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Publication Date
2019

Peer reviewed|Thesis/dissertation
Essays on Macroeconomics

A dissertation submitted in partial satisfaction
of the requirements for the degree
Doctor of Philosophy in Economics

by

Matías Vieyra

2019
ABSTRACT OF THE DISSERTATION

Essays on Macroeconomics

by

Matías Vieyra
Doctor of Philosophy in Economics
University of California, Los Angeles, 2019
Professor Ariel Tomás Burstein, Chair

This dissertation presents three contributions to the field of macroeconomics.

In the first chapter, I study the redistributive effects of monetary policy generated by differences in expenditure choices over goods. I show that necessities, which feature low expenditure elasticities, and are therefore consumed relatively more by low-expenditure households, have a higher frequency of price adjustments relative to luxuries. I develop a multisector monetary model with incomplete markets to quantify the effects of monetary shocks on consumption for households with different levels of labor income and financial wealth. The model can replicate the allocation of expenditure over goods observed in the US. Following an expansionary monetary shock, I find that households that are borrowing-constrained and have low labor income face inflation rates higher than the aggregate, and therefore increase their consumption less than the increase predicted by a model that doesn’t account for expenditure heterogeneity.

The second chapter, joint with Juliane Begenau, Saki Bigio, and Jeremy Majerovitz, presents five facts on the behavior of U.S. banks between 2007 and 2015 that impose useful restrictions on the formulation of a bank problem. (1) Market to book leverage ratio diverged significantly during the crisis. (2) Book values appear to be backward looking. There is more information content about future bank profitability and loan losses in market values than
in book values. (3) Neither market nor regulatory constraints are strictly binding for most banks. (4) Banks operate with a target market leverage ratio. (5) The adjustment behavior back to the target changed fundamentally after the crisis.

In the third chapter, also joint with Juliane Begenau, Saki Bigio, and Jeremy Majerowitz, we present ongoing work on a model that rationalizes the five facts described earlier. The goal is to produce a reduced-form partial equilibrium model that illustrates the main features that a quantitative general equilibrium model would need to match these facts. We develop an heterogeneous-bank model that rationalizes these facts and can serve as a building block for future work. An estimated version of the model successfully replicates the behaviour of market leverage and liabilities observed before and after the Great Recession.
The dissertation of Matías Vieyra is approved.

Matthew Saki Bigio Luks
Pablo David Fajgelbaum
Nico Voigtländer

Ariel Tomás Burstein, Committee Chair

University of California, Los Angeles

2019
To my wife Victoria,

for my father Raúl,

and in loving memory of Celina.
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I am deeply indebted to Ariel Burstein and Pablo Fajgelbaum for their invaluable feedback in all stages of the project. I am also eternally grateful to Saki Bigio for his guidance and support.

My work has benefited from suggestions from Andy Atkeson, Maximiliano Dvorkin, Alessio Galluzzi, Javier Garcia Cicco, François Geerolf, Gary Hansen, Rody Manuelli, Emi Nakamura, Lee Ohanian, Ernesto Pastén, Juan Sánchez, Matías Tapia, Jonathan Vogel, Gabriel Zaourak, and seminar participants at UCLA.

I gratefully acknowledge financial support from the Department of Economics and the Graduate Division, and I am thankful to the Central Bank of Chile and the Federal Reserve Bank of St. Louis, where I spent two productive summers. I also want to thank the staff of the Department of Economics at UCLA for their unwavering support, specially Jessica Perez, Grace Fransisca, Juliana Smith, and Chiara Paz.

Finally, I could not have done this without my first co-author, Victoria, a necessary and sufficient condition for convergence of this dissertation.
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CHAPTER 1

The Expenditure Channel of Monetary Policy

1.1 Introduction

Recent papers in monetary economics have turned their attention to the heterogeneous responses across agents to monetary policy. Differences in consumption plans and portfolio choices (Auclert (2017); Coibion et al. (2017); Wong (2016)) expose agents to monetary shocks that change the relative price of consumption over time or the valuations of assets. Understanding the mechanisms through which monetary policy impacts real variables is important in order to inform policy and derive robust out-of-sample predictions. Moreover, a detailed specification of these mechanisms grounded on disaggregated empirical evidence allows us to quantify the welfare implications of monetary policy.

In this paper I provide a new channel through which heterogeneity matters for the effects of monetary policy: poorer households buy relatively more of goods whose prices adjust more frequently. I build on the applied literature of demand estimation to isolate the relationship between expenditure choices across goods and a household’s level of expenditure. This helps control for demographic variables or heterogeneity of preferences. Using the estimated parameters of the demand system I find a negative correlation between expenditure elasticities for nondurable goods and services and the frequency with which prices adjust, as measured by Nakamura and Steinsson (2008).

Motivated by this finding, I quantify the effects of a monetary shock on consumption at the household level through the lens of a model that incorporates these two characteristics of goods. I build a model of heterogeneous consumers that face borrowing constraints and have preferences over goods that feature nonhomotheticity. These preferences generate a
relationship between a household’s income and wealth and her expenditure choices over goods, allowing expenditure shares to vary over the distribution. The incompleteness of financial markets has been recently emphasized by Kaplan, Moll, and Violante (2018) as a feature necessary to bridge the gap between predictions of standard monetary models for marginal propensities to consume and observed counterparts. On the supply side, I assume that firms must pay an adjustment cost in order to adjust their prices, and these nominal frictions generate heterogeneity in the responses of prices to monetary shocks.

My model predicts that, following a 0.25 percentage points expansionary monetary shock, households with low labor income and wealth experience an accumulated increase in their consumption of 0.02 percentage points less than in a model that doesn’t account for expenditure heterogeneity. This response can be decomposed as coming from two forces. First, these households face a higher inflation rate than the aggregate, because they buy relatively more of goods whose inflation rate is more responsive to the monetary shock, and therefore higher. Second, because of the borrowing constraint, they are unable to borrow in order to smooth consumption over time.

Other studies have documented the redistributive effects of monetary policy coming from different expenditure choices. Cravino, Lan, and Levchenko (2018) compute price indices along the distribution of income and find that households in the middle of the distribution tend to buy relatively more of goods whose prices adjust more frequently, whereas households at the top and bottom deciles buy goods of relatively similar rigidity. My results differ from theirs in that, instead of computing the unconditional shares of expenditure, I estimate a demand system and use a model that generates the same predicted shares. This has the advantage of been able to focus exclusively on the heterogeneity of expenditure coming from expenditure levels, as opposed to other confounding factors that can be influencing expenditure choices, such as age, family size, race, genre, number of children or homeownership. My estimates reveal a negative correlation between the average frequency of price adjustments and the level of expenditure of a household, with poorer households buying more flexibly priced goods. The main category explaining the difference between my result
and theirs is motor fuel, which they find is bought predominantly more by middle-income households, whereas I estimate to be a necessity. My estimate of this expenditure elasticity is in line with previous work in the applied literature, such as in Banks, Blundell, and Lewbel (1997).

Cravino, Lan, and Levchenko (2018) also build a model to derive structural estimations of the causal effect of monetary shocks on households inflation rates. In their model the expenditure shares are fixed, and households have homothetic preferences, so in the aggregate the economy delivers predictions that are very close to those of a representative agent New Keynesian model. Moreover, because there are no financial frictions, there is a direct mapping between these inflation rates and the heterogeneous effects on consumption. In my model, poorer households are more likely to be borrowing constrained, so the drop in consumption generated by the contractionary monetary shock cannot be smoothed.

Nonhomothetic preferences have been used at the two extremes of the frequency of analysis: static trade models and long-run growth models. My paper fills a gap by studying the implications of nonhomotheticity at the business cycle frequency. An exception to this is Ravn, Schmitt-Grohe, and Uribe (2010), who use Stone-Geary preferences to generate countercyclical markups.

Models of trade that incorporate nonhomothetic preferences include Fajgelbaum, Grossman, and Helpman (2011), Fieler (2011) and Matsuyama (2000), who study how income differences across countries can influence trade. When preferences are homothetic, gravity equations depend on income. However, income per capita is a strong predictor of trade flows and quality differentiation, whereas population is not. Nonhomotheticity allows for demand-induced patterns of specialization. Moreover, Fajgelbaum and Khandelwal (2016) use the same demand system as in this paper to quantify the gains from trade along the distribution of income.

Structural change can arise when economies develop if income elasticities differ from 1, so that as income grows, consumers spend larger fractions on services, at the expense of agriculture. Recent papers that use nonhomothetic demands to account for this relationship
between sectorial demands and income include Mestieri, Lashkari, and Comin (2015), and Boppart (2014).

On the empirical side, Hobijn and Lagakos (2005) and Hobijn et al. (2009) use differences in expenditure weights across households to document heterogeneity of inflation rates. Using scanner level data, Kaplan and Schulhofer-Wohl (2017) construct inflation rates at the household level. They show that variation in prices paid for the same goods is the main source of heterogeneity, and that studies using aggregate price indices are underestimating the dispersion of inflation rates. Finally, Coibion et al. (2017) study the effects of monetary policy shocks on consumption and income inequality, but do not account for heterogeneity of inflation rates, the channel explored in this paper. My model-derived impulse responses are consistent with their finding of regressive effects of monetary shocks.

In monetary economics, I provide a new channel through which heterogeneity matters for the effects of monetary policy: nonhomotheticity yields differences in inflation rates across households. Previous papers emphasizing the role of heterogeneity for monetary policy include Auclert (2017), who decomposes the effects of monetary shocks into thee channels (earnings heterogeneity, a Fisher effect and balance sheet redistributions), and shows how the marginal propensity to consume differs along these; and Kaplan, Moll, and Violante (2018), who show how limited participation in asset markets can generate heterogeneity of marginal propensities to consume. None of these papers account for heterogeneity of expenditure patterns across households, which I show to be relevant to understand how households are impacted differentially.

The rest of the paper is organized as follows. Section 1.2 introduces the general equilibrium model. Section 1.3 provides a simplified framework that looks at how the assumption of nonhomothetic preferences has implications for the savings decisions of the households, and discusses how it is mapped to the household level data on expenditures. Section 1.4 provides a discussion of the quantitative predictions of the model. Section 1.5 concludes.
1.2 A Monetary Model of Inflation Heterogeneity

1.2.1 Outline

I develop a New Keynesian model with multiple sectors with heterogeneity in price stickiness. I use a continuous time Hugget-Aiyagari model, so households are constrained in their ability to borrow. This implies that poorer households are less able to substitute consumption intertemporally, and so inflation has a proportional effect on the purchasing power of their income. Firms face sector-specific nominal rigidities. There is a continuum of households with distribution $\mu_t$ over state variables $(a,z)$. Each household has preferences over $G$ goods, supplies labor, receives profits from the firms and saves in one risk free bond paying real interest rate $r(t)$. A monetary authority follows a Taylor rule that aims to stabilize the aggregate inflation rate. All prices are expressed relative to this aggregate price index. I consider the perfect foresight response to an unanticipated monetary shock, understood as a zero-probability shock to the intercept of the Taylor rule.

1.2.2 Households

A household characterized by assets and idiosyncratic labor productivity $(a,z)$ solves a problem given by:

$$
\rho v_t(a,z) \equiv \max_{c,\ell} U(c,\ell) + \left[ w_t z \ell + r_t a + T_t - x(c, p_t) \right] v_{a,t}(a,z) \\
+ \mu(z) v_{z,t}(a,z) + \frac{1}{2} \sigma^2(z) v_{zz,t}(a,z) + \dot{v}(a,z)
$$

subject to:

$$
a \geq a, \ell \in [0,1], c \geq 0
$$

$$
dz_t = \mu(z_t) dt + \sigma(z_t) dB_t, \quad z \in [\underline{z}, \overline{z}]
$$

The budget constraint is composed of labor income, given by the product of the wage per efficiency units $w_t$, the idiosyncratic labor productivity $z$ (which because there’s only
one uncontingent asset, is uninsurable) and hours worked \( \ell \). Households also earn interest \( r_t \) on their assets \( a \), receive transfers \( T_t \) generated from the ownership of the firms, and pay the monetary value \( x(c, p_t) \) of their real consumption \( c \). The idiosyncratic labor productivity \( z \) follows an Ito Process with \( B_t \) a standard Brownian motion. I have generalized the household’s problem by allowing her expenditure function to be a nonlinear function of consumption \( c \), whereas with homothetic preferences typically used in the literature one would have \( x(c, p_t) \equiv P_t c \), as for example in Kaplan, Moll, and Violante (2018). Households in this economy buy \( G \) different goods, and relative prices play a nontrivial role in savings decisions, as explained in the next section. The static Hicksian demands \( \{q_{g,t}(a, z)\}_{1 \leq g \leq G} \) can be derived using Shephard’s Lemma. The next section provides a more detailed discussion of the expenditure function.

The first order conditions are:

\[
U_c(c, \ell) = v_{a,t} x_c(c, p_t) \tag{1.1}
\]

\[
-\frac{U_c(c, \ell)}{U_{\ell}(c, \ell)} = \frac{w_t z}{x_c(c, p_t)} \tag{1.2}
\]

The first condition says that the marginal utility of real consumption \( c \) has to equal the marginal utility of wealth, \( v_{a,t} \), multiplied by the relative price between wealth and consumption, given by \( x_c(c, p_t) \), which is the monetary cost of an extra unit of real consumption \( c \). Note that for homothetic preferences, \( x_c(c, p_t) = P_t \) and we get the standard formula. In this sense, \( x_c(c, p_t) \) plays the role of a household-specific deflator, because it depends on the level of real consumption \( c \). The second equation is the static condition that equalizes the marginal rate of substitution between labor and consumption to their relative price, with the caveat that the price of consumption is \( x_c(c, p_t) \).

Finally, the value function needs to satisfy the boundary conditions:

\[
v_{a,t}(a, z) \geq \frac{U_c(c(a), \ell(a))}{x_c(c(a), p_t)} \quad \text{with} \quad x(c(a), p_t) = w_t z \ell(a) + r_t a + T_t \quad \forall z, t
\]
\[ v_{z,t}(a, z) = v_{z,t}(a, z) = 0 \quad \forall a, t \]

The first condition states that, when the household’s wealth is at the constraint \( a \), the first order condition for consumption does not hold with equality. The second condition is derived from the process for \( z \) being reflected at \( z \) and \( \bar{z} \).

### 1.2.3 Firms

Goods in this economy are produced in a nested way. In each sector \( g \in \{1, ..., G\} \) there is a final producer that uses a CES technology function and sells to the households. In order to produce output \( Y_{g,t} \) it buys intermediate inputs from a continuum of monopolistic competitors, which are sector-specific.

A final good producer solves:

\[
\max \{ y_{g,t}(j) \} \quad \text{subject to} \quad Y_{g,t} = \left( \int_0^1 y_{g,t}(j) \, dj \right)^{\frac{1}{\epsilon}} \quad (1.3)
\]

where \( P_{g,t} = \left( \int_0^1 p_{g,t}(j) \, dj \right)^{\frac{1}{\epsilon}} \) is the price charged to households, and \( p_{g,t}(j) \) the price paid to the intermediate producer \( j \) of sector \( g \) for input \( y_{g,t}(j) \). This yields the demand:

\[
y_{g,t}(j) = \left( \frac{p_{g,t}(j)}{P_{g,t}} \right)^{\epsilon - 1} Y_{g,t} \quad (1.4)
\]

An intermediate producer in sector \( g \) produces using a linear production technology given by \( y_{g,t}(j) = l_{g,t}(j) \), and solves the problem:

\[
\max \{ p_{g,t}(j) \} \quad \text{subject to} \quad \Pi(p_{g,t}(j)) = \left( p_{g,t}(j) - w_t \right) y_{g,t}(j) - \frac{\lambda_g}{2} \left( \pi_{g,t} + \pi_t \right)^2 Y_{g,t} \quad (1.5)
\]

subject to:

\[
\Pi(p_{g,t}(j)) = (p_{g,t}(j) - w_t) y_{g,t}(j) - \frac{\lambda_g}{2} \left( \pi_{g,t} + \pi_t \right)^2 Y_{g,t}
\]

and the demand given by (1.4). Since the production technology is linear in labor, the
intermediate firm’s marginal cost is given by the wage $w_t$. Following Rotemberg (1982), in order to adjust the price, the firm must pay a quadratic cost, expressed in units of the final good of sector $g$. The parameter $\lambda_g$ governs the degree of price rigidities in sector $g$. Since $p_{g,t}(j)$ is expressed in units of the aggregate price index, the total inflation rate is the sum of the change in the price relative to the aggregate, given by $\pi_{g,t} \equiv \frac{p_{g,t}(j)}{p_{g,t}(j)}$, and the aggregate inflation rate, $\pi_t$. Moreover, since $p_{g,t}(j)$ is expressed in units of the aggregate price index, the firm uses the real interest rate $r_t$ (that is, the nominal rate minus the aggregate inflation rate) to discount profits. In a symmetric equilibrium we have $p_{g,t}(j) = P_{g,t}$ and $y_{g,t}(j) = Y_{g,t} = L_{g,t}$, so the pricing policy of intermediate producers of sector $g$ is:

$$
\frac{\epsilon}{\lambda_g} \left( \frac{w_t}{P_{g,t}} - \frac{\epsilon - 1}{\epsilon} \right) + \pi_{g,t} + \pi_t = \left( \pi_{g,t} + \pi_t \right) \left( r_t - \pi_{g,t} - \frac{\dot{Y}_{g,t}}{Y_{g,t}} \right)
$$

(1.6)

See Appendix 1.6.1 for the derivation. As $\lambda_g \to 0$ prices become perfectly flexible and the firms in sector $g$ charge a constant markup over the marginal cost: $P_{g,t} = \frac{\epsilon}{\epsilon-1} w_t$.

### 1.2.4 Monetary Authority

The monetary authority sets the real interest rate following a Taylor rule that depends on the aggregate inflation rate:

$$
r_t = r + (\varphi_\pi - 1) \pi_t + \varepsilon_t
$$

where $r$ is the real interest rate in the stationary distribution, $\varphi_\pi > 1$ governs the response of the nominal interest rate to aggregate inflation, and $\varepsilon_t = 0$ in the stationary distribution. The aggregate inflation rate puts weights $\omega_g$ on the inflation rates, so the weighted average of inflation rates $\pi_g$ must satisfy:

$$
\sum_{g=1}^{G} \omega_g \pi_{g,t} = 0
$$

with $\omega_g \geq 0$, $\sum_{g=1}^{G} \omega_g = 1$, since $p_{g,t}$ are relative prices (in units of the aggregate price index).
1.2.5 Equilibrium

An equilibrium in this economy is a set of paths for prices \( \{P_{g,t}\}_{1 \leq g \leq G, t \geq 0} \), quantities \( \{Y_{g,t}, L_{g,t}\}_{1 \leq g \leq G, t \geq 0} \), asset, labor, the aggregate inflation rate \( \{\pi_t\}_{t \geq 0} \), and the distribution \( \{\mu_t\}_{t \geq 0} \) such that: (1) households and firms maximize their objectives taking prices and the distribution as given; (2) the sequence of distributions satisfies aggregate consistency conditions; (3) markets clear:

- the good \( g \) market clearing condition is:
  \[
  \int q_{g,t}(a, z) d\mu_t = Y_{g,t}
  \]

- the asset market clearing condition is:
  \[
  \int a d\mu_t = 0
  \]

- the labor market clearing condition is:
  \[
  \int z \ell_t(a, z) d\mu_t = \sum_{g=1}^{G} L_{g,t}
  \]

1.3 Savings, Demand System and Estimation

Before discussing the calibration of the model, this section discusses how aggregators over goods can influence the savings decision of a household. I first show theoretically that the inflation rate that a household uses to discount real consumption in the Euler equation depends on static income elasticities, and so does the intertemporal elasticity of substitution. Next, I specify preferences using a demand system that has a long tradition in the applied literature, and discuss the data used and estimation procedures for the parameters of the demand system.
1.3.1 A Generalized Euler Equation

Consider a simplified version of the model of Section 1.2, where $a \to -\infty$, so there are no borrowing constraints and the Euler equation holds with equality for all households. Moreover, I assume that households supply all their labor inelastically and denote by $W_h^0$ the present discounted value of their human and financial wealth. Given the assumption of time-separable preferences, household $h$’s utility maximization problem can be decomposed into two stages. In the first stage, the household solves a static expenditure minimization problem:

$$x(c_h, p_t) \equiv \min_{q_h^t = \{q_{h,s}^t\}, 1 \leq s \leq G} \sum_{g=1}^{G} p_{g,t} q_{g,t}^h \quad \text{subject to } u(q_t^h) = c_h$$

where $c_h^t$ is household $h$’s real consumption at period $t$. The utility function $u$ represents preferences over goods $g \in \{1, ..., G\}$. In the second stage, household $h$ chooses it’s intertemporal allocation of consumption by solving:

$$\max_{\{c_t^h\}_{t \geq 0}} \int_{0}^{\infty} e^{-\rho t} U(c_t^h) dt$$

subject to:

$$\int_{0}^{\infty} \exp \left( - \int_{0}^{\tau} i_t d\tau \right) x(c_t^h, p_t) dt = W_h^0$$

where $i_t$ is the nominal interest rate. The utility function $U$ represents preferences over real consumption over time $c_t^h$. Note that $U$ is irrelevant for the static minimization problem, but separating preferences in this nested fashion using $u$ and $U$ facilitates discussion.

Consider the tradeoff that household $h$ faces when deciding how much to consume in periods $t$ and $t + \Delta$. A discrete time Euler equation reads:

$$\frac{U_c(c_{t+\Delta}^h)}{U_c(c_t^h)} = e^{-\rho \Delta} \frac{1 + i_t}{x_c(c_{t+\Delta}^h, p_{t+\Delta})/x_c(c_t^h, p_t)}$$

We can see that the nominal interest rate is discounted by the change in the marginal cost of consumption, $x_c(c_t^h, p_t)$, which measures the monetary cost of acquiring an extra unit
of real consumption $c_t^h$.

To settle ideas, consider the case where $u$ represents constant elasticity of substitution (CES) preferences over goods. Then,

$$x(c_t^h, p_t) = \left[ \sum_{g=1}^{G} (p_{g,t})^{1-\sigma} \right]^{\frac{1}{1-\sigma}} c_t^h \equiv P_t c_t^h$$

In this case, real consumption $c_t^h = x(c_t^h, p_t)/P_t$ has the usual expression of expenditure divided by a price index. The Euler equation reads:

$$\frac{U_c(c_t^h + \Delta)}{U_c(c_t^h)} = e^{-\rho \Delta} \frac{1 + i_t}{P_{t+\Delta}/P_t}$$

and we see that the inflation rate used to discount the nominal interest rate is the same for all households. Moreover, the marginal cost of consumption $x_c(c_t^h, p_t)$ and the ideal price index (which measures the average cost of real consumption) are both equal to $P_t$, and there is no difference between the average and marginal dollars spent. The following lemma generalizes the Euler equation for any preference specification.

**Lemma 1.1.** The Euler equation can be characterized in terms of expenditure elasticities as:

$$\frac{\dot{c}_t^h}{c_t^h} = \frac{i_t - \rho - \frac{1}{\varepsilon_x} \pi_t^h}{-U''(c_t^h) c_t^h / U'(c_t^h) - \sigma(c_t^h, p_t)}$$

where:

$$\pi_t^h \equiv \sum_{g=1}^{G} \omega_g(c_t^h, p_t) \varepsilon_x^g(c_t^h, p_t) \frac{\dot{p}_{g,t}}{p_{g,t}}$$

$$\sigma(c_t^h, p_t) \equiv \sum_{g=1}^{G} \omega_g(c_t^h, p_t) \sum_{j=1}^{G} \frac{-u_{gj}(q_{j,t}) q_{j,t}^h}{u_g(q_t^h)} \varepsilon_x^g(c_t^h, p_t)$$

the budget shares are $\omega_t^h = p_{g,t} q_{g,t}^h / \sum_{j=1}^{G} p_{j,t} q_{j,t}^h$, and $\varepsilon_x^y$ represents the elasticity of $x$ with respect to $y$.

We can see that the inflation rate relevant for household $h$ is a weighted average of the
changes in the good prices, with the weights equal to the product of the budget shares and the elasticities of the Hicksian demands with respect to real consumption. Note that, with homothetic preferences, these elasticities are all 1, and the budget shares are independent of the level of \( c_t \), so the inflation rates are identical across households. The consumption elasticities are an adjustment that accounts for the fact that, when deciding how much to consume in a given period, the household must compute the cost of the goods that will be bought in the margin, and not that of those bought on average. In turn, the intertemporal elasticity of substitution (IES) is a weighted average of the real consumption elasticities, with weights depending on the good-specific IES and the budget shares.

A simple example helps to illustrate the distinction between the average and the marginal cost of consumption. Consider a household whose preferences are quasilinear. This implies that, at any given period \( t \), every extra dollar will be spent entirely on the linear good (because the income effect for all other goods is 0)\(^1\). Then, this household’s Euler equation will only use the inflation rate of the linear good to discount the interest rate. However, if one were to use the budget shares as weights of the inflation rate, then all inflation rates would enter the Euler equation, because the household is buying positive amounts of all the \( G \) goods.

Quasilinear preferences represent a case where the distinction between the marginal and the average cost of consumption is most extreme. The relevance of this distinction is ultimately quantitative and addressed in the next subsection.

1.3.2 Estimating A Demand System

For the remainder of the paper, I assume that the expenditure function is derived from the Almost Ideal Demand System, given by:

\[
x(c, p_t) = a(p_t)c^{b(p_t)}
\]

\(^1\)Provided that the solution is interior, which I assume here for the sake of the argument
These preferences were first introduced in Deaton and Muellbauer (1980), and have convenient properties. First, the homothetic case is nested by setting $b(p_t) = 1$, and so the linearity of the Engel curves can be tested. Second, they are a flexible functional form, in that any other expenditure function will coincide up to a second order approximation. Third, the demands written in budget share form are approximately linear in the parameters of the system.

As is standard in the applied literature, $a(p_t)$ and $b(p_t)$ are parametrized as Translog price indices that depend on the entire vector of prices:

$$\ln a(p_t) = \alpha_0 + \sum_{g=1}^{G} \tilde{\alpha}_g \ln p_{g,t} + \frac{1}{2} \sum_{g=1}^{G} \sum_{j=1}^{G} \gamma_{gj} \ln p_{j,t} \ln p_{g,t}$$

$$\ln b(p_t) = \sum_{g=1}^{G} \beta_g \ln p_{g,t}$$

with $\sum_{g=1}^{G} \alpha_g = 1$, $\sum_{j=1}^{G} \gamma_{gj} = 0$, $\gamma_{gj} = \gamma_{jg}$, $\sum_{g=1}^{G} \beta_g = 0$. The budget shares derived from these preferences are:

$$\omega_{gh}(c_{ht}, p_t) = \tilde{\alpha}_g + \sum_{j=1}^{G} \gamma_{gj} \ln p_{j,t} + \beta_g \ln \frac{x(c_{ht}, p_t)}{a(p_t)}$$

and the inflation rate in the Euler equation with these preferences reads:

$$\pi_t^h = \sum_{j=1}^{G} \left[ \omega_{gh}(c_{ht}', p_t) + \beta_g \right] \frac{\dot{p}_{g,t}}{p_{g,t}}$$

Note that $\beta_g$ controls the slope of the Engel curves. If they are zero, budget shares are independent of expenditure, the second term in the weights of inflation drop, and the budget shares become independent of consumption, as with homothetic preferences.

In order to estimate the parameters of this demand system, I allow $\tilde{\alpha}_g$ to depend on demographic characteristics $D_t^h$:

$$\tilde{\alpha}_g \equiv \alpha_g + D_t^h \eta_g$$
where demographic characteristics include age, race, marital status, education level, and sex of the head of the household, family type (single parent, married couple, retired, etc.) and size, age of the children, and dummies for homeowners, vehicle owners, quarters and years.

As discussed below, the CES does not provide information on prices paid by households, and so I am constrained to use aggregate price indices. This generates imprecise estimates of the price elasticities, because of the limited variation of these indices. Moreover, without any restrictions on \(\{\gamma_{gj}\} \) one would have to estimate \(G(G + 1)\) parameters, generating what is known in the literature as the "too many parameters" problem. For these reasons, I assume:

\[
\gamma_{gj} = \begin{cases} 
(1 - \frac{1}{G}) \gamma_g & \text{if } g = j \\
-\frac{\gamma_g}{G} & \text{if } g \neq j
\end{cases}
\]

With these assumptions, the estimating equation is:

\[
\omega_{g,t}^h = \alpha_g + D_t^h \eta_g + \gamma_g \left[ \ln p_{g,t} - \frac{1}{G} \sum_{j=1}^{G} \ln p_{j,t} \right] + \beta_g \ln \frac{x_t^h}{a(p_t)} + \nu_t
\]

where \(\nu_t\) is measurement error with mean zero. I use an iterative feasible generalized least squares procedure, which in the context of seemingly unrelated equations is equivalent to maximum likelihood estimation. For the first iteration I approximate \(a(p_t)\) with Stone’s Price Index, which takes the form \(\sum_{g=1}^{G} \omega_g \ln p_{g,t}\), with \(\omega_g\) the aggregate shares for category \(g\). Subsequently, I replace \(a(p_t)\) with the values for \(\{\alpha_g, \eta_g, \gamma_g\}\) from the previous iteration.

1.3.2.1 Data

I use household-level expenditure data from the Consumer Expenditure Survey (CES). This is a rotating panel where each quarter about 6,000 households are asked to report expenditure, income and demographic characteristics. The data are composed of two separate surveys, Interview (which focuses on large expenditures) and Diary (which tracks smaller, more frequent expenditures). No household reports information for both surveys, yet esti-
mation of (1.7) requires household-level variables, such as expenditure and the demographic controls. For this reason, I use exclusively the Interview Survey, which nonetheless covers about 95% of expenditures. Moreover, I drop categories of good corresponding to durables, since these are purchased infrequently and may not be so responsive to transitory changes in prices generated by monetary shocks. These leaves about 80% of all expenditures. As in Aguiar and Bils (2015), I restrict the sample to urban households whose head is between 25 and 64 years old, and similar to them, I require that households report positive expenditures in at least 15 out of 36 categories. I also truncate the sample by dropping households at the top and bottom 5% of the distribution of expenditure, to avoid the problems of outliers and top-coding. The final sample is composed of 57,961 quarter-household observations over the period 2010-2016.

Estimation of (1.7) requires prices, which are not reported in the CES. In order to match goods in the CES to the price indices provided by the Bureau of Labor Statistics (BLS), I manually match Classes in the CES and the BLS. Even though the frequencies of price adjustments reported by Nakamura and Steinsson (2008) are disaggregated by Entry Level Item (ELI), price indices reported by the BLS are not found at this level of disaggregation. Moreover, using a very disaggregated level increases substantially the percentage of households that report zero expenditure in a large number of categories. Finally, I restrict the sample to 2010-2016 because prior to 2010 some price indices are not published by the BLS.

1.3.2.2 Results

Table 1.1 reports the estimates for the expenditure semi-elasticities $\beta_g$ for the $G = 36$ categories of goods, sorted by the share of aggregate expenditure. Recall that goods with $\beta_g < 0$ are necessities, and therefore consumed relatively more by households with low expenditure, whereas goods with $\beta_g > 0$ are luxuries, and consumed relatively more by households with high expenditure. Among the largest categories as measured by the share of aggregate expenditure, some of the categories that are estimated to be necessities are Motor fuel, Gas (piped) and electricity, and Telephone services. On the other hand, categories
Table 1.1: Expenditure Semi-elasticities $\beta_g$ estimated from Equation (1.7)

<table>
<thead>
<tr>
<th>Description</th>
<th>$\beta_g \times 100$</th>
<th>p-value</th>
<th>Agg. Share</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food away from home</td>
<td>0.136</td>
<td>0.00</td>
<td>11.2%</td>
</tr>
<tr>
<td>Motor fuel</td>
<td>-0.424</td>
<td>0.00</td>
<td>11.0%</td>
</tr>
<tr>
<td>Hospital and related services</td>
<td>0.024</td>
<td>0.40</td>
<td>10.2%</td>
</tr>
<tr>
<td>Gas (piped) and electricity</td>
<td>-0.381</td>
<td>0.00</td>
<td>7.4%</td>
</tr>
<tr>
<td>Telephone services</td>
<td>-0.283</td>
<td>0.00</td>
<td>5.7%</td>
</tr>
<tr>
<td>Tuition, other school fees, and childcare</td>
<td>0.547</td>
<td>0.00</td>
<td>5.1%</td>
</tr>
<tr>
<td>Professional services</td>
<td>0.094</td>
<td>0.00</td>
<td>4.8%</td>
</tr>
<tr>
<td>Motor vehicle insurance</td>
<td>-0.157</td>
<td>0.00</td>
<td>4.4%</td>
</tr>
<tr>
<td>Recreation services</td>
<td>0.148</td>
<td>0.00</td>
<td>3.5%</td>
</tr>
<tr>
<td>Household operations</td>
<td>0.271</td>
<td>0.00</td>
<td>3.4%</td>
</tr>
<tr>
<td>Information technology, hardware and services</td>
<td>-0.141</td>
<td>0.00</td>
<td>3.1%</td>
</tr>
<tr>
<td>Public transportation</td>
<td>0.126</td>
<td>0.00</td>
<td>2.9%</td>
</tr>
<tr>
<td>Motor vehicle maintenance and repair</td>
<td>0.004</td>
<td>0.72</td>
<td>2.8%</td>
</tr>
<tr>
<td>Water and sewer and trash collection services</td>
<td>-0.124</td>
<td>0.00</td>
<td>2.4%</td>
</tr>
<tr>
<td>Lodging away from home</td>
<td>0.149</td>
<td>0.00</td>
<td>2.3%</td>
</tr>
<tr>
<td>‘Tenants’ and household insurance</td>
<td>-0.063</td>
<td>0.00</td>
<td>2.0%</td>
</tr>
<tr>
<td>Women’s apparel</td>
<td>0.003</td>
<td>0.54</td>
<td>1.9%</td>
</tr>
<tr>
<td>Miscellaneous personal services</td>
<td>0.118</td>
<td>0.00</td>
<td>1.8%</td>
</tr>
<tr>
<td>Pets, pet products and services</td>
<td>0.003</td>
<td>0.75</td>
<td>1.7%</td>
</tr>
<tr>
<td>Personal care services</td>
<td>0.004</td>
<td>0.31</td>
<td>1.6%</td>
</tr>
<tr>
<td>Motor vehicle fees</td>
<td>-0.010</td>
<td>0.02</td>
<td>1.3%</td>
</tr>
<tr>
<td>Tobacco and smoking products</td>
<td>-0.088</td>
<td>0.00</td>
<td>1.3%</td>
</tr>
<tr>
<td>Alcoholic beverages at home</td>
<td>-0.016</td>
<td>0.00</td>
<td>1.1%</td>
</tr>
<tr>
<td>Men’s apparel</td>
<td>0.018</td>
<td>0.00</td>
<td>1.0%</td>
</tr>
<tr>
<td>Window and floor coverings and other linens</td>
<td>0.019</td>
<td>0.03</td>
<td>0.9%</td>
</tr>
<tr>
<td>Footwear</td>
<td>-0.007</td>
<td>0.01</td>
<td>0.9%</td>
</tr>
<tr>
<td>Alcoholic beverages away from home</td>
<td>0.043</td>
<td>0.00</td>
<td>0.9%</td>
</tr>
<tr>
<td>Fuel oil and other fuels</td>
<td>-0.015</td>
<td>0.02</td>
<td>0.6%</td>
</tr>
<tr>
<td>Jewelry and watches</td>
<td>0.023</td>
<td>0.00</td>
<td>0.5%</td>
</tr>
<tr>
<td>Recreational reading materials</td>
<td>-0.007</td>
<td>0.00</td>
<td>0.5%</td>
</tr>
<tr>
<td>Girls’ apparel</td>
<td>-0.002</td>
<td>0.42</td>
<td>0.4%</td>
</tr>
<tr>
<td>Boys’ apparel</td>
<td>-0.005</td>
<td>0.03</td>
<td>0.4%</td>
</tr>
<tr>
<td>Educational books and supplies</td>
<td>0.004</td>
<td>0.10</td>
<td>0.4%</td>
</tr>
<tr>
<td>Infants’ and toddlers’ apparel</td>
<td>-0.010</td>
<td>0.00</td>
<td>0.3%</td>
</tr>
<tr>
<td>Miscellaneous personal goods</td>
<td>0.001</td>
<td>0.18</td>
<td>0.1%</td>
</tr>
<tr>
<td>Personal care products</td>
<td>-0.002</td>
<td>0.01</td>
<td>0.1%</td>
</tr>
</tbody>
</table>
estimated to be luxuries are Food away from home, Hospital and related services, and Tuition, other school fees, and childcare. Most of the $\beta$'s are statistically significant.

1.4 Quantitative Results

I now embed the Almost Ideal Demand System into the model of Section 1.2, and study the heterogeneous effects of a monetary shock on consumption across households.

1.4.1 Calibration

The flow utility is specified as in Greenwood, Hercowitz, and Huffman (1988):

$$u(c, \ell) = \frac{1}{1-\gamma} \left( c - \psi z \frac{\ell^{1+\varphi}}{1 + \varphi} \right)^{1-\gamma}$$

so that assumptions about preferences over goods generates no income effects for labor supply. As in Kaplan, Moll, and Violante (2016), I set $\gamma = 1$ and $\varphi = 1$, so the elasticity of labor supply is 1. Note however that because of preferences being GHH, the IES is lower than 1, and close to 0.55 on average in the stationary distribution.

The disutility of labor is scaled by $z$, the uninsurable idiosyncratic labor productivity, so that the choice of hours worked is not affected by it. This allows to have a direct mapping from $z$ to income in the data. Henceforth, $z$ is assumed to follow an Ohrstein Ullenbeck stochastic process, the continuous time equivalent of an AR(1):

$$dz_t = -\mu \ln z_t dt + \sigma dB_t$$

where $B_t$ is a standard Brownian Motion. As in Aiyagari (1994), I set $\mu = 0.15$ so that annual incomes have an autocorrelation of 0.86 and $\sigma = 0.01$ so that the annual standard deviation of incomes is 0.15.

In the stationary distribution we have $\pi_g = \pi = 0$, so $P_g = \frac{\epsilon}{\epsilon - 1} w$. Normalizing $P_g = 1$
we get $w = \frac{\epsilon - 1}{\epsilon}$. Henceforth, as is standard in the New Keynesian literature, I set $\epsilon = 0.1$ to generate markups of 11%. In order to calibrate the parameters $\lambda_g$ governing the degrees of price rigidities, note that to a first order approximation, the pricing policy using quadratic adjustment costs coincides with the one derived under Calvo price stickiness (where firms face Poisson arrival opportunities to adjust prices). Therefore, I match the coefficients associated with the marginal cost in both pricing equations. Let $\theta_g$ be the frequency of price adjustments under Calvo pricing. Then,

$$\frac{\epsilon}{\lambda_g} = \frac{\theta_g (1 - e^{-\rho} (1 - \theta_g))}{1 - \theta_g}$$

I obtain $\theta_g$ from the frequencies of price adjustments for regular prices estimated by Nakamura and Steinsson (2008). They report these frequencies by ELI, whereas my 36 categories correspond to classes. To compute class-level frequencies I average ELI-level frequencies weighting them by the shares of expenditure of each ELI.

Given the normalization $P_g = 1$, we have $a(p) = b(p) = 1$ and $x(c, p) = c$. The first order condition (1.2) for the marginal rate of substitution between consumption and labor in the stationary distribution becomes $\psi \ell^c = w$. Given $\varphi = 1$, I set $\psi = 2.7$ so that households work $1/3$ of their unit endowment of time in the stationary equilibrium. I set $\rho = 0.0052$ so that the annual risk free real rate is 2%. As in Kaplan, Moll, and Violante (2018), I make transfers $T_t$ proportional to labor productivity $z$ and set the borrowing limit $a$ equal to average quarterly labor income.

For the Taylor rule, I assume that $\varphi_\pi = 1.25$, a standard value used in the literature, and that the weights $\omega_g$ used by the monetary authority to compute aggregate inflation correspond to the aggregate expenditure shares observed in the stationary equilibrium. The monetary shock to be studied is $\varepsilon_0 = -1\%$ with $\varepsilon_t = e^{-0.5t}\varepsilon_0$, as in Kaplan, Moll, and Violante (2018).
1.4.2 Frequency of Price Adjustments and Expenditure Elasticities

The degree to which households along the distribution will be impacted by the monetary shock depends on how exposed they are to the goods whose prices are more sensitive to it. In this model, the elasticity of the inflation rate with respect to the monetary shock is governed by the parameter $\lambda_g$, which in turn is inversely related to the frequency of price adjustment $\theta_g$. A poorer household will spend relatively more on goods with $\beta_g < 0$, so we should expect to see regressive effects from monetary shocks if the expenditure semi-elasticities $\beta_g$ are negatively correlated with the frequencies of price adjustments $\theta_g$. Figure (1.1) shows that this is precisely the case:

Figure 1.1: Frequency of Price Adjustments and Expenditure Elasticities

The horizontal axis shows the expenditure semi-elasticities $\beta_g$ estimated using the procedure from Subsection (1.3.2.2). The vertical axis shows the frequencies of price adjustment for regular prices $\theta_g$ estimated by Nakamura and Steinsson (2008). A value of 50 means that, for that category, 50% of prices are changing any given month. The size of the circle is determined by the aggregate share of expenditure. Labels are reported for categories with shares above 2.5%.

Necessities such as Motor fuel and Gas and electricity have a high frequency of price adjustments, whereas luxuries such as Tuition, other school fees, and childcare, and Household operations (e.g., home maintenance and repair services, domestic services) are infrequently adjusted.
1.4.3 Inflation Heterogeneity and Consumption Responses

The response of aggregate variables to the monetary shock is in line with the results of the literature. After the 0.25% expansionary shock, aggregate expenditure, labor and inflation, and the real wage increase.

I now turn attention to the model’s prediction for inflation rates across goods. Figure 1.2 shows the response of inflation rates $\pi_g$ after the shock. Recall that these inflation rates are deviations from the aggregate inflation rate $\pi$, since prices are expressed in units of the aggregate price index. As expected, the goods with lower $\lambda_g$ values, corresponding to lower costs of price adjustments, have higher inflation rates $\pi_g$. Goods with inflation rates higher than the aggregate rate include Lodging away from home, Fuel oil, Gas and electricity, Motor fuel and Public transportation (which includes airline fares).

Figure 1.2: Heterogeneity of Inflation Rates $\pi_g$ After a 1% Expansionary Shock

How are households with different levels of wealth $a$ and productivity $z$ impacted by the monetary shock? In order to answer this question I track the dynamics of real consumption $c$ for four households: (1) a household that has wealth $a = a_0$ and productivity $z = z_0$; (2) a household that has wealth $a = a_0$ and productivity $z = \overline{z}$; (3) a household that has high wealth $a > 0$ and productivity $z = z_0$; (4) and a household that has high wealth $a > 0$ and productivity $z = \overline{z}$. 

20
\( a > 0 \) and productivity \( z = \tau \). This categorization allows to separately understand the roles played by income and the borrowing constraint on the cross sectional response of consumption. Moreover, in order to understand the role of expenditure heterogeneity, I consider the difference in the response of consumption for two models: the one outlined in this section, and one where preferences are homothetic, so \( \beta_g = 0 \ \forall \ g \). Figure 1.3 shows this difference divided by the household’s consumption in the stationary equilibrium. Formally, it plots the paths of:

\[
\frac{c_t^{\{\beta_g \neq 0\}}(a, z) - c_t^{\{St\}}(a, z)}{c_t^{\{St\}}(a, z)} - \frac{c_t^{\{\beta_g = 0\}}(a, z) - c_t^{\{St\}}(a, z)}{c_t^{\{St\}}(a, z)}
\]

where \( c_t^{\{\beta_g \neq 0\}}(a, z) \) is the value of consumption at period \( t \) for a household with state variables \( (a, z) \) in the model with nonhomothetic preferences, \( c_t^{\{\beta_g = 0\}}(a, z) \) corresponds to the model with homothetic preferences, and \( c_t^{\{St\}}(a, z) \) to the stationary equilibrium.

Figure 1.3: Differential Response of Consumption with/without Nonhomothetic Preferences

The model with nonhomothetic preferences delivers the prediction that households with low wealth and low productivity are worse off throughout the entire transition back to the stationary equilibrium.

---

\(^2\)In the stationary equilibrium we have \( a(p) = b(p) = 1 \), so both models with homothetic and nonhomothetic preferences deliver the same outcome.
stationary equilibrium, relative to a model with homothetic preferences. This is explained by the fact that these households suffer higher inflation, and cannot smooth out over time the expansionary effects of the monetary shock because of the borrowing constraint. In contrast, households that either have high wealth or high productivity (or both) are better suited to smooth consumption over time. In fact, the difference in the response of consumption across models, with consumption in the model with nonhomothetic preferences lower in the first quarters and higher thereafter, resembles the difference in aggregate expenditure across models. In other words, these richer households react similar to aggregate variables.

1.5 Conclusion

I have relaxed the assumption of homothetic preferences over goods, and estimated expenditure and price elasticities using a flexible demand system. I then showed how these elasticities influence the response of expenditure and savings to a monetary shock. A key concept, previously overlooked in the literature, is the distinction between the valuation of the average and the marginal dollars spent. Thinking about savings decisions in the margin has the implication that one needs to adjust the weights on goods inflation rates by expenditure elasticities, and this uncovers heterogeneity in inflation rates across households after monetary shocks.

This model can be used to address normative and positive problems associated with the effects of monetary policy. How should a central bank that has redistributive concerns conduct monetary policy? And in particular, should it target an inflation rate that puts weights on good prices different from those of the aggregate price index? On the positive side, what would be the effect of monetary policy on aggregate expenditure if the distribution of wealth were more/less unequal than the one observed? How would monetary policy impact aggregate variables if services represented a larger share of the economy?

An alternative way to incorporate expenditure heterogeneity is explored in Cravino, Lan, and Levchenko (2018). They assume that there are many households with heterogeneous
preferences, with the parameters governing the expenditure shares jointly distributed with the household’s labor productivity. This provides a computational challenge in the context of models where households have endogenous state variables, as one would have to solve an HJB equation for each type of preferences. My model provides an alternative that circumvents this complication.

The predicted differences in consumption across the distribution of households are admittedly small, given the small effects on relative prices generated by the monetary shock. Future work could address the welfare implications of large variations in exchange rates, which are typically associated with significant changes in relative prices, in particular between tradable and nontradable goods. Moreover, in this model I have muted the effects of demographic characteristics by aggregating over them, yet one could be interested in understanding how households with, for instance, different age profiles, are affected by monetary policy.
1.6 Model Appendix

1.6.1 Intermediate Firm’s Problem

This section derives the pricing policy of an intermediate firm. Firm \(j\) in sector \(g\) faces nominal rigidity \(\lambda_g\) and solves the problem:

\[
\max_{\{p_{g,t}(j)\}_{t \geq 0}} \int_0^\infty e^{-\int_0^t r_s ds} \Pi (p_{g,t}(j)) \, dt
\]

subject to:

\[
\Pi (p_{g,t}(j)) = (p_{g,t}(j) - w_t) y_{g,t}(j) - \frac{\lambda_g}{2} (\pi_{g,t} + \pi_t)^2 Y_{g,t}
\]

and the demand given by (1.4). The Hamiltonian is:

\[
H (p_g(j), \dot{p}_g(j), \eta) \equiv (p_g(j) - w) \left( \frac{p_g(j)}{P_g} \right)^{-\epsilon} Y_g - \frac{\lambda_g}{2} \left( \frac{\dot{p}_g(j)}{p_g(j)} + \pi \right)^2 P_g Y_g + \eta \dot{p}_g(j)
\]

The first order conditions are:

\[
\left( \frac{p_g(j)}{P_g} \right)^{-\epsilon} Y_g - \epsilon (p_g(j) - w) \left( \frac{p_g(j)}{P_g} \right)^{-\epsilon - 1} Y_g P_g + \lambda_g \left( \frac{\dot{p}_g(j)}{p_g(j)} + \pi \right) \frac{\dot{p}_g(j)}{p_g(j)} P_g Y_g = r \eta - \dot{\eta}
\]

\[
\eta = \lambda_g \left( \frac{\dot{p}_g(j)}{p_g(j)} + \pi \right) \frac{P_g Y_g}{p_g(j)}
\]

In a symmetric equilibrium \(p_g(j,t) = P_g(t)\), so:

\[
Y_g \left[ 1 - \epsilon (P_g - w) \frac{1}{P_g} + \lambda_g \pi_g (\pi_g + \pi) \right] = r \eta - \dot{\eta}
\]

\[
\eta = \lambda_g (\pi_g + \pi) Y_g \Rightarrow \dot{\eta} = \lambda_g (\dot{\pi}_g + \dot{\pi}) Y_g + \lambda_g (\pi_g + \pi) \dot{Y}_g
\]
Substituting out $\eta$ and dividing by $\lambda g Y_g$ yields equation (1.6).

1.7 Numerical Appendix

1.7.1 Stationary Equilibrium

This describes the algorithm used to solve the stationary equilibrium. Both here and for the transition the HJB of the household and the Kolmogorov forward equation are computed using the methods described in Achdou et al. (2017).

1. Set $P_g = 1\forall g$ and $w = ((\varepsilon - 1)/\varepsilon)$. Guess $r$.
2. Compute aggregate labor:
   \[ L = [(w/\psi)]^{(1/\varphi)} \]
3. Compute transfers:
   \[ T = \frac{1}{\varepsilon} L \]
4. Solve the household’s problem.
5. Aggregating individual’s policy function compute implied savings:
   \[ S_t = \int a d\mu_t \]
6. If $S_t$ is close to 0, end. Else, update $r$ using a relaxation method and return to step 4.

1.7.2 Transitional Dynamics

1. Guess $r_{tT}^T$, which is updated in every iteration, and $\{P_{g,t}\}, T_{tT}^T$, which are only used in the first iteration.
2. Use the Taylor rule to compute the aggregate inflation rate $\pi_t$. 

25
3. Use an aggregate version of the pricing policies to compute $w_t$. To derive it, multiply the pricing policies (1.6) by $\omega_g$ and sum over $g$. Using $\sum_{g=1}^{G} \omega_g \pi_{g,t} = \sum_{g=1}^{G} \omega_g \dot{\pi}_{g,t} = 0$ yields:

$$0 = \sum_{g=1}^{G} \omega_g \frac{\epsilon}{\lambda_g} \left( \frac{w_t}{P_{g,t}} - \frac{\epsilon - 1}{\epsilon} \right) + \dot{\pi} - \pi r$$

which can be solved backwards with terminal condition $w_T = \bar{w}$.

4. Use the pricing policies of the firms to compute the inflation rates $\pi_g$.

5. Solve the household’s problem.

6. Aggregating individual’s policy function compute implied savings and transfers.

7. If $S_t$ is close to 0, end. Else, update $r_t$ using:

$$r_t = r + \zeta_S dS_t$$

where $\zeta_S > 0$, and return to step 2.
CHAPTER 2

Banks Adjust Slowly: Evidence and Lessons for Modelling

2.1 Introduction

Since the Great Financial Crisis, policy makers and academics alike are reassessing their models about banks. This paper studies how to set up the problem of a bank such that it reflects its essential features. We report five empirical facts that inform us about the objective and constraints of banks using U.S. bank holding company data.

Our study of bank data is summarized by the following five facts:

1. Banks’ book and market leverage ratios behaved very differently during the 2008-2009 crisis. Market leverage rose dramatically during the crisis whereas book leverage remained constant. Between 2007Q3 and 2014Q4, bank holding companies lost 54% of their market capitalization. Book equity losses represented only 7% and were entirely made up by equity issuances.

2. Market values capture information that book values do not, and book values do not fully respond to shocks.

3. Neither regulatory nor market constraints bind strictly for most banks. The cross-section of banks shows a large dispersion in individual market leverage. Few banks are close to their regulatory constraints.

4. Banks appear to operate with a target leverage ratio. In response to an unexpected negative stock-return shock, market-leverage increases and adjusts slowly to return to
its initial level.

5. Prior to the crisis, in response to an unexpected negative stock-return shock, banks would primarily sell assets. Post-crisis, banks intensified the use of retained earnings and equity issuances, and reduced the extent of adjustment through asset sales.

These facts highlight several essential features of the data that place restrictions on the formulation of banks’ problem. Fact (1) suggests that it is not innocuous which state variable one picks for the formulation of the bank problem as book and market leverage ratios diverged dramatically during the crisis. Fact (2) suggests that during the crisis, market values depicted a more accurate picture of banks’ health. Banks, however, do face regulatory constraints that are formulated in book values. Even though – as suggested by fact (3) – market and regulatory constraints do not bind exactly they still can affect banks’ decisions in a dynamic way.

Fact (4) suggests that banks target a market-based leverage ratio. When a shock moves banks away from this ratio, they take time to return to it. This finding is based on cross-sectional variation in market returns to individual bank stock returns—we call those deviations from the mean return shocks. The idea behind our identification strategy is that bank stock returns pick-up information about the effective value of a bank—information not contained in their books. If markets are efficient, return shocks should be unpredictable. Thus, a negative market-return shocks, that is not accompanied by a change in book values, is an indication of bank losses that have not been accounted in accounting books. Hence, return shocks allow us to investigate how a position on a bank’s balance sheet evolves after the bank suffers losses that are not yet written down. We estimate that in response to a negative return shock, which mechanically pushes market-leverage on impact, banks take actions to slowly revert back to the same market leverage target they had prior to the shock. Banks reverse their increase in market leverage by operating on all possible margins of adjustment: they tend to retain earnings, increase external equity, and reduce their liabilities. However, the process is gradual.

Fact (5) summarizes the margins used by banks to delever since the crisis. Pre-crisis,
a bank that experienced a negative return shock relied mostly on asset sales to gradually reduce their liabilities. In the post-crisis, banks with negative excess-returns relied relatively more on alternative measures to delever: either through external equity issuances or retained earnings, with many banks paying zero dividends.

Section 2.2 presents our set of five facts and Section 2.3 concludes.

2.2 Five Facts

We base our five facts on a panel of bank-level data, focusing on top-tier United States Bank Holding Companies (BHCs).\(^1\) BHCs provide a comprehensive picture of the activities of a financial organization rather than the narrower accounts of their commercial bank subsidiaries—for example, we study Citigroup rather than Citibank. BHCs can also be matched to market data, which is an important part of the analysis. Book data is obtained from the FR Y-9C regulatory reports that BHCs file with the Federal Reserve, and merged with market data from the Center for Research in Security Prices (CRSP). We analyze the data from 2000 Q1 to 2015 Q4, and extend the sample when we estimate impulse response functions. The FR Y-9C filed by BHCs with total assets above $500 million.\(^2\) Since the banking industry is highly concentrated, our sample is representative of the industry. We drop entrants to correct for the entry of major financial institutions.\(^3\)

2.2.1 The Financial Crisis in the Time Series

Aggregate Balance Sheet Components. To get a sense about how the crisis affected banks, we begin by reporting the evolution of key balance sheet components in Figure 2.1.

---

\(^1\)A bank holding company is an umbrella company which holds banks, and other financial institutions, while a commercial banks is a single bank which provides traditional banking services like deposits and loans. For example, Citibank is a commercial bank, which is held by Citigroup, which is a BHC which holds Citibank and other banks, including non-commercial banks.

\(^2\)Prior to 2006 Q1, this threshold was $100 million, and the threshold became $1 Billion in March 2015.

\(^3\)Without this correction, we see a spurious increase in the assets of the traditional industry due to the reclassification of large actors such as Morgan Stanley and Goldman Sachs into bank holding companies.
Notes: These figures show data on assets, liabilities, loans, and loans net of ALL for BHCs. Data come from the FR Y-9C. Loans net of ALL refers to loans minus the allowance for loan losses (this subtracts out “probable and estimable” future losses on the current stock of loans). All variables converted to 2012 Q1 dollars using the seasonally-adjusted GDP deflator. Left panel shows aggregate series, dropping new entrants. Right panel shows data for the “Big Four” largest BHCs. Note that the spike in the balance sheet of Wells Fargo is due to its acquisition of Wachovia. Similarly, JP Morgan purchased and took on Bear Stearns and WaMu, while Bank of America took on Merrill Lynch and the remainder of CountryWide.

This figure shows total assets, liabilities, and loans—not netting out the allowance for loan losses—for the aggregate banking sector (left panel) and the four largest BHCs in terms of assets. The banking industry is highly concentrated: the “Big Four” largest BHCs account for roughly 50 percent of aggregate assets. At the onset of the crisis, the growth of bank assets, loans, and liabilities slowed down, but never dropped as dramatically as bank equity market valuations (see below). Loans, the largest component of bank assets, stagnated during the crisis and eventually fell. By 2009 Q4, the book value of loans net of the allowance for loan losses had fallen by $361 billion, a drop of only 6.84%. This number is driven only in part by losses as banks also slowed down the issuances of new loans.

Figure 2.2 shows that provisions for loan losses and net charge-offs only reached their peak in 2009 and 2010 respectively, and remained quite elevated at least through 2011, well after the recession had ended and many years after market values had crashed (see below).

---

4 The allowance for loan losses is an estimate of probable loan losses for the loans currently on the balance sheet. The next subsection will provide more detail on how bank accountants come up with this number.
Notes: These figures show data coming from the FR Y-9C. The data are aggregate time series, dropping new entrants. The left panel shows net trading revenue and profits/losses on credit exposures and interest rate exposures. The right panel shows net charge-offs of loans (charge-offs minus recoveries), and decomposes this into loans backed by real estate, commercial and industrial (C&I) loans, loans to individuals (these loans are for consumption purposes and are not secured by real estate), and all other loans (e.g. interbank loans, agricultural loans, and loans to foreign governments).

The decomposition of net charge-offs shows that these losses were heavily driven by real estate, suggesting they were associated with the housing crisis.\(^5\)

Net Worth Measures: Market Equity versus Book Equity. To get a sense about how the crisis affected banks, we report the changes in select aggregate balance sheet components and aggregate bank equity return data since the beginning of the Great Recession in 2007 Q3 in Table 2.1. We do so in two ways. We first fit a linear trend to the logged real series and report deviations from that trend in the first three columns.\(^6\) We estimate the trend using the data through 2007 Q3 and report changes since at that trend.\(^7\) Second, we report

\(^5\)When a bank has a loss that is estimable and probable, it first provisions for loan losses, which shows up as PLL. Later when the loss occurs, the asset is charged off and thus taken off the books, which shows up as charge-offs, although occasionally the bank can recover the asset later. Net charge-offs is charge-offs minus recoveries. We show a decomposition by category for net charge-offs but not for PLL because the FR Y-9C does not provide information on PLL by loan category.

\(^6\)We use the seasonally-adjusted GDP deflator to adjust for inflation, and report all values in 2012 Q1 dollars.

\(^7\)Since market return and book ROE are flows rather than levels, we detrend by simply subtracting the pre-crisis average. Also, since flows can be negative, we use \(\log(1 + r)\) instead of \(\log(r)\). A concern with
simply the real changes since 2007 Q3 in the last three columns. Each column computes the change until the fourth quarter of the year indicated by the column. For aggregate bank balance sheet quantities, we focus our attention on the aggregate series of loans and different measures of equity since these are the quantities that are at the heart of macro-finance models. We also report the changes in bank equity return data to provide a summary of shareholder losses and in the S&P stock market index for comparison. The striking fact emerging from Table 2.1 is the difference between the aggregated series of market and book valuations. The market data—market capitalization and market equity returns—shows that banks suffered large valuation losses during the crisis. The book data—book equity and common book equity and book equity return—shows only small changes. Between 2007 Q3 and 2008 Q4, market capitalization dropped by 54 per cent ($705 billion) and by the fourth quarter of 2010 the gap was still 30 per cent ($378 billion). Much of this rebound followed from new equity issuances. By contrast, real book equity did not fall during the crisis. It actually increased substantially post crisis. In fact, book losses were entirely made up for by equity issuances.\textsuperscript{8} In comparison, capitalized book equity return losses amounted to only $66 billions in 2009 Q4 that is less than one tenth of the capitalized market equity losses.

Figure 2.3 plots market equity (market capitalization), book equity, and preferred equity for the aggregate banking sector (left panel) and the four largest banks (right panel) in terms of assets.\textsuperscript{9} In the appendix, we also provide a similar analysis of aggregate issuances and dividends. The figure shows that the discrepancy between market- and book equity cannot be explained by preferred equity, which is included in book equity but not in market

\textsuperscript{8}We measure market valuation of banks in terms of market capitalization and not prices. Therefore, share dilutions cannot explain the difference with book values.

\textsuperscript{9}The fact that book equity for public BHCs is so close to book equity for all BHCs is a result of the high concentration of equity in the largest banks.
Table 2.1: Aggregate Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>Log-Linear</th>
<th>Real Change</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2008</td>
<td>2009</td>
</tr>
<tr>
<td>Market Cap.</td>
<td>-61.21%</td>
<td>-49.98%</td>
</tr>
<tr>
<td></td>
<td>(-$945B)</td>
<td>(-$790B)</td>
</tr>
<tr>
<td>Book Equity</td>
<td>-3.46%</td>
<td>-1.50%</td>
</tr>
<tr>
<td></td>
<td>(-$32B)</td>
<td>(-$15B)</td>
</tr>
<tr>
<td>Common Equity</td>
<td>-28.44%</td>
<td>-11.69%</td>
</tr>
<tr>
<td></td>
<td>(-$275B)</td>
<td>(-$120B)</td>
</tr>
<tr>
<td>Loans Net of ALL</td>
<td>2.68%</td>
<td>-10.41%</td>
</tr>
<tr>
<td></td>
<td>($141B)</td>
<td>(-$571B)</td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>-25.55%</td>
<td>-7.01%</td>
</tr>
</tbody>
</table>

Notes: Top row shows cyclical deviations in percentage points since 2007 Q3; bottom row shows deviations converted into raw values. Book equity refers to book equity of publicly traded BHCs. Loans net of ALL refers to loans minus the allowance for loan losses (this subtracts out “probable and estimable” future losses on the current stock of loans). All variables deflated using the seasonally-adjusted GDP deflator. Level variables are converted to 2012 Q1 dollars, flow variables are deflated by subtracting inflation. Bank market return deviations and book return on equity are cumulated since the end of 2007 Q3, and dollar values are obtained by multiplying the cumulative percentage point deviation by real market capitalization and real book equity at the end of 2007 Q3, respectively.
Notes: These figures show data on book equity, market capitalization, and preferred equity for BHCs. Book equity and preferred equity data come from the FR Y-9C, and market capitalization data is based on CRSP data. All variables converted to 2012 Q1 dollars using the seasonally-adjusted GDP deflator. The left panel shows aggregate series, dropping new entrants, with “Equity” referring to book equity for all BHCs in sample and “Equity (Public BHCs)” referring to only publicly-traded BHCs that can be matched to CRSP data. The right panel shows data for the “Big Four” largest BHCs, with “Equity” referring to book equity.

capitalization.\(^\text{10}\) The pattern for the largest banks is very similar.\(^\text{11}\) Citigroup is an extreme example of the discrepancy between book and market values. The bank lost 90% in terms of its market capitalization. But according to its book equity measure, Citigroup did fine. Its book equity continued to grow as it had before the crisis.\(^\text{12}\)

We summarize this section with our first fact:

Fact 2.1. Book values and market values diverged during the crisis. Between 2007 Q3 and 2008 Q4, BHCs lost $705 billion in market capitalization, a decline of 54%. Book equity losses were only $66 billion (7.84%) and were entirely made up for with new equity issuances.

The large discrepancy between market and book equity suggests an outsized role for bank

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\(^\text{10}\)Preferred equity rose temporarily during the crisis due to TARP.

\(^\text{11}\)The discontinuities in the individual bank series reflect mergers and acquisitions, e.g. the acquisition of Wachovia by Wells Fargo during the crisis.

\(^\text{12}\)Citigroup suffered heavy losses during the crisis and did not undergo any major mergers or acquisitions, making it a particularly clean test case.
accounting rules.\textsuperscript{13} For example, banks have some flexibility regarding when to acknowledge book losses. Changes in the underlying market value (see filing instructions for FR-Y-9C BHCs regulatory reports) are not included by any loan income measure for loans held on books. A bad loan is only written off once the loss has occurred as opposed to when the loss is expected. Thus book measures are bound to be backward looking. This is also evident from Figure 2.2. It shows that loan charge-off peaked in 2010 when the crisis had been already called off. The next section analyzes this issue more deeply. It shows that market values contain information that are not contained in books.

\subsection{2.2.2 Cross-Sectional Analysis}

**Information Content in Book Values and Market Capitalization.** The nature of book values implies that they tend to be backward looking. The fact that loan charge-offs peaked well after the crisis (see Figure 2.2) is suggestive evidence for this very idea. Perhaps banks’ market capitalizations are better in capturing available information about their net worth. We test this notion using cross-sectional regressions of market capitalization on book equity and profitability measures. Thereby we rely on the efficient markets hypothesis that suggests that market values reflect all available information about future dividends, and, by extension, about banks future profits and net worth today. If this is true, and market capitalization contains additional information about bank profitability not captured by book values, then market capitalization will be correlated with other variables that reflect profitability, even after we condition on book equity. We run cross-sectional regressions of the following form:

\[
\log (\text{Market Cap}_i) = \alpha + \beta \log (\text{Book Equity}_i) + f(X_i) + \epsilon_i,
\]

\textsuperscript{13}Another natural candidate to explain the differences between market and book valuations are movements in risk premia. For this reason, we compare the percentage drop in the stock market index, the S&P 500, to drop in bank valuations (see Table 2.1). It is substantially smaller (a drop of 28.83\% by 2009 Q4) than the cumulative drop in the market return of BHCs (55.28\% over the same time period). Hence, the discrepancy is unlikely to be driven by differences in risk premia alone.
where \( f(X) \) represents polynomials in our variables of interest, and \( i \) indexes banks. The regression is run in the cross-section for a single time period, so there is no \( t \) subscript. Each observations is thus a bank in the selected quarter. We then test whether \( f(X) \) adds substantial predictive power to the regression. If it does, this suggests that market capitalization captures information in \( f(X) \) that book equity does not fully capture.\(^{14}\)

For example, one variable we will examine is the (logged) ratio of delinquent loans to total loans.\(^{15}\) If books are slow to reflect true conditions, then we would expect that, during the crisis, market capitalization will be decreasing in the delinquent loans ratio, controlling for book equity. Market participants will incorporate loan delinquencies into their valuation of the bank, while books will not have adjusted yet. Because of delayed acknowledgment of losses, book values will be over-optimistic about how many delinquent loans will eventually be repaid, and will also fail to incorporate the degree to which loan delinquencies today may indicate more delinquencies tomorrow. This pattern is what we will find in the data: during the crisis, market capitalization is declining in the delinquent loans ratio, controlling for book equity.

Table 2.2 shows the results of our analysis for a pre-crisis period (2006 Q1) and a post-crisis period (2009 Q1). We maintain a consistent sample by running our analysis on a cross-section of banks for which all of the variables we use are available. In both periods, return on book equity (RoE) over the past year plays a substantial role in explaining market capitalization. Moreover, future RoE, over the next year and over the year after that, also have a reasonable degree of predictive power, even after controlling for the current RoE. This suggests that market participants care about profits and are forward looking: they have some ability to predict future profitability and incorporate this into their valuation. Moreover, although this information is known to market participants, it is not captured by book values. The delinquent loans ratio has sizable predictive power during the post-crisis

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\(^{14}\)Of course, an alternative explanation is that market cap over-reacts to the information in \( f(X) \). However, at least the direction of the effects are what we would expect if books are sluggish and market capitalization reflects the truth.

\(^{15}\)We define a loan as delinquent if it past due more than 30 days or if it is in non-accrual.
period, reflecting the importance of concerns about loan performance during the post-crisis period. Notably, the predictive power of the delinquent loans ratio goes away once we add in controls for present and future RoE, which suggests, unsurprisingly, that delinquent loans affect market values primarily through profits. Log liabilities, which represents leverage, also play a modest role in explaining market capitalization (since we control for log equity, adding log liabilities to the regression is equivalent to adding in log liabilities minus log equity, which is a measure of log book leverage). We also learn something from the root mean squared error of these regressions. It is much higher post-crisis, consistent with the divergence of book and market values and suggesting that books had less informational content post-crisis. To help us visualize the additional information content of market values over and above book values, we also construct graphs. To do this, we first run the regression, \( y = \alpha + f(X) + \delta W + \epsilon \), where \( y \) is the outcome variable, \( W \) are the control variables, and \( f(X) \) is a polynomial in the regressor of interest. We then construct \( y - \alpha - \delta W \) and plot this on the vertical axis. We plot the regressor of interest, \( X \), on the horizontal axis. By construction, the polynomial \( f(X) \) that best fits \( y \) will also be the polynomial that best fits \( y - \alpha - \delta W \), so this graph allows us to plot \( f(X) \) and assess goodness of fit. In Figure 2.4, we show these graphs, with log market capitalization as the outcome, for a quartic in log RoE over the past year (controlling for log book equity), and for a quartic in log RoE over the next year (controlling for log book equity and a quartic in log RoE over the past year).\(^{16}\) These graphs confirm that market capitalization, controlling for book equity, is increasing in both RoE over the past year and in RoE over the next year. The non-linear regression specification is important. For example, in the post-crisis period, there is a left tail of banks with very negative RoE; in this region the marginal effect of RoE on market capitalization is much smaller. Our second fact is the take-away from this analysis:

**Fact 2.2.** Market values capture information that book values do not, and book values do not fully respond to shocks.

\(^{16}\)For improved visibility, we exclude outliers from the graph window by limiting the graph’s horizontal axis to values within ±3 standard deviations from the mean.
Table 2.2: Partial $R^2$ for Different Predictors of Market Capitalization.

### 2006 Q1

<table>
<thead>
<tr>
<th>Predictor</th>
<th>2006 Q1</th>
<th>2006 Q1</th>
<th>2006 Q1</th>
<th>2006 Q1</th>
<th>2006 Q1</th>
<th>2006 Q1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log Book Equity</td>
<td>0.964</td>
<td>0.442</td>
<td>0.964</td>
<td>0.984</td>
<td>0.985</td>
<td>0.710</td>
</tr>
<tr>
<td>Log Liabilities (Quadratic)</td>
<td>0.102</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log Delinquent Loans Ratio (Quartic)</td>
<td>0.020</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log RoE over Past Year (Quartic)</td>
<td></td>
<td>0.593</td>
<td>0.209</td>
<td>0.220</td>
<td>0.203</td>
<td></td>
</tr>
<tr>
<td>Log RoE over Next Year (Quartic)</td>
<td></td>
<td>0.105</td>
<td>0.062</td>
<td>0.086</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log RoE Year After Next (Quartic)</td>
<td></td>
<td>0.036</td>
<td>0.025</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Root Mean Squared Error</td>
<td>0.307</td>
<td>0.292</td>
<td>0.305</td>
<td>0.197</td>
<td>0.187</td>
<td>0.185</td>
</tr>
<tr>
<td>Number of Observations</td>
<td>323</td>
<td>323</td>
<td>323</td>
<td>323</td>
<td>323</td>
<td>323</td>
</tr>
</tbody>
</table>

### 2009 Q1

<table>
<thead>
<tr>
<th>Predictor</th>
<th>2009 Q1</th>
<th>2009 Q1</th>
<th>2009 Q1</th>
<th>2009 Q1</th>
<th>2009 Q1</th>
<th>2009 Q1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log Book Equity</td>
<td>0.837</td>
<td>0.255</td>
<td>0.872</td>
<td>0.909</td>
<td>0.910</td>
<td>0.912</td>
</tr>
<tr>
<td>Log Liabilities (Quadratic)</td>
<td>0.027</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log Delinquent Loans Ratio (Quartic)</td>
<td>0.307</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log RoE over Past Year (Quartic)</td>
<td></td>
<td>0.473</td>
<td>0.267</td>
<td>0.244</td>
<td>0.204</td>
<td></td>
</tr>
<tr>
<td>Log RoE over Next Year (Quartic)</td>
<td></td>
<td>0.057</td>
<td>0.031</td>
<td>0.014</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log RoE Year After Next (Quartic)</td>
<td></td>
<td>0.068</td>
<td>0.057</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Root Mean Squared Error</td>
<td>0.694</td>
<td>0.687</td>
<td>0.580</td>
<td>0.508</td>
<td>0.497</td>
<td>0.483</td>
</tr>
<tr>
<td>Number of Observations</td>
<td>280</td>
<td>280</td>
<td>280</td>
<td>280</td>
<td>280</td>
<td>280</td>
</tr>
</tbody>
</table>

Notes: These tables show results from a cross-sectional regression of log market capitalization on log book equity and other variables. The first six rows of each table show the partial $R^2$ associated with that group of variables (how much of the remaining variance in the outcome is explained by that group of variables, after controlling for the other variables). Log book equity enters linearly into the regression, log liabilities enters quadratically, and all other groups of variables enter with a quartic, in order to capture the nonlinearity present in the data. The regressions in the first table are run for 2006 Q1 on the cross-section of banks with all variables available, the regressions in the second table are run for 2009 Q1 on the cross-section of banks with all variables available. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The delinquent loans ratio is the ratio of loans past due 30 days or more plus loans in non-accrual, over total loans. Log RoE is defined as log (1 + RoE). RoE over the past year is defined as book net income over the last four quarters divided by book equity four quarters ago; RoE over the next year and RoE over the year after next are defined as the one and two year leads of this variable.
Figure 2.4: Effect of Variables on Market Capitalization

Notes: These figures show results from a cross-sectional regression of log market capitalization on assorted variables. The top row shows results from a regression of log market capitalization on log book equity and a quartic in log RoE over the past year. The bottom row shows results from a regression of log market capitalization on log book equity, a quartic in log RoE over the past year, and a quartic in log RoE over the next year. The horizontal axis shows the regressor of interest, and the vertical axis shows the outcome minus the effect of the controls (for the top row, the controls are a constant and log book equity, for the bottom row, the controls are a constant, log book equity, and a quartic in log RoE over the past year). The left column shows results for 2006 Q1, the right column shows results for 2009 Q1. Regressions are run on the cross-section of banks with all variables available, but the horizontal axis of the graph window is restricted to ±3 standard deviations from the mean to improve visibility. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. Log RoE is defined as log (1 + RoE). RoE over the past year is defined as book net income over the last four quarters divided by book equity four quarters ago; RoE over the next year is defined as the one year lead of this variable.
Figure 2.5: Book and Market Leverage of Bank Holding Companies

Notes: These figures show data on book and market leverage for BHCs. Book data (book equity and liabilities) come from the FR Y-9C, and market capitalization data is based on CRSP data. The left panel shows aggregates from BHC balance sheets (dropping new entrants), and the right panel shows data for the “Big Four” largest BHCs. Book leverage is computed as assets/book equity, and market leverage is computed as (liabilities + market capitalization)/market capitalization. The aggregate leverage ratios are computed as (aggregate liabilities + aggregate equity)/aggregate equity.

Cross-Section of Book Leverage and Regulatory Constraints.

During the crisis, liabilities were relatively flat (Figure 2.1), book equity continued to rise, and market equity crashed (Figure 2.3). This results in the pattern for leverage in Figure 2.5. Book leverage rose moderately before the crisis and actually fell after the crisis. The smooth patterns in book leverage were hardly reflective of a major banking crisis. Market leverage, by contrast, spiked dramatically during the crisis, and remained almost twice as high compared to its pre-crisis level for at least four years. Given the dramatic rise in market leverage and the divergence of market and book leverage during the crisis, we next turn to analyze the extent to which banks faced binding constraints on their leverage. The aggregate time series results suggest that there was not a binding cap on market leverage, since market leverage rose dramatically during the crisis. The time series cannot so easily rule out regulatory constraints on book leverage: book leverage did not rise dramatically, and regulatory constraints changed over time after the crisis.

To further examine the degree to which regulatory capital constraints did or did not bind during the crisis, we turn to the cross-section. Even if the constraints did not bind on average,
they may have been binding for some banks. Under Basel II (the regulatory standard in place during the crisis), bank holding companies were subject to regulatory minimums on their total capital ratio and their tier 1 capital ratio. These capital ratios are computed as Qualifying Capital/Risk-Weighted Assets, and thus a bank with a higher capital ratio has lower leverage. Basel II required banks hold a minimum tier 1 capital ratio of 4% and a minimum total capital ratio of 8%. In order to be categorized as “well-capitalized,” banks had to meet minimum capital ratios that were two percentage points higher (6% and 10% respectively); being categorized as well-capitalized is helpful because banks that are not well-capitalized are subject to additional regulatory scrutiny (Basel Committee on Banking Supervision 1998; Basel Committee on Banking Supervision 2006). After the crisis, tighter capital requirements were phased in under Basel III. The minimum total capital ratio stayed at 8% throughout our sample period, but the Tier 1 capital ratio rose to 4.5% in 2013, 5.5% in 2014, and finally settled at 6% starting in 2015. Also under Basel III, additional capital ratios (e.g. tier 1 leverage and common equity capital ratio) began being monitored (however these ratios are quite similar to the pre-existing tier 1 and total capital ratios), and starting in 2016, a “capital conservation buffer” and special requirements for systemically important financial institutions were introduced (Basel Committee on Banking Supervision 2011).

Figure 2.6 shows the share of BHCs close to the regulatory minimum for different capital ratios. The panels show that although some banks were close to the regulatory minimum, and this share rose during the crisis, the vast majority of banks were not near the regulatory constraint. Interestingly, the share rose to its peak in 2010 similar to when loan losses peaked, suggesting delay in the accounting of losses. Thus, although regulatory constraints may have been relevant for some banks, they do not bind strictly for most banks, especially since even banks that are near the regulatory constraint can often avoid activating the constraint by manipulating their books.

This yields our third fact:

**Fact 2.3.** _Neither regulatory nor market constraints bind strictly for most banks._

Of course, just because the constraint does not bind directly does not mean it could never
Notes: These figures show data on regulatory capital ratios for BHCs from the FR Y-9C. The left panel shows data on the distribution of the tier 1 capital ratio, and the right panel shows data on the distribution of the total capital ratio. The figures plot the share of banks whose regulatory capital ratio falls below a given level, computed using the full, unweighted sample. The regulatory capital requirements are shown on the graph and described in the text.

bind in the future. For example, banks could choose to keep leverage low in order to avoid a future shock that causes the constraint to bind. Since a small but non-negligible share of banks do find themselves below the regulatory minimum during the crisis, and a larger share find themselves below the threshold to be considered well-capitalized, this is a real concern for banks. However, since regulatory constraints do not bind for most banks even in the crisis, and since banks can manipulate their books, a model in which regulatory constraints binds directly for most banks will not accurately describe bank behavior.

2.2.3 Target Leverage and Adjustment Costs

Once we add leverage adjustment costs (e.g. equity issuance costs and balance sheet stickiness) the choice of leverage becomes a dynamic problem. With adjustment costs, banks will have some long-run “target leverage,” but when they are hit by a shock, they will only gradually adjust back to the target level. This is in contrast to a simple model without adjustment costs: without adjustment costs, we would expect immediate adjustment and no dynamics.
In order to investigate whether a target leverage level with adjustment costs determines bank behavior, we explore how banks respond to shocks. We focus on excess-return shocks as a measure for ex-ante unpredictable return innovations. For the rest of the paper we refer to these innovations as return shocks. These return shocks can be interpreted as a linear transformation of default shocks to banks’ assets. We will also use this interpretation in our model. We will find that the target leverage theory with adjustment costs seems to describe the data well, and provides insights into potential ways to endogenize the frictions that banks face, which we will discuss in the next section.

Econometric Specification. We estimate the following panel regressions:

$$
\Delta \log(y_{i,t}) = \alpha_t + \sum_{h=0}^{k} \beta_h \cdot \log(1 + r_{i,t-h}) + \gamma_h \cdot Post_t \cdot \log(1 + r_{i,t-h}) + \epsilon_{i,t}
$$

where $i$ indexes over banks, $t$ indexes over quarters, $r_{i,t}$ indicates the market return over the past quarter for bank $i$ in quarter $t$, $\alpha_t$ is a time fixed effect, and $Post_t$ is an indicator variable equal to one if the current quarter is post-crisis (we treat 2007 Q4 as first quarter for which $Post_t = 1$), and $y_{i,t}$ is the outcome of interest.\footnote{Since market returns are changes in equity valuations, taking first differences in logs provides a tight conceptual link between the outcome and the regressor. Using levels would mean that the outcome was highly correlated with bank size. This would raise concerns about stationarity, and if the market returns of banks exhibit a size premium then this would also lead to omitted variables bias. Using levels could also result in a regression that was heavily influenced by a few large banks, given the highly skewed bank size distribution. For the same reason we do not weight our regressions: the bank size distribution is highly skewed, and so a weighted regression would be equivalent to a regression with only the handful of largest banks. If the variance of the residuals were lower for larger banks, then using weights would yield a more efficient estimator. Empirically however, the variance of the residuals does not appear to vary substantially by bank size.}

\footnote{One might favor an alternative specification which includes lags of the dependent variable in addition to contemporaneous and lagged returns. This faces two issues: Nickell bias and bad control. Including the dependent variable as a lag will induce bias, as documented by Nickell (1981). Dealing with this bias is challenging, and may result in poor precision. Perhaps more importantly, the lagged dependent variable is a "bad control," in that it is endogenous to the regressor. We wish to back out the effect of a return shock in $t - 3$ on the change in liabilities in $t$: if we condition on liabilities in $t - 1$, which is itself also affected by the past return shock, then we will not identify our parameter of interest.}

Time-fixed effects soak up aggregate shocks (e.g. changes in the price of loans due to demand shocks) giving us a partial equilibrium, supply-side impulse response, estimated off of the cross-sectional variation. In
all specifications, we use \( k = 20 \). Given the high number of lags, we extend our data to 1990 Q3. This is the first quarter in which we can identify which banks are top-tier BHCs from the FR Y-9C. This extension is necessary to obtain precise pre-crisis estimates. We cluster standard errors by bank. Finally, to report the impulse response function, we cumulate the coefficients: the pre-crisis contemporaneous response is \( \beta_0 \), the next period is \( \beta_0 + \beta_1 \), and so on. For post-crisis, we also add the corresponding \( \gamma \) terms.

The choice of market return shocks instead of equity shocks is important for our identification strategy: market capitalization is a choice variable (banks can affect their equity by issuing equity or lowering dividends), and is thus endogenous. On the other hand, under the efficient-markets hypothesis variation in excess returns should be unpredictable \textit{ex ante} after adjusting for the risk-premium. This forms the basis of our identification strategy: we treat cross-sectional variation in returns as unanticipated shocks that perturb bank equity.

For our main results, we employ a risk-adjustment procedure, described in the appendix. This procedure isolates variation in risk-adjusted returns, and uses this variation to estimate the impulse response. However, the results for unadjusted returns are qualitatively and quantitatively similar, and are also reported in the appendix.

**Estimated Responses.** How do shocks affect banks’ balance sheet, financing, and payout choices? We estimate impulse response functions for logged stock variables (liabilities, market capitalization, book equity, and market leverage), as well as for log flows (issuance rates, common dividend rates, and book return). Results are shown in Figure 2.7 for stocks and in Figure 2.8 for flows. Importantly, we are showing the response to a negative return shock, since we find this easier to think about in the context of the financial crisis. The x-axis of our plots shows the contemporaneous response (\( -\beta_0 \) for pre-crisis and \( -\beta_0 - \gamma_0 \) for post-crisis) as quarter 1, the response one quarter later (\( -\beta_0 - \beta_1 \) and \( -\beta_0 - \beta_1 - \gamma_0 - \gamma_1 \)) as quarter 2, and so on.

**Responses of stock variables.** Suppose banks have a target leverage ratio, we would expect banks to respond to a negative wealth shock (which mechanically increases leverage)
Notes: These figures show estimated impulse response functions for BHCs. The figures show the estimated impulse response to a one unit negative returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The “post-crisis” period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The panels display the impulse responses of log liabilities, log market capitalization, log market leverage, and log book equity. Market leverage is defined as log(Liabilities/Market Capitalization), so that it represents the difference between the response of log liabilities and log market capitalization (results using log(Liabilities + Market Capitalization)/Market Capitalization) are extremely similar.)
by moving back towards their target leverage. The data is consistent with such an adjustment in response to return shocks. The data also reveals that this response is slow, suggesting adjustment costs play an important role. The impulse response of log market leverage, defined here as log(Liabilities/Market Capitalization), is simply the difference between the response of log liabilities and log market capitalization. In the initial quarter, the mechanical effect on the denominator dominates. Thanks to the great deal of post-crisis adjustment on the equity side, banks adjusted their market leverage faster in response to cross-sectional shocks post-crisis than pre-crisis. However, the effect of return shocks on market leverage does not vanish, even five years out, suggesting adjustment costs are quite important.

This yields our fourth fact:

**Fact 2.4.** Banks appear to operate with a target leverage ratio to which they only return slowly after shocks, suggesting adjustment costs.

We can decompose the impulse response of market leverage into the responses of liabilities and of market capitalization. After a 10% return shock, five years out, there is a decrease of 3-4% in liabilities in the pre-crisis environment and of 2-3% in the post-crisis. Although banks appear to adjust liabilities more slowly post-crisis, this is reversed for market capitalization. In the pre-crisis, market equity falls mechanically in response to the shock (a return shock automatically lowers equity one-for-one), with little further response of equity afterwards. By contrast, in the post-crisis, five years out, a 10% shock to returns yields only a roughly 5% decrease in market capitalization. These impulse responses show that banks switched from responding exclusively by decreasing assets and liabilities, in the pre-crisis period to a combination of balance sheet and equity adjustment in the post-crisis period, with equity adjustments being more important. When we say equity was the more important margin of adjustment, we mean that the adjustment of log market equity is larger, which means that the equity adjustment is more important for adjusting leverage. Of course, if we did not take logs, the balance sheet response would be larger since banks are highly levered.

This yields our fifth and final fact:
Fact 2.5. Prior to the crisis, banks adjusted leverage primarily by reducing debt keeping equity unchanged. Post-crisis, banks also raised equity through retained earnings and issuance.

Finally, our analysis in the previous section suggested that book values respond only slowly to losses that are reflected quickly in market returns. Thus, book equity (see Figure 2.7) and book returns on equity (see Figure 2.8) should exhibit a delayed response to market returns. This is exactly what we see in the data.

Response of Flow Variables. We now describe movements in flow variables such as the dividend rate, the return on book equity (ROE), and equity issuance rate. For flows, we modify the methodology. We estimate the following equation:

\[
\log(1 + y_{i,t}) = \alpha_t + \sum_{h=0}^{k} \beta_h \cdot \log(1 + r_{i,t-h}) + \gamma_h \cdot Post_t \cdot \log(1 + r_{i,t-h}) + \epsilon_{i,t}.
\]

where now \( y_{i,t} \) now represents the dividend rate, equity issuance rate, or book returns. Thus, when we report the impulse responses, rather than tracing out the rate, we are summing up cumulative deviations from the mean.\(^{19}\)

According to the pecking-order theory ((Myers and Majluf 1984)), internal equity financing (retaining earnings by reducing dividends) should be cheaper than outside equity finance and thus be preferred by banks. The data does not provide clear evidence for this theory. The cumulative response of the common dividend rate to a negative return shock is small and actually positive pre-crisis\(^{20}\) and the issuance rate response is small and negative. During the pre-crisis periods, banks chose to not recapitalize, consistent with the response in market leverage in Figure 2.7. During the post-crisis periods, banks recapitalized by both reducing

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\(^{19}\)We do this because for flows we are interested in how the flows cumulate over time to affect the stocks: elevated issuance rates cumulate to an increase in equity, book returns cumulate to a change in book equity, etc. In practice this is also useful econometrically: we are able to get precise estimates when we plot the cumulative response of these flow variables, while attempting to trace out the path of the flows gives us estimates that are mostly noise. Moreover, since our flow variables do not depend on the size of the bank, there is no need to take first differences as we do for the stock variables.

\(^{20}\)Some of this effect may be mechanical: dividend per share tends to be fixed, and so a return shock leading to a fall in market capitalization will automatically raise the dividend rate, and this effect will cumulate until the dividend per share is newly set.
Notes: These figures show estimated impulse response functions for BHCs. The figures show the estimated impulse response to a one unit negative returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The “post-crisis” period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The panels display the impulse responses of the common dividend rate, equity issuance rate, and book returns. The logged common dividend rate is defined as $\log(1 + \text{Common Dividends} / \text{Market Capitalization})$, the logged issuance rate is defined as $\log(1 + \text{Equity Issuances} / \text{Market Capitalization})$, and logged book returns are defined as $\log(1 + \text{Book Net Income} / \text{Book Equity Last Quarter})$. 
dividends\textsuperscript{21} and increasing issuances. Most importantly, the cumulative response of issuance rates is much larger than that of dividend rates: the post-crisis response of issuance rates is three times the magnitude of the dividend rate response. In other words, when shocks were large and profits low, recapitalization via retained earnings would have taken a lot of time.

**Identification Discussion and Robustness.** There are a few threats to identification. Returns in a given period may be correlated with other variables \textit{ex-post} — e.g., banks with higher exposure to subprime mortgages should suffer heavier losses during the crisis. This means that returns in one period may be correlated with returns in another period \textit{ex post}. This correlation will cause omitted variables bias if the outcome variable is affected by returns in both periods, but one of the periods is excluded. To deal with this issue, we include twenty lags of market returns. We also implement a placebo test where we add leading values of returns into the specification — future return shocks should not affect the present, otherwise they aren’t exogenous “shocks”. Our placebo tests, shown in the appendix, suggest that the bias in our estimates is small.

Our estimates could also be polluted by omitted variables bias from other sources. Cross-sectional return shocks might incorporate idiosyncratic information not just about a bank’s present portfolio (e.g. the default rate of mortgages on Citi’s balance sheet will rise relative to those of other banks) but also about the relative profitability of its future portfolio (e.g. the default rate on the mortgages that Citi will issue in the future will rise relative to those of other banks) and thus affect the bank’s problem through channels other than perturbing equity (if expected returns on future assets fall, then it could make sense to lower leverage). We do not think this problem is severe since we think most banks have access to fairly similar assets and thus the idiosyncratic element of shocks is mainly a function of the present portfolio (since we control for time fixed effects, our estimates are identified only off of the idiosyncratic component of the shocks). Our estimates could also be biased to the extent

\textsuperscript{21}Some of this may be regulatory: undercapitalized banks are not allowed to distribute dividends.
that the efficient markets hypothesis does not hold perfectly.\textsuperscript{22} We acknowledge that our identification strategy is imperfect, but we think these issues are not crucial. Since it is infeasible for us to run a randomized controlled trial, we believe our approach provides valuable information about the dynamic behavior of banks before and after the crisis.

\subsection*{2.2.4 Interpretation of the Facts}

These facts can be microfounded with common bank accounting practices and capital reallocation costs.

**Bank Accounting Practices.** The discrepancy between book and market equity reflects bank accounting practices. Banks can delay acknowledging losses on their books (e.g. Laux and Leuz 2010), because banks are not required to mark-to-market the majority of their assets. There are many incentives to delay book losses. In practice, a key metric for measuring success of a bank is the book return on equity (ROE).\textsuperscript{23} Given that ROE is a measure of success, manager compensation is linked to book value performance. Moreover, shareholders and other stakeholders may base their valuations on information from book data. Finally, banks are required to meet capital standards based on book values (if Citi had acknowledged a 90\% loss of equity on the books, it would have been severely undercapitalized).

Banks’ ability to “massage” their accounts is studied extensively in the accounting literature (Bushman (2016) and Acharya and Ryan (2016) review the literature on this issue, Francis, Hanna, and Vincent (1996) studies the same issue for non-financial firms). In practice, banks can record securities on the books using two methodologies: either amortized historical cost (the security is worth what it cost the bank to buy it with appropriate amor-

\textsuperscript{22}The main concern here would be that the shocks were actually anticipated, perhaps because banks have access to information that doesn’t reach the market until later. This would lead to pre-shock trends, which our placebo test is designed to test for. Our placebo tests find that any such bias is small.

\textsuperscript{23}For example, JP Morgan’s 2016 annual report states “the Firm will continue to establish internal ROE targets for its business segments, against which they will be measured” (on page 83 of the report).
tization) or fair value accounting. In addition to mis-pricing securities, another degree of freedom is the extent to which banks can acknowledge impairments: banks have the right to delay acknowledging impairments on assets held at historical cost, if they deem those impairments as temporary (i.e. they believe the asset will return to its previous price). This gives banks substantial leeway, and led banks to overvalue assets on the books during the crisis. Huizinga and Laeven (2012) find that banks used discretion to hold real-estate related assets at values higher than their market value. (Laux and Leuz 2010) note some notable cases of inflated books during the crisis: Merrill Lynch sold $30.6 billion dollars of CDOs for 22 cents on the dollar while the book value was 65 percent higher than its sale price. Similarly, Lehman Brothers wrote down its portfolio of commercial MBS by only three percent, even when an index of commercial MBS was falling by ten percent in the first quarter of 2008. Laux and Leuz 2010 also document substantial underestimation of loan losses in comparison to external estimates.

This shows up in our own analysis as well: Figure 2.2 shows that provisions for loan losses and net charge-offs only reached their peak in 2009 and 2010 respectively, and remained quite elevated at least through 2011, well after the recession had ended. The decomposition of net charge-offs shows that these losses were heavily driven by real estate, suggesting they were associated with the housing crisis. Banks’ books were only acknowledging in 2011 losses

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24 Fair value accounting can be done at three levels: Level 1 accounting uses quoted prices in active markets. Level 2 uses prices of similar assets as a benchmark to value assets that trade infrequently. Level 3 is based on models that do not involve market prices (e.g. a discounted cash flow model). Banks are required to use the lowest level possible for each asset. In practice, most assets are recorded at historical cost. The majority of fair value measurements are Level 2 (Goh et al. 2015; Laux and Leuz 2010). Recent work has shown that the stock market values fair value assets less if they are measured using a higher level of fair value accounting. This leaves room to mis-price assets on books. Particularly during 2008, Level 2 and Level 3 measures of assets were valued substantially below one (Goh et al. 2015; Kolev 2009; Song, Thomas, and Yi 2010). Laux and Leuz (2010) document sizable reclassifications from Levels 1 and 2 to Level 3 during this period. They highlight the case of Citigroup, which moved $53 billion into Level 3 between the fourth quarter of 2007 and the first quarter of 2008 and reclassified $60 billion in securities as held-to-maturity which enabled Citi to use historical costs.

25 When a bank has a loss that is estimable and probable, it first provisions for loan losses, which shows up as PLL. Later when the loss occurs, the asset is charged off and thus taken off the books, which shows up as charge-offs, although occasionally the bank can recover the asset later. Net charge-offs is charge-offs minus recoveries. We show a decomposition by category for net charge-offs but not for PLL because the FR Y-9C does not provide information on PLL by loan category.
that the market had already predicted when the crisis hit.

Harris, Khan, and Nissim (2013) construct an index, based on information available in the given time period, that predicts future losses substantially better than the allowance for loan losses.\footnote{The ALL is the stock variable corresponding to the PLL.} This implies that the allowance for loan losses is not capturing all of the available information to estimate losses. This may in part be strategic manipulation, but there may also be a required delay in acknowledging loan losses. Under the “incurred loss model” that was the regulatory standard during the crisis, banks are only allowed to provision for loan losses when a loss is “estimable and probable” (Harris, Khan, and Nissim 2013). Thus, even if banks know that many of their loans will eventually suffer losses, they were not supposed to update their books until the loss was imminent.

Thus, any theory of regulatory constraints on bank leverage should “account for accounting.” Bank accounting rules enable banks to delay acknowledging losses and avoid the regulatory constraint. Some existing work has begun to do this: Milbradt (2012) analyzes the distorted incentives brought about by Level 3 fair-value accounting, and Caballero, Hoshi, and Kashyap (2008) note that regulatory constraints may be one factor contributing to evergreening during Japan’s stagnation, because evergreening allows banks to delay acknowledging losses on the books.

**Capital Reallocation Costs.** The data is consistent with banks targeting a leverage level and facing adjustment costs. We now examine what form these adjustment costs might take and what margin they operate along. We need adjustment costs on both liabilities and equity to explain the slow adjustment of leverage.\footnote{This assumes that the bank’s problem is homothetic. Delevering by raising equity will increase size and delevering by shrinking assets and liabilities will lower size. Thus, non-homotheticity could serve the role of an adjustment cost, and the bank would have both a target leverage and a target size.} Without adjustment costs, banks could simply expand or shrink assets and liabilities to meet the target leverage ratio. Without equity adjustment costs, banks could simply expand or shrink equity to the desired level.

Adjustment costs can come in the form of illiquid assets or illiquid liabilities. Asset illiq-
uidity could arise for a number of reasons, such as relational capital, bank specific expertise, asymmetric information, or the incentives to avoid acknowledging book losses on the books. We will discuss some of these in more detail in the next section. Liabilities can also be illiquid: in particular, banks could only have long term assets like deposits that are costly to terminate early (turning away depositors could be harmful if the bank wants to promote depositor loyalty for the future, and in many cases it is impossible to terminate these liabilities early). We can examine this issue empirically by looking at the composition of bank liabilities in the cross-section.

In Figure 2.9, we present kernel density estimates of the distribution of deposits (notoriously sticky) as a share of total liabilities, and of the sum of repurchase agreements (Repo) and federal funds purchased (Fed Funds) as a share of total liabilities. We think of Repo and Fed Funds as liquid liabilities because they mature very quickly, i.e., mostly overnight. The figures show that, both before and after the crisis, most banks had some Repo on their balance sheet and not all of their liabilities were deposits. This is particularly true for the larger banks that drive aggregate leverage. This suggests that banks indeed had room to pay off their liquid liabilities, so liquid liabilities were probably not a crucial friction for most banks.

Equity adjustment costs can come on the issuance or dividend margin. For issuances, agency theories suggest that banks pay a premium on equity issuances due to asymmetric information. Yet, in the post-crisis, banks actually responded to negative shocks mainly with increased issuances rather than with lowering dividends. This may be in part because dividends are constrained to be non-negative, and many banks were up against the zero dividends constraint. To investigate this further, the left panel of Figure 2.10 shows kernel density estimates of the distribution of common dividend rates.

The kernel density estimates and accompanying summary statistics indicate that a divi-

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28Of course, the path of bank equity also depends on profits. However, since in our framework equity is measured as market capitalization and thus profits are measured as market returns, the efficient market hypothesis tells us that, setting aside the issue of risk adjustment, there should be no effect of past variables on market profits.
dend rate of zero is common, and became more common during the crisis. The right panel shows a new impulse response function for the common dividend rate estimated via a Tobit model with left censoring of dividend at zero. The alternative estimate suggests that the cumulative response of post-crisis common dividend rates to return shocks would have been much larger had dividends not been constrained by zero. Whether this constraint is sufficient to fully explain the disparity between the response of dividends and issuances is unclear: the Tobit model imposes a particular assumption about the censoring process (e.g. the error term is normally distributed), and it is also not obvious how issuances would have behaved if dividends had not been constrained. We do not attempt to develop a full structural model of bank behavior in this paper. However, it is clear that the requirement that dividends be non-negative is an important constraint.

2.3 Conclusion

This paper summarizes five empirical facts about the behavior of banks during the US the financial crisis of 2008. We use these facts to explore what features banking models need in order to get closer to the data. Our empirical findings suggest a theory wherein banks target market leverage, but where adjustments to that target are gradual due to adjustment costs. A comparison between pre- and post-crisis responses suggest that, in contrast to the pre-crisis period, in the post-crisis banks relied more on retained earnings than on assets sales to readjust market leverage back to target.
2.4 Additional Figures

Figure 2.9: Liquidity of Deposits for Bank Holding Companies

Notes: These figures show kernel density estimates of liability composition ratios for BHCs. Data are from the FR Y-9C. We use 2006 Q1, 2009 Q1, and 2012 Q1 as reference quarters, and restrict to banks that are present in the data in all three reference quarters, in order to ensure comparability. The top panels show deposits as a share of total liabilities, and the bottom panels show repo (securities sold under agreement to repurchase) plus federal funds purchased as a share of total liabilities. The left panels are unweighted estimates, and the right panels are weighted by the total liabilities of the bank. The bottom panels are trimmed at 0.3 in order to improve legibility.
Figure 2.10: The Effects of the Zero Lower Bound for Common Dividends

Notes: The left panel shows kernel density estimates of common dividend rates (with book equity as the denominator) for BHCs. Data are from the FR Y-9C. We use 2006 Q1, 2009 Q1, and 2012 Q1 as reference quarters, and restrict to banks that are present in the data in all three reference quarters, in order to ensure comparability. The panel is trimmed at 0.5 in order to improve legibility. The right panel shows the estimated impulse response of log common dividends to a one unit negative returns shock. The impulse response is estimated using a Tobit model with left censoring at zero. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The “post-crisis” period begins in 2007 Q4. The logged common dividend rate for the right panel is defined as log(1 + Common Dividends/Market Capitalization), to maintain consistency with the rest of the impulse response section.
2.4.1 Risk Adjustment Procedure

For our main impulse response results, we wish to use risk-adjusted returns, rather than raw returns. More formally, we assume that the market returns of bank $i$ at time $t$ are given by

$$r_{it} - r_{ft}^f = \alpha_i + \beta_i \left( r_{mt}^m - r_{ft}^f \right) + \eta_{it}$$

where $r_{ft}^f$ is the risk free-rate (which we measure using the rate on 3-Month Treasuries), $r_{mt}^m$ is the market return (which we measure as the value-weighted return, including dividends, on the S&P 500), and $r_{it}$ is the market return of bank $i$ at time $t$. All returns are logged, e.g. $r_{mt}^m$ refers to log $(1 + \text{Market Return})$. We wish to isolate variation in the idiosyncratic shocks, $\eta_{it}$, and use this variation to estimate the impulse responses.

A natural, but naive, approach would be to estimate the above model for each bank $i$ using OLS, and then use the estimated residuals, $\hat{\eta}_{it}$, as the regressors in the impulse response estimation. The problem here is that it induces bias: $\hat{\eta}_{it}$ is a noisy measure of the true regressor $\eta_{it}$, which leads to bias as long as $T$ is finite (the bias will shrink as $T$ grows large, because $\hat{\eta}_{it}$ will converge to the true $\eta_{it}$).

Fortunately, there is a simple solution: we estimate $\hat{\eta}_{it}$ using OLS, and then we use $\hat{\eta}_{it}$ as an instrument for the unadjusted return. Since our main regressions use contemporaneous returns, twenty lags, and their interaction with a post-crisis dummy, this means we use contemporaneous $\hat{\eta}_{it}$, twenty lags of $\hat{\eta}_{it}$, and their interaction with a post-crisis dummy as instruments. Instrumental variables does not suffer from the same problem of bias under classical measurement error. Instead, to get identification under the assumed model for returns, we need our instrument to be correlated with the “good variation”, $\eta_{it}$, and uncorrelated with the “bad variation,” $\alpha_i + \beta_i \left( r_{mt}^m - r_{ft}^f \right)$. This is mechanically what we are doing when we run OLS at the bank level, and if the assumed model for returns is correct, then we have $E \left[ \hat{\eta}_{it} \left( \alpha_i + \beta_i \left( r_{mt}^m - r_{ft}^f \right) \right) \right] = \alpha_i E \left[ \hat{\eta}_{it} \right] + \beta_i E \left[ \hat{\eta}_{it} \left( r_{mt}^m - r_{ft}^f \right) \right] = 0 + 0$. Thus, our instrumental variables strategy will give us a consistent estimator of the true impulse response, under the assumption that we have the correct model of returns. Since the OLS
regression estimating $\hat{\eta}_{it}$ is conducted at the bank level, we cluster our standard errors at the bank level (clustering at the bank level is already a good idea).

### 2.4.2 Results without Risk Adjustment

While we favor the risk-adjusted results, we also have computed “unadjusted results” for the impulse responses, which we report here for completeness. We also have computed unadjusted results which control for bank fixed effects. The results with bank fixed effects are a middle ground between unadjusted and adjusted results: they allow for heterogeneity in $\alpha_i$ (this gets absorbed by the bank fixed effect), but not in $\beta_i$ (the time fixed effect will absorb $r^f_t + \beta \left(r^m_t - r^f_t\right)$, with a homogeneous $\beta$, but will not deal with heterogeneous $\beta_i$). The results are qualitatively and quantitatively similar across the methods. However, compared to the other methods, the bank fixed effect method yields a slower response of liabilities in the pre-crisis. As a result, the results from the bank fixed effect method find responses of liabilities pre-crisis vs. post-crisis that are not meaningfully different from each other. Nonetheless, since two out of three methods find a larger response of liabilities pre-crisis than post-crisis, since we favor the risk-adjusted results for a priori reasons, and given the non-negligible standard errors on the pre-crisis response, we prefer the interpretation that the pre-crisis response of liabilities is larger than the post-crisis response.

### 2.4.3 Placebo Tests

To test the validity of our identification strategy, we conduct placebo tests where we include ten leads of returns (in addition to the contemporary value and twenty lags as before). If the returns really are unanticipated shocks, then the leading values should not affect current behavior. This is similar to testing for pre-trends: we are testing whether the banks that will experience higher returns in the future are already acting differently today. Overall, the placebo test are encouraging, and suggest that our results are not driven by prior differences in the behavior of banks which experience return shocks.
Notes: These figures show estimated impulse response functions for BHCs. The figures show the estimated impulse response to a one unit negative returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The “post-crisis” period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The panels display the impulse responses of log liabilities, log market capitalization, log market leverage, and log book equity. Market leverage is defined as log(Liabilities/Market Capitalization), so that it represents the difference between the response of log liabilities and log market capitalization (results using log(Liabilities + Market Capitalization)/Market Capitalization) are extremely similar.)
Figure 2.12: Estimated Impulse Responses for Flow Variables (No Risk Adjustment)

Notes: These figures show estimated impulse response functions for BHCs. The figures show the estimated impulse response to a one unit negative returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The “post-crisis” period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The panels display the impulse responses of the common dividend rate, equity issuance rate, and book returns. The logged common dividend rate is defined as log(1 + Common Dividends/Market Capitalization), the logged issuance rate is defined as log(1 + Equity Issuances/Market Capitalization), and logged book returns are defined as log(1 + Book Net Income/Book Equity Last Quarter).
Figure 2.13: Estimated Impulse Responses for Stock Variables (Bank Fixed Effects)

Notes: These figures show estimated impulse response functions for BHCs. The figures show the estimated impulse response to a one unit negative returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The “post-crisis” period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The panels display the impulse responses of log liabilities, log market capitalization, log market leverage, and log book equity. Market leverage is defined as log(Liabilities/Market Capitalization), so that it represents the difference between the response of log liabilities and log market capitalization (results using log(Liabilities + Market Capitalization)/Market Capitalization) are extremely similar.)
**Figure 2.14: Estimated Impulse Responses for Flow Variables (Bank Fixed Effects)**

![Graphs showing estimated impulse responses for flow variables](image)

**Notes:** These figures show estimated impulse response functions for BHCs. The figures show the estimated impulse response to a one unit negative returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The “post-crisis” period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The panels display the impulse responses of the common dividend rate, equity issuance rate, and book returns. The logged common dividend rate is defined as log(1 + Common Dividends/Market Capitalization), the logged issuance rate is defined as log(1 + Equity Issuances/Market Capitalization), and logged book returns are defined as log(1 + Book Net Income/Book Equity Last Quarter).
Figure 2.15: Estimated Impulse Responses for Stock Variables (Risk-Adjusted, with Placebo)

Notes: These figures show estimated impulse response functions for BHCs. The figures show the estimated impulse response to a one unit negative returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The “post-crisis” period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The panels display the impulse responses of log liabilities, log market capitalization, log market leverage, and log book equity. Market leverage is defined as log(Liabilities/Market Capitalization), so that it represents the difference between the response of log liabilities and log market capitalization (results using log(Liabilities + Market Capitalization)/Market Capitalization) are extremely similar.)
Figure 2.16: Estimated Impulse Responses for Flow Variables (Risk-Adjusted, with Placebo)

Notes: These figures show estimated impulse response functions for BHCs. The figures show the estimated impulse response to a one unit negative returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The “post-crisis” period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The panels display the impulse responses of the common dividend rate, equity issuance rate, and book returns. The logged common dividend rate is defined as \( \log(1 + \text{Common Dividends}/\text{Market Capitalization}) \), the logged issuance rate is defined as \( \log(1 + \text{Equity Issuances}/\text{Market Capitalization}) \), and logged book returns are defined as \( \log(1 + \text{Book Net Income}/\text{Book Equity Last Quarter}) \).
Figure 2.17: Estimated Impulse Responses for Stock Variables (No Risk Adjustment, with Placebo)

Notes: These figures show estimated impulse response functions for BHCs. The figures show the estimated impulse response to a one unit negative returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The “post-crisis” period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The panels display the impulse responses of log liabilities, log market capitalization, log market leverage, and log book equity. Market leverage is defined as log(Liabilities/Market Capitalization), so that it represents the difference between the response of log liabilities and log market capitalization (results using log(Liabilities + Market Capitalization)/Market Capitalization) are extremely similar.)
Figure 2.18: Estimated Impulse Responses for Flow Variables (No Risk Adjustment, with Placebo)

Notes: These figures show estimated impulse response functions for BHCs. The figures show the estimated impulse response to a one unit negative returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The “post-crisis” period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The panels display the impulse responses of the common dividend rate, equity issuance rate, and book returns. The logged common dividend rate is defined as log(1 + Common Dividends/Market Capitalization), the logged issuance rate is defined as log(1 + Equity Issuances/Market Capitalization), and logged book returns are defined as log(1 + Book Net Income/Book Equity Last Quarter).
Figure 2.19: Estimated Impulse Responses for Stock Variables (Bank Fixed Effects, with Placebo)

Notes: These figures show estimated impulse response functions for BHCs. The figures show the estimated impulse response to a one unit negative returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The “post-crisis” period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The panels display the impulse responses of log liabilities, log market capitalization, log market leverage, and log book equity. Market leverage is defined as log(Liabilities/Market Capitalization), so that it represents the difference between the response of log liabilities and log market capitalization (results using log(Liabilities + Market Capitalization)/Market Capitalization) are extremely similar.)
Figure 2.20: Estimated Impulse Responses for Flow Variables (Bank Fixed Effects, with Placebo)

Notes: These figures show estimated impulse response functions for BHCs. The figures show the estimated impulse response to a one unit negative returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The “post-crisis” period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The panels display the impulse responses of the common dividend rate, equity issuance rate, and book returns. The logged common dividend rate is defined as log(1 + Common Dividends/Market Capitalization), the logged issuance rate is defined as log(1 + Equity Issuances/Market Capitalization), and logged book returns are defined as log(1 + Book Net Income/Book Equity Last Quarter).
CHAPTER 3

Modelling Banks Adjustment

3.1 Introduction

The data presented in Chapter 2 implies that bank models should incorporate (1) meaningful differences between book accounting and market values, (2) a target for market-based leverage, (3) a slow adjustment to the market-leverage target, (4) a rich cross-section of book and market leverage ratios, and (5) time variation in frictions. In this chapter we present a partial-equilibrium model of banks with these features. The model is admittedly reduced form, but tractable and consistent with the restrictions placed by the data. If a model with proper microfoundations can deliver similar portfolio tradeoffs, its mechanics are likely consistent with the data.

The model works as follows. As in O’Hara (1983), banks want to smooth dividends. The model distinguishes between book and market value of assets, because banks can (as in the data) choose to delay acknowledging losses. This drives the differences between the book and market value of equity. Banks face both book (regulatory) and market leverage constraints. If they violate either constraint banks are liquidated. Because loans are exposed to idiosyncratic default shocks, banks cannot control their leverage perfectly. When banks sell loans rather than keeping them on their balance sheet until they mature, they have to sell them at a discount (as in Shleifer and Vishny 1992). Since cutting back dividends is costly, these frictions produce a target for market leverage. Furthermore, they cause leverage to slowly revert back to the target if it starts off target. We calibrate some model parameters and estimate the both the cost of selling assets pre-crisis and post-crisis. The simulated model generates moments that are consistent with the five facts. The estimation
finds higher adjustment costs post crisis.

The model allows us to decompose market leverage into cash flow fundamentals (low dividend rate) captured by book values, cash flow fundamentals not captured by book values (delayed loan losses), and market discount rates.\footnote{Atkeson et al. (2018) decompose banks’ market to book ratio of equity to draw conclusions about changes in banks’ government guarantees.}

Although the model does not feature microfoundations for the components that slow down the balance sheet adjustments, it produces patterns that can help us distinguish between alternative microfoundations. We can find those microfoundations in some studies. We argue that a recent wave of banking models should bring those microfoundations into macroeconomic models with banks. An example of a model with where banks have accounting flexibility is Caballero, Hoshi, and Kashyap (2008). That paper argues that in the early nineties, Japanese banks avoided the recognition of loan losses by rolling-over loans that would have otherwise defaulted. This phenomenon is called evergreening and was possibly also important in the recent crisis in the US. Another microfoundation for book-market accounting differences is found in Milbradt (2012). In this model banks do not trade assets after their price falls below their purchase price to avoid recognizing losses that reduce their regulatory capital. Many corporate finance models produce a target for leverage that emerges from a trade-off theory—see for example, Kraus and Litzenberger (1973), Myers (1984), Hennessy and Whited (2005) and Frank and Goyal (2011). Some recent work has provided further micro-foundations for slow reversals of leverage. In DeMarzo and He (2016) and Gomes, Jermann, and Schmid (2016), a slow adjustment to a leverage target is produced by a combination of long-term debt, default, and lack of commitment to a debt policy. Those models could be adapted to account for deposit insurance and implicit government subsidies. Equity issuance costs are rationalized by agency frictions, like for example in Myers (1977, a debt overhang model) and Myers and Majluf (1984, a private information model).\footnote{In the midst of the crisis, Acharya et al. (2010) argues that banks receiving capital infusions where deliberately paying dividends to shareholders.}

Fact 4 also suggests that the costly liquidation of bank assets is an important feature of
the problem faced by banks. Adjustment costs on asset sales arise naturally when banks hold information sensitive assets, typically viewed as a specialty of banks (e.g. Leland and Pyle (1977), Diamond (1984a), Williamson (1986), Tirole (2011), and Dang et al. (2017)).\footnote{This was the topic of Darrell Duffie’s presidential address to the American Finance Society in 2010 (Duffie 2010).}

Another interpretation is that this reflects an aggravated cost of selling assets (as conceived in Shleifer and Vishny 1992). Fact 4 is suggestive that asset liquidations became more costly during the crisis. Gorton and Ordonez (2011) and Dang, Gorton, and Holmström (2010) are both models where assets become more illiquid during crises due to adverse selection.\footnote{In Gorton and Ordonez (2011) and Dang, Gorton, and Holmström (2010) the equity of constrained agents determines their incentives to acquire information: Thus, equity losses may trigger adverse selection because the economy swings from states where information is symmetric and assets are liquid to states where information is asymmetric and assets illiquid.}

In a related paper, Adrian, Boyarchenko, and Shin (2016) take a long-run perspective also to discuss which theories of financial intermediation are supported by the data. Consistent with their analysis, our model captures that book-leverage matters because losses cannot be delayed forever.

Section 3.2 presents the model and Section 3.3 concludes.

3.2 A Model to Rationalize the Facts

We now present a model that rationalizes the five facts described earlier. The goal is to produce a reduced-form partial equilibrium model that illustrates the main features that a quantitative general equilibrium model would need to match these facts. Though many of the model’s features are ad-hoc, the features point toward deeper theories that could microfound them.

3.2.1 The Model

We study a collection of heterogeneous banks in partial equilibrium. That is, banks take loan and deposit rates as given. From the solution to banks’ optimization problem, we simulate
a panel series of banks that we compare to the cross-sectional data.

**Environment.** Time is discrete, infinite, and indexed by $t$. The only source of risk is loan default risk. The bank maximizes the expected discounted value of dividends $div_t$. Banks have preferences over dividends defined recursively by:

$$
  u_t = U(div_t) + \beta U(CE_t[U^{-1}(u_{t+1})])
$$

where:

$$
  U(div_t) = \frac{div_t^{1-\theta}}{1 - \theta} \quad \text{and} \quad CE_t[\nu_{t+1}] = \left( E_t[\nu_{t+1}^{1-\psi}] \right)^{\frac{1}{1-\psi}}.
$$

The term $div_t$ denotes dividend payouts at time $t$ and $u$ is an Epstein-Zin utility function with a risk aversion coefficient of $\psi$ and an intertemporal elasticity of substitution of $1/\theta$. We introduce curvature to the bank’s objective function for two reasons: (a) to deliver dividend smoothing, and (b) to study the degree of risk-aversion needed to match the data.

**State Variables.** We distinguish between economic values and accounting values. At $t$, a bank holds stocks of outstanding loans $\ell_t$ and liabilities (deposits) $d_t$. The economic (or real) value of loans is denoted by $\ell_t$, and the accounting (or book) value of loans is denoted by $\bar{\ell}_t$. Whereas the economic value is a state variable that captures the future actual income stream, the accounting value equals the face value of loans owed to the bank. The distinction appears because some loans default, but this information is not captured in the accounting value of the books automatically. However, the bank’s actions are affected by both, the economic and accounting values. The real value of loans, is not the market value, because loans are not sold at that price. However, the market valuation of loans, and therefore equity, is based on the real value of loans. We introduce this distinction in order to speak to Facts 1 through 3, because we assume that markets observe $\ell_t$, while an econometrician that uses bank regulatory data observes only $\bar{\ell}_t$. We can however, interpret return shocks as conveying information about $\ell_t$.

Loans are long term and mature at rate $\delta$. Long-term debt is necessary to produce slow-moving leverage. Loans are also risky. In particular, every period, a fraction $1 - \varepsilon$ of loans
default. The distribution of $\varepsilon$ has a c.d.f. $F(\varepsilon)$. The stock of loans $\ell_t$ is the stock net of defaults. The stock of book loans, $\bar{\ell}_t$, is similar but does not account for losses.\(^5\) The laws of motion for $\ell_t$ and $\bar{\ell}_t$ are linked but not identical, as we show below. The state variables for a bank are the triplet $\{\ell_t, d_t, \bar{\ell}_t\}$.

**Loan creation.** The loan market is simplified to keep the model tractable. Each period, a bank chooses a flow of new loans, $I_t$. Banks fund new loans $I_t$ by issuing deposits. Banks issue new deposits at the price $p(I_t, \ell_t, d_t)$ given by:

$$p(I_t, \ell_t, d_t) = 1 + \gamma I_t / (\ell_t - d_t).$$

Since $\ell_t - d_t$ is the bank’s (real) equity, the price of deposits depends on the ratio of newly issued loans relative to equity. This functional form introduces quadratic adjustment costs to the bank’s portfolio. As the bank issues more loans relative to its equity, it issues more liabilities. Likewise, as it sells more loans, it receives a lower price.\(^6\) The combination of an adjustment cost and long-term loans will induce slow-moving leverage. We assume that banks cannot issue new equity, so we focus on the dividend or asset sales margin when we discuss the connection to Facts 4 and 5. Every period, banks choose how much to payout $\text{div}$ and how many loans to issue $I$.

**Laws of motion.** From now on, we represent the problem recursively. We denote by $x'$ the variables chosen at the end of period $t$. The law of motion for the real (the economic value) of loans is:

$$\ell' = (1 - \delta) \ell + I. \quad (3.1)$$

The bank enters the period with a pre-determined amount of loans $\ell$. Over the period, a

---

\(^5\)This is a novel feature of our model and critical to capture banks’ ability to engage in evergreening and to avoid the recognition of losses immediately (see empirical evidence in Blattner, Farinha, and Rebelo (2017)). Evergreening as described in Caballero, Hoshi, and Kashyap (2008) occurs when banks roll over a loan that won’t be paid. The objective is to avoid registering losses. Since rolling over a loan does not require new funds, evergreening allows the bank to reduce its accounting equity without a cost.

\(^6\)These costs reflect the notion that banks specialize in assets that have asymmetric information problems between borrowers and lenders (e.g. Leland and Pyle (1977), Diamond (1984b), and Dang et al. (2017)).
fraction \( \delta \) matures and new loans \( I \) are added. Note that \( \ell' \) is not necessarily the same as \( \ell \) at
the beginning of \( t+1 \) because the bank cannot condition its choice of \( \ell' \) on the default shock.
Between the end of period \( t \) and \( t+1 \), loans receive a default shock and only a fraction \( \varepsilon \) of
loans survive. Therefore, by the beginning of next period the bank inherits \( \varepsilon \ell' \) loans. This
term will be captured in the value function.

The law of motion for deposits is:

\[
d' = (1 + r^d) d - (r^\ell + \delta) \ell + p(I, \ell, d) I + \text{div},
\]

where \( r^\ell \) denotes the loans rate that applies to all outstanding loans, and \( r^d \) denotes interest
on deposits. When loans mature, the bank receives the repayment value of matured loans
in deposits \( \delta \ell \). New loans cost \( p(I, \ell, d) \) deposits per unit of new loan \( I \).

The law of motion for loans on books is:

\[
\bar{\ell}' = (1 - \delta) \bar{\ell} + I
\]

The idea behind this law of motion is that the book value of loans does not recognize losses
immediately and therefore it follows the same law of motion as the market value of loans
\( \ell \), but with \( \varepsilon = 1 \). Although, in the model losses are never accounted, they are partially
recognized in the sense that deposits do not evolve as book loans would predict. Deposits
move only based on the repayment of the real value of loans.

**Regulation.** The book value of loans is subject to a regulatory constraint. We model
capital requirements as \( d' \leq \kappa (\phi \bar{\ell}' - d') \).

Here, \( \kappa \) is a capital requirement coefficient and
\( \phi \) a risk-weight on loans. This formulation allows banks to adjust their portfolio at the
beginning of the period. It also implies that some banks would violate the constraint if the
constraint were written in loan terms. We define \( \rho \equiv \phi \kappa / (1 + \kappa) \) and rewrite the constraint

\text{\footnotesize\textsuperscript{7}} We can also express the constraint in terms of equity capital.
as:

\[ d' \leq \rho \bar{\ell}'. \]

**Bank Liquidation.** A bank is shut down for two reasons: If the bank enters a period with leverage above \( \lambda \), the bank is liquidated. We interpret this as a market-based liquidation. Second, regulatory liquidiation occurs when there are no values of \((div, I)\) such that the regulatory capital constraint can be satisfied. Notice that a bank is subject to random default shocks, and due to the adjustment costs, it cannot control the market value of its loans at the beginning of the following period. An important observation is that the regulatory constraint operates on the bank’s control variables, and not on its states—the bank chooses \((div, I)\) to avoid violating the constraint. Thus, with a default shock, the bank has time to adjust its books after a shock to avoid violating the constraint. This is not true about the market-based constraint, that applies to the bank’s state variables, at the beginning of the period.

We define liquidation sets. Let \( \Gamma_r \) be the set of states of regulatory defaults, i.e. the states \( \{\ell, d, \bar{\ell}\} \) such that \( d' > \rho \bar{\ell}' \) for every \( \{div, I\} \). Let \( \Gamma_m \) be the states where \( \ell / (\ell - d) > \lambda \). The full set of liquidation states is \( \Gamma = \Gamma_m \cup \Gamma_r \). If the bank enters either liquidation state, the value of the bank is set to zero.

The bank’s policy functions are the solutions to the following profit-maximization problem:

\[
V(\ell, d, \bar{\ell}) = \max_{\{div, I\}} U(div) + \beta U(CE_t[U^{-1}(V(\varepsilon \ell', d', \bar{\ell}'))])
\]

subject to: (3.1), (3.2), (3.3) and (3.4) and \( V(\ell, d, \bar{\ell}) = 0 \) if \( \{\ell, d, \bar{\ell}\} \in \Gamma \).

### 3.2.2 Analysis

We show that the model is scale invariant. We define three useful objects. The first is the real value of bank equity, \( W \equiv \ell - d \), the state variable used by markets to price and give a market value for the bank. The second is real leverage: \( \lambda \equiv \ell / W \). The third is the ratio of the real to book value of loans: \( q \equiv \ell / \bar{\ell} \). One proposition summarizes the main result:
Proposition 3.1. Given \( \{ \lambda, q \} \) the value function is homothetic in wealth \( W \):

\[
V(\ell, d, \bar{\ell}) = \bar{V}(\lambda, q) W^{1-\theta}
\]

where \( \text{div} = cW \) and \( I = \nu \ell \), and \( \{ \bar{V}, \nu, c \} \) solve the following Bellman equation:

\[
\bar{V}(\lambda, q) = \max_{\{c, \nu, I\}} U(c) + \beta \left[ U^{-1} \left( \frac{\varepsilon}{\lambda' - (1 - \varepsilon)} \right) \Omega(\varepsilon^{1-\theta}) \right]
\]

subject to:

(a) the law of motion for leverage:

\[
\lambda' = \frac{1 - \delta + \nu}{\Omega(1)} \lambda
\]

(b) the law of motion for the market to book ratio of loans:

\[
q' = \frac{1 - \delta + \nu}{(1 - \delta) \frac{1}{q} + \nu}
\]

(c) the portfolio return:

\[
\Omega(\varepsilon) \equiv \left[ \varepsilon (1 - \delta) + \delta + r^d \right] \lambda - \left( 1 + r^d \right) (\lambda - 1) - c - (1 - \varepsilon) \nu \lambda - \gamma (\nu \lambda)^2
\]

(d) the regulatory constraint:

\[
\frac{1}{\lambda'} \geq 1 - \frac{\rho}{q'} \text{ for } q' \geq \rho
\]

(e) liquidation states:

\[
\bar{V}(\lambda, q) = 0 \text{ if } (\lambda, q) \in \Gamma
\]
and the default region defined in terms of $(\lambda, q)$ is:

$$
\Gamma \equiv \left\{ (\lambda, q) : \lambda > \bar{\lambda} \cup (1 - \rho)^2 + 4\gamma \left\{ \left[ \rho (1 - \delta) \frac{1}{q} + \delta + r^t \right] \lambda - (1 + r^d) (\lambda - 1) \right\} < 0 \right\}.
$$

A proof is contained in the Appendix. The proposition merits discussion. First, we factor out bank equity from the objective and transform the problem into a scale-free problem. The normalized problem has two state variables: $\lambda = \ell / W$ (real leverage), and $q = \ell / \bar{\ell}$ (the Tobin q for loans). The Bellman equation $\bar{V}(\lambda, q)$ is a standard consumption-portfolio problem augmented with regulatory and market-solvency constraints, and a price impact for selling or buying loans. Constraints (a) and (b) are the laws of motion for the state variables. Equation (c) is the gross return on real equity of the bank, and is composed of three terms: $\varepsilon (1 - \delta) + \delta + r^t$ is the gross return on loans, and multiplied by $\lambda$ gives the return on assets per unit of equity; $1 + r^d$ is the gross cost of deposits, and multiplied by $\lambda - 1$ gives the cost of deposits per unit of equity; and $c + (1 - \varepsilon) \ell \lambda + \gamma (\ell \lambda)^2$ are the payouts of the bank per unit of equity in the form of dividends and adjustment costs. The inequality (d) and the condition in (e) are the regulatory constraint and liquidation values re-written in terms of the states $\{\lambda, q\}$. Once the normalized problem is solved, we recover the original policy functions and laws of motion.

**Model Properties.** The model allows us to reproduce Facts 1 through 5. The price impact on the value of loans implies that banks cannot adjust their portfolio immediately. Leverage $\lambda$ is a double-edged sword: on the one hand, it enhances returns—this is captured by equation (3.7) since leverage increases $\Omega (\varepsilon)$. On the other hand, leverage exposes the bank to the risk of liquidation. Given a choice of $\lambda'$, a bad draw of $\varepsilon$ increases leverage at the beginning of next period. Moreover, the default set $\Gamma(\lambda, q)$ is increasing in $\lambda$. Higher leverage makes it more likely to violate the market leverage constraint $\bar{\lambda}$. It also tightens the regulatory constraint—Figure 3.1, left panel.

Even if a single shock does not force liquidation, leverage cannot be adjusted automatically, so a series of negative shocks will induce greater leverage and lead to liquidation. The
combination of a concave objective and the risk of default leads to a target level for leverage.

Regulatory constraints enter the problem in a novel way. In the constraint in equation (3.8), the coefficient $\rho < 1$ makes the constraint set convex and generates a trade-off between dividend payouts and growth in loans. The constraint is tighter the greater $q$. The right panel of Figure 3.1 shows the regulatory constraint in the ($\lambda', q'$) space:

Figure 3.1: Default and Constraint Sets. 

(a) Default Set (red)

(b) Regulatory Constraint Set (green)
3.2.3 Model Implied Return Shocks and Impulse Responses

We now construct a pricing equation for bank stocks and map the underlying default shocks to a return shock. The goal is to produce the model analogue to the impulse responses to return shocks. Consider a representative outside investor with a constant discount factor $\mu$. We abstract from aggregate shocks and shocks to risk premia. We assume that the investor observes $\varepsilon$. Hence, information about $\varepsilon$ is contained in market prices, but not in book values.

The investor values a share of bank stock according to:

$$ S (\ell, d, \bar{\ell}) = div + \mu E [S (\varepsilon \ell', d', \bar{\ell'})]. $$

Now, this value function can also be written recursively:

$$ S (\ell, d, \bar{\ell}) \equiv s (\lambda, q) W = cW + \mu E [s (\lambda', q') \Omega (\varepsilon)] W. $$

We interpret $S (\ell, d, \bar{\ell})$ as the market capitalization of a bank. The expected return on bank shares can be defined as:

$$ \bar{R} \equiv (div + E [S (\varepsilon \ell', d', \bar{\ell'})]) / S (\ell, d, \bar{\ell}) = (c + E [s (\lambda', q') \Omega (\varepsilon)]) / s (\lambda, q). $$

Realized returns for a given bank are given by:

$$ R (\varepsilon) = (c + s (\lambda', q') \Omega (\varepsilon)) / s (\lambda, q). $$

The model analogue to the return shocks in Section (2.2) is then given by:

$$ \Delta R (\varepsilon) = R (\varepsilon) - \bar{R} = (s (\lambda', q') \Omega (\varepsilon) - E [s (\lambda', q') \Omega (\varepsilon)]) / s (\lambda, q). $$

which is a mean-zero random variable by construction. In the following section, we report the impulse responses of bank variables to changes in $\Delta R (\varepsilon')$. These are precisely the analogue of the impulse responses in Chapter 2. Before we get to that, we present the calibration and
3.2.4 Calibration and Estimation

Calibration. We match the model to quarterly data. The parameters \( \{ \theta, \psi, \beta, \mu, r^\ell, r^d, \delta, \bar{\lambda}, \phi, \kappa \} \) are calibrated.

The values of all parameters are listed in Table 3.1. The returns on bank assets \( r^\ell \) and bank liabilities \( r^d \) are respectively set to 1.5% and 1.0% to yield corresponding annualized returns of 6% and 4%.

We set the capital constraint to \( \kappa = 9 \) to have a capital requirement ratio in line with Basel III. We set \( \phi = 0.85 \) to fit the risk-weights of loans, also in line with Basel III.\(^8\) We set \( \delta \) to 4.5% to obtain an average loan maturity of 5 years.

The rest of calibrated parameters is chosen to produce target moments. We set the risk-aversion coefficient \( \psi \) to zero because higher levels of risk aversion produce lower leverage rates. The model already falls short of the market leverage of the data. We set the inverse IES \( \theta = 1 \) to obtain an elasticity of dividends to expected returns of 1. We calibrate \( \beta \) to 0.99 to get as close as possible to the dividend rate in the data. The price/dividend ratio is determined by \( \mu \) once we set all other parameters.

---

\(^8\)Basel III features various capital requirements that banks simultaneously need to satisfy, some of which feature different risk weights when computing the value of banks’ assets. We see 85% percent as appropriate given these different requirements.
Table 3.1: Parametrization

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta = 1$</td>
<td>Inverse IES</td>
<td>Dividend Elasticity</td>
</tr>
<tr>
<td>$\psi = 0$</td>
<td>Risk aversion</td>
<td>Mean market leverage</td>
</tr>
<tr>
<td>$\beta = 0.99$</td>
<td>Banker’s discount factor</td>
<td>Mean dividend rate</td>
</tr>
<tr>
<td>$\mu = 0.98$</td>
<td>Investor’s discount factor</td>
<td>Price-dividend ratio</td>
</tr>
<tr>
<td>$r^f = 0.015$</td>
<td>Loan yield</td>
<td>BHC data: interest income / loans</td>
</tr>
<tr>
<td>$r^d = 0.010$</td>
<td>Bank debt yield</td>
<td>BHC data: interest expense / debt</td>
</tr>
<tr>
<td>$\delta = 0.045$</td>
<td>Loan maturity</td>
<td>BHC data: average loan maturity</td>
</tr>
<tr>
<td>$\bar{\lambda} = 50$</td>
<td>Market leverage constraint</td>
<td>BHC data: maximum leverage</td>
</tr>
<tr>
<td>$\phi = 0.85$</td>
<td>Risk weight on loans</td>
<td>FR-Y-9C instructions</td>
</tr>
<tr>
<td>$\kappa = 9$</td>
<td>Capital requirement</td>
<td>FR-Y-9C instructions</td>
</tr>
</tbody>
</table>

**Estimation.** To parametrize the c.d.f. for $F(\varepsilon)$ we set the support of $\varepsilon$ to $[\varepsilon, 1]$ and assume that $\ln(1 - \varepsilon)$ is distributed lognormal with mean 0 and variance $\sigma$. We truncate the support below by $\varepsilon = 0.9$, although for the values of $\sigma$ we obtain this is numerically inconsequential.

We estimate two values for the adjustment cost of asset sales $\gamma$, one for the pre-crisis level and one for the post-crisis level: $\{\gamma_{\text{pre}}, \gamma_{\text{crisis}}\}$. We also estimate two values for the volatility of default shocks $\sigma$, one for the pre-crisis level and one for the post-crisis level: $\{\sigma_{\text{pre}}, \sigma_{\text{crisis}}\}$.

We use a GMM estimation procedure that uses the market leverage and impulse responses to return shocks as the data moments. In order to make the estimation computationally feasible, we resort to an auxiliary model where $q$ is kept fixed at 1. The description of this auxiliary model, which features just one state variable, can be found in the Appendix. We show below that the auxiliary model generates similar predictions for the variables that we use for estimation as the full model. We generate data from the model by simulating the paths of banks and running the same specification as in Section (2.2).  

We estimate $\{\gamma_{\text{pre}}, \gamma_{\text{crisis}}, \sigma_{\text{pre}}, \sigma_{\text{crisis}}\}$ to match three sets of moments from the empirical section. We target two impulse responses to returns shocks from Section 2.2. We focus on

---

We simulate the paths for 10,000 banks for 200 periods for the pre-crisis regime to guarantee that the cross-sectional distribution of the state variables is stationary. We then keep the same number of quarters for which we have data before and after the crisis.
the response to market leverage, for which we use \((\lambda - 1)/s(\lambda, 1)\) as the model analogue, and on liabilities, which in the model equal \((\lambda - 1)W\). We also target the mean log market leverage observed from 1990Q3 to 2006Q4, and in 2009Q1. We then compute the distance between the coefficients estimated from the model and those from the empirical section, using as weights a diagonal matrix formed by the inverse of the variances of the estimated coefficients, as in Christiano et al. 2005. We select the values of \(\{\gamma^{\text{pre}}, \gamma^{\text{crisis}}, \sigma^{\text{pre}}, \sigma^{\text{crisis}}\}\) that minimize this distance.

We choose the target moments in order to help the model match Facts 4 and 5. In order to accommodate Fact 4, we target the empirical response of market leverage to return shocks. We do the same for the response of liabilities to return shocks in order to account for Fact 5; namely, that the composition of the adjustment of market leverage differs pre- and post-crisis. In the appendix, we discuss in more detail how the parameters \(\sigma\) and \(\gamma\) are identified using these moments and the mean market leverage.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\gamma^{\text{pre}})</td>
<td>Adjustment cost pre-crisis</td>
<td>0.39</td>
</tr>
<tr>
<td>(\gamma^{\text{crisis}})</td>
<td>Adjustment cost post-crisis</td>
<td>0.62</td>
</tr>
<tr>
<td>(\sigma^{\text{pre}})</td>
<td>Volatility of default shocks pre-crisis</td>
<td>0.0613</td>
</tr>
<tr>
<td>(\sigma^{\text{crisis}})</td>
<td>Volatility of default shocks post-crisis</td>
<td>0.0665</td>
</tr>
</tbody>
</table>

**Results.** Figure 3.2 presents the numerical solution to the bank’s problem and the investor’s valuation using the calibration and estimation above. The figure reports the bank value \(\bar{V}\), the stock price \(s\), the dividend rate \(c\), and the loan issuance rate \(i\). These objects are normalized to a unit level of \(W\). The plots are functions of \(\lambda\) and \(q\).

It is important to discuss the shape of the value function \(\bar{V}\). There are regions of the state space where the bank value and stock price drop to zero. These are the areas in the liquidation set \(\Gamma\). A second observation is that the value function is non-monotone in leverage. The reason is that the value inherits the properties of a static trade-off between
Figure 3.2: Behavior of Bank Variables as a Function of the State Variables: Leverage and Market-to-Book Value of Loans

Note: The figure shows the value function of the bank, the stock value, the optimal dividend payout and lending rates of banks as functions of the individual bank state variables ($\lambda, q$).

Risk and return. In this model, the bank cannot control its leverage perfectly because of the shocks and costs to adjust its leverage. When leverage increases, the bank increases its immediate return by exploiting the interest spread. However, excessive leverage puts the bank at risk of liquidation. A third observation is that as $q$ increases the capital requirement of the bank tightens the constraint set, and this lowers the bank’s value.

The value of bank shares inherits the shape of the bank’s value function. The only difference is in scale. The scale and shapes are different because the outside investor values cash-flows linearly. Importantly, the value of bank shares captures changes in real wealth that book loans don’t. Thus, return shocks in the model convey information about the future behavior of the bank, something that rationalizes our identification strategy. The behavior of loan growth and dividends are determined by the desire to grow leverage and the costs of delaying dividends.

Portfolio Adjustments Before and After after a Loan Liquidity Crisis. We now study the impulse response functions (IRFs) to return shocks produced by the auxiliary
model. We present the impulse responses for \( \{\gamma^{\text{pre}}, \gamma^{\text{crisis}}, \sigma^{\text{pre}}, \sigma^{\text{crisis}}\} \) keeping all other parameters constant. Namely, we compare the responses to a 1% negative realization of \( R(\varepsilon) \) for the high and low value of \( \gamma \) and \( \sigma \).

The blue curves in Figure (3.3) are the model-simulated responses, and the black lines are the empirical estimates from Chapter 2. The top panel reports the IRF of market leverage and total liabilities pre-crisis, whereas the bottom panel reports the IRF of market leverage and total liabilities post-crisis. After the shock, the market leverage of the banks jumps mechanically on impact. Market leverage remains above trend for 2.5 years. The estimated value of \( \sigma \) helps match the level of market leverage, whereas the \( \gamma \) parameters trade off a good fit for the response of market leverage versus total liabilities. The model has an almost perfect fit when only one of these two variables is targeted, and settles for an intermediate fit of the two when the unrestricted GMM procedure is used.
These responses show that this model is capable of replicating the five facts. Fact 1 notes the divergence of book and market values, and that market values capture information not captured by the books. The divergence of book and market values is captured by the decline in $q$ in the model after the shock. By construction, market values contain more information about the true state of the bank than book values, consistent with Fact 2. Consistent with Fact 3, banks in the model, are typically not up against the regulatory constraint nor the market constraint on leverage. However, bank decisions take into account the consequences of hitting these constraints. Fact 4 is evidence on a leverage target and slow reversion to that target. The responses produced by the model are consistent with that pattern. Fact 5 notes the difference in the impulse responses pre- and post-crisis. In both the model and the data, the pre-crisis impulse response shows banks responding to a negative shock by
reducing their liabilities, with little action on the equity finance margins. Post-crisis, banks reduce leverage less actively through the use of asset sales.\textsuperscript{10}

In addition to producing the five stylized facts of the data, the model reveals some additional insights. Within the model, adjustment costs serve as an amplification mechanism for shocks. The bank not only faces direct losses from defaults, but faces additional losses due to increased adjustment costs to reach their target leverage. Another insight is that books are slow to reflect true equity losses; this delay provides an immediate regulatory slack. An immediate adjustment of books could be very costly. Allowing banks to smooth out the necessary adjustment over time can limit the consequences of their equity losses and work as a dampening effect.

Another insight is that the slow adjustment of bank balance sheets is a mechanism for the persistence of a credit crunch. If banks cannot adjust their leverage immediately, they will attempt to cut back on the flow of new loans. Thus, a contraction in the credit supply will persist until banks return to their target leverage.

Although we have not incorporated general equilibrium effects into the model, we can anticipate there could be contagion. Forced asset sales by one bank can lower the price of loans for other banks.

3.3 Conclusion

We present a heterogeneous-bank model that distinguishes book from market values. In our model, both measures of equity matter for banking decisions. A novel feature is that banks have the ability to delay the acknowledgement of loan losses on their books. The model produces an endogenous target for leverage and features adjustment costs to the resale of assets. The model reproduces the impulse responses that we estimated from the cross-sectional data. When we interpret the crisis as a period of greater adjustment costs

\textsuperscript{10}In the data we also examined the breakdown between raising equity through new issuances vs. through lower dividends (retained earnings). In the model, we abstract from this distinction for simplicity.
and larger realization of the default shock, the model is able to reproduce the shift in the method of adjustment of market equity that we see in the data.

The model has many reduced-form assumptions and is studied in partial equilibrium. However, it highlights the essential theoretical features that are necessary to reproduce the five empirical facts. Our hope is that we can use this model to search for the microfoundations that can deliver similar reduced form dynamics and, therefore, can also reproduce the empirical facts.
3.4 Appendix

3.4.1 Proof of Proposition 3.1

Preliminary Definitions. Define the net investment rate of the bank as:

\[ \iota \equiv I / \ell. \]

and express the dividend-to-equity ratio as:

\[ c \equiv \text{div}/W. \]

Note the following identities:

\[ \ell = \lambda W \]
\[ d = (\lambda - 1)W \]
\[ \bar{\ell} = q^{-1} \lambda W. \]

We present some observations that aid the proof of the proposition.

Observation 1: homogeneity of \( p \). Observe that:

\[
\begin{align*}
p(I, \ell, d) &= 1 + \gamma \frac{I}{\ell} \frac{\ell}{\ell - d} \\
&= 1 + \gamma \frac{I}{\ell} \frac{\ell}{W} \\
&= 1 + \gamma \iota \lambda.
\end{align*}
\]

Thus, we can express \( p(I, \ell, d) \) as:
which is a function independent of the size of the bank, and only depends on the composition of its assets and its investment rate.

**Observation 2: homogeneity of regulatory constraint.** We want to express the capital requirement in terms of the end-of-period choices \((\lambda', q')\). First, notice that we can express the regulatory constraint as:

\[
d' \leq \frac{\kappa}{1+\kappa} \phi \bar{\ell}.
\]

Let \(\rho \equiv \phi \kappa / (1+\kappa)\). Then, this constraint is equivalent to:

\[
\frac{d'}{W'} \leq \rho \frac{\ell'}{W'}
\]

Using the definitions of \(\lambda\) and \(q\):

\[
\lambda' - 1 \leq \frac{\rho}{q'} \lambda'
\]

and rearranging:

\[
\lambda' \leq \frac{1}{1 - \frac{\rho}{q'}}
\]

which is operational for \(q' \geq \rho\). Note that the constraint is independent of \(W\).

**Observation 3: growth independence.** Next, we denote by \(W'(\varepsilon)\) the bank’s equity at the beginning of period \(t + 1\), as a function of the realization of the default shock, and express it as a linear function of wealth:

\[
W'(\varepsilon) = \varepsilon \ell' - d'
\]

\[
= \varepsilon ((1 - \delta) \ell + I) - ((1 + r^d) d - (\delta + r^f) \ell + p(I, \ell, d) I + \text{div})
\]

\[
= (\varepsilon (1 - \delta) + \delta + r^f) \ell - (1 + r^d) d - \text{div} + (\varepsilon - \bar{p}(\iota, \lambda)) I
\]
Now, we factor $W$ from the expression above to obtain:

$$W' (\varepsilon) = \left[ (\varepsilon (1 - \delta) + \delta + \ell \lambda) - (1 + r^d) (\lambda - 1) - c + (\varepsilon - \bar{p} (\iota, \lambda)) \iota \lambda \right] W$$

$$= \Omega (\varepsilon) W.$$

Here, we implicitly defined the return on equity as:

$$
\Omega (\varepsilon) \equiv (\varepsilon (1 - \delta) + \delta + \ell \lambda) - (1 + r^d) (\lambda - 1) - c + (\varepsilon - \bar{p} (\iota, \lambda)) \iota \lambda.
$$

Substituting in the price of deposits:

$$
\Omega (\varepsilon) \equiv (\varepsilon (1 - \delta) + \delta + \ell \lambda) - (1 + r^d) (\lambda - 1) - c - (1 - \varepsilon) \iota \lambda - \gamma (\iota \lambda)^2.
$$

**Observation 4: recursive leverage.** Next, from the law of motion of $\ell$ we obtain a recursive expression for the law of motion of leverage $\lambda$ given any choice of $\iota$ and $c$:

$$
\lambda' = \frac{\ell'}{W'} = \frac{(1 - \delta + \iota) \ell}{W'} = (1 - \delta + \iota) \lambda \frac{W}{W'}.
$$

But note that $W' = W' (1) = \Omega (1) W$. Therefore:

$$
\lambda' = \frac{1 - \delta + \iota}{\Omega (1)} \lambda.
$$

Similarly, the value of $\lambda$ at the beginning of period $t + 1$ after the realization of the default...
shock is:

\[
\lambda'(\varepsilon) = \frac{\varepsilon\ell'}{W'(\varepsilon)} = \frac{\varepsilon(1 - \delta + \iota)\ell}{W'(\varepsilon)} = (1 - \delta + \iota)\lambda' \frac{\varepsilon W}{W'(\varepsilon)} = \frac{\varepsilon\Omega(1)}{\Omega(\varepsilon)} \lambda' = \frac{\varepsilon}{1/\lambda' - (1 - \varepsilon)}
\]

To derive the last equality we use the fact that, from the law of motion for \(\lambda'\) and the definition for \(\Omega(\varepsilon)\):

\[
(1 + r^f)\lambda - (1 + r^d)(\lambda - 1) - c - \gamma(\mu\lambda)^2 = \Omega(1) = \frac{1 - \delta + \iota}{\lambda'} \lambda
\]

\[
\Rightarrow c = (1 + r^f)\lambda - (1 + r^d)(\lambda - 1) - \gamma(\mu\lambda)^2 - (1 - \delta + \iota)\frac{\lambda}{\lambda'}
\]

and substituting this expression in \(\Omega(\varepsilon)\):

\[
\frac{\Omega(1)}{\Omega(\varepsilon)} \varepsilon \lambda' = \frac{(1 - \delta + \iota)\frac{\lambda}{X}}{(\varepsilon(1 - \delta) + \delta + r^f)\lambda - (1 + r^d)(\lambda - 1) - c - (1 - \varepsilon)\iota\lambda - \gamma(\mu\lambda)^2} \varepsilon \lambda'
\]

\[
= \frac{(1 - \delta + \iota)\frac{\lambda}{X}}{(1 - \delta + \iota)\lambda}
\]

\[
= \frac{(\varepsilon(1 - \delta) + \delta + r^f)\lambda - (1 + r^d)\lambda + (1 - \delta + \iota)\frac{\lambda}{X} - (1 - \varepsilon)\iota\lambda}{(1 - \delta + \iota)\lambda}
\]

\[
= \frac{\varepsilon}{1/\lambda' - (1 - \varepsilon)}
\]

**Observation 5: recursive expression for \(q\).** Next, we show how to write \(q\) in a recursive way:
\[
q' = \frac{\ell'}{\ell} = \frac{(1 - \delta) \ell + I}{(1 - \delta) \ell + I} = \frac{(1 - \delta) \lambda + \iota \lambda}{(1 - \delta) \frac{1}{q} + \iota} W
\]

\[
q' = \frac{1 - \delta + \iota}{(1 - \delta) \frac{1}{q} + \iota}.
\]

The value of \( q \) at the beginning of period \( t + 1 \) after the realization of the default shock is simply \( q'(\varepsilon) = \varepsilon q' \).

**Observation 6: default set.** The regulatory constraint can be re-expressed as:

\[
1 - \rho \frac{q'}{q} \leq \frac{1}{\lambda'}.
\]

From here, we employ the laws of motion for \( \lambda' \) and \( q' \) to obtain:

\[
1 - \rho \frac{1 - \delta + \iota}{(1 - \delta) \frac{1}{q} + \iota} \leq \frac{\Omega (1)}{(1 - \delta + \iota) \lambda}
\]

\[
1 - \frac{\Omega (1)}{(1 - \delta + \iota) \lambda} \leq \frac{\rho \left( (1 - \delta) \frac{1}{q} + \iota \right)}{1 - \delta + \iota}
\]

\[
1 - \frac{(1 + r^\ell) \lambda - (1 + r^d) (\lambda - 1) - c - (1 - \bar{p}(\iota, \lambda)) \iota \lambda}{(1 - \delta + \iota) \lambda} \leq \frac{\rho \left( (1 - \delta) \frac{1}{q} + \iota \right)}{1 - \delta + \iota}
\]

\[
(\iota - \delta - r^\ell) \lambda + (1 + r^d) (\lambda - 1) + c + (1 - \bar{p}(\iota, \lambda)) \iota \lambda \leq \rho \left( (1 - \delta) \frac{1}{q} + \iota \right) \lambda
\]

Substituting for \( \bar{p} \) and re-arranging we obtain:

\[
(1 - \rho) \iota \lambda + \gamma (\iota \lambda)^2 + c \leq \left( \rho (1 - \delta) \frac{1}{q} + \delta + r^\ell \right) \lambda - (1 + r^d) (\lambda - 1).
\]

So long as \( \rho < 1 \), the constraint set is convex, for any \((\lambda, q)\). Note again that the constraint is independent of \( W \). Default occurs whenever there are no possible choices of \((c, \iota)\) such that this inequality can be satisfied. Note that the left hand side is minimized for \( c = 0 \) and...
\( \nu = -(1 - \rho) / 2\gamma \lambda \). Evaluating the left hand side at the minimizers, default occurs if \((\lambda, q)\) belong to the region where the minimized left hand side is larger than the right hand side:

\[
-\frac{(1 - \rho)^2}{4\gamma} > \left( \rho (1 - \delta) \frac{1}{q} + \delta + r^\ell \right) \lambda - \left( 1 + r^d \right) (\lambda - 1)
\]
or:

\[
(1 - \rho)^2 + 4\gamma \left[ \left( \rho (1 - \delta) \frac{1}{q} + \delta + r^\ell \right) \lambda - \left( 1 + r^d \right) (\lambda - 1) \right] < 0
\]

The default set \( \Gamma^m \) is simply the values of \( \ell \) and \( d \) such that \( \ell / (\ell - d) = \lambda > \bar{\lambda} \).

**Observation 7: homogeneity of the value function.** We guess and verify that:

\[
V (b, l, \bar{b}) = \bar{V} (\lambda, q) W^{1 - \theta}
\]

for Epstein-Zin preferences with IES \( 1/\theta \). Next, we work with our guess for the value function:

\[
\bar{V} (\lambda, q) W^{1 - \theta} = \max_{\{\text{div}, I\}} U \left( \frac{\text{div}}{W} \right) W^{1 - \theta} + \beta U \left( CE_t \left[ U^{-1} \left( V (\varepsilon \ell', d', \bar{\ell}') \right) \right] \right)
\]

subject to:

\[
\ell' = (1 - \delta) \ell + I \\
\bar{\ell}' = (1 - \delta) \bar{\ell} + I \\
d' = (1 + r^d) d - (\delta + r^\ell) \ell + p (I, \ell, d) I + \text{div} \\
d' \leq \kappa (\phi \bar{\ell}' - d').
\]

First, we transform the constraint set. We use the law of motion for wealth to express every equation in terms of past wealth and current decisions:

\[
W' (\varepsilon) = \Omega (\varepsilon) W.
\]
Then, observations 4 and 5 imply that once we know $\lambda'$ and $q'$ and $W'$, we know $\{\ell', \bar{\ell}', d'\}$ and the first equations are satisfied.

Second, we divide both sides of the regulatory constraint, again to obtain:

$$(1 - \rho) \iota \lambda + \gamma (\iota \lambda)^2 + c \leq \left( \rho (1 - \delta) \frac{1}{q} + \delta + \rho^{\ell} \right) \lambda - (1 + r^d) (\lambda - 1).$$

If the conjecture is right, we can replace the law of motion into the objective and obtain:

$$\bar{V} (\lambda, q) W^{1-\theta} = \max_{\{c, \iota\}} U (c) W^{1-\theta} + \beta U \left( CE_t \left[ U^{-1} \left( \bar{V} \left( \frac{\varepsilon}{1/\lambda' - (1 - \varepsilon)}, \varepsilon q' \right) \Omega (\varepsilon)^{1-\theta} \right) \right] \right) W^{1-\theta}.$$ 

This shows that the objective is indeed homothetic in wealth. Hence, we have to solve the following:

$$\bar{V} (\lambda, q) = \max_{\{c, \iota\}} U (c) + \beta U \left( CE_t \left[ U^{-1} \left( \bar{V} \left( \frac{\varepsilon}{1/\lambda' - (1 - \varepsilon)}, \varepsilon q' \right) \Omega (\varepsilon)^{1-\theta} \right) \right] \right)$$

subject to the conditions we described above:

(a) Law of motion for leverage:

$$\lambda' = \frac{(1 - \delta + \iota) \lambda}{\Omega (1)}.$$

(b) Law of motion for books:

$$q' = \frac{1 - \delta + \iota}{(1 - \delta) \frac{1}{q} + \iota}.$$ 

(c) Loan price:

$$\bar{p} (\iota, \lambda) = 1 + \gamma \iota \lambda$$

(d) Return on equity:

$$\Omega (\varepsilon) = (\varepsilon (1 - \delta) + \delta + \rho^{\ell}) \lambda - (1 + r^d) (\lambda - 1) - c - (1 - \varepsilon) \iota \lambda - \gamma (\iota \lambda)^2.$$ 

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(e) Regulatory constraint:
\[ \frac{1}{\lambda'} \geq 1 - \frac{\rho}{q'} \quad \text{for} \quad q' \geq \rho \]

This verifies the proposition.

### 3.4.2 Computational Algorithm

To solve numerically the problem of the bank we re-write it in terms of the control variables \( \lambda' \) and \( q' \). To solve for \( c \) and \( \iota \) in terms of \( \lambda' \) and \( q' \), first note that the law of motion for \( q \) implies:
\[ q' = \frac{1 - \delta + \iota}{(1 - \delta) \frac{1}{q' + \iota}} \Rightarrow \iota = - (1 - \delta) \frac{1}{1 - q'} \]

and, using the definition of \( \Omega (1) \), the law of motion for \( \lambda \) implies:
\[ \lambda' = \frac{1 - \delta + \iota}{(1 + r^\ell) \lambda - (1 + r^d) (\lambda - 1) - c - \gamma (\iota \lambda)^2} \frac{1}{\lambda} \]

\[ \Rightarrow c = (1 + r^\ell) \lambda - (1 + r^d) (\lambda - 1) - \gamma (\iota \lambda)^2 - (1 - \delta + \iota) \frac{\lambda}{\lambda'} \]

Another useful observation is that, with this expression for \( c \), the return on equity can be expressed as:
\[ \Omega (\varepsilon) = (1 - \delta + \iota) \lambda \left( \frac{1}{\lambda'} - (1 - \varepsilon) \right) \]

This rearrangement is useful because the first part, \((1 - \delta + \iota) \lambda\), is independent of \( \varepsilon \), and can therefore be factored out of the continuation value in the Bellman equation:
\[ U \left( C E_t \left[ U^{-1} \left( \bar{V} \left( \frac{\varepsilon}{1/\lambda' - (1 - \varepsilon)}, \varepsilon q' \right) \Omega (\varepsilon)^{1-\theta} \right) \right] \right) = \ldots \]

\[ ((1 - \delta + \iota) \lambda)^{1-\theta} U \left( C E_t \left[ U^{-1} \left( \bar{V} \left( \frac{\varepsilon}{1/\lambda' - (1 - \varepsilon)}, \varepsilon q' \right) \left( \frac{1}{\lambda'} - (1 - \varepsilon) \right)^{1-\theta} \right) \right] \right) \]

The second term in the continuation value, \[ U \left( C E_t \left[ U^{-1} \left( \bar{V} \left( \frac{\varepsilon}{1/\lambda' - (1 - \varepsilon)}, \varepsilon q' \right) \left( \frac{1}{\lambda'} - (1 - \varepsilon) \right)^{1-\theta} \right) \right] \right), \]
does not depend on the state variables \((\lambda, q)\), and hence only needs to be computed once per iteration (as opposed to once for every point in the state space).

The main computational challenge to solve the model is that we need to construct a mapping between the grids for \(\lambda'\) and \(q'\) and the values of \(\lambda\) and \(q\) at the beginning of period \(t + 1\), \(\frac{\varepsilon}{1/\lambda' - (1 - \varepsilon)}\) and \(\varepsilon q'\), which need to belong to the grids for \(\lambda\) and \(q\). However, because of the factorization of the continuation value we just did, this mapping only needs to be computed once at the beginning of the algorithm.

To reduce the numerical error generated by the discrepancies between \(\frac{\varepsilon}{1/\lambda' - (1 - \varepsilon)}\) and the value assigned in the grid for \(\lambda\), and between \(\varepsilon q'\) and the value assigned in the grid for \(q\), the mapping we use assigns the two closest values in the state space, with proportions given by 1 minus the size of the discrepancy. For instance, if \(\frac{\varepsilon}{1/\lambda' - (1 - \varepsilon)} = 5.4\) and the two closest values in the grid for \(\lambda\) are 5.2 and 5.7, then we weight the continuation value with \(\lambda = 5.2\) by \(1 - 0.2/0.5\) and the continuation value with \(\lambda = 5.7\) by \(1 - 0.3/0.5\).

We solve the model using value function

\[
\Omega(\varepsilon) = (1 - \delta + \iota) \lambda \left( \frac{1}{\lambda'} - (1 - \varepsilon) \right)
\]

iteration over a discretized state space of 2000 nodes for \(\lambda\) and 500 nodes for \(q\). The control variables \(\lambda'\) and \(q'\) are also discretized over 2000 and 500 nodes, respectively. The exogenous state variable \(\varepsilon\) is discretized over 1000 nodes.

Given the parameters and prices (i.e. the interest rates on loans and deposits), the bank solves its optimization problem. The following computational algorithm solves the bank’s problem by updating the value function of the bank until a fixed point is reached.

1. Use the value function from the last iteration (or a guess if it is the first).

2. For each value of the state variables, check whether the constraints are not violated.

3. Conditional on a given endogenous state variable combination

(a) for each possible value of the control variables \((\lambda', q')\)
i. Update the values of the variables \((c, \iota)\) as well as \((1 - \delta + \iota) \lambda\).

ii. Compute the new value function, using the Bellman equation.

(b) Find the value of the control variables \((\lambda', q')\) that maximizes the value function for a given state variable combination.

4. The resulting updated value function is compared to the initial value function. Until convergence is reached, repeat steps 1 to 4.

3.4.3 Auxiliary Model

In order to estimate the adjustment costs and volatility of default shocks pre- and post-crisis, we use a simplified version of the model that keeps the value of the Tobin q for loans fixed at \(q = 1\). Note that for \(q = 1\) the optimal choice is \(q' = 1\). We abstract from \(\varepsilon\) shocks to \(q\) for this simplified case, so the state \(q = 1\) becomes absorbent. The problem of the bank is:

\[
V^{aux}(\lambda) = \max_{\{\lambda', \iota\}} \left\{ U(c) + \beta U \left( C E_t \left[ U^{-1} \left( V^{aux} \left( \frac{\varepsilon}{1/\lambda' - (1 - \varepsilon)} \right) \Omega(\varepsilon)^{1 - \theta} \right) \right] \right) \right\}
\]

subject to:

\[
c = (1 + r^d) \lambda - (1 + r^d) (\lambda - 1) - \gamma (\iota \lambda)^2 - (1 - \delta + \iota) \frac{\lambda}{\lambda'}
\]

\[
\Omega(\varepsilon) = (1 - \delta + \iota) \lambda \left( \frac{1}{\lambda} - (1 - \varepsilon) \right)
\]

\[
\lambda' \leq \frac{1}{1 - \rho}
\]


Harris, Trevor, Urooj Khan, and Doron Nissim (2013). “The Expected Rate of Credit Losses on Banks’ Loan Portfolios”.


Song, Chang Joon, Wayne B. Thomas, and Han Yi (2010). “Value relevance of FAS No. 157 Fair Value hierarchy information and the impact of corporate governance mechanisms”.

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