b q_a)_{V+A}
\end{aligned}
\qquad (4.3)
$$

where $(\bar q q)_{V\pm A}$ is shorthand for $\bar q \gamma_\mu (1 \pm \gamma_5) q$. Both the coefficients C_i and the matrix elements of the operators Q_i depend on an arbitrary renormalization point μ but their combination in the Hamiltonian, Eq. (4.2), is μ-independent. QCD-penguins arising from u and c quark loops combine into terms precisely of the form of top-quark penguins, since $\lambda_c + \lambda_u = -\lambda_t$. We have also neglected electroweak penguins (EWP), operators Q_{7-10} in Ref. [71]. These introduce new isospin triplets into the Hamiltonian with a λ_t coefficient, suppressed relative to the top-penguins by α/α_s. We have ignored EWP contributions out of pragmatism: were we to include their effects in our fits the number of unknown matrix elements would exceed the number of measured data. But our pragmatism is informed: the coefficients of EWP in the effective Hamiltonian are suppressed relative to QCD penguins roughly by a factor of α/α_s, or about 7% if evaluated at $\mu = M_Z$ and smaller at m_b. As will become evident, the approximation is supported by the very good fit of the model to both $B \to K\pi$ and $B \to \pi\pi$ processes.

As far as the group theory analysis of rates and CP asymmetries is concerned, different four-quark operators contributing to the Hamiltonian can be distinguished solely by their isospin quantum numbers and CKM factors. The Hamiltonian can therefore be compactly written in terms of the isospin representations in the following way:

$$
H = V_{ub}V_{us}^* \left(\mathbf{1} + [3]^1_1 \right) + \frac{\alpha_s}{8\pi} V_{tb}V_{ts}^* \, \mathbf{1}^0 ; \qquad (4.4)
$$

Table 4.1: Data available in $B \to K\pi$ decays [1]. The C and S parameters are measured for decays into the final CP eigenstate $B_d^0 \to K_S^0 \pi^0$. The amplitude for $B_d^0 \to K^0\pi^0$ on the other hand is given as $A(B_d^0 \to K^0\pi^0) = \sqrt{2}\, A(B_d^0 \to K_S^0\pi^0)$:

Mode	$\mathcal{B}\ (10^{-6})$	A_{CP}	C_f	S_f
$B^+ \to K^+\pi^0$	12.9 ± 0.5	0.037 ± 0.021	–	–
$B^+ \to K^0\pi^+$	23.8 ± 0.7	0.014 ± 0.019	–	–
$B_d^0 \to K^0\pi^0$	9.9 ± 0.5	–	0.00 ± 0.13	0.58 ± 0.17
$B_d^0 \to K^+\pi^-$	19.6 ± 0.5	0.087 ± 0.008	–	–

where $\mathbf{1}$ ($\mathbf{1}'$) denotes the singlet coming from the tree (penguin) operators [3]; $\mathbf{3}_1$ represents the triplet operator, and α_s the strong coupling constant evaluated at M_Z. We choose to normalize the singlet penguin operator with an agnostic factor of $\alpha_s/(8\pi)$ to make explicit the loop factor associated with it. This normalization does not affect the results of this paper, but it is a useful choice that, naïvely, would give reduced matrix element values of the same order of magnitude for every contribution. We introduce shorthand for the reduced matrix elements, as follows:

$$\langle 2|\mathbf{1}|B\rangle \equiv P_b; \quad \langle 2|\mathbf{1}'|B\rangle \equiv P_a;$$
$$\langle 2|\mathbf{3}|B\rangle \equiv T; \quad \langle 4|\mathbf{3}|B\rangle \equiv S\,. \tag{4.5}$$

While we cannot compute P_a, P_b, S and T from first principles, we can determine them by fitting to experimental measurements of decay rates and CP asymmetries.

In terms of the reduced matrix elements in Eq(4.5), the isospin decomposition of the decay amplitudes is

$$A(B^+ \to K^+\pi^0) = V_{ub}V_{us}^*\,\frac{1}{\sqrt{2}}(P_b + T + 2S) + \frac{\alpha_s}{8\pi}V_{tb}V_{ts}^*\,\frac{P_a}{\sqrt{2}}\,;$$
$$A(B^+ \to K^0\pi^+) = V_{ub}V_{us}^*\,(P_b + T - S) + \frac{\alpha_s}{8\pi}V_{tb}V_{ts}^*\,P_a\,;$$
$$A(B^0 \to K^0\pi^0) = V_{ub}V_{us}^*\,\frac{1}{\sqrt{2}}(-P_b + T + 2S) - \frac{\alpha_s}{8\pi}V_{tb}V_{ts}^*\,\frac{P_a}{\sqrt{2}};$$

$$A(B^0 \to K^+\pi^-) = V_{ub}V_{us}^*(P_b - T + S) + \frac{\alpha_s}{8\pi} V_{tb}V_{ts}^* P_a : \qquad (4.6)$$

There is a contribution proportional to $V_{ub}V_{us}^*$ to the amplitude $A(B^+ \to K^0\pi^+)$. The only contribution to this process stems from the annihilation diagram, shown in Fig. 4.1. There is extensive literature on annihilation diagram suppression with respect to W-emission diagrams [73, 74]. To evaluate this expectation, denote the matrix element associated with the annihilation diagram by $M \equiv P_b + T - S$ and let $|M| = x|P_a|$ so that x measures the relative importance of annihilation in comparison to the top-loop penguin. The value of x for which the annihilation and penguin contributions to $B^+ \to K^0\pi^+$ are of the same order can be estimated as

$$x = \frac{\alpha_s}{8\pi}\left|\frac{V_{tb}V_{ts}}{V_{ub}V_{us}}\right| \simeq 0.24 : \qquad (4.7)$$

Figure 4.1: Leading order diagram contributing to the $B^+ \to K^0\pi^+$ process.

Results of the fit

The available decay data for $B \to K\pi$ are collected in Table 4.1; the observables are defined in Appendix B. Performing a χ^2 fit of matrix elements in Eq. (4.6) to the data, we find values for the matrix elements that match the observed data with a 95% confidence level. These minima are illustrated with 68% and 95% confidence levels in the $|P_a|$ vs. $|P_b|$ and $|T|$ vs. $|S|$ planes, respectively, in Fig. 4.2. The best fit has $\{|P_a|; |P_b|; |T|; |S|\} \simeq \{0.237; 7.2 \times 10^{-3}; 8.4 \times 10^{-3}; 2.2 \times 10^{-3}\}$ MeV with a chi-squared of $\chi^2 = 1.70$ for two degrees of freedom (a common phase in the reduced matrix elements is unobservable). The $\Delta I = 0$ contribution to the amplitudes, from the Hamiltonian in the singlet representation, is given by the

(a) (b)

Figure 4.2: Fit to data of the reduced matrix elements for B ! K . The gures show the 68% (green) and 95% (yellow) CL regions in the $|P_a|$ vs $|P_b|$ and $|T|$ vs $|S|$ planes. These two pairs of variables are what dictate the enhancements in Eqs. (4.10) and (4.11). The corresponding minima are labeled in each plot. The raggedness of the contours is an artifact of the numerical computation.

quantity

$$a_{I=0} = P_b + \frac{s}{8}\,\frac{V_{tb}V_{ts}}{V_{ub}V_{us}}P_a \qquad (4.8)$$

and the I = 1 contribution, from the triplet Hamiltonian, by

$$a_{I=1} = \{T + 2S;\ T \quad S\}. \qquad (4.9)$$

for (B^+ ! K^+ ^{0}; B^0 ! K^0 $^+$) and (B^+ ! K^0 $^+$; B^0 ! K^+) respectively. For the best t then, we nd

$$\frac{a_{I=0}}{a_{I=1}} = \{4.8;\ 9.9\} \qquad (4.10)$$

which is reminiscent of the I = $\frac{1}{2}$ rule from K ! decays.

A second, slightly higher ^{2}-minimum has $\{|P_a|; |P_b|; |T|; |S|\} \simeq \{$ 0.075;

0:052; 7:3 10 ^{3}; 2:4 10 ^{3}g MeV with a chi-squared of ^{2} = 1:80 and

$$\frac{a_{\ I=0}}{a_{\ I=1}} = f\,5{:}2;\ 12{:}6g : \qquad (4.11)$$

Both of these minima have signi cant enhancement of the penguin singlet, P_a, over the triplet matrix elements, T and S. In the best t case, however, the other singlet matrix element, P_b, does not show signi cant enhancement over the triplet matrix elements. Consequently, the annihilation diagram contribution is negligible in the best t (jM j = 0:013 MeV or, equivalently, x = jM=P_aj = 0:055, to be compared with Eq. (4.7)) but provides a larger contribution than that of the penguin diagram in the second best t (where jM j = 0:055 MeV or, equivalently, x = 0:732).

For completeness we note that there are two additional minima corresponding to ^{2} = 3:04 and 4.34. These two minima are less favorable, so we ignore them in the rest of our study.

In all but the least favored minimum, there is signi cant enhancement of jP_aj over the triplet Hamiltonian matrix elements. Moreover, the total contribution from the I = 0 Hamiltonian, $a_{\ I=0}$, enjoys an enhancement over the I = 1 contribution, $a_{\ I=1}$. More precise data will be welcomed to distinguish between these minima, which would also decide the role of the annihilation diagram in these decays.

4.2.2 B !

The isospin analysis for nal states is analogous to that for K decays, where the I = $\frac{1}{2}$ rule was discovered. Operator contributions are of the form in (4.3), but for S = 0 processes. The Hamiltonian decomposes under isospin as 2 2 2 = 2 + 2 + 4 so that

$$H = V_{ub}V_{ud} \quad [2]^{2} + [4]^{12}_{1} + \frac{s}{8} V_{tb}V_{ts}\ [2^{0}]^{2} ; \qquad (4.12)$$

(a) (b)

Figure 4.3: Fit to data of the reduced matrix elements for $B \to$. The gures show the 68% (green) and 95% (yellow) CL regions in the $|Q_a|$ vs $|Q_b|$ and $|Q_a|$ vs $|U|$ planes. The raggedness of the contours is an artifact of the numerical computation.

The nal states transform as $(3 \quad 3)_S = 1 + 5$, so the non-vanishing reduced matrix elements are

$$\langle 1|2^0|B\rangle = Q_a\,;\ \langle 1|2|B\rangle = Q_b\,;\ \langle 5|4|B\rangle = U \qquad (4.13)$$

and the decay amplitudes relevant to the processes in Table 4.2 are

$$\begin{aligned} A(B^+ \to \ ^+\ ^0) &= \sqrt{\frac{3}{2}}\, V_{ub} V_{ud}\ U\,; \\ A(B^0 \to \ ^0\ ^0) &= V_{ub} V_{ud} \left(\frac{1}{\sqrt{3}}\ Q_b \quad \sqrt{2}\, U\right) + \frac{s}{8} V_{tb} V_{td} \ \frac{1}{\sqrt{3}} Q_a\,; \\ A(B^0 \to \ ^+\ \ ) &= V_{ub} V_{ud} \left(\frac{1}{\sqrt{3}} \quad \sqrt{2}\, Q_b + U\right) + \frac{s}{8} V_{tb} V_{td} \quad \sqrt{\frac{2}{3}}\, Q_a\,: \end{aligned} \qquad (4.14)$$

Results of the t

The data available in this decay channel are listed in Table 4.2. We perform a ^{2}- t of the model, Eq. (4.14), to the data. The result of the t is illustrated

Table 4.2: Data available in $B \to \pi\pi$ decays from Ref [1].

Mode	$\mathcal{B}$ (10^{-6})	A_{CP}	C_f	S_f
$B^+ \to \pi^+\pi^0$	5.5 ± 0.4	0.03 ± 0.04	–	–
$B^0 \to \pi^0\pi^0$	1.91 ± 0.22	–	0.43 ± 0.24	–
$B^0 \to \pi^+\pi^-$	5.12 ± 0.19	–	0.38 ± 0.15	0.65 ± 0.07

with 68% (green) and 95% (yellow) CL regions in the $|Q_a|$ vs $|Q_b|$ and $|Q_a|$ vs $|U|$ planes, respectively, in Fig. 4.3. For the best fit to the data we obtain $\{|Q_a|, |Q_b|, |U|\} \simeq \{0.35, 8.8 \times 10^{-3}, 5.8 \times 10^{-3}\}$ MeV with a chi-squared of $\chi^2 \simeq 1.39$ for 2 degrees of freedom. Two additional regions with a good fit to the data are found, one with $\{|Q_a|, |Q_b|, |U|\} \simeq \{0.82, 3.9 \times 10^{-3}, 5.8 \times 10^{-3}\}$ MeV for a chi-squared of $\chi^2 \simeq 2.07$ and the other with $\{|Q_a|, |Q_b|, |U|\} \simeq \{0.82, 7.7 \times 10^{-3}, 5.8 \times 10^{-3}\}$ MeV for a chi-squared of $\chi^2 \simeq 3.38$. Since the last minimum is less favorable, we will ignore it. The contribution to the amplitudes from the Hamiltonian in the doublet representation is

$$a_{\Delta I=1/2} = Q_b + \frac{\alpha_s}{8\pi}\frac{V_{tb}V_{td}}{V_{ub}V_{ud}} Q_a \tag{4.15}$$

and from the fourplet Hamiltonian

$$a_{\Delta I=3/2} = U. \tag{4.16}$$

We find no enhancement of the $\Delta I = 1/2$ amplitude with respect to the $\Delta I = 3/2$ amplitude. To wit, for the best fits (next favorable minimum) we find

$$\frac{a_{\Delta I=1/2}}{a_{\Delta I=3/2}} = 1.04\,(1.05). \tag{4.17}$$

There is little enhancement of the reduced matrix element corresponding to the tree-level doublet Hamiltonian, Q_b, with respect to the tree-level quadruplet U. However, the large enhancement of the penguin doublet reduced matrix element

Q_a over U is analogous to that in the $K \to$ decays, which has identical isospin analysis to the $B \to$ case. That a similar enhancement exists in the B system |both in K and nal states| is striking, and cries out for a dynamical explanation of the role of avor symmetries in these enhancements.

4.3 Short distance QCD e ects

How much of the enhancement in the lower dimensional isospin representation matrix elements can be attributed to computable short distance QCD e ects? Comparing the e ective Hamiltonian in Eq. (4.2) against the decay amplitudes in Eq. (4.6), we see that

$$\frac{s}{8} P_a = \langle K \, | \sum_{i=3}^{6} C_i(m_b) Q_i | B \rangle = |C_6(m_b)| \langle 2 | 1^0 | 2 \rangle : \qquad (4.18)$$

Our analysis cannot yield information about the matrix elements of each of the operators $Q_{3;:::;6}$. The last step in (4.18) de nes the matrix element of the sum of the operators, $\langle 2|1^0|2\rangle$, after extracting the magnitude of the largest Wilson coe cient, $|C_6|$.

Similarly we can de ne

$$\begin{aligned} P_b &= \langle K \, | \sum_{i=1,2} C_i(m_b) Q_i | B \rangle = C \ (m_b) \langle 2|1|2 \rangle ; \\ T &= \langle K \, | \sum_{i=1,2} C_i(m_b) Q \ | B \rangle = C \ (m_b) \langle 2|3|2 \rangle ; \qquad (4.19) \\ S &= \langle K \, | \sum_{i=1,2} C_i(m_b) Q \ | B \rangle = C \ (m_b) \langle 4|3|2 \rangle ; \end{aligned}$$

where $C = C_1 \ \ C_2$ and $Q = Q_1 \ \ Q_2$. The Q operators do not have de nite isospin. However, for the $B \to$ case the corresponding operator Q is pure $I = 1{=}2$, so using the Q basis is natural. Moreover, at 1-loop the operators Q do not mix among themselves. Hence, to estimate the matrix elements of the \tree" operators we have extracted the coefficient C. In any case, since C are of order 1, this introduces little bias in our analysis.

For our analysis we take the numerical value of Wilson coefficients at NLO in the NDR scheme for $\Lambda^{(5)}_{\overline{MS}} = 225$ MeV from table 8 of [71]. We find that, for matrix elements from our best fit,

$$\begin{aligned} |\langle 2|1'|2\rangle| &\approx 0.028 \text{ MeV}, & |\langle 2|3|2\rangle| &\approx 0.007 \text{ MeV}, \\ |\langle 2|1|2\rangle| &\approx 0.006 \text{ MeV}, & |\langle 4|3|2\rangle| &\approx 0.002 \text{ MeV}. \end{aligned} \tag{4.20}$$

while for the secondary χ^2 minimum

$$\begin{aligned} |\langle 2|1'|2\rangle| &\approx 0.009 \text{ MeV}, & |\langle 2|3|2\rangle| &\approx 0.006 \text{ MeV}, \\ |\langle 2|1|2\rangle| &\approx 0.041 \text{ MeV}, & |\langle 4|3|2\rangle| &\approx 0.002 \text{ MeV}. \end{aligned} \tag{4.21}$$

The $\Delta I = 0$ enhancement for both of these sets of matrix elements, Eqs. (4.10) and (4.11), corresponds to an enhancement of one or the other singlet matrix element relative to the largest triplet by a factor of between 4 and 7.

An analogous analysis can be performed for $B \to$ decays. We define

$$\begin{aligned} \frac{\alpha_s}{8\pi} Q_a &= \langle \ | \sum_{i=3}^{6} C_i(m_b) Q_i |B\rangle = |C_6(m_b)| \langle 1|2'|2\rangle . \\ Q_b &= \langle \ | \sum_{i=1,2} C_i(m_b) Q_i |B\rangle = C(m_b) \langle 1|2|2\rangle , \\ U &= \langle \ | C_+(m_b) Q_+ |B\rangle = C_+(m_b) \langle 5|4|2\rangle , \end{aligned} \tag{4.22}$$

The matrix element of the operator Q_+ can be determined because it is the only "tree" contribution to a $\Delta I = 3/2$ transition. We find that, for matrix elements from our best fit,

$$\begin{aligned} |\langle 1|2'|2\rangle| &\approx 0.040 \text{ MeV}, \quad |\langle 1|2|2\rangle| \approx 0.007 \text{ MeV}, \\ |\langle 5|4|2\rangle| &\approx 0.006 \text{ MeV}, \end{aligned} \tag{4.23}$$

while for the secondary χ^2 minimum

$$|\langle 1|2^0|2\rangle| \approx 0.094\ \text{MeV}, \quad |\langle 1|2|2\rangle| \approx 0.003\ \text{MeV}, \quad |\langle 5|4|2\rangle| \approx 0.006\ \text{MeV}. \tag{4.24}$$

4.4 Discussion and Conclusions

There is a striking consistency in the reduced matrix element enhancement that persists in the B decay channels studied. As suggested at the end of Section 4.2.2, this may be indicative of the importance of flavor symmetries in non-perturbative regimes in QCD, or perhaps in new physics contributions (note we have only assumed the quark model, CKM parametrization, *etc.* of the Standard Model). The enhancement of matrix elements with effective Hamiltonians in lower-dimensional isospin representations is only present when penguin diagrams can compete against tree level weak exchanges, which are also the processes where CP violation is predicted at lowest order. These are the $B \to K\pi$ and $B \to \pi\pi$ channels in this work.

In our estimates for hadronic matrix elements in Eqs. (4.20), (4.23) and (4.24), but not (4.21), it is the penguin contributions to the lowest isospin change operator ($\Delta I = 0$ for $B \to K\pi$ and $\Delta I = 1/2$ for $B \to \pi\pi$), rather than both penguin and tree contributions, that are enhanced. While we cannot select among the fits *a priori*, in the best fits for both $B \to K\pi$ and $B \to \pi\pi$ the penguin dominates the total enhancement, giving a factor of between 4 and 7. The precise value of the enhancement is immaterial: we have made plausible assumptions to remove the short distance QCD effects, but we don't have the means to do this precisely and unambiguously. Moreover, the matrix elements $E_a, \ldots, U$ are defined with convenient factors of $\sqrt{2}$ and $\sqrt{3}$ which further adds to the ambiguity. But the enhancement of *amplitudes*, Eqs. (4.10) (or (4.11)), is unambiguous. Comparable enhancements in the penguin matrix elements for $B \to K\pi$ and $B \to \pi\pi$ lead to a significant amplitude enhancement in $B \to K$ but very little enhancement in $B \to$, but only because the latter is CKM-suppressed relative to the former.

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Chapter 5

Studies of degrees of freedom in quantum field theories I: Consequences of Weyl consistency conditions

The running of quantum field theories can be studied in detail with the use of a local renormalization group equation. The usual beta-function effects are easy to include, but by introducing spacetime-dependence of the various parameters of the theory one can efficiently incorporate renormalization effects of composite operators as well. An illustration of the power of these methods was presented by Osborn in the early 90s, who used consistency conditions following from the Abelian nature of the Weyl group to rederive Zamolodchikov's c-theorem in $d = 2$ spacetime dimensions, and also to obtain a perturbative a-theorem in $d = 4$. In this work we present an extension of Osborn's work to $d = 6$ and to general even d. We compute the full set of Weyl consistency conditions, and we discover among them a candidate for an a-theorem in $d = 6$, similar to the $d = 2, 4$ cases studied by Osborn. Additionally, we show that in any even spacetime dimension one finds a consistency condition that may serve as a generalization of the c-theorem, and that the associated candidate c-function involves the coefficient of the Euler term in the trace anomaly. Such a generalization hinges on proving the positivity of a certain "metric" in the space of couplings.

5.1 Introduction

When a symmetry of a quantum field theory (QFT) is broken by quantum corrections, then the corresponding anomaly can be reproduced by a contribution to the generating functional of the theory [75, 76]. The algebra of the symmetry that is violated constrains the symmetry-breaking parameters that appear in these anomalous contributions, which are thus forced to satisfy the so-called Wess–Zumino consistency conditions [75].

The study of the Wess–Zumino consistency conditions for the Weyl anomaly was undertaken by Osborn in the early 90s and produced remarkable results [77]. In $d = 2$ spacetime dimensions, for example, Osborn obtained an independent proof of Zamolodchikov's c-theorem [11]. Furthermore, an extension of the c-theorem to 4d, commonly referred to as the a-theorem, was demonstrated perturbatively [15, 77], establishing in perturbation theory the intuition that the number of massless degrees of freedom of a QFT decreases under renormalization-group (RG) flow.

This perturbative 4d result was based on previous work by Jack and Osborn [15], who computed the local RG equation for a general renormalizable QFT in a curved background using dimensional regularization. To account for effects of renormalization of composite operators, Jack and Osborn used spacetime-dependent coupling constants, a trick that allows for straightforward computations of Green functions of composite operators (at least of those that appear in the Lagrangian) and the stress-energy tensor. Their candidate c-function agrees with Cardy's suggestion [14]: it is equal to the coefficient a of the Euler term in the trace anomaly at fixed points of the RG-flows. Nevertheless, it differs from a if the corresponding flat-space theory is not a conformal field theory (CFT). In subsequent work Osborn reproduced the main results of [15] by requiring that two successive Weyl variations of the effective action commute (since the Weyl group is Abelian), and also showed that the main results of the analysis are scheme-independent [77].

Invariance under the Weyl group in the flat background limit is *a priori* a stronger requirement than that of scale invariance of a theory on a flat background. The former imposes the vanishing of the trace of the stress-energy tensor, while the latter requires that the trace be the divergence of a suitable vector operator [78]. This suggests that the study of Weyl deformations may elucidate the relation between scale and conformally invariant theories in flat backgrounds. Indeed, these methods have recently been used to show, in perturbation theory, that a unitary 4d QFT invariant under the Poincaré group extended by the generator of scale transformations is automatically invariant under the four generators of special conformal transformations [79], even though the parameters of the theory may display cyclic behavior [80].[1]

In this paper we undertake an investigation of the response of QFTs to Weyl transformations in $d = 6$.[2] In particular, we determine the Weyl consistency conditions and general RG properties of six-dimensional QFTs. It is evident that this line of inquiry is interesting if it leads to results similar to those already obtained in $d = 4$, say, a perturbative extension of the 2d c-theorem and a proof that scale implies conformal invariance in 6d. But the investigation is also interesting in light of the advent of strongly coupled conformal CFTs that lack a Lagrangian description in $d = 6$, like the famous $(2,0)$ theory.[3] This suggests the existence of flows, i.e. families of non-conformal QFTs, between such theories, with an associated flow of a presumed c-function. The class of perturbative, renormalizable, $d = 6$

[1] The same result was also obtained using different methods in [81]. In renormalizable theories with $\mathcal{N} = 1$ supersymmetry it was shown perturbatively that cyclic behavior of parameters does not arise [82, 83]. The situation in $d = 4$ needs further investigation [84, 85, 86].

[2] Weyl consistency conditions in $d = 3$ were studied recently in [87].

[3] The consistency conditions we derive can be seen as relations among correlation functions involving composite operators.

models is restricted to scalar fields with cubic interactions for which the general analysis is of limited interest. They could, however, be put to use as a check of results using standard methods of perturbation theory.

As we will see, the consistency conditions bring us very close to an extension of the c-theorem to $d = 6$. More specifically, one can define a quantity $\tilde{a}$, which is a function of the dimensionless coupling constants g^i, that satisfies the equation

$$\frac{d\tilde{a}}{dt} = \frac{1}{6} H_{ij}\beta^i\beta^j; \qquad (5.1)$$

where the RG time is $t = -\ln(\mu/\mu_0)$, taken here to increase as we flow to the IR, and $\beta^i = -dg^i/dt$, as usual. The quantity $\tilde{a}$ agrees with the coefficient of the Euler term in the 6d trace anomaly at fixed points. The symmetric tensor H_{ij} can be viewed as a metric in the space of couplings. A proof of the a-theorem would be immediate if H_{ij} were shown to be positive-definite. This is analogous to the situation in $d = 4$ where perturbative positivity of the analogous "metric" has been shown by explicit computation in a generic QFT—it is here and only here that perturbation theory is used in proving the a-theorem and that scale implies conformal invariance in $d = 4$.

The analysis of the Weyl consistency conditions in $d = 6$ is significantly more complicated than in $d = 4$. This analysis reveals generic features that were not apparent in Osborn's treatment, and actually allows us to demonstrate the validity of (5.1) for QFTs in any even-dimensional spacetime. The only ingredient missing for a generalization of Zamolodchikov's c-theorem to any even dimension is a demonstration that the "metric" H_{ij} is positive-definite. As already mentioned, H_{ij} is positive-definite in $d = 4$ at lowest order in perturbation theory. It would be interesting to extend this result to higher even dimensions. Of course, a non-perturbative proof of the positivity of H_{ij} in even $d \geq 4$ is the ultimate goal of this line of research.

To address questions similar to those that motivated this work, Komargodski and Schwimmer have put forward an argument that gives a non-perturbative physicist's proof of the weak version[4] of the a-theorem in $d = 4$ [16]. More specifically, they provided a compelling argument that in the flow from a UV CFT to an IR CFT the inequality $a_{UV} > a_{IR}$ is satisfied, without however providing a monotonically-decreasing c-function. Attempts to extend that line of reasoning to the $d = 6$ case have been unsuccessful, although in explicit examples the validity of $a_{UV} > a_{IR}$ was demonstrated [17]. The method we use in this work is very different from that used in [16, 17], and allows us to obtain results local in the RG scale.

The organization of the paper is as follows. In the next section we summarize the results of Osborn in 2d and 4d. We continue in Section 5.3 by describing the 6d case in detail, and we then illustrate in Section 5.4 the ingredients that allow us to genaralize our analysis regarding the a-theorem to all even spacetime dimensions. In Appendices C and D we present details regarding our conventions as well as the terms that participate in the consistency conditions in 6d. A Mathematica file that contains all the consistency conditions in 6d is included with our submission.

5.2 Summary of the 2d and 4d cases

To begin, let us introduce the basic setting. More details can be found in [77, 89]. We are working in Euclidean space and we define the generating functional W of connected Green functions via

$$e^{W} = \int [d\phi]\, e^{-S};$$

where S is the Euclidean action with all required counterterms. S contains a potential of the form $g^i \mathcal{O}_i$, where g^i are parameters which can be taken to be dimensionless, and $\mathcal{O}_i$ are scaling-dimension-d operators where d is the spacetime

[4]For a discussion of the various versions of the a-theorem see [88].

dimension.[5] W is a function of the renormalized couplings g^i and the metric $\gamma_{\mu\nu}$.

Consider now the RG flow as a flow in the space of theories as parametrized by their couplings g^i. The arbitrary RG parameter μ has to be introduced, and the flow is then generated by

$$D = \mu\frac{\partial}{\partial\mu} + \beta^i\frac{\partial}{\partial g^i},$$

where $\beta^i = \mu\, dg^i/d\mu$ is the beta function. W is a finite scalar function, since it is derived from S which includes all necessary counterterms, and it is thus invariant under the RG flow:

$$DW = 0. \tag{5.2}$$

This is simply the Callan–Symanzik equation.

To define a local RG equation we let the parameters of S as well as the spacetime metric be arbitrary functions of spacetime. New counterterms involving derivatives on the metric and the couplings are then necessary for finiteness. With their inclusion in S functional differentiations of W are guaranteed to produce finite operator-insertions in Green functions. Local rescalings of length are described by

$$\gamma_{\mu\nu}(x) \to e^{2\sigma(x)}\gamma_{\mu\nu}(x)$$

and they form the Weyl group.

We define the quantum stress-energy tensor and finite composite operators using

$$T_{\mu\nu}(x) = 2\frac{\delta S}{\delta\gamma^{\mu\nu}(x)}; \qquad [O_i(x)] = \frac{\delta S}{\delta g^i(x)};$$

where functional derivatives are defined in d spacetime dimensions by

$$\frac{\delta}{\delta\gamma^{\mu\nu}(x)}\gamma_{\kappa\lambda}(y) = \tfrac{1}{2}\delta_{(\kappa}^{\mu}\delta_{\lambda)}^{\nu}\,\delta^d(x;y); \qquad \frac{\delta}{\delta g^i(x)}g^j(y) = \delta_i^j\,\delta^d(x;y);$$

with $X_{(i}Y_{j)} \equiv X_iY_j + X_jY_i$, $\delta^d(x;y) = \delta^d(x-y)/\sqrt{\gamma(x)}$, γ is the determinant of

[5] Non-marginal operators can also be included, see [77].

the metric, and $\delta^d(x)$ the usual delta function in d dimensions. At the level of the generating functional we implement infinitesimal local Weyl transformations with the generators

$$\Delta^W_\sigma = 2\int d^dx \sqrt{g}\, \sigma\, g^{\mu\nu} \frac{\delta}{\delta g^{\mu\nu}}\,; \qquad \Delta_\beta = \int d^dx \sqrt{g}\, \beta^i \frac{\delta}{\delta g^i}\,.$$

With these definitions it is obvious that

$$\Delta^W_\sigma W = \int d^dx \sqrt{g}\, \sigma \langle T^\mu{}_\mu \rangle\,; \qquad \Delta_{\sigma\beta} W = \int d^dx \sqrt{g}\, \sigma \beta^i \langle [O_i] \rangle\,.$$

It is known that the Weyl variation of W, $\Delta^W_\sigma W$, is anomalous in curved space [12], even when the flat-space theory is a CFT.

In general, one can write

$$\Delta^W_\sigma W = \Delta_{\sigma\beta} W + \int d^dx \sqrt{g}\, (\text{terms with derivatives on } \sigma, g^i, g_{\mu\nu})\,. \tag{5.3}$$

For a classically scale invariant theory we also have

$$\left(\mu \frac{\partial}{\partial \mu} + 2\int d^dx \sqrt{g}\, g^{\mu\nu} \frac{\delta}{\delta g^{\mu\nu}} - \Delta_\beta\right) W = 0\,;$$

and so if the integral in (5.3) is neglected, then (5.3) reduces to the Callan–Symanzik equation (5.2). Therefore, (5.3) serves as a local version of the Callan–Symanzik equation. It is straightforward to see that (5.3) is equivalent to

$$T^\mu{}_\mu = \beta^i [O_i] + (\text{curvature}, \partial g)\text{-terms.} \tag{5.4}$$

This is the most general form of the trace anomaly.[6] Consistency conditions follow from requiring that $[\Delta^W_\sigma - \Delta_{\sigma\beta}, \Delta^W_{\sigma'} - \Delta_{\sigma'\beta}]W = 0$, as imposed by the fact that the

[6] Actually there is a more general form that includes renormalization effects of a specific vector operator of classical scaling dimension $d-1$. In $d = 4$ such operators were considered by Osborn [77], and were also found in dimensional regularization in [15]. For the significance of such contributions the reader is referred to [79].

Weyl group is Abelian.

5.2.1 The 2d case

In two dimensions, the number of terms that are diﬀeomorphism and scale invariant that can contribute to the trace anomaly is small, and the elegance of the c-theorem is manifest. There is one curvature term and two terms with derivatives on spacetime-dependent couplings one can write down. The trace anomaly is reproduced by

$$^{W}W = \quad W \int d^2x\sqrt{g}\,(\tfrac{1}{2}\quad R \quad \tfrac{1}{2}\ {}_{ij}\,\partial g^i \partial g^j) + \int d^2x\sqrt{g}\,\partial \quad w_i \partial g^i; \qquad (5.5)$$

where R is the Ricci scalar and $\ ,\ {}_{ij}$, and w_i are functions of the couplings. From $[\ ^{W}\quad ;\ ^{W}_{0}\quad {}_{0}]W = 0$ with (5.5) we obtain a single consistency condition, namely

$$\partial \quad + w_i\, \partial \ ^i + \ ^i \partial_i w_j\, \partial g^j \quad {}_{ij}\ ^i\, \partial g^j = 0;$$

where $\partial_i \quad \partial = \partial g^i$. Since this has to be true for arbitrary ∂g^i, we conclude that

$$\partial_i \tilde{\ } = \ {}_{ij}\ ^j + \partial_{[i} w_{j]}\ ^j; \qquad \tilde{\ } = \quad + w_i\ ^i; \qquad (5.6)$$

where $X_{[i}Y_{j]} \quad X_iY_j \quad X_jY_i$. By multiplying (5.6) by $\ ^i$ we get,

$$\frac{d\tilde{\ }}{dt} = \quad {}_{ij}\ ^i\ ^j;$$

which is equivalent to Zamolodchikov's c-theorem if $\ {}_{ij}$ is positive-deﬁnite.

One should note here that there is a degree of arbitrariness in the deﬁnition of the various coeﬃcients in (5.5), corresponding to the addition of allowed terms in the generating functional of the original 2d theory. Indeed, if we shift $W \to W + \quad W$, where

$$W = \int d^2x\sqrt{g}\,(\tfrac{1}{2}bR \quad \tfrac{1}{2}b_{ij}\,\partial g^i \partial g^j);$$

with arbitrary functions b, b_{ij}, then

$$\chi = \beta^i \partial_i b = L_\beta b; \qquad w_i = \partial_i b + b_{ij}\beta^j$$
$$\chi_{ij} = \beta^k \partial_k b_{ij} + \partial_i \beta^k b_{kj} + \partial_j \beta^k b_{ik} = L_\beta b_{ij};$$

where L_β is the Lie derivative along the beta-function vector. Nevertheless, the consistency condition is invariant under this arbitrariness.[7] Osborn then establishes that there is a choice of the arbitrariness so that the corresponding χ_{ij} is positive-definite, essentially equal to Zamolodchikov's metric $G_{ij} = (x^2)^2 \langle [O_i(x)][O_j(0)] \rangle$. With that choice $\tilde{\beta}$ becomes Zamolodchikov's c-function C. As a final remark let us point out here that possible dimension-one vector operators are neglected in the treatment of Osborn—such operators were considered in [90].

5.2.2 The 4d case

In four dimensions the elegance of the two-dimensional case is obfuscated by the fact that there exist four curvature invariants (that conserve parity) and quite a few terms that involve derivatives on the couplings. The terms that account for the trace anomaly may be written as

$$\Delta^W_\sigma W = \Delta_\beta W + \int d^4x \sqrt{g}\, \sigma T + \int d^4x \sqrt{g}\, \partial_\mu \sigma Z^\mu ; \tag{5.7}$$

where

$$\begin{aligned} T = {} & \beta_a I + \beta_b E_4 + \tfrac{1}{9}\beta_c R^2 + \tfrac{1}{3}\chi^e_i \partial_\mu g^i \partial^\mu R + \tfrac{1}{6}\chi^f_{ij} \partial_\mu g^i \partial^\mu g^j R + \tfrac{1}{2}\chi^g_{ij} \partial_\mu g^i \partial_\nu g^j G^{\mu\nu} \\ & + \tfrac{1}{2}\chi^a_{ij} \nabla^2 g^i \nabla^2 g^j + \tfrac{1}{2}\chi^b_{ijk} \partial_\mu g^i \partial^\mu g^j \nabla^2 g^k + \tfrac{1}{4}\chi^c_{ijkl} \partial_\mu g^i \partial^\mu g^j \partial_\nu g^k \partial^\nu g^l ; \end{aligned} \tag{5.8}$$

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[7] The arbitrariness we are discussing here is analogous to the arbitrariness that affects the coefficient of $\Box R$ in the 4d trace anomaly at the fixed point. In 2d we see that outside the fixed point $\tilde{\beta}$ has a degree of arbitrariness. Of course when $\beta^i = 0$ the well-defined $\tilde{\beta}$ is the central charge of the corresponding CFT (up to normalization).

and

$$Z = G \; w_i \partial g^i + \tfrac{1}{3}\partial(b' R) + \tfrac{1}{3} R Y_i \partial g^i$$
$$+ \partial(U_i \nabla^2 g^i + \tfrac{1}{2} V_{ij} \partial g^i \partial g^j) + S_{ij} \partial g^i \nabla^2 g^j + \tfrac{1}{2} T_{ijk} \partial g^i \partial g^j \partial g^k; \qquad (5.9)$$

up to terms with vanishing divergence. Definitions of the various curvatures can be found in (C.3). G is the Einstein tensor and the various coefficients are functions of the couplings.

Here one finds six consistency conditions (which can be further decomposed). Two of them are particularly interesting. First, there is a consistency condition like (5.6) involving β_b:

$$\partial_i \tilde{\beta}_b = \tfrac{1}{8}\chi^g_{ij}\beta^j + \tfrac{1}{8}\partial_{[i} w_{j]}\beta^j; \qquad \tilde{\beta}_b = \beta_b + \tfrac{1}{8} w_i \beta^i: \qquad (5.10)$$

The consistency condition (5.10) can lead to an extension of Zamolodchikov's result to 4d if the metric χ^g_{ij} can be shown to be positive-definite. Of course, just like in 2d, there is an arbitrariness in the definition of χ^g_{ij} as well as in other coefficients in (5.8) and (5.9). To get an a-theorem it suffices to show that there is a choice of the arbitrariness so that χ^g_{ij} is positive-definite. This relies on the fact that (5.10) is invariant under the arbitrariness.

The other consistency condition we would like to draw attention to is

$$\beta_c = \tfrac{1}{4}(\partial_i b' - \chi^e_i)\beta^i:$$

This shows that the coefficient of R^2 in the trace anomaly is generally non-zero outside the fixed point. It also motivates the use of the term "vanishing anomalies" for contributions to the trace anomaly like R^2 in $d = 4$: these are anomalies that are present along the flow but vanish at the fixed point.

In our treatment so far we have neglected relevant operators with classical scaling dimension three or two that may be present in a four-dimensional theory.

Osborn has considered such operators in [77], and has shown that the condition (5.10) is actually unaffected by their presence, except for a shift of ^{i} due to the presence of dimension-three vector operators. This shift played an important role in [79], where it was calculated at three loops in the most general renormalizable 4d QFT, and was used to show that at the perturbative level scale implies conformal invariance in unitary renormalizable 4d QFTs.

5.3 The 6d case

As we saw in the previous section the elegance of the consistency conditions rapidly disappears in the jump from 2d to 4d. Nevertheless, a consistency condition similar to (5.6) remains, and it is interesting to see if this is an accident or if such a consistency condition can be obtained in higher (even) dimensions. This is the main motivation behind this work, and the treatment of the highly nontrivial 6d case gives us valuable intuition that actually applies to all even dimensions. We postpone the discussion of the general even-d case until the next section, and we turn now to the consistency conditions in $d = 6$. Appendices C and D contain information on conventions, basis choices, as well as the terms that appear in the trace anomaly in 6d away from the fixed point.

5.3.1 Basis of curvature tensors

It is clear from the complexity of the 4d case that the situation in 6d is significantly more challenging. As a first step we have to classify the curvature tensors that can be used in the anomaly terms. Of course terms without curvatures also need to be considered.

To begin, note that for the various contributions to (^{W})W we are only constrained by diffeomorphism invariance and power counting. Let us look at a consequence of this in 4d: in anomaly terms with one power of curvature one cannot involve the Riemann tensor (without contracting its indices). Indeed, the

Riemann tensor has four free indices, for which we would need four derivatives on one or more couplings. This would result in a term with mass dimension six. Therefore, in 4d, one can only include curvature tensors with up to two free indices, and those are R, R, and R.[8] Since the variation of the 4d Euler density in $d = 4$ is $(\sqrt{\ } E_4) = 8\sqrt{\ } G \ r \ @$, it is preferable to include the Einstein tensor instead of the Ricci tensor. This choice produces the consistency conditions in a convenient form, but it is not essential. Indeed, the consistency conditions in a speci c basis can be recast to the form obtained in any other basis by a rede nition of the coe cients of the various anomaly terms.

In 6d a similar choice is dictated by the fact that the Weyl variation of the 6d Euler density is

$$(\sqrt{\ } E_6) = 12\sqrt{\ }(3E_4 \quad 2RR \; + 4R \; R \; + 4R \; R \quad 2R \; R \;) r \; @ ;$$

where E_4 is given in even $d > 2$ by $E_4 = \frac{2}{(d \ 2)(d \ 3)}(R \; R \quad 4R \; R \; + R^2)$. The tensors quadratic in curvature that we have to consider can be found in (C.3); the tensor H_1 is chosen so that in $d = 6$ the variation of the Euler density is $(\sqrt{\ } E_6) = 6\sqrt{\ } H_1 \; r \; @$. As far as terms quadratic in curvature are concerned, we also have to include the terms (C.4), which are basically derivatives of the terms in (C.3). Terms linear in curvature include (C.1) and (C.2). In writing down the various curvature tensors one has to identify a complete but not over-complete basis, a problem complicated by the symmetries of the Riemann tensor and the Bianchi identities.

As far as scalar terms cubic in curvature are concerned [91, 92], the situation is slightly more subtle. We have to include the terms in (C.5), but among them there are trivial anomalies, i.e. the terms $J_{1;:::;6}$ whose coe cient can be varied at will by a choice of local counterterms. These are not genuine anomalies, but they

[8] Incidentally, using the same argument one sees that G, where G is the Einstein tensor, can also not be included in the anomaly terms.

nevertheless appear in the trace anomaly, even at the fixed point. The well-known example is the term R in 4d. In (C.5) there are also vanishing anomalies, i.e. curvature terms that have to be included outside the fixed point, but that do not satisfy the consistency conditions at the fixed point and thus their coefficient has to be set to zero there. These are the terms $L_{1;\ldots;7}$ in (C.5). As we already mentioned there is only one such term in 4d, namely R^2. Here, the form of $L_{1;\ldots;6}$ is chosen based on the fact that these are the terms that shift the coefficients of the trivial anomalies at the fixed point, i.e. $\int d^6x\sqrt{-g}\,L_{1;\ldots;6} = \int d^6x\sqrt{-g}\,J_{1;\ldots;6}$.

While $J_{1;\ldots;6}$ can be included in the basis of terms cubic in curvature, there is a more convenient choice based on the fact that in order to show that $\int d^6x\sqrt{-g}\,L_{1;\ldots;6} = \int d^6x\sqrt{-g}\,J_{1;\ldots;6}$ one has to integrate by parts. But since total derivatives can be neglected in our considerations (for can be taken to have local support), this implies that we don't have to include the trivial anomalies $J_{1;\ldots;6}$ in $(^W)W$, so long as we include terms arising from $\int d^6x\sqrt{-g}\,z_{1;\ldots;6}L_{1;\ldots;6}$ before any integrations by parts. (Here $z_{1;\ldots;6}$ are arbitrary functions of the couplings.) Consequently, terms cubic in curvature that we need to consider are the three terms $I_{1;2;3}$ that lead to Weyl-invariant densities, the 6d Euler term E_6, and the seven vanishing anomalies $L_{1;\ldots;7}$. As we explained, this relies on the ability to discard total derivatives.

5.3.2 Contributions to the anomaly

Now that we have a complete basis of curvature tensors we are ready to write down the most general anomaly functional $(^W)W$. It takes the form

$$^W W = W + \sum_{p=1}^{65}\int d^6x\sqrt{-g}\,T_p + \sum_{q=1}^{30}\int d^6x\sqrt{-g}\,\partial Z_q\,;$$

where the T_p and Z_q are dimension-six and dimension-five terms respectively, that can involve curvatures as well as derivatives on the couplings g^i (see Appendix D).

Much like Osborn did in $d = 2; 4$ we split the anomaly contributions into terms with σ and $\partial\sigma$. This splitting may seem mysterious—as we could also introduce terms of the form $\int d^6x \sqrt{g}\, \sigma V$, for example—but it is used here in order to get the consistency conditions in a most convenient form. We can obtain the desired form of the consistency conditions even without the splitting, if we carefully choose the coefficients of the various terms in the anomaly. This can be seen by integrating by parts to rewrite the Z_q terms in the form of the T_p terms, which would lead to some new T_p terms but also to shifts of coefficients of existing T_p terms.

Let us illustrate this point more clearly in the 2d case. Suppose that instead of (5.5) we started with the equivalent

$$\Delta^W_\sigma W = \Delta_\sigma W - \int d^2x \sqrt{g}\, \sigma \left(\tfrac{1}{2}\beta R - \tfrac{1}{2}\chi^0_{ij}\partial g^i \partial g^j + w_i \Box g^i\right). \qquad (5.11)$$

After an integration by parts of the $\Box g^i$ term this amounts simply to the definition $\chi_{ij} = \chi^0_{ij} + 2\partial_{(i} w_{j)}$ in (5.5). This can also be seen by computing the Weyl consistency condition from (5.11) directly. We get

$$\partial_i \tilde{\beta} = (\chi^0_{ij} + 2\partial_i w_j)\beta^j = (\chi^0_{ij} + \partial_{(i} w_{j)})\beta^j + \partial_{[i} w_{j]}\beta^j. \qquad (5.12)$$

Clearly, (5.12) is equivalent to (5.6) with the proper definition of χ_{ij}.

5.3.3 Some consistency conditions

Here we include some consistency conditions and we comment on the most interesting ones. A Mathematica file with all the consistency conditions is included with our submission.

Just like in 2d and 4d we obtain consistency conditions simply by the requirement $[\Delta^W_\sigma, \Delta^W_{\sigma'}]W = 0$. In our case we find a total of forty one consistency conditions[9]. For example, consistency requires that terms proportional to $\partial \quad ^{2} \, ^{0} \quad \partial \quad ^{0} \, ^{2}$ add up to zero, which leads to

$$\partial(4b_{11} \quad 3A_i \quad ^i) + (4A_i + 2G_i^4 + 5H_i^5 + 2H_i^6 \quad 2I_i^4) \, \partial g^i + 6A_i \, \partial \quad ^i \quad A^0_{ij} \quad ^j \, \partial g^i = 0;$$

which implies that

$$\partial_i(4b_{11} \quad 3A_j \quad ^j) + 4A_i + 2G_i^4 + 5H_i^5 + 2H_i^6 \quad 2I_i^4 + 6A_j \, \partial_i \quad ^j = A^0_{ij} \quad ^j:$$

Among the forty one consistency conditions in 6d the most interesting is the one similar to (5.6), obtained from terms proportional to $(\partial \quad ^0 \quad ^0\partial \quad)H_1$. It reads

$$\partial(6a + b_1 \quad \tfrac{1}{15}b_3) + H_i^1 \, \partial \quad ^i + \quad ^i \partial H_j^1 \, \partial g^j \quad H^1_{ij} \quad ^i \, \partial g^j = 0;$$

which can be brought to the form

$$\partial_i \tilde{a} = \tfrac{1}{6}H^1_{ij} \quad ^j + \tfrac{1}{6}\partial_{[i}H^1_{j]} \quad ^j; \qquad \tilde{a} = a + \tfrac{1}{6}b_1 \quad \tfrac{1}{90}b_3 + \tfrac{1}{6}H_i^1 \quad ^i: \tag{5.13}$$

The consistency condition (5.13) has a new feature compared to the 2d and 4d cases, i.e. that the function $\tilde{a}$ contains the coefficients b_1 and b_3 of the vanishing anomalies L_1 and L_3 respectively. This is of no consequence as far as the value of $\tilde{a}$ at the fixed point is concerned: there $\tilde{a} = a$, for $b_1 = b_3 = 0$ at the fixed point. This fact is actually made explicit by three consistency conditions. More specifically, from terms proportional to $(\partial \quad r \quad \partial \quad ^0 \quad \partial \quad ^0 r \quad \partial \quad)r \quad G$, $(\partial \quad ^0 \quad ^0\partial \quad)r \quad H_4$, and $(\partial \quad ^0 \quad ^0\partial \quad)r \quad H_3$ we find

$$b_7 = \tfrac{1}{8}F_i \quad ^i; \tag{5.14a}$$

[9] Some of these consistency conditions can be further decomposed as a result of the variety of ways with which spacetime derivatives can act on couplings.

$$3b_1 \quad 8b_7 = \tfrac{1}{4}(\partial_i b_{14} + I^7_i)\ {}^i; \qquad (5.14b)$$

$$12b_1 \quad b_3 \quad 16b_7 = (\partial_i b_{13} + I^6_i)\ {}^i; \qquad (5.14c)$$

respectively. From similar consistency conditions we can verify that $b_2; b_4; b_5$ and b_6 are also zero at the xed point, as expected since they are coe cients of vanishing anomalies.

5.3.4 Possibility for an a-theorem in 6d

The consistency condition (5.13) has the potential to lead to a result similar to that of Zamolodchikov in 2d. Indeed, contracting with the beta function it follows that (5.13) implies

$$\frac{d\tilde{a}}{dt} = \frac{1}{6}H^1_{ij}\ {}^i\ {}^j: \qquad (5.15)$$

Note that here the conditions (6.20) allow us to absorb the b_1 and b_3 contributions in $\tilde{a}$ to a shift of H^1_i. Of course what is missing is a proof of the positive-de niteness of H^1_{ij}.

It is important to point out that the consistency condition (5.13) is actually stronger than (5.15). Indeed, (5.13) also contains information about the possibility of a gradient ow interpretation of the RG ow. For that, it has to be that $\partial_{[i}H^1_{j]} = 0$, in which case $\tilde{a}$ is the \potential" whose gradient produces the RG ow.

Let us now concentrate on a technical but important point. It turns out that the tensor H_1, which appears in $(\sqrt{\ }E_6) = 6\sqrt{\ }H_1\ \nabla\ \partial$, is divergenceless. A similar statements holds in two, $(\sqrt{\ }R) = 2\sqrt{\ }\ \nabla\ \partial$, and four dimensions, $(\sqrt{\ }E_4) = 8\sqrt{\ }G\ \nabla\ \partial$. This is actually crucial for the coe cient of the Euler term to be involved in a consistency condition like (5.13), which has the chance to lead to an a-theorem. This is not so easy to see in 2d and 4d, but it is clear in 6d.

Indeed, consider, for example, the consistency condition arising from terms proportional to $(\partial\,{}^{0}\;{}^{0}\partial\,)H_4$. It reads

$$\partial_i \tilde{b}_1 = \tfrac{1}{12}(H^4_{ij} + \tfrac{1}{2}F_{ij})\,{}^j + \tfrac{1}{12}\partial_{[i}H^4_{j]}\,{}^j + \tfrac{1}{6}I^7_i; \qquad \tilde{b}_1 = b_1 + \tfrac{2}{3}b_7 + \tfrac{1}{12}H^4_i\,{}^i. \quad (5.16)$$

The contribution $\frac{1}{6}I^7_i$ does not allow (5.16) as a candidate for the generalization of Zamolodchikov's result.[10] This contribution in fact arises from the term $T_{18} = I^7_i\,\partial g^i\,\nabla H_4$. Were ∇H_1 non-vanishing, we would not be able to find a consistency condition like (5.13). It can be verified by explicit computations that H_1 is the only divergenceless symmetric two-index tensor quadratic in curvature. It is thus a generalization of the Einstein tensor. As we will see in the next section Lovelock has constructed all such generalizations a long time ago [93], something that will allow us to argue for a consistency condition similar to (5.13) in all even d.

5.3.5 Arbitrariness

Just like the coefficient of $\Box R$ in the four-dimensional trace anomaly, the various coefficients in T_p and Z_q are affected by the choice of additive, quantum-field-independent counterterms. Indeed, calculations in curved space and with x-dependent couplings will result in infinities that will need to be renormalized via counterterms whose finite part is arbitrary. Therefore, different subtraction schemes will result in different coefficients for T_p and Z_q.

The most general addition to the generating functional of our theory is

$$W = \sum_{p=1}^{65}\int d^6x\sqrt{-g}\,X_p;$$

where the X_p terms have the same form as the T_p terms but with arbitrary coefficients. There is no arbitrariness introduced by terms X_q similar in form to

[10] Of course this can also be seen from the fact that $\tilde{b}_1$ becomes zero at fixed points, and so it cannot possibly be monotonically-decreasing along an RG flow.

the Z_q terms, for those are total derivatives. Now, the consistency conditions are invariant under the shift $W \to W + W$, although the coefficients in the consistency conditions will shift. Let us see how this works for (5.13).

The relevant terms are

$$\int d^6x \sqrt{-}\,(z_a E_6 + z_{b_1} L_1 + z_{b_3} L_3 \quad \tfrac{1}{2} z^{H^1}_{ij} @g^i @g^j H_1 \,) \quad W \qquad (5.17)$$

and one can verify that their inclusion leads to shifts

$$\begin{aligned} a &= L \;\; z_a; \qquad b_1 = L \;\; z_{b_1}; \qquad b_3 = L \;\; z_{b_3}; \\ H^1_i &= \;\; 6@(z_a + \tfrac{1}{6} z_{b_1} \;\; \tfrac{1}{90} z_{b_3}) + z^{H^1}_{ij} \;\; {}^j; \qquad H^1_{ij} = L \;\; z^{H^1}_{ij}; \end{aligned} \qquad (5.18)$$

under which (5.13) is invariant. Note that a, which is of course well-defined at the fixed point, is arbitrary along the flow, while H^1_i and H^1_{ij} have a degree of arbitrariness even at the fixed point. Also note that the shifts (5.18) cannot be used to set the corresponding coefficients to zero, except for H^1_i if $H^1_i = @X$ for some X.

This observation leads to an important point, which we have already emphasized: regarding the a-theorem, one should not be able to prove that the metric H^1_{ij} is positive-definite in all generality. Instead, one ought to be able to show that there is a choice for the arbitrariness (5.17) such that H^1_{ij} is positive-definite. That specific choice then gives us the quantity a whose flow is monotonic, through the dependence of H^1_i on $z^{H^1}_{ij}$. Recall that in 2d arbitrariness similar to the one described here was used by Osborn to rederive Zamolodchikov's c-theorem (see [7] for details).

5.4 Consistency conditions in even spacetime dimensions

In this section we identify the ingredients that allow us to conclude that a consistency condition like (5.13) appears in all even spacetime dimensions. Of course non-trivial CFTs in $d > 6$ are not known, but it is still interesting to consider the generalization of our results.

According to the classification of [94], for a CFT in any even spacetime dimension lifted to curved space the conformal anomaly consists of a unique Euler term (type-A anomaly), a number of terms that lead to locally Weyl invariant densities (type-B anomalies), as well as a number of trivial anomalies. Outside the fixed point we also have a number of vanishing anomalies. As for the trivial anomalies, these can always be accounted for by terms with $d/2-1$ powers of curvature.

Now, in any even spacetime dimension, $d = 2n$, it is easy to see that the Weyl variation of the Euler density $\sqrt{-g}E_{2n}$, where

$$E_{2n} = \frac{1}{2^n} R_{i_1j_1k_1l_1}\cdots R_{i_nj_nk_nl_n}\,\epsilon^{i_1j_1\ldots i_nj_n}\,\epsilon^{k_1l_1\ldots k_nl_n}\,,$$

gives

$$\delta_\sigma(\sqrt{-g}E_{2n}) = \sqrt{-g}\,H^{\mu\nu}\nabla_\mu\partial_\nu\sigma\,,$$

for some symmetric tensor $H^{\mu\nu}$ with $n-1$ powers of the curvature. As Lovelock showed [93], this tensor $H^{\mu\nu}$ is the unique tensor with the properties of the Einstein tensor—in particular, it is the only two-index symmetric tensor with $n-1$ powers of the curvature that is divergenceless:

$$\nabla_\mu H^{\mu\nu} = 0\,.$$

Regarding the consistency condition similar to (5.13), this observation allows us to conclude that the only relevant terms among the various contributions to the anomaly $(\Delta^W_\sigma)W$ are

$$\int d^{2n}x\sqrt{-g}\,\sigma\left[(-1)^n aE_{2n} + \sum_p b_pL_p + \tfrac{1}{2}H_{ij}\,\partial_\mu g^i\partial_\nu g^j H^{\mu\nu}\right] + \int d^{2n}x\sqrt{-g}\,\partial_\mu\sigma\, H_i\,\partial_\nu g^i H^{\mu\nu}\,;$$

where L_p are some vanishing anomalies. A consistency condition similar to (5.13) is thus easily found, and is of course invariant under arbitrariness generated by contributions similar to (5.17).

A relation of the metric H_{ij} to a positive-definite metric is currently only known in 2d [77]. A similar relation in higher even d is lacking, but its possible existence would immediately imply the generalization of Zamolodchikov's result. To summarize, in any even spacetime dimension one can find a scalar quantity $\tilde{a}$ such that

$$\partial_i \tilde{a} = H_{ij}\beta^j + \partial_{[i}H_{j]}\beta^j . \tag{5.19}$$

The quantity $\tilde{a}$ becomes the coefficient of the Euler term in the trace anomaly at the fixed point, but more generally it includes a linear combination of the b_ps and a term $H_i\beta^i$. The relation (5.19) immediately implies that

$$\frac{d\tilde{a}}{dt} = H_{ij}\beta^i\beta^j ;$$

which, if H_{ij} can be related to a positive-definite metric via the arbitrariness $\Delta H_{ij} = \mathcal{L}_\beta z^H_{ij}$ with z^H_{ij} an arbitrary symmetric tensor, is the generalization of the 2d c-theorem.

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Chapter 6

Studies of degrees of freedom in quantum field theories II: An unexpected surprise: The a-theorem in six dimenisions

In the last chapter we established the relation that permits an a-theorem in d = 2 and d = 4 dimensions: an equation that could imply the monotonic increase or decrease of a quantity that is stationary at critical points and equal to the coefficient of the Euler density, a, at those critical points. Here we undertake the computation of the quantity $_{IJ}$ in that equation. In the $d = 2$ case this was shown to be positive definite, and in the $d = 4$ case it was shown positive definite in perturbation theory. In six dimensions, we find that $_{IJ}$ is actually negative definite in perturbation theory. We discuss the implications of this computation.

6.1 Introduction

The counting of degrees of freedom in quantum field theories (QFTs) is of paramount importance in understanding their structure and phases. In particular, it is often of interest to understand how low-energy, long-range "IR" degrees of freedom might be related to the underlying microscopic "UV" degrees of freedom. For example, in quantum chromodynamics we observe protons, pions, etc. at low energies, but believe them to be made up of quarks and gluons that make up the microscopic theory.

The most complete understanding of these degrees of freedom exists in two dimensional QFTs. There, a quantity exists that can be shown to undergo a monotonically decreasing renormalization group flow from a critical point in the UV to a critical point in the IR. At the critical points the quantity is stationary with respect to variations in scale and becomes the central charge c of the Virasoro algebra that describes the critical point's conformal field theory, which is also the coefficient of the topological term (Ricci scalar) in the two-dimensional Weyl anomaly. This is the result of Zamolodchikov [11].

In the four-dimensional case, which is of great interest to particle physicists, results are not so definitive. Cardy suggested [14] that the four-dimensional analog of c is the coefficient of the Euler density in the four-dimensional Weyl anomaly, a. In fact, it was shown, using heat kernel methods for field theories on curved backgrounds and Weyl consistency conditions, that a perturbative version of Zamolodchikov's result holds [15]. More recently, non-perturbative methods have made headway into the so-called weak version of the a-theorem, where, instead of establishing a monotonic flow, a relation between the value of a at the critical points is established, namely that $a_{UV} - a_{IR} > 0$.

In this paper we investigate the possibility of an a-theorem in six dimensions (6D). The six dimensional case is of interest in clarifying the basic structure of any QFT. For example, in string theoretic constructions of 6D theories from the low energy dynamics of M5 branes, the spectrum of operators may be classified without any knowledge of what Lagrangian "describes" the theory. It may be that the degrees of freedom of any QFT do not rely on a Lagrangian description, and that in fact there are more fundamental features that may be elucidated with any such description. It is with this type of motivation that we pursue a six-dimensional a-theorem.

In fact, we find that the opposite conclusion of two- and four-dimensional a-theorems may be drawn in six dimensions, at least in perturbation theory. Namely, we find that the candidate for an a-theorem singled out by the Weyl consistency conditions *increases* monotonically along the renormalization group flow. To come to such a conclusion, we use the methods developed in [77] and [15]. This involves constraining the form of the Weyl anomaly from the Abelian nature of the Weyl group—because the Weyl group is related to a change of scale, this imposes a particular constraint on the renormalization group properties of quantities in the anomaly and, in particular, produces a candidate for an a-theorem. In section 6.2 we explain this method, compute the beta functions and anomalous dimensions in section 6.3, and in section 6.4 we show that the quantity that becomes a at critical points *increases* monotonically along the renormalization group flow, at least in perturbation theory. We discuss the implications of this result in section 6.5.

6.2 The trace anomaly in position space

In this section we outline the method of calculation employed in this paper. For more details the reader is referred to [95, 96, 97, 98], where such computations have been thoroughly explained and demonstrated.

In this work we will study quantum field theories defined in spacetime dimension d by a set of couplings g^I and fields ϕ. For our computations we will use dimensional regularization with minimal subtraction so that

$$g^I_{Bare} = \quad ^{k^I}\ (g^I + L^I(g)) ; \qquad {}_{Bare} = \quad Z^{1=2}(g) ; \tag{6.1}$$

for some numbers k^I and ,[1] and with L^I and $Z^{1=2}$ containing series of poles in ($Z^{1=2}$ also contains the unity, $Z^{1=2} = 1 +$ -poles). Then, the beta function and anomalous dimension are given by

$$\hat{}^I \quad \frac{dg^I}{d} = \quad k^I g^I + \quad ^I \quad \text{and} \quad \hat{} \quad + Z^{1=2} \frac{dZ^{1=2}}{d} = \quad + \quad ;$$

respectively, where and are the quantum beta function and anomalous dimension respectively.

Now, in quantum eld theory in at spacetime, wavefunction and coupling renormalization are enough to render nite correlation functions involving fundamental elds. When correlation functions involving composite operators are included, further counterterms are necessary. A convenient way to deal with these is by introducing sources for the composite operators, and including counterterms proportional to spacetime derivatives on those sources. For operators that appear in the Lagrangian it is enough to take their couplings as spacetime-dependent sources, $g^I\ ! \quad g^I(x)$, and introduce counterterms proportional to $@g^I(x)$ [15, 99]. Finally, when a at-space eld theory is lifted to curved space with metric , new divergences proportional to the curvatures de ned from appear, and thus further counterterms involving the curvatures are required for niteness.

In [15] a systematic treatment of such e ects was undertaken, and the general expression

$$\tilde{L}(\,; g; \quad) = L \ (\,; g; \quad) \qquad R \tag{6.2}$$

was proposed, where R includes all eld-independent counterterms, proportional

[1] Note that the index carried by k is not subject to the summation convention.

only to curvatures and $@g^I(x)$.[2] Also, L already contains wavefunction and coupling renormalization factors $Z^{1=2}$ and L^I as in (6.1).

The RGE one derives from (6.2) is

$$\hat{}^I \frac{@}{@g^I} + (\hat{}\ \ )\frac{@}{@} \quad \tilde{L} = \quad R\,; \qquad (6.3)$$

which, by (6.2) and the Callan{Symanzik equation, requires

$$\hat{}^I \frac{@}{@g^I} \quad R = \quad R: \qquad (6.4)$$

As explained in [15] and we will review in the following, the terms R de ned by (6.4) contribute, among others, to the trace anomaly of the theory in curved space.

6.2.1 Background eld method

The contribution R in (6.2) is straightforward to compute in perturbation theory. Employing the background eld method one simply computes the e ective action starting from L , which thus dictates the form of the counterterms R . More speci cally, we start by splitting the eld into an arbitrary background part $_b$ and a uctuation f ,

$$= {}_b + f:$$

We can also introduce a source J , and obtain the e ective action $W[\ _b; J;\ ]$ (generating functional of connected graphs) after we integrate out f:

$$e^{W[\ _b;J;\ ]} = \int Df\, e^{\ \tilde{S}[\ ;\ ] + \int d^dx \sqrt{\ } J(x) f(x)}\,; \qquad \tilde{S} = \int d^dx \sqrt{\ }\,\tilde{L}\,; \qquad (6.5)$$

where is the determinant of the metric .

To continue, let us denote by $S^{(0)}$ the action without any counterterms.

[2] A term F ( ) that includes all eld- curvature- and $@g^I(x)$-dependent counterterms also has to be included for niteness in (6.2), but since it will not be important for our considerations we will neglect it.

Then, we expand $S^{(0)}[\ ;\ \ ]$ in uctuations,

$$S^{(0)}[\ ;\ \ ] = S^{(0)}[\ _b;\ \ ] + \int d^dx\, p\ \frac{S^{(0)}}{}\Big|_b f + \frac{1}{2}\int d^dx\, p\ f M f + S_{int}[f]; \quad (6.6)$$

where $M = r^{\ \ 2} + d^2V{=}d^{\ 2}$, with V the potential in L . Then, by expanding (6.5) we nd that, at the zeroth order,

$$W^{(0)}[\ _b;\ \ ] = \ \ S^{(0)}[\ _b;\ \ ];$$

and at the one-loop order,

$$W^{(1)}[\ _b;\ \ ] = \ \ S^{(1)}[\ _b;\ \ ] \ \ \tfrac{1}{2}\ln\det M; \quad (6.7)$$

after we perform in (6.5) the Gaussian integral over f in the third term in the right-hand side of (6.6) and take $J = 0$. Here, $S^{(1)}$ contains the one-loop contributions to $Z^{1=2}$ and L of (6.1), which are chosen to absorb the in nities coming from the term $\ \tfrac{1}{2}\ln\det M$ so that $W^{(1)}$ is nite. In addition, with the extension (6.2) it is clear from (6.7) that the one-loop contribution to $\ \ R$ is given by the negative of the appropriate simple-pole part of $\tfrac{1}{2}\ln\det M$:

$$\int d^dx\, p\ \ \ ^{(1)}\ R \ \ \ (\ \tfrac{1}{2}\ln\det M\,)^{pole}: \quad (6.8)$$

Then, from (6.4) and (6.8) we can evaluate $^{(1)}\ \ R$.

At higher loops the interaction term $S_{int}[f]$ in (6.6) is considered and vacuum-bubble diagrams as well as diagrams with counterterm insertions are constructed. The counterterms are of course xed here by the previous loop order, i.e. by $S^{(1)}$. These diagrams can be evaluated in position space, using coincident limits of propagators according to the diagram topology. With these methods no loop integrations need to be performed. If we denote by $S^{(2)}$ the contribution of all such diagrams, we nd

$$W^{(2)}[\ _b;\ \ ] = \quad \tilde{S}^{(2)}[\ _b;\ \ ] + S^{\ (2)};$$

which, by (6.2), implies that

$$\int d^dx \sqrt{\ } \quad ^{(2)}\ R = \quad (S^{\ (2)})^{pole};$$

From the simple poles in $^{(2)}$ R it is straightforward to evaluate $^{(2)}$ R using the RGE (6.4). Clearly these computations can be carried out order by order in perturbation theory.

6.2.2 Heat kernel

Using heat-kernel techniques the evaluation of $(\frac{1}{2} \ln \det M\ )^{pole}$ is straightforward.

At two loops we have to consider the diagrams in Fig. 6.1, where in the diagram on the right $\otimes$ denotes the one-loop counterterm. Note that these are

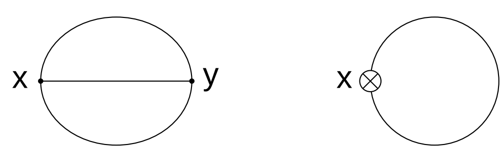


Figure 6.1: The diagrams that need to be considered at the two-loop level.

graphs in position space, and that short distance singularities arise here from the coincident limit of products of position-space propagators.

6.2.3 Trace anomaly

We can de ne the quantum stress-energy tensor and nite composite operators by

$$T\ \ (x) = 2 \frac{\tilde{S}}{\ \ (x)}; \qquad [O_I(x)] = \frac{\tilde{S}}{g^I(x)};$$

where functional derivatives are de ned in d spacetime dimensions by

$$\frac{}{(x)}(y) = \tfrac{1}{2}\,(\quad)\ {}^{d}(x;y); \qquad \frac{}{g^{I}(x)}g^{J}(y) = {}_{I}^{J}\ {}^{d}(x;y);$$

with $X_{(I}Y_{J)} \quad X_I Y_J + X_J Y_I$, ${}^{d}(x;y) = {}^{d}(x \quad y)/\sqrt{\ \ (x)}$, where ${}^{d}(x)$ is the usual delta function in d dimensions. At the level of the generating functional we implement in nitesimal local Weyl transformations with the generators

$$^{W} = 2\int d^{d}x\sqrt{-}\ \frac{}{}; \qquad = \int d^{d}x\sqrt{-}\ \ ^{I}\frac{}{g^{I}}:$$

With these de nitions it is obvious that

$$^{W}W = \int d^{d}x\sqrt{-}\ \ \langle T\ \rangle; \qquad W = \int d^{d}x\sqrt{-}\ \ \langle\ ^{I}[O_I]\rangle:$$

Now, it is easy to see that the term $\tilde{L}$ in $\quad T\quad$ results, after we use (6.3), in

$$\langle T\ \rangle\ \langle\ \ T\ \rangle \qquad R; \qquad T\ \ = \ ^{I}O_I: \tag{6.9}$$

Equivalently, we can write

$$^{W}W \qquad W \quad \int d^{d}x\sqrt{-}\qquad R: \tag{6.10}$$

Terms in the right-hand side of (6.9) have been computed in [15] for eld theories in $d = 4$. In this work we will compute such terms for a wide class of eld theories in $d = 6$. As we just saw, these computations give results on the various terms that appear in the consistency conditions derived from (6.10) [100].

6.3 Beta functions and anomalous dimensions

The theory we will work with has Lagrangian

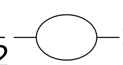
$$\mathcal{L} = \tfrac{1}{2}Z_{ij}\,\partial\phi_i\,\partial\phi_j + \tfrac{1}{3!}\,\mu^{\epsilon/2}(gZ_g)_{ijk}\,\phi_i\phi_j\phi_k . \tag{6.11}$$

Using background-field and heat-kernel methods the computation of Z_{ij} and $(Z_g)_{ijk;lmn}$ is easily done in position space and does not require the calculation of any integrals.[3] These methods have been developed and applied to four-dimensional theories in [95, 96, 97, 98]. They have also been used in six-dimensions in [101, 102, 103, 104]. In dimensional regularization with $d = 6 - \epsilon$ the Z's are of course a series of poles in ϵ, and the residues of the simple-poles determine the anomalous dimension γ_{ij} of ϕ_i and the beta function β_{ijk}.

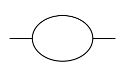
The anomalous dimension is defined by

$$\gamma = -Z^{-1/2}\frac{dZ^{1/2}}{dt}; \qquad t = -\ln(\mu/\mu_0);$$

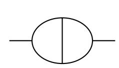
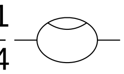
where the RG time t is defined to increase as we flow to the IR. At one loop we find

$$\gamma^{(1)} = -\frac{1}{64\pi^3}\frac{1}{12}$$  ; (6.12)

where we use the diagram to denote the corresponding contraction of the couplings, i.e.

 $= g_{ikl}\,g_{jkl}\,.$

The two-loop anomalous dimension is

$$\gamma^{(2)} = \frac{1}{(64\pi^3)^2}\frac{1}{18}$$  $-\dfrac{11}{24}$  . (6.13)

For the case of a single field our results (6.12) and (6.13) reduce to the results

[3] Comparing with (6.1) we can write $gZ_g = g + L$, with a L a series of poles in ϵ.

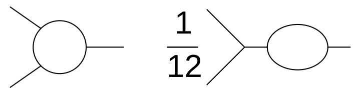
of [105] (see also [101, 102, 103, 104]).

The beta function is defined by

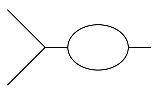
$$\beta = \mu \frac{dg}{d\mu} = \frac{dg}{dt}.$$

At one loop we find

$$\beta^{(1)} = \frac{1}{64\pi^3} \left(\ldots - \frac{1}{12} \ldots \right); \qquad (6.14)$$



where permutations of the free indices in the wavefunction-renormalization correction are understood, i.e.

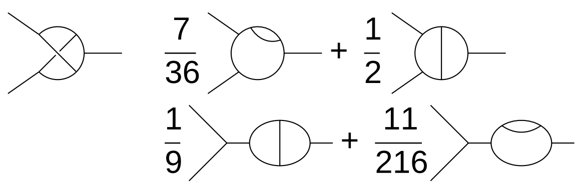


$$= g_{ijl}\, g_{lmn}\, g_{kmn} + \text{permutations}.$$

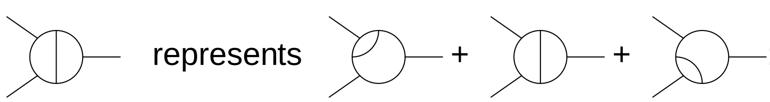
Eq. (6.14) reproduces the result of [105] (see also [101, 102, 103, 104]) in the case of a single field . In that case $\beta^{(1)}$ has a negative sign, and hence the corresponding theory is asymptotically-free.

The two-loop beta function is

$$\beta^{(2)} = \frac{1}{(64\pi^3)^2} \left(\frac{1}{2} \ldots - \frac{7}{36} \ldots + \frac{1}{2} \ldots - \frac{1}{9} \ldots + \frac{11}{216} \ldots \right) \qquad (6.15)$$



The first contribution to (6.15) is non-planar. For the seemingly asymmetric vertex corrections in (6.15) (second and third term) a symmetrization is understood; for example,



represents ... + ... + ...

In the single-field case (6.15) reproduces the result of [105] (see also [102, 103, 104][4]),

[4] There is a typo in the relevant equation in [104].

which, just like $^{(1)}$, is also negative.

6.4 The metric in coupling space

Using heat kernel techniques we have computed

$$^{(2)} \quad R \qquad \frac{1}{(64\ \ ^3)^2}\,\frac{1}{12960} H_1 \ \ g_{ijk}\ r \quad @g_{ijk}\ ; \tag{6.16}$$

where

$$H_1 \quad = 2(3E_4 \qquad 2RR \quad + 4R \quad R \quad + 4R \quad R \qquad 2R \quad R \quad);$$

with E_4 given in even $d > 2$ by $E_4 = \frac{2}{(d\ \ 2)(d\ \ 3)}(R \quad R \qquad 4R \quad R \quad + R^2)$. From (6.16) and (6.10) we can extract

$$H^1_{IJ} \ = \quad \frac{1}{(64\ \ ^3)^2}\,\frac{1}{3240}\ {}_{IJ} \qquad \text{and} \qquad H^1_I = \quad \frac{1}{(64\ \ ^3)^2}\,\frac{1}{6480} g_I\ ; \tag{6.17}$$

where we use notation of [100] and denote $I = (ijk)$. The results (6.17) are unambiguous and scheme-independent. As we observe, the leading, two-loop contribution to the metric is negative, and so the consistency condition

$$@\tilde{a} = \tfrac{1}{6} H^1_{IJ} \quad {}^J + \tfrac{1}{6} @_{[} H^1_{J]} \quad {}^J; \qquad \tilde{a} = a + \tfrac{1}{6} b_1 \qquad \tfrac{1}{90} b_3 + \tfrac{1}{6} H^1_I \quad {}^I : \tag{6.18}$$

derived in [100], and its consequence

$$\frac{d\tilde{a}}{dt} = \quad \tfrac{1}{6} H^1_{IJ} \quad {}^I \quad {}^J ; \tag{6.19}$$

cannot possibly lead to a strong a-theorem for $\tilde{a}$.

Now, the theory (6.11) has only the Gaussian xed point in perturbation theory. Non-perturbatively there may be a non-trivial xed point, but our results (6.17) cannot be used to extract any consequences of (6.19) beyond perturbation theory. Nevertheless, as long as the flow of our theory can be described perturbatively, the quantity $\tilde{a}$ is monotonically increasing.

6.4.1 Perturbative contributions to $\tilde{a}$

Another use of the consistency conditions is the evaluation of some quantities at higher loop orders. From the consistency conditions

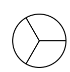
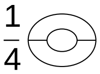
$$b_1 = \tfrac{1}{3}(F_I - \tfrac{1}{4}\partial_I b_{14} - \tfrac{1}{4}I^{7}_{I})\,{}^{I}; \tag{6.20a}$$

$$b_3 = (2F_I + \partial_I b_{13} - \partial_I b_{14} + I^{6}_{I} - I^{7}_{I})\,{}^{I}; \tag{6.20b}$$

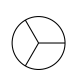
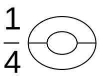
it is clear that at two loops $b_1^{(2)} = b_3^{(2)} = 0$. This, in conjunction with (6.18) and (6.17), implies that $a^{(2)} = 0$. These results have been verified by our explicit computations. Now, at two loops we have also found

$$F^{(2)}_{ijk} = \frac{1}{(64\pi^3)^2}\frac{1}{1080}g_{ijk}\,; \qquad b^{(2)}_{13} = b^{(2)}_{14} = 0\,; \qquad I^{6\,(2)}_{ijk} = I^{7\,(2)}_{ijk} = 0\,;$$

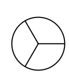
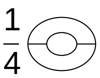
and so we can compute

$$b_1^{(3)} = \tfrac{1}{6}b_3^{(3)} = \frac{1}{(64\pi^3)^3}\frac{1}{3240}$$  $$-\frac{1}{4}$$  :

With these results and using (6.18) with (6.14) and (6.17) we find that the three-loop contribution to $\tilde{a}$ is

$$a^{(3)} = \frac{1}{(64\pi^3)^3}\frac{7}{388\,800}$$  $$-\frac{1}{4}$$  (6.21)

and for the quantity $\tilde{a}$

$$\tilde{a}^{(3)} = \frac{1}{(64\pi^3)^3}\frac{1}{77\,760}$$  $$-\frac{1}{4}$$  : (6.22)

6.5 Discussion

Using the result of our computation, eq. (6.17), in the evolution eq. (6.19), or equivalently, the explicit form of $\tilde{a}$ in eq. (6.22), it is apparent that, in perturbation theory, the quantity $\tilde{a}$ in eq. (6.19) actually increases as one decreases the renormalization scale, contrary to intuition developed in $d = 2, 4$ dimensions where $\tilde{a}$ seemed to count the degrees of freedom in a QFT.

This result should be taken with two comments in mind. Firstly, that the result is a perturbative one, and we cannot say anything about non-perturbative regimes of six-dimensional QFTs. And secondly, that there are no known perturbative critical points other than the single, trivial one at $\lambda_{ijk} = 0$ so in this context renormalization group flows do not connect pairs of critical points[5]. However, it is still true that, with eq. (6.19) identical in $d = 2, 4$, and 6 dimensions, the strong version of the a-theorem holds perturbatively in $d = 2, 4$ but not in $d = 6$.

We do not know what is the reason for this failure. One possibility may be the unstable nature of the theory we are considering. After all, a cubic potential is unbounded below. However, the state $\langle\phi_i(x)\rangle = 0$ is perturbatively stable and our computations are valid only in the perturbative regime. Moreover, the analogous case in 4D, namely an inverted quartic potential, is also perturbatively stable and, however, does satisfy a perturbative a-theorem (since the metric in theory space, χ_{IJ}, is perturbatively positive in 4D regardless of the sign of the quartic coupling constants). Another possibility is that a flow between critical points is required for an a-theorem to hold, but the only critical point in the class of theories in eq. (6.11) is the Gaussian UV-fixed point at $\lambda_{ijk} = 0$. But, again comparing to known cases, a perturbative strong a-theorem holds for scalar-plus-spinor theories in 4D in spite of only having a Gaussian IR fixed point at the origin of coupling constant space.

a-theorems can be used to restrict proposed dynamics of strongly interacting

---

[5] This does not mean that they do not exist. Non-trivial, perturbative flows between a UV and IR critical point have been studied in $6-\epsilon$ dimensions in the $O(N)$ model recently, as in [106]. It is an open question as to whether or not such results could be extended to 6 dimensions.

models [14]. If our result that a increases in flows towards the IR holds even non-perturbatively, one could envision using it to restrict putative dynamics of strongly interacting QFT in 6D. In this sense, the existence of an "anti-a-theorem" may be just as useful as a normal one. It is therefore of interest to investigate renormalization group flows in the vicinity of non-Lagrangian critical QFTs that have been formulated through studies of low energy dynamics of M5 branes. Of course, another avenue of research is the establishment of the theorem non-perturbatively.

We would like to thank Ken Intriligator and Jaewon Song for valuable discussions. This work was supported in part by the US Department of Energy under contract DE-SC0009919.

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Appendix A

Mass Splitting in the 50 of SU(5)

In this appendix we show that the $(6,1)_{4/3}$ and $(6,1)_{1/3}$ of the 50 ( ) can be made arbitrary light while the masses of the other components remain close to the GUT scale. The splitting happens during the spontaneous breaking of $SU(5) \to SU(3)_C \; SU(2)_L \; U(1)_Y$ by the vev of the adjoint scalar, $\langle H_{24} \rangle = v_{24}\text{diag}(2, 2, 2, 3, 3)$. For convenience, we label each component of the 50 by $(_1, _2, _3, _4, _5, _6) = ((6,3)_{1/3}, (6,1)_{4/3}, (8,2)_{1/2}, (3,2)_{7/6}, (3,1)_{1/3}, (1,1)_2)$. To see that the multiplet does indeed split, consider the renormalizable scalar potential of the form

$$V(\;) = m_1^2 \; {}^{ABCD} \; {}^{y}_{ABCD} + \frac{m_2^2}{v_{24}} \; {}^{ABCD} \; {}^{y}_{ABCE} (H_{24})^E_D + \frac{m_3^2}{v_{24}^2} \; {}^{ABCD} \; {}^{y}_{ABEF} (H_{24})^E_C (H_{24})^F_D ; \tag{A.1}$$

where m_is are at their natural value around the GUT scale, ^{ABCD} is antisymmetric in $A \leftrightarrow B$, $C \leftrightarrow D$, symmetric in $(AB) \leftrightarrow (CD)$ with ${}_{EABCD} \; {}^{ABCD} = 0$ and $A, \ldots, F$ are SU(5) indices. Expanding around $H_{24} = \langle H_{24} \rangle$, the mass of each component of   is

$$
\begin{aligned}
m^2_{\ 1} &= m_1^2 + \frac{1}{2}m_2^2 \quad 6m_3^2; & m^2_{\ 4} &= m_1^2 + \frac{7}{4}m_2^2 + \frac{3}{2}6m_3^2; \\
m^2_{\ 2} &= m_1^2 \quad m_2^2 + 4\,m_3^2; & m^2_{\ 5} &= m_1^2 + \frac{1}{2}m_2^2 + \frac{1}{4}m_3^2; \\
m^2_{\ 3} &= m_1^2 \quad \frac{3}{4}m_2^2 \quad m_3^2; & m^2_{\ 6} &= m_1^2 + 3\,m_2^2 + 9\,m_3^2:
\end{aligned}
\qquad \text{(A.2)}
$$

Thus by tuning m_1, m_2 and m_3, the masses of $(_1; \ \ _2) = ((\ 6; 3)\ \ _{1=3}, (6; 1)_{4=3})$ can be made arbitrary light while other the components remain heavy.

Appendix B

Relevant Observables in B Decays

Here we review the definition of various decay observables employed in our analysis. We will follow the convention of Ref. [1]. We denote an amplitude for the B-meson, B, decaying to final state f by A_f. The CP-conjugated decay is denoted by $\bar{A}_f$. Since we are interested in the s-wave 2-body decay of the B, the partial decay width is given by

$$ {}_f = \frac{1}{8}\frac{p}{m_B^2}|A_f|^2 \qquad \text{(B.1)}$$

where p is the magnitude of the 3-momentum of one of the daughter particles. The branching ratio, B, can then be computed from the above partial width.

We are also interested in the CP-violating properties of the decays. For decays of charged Bs we can define the direct CP-violation as

$$A_{CP} \quad \frac{|\bar{A}_f|^2 \quad |A_f|^2}{|\bar{A}_f|^2 + |A_f|^2}. \qquad \text{(B.2)}$$

In the case of the neutral B^0 decay where the final state f is common to both B^0 and $\bar{B}^0$ decays, we have to take into account $B^0 \quad \bar{B}^0$ mixing in defining CP-violating parameters. This occurs when f is a CP eigenstate, i.e. $f = \quad f$. The two CP-violating parameters can be defined as[1]

$$C_f \equiv \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}; \quad S_f \equiv \frac{2\mathrm{Im}(\lambda_f)}{1+|\lambda_f|^2}; \tag{B.3}$$

where

$$\lambda_f = \eta_f \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \frac{\bar{A}_f}{A_f}. \tag{B.4}$$

In case of $B^0 \to K^0 \pi^0$ decay, neutral kaon mixing contributes an extra factor of $V_{cd} V_{cs}^* / V_{cd}^* V_{cs}$ in the definition of λ_f.

[1]Here we ignore the effect of CP-violation in $B^0 \to \overline{B}^0$ mixing which is less than 1%.

Appendix C

Conventions and a basis for curvatures in six-dimensions

Throughout this paper we follow the conventions of Misner, Thorne and Wheeler [107] for the Riemann tensor. For the Weyl variation of the metric we choose

$$! \quad e^{2} \quad :$$

In nitesimally, then, $= \; 2$ and so $= 2$ (we do not use for an in nitesimal , since no confusion can arise).

It is important to classify the curvature terms of various mass dimensions. These will be used subsequently to construct all possible terms that can appear in $(\; ^{W} \quad)W$. In two and four spacetime dimensions this is very easy, but in six it becomes a rather cumbersome problem, plagued by complications due to the large number of monomials and the identities of the Riemann tensor.

A complete basis B_2 of dimension-two curvature terms that can be used in ^{W}W is given by the Ricci scalar, the Einstein tensor, and the Riemann tensor,

$$\frac{1}{d \;\; 1} R; \qquad G \;\; ; \qquad R \qquad ; \tag{C.1}$$

where we de ne the Einstein tensor as

$$G \quad = \tfrac{2}{d\ \ 2}(R \qquad \tfrac{1}{2} \quad R) \qquad (d \quad 3);$$

where R is the Ricci tensor. Taking a derivative leads to three dimension-three terms, but, by di eomorphism invariance and simple power counting, only two can be used in ${}^{W}W$, namely

$$\tfrac{1}{d\ \ 1}@R \qquad \text{and} \qquad r \quad G \quad : \tag{C.2}$$

These form the basis B_3.

At the level of dimension-four curvature terms only terms with up to two free indices are allowed in ${}^{W}W$. We consider the basis B_4 with elements

$$\begin{aligned}
E_4 &= \tfrac{2}{(d\ \ 2)(d\ \ 3)}(R \quad R \qquad 4R \quad R \quad + R^2); \\
I &= R \quad R \qquad \tfrac{4}{d\ \ 2}R \quad R \quad + \tfrac{2}{(d\ \ 1)(d\ \ 2)}R^2; \qquad \tfrac{1}{(d\ \ 1)^2}R^2; \qquad \tfrac{1}{d\ \ 1} \quad R; \\
H_1 &= \tfrac{(d\ \ 2)(d\ \ 3)}{2}E_4 \qquad 4(d \quad 1)H_2 \quad + 8H_3 \quad + 8H_4 \qquad 4R \quad R \quad ; \\
H_2 &= \tfrac{1}{d\ \ 1}RR \quad ; \qquad H_3 = R \quad R \quad ; \qquad H_4 = R \quad R \quad ; \\
H_5 &= \quad R \quad ; \qquad H_6 = \tfrac{1}{d\ \ 1}r \quad @R:
\end{aligned} \tag{C.3}$$

All $H_{1;:::;6}$ are symmetric. I is the Weyl tensor squared, $I = W \quad W$, and $\sqrt{}E_4$ is the four-dimensional Euler density. In our conventions the Weyl tensor is given by

$$W \quad = R \quad + \tfrac{2}{d\ \ 2}(\ {}_{[}\, R\, {}_{]} \ + \ {}_{[}\, R\, {}_{]}\ ) + \tfrac{2}{(d\ \ 1)(d\ \ 2)} \ {}_{[} \quad {}_{]}\, R \qquad (d \quad 3):$$

The dimension- ve curvature terms we need to consider are given by

$$@E_4; \qquad @I; \qquad \tfrac{1}{(d\ \ 1)^2}R\,@R; \qquad \tfrac{1}{d\ \ 1}@ \quad R; \qquad r \quad H_{(2;3;4)} \quad ; \tag{C.4}$$

and they form the basis B_5. Note that we do not need $r \quad H_1$, for $r \quad H_1 = 0$.[1] Similarly, $r \quad H_{(5;6)}$ are not necessary, for

$$r \quad H_5 = r \quad f(d \quad 1)H_2 \quad 2H_4 \quad \frac{1}{2} \quad [\tfrac{1}{8}(d \quad 2)^2 E_4 \quad \tfrac{d \quad 2}{4(d \quad 3)} I + \tfrac{d \quad 2}{4(d \quad 1)} R^2 \quad R]g$$

and

$$r \quad H_6 = r \quad [H_2 \quad \frac{1}{d \quad 1} \quad (\tfrac{1}{4}R^2 \quad R)]:$$

The corresponding matrix of coe cients of the remaining terms in $r \quad B_4$ has full rank, which shows that B_5 is a good basis.

Finally, a complete basis of scalar dimension-six curvature terms was constructed in [91]. Its building blocks are $K_{1;\ldots;17}$ given by

$$K_1 = R^3; \quad K_2 = RR \quad R \quad; \quad K_3 = RR \quad R \quad; \quad K_4 = R \quad R \quad R \quad;$$
$$K_5 = R \quad R \quad R \quad; \quad K_6 = R \quad R \quad R \quad; \quad K_7 = R \quad R \quad R \quad;$$
$$K_8 = R \quad R \quad R \quad; \quad K_9 = R \quad R; \quad K_{10} = R \quad R \quad;$$
$$K_{11} = R \quad R \quad; \quad K_{12} = R \quad r \quad @R; \quad K_{13} = r \quad R \quad r \quad R \quad;$$
$$K_{14} = r \quad R \quad r \quad R \quad; \quad K_{15} = r \quad R \quad r \quad R \quad;$$
$$K_{16} = \quad R^2; \quad K_{17} = \quad {}^2R:$$

At the xed point we can express the trace anomaly in the basis K's, and the consistency condition implies that there are seven combinations of K's whose coe cient has to be set to zero [91, 92]. Thus, we can arrange the K's in the basis of [92],

$$I_1 = \tfrac{19}{800}K_1 \quad \tfrac{57}{160}K_2 + \tfrac{3}{40}K_3 + \tfrac{7}{16}K_4 \quad \tfrac{9}{8}K_5 \quad \tfrac{3}{4}K_6 + K_8;$$
$$I_2 = \tfrac{9}{200}K_1 \quad \tfrac{27}{40}K_2 + \tfrac{3}{10}K_3 + \tfrac{5}{4}K_4 \quad \tfrac{3}{2}K_5 \quad 3K_6 + K_7;$$

---

[1] In any even dimension d, the Weyl variation of $\sqrt{\;}E_d$ is $\sqrt{\;}H \quad r \quad @$ for some symmetric tensor H. Since $\int d^dx \sqrt{\;}E_d = 0$, it follows that $r \quad r \quad H = 0$. In [93] it was shown, however, that H is actually divergenceless, i.e. $r \quad H = 0$.

$$
\begin{aligned}
I_3 = & -\tfrac{11}{50}K_1 + \tfrac{27}{10}K_2 - \tfrac{6}{5}K_3 - K_4 + 6K_5 + 2K_7 - 8K_8 \\
& + \tfrac{3}{5}K_9 - 6K_{10} + 6K_{11} + 3K_{13} - 6K_{14} + 3K_{15}; \\
E_6 = & \ K_1 - 12K_2 + 3K_3 + 16K_4 - 24K_5 - 24K_6 + 4K_7 + 8K_8; \\
J_1 = & \ 6K_6 - 3K_7 + 12K_8 + K_{10} - 7K_{11} - 11K_{13} + 12K_{14} - 4K_{15}; \\
J_2 = & -\tfrac{1}{5}K_9 + K_{10} + \tfrac{2}{5}K_{12} + K_{13}; \\
J_3 = & \ K_4 + K_5 - \tfrac{3}{20}K_9 + \tfrac{4}{5}K_{12} + K_{14}; \\
J_4 = & -\tfrac{1}{5}K_9 + K_{11} + \tfrac{2}{5}K_{12} + K_{15}; \\
J_5 = & \ K_{16}; \\
J_6 = & \ K_{17};
\end{aligned}
$$

which makes manifest the splitting of anomalies at the fixed point into type A (E_6) and B ($I_{1;2;3}$) according to the classification of [94], and also trivial ($J_{1;\ldots;6}$). To be more specific, $I_{1;2;3}$, which can be expressed as

$$
\begin{aligned}
I_1 = & \ W \quad W \quad W \quad ; \\
I_2 = & \ W \quad W \quad W \quad ; \\
I_3 = & \ W \quad (\quad + 4R \quad \tfrac{6}{5} \quad R)W \quad - \tfrac{2}{3}J_1 - \tfrac{13}{3}J_2 + 2J_3 + \tfrac{1}{3}J_4;
\end{aligned}
$$

lead to locally Weyl-invariant densities, while $J_{1;\ldots;6}$ can be set to zero in the trace anomaly by a choice of local counterterm, just like $\Box R$ in four dimensions. The piece $-\frac{2}{3}J_1 - \frac{13}{3}J_2 + 2J_3 + \frac{1}{3}J_4$ in I_3 is necessary for $\sqrt{\ } I_3$ to be locally Weyl-invariant [92].

For our purposes all K's are needed, since we are interested in consistency conditions valid along the RG flow. It is convenient to work in the basis B_6 given by

$$
\begin{gathered}
I_1, \qquad I_2, \qquad I_3, \qquad E_6, \\
L_1 = \tfrac{1}{30}K_1 + \tfrac{1}{4}K_2 \quad K_6, \qquad L_2 = \tfrac{1}{100}K_1 + \tfrac{1}{20}K_2, \\
L_3 = \tfrac{37}{6000}K_1 + \tfrac{7}{150}K_2 \quad \tfrac{1}{75}K_3 + \tfrac{1}{10}K_5 + \tfrac{1}{15}K_6, \qquad L_4 = \tfrac{1}{150}K_1 + \tfrac{1}{20}K_3, \\
L_5 = \tfrac{1}{30}K_1, \qquad L_6 = \tfrac{1}{300}K_1 + \tfrac{1}{20}K_9, \qquad L_7 = K_{15}, \\
J_1, \qquad J_2, \qquad J_3, \qquad J_4, \qquad \tfrac{1}{25}J_5, \qquad \tfrac{1}{25}J_6.
\end{gathered}
\tag{C.5}
$$

The form of $L_{1,\dots,6}$ is chosen based on the fact that these are the terms that shift the coe cients of the trivial anomalies at the xed point, i.e. $\int d^6x\sqrt{\bar{g}}\, L_{1,\dots,6} = \int d^6x\sqrt{\bar{g}}\, J_{1,\dots,6}$.

The choice of bases is arbitrary, and the form of the consistency conditions depends on the choice. Nevertheless, the essential conclusions derived from the consistency conditions are basis-independent.

Appendix D

Terms in the six-dimensional Weyl anomaly

In six spacetime dimensions there are ninety five independent terms that can contribute to $(\,^{W}\,)W$. We include them in this appendix for easy reference.

In general, we can write

$$^{W}W = W + \sum_{p=1}^{65}\int d^6x\sqrt{-g}\,T_p + \sum_{q=1}^{30}\int d^6x\sqrt{-g}\,\partial\,Z_q\,.$$

Clearly, the T_p and Z_q are dimension-six and dimension-five terms respectively, that can involve curvatures as well as derivatives on the couplings g^i. In writing down the various terms below, we neglect total derivatives, and we keep in mind the convenient form in which we want to obtain the consistency conditions.

If only curvatures are included, then we have the terms

$$T_1 = c_1 I_1;\qquad T_2 = c_2 I_2;\qquad T_3 = c_3 I_3;$$
$$T_4 = aE_6;\qquad T_{5,\ldots,11} = b_{1,\ldots,7} L_{1,\ldots,7}.$$

We call these the (0,6) terms, for only curvatures and derivatives on curvatures contribute to the power counting. We also have $(0;5)$ terms, given by

$$Z_1 = b_8 @E_4; \quad Z_2 = b_9 @I; \quad Z_3 = \tfrac{1}{25} b_{10} R @R;$$
$$Z_4 = \tfrac{1}{5} b_{11} @ \ R; \quad Z_{5;6;7} = b_{12;13;14} r \ H_{2;3;4}:$$

Next, we can allow one power of $@g^i$ to get

$$T_{12} = I_i^1 @g^i @E_4; \quad T_{13} = I_i^2 @g^i @I; \quad T_{14} = \tfrac{1}{25} I_i^3 @g^i R @R;$$
$$T_{15} = \tfrac{1}{5} I_i^4 @g^i @ \ R \quad T_{16;17;18} = I_i^{5;6;7} @g^i r \ H_{2;3;4};$$

which are the $(1;5)$ terms. The $(1;4)$ terms are

$$Z_8 = G_i^1 @g^i E_4; \quad Z_9 = G_i^2 @g^i I; \quad Z_{10} = \tfrac{1}{25} G_i^3 @g^i R^2;$$
$$Z_{11} = \tfrac{1}{5} G_i^4 @g^i \ R; \quad Z_{12;:::;17} = H_i^{1;:::;6} @g^i H_{1;:::;6}:$$

The $(2;4)$ terms are given by

$$T_{19} = \tfrac{1}{2} G_{ij}^1 @g^i @g^j E_4; \quad T_{20} = \tfrac{1}{2} G_{ij}^2 @g^i @g^j I; \quad T_{21} = \tfrac{1}{50} G_{ij}^3 @g^i @g^j R^2;$$
$$T_{22} = \tfrac{1}{10} G_{ij}^4 @g^i @g^j \ R; \quad T_{23;:::;28} = \tfrac{1}{2} H_{ij}^{1;:::;6} @g^i @g^j H_{1;:::;6};$$

while the $(2;3)$ terms are

$$Z_{18} = F_i r \ @g^i r \ G \ ; \quad Z_{19} = \tfrac{1}{5} E_i \ g^i @R; \quad Z_{20} = \tfrac{1}{5} E_{ij} @g^i @g^j @R:$$

The $(3;3)$ terms are

$$T_{29} = F_{ij} @g^i r \ @g^j r \ G \ ; \quad T_{30} = F_{ij}^0 @g^i r \ @g^j r \ G \ ;$$
$$T_{31} = \tfrac{1}{2} F_{ijk} @g^i @g^j @g^k r \ G \ ; \quad T_{32} = \tfrac{1}{5} \hat{E}_{ij} @g^i \ g^j @R;$$
$$T_{33} = \tfrac{1}{10} E_{ijk} @g^i @g^j @g^k @R;$$

and the $(3;2)$ terms are

$$Z_{21} = D_{ij}\ \partial g^i \nabla\ \partial g^j\ R\ ; \qquad Z_{22} = C_i\ \partial\ g^i\ G\ ;$$

$$Z_{23} = C_{ij}\ \partial g^i \nabla\ \partial g^j\ G\ ;$$

$$Z_{24} = C^0_{ij}\ \partial g^i\ g^j\ G\ ; \qquad Z_{25} = \tfrac{1}{5} B_{ij}\ \partial g^i\ g^j\ R:$$

The $(4;2)$ terms are

$$T_{34} = D_{ijk}\ \partial g^i \partial g^j \nabla\ \partial g^k\ R\ ; \qquad T_{35} = \tfrac{1}{4} D_{ijkl}\ \partial g^i \partial g^j \partial g^k \partial g^l\ R\ ;$$

$$T_{36} = \hat{C}_{ij}\ \nabla\ \partial g^i\ g^j\ G\ ; \qquad T_{37} = \tfrac{1}{2} \hat{C}^0_{ij}\ \nabla\ \partial g^i \nabla\ \partial g^j\ G\ ;$$

$$T_{38} = \tfrac{1}{2} C_{ijk}\ \partial g^i \partial g^j\ g^k\ G\ ;$$

$$T_{39} = C^0_{ijk}\ \partial g^i \partial g^j \nabla\ \partial g^k\ G\ ; \qquad T_{40} = \tfrac{1}{2} C^{00}_{ijk}\ \partial g^i \partial g^j \nabla\ \partial g^k\ G\ ;$$

$$T_{41} = \tfrac{1}{4} C_{ijkl}\ \partial g^i \partial g^j \partial g^k \partial g^l\ G\ ; \qquad T_{42} = \tfrac{1}{5} B_i\ {}^2 g^i\ R;$$

$$T_{43} = \tfrac{1}{10} \hat{B}_{ij}\ g^i\ g^j\ R;$$

$$T_{44} = \tfrac{1}{10} \hat{B}^0_{ij}\ \nabla\ \partial g^i \nabla\ \partial g^j\ R; \qquad T_{45} = \tfrac{1}{10} B_{ijk}\ \partial g^i \partial g^j\ g^k\ R;$$

$$T_{46} = \tfrac{1}{10} B^0_{ijk}\ \partial g^i \partial g^j \nabla\ \partial g^k\ R; \qquad T_{47} = \tfrac{1}{20} B_{ijkl}\ \partial g^i \partial g^j \partial g^k \partial g^l\ R;$$

and the $(5;0)$ terms are

$$Z_{26} = A_{ij}\ \partial\ g^i \nabla\ \partial g^j ; \qquad Z_{27} = A^0_{ij}\ \partial g^i\ {}^2 g^j ; \qquad Z_{28} = A_{ijk}\ \partial g^i \nabla\ \partial g^j\ g^k;$$

$$Z_{29} = A^0_{ijk}\ \partial g^i \nabla\ \partial g^j \nabla\ \partial g^k; \qquad Z_{30} = \tfrac{1}{2} A_{ijkl}\ \partial g^i \partial g^j \partial g^k\ g^l:$$

Finally, the $(6;0)$ terms are

$$T_{48} = A_i \;\; {}^3 g^i ; \qquad T_{49} = \hat{A}_{ij} \;\; {}^2 g^i \;\; g^j ; \qquad T_{50} = \tfrac{1}{2} \hat{A}^0_{ij} \; @ \;\; g^i @ \;\; g^j ;$$

$$T_{51} = \tfrac{1}{2} \hat{A}^{00}_{ij} r \;\; r \;\; @ g^i r \;\; r \;\; @ g^j ; \qquad T_{52} = \tfrac{1}{8} \hat{A}_{ijk} \;\; g^i \;\; g^j \;\; g^k ;$$

$$T_{53} = \tfrac{1}{2} \hat{A}^0_{ijk} \; r \;\; @ g^i r \;\; @ g^j r \;\; @ g^k ; \qquad T_{54} = \hat{A}^{00}_{ijk} \; @ g^i \;\; g^j @ \;\; g^k ;$$

$$T_{55} = A_{ijk} \; @ g^i r \;\; @ g^j @ \;\; g^k ; \qquad T_{56} = \tfrac{1}{2} A^0_{ijk} \; @ g^i @ g^j \;\; {}^2 g^k ;$$

$$T_{57} = \tfrac{1}{2} A^{00}_{ijk} \; @ g^i @ g^j r \;\; @ \;\; g^k ; \qquad T_{58} = \tfrac{1}{4} \hat{A}_{ijkl} \; @ g^i @ g^j \;\; g^k \;\; g^l ;$$

$$T_{59} = \tfrac{1}{4} \hat{A}^0_{ijkl} \; @ g^i @ g^j r \;\; @ g^k r \;\; @ g^l ; \qquad T_{60} = \tfrac{1}{2} \hat{A}^{00}_{ijkl} \; @ g^i @ g^j r \;\; @ g^k r \;\; @ g^l ;$$

$$T_{61} = \tfrac{1}{2} A_{ijkl} \; @ g^i @ g^j r \;\; @ g^k \;\; g^l ; \qquad T_{62} = \tfrac{1}{2} A^0_{ijkl} \; @ g^i @ g^j @ g^k r \;\; r \;\; @ g^l ;$$

$$T_{63} = \tfrac{1}{4} A_{ijklm} \; @ g^i @ g^j @ g^k @ g^l \;\; g^m ; \qquad T_{64} = \tfrac{1}{4} A^0_{ijklm} \; @ g^i @ g^j @ g^k @ g^l r \;\; @ g^m ;$$

$$T_{65} = \tfrac{1}{8} A_{ijklmn} \; @ g^i @ g^j @ g^k @ g^l @ g^m @ g^n :$$

Note that when we have more than two derivatives on a coupling only one ordering of the derivatives is independent. That is because all other orderings can be produced by commuting covariant derivatives, a process which introduces Riemann tensors or its contractions. This leads to terms with curvature tensors that we have already included.

The scalar quantities $a; b_{1;:::;14}$ and $c_{1;2;3}$ are functions of the couplings g^i. All $A; :::; I$ are also functions of the couplings, but not all of them are tensors under reparametrizations in the space of couplings, owing to the fact that $\;\; g^i$ transforms inhomogeneously under $g^i \,!\;\; g^i(g)$; more speci cally, $\;\; g^i =$ $@_j g^i \;\; g^j + @_j @_k g^i \; @ g^j \; @ g^k$. E_{ijk} in T_{33} is an example of this, because of the g^j in T_{32}.

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