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ABSTRACT

Assuming that Regge poles control high energy scattering at small momentum transfers, a firm dynamical prediction is made for the magnitude of the total $\pi\pi$ cross section and an estimate given for the corresponding width of the forward diffraction peak, starting from a knowledge of the ρ and f^0 masses and the lifetime of the ρ. The results are in satisfactory agreement with numbers inferred from high energy πN and NN scattering through the factorization theorem for residues.

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I. INTRODUCTION

Skepticism about the dominance of Regge poles in high energy scattering at small momentum transfers, in particular for the forward elastic diffraction peak, has been expressed for three reasons: (1) The widths of the diffraction peaks observed in the 10-20 GeV range have not exhibited a consistent tendency to shrink with increasing energy.[1] (2) Mandelstam has shown that there are probably branch points to the right of the poles in the angular momentum complex plane.[2] (3) When diffraction scattering is described phenomenologically in terms of the optical model the unitarity condition is found to play a major role; Regge pole parameters seem unrelated to such a constraint. It should be realized that no straightforward connection exists between these three arguments. Because the Pomeranchuk trajectory has a small slope, secondary trajectories have been shown capable at 10-20 GeV lab energies of suppressing or enhancing the slow shrinkage associated with the Pomeranchuk trajectory alone.[3] To see this logarithmic shrinkage enormously higher energies will be required, where branch points in angular momentum may well be important since their role also increases logarithmically with energy. Branch points are not needed to explain the observations at currently accessible energies. In this connection and also in relation to point (3) it should be emphasized that the Mandelstam branch points do not arise directly from the unitarity constraint but from singularities in the region where both energy and momentum transfer are large, singularities that are weak when evaluated in the strip approximation.[4,5] The gross features of high energy unitarity, such as those included in the optical model, ought to be achievable without invoking branch points in angular momentum.[6]

The extremely important possibility therefore remains open that small momentum-transfer ($\lesssim$ 2 GeV) reactions in the 10-100 GeV range of lab energies are controlled to a good approximation by a modest number of high ranking Regge poles. We present here evidence that such is in fact the case for elastic π-π scattering. Our main result is a dynamical prediction, having an estimated uncertainty $\lesssim$ 30%, of the total $\pi\pi$ cross section at high energies; the result agrees with experiment. A secondary result, less precise, is an estimate of the width of the $\pi\pi$ diffraction peak; again the experimental comparison is satisfactory.

Our calculations are based on a recent version of the strip approximation that assumes both high and low energy phenomena at low momentum transfers ($\lesssim$ 2 GeV) to be controlled by the top ranking Regge trajectories.[5] An attempt is being made to solve bootstrap equations that will generate these trajectories and the associated residues. This program is still in an early stage but we show here that if the strip approximation succeeds in calculating the masses and widths of the ρ and f^o mesons it will correctly predict both the high energy $\pi\pi$ total cross section and the width of the diffraction peak.

Since the strip approximation does not explicitly impose unitarity at high energy, how can it possibly predict the scattering there? We believe the answer to lie in the redundancy of requiring unitarity in each of the different reactions described by the analytic continuation of a connected part; unitarity in any one reaction is sufficient. Alternatively it may be sufficient to impose only low energy unitarity if this be done for all the different reactions; such is the basis of the strip approach.[7] The results of this paper show that solutions of low energy equations, when analytically continued, seem automatically to conform to high energy unitarity limitations and to generate diffraction scattering compatible with experiment. Such a circumstance may appear miraculous, but current bootstrap dynamics unavoidably is based on apparent miracles, which will become understandable only when our viewpoint is broadened in a fundamental way.

II. THE EXPERIMENTAL SITUATION

What are the facts to be explained? At the simplest level they are the magnitudes of high energy total cross sections[8] and the widths of the forward peaks in elastic scattering.[1] Unitarity, as expressed for example through the optical model, constrains the total cross section to be $\lesssim 2\pi R^2$, where $R^2 \sim (\Delta t)^{-1}$, Δt being the width of the forward peak. (Precisely, we define $(\Delta t)^{-1}$ as the logarithmic derivative at $t = 0$ of $d\sigma/dt$, if t is the negative square of momentum transfer.) It is well known that this limit is closely approached in all the systems experimentally studied. For nucleon-nucleon scattering near 20 GeV lab energy $(\Delta t)_{NN} \approx \frac{1}{10}\,\mathrm{GeV}^2$, with $\sigma_{NN}^{tot} \approx 40$ mb, while for pion-nucleon scattering[9] the total cross section is half as large and the peak slightly broader; $(\Delta t)_{\pi N} \approx \frac{1}{7}\,\mathrm{GeV}^2$.[10] There are of course differences between π^+p and π^-p and between pp and $p\bar{p}$, but such differences appear to be diminishing as the energy increases and may be ignored in a first approximation. Similarly we may temporarily ignore the small and erratic variations in energy observed[11] for the peak widths.

It is unfortunate that $\pi\pi$ scattering cannot directly be measured because the dynamical equations here are the simplest. Nevertheless if the Regge pole representation is tentatively accepted then the factorizability of the residues, as pointed out by Gell-Mann and by Gribov and Pomeranchuk,[11] allows the inference of the following high energy $\pi\pi$ total cross section and forward peak width:

$$\sigma_{\pi\pi}^{tot} = (\sigma_{\pi N}^{tot})^2/\sigma_{NN}^{tot} = 10 \text{ mb}$$

$$(\Delta t)_{\pi\pi}^{-1} = 2(\Delta t)_{\pi N}^{-1} - (\Delta t)_{NN}^{-1} \approx 4\,\mathrm{GeV}^{-1},$$

a combination which again is near the unitarity limit. The first task of the theory is to explain these two numbers, and it is in this connection that we have results to report.[12]

III. AN APPROXIMATE FORMULA FOR $\pi\pi$ RESIDUES

We proceed immediately to derive an approximate formula for $\pi\pi$ residues, appropriate to both the Pomeranchuk and ρ trajectories. The derivation employs the strip concept but does not neglect inelastic scattering, even in the low energy resonance region.

Let $A_\ell^I(s)$ be the partial wave amplitude for elastic $\pi\pi$ scattering with isotopic spin I at energy squared s, normalized to $\left[s/(s - 4m_\pi^2)\right]^{\frac{1}{2}} \exp(i\,\delta_\ell^I) \sin \delta_\ell^I$, where δ_ℓ^I is the (complex) phase shift. Following Ref. 5 we write

$$\frac{A_\ell^I(s)}{q_s^{2\ell}} = \frac{N_\ell^I(s)}{D_\ell^I(s)} = B_\ell^{I(V)}(s) + \frac{1}{\pi}\int_{s_0}^{s_1} \frac{ds'}{s' - s}\,\frac{\operatorname{Im} A_\ell^I(s')}{q_{s'}^{2\ell}}, \tag{1}$$

where $D_\ell^I(s)$ is cut only across the strip between $s_0 = 4m_\pi^2$ and s_1, $N_\ell^I(s)$ carrying all the remaining cuts. As explained in Ref. (5) the term $B_\ell^{I(V)}$ plays somewhat the role of a "potential" and may be calculated from the trajectories and residues of the leading Regge poles.

A Regge pole occurs at a zero of $D_\ell^I(s)$, i.e.

$$D^I_{\alpha_i(s)}(s) = 0, \tag{2}$$

the residue in ℓ of $A_\ell^I(s)$ being given by

$$\beta_i(s) = (q_s^{\,2})^{\alpha_i(s)} \gamma_i(s) = (q_s^{\,2})^{\alpha_i(s)} \frac{N_{\alpha_i(s)}(s)}{\left[\frac{\partial}{\partial s} D_\ell^I(s)\right]_{\ell=\alpha_i(s)}} \left(-\frac{d\alpha_i(s)}{ds}\right) . \tag{3}$$

From formula (1) it is possible to show that

$$N^I_{\alpha_i(s)}(s) = -\frac{1}{\pi} \int_{s_0}^{s_1} \frac{ds'}{s' - s} B^{I(V)}_{\alpha_i(s)}(s') \operatorname{Im} D^I_{\alpha_i(s)}(s') , \tag{4}$$

while

$$\left[\frac{\partial}{\partial s} D_\ell^I(s)\right]_{\ell=\alpha_i(s)} = \frac{1}{\pi} \int_{s_0}^{s_1} \frac{ds'}{(s' - s)^2} \operatorname{Im} D^I_{\alpha_i(s)}(s') . \tag{5}$$

Note that inelastic scattering inside the strip has not been neglected.

Now it turns out in all approximations studied so far that for both $I = 0$ and $I = 1$, when ℓ is near 1, the functions $B_\ell^{I(V)}$ and $(-\operatorname{Im} D_\ell^I)$ are all positive across the strip. Furthermore the "potential" $B_\ell^{I(V)}(s)$ varies slowly. Thus we are led to the basic approximation

$$\frac{\gamma_i(s)}{\alpha_i'(s)} \approx (\bar{s}_1 - s)\, B^{I(V)}_{\alpha_i(s)}(\bar{s}_1) , \qquad s \ll \bar{s}_1 , \tag{6}$$

where $\bar{s}_1$ is some average energy inside the strip. Calculations with plausible choices for $B_\ell^{I(V)}$ support formula (6) and suggest that $\bar{s}_1 \gtrsim 2\ \mathrm{GeV}^2$, but a precise value for $\bar{s}_1$ will not be required immediately.

IV. THE TOTAL $\pi\pi$ CROSS SECTION

Many applications of formula (6) are possible. Our first application does not require the form of $B_\ell^{I(V)}(s)$ but only the relation

$$B_\ell^{I=0(V)}(s) \approx 2\ B_\ell^{I=1(V)}(s)\ , \qquad s \lesssim s_1\ , \tag{7}$$

a result that follows from the crossing matrix if the high energy "potential" is dominated by $I = 1$ exchange. A study of $I = 0$ exchange, to be published elsewhere, confirms the usual assumption that this contribution to the force for $s < s_1$ is less important than that of $I = 1$. It is taken for granted that $I = 2$ exchange is negligible.

Assumption (7) leads to the circumstance that for the Pomeranchuk trajectory and the ρ trajectory the values of $\bar{s}_1$ are roughly the same. Noting that $\alpha_P(0) = 1$ while $\alpha_\rho(m_\rho^2) = 1$, we may combine (6) and (7) to obtain

$$\frac{\gamma_P(0)}{\alpha_P'(0)}\ \frac{\alpha_\rho'(m_\rho^2)}{\gamma_\rho(m_\rho^2)} \approx 2\,\frac{\bar{s}}{\bar{s} - m_\rho^2} \approx 2 \tag{8}$$

if $\bar{s} \gg m_\rho^2$. It is easy to verify that

$$\frac{\gamma_\rho(m_\rho^2)}{\alpha_\rho'(m_\rho^2)} \approx \frac{4\,\Gamma_\rho}{m_\rho} , \tag{9}$$

where Γ_ρ is the full width of the ρ, so we find

$$\gamma_P(0) \approx 8\,\frac{\Gamma_\rho}{m_\rho}\,\alpha_P'(0) . \tag{10}$$

The high energy $\pi\pi$ total cross section, if it is in fact controlled by the Pomeranchuk-Regge pole, is given by

$$\sigma_{\pi\pi} = 8\pi^2\,\gamma_P(0) , \tag{11}$$

so we predict finally that

$$\sigma_{\pi\pi} \approx 64\,\pi^2\,\frac{\Gamma_\rho}{m_\rho}\,\alpha_P'(0) . \tag{12}$$

The slope of the Pomeranchuk trajectory, assuming that it passes through $\ell = 2$ at the mass of the f^0, has been estimated by Ahmadzadeh and Sakmar as $\frac{1}{3}$ GeV^{-2}.[13] Taking Γ_ρ = 100 MeV and m_ρ = 750 MeV, we then find from formula (12):

$$\sigma_{\pi\pi} \approx 11 \text{ mb}.$$

V. THE WIDTH OF THE $\pi\pi$ DIFFRACTION PEAK

To estimate the width of the diffraction peak, a more specific assumption must be made about the "potential" $B_\ell^{I=0(V)}$. In general, high energy dominance of the Pomeranchuk-Regge pole leads to

$$\text{Im}\, A_{\pi\pi}(s,t) \xrightarrow[s \to \infty]{} \frac{\pi}{3}\,(2\alpha_P(t)+1)\,\gamma_P(t)\left(q_t^{\,2}\right)^{\alpha_P(t)} P_{\alpha_P(t)}\left(\frac{s}{2q_t^{\,2}}\right),$$

$$\approx \frac{\pi}{3}\,(2\alpha_P(t)+1)\,\gamma_P(t)\left(\frac{s}{2}\right)^{\alpha_P(t)}, \qquad (13)$$

for $\alpha_P(t)$ near 1.[14] Using Formula (6), we then have

$$\text{Im}\, A_{\pi\pi}(s,t) \;\propto\; (2\alpha_P(t)+1)\,\alpha_P{}'(t)\,(\bar{s}-t)\,B^{I=0(V)}_{\alpha_P(t)}(\bar{s})\left(\frac{s}{2}\right)^{\alpha_P(t)} \qquad (14)$$

The form most commonly used for $B_\ell^{I=0(V)}(s)$ is that based on exchange of a (fixed spin) ρ :

$$B_\ell^{I=0(V)}(s) \;\propto\; \left(1+\frac{s}{2q_\rho^{\,2}}\right)\frac{Q_\ell\left(1+\dfrac{m_\rho^{\,2}}{2q_s^{\,2}}\right)}{\left(q_s^{\,2}\right)^{\ell+1}}\,. \qquad (15)$$

Although we expect important deviations from this behavior when the potential is carefully calculated, the form (15) may serve to indicate the ℓ dependence of the potential, which is all we need for the shape of the diffraction peak.

For $s \sim 2\ \text{GeV}^2$, most of the ℓ dependence of (15) resides in the factor $(q_s^{\,2})^{\ell}$, and the weak remaining ℓ dependence in Q_ℓ near $\ell = 1$ is conveniently almost proportional to $(2\ell+1)^{-1}$. Thus (14) becomes

$$\text{Im}\, A_{\pi\pi}(s,t) \;\propto\; \alpha_P{}'(t)(\bar{s}-t)\left(\frac{s}{2q_s^{\,2}}\right)^{\alpha_P(t)}. \qquad (16)$$

At this point, evidently, an estimate of $\bar{s}$ is required as well as an estimate of the shape of the Pomeranchuk trajectory near $t = 0$. From preliminary calculations of trajectories and residues with a variety of "potentials" and strip widths, when the potential and strip width are adjusted to give $\alpha_P(0) = 1$, $\alpha_P'(0) \approx \frac{1}{3}\ \mathrm{GeV}^{-2}$, we find $\bar{s} \sim 2\ \mathrm{GeV}^2$. Furthermore the trajectory for $t \ll \bar{s}$, is represented roughly by the form

$$\alpha_P(t) \approx c + \frac{1-c}{1 - t/\bar{s}} , \tag{17}$$

so

$$\alpha_P'(t) \approx \frac{\alpha_P'(0)}{(1 - t/\bar{s})^2} . \tag{18}$$

Taking the logarithmic derivative of (16) at $t = 0$ we then calculate

$$a_{\pi\pi} \equiv \frac{1}{2}\,(\Delta t_{\pi\pi})^{-1} \approx (\bar{s})^{-1} + \alpha_P'(0)\,\ln\left(\frac{2s}{\bar{s}}\right) . \tag{19}$$

As explained in section II above we expect $(\Delta t_{\pi\pi})^{-1}$ to be $\sim 4\ \mathrm{GeV}^{-2}$ at an s corresponding to 20 GeV lab energy for NN scattering. This is $s \sim 40\ \mathrm{GeV}^2$, so taking $\bar{s} = 2\ \mathrm{GeV}^2$ we have from (19)

$$\begin{aligned} a_{\pi\pi} &\approx \left(0.5 + \frac{1}{3}\ln 40\right)\mathrm{GeV}^{-2} \\ &= 1.7\ \mathrm{GeV}^{-2} , \end{aligned} \tag{20}$$

not far from the expected $2\ \mathrm{GeV}^{-2}$.

It thus appears that if the strip approximation succeeds in explaining the masses and widths of the ρ and f^0 mesons it will correctly predict both the high energy $\pi\pi$ total cross section and the width of the diffraction peak.[15]

An additional result not immediately subject to experimental test is the effect of the ρ trajectory on the high energy $\pi\pi$ amplitude. Using the same approximations as above one merely adds to formula (16) a factor

$$\begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \end{pmatrix} + \begin{pmatrix} 1 \\ 1/2 \\ -1/2 \end{pmatrix} \frac{1}{2} \left(\frac{2s}{\bar{s}} \right)^{\alpha_\rho(t)-\alpha_P(t)} , \qquad (21)$$

where the column vectors have elements corresponding to $I = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$. Notice that if $\alpha_P - \alpha_\rho \lesssim 0.5$ the influence of the ρ will persist to rather high energies.

FOOTNOTES AND REFERENCES

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5. G. F. Chew, Phys. Rev. 129, 2363 (1963); G. F. Chew and C. E. Jones, (Lawrence Radiation Laboratory Report UCRL-10992, Aug. 1963), submitted to Phys. Rev.
6. We are indebted to S. Mandelstam for discussions on this point.
7. Our equations do not correspond to a neglect of inelastic discontinuities at high energy. We simply manage to avoid in this region any explicit statement about unitarity.
8. S. J. Lindenbaum, W. A. Lane, J. A. Niederer, S. Ozaki, J. J. Russell and L. C. L. Yuan, Phys. Rev. Letters 7, 184 (1961); G. von Dardel, D. Dekkers, R. Mermod, M. Vivargent, G. Weber, and K. Winter, Phys. Rev. Letters 8, 173 (1962).

9. In view of the success of the eightfold way it seems safe to assume that KN scattering will be understandable if success is achieved for the πN system.

10. This number contains a correction by Ahmadzadeh and Sakmar (Ref. 3) and by W. Rarita (private communication, Berkeley, 1964) to remove the effect of secondary trajectories. The corresponding correction in the NN case happens to be negligible.

11. M. Gell-Mann, Phys. Rev. Letters 8, 263 (1962); V. Gribov and I. Pomeranchuk, Phys. Rev. Letters 8, 343 (1962).

12. A second task will be to explain why $\sigma_{\pi N}^{tot} \approx 2\, \sigma_{\pi\pi}^{tot}$ and why $(\Delta t)_{\pi N} \approx \frac{1}{2}(\Delta t)_{\pi\pi}$. Also the tendency of the peak shape to appear exponential must be explained. If we fail on these points our success with the $\pi\pi$ system must be counted as accidental. After such gross features are dealt with, the variations of peak widths with energy and the differences between π^+p and π^-p, as well as between pp and $p\bar{p}$, requires explanation, as does the magnitude and shape of charge exchange scattering. All such questions involve secondary trajectories, and the phenomenological studies of Ref. 3, lead one to expect that insuperable difficulties will not arise. The greatest challenge in this area is to explain why the $N\bar{N}$ residue of at least one secondary pole changes sign; from our dynamical equations such a circumstance seems entirely possible.

13. A. Ahmadzadeh and I. Sakmar, Physics Letters 5, 145 (1963).

14. The real part of the amplitude is small near $t = 0$.

15. Note that even to get the correct sign for $\sigma_{\pi\pi}$ and $a_{\pi\pi}$ from a dynamical calculation of the Regge parameters is a non-trivial achievement.