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Journal
Social Science Research, 5(4)

ISSN
0049-089X

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Publication Date
1976-12-01

DOI
10.1016/0049-089x(76)90025-9

Peer reviewed

SOCIAL SCIENCE RESEARCH 5, 385-404 (1976)

Why the Trade/GNP Ratio Decreases with Country Size[1]

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Several rational models are constructed to show why the ratio of a country's foreign trade to its GNP should decrease with increasing country size. A physics-inspired model of absorption of a quasi-continuous particle flow by a quasi-homogeneous space results in detailed agreement with the observed contemporary world-wide pattern of import/GNP vs. population size. On the average, world imports behave as if 63% of a product flow were absorbed by the nearest 730,000 people or by people within 37 miles of the production center, whichever number of people is higher. "Characteristic transport distances" (over which 63% of flow is absorbed) are given for 110 countries and range from 10 to 300 miles. The corresponding "characteristic absorption numbers" range from 160 to 1700 people; they indicate the number of people, linearly arranged, which absorbs 63% of a flow.

The ratio of a country's foreign trade (i.e., exports and imports) to its GNP has a known tendency to decrease with the size of the country (Chenerey, 1960; Deutsch and Eckstein, 1961; Deutsch *et al.*, 1962). Recent analysis of more extensive data (Taagepera and Hayes, 1977) has clarified *how* the trade/GNP ratio (in %) varies with population size (P, in millions). The relation is especially clear-cut for imports where the empirical equation

$$\text{Imp/GNP} = 40\,P^{-1/3} \qquad (1)$$

holds within a factor of 2, for more than 90% of contemporary countries (see dashed line in Fig. 1 which is plotted on log-log scale so that a cubic relation yields a straight line), and also for the United States ever since 1799. The actual least square fit (of logarithms) for data in Fig. 1 is Imp/GNP = $37.6\,P^{-1/3.10}$, with correlation coefficient $r = -.83$, which means that P accounts for 69% of the observed variation in the Imp/GNP ratio. In Eq. 1, the regression coefficients (37.6 and 3.10) have been rounded off to a single significant figure, since the data scatter does not warrant more precision. (Fake overprecision is a major scourge of today's social science literature.) For exports, the analogous equation

[1]This research was partly supported by the Irvine branch of the Institute of Transportation and Traffic Engineering, University of California.

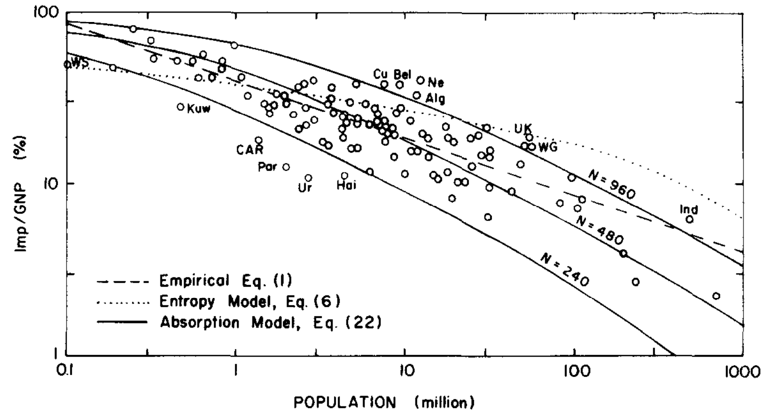

Fig. 1. Import/GNP ratio versus population, on log-log scale. Data from Taagepera and Hayes (1977).

$$\text{Exp/GNP} = 30\,P^{-1/3} \tag{2}$$

holds less well, mainly due to the exclusion of nonmonetary exports (labor, services, military-political concessions and favors) which help to compensate for trade deficits. Because of these exclusions (which are discussed in more detail by Taagepera and Hayes, 1977), import figures are more meaningful trade indicators than are export figures. Note that the Exp/Imp ratio which results from Eqs. 1 and 2 is 3/4, irrespective of country size.

It is now time to proceed beyond the empirical description, and explain in terms of a theoretical model *why* the trade/GNP ratio should change with country size the way it is observed to do. In so doing, we will gain insight into the nature of the phenomenon. In particular, it will be seen that the model proposed results in an interesting parameter, the characteristic transport distance, for the world and for individual countries. This parameter may have not only intellectual but also practical importance.

What is a model? In physical sciences it usually means a construct of mind which uses reasoning to explain why things interact the way they do. In social sciences results of mechanical data analysis, such as regression equations, are also sometimes called models. This confusion of description with explanation is regrettable. Calling an empirical equation a model often masks the need for a truly explanatory model. In this paper the term "model" always designates an explanatory one. Thus, Eq. 1 is *not* a model but an empirical product of data analysis begging for a rational justification of its existence.

Several semiquantitative or otherwise inadequate approaches to the trade-size relationship will be discussed briefly first, in order to delineate the nature of the problem. Then a model that fits the data is presented. It is based on the notion of gradual absorption of a "flux" (e.g., of neutrons or of goods) in a medium, an approach that has been used extensively to describe physical phenomena. Although such phenomena take place in a multidimensional space, the problem is usually first solved for the simpler one-dimensional case. This will be done here, too. The model will then be extended to the more life-like case of two-dimensional countries. It will be tested against contemporary trade/GNP data, and the resulting characteristic transport distances will be determined.

In all these models exports are assumed to balance the imports although actual export figures reported tend to be lower (cf. Eqs. 1 and 2). The resulting GNP-normalized fractions will be designated by F:

$$F = \text{Imp/GNP} = \text{Exp/GNP}. \tag{3}$$

The theoretical argument will be carried out in terms of imports or exports, whichever is easier to visualize. Testing with data will use imports, because of the aforementioned shortcomings of export figures.

PRELIMINARY APPROACHES

Three approaches will be discussed, all of which indicate that the trade/GNP ratio should be expected to decrease with country size. These approaches either do not describe the exact pattern of change or, if they do, this pattern differs from the observed one. But they are nonetheless useful in specifying the problem and giving one an intuitive feeling about the main trend.

The Merger Approach

Suppose that two countries with equal GNP (G) and exports (E) decide to merge. (An approximate actual example would be the Egypt-Syria union around 1960.) Prior to merger, both countries have an F ratio of E/G. Designate *by* E'' their reciprocal exports which will no more be considered part of exports after the merger. The F ratio of the larger country resulting from the merger will be $2(E - E'')/2G = E/G - E''/G$ which is smaller than the F ratio of either of the original components. The merger is, of course, likely to alter the commercial pattern. But it is most likely to increase the reciprocal interaction of the two components at the expense of outside trade, thus reducing the trade/GNP ratio even more. The outcome of merger becomes ambiguous when the two components are not of equal size.

Conversely, if a state loses territory, the earlier internal trade between the separating parts becomes part of foreign trade of each part, and relative foreign trade increases. This approach was used by Deutsch and Eckstein (1961) to explain qualitatively an increase in German trade/GNP ratio from

1910 to 1950: "If one could deduct from these foreign trade figures the amounts representing commerce with former German territories such as Alsace-Lorraine since 1918 and East Germany since 1945, and if one allows for the increased need for foreign trade in a truncated Germany, it seems likely that the actual trend of the foreign ratio would have been downward from the first decade of the twentieth century to the mid-1950's, had the relevant German territory remained constant."

The Boundary Conditions Approach

Reasoning and calculations using boundary conditions pervades physics (see, e.g., Churchill, 1941). However, there is nothing physical about the method itself. Since this reasoning tool seems to be little known and accepted among many social scientists, its basic philosophy will be briefly presented. Suppose the values of some property or characteristic are known at the boundaries of a region but not inside the region. Assuming that sharp discontinuities are forbidden, the boundary values may give us some indication about values inside the region. If the rules that govern the local rate of change of the property with distance are also specified, through appropriate differential equations, the values at any point within the region may be uniquely determined. The region and its boundaries need not involve physical space–it may be a region in time, population, etc. Boundary conditions are used to solve major problems in electrodynamics (Jackson, 1962, pp. 15ff; and Halliday and Resnick, 1962, p. 668), quantum mechanics (Schiff, 1955, pp. 29ff), nuclear physics (Evans, 1955, p. 67), solid state physics (Kittel, 1956, pp. 246ff), nuclear reactor design (Glasstone and Edlund, 1952, pp. 102ff), and in various other engineering problems (Karman and Biot, 1940, pp. 309ff). For a discussion of boundary conditions as a subgroup of non-holonomic constraints, see Goldstein (1959, p. 11).

Let us now establish the conceptual boundary conditions for the dependence of ratio F on the country's population size. This ratio can, in principle, take values ranging from 0, when nothing is exported and imported, to 1, when everything produced is exported and everything consumed is imported, excluding nonmonetary production and exchanges.

The largest conceivable population size a country could reach is the population of the whole system within which the trading is carried out, i.e., the world population (P_W). Such a country's foreign trade would perforce be zero, since there is nobody left outside the country with whom to trade. Note that we are not at all concerned here with the practical feasibility of a world state–the conceptual limiting value may or may not be achievable in reality. All we say is that no state can be larger than the world, and if a state ever reached this upper limit on size, then its foreign trade would be zero.

The smallest conceivable population a country could have is one person, if we define a country to be a territory involving people. Again, we are not concerned with the practical viability of so small a country–it may be an unrealizable limit case. But if such a country came to exist, its ratio F would have to be unity, since everything it produced and sold for money would have to be sold abroad. (This would not be true, if the GNP included the nonmonetary subsistance economy products. But the GNP as presently defined and measured includes only goods and services sold to other people for money.)

The boundary conditions are thus the following:

$$1 < P < P_W \quad \text{and} \quad 0 < F < 1;$$
$$F = 1 \quad \text{for} \quad P = 1;$$
$$F = 0 \quad \text{for} \quad P = P_W. \tag{4}$$

Because of general considerations of continuity and simplicity, we should expect F to decrease gradually and smoothly from 1 to 0 when a country's populations size increases from 1 to P_W, unless we have additional conceptual or observational reasons that require discontinuities or fluctuations in F as P increases. In the absence of such reasons, we can conclude from the boundary conditions that F should be a monotonically decreasing function of population size:

$$dF/dP < 0. \tag{5}$$

However, it is not enough to show that the rate of change of F with P is negative; a complete explanation requires that F (or its derivative) be given explicitly as a function of P.

The Entropy Approach

People trade because different people produce different things. The more people a country has, the more different things are likely to be produced in this country and, therefore, the fewer items need to be imported from abroad. The variety of products a country makes depends of course on many other geographical and social factors. But all other factors being equal, a more populous country can be expected to have a larger variety of products. The same expectation prevails, on the average, if the effect of other factors is unknown.

A systematic way to measure variety is supplied by the notion of entropy (see, e.g., Theil, 1967; or Taagepera and Ray, 1977). For P possible states or components of a given system, maximum entropy equals the natural logarithm of P. In the present context, the number of components is the number of people. The number of distinguishable products made by a country with population P, might thus be expected to be proportional to $\ln P$, while that of the whole world would be proportional to $ln\ P_w$. Then the country considered would produce internally a fraction $ln\ P/ ln\ P_w$ of what the world can offer, and it might want to import the rest. Assuming balanced trade, this means that a fraction $ln\ P/ ln\ P_w$ of GNP is *not* exported, so that

$$F = 1 - ln\ P/ln\ P_w. \qquad (6)$$

This equation satisfies the previously established conditions (4) and (5). But the resulting curve (for P_w = 4 billion) differs markedly from the actual pattern (see dotted curve in Fig. 1). Equation 6 predicts rather correctly the F ratios of middle-size countries, but it overestimates F for the large countries and underestimates it for the small countries. Even this limited degree of agreement is quite remarkable, if one bears in mind that Eq. 6 contains not a single freely adjustable parameter, once the boundary conditions are established. Its major conceptual flaw is that it ignores the spatial aspect of the problem: products made nearby may be substituted for more desirable products made further away, because of information and transport costs. This spatial aspect dominates the preferred model which is presented next.

THE ABSORPTION MODEL FOR FOREIGN TRADE IN HOMOGENEOUS MEDIA

The basic model is the one used in physics for absorption of any flux of fluids or particles in homogeneous surroundings. It is used for radiation absorption (e.g., Richards *et al.*, 1960, p. 811), and for neutron absorption in nuclear reactors (e.g., Glasstone and Edlund, 1952, p. 45). However, there is nothing about the model that is specific to inanimate physical objects. It deals with properties of space, rather than those of particular fluids or particles. To the extent that trade takes place in physical space it is subject to the same laws of space as any other flow.

Consider a "point source," i.e., a small region that produces, at a constant rate of I_0 units per day, certain items with uniform characteristics, be they monoenergetic neutrons or a certain brand of radio sets. These items "flow out" from the source into the surrounding space. This space contains objects that tend to absorb the flow. A neutron may encounter an atomic nucleus with suitable characteristics, and a radio set may encounter a customer–and both stop there. Those readers who may object to the gross oversimplification of the trade pattern should keep in mind that the neutron absorption process, too, is more complex than described above, yet the simple basic model has been found to be indispensable in their case, as a starting point.

Assume that such absorption centers are evenly distributed in space (which may be three-dimensional in the case of neutrons and two-dimensional in the case of radio sets). Then the probability of any particular item being absorbed during a short distance increment is the same everywhere. Consider the flow intensity I (i.e., the number of items which have not yet been absorbed) at distance r from the source. Designate by dI the change in intensity over a further infinitesimal increment dr in distance. (Note that we disregard the discontinuous nature of particle flow when we use the calculus notation dI; this is another approximation that has been useful in physics, as long as the number of particles is large.) For space with homogeneous absorption characteristics dI is proportional to I and to dr, so that

$$dI/dr = -k\,I, \tag{7}$$

where the constant k reflects the absorptive ability of the space (cf. Glasstone and Edlund, 1952, p. 45). This equation says that the rate of absorption depends on the absorption coefficient k of the surrounding matter (be it made of atoms or customers) and on the number (I) of items offered for potential absorption. (We assume that the neutrons or the radio sets come in limited numbers so that the market is not saturated.) Equation 7 expresses the properties of homogeneous space rather than of any specific medium or flow. It integrates into

$$I = I_0 \exp(-kr), \tag{8}$$

where I_0 is the intensity at the source. The inverse ($L = 1/k$) of the absorption coefficient is a characteristic absorption distance; it is the distance over which I is reduced to a fraction $1/e$ of its original value.

For the reader unaccustomed to model-building, the assumptions made (and even more, those that will soon be made) may seem to naively oversimplify the complex reality of economic production and trade. They do. But it should not be thought that this oversimplification does not arise in physics models. It does; and yet such simple models have been found to be useful, nay, indispensable, for the development of the science of physics.

The One-Dimensional Case

Many problems involving multidimensional space are first solved in one dimension, so that the conceptual problems can be settled before one has to worry about the complex integrations that multidimensional space inevitably introduces. Once the one-dimensional case is solved, all attention can be directed to the calculus needed for the two-dimensional geometry of our basically flat countries or the three-dimensional geometry needed in most physical problems. In social sciences, Kochen and Deutsch (1969) have used the one-dimensional approach in modeling of optimal decentralization.

Consider a country of length $2R$, in a one-dimensional homogeneous infinite-length world (Fig. 2). A production center of intensity I_0 at distance

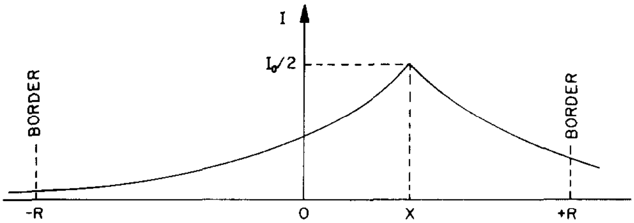

Fig. 2. One-dimensional absorption model.

x from the center sends half of its production flow in each direction. The export (E') of the production from this center corresponds to the sum of flow intensities at the two border points, and is thus given by Eq. 8 as

$$E'(x) = (I_0/2)\ [e^{-k(R-x)} + e^{-k(R+x)}]. \tag{9}$$

Assuming now a continuous and homogeneous distribution of production throughout the country, with all products having the same absorption coefficient k, the country's total production (G) is

$$G = \int_{-R}^{R} i\ dx = 2iR, \tag{10}$$

where i is the constant "production density" per unit distance, i.e., the amount produced per unit distance. The country's total exports (E) are obtained by replacing I_0 by $(i\,dx)$ in Eq. 9, and by integrating for all production centers, from border to border:

$$E = \int_{-R}^{R} (i/2)\ [e^{-k(R-x)} + e^{-k(R+x)}]\ dx = (i/k)\ (1 - e^{-2kR}). \tag{11}$$

The country's export/GNP ratio F_1 (where the subscript "1" refers to the one-dimensional model) is then

$$F_1 = E/G = (1 - e^{-2kR})/2kR. \tag{12}$$

Since we assume that the same i and k values prevail throughout the world, the levels of production and absorption must be the same everywhere. Therefore, in our particular country, exports must be compensated exactly by imports from the rest of the world, so that Eq. 3 is upheld.

These results are not altered, if we replace the continuum of production and absorption by uniformly spaced discrete production and absorption centers, as long as the number of such centers is large. Assume that individual persons are such centers and designate by D' the linear population density (i.e., the number of people per unit length). The total population of the country is $P = 2RD'$. Then F_1 can be expressed as a function of population; with $N = D'/k$, (12) becomes

$$F_1 = (1 - e^{-P/N})N/P. \tag{13}$$

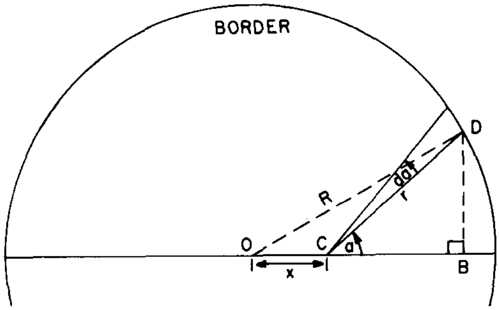

Fig. 3. Two-dimensional absorption model.

Just as $L = 1/k$ represents a characteristic distance, N is a characteristic linear population: it is the number of people (absorption centers) that a flow encounters before its intensity is reduced to a fraction $1/e$ of its original intensity. Mathematically N represents a rescaling of the import/GNP ratio in the light of the absorption model, followed by standardization for population.

The Two-Dimensional Case

Consider next a two-dimensional flat homogeneous world. The same line of reasoning can be used as for the one-dimensional case, but the calculations involved are more complex. The reader unfamiliar with calculus might want to go directly to the final integration (21) which is analogous to the first part of Eq. 11 in one dimension.

Consider a circular country of radius R, and one of its production sources, at distance x from the country's center (see Fig. 3). Equation 8 gives the total flow intensity $I(r)$ for this source, at a distance r from the source, as it radiates out evenly in all directions. Within a small angle da, the flow intensity is $da/2\pi$ of the total intensity $I(r)$ at the given distance. The source's contribution (E') to the country's export is obtained by integrating Eq. 8 over all angles along the country borders, keeping in mind that r changes with the angle:

$$E'(x) = \int_0^{2\pi} \frac{I(r)}{2\pi} da = \frac{I_0}{\pi} \int_0^{\pi} \exp(-kr)\, da, \tag{14}$$

where $r(a)$ is determined as follows. In Fig. 3,

$$(OD)^2 = (OC + CB)^2 + (BD)^2. \tag{15}$$

Since $OD = R$, $OC = x$, $CB = r \cos a$, and $BD = r \sin a$, this leads to

$$R^2 = x^2 + 2xr \cos a + r^2. \tag{16}$$

Solving for r, and accepting the positive solution only:

$$r = -x \cos a + (R^2 - x^2 \sin^2 a)^{1/2}. \tag{17}$$

Assume now a continuous and homogeneous distribution of production throughout the country, with production density i per unit area. The total production of the country is

$$G = i\pi R^2. \tag{18}$$

Also assume that all products have the same absorption coefficient k. Then the export from all sources at distance x from the country's center is the same, and is given by (14). The area within a thin shell of thickness dx at distance x from the country's center is $2\pi x dx$; its exports are obtained by replacing I_0 in (14) by $2\pi i x dx$. The country's total exports (E) are obtained by then integrating (14) over all values of x, from 0 to R:

$$E = 2i \int_0^R x \int_0^\pi \exp(-kr)\, da\, dx. \tag{19}$$

Introducing the value of r from (17), and combining (19) and (18) yields an expression for the export/GNP ratio F_2 where the subscript "2" refers to the two-dimensional model:

$$F_2 = E/G = \frac{2}{\pi R^2} \int_0^R \int_0^\pi x \exp\,[k(x \cos a - (R^2 - x^2 \sin^2 a)^{1/2})]\, da\, dx. \tag{20}$$

Normalize with respect to country radius R, by introducing $y = x/R$. Then F_2 is given as a function of kR only:

$$F_2 = \frac{2}{\pi} \int_0^1 \int_0^\pi y \exp\,[kR(y \cos a - (1 - y^2 \sin^2 a)^{1/2})]\, da\, dy. \tag{21}$$

This equation cannot be solved analytically. A numerical approximation was obtained by subdividing the range of y into 20 equal zones of 0.05 each and the range of a into 18 equal zones of 10° each. Throughout each of the 360 integration regions thus defined, y and a were assumed to have a constant value equal to the value at the center point of this region. Integration is then replaced by summation:

$$F_2 \approx \frac{1}{180^\circ} \sum_{m=1}^{20} \sum_{n=1}^{18} y_m \exp\,[kR(y_m \cos a_n - (1 - y_m^2 \sin^2 a_n)^{1/2})], \tag{22}$$

where $y_m = 0.05m - 0.025$ and $a_n = (10n - 5)^\circ$. The summation was repeated for various values of kR, and the results are shown in Table 1. Repeating the calculations with fewer zones gave essentially the same results: this suggests that the approximate F_2 values obtained through summation are close to the actual values. In view of the uncertainties of trade data to be tested, even a summation error ±0.01 for F_2 would be acceptable, but we are more likely within ±0.001.

Table 1 also lists values of $F_1(0.83kR)$, i.e., the F ratios calculated from

TABLE 1

Trade/GNP Ratio F as a Function of Normalized Country Size, for One- and Two-Dimensional Models

kR	$F_2(kR)$ from Eq. 22	$F_1(0.83kR)$ from Eq. 13	$k^2(\pi R^2)$
0	1.000	1.000	0
.005	.996	.996	.000079
.01	.992	.992	.000314
.02	.983	.984	.00126
.05	.959	.960	.0079
.1	.920	.921	.0314
.2	.848	.851	.126
.5	.677	.679	.79
1	.487	.488	3.14
2	.294	.290	12.6
5	.125	.120	79
10	.061	.060	314
20	.029	.030	1,260
50	.008	.012	7,900
100	.002	.006	31,400

the one-dimensional model (Eq. 12) using 0.83 times the kR value listed. The functions $F_2(kR)$ and $F_1(0.83kR)$ agree within ±0.005, for any value of kR. In other words, for our purposes a two-dimensional circular country of radius R is equivalent to a one-dimensional country of length $1.66R$. This insensitivity of the outcome to a major change such as dimensionality of space suggests that it should also be insensitive to smaller changes in the geometrical shape of the country. Thus we will not have to be concerned about the exact contours of the actual countries but could treat them as circular areas equal to the actual areas.

Recall that $L = 1/k$ is the characteristic absorption distance over which flow intensity is reduced to $1/e$ of its original value. The values of $kR = R/L$ in Table 1 can be viewed as values of R when the characteristic length L is taken as unit length. Similarly, $\pi k^2 R^2$ (also shown in Table 1) can be viewed as the area ($A = \pi R^2$) of the circular country when L^2 is taken as unit area. Furthermore, if D is the population density of the homogeneous world, then $\pi k^2 R^2$ can also be viewed as the country's population ($P = AD$) when $N = DL^2$ is taken as unit population:

$$\pi k^2 R^2 = A/(L^2) = P/(N^2). \tag{23}$$

Just as L is the characteristic absorption distance,

$$N = L\,(D)^{1/2} = (D)^{1/2}/k \tag{24}$$

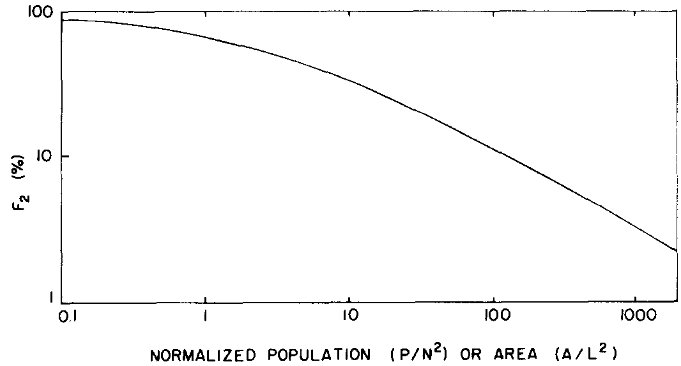

Fig. 4. Dependence of import/GNP or export/GNP ratio F on normalized population or area, according to the two-dimensional absorption model, on log-log scale. Based on Table 1 and on normalization Eq. 23.

is the characteristic linear population number: it is the number of people (absorption centers) that a flow encounters before its intensity is reduced to $1/e$ of its original intensity. Mathematically N is obtained by rescaling the import/GNP ratio in the light of the absorption model and standardizing for population.

Figure 4 shows the curve of F_2 versus P, on log-log scale, when $N = 1$. A change in the value of N would merely shift the curve along the log P axis, by a distance of 2 log N, without changing the curve shape.

TESTING THE ABSORPTION MODEL

The absorption model was developed in terms of an infinite flat two-dimensional world with homogeneous production and absorption characteristics, and in terms of a circular country within this world. A relationship between such a country's area and export/GNP ratio resulted. Instead of physical area, the population size or the GNP could be used, since production and absorption homogeneity suggests uniform population density and productivity. Instead of exports, imports could be used, since in a homogeneous world exports must equal imports.

The actual world deviates from the model in a number of ways of which only some of the most important will be mentioned here. Production centers are not uniformly and continuously distributed, and absorption by space is not uniform and continuous, due to geographical and sociocultural factors. Different products have different production patterns and absorption distances. Countries are not circular, and foreign trade is not carried out uniformly along the periphery. National borders may offer barriers to trade.

For the largest countries, the infinite world approximation may not be appropriate. The effect of maritime borders is not included in the model.

The only factor that the model does include is the effect of distance, and hence of country size. But this factor is of overriding importance both conceptually and according to previous empirical analysis. (Recall that population accounts for 69% of observed variation in the Imp/GNP data shown in Fig. 1.) It remains to test whether the detailed shape of the predicted relationship bears any resemblance to the observed pattern. To the extent that this is the case, the simple model would have some merit.

In the case of homogeneous population and production distribution, the test could be carried out equally well in terms of area, population, or GNP. This is no more true for the actual inhomogeneous world. Physical extent of a country may become less important than population size, since production and absorption of goods can be expected to depend more on the number of people than of square miles. Plotting the trade/GNP ratio data from Taagepera and Hayes (1977) versus area confirms this expectation: scattering of points increases considerably, compared to plots using population, although the general inverse cube root trend (as in Eqs. 1 and 2) is preserved. The GNP of a country might be an even more suitable parameter than population, since it corrects for productivity inhomogeneities. However, the reported GNP values depend on currency exchange rates used, a rather floating basis, especially if one should want to compare different historical periods. Population size offers a steadier comparison basis and was chosen as the basic size variable.

A similar question arises with regard to exports and imports which have to balance each other in the case of a homogeneous world model. As mentioned in the introduction, export figures tend to fall short of import figures because some export forms are not included, and the export/GNP ratio is less well correlated with population than is the case for the import/GNP ratio. Imports seem to reflect better the actual extent of exchange of goods and services; therefore, the import/GNP ratio will be used in the testing of the model, although it was more convenient to visualize the homogeneous world model in terms of exports.

In Fig. 1 the theoretical $F_2(P/N^2)$ curve from Fig. 4 has been fitted to data by translation along the P/N^2 axis. This procedure is equivalent to changing the value of N; the curves shown correspond to characteristic absorption numbers N = 240, 480, and 960 persons, respectively. The implications of these numbers will be discussed later. The theoretical curve for N = 480 largely agrees with the previous empirical cube root expression (Eq. 1). For population sizes above 200 million where the two curves diverge, the absorption model yields a somewhat better visual fit than the cube root equation. The model also satisfies the basic boundary conditions (4) except that it assumes an infinite world population. For a country of 4 billion people (the present world population), the model (with N = 480) indicates an F_2 of 0.4% instead of the required zero. (Equation 1 yields 2.5%.)

It should be reminded that we are not comparing here two competing models–Eq. 1 is not a conceptual model but an empirical expression. To the extent that it agrees with the absorption model the latter is validated and, in turn, lends rational legitimacy to the empirical expression.

One may wonder whether agreement with data might not be an artifact. It should be asked whether any model that satisfies the basic boundary and continuity conditions (4) and (5) and includes an adjustable multiplicative size parameter (like N in our case) could not be fitted as well, given the scattering of data. An inspection of curve (6) in Fig. 1 shows that this is not always the case: no amount of shifting this curve along the log P axis could achieve a degree of fit comparable to that of curve (22). In fact, the absorption model agrees with data pattern much better than could be expected in view of the large number of factors that the model neglects.

Data deviation from the best fit of the model ($N = 480$) in Fig. 1 could be due to three types of factors.

1. *Space-independent factors,* such as historical traditions and present policies for which a space model cannot account by its very nature.

2. *Space inhomogeneity factors,* such as product variety, uneven population density, transport barriers due to mountains or seas, and irregular country shape. In terms of the model, the production density i and absorption constant k would not, in contrast to assumptions used, be constant over space. In principle, the whole world could be subdivided into regions so small that i and k would be constant within each region. The total effect on a country could then be summed up. As a first step in this direction, the average k could be calculated separately for each country, from its actual import/GNP ratio and size, assuming constant i throughout the country (but not throughout the world). This will be done in the next section.

3. *Border barrier effect.* While the basic absorption Eq. 8 for homogeneous space is well tested in physics, the nature of national borders is open to question. They are not mere curves drawn on the map, without any impact on trade, as the model implies. They could be viewed as semitransparent barriers, i.e., barriers that transmit a certain fraction of flow that reaches them, and reflect the rest. (For previous use of semitransparent barrier model in physics, see e.g., Schiff, 1955, p. 92.) The resulting foreign trade would be lower than is the case without barrier, and the effect might be expected to be strongest in small countries where all production centers are close to some border; thus the very shape of the F vs. P curve in Fig. 4 should be altered. In the case of the one-dimensional model, previous calculations were repeated, including this time a border barrier of uniform transparency. As expected, the whole F_1 vs. P curve is lowered; however, the curve shape is not appreciably altered (unless the transmitted fraction is taken as an unrealistically low 1% or less). The whole curve is shifted to the left so that the semitransparent barrier effect cannot be distinguished from that of a world-wide increase in absorp-

tion constant k. In view of this inconclusive outcome, detailed calculations are not shown here.

CHARACTERISTIC ABSORPTION NUMBER AND TRANSPORT DISTANCE

It would seem from Fig. 1 that the worldwide average characteristic absorption number N is around 480 persons; i.e., a flux of goods could be visualized as being reduced to $1/e$ or about 37% of its original intensity after flowing past 480 people, in a radial direction from the production center; the remaining 63% have been absorbed. In two-dimensional space, it would mean that the πN^2 = 730,000 people closest to a production source absorb, on the average, 63% of its production. In the case of a few countries (those above the N = 960 curve) the same degree of absorption is achieved by the nearest three million or more people, while in a few other countries (those below the N = 240 curve) less than 180,000 nearest people absorb 63% of the production. The large majority of countries fall inbetween these extremes.

The same results can be expressed in terms of the characteristic transport distance L. For the present world average population density of 70 persons per square mile, Eq. 24 with N = 480 leads to L = 57 miles. Thus, 67% of production is consumed, on the average, within 57 miles of the production center, according to the model. For individual countries, their actual population densities would have to be taken into account.

In the previous section, all the available import/GNP ratios of contemporary countries were used to test the model, although different countries can be expected to have different characteristic absorption numbers and transport distances. The procedure can now be reversed: assuming that the model is valid, the actual import/GNP ratio and population size can be used to determine the characteristic absorption number for each country. Using the area instead of population, the characteristic transport distance also can be determined. The world-wide results are shown in Table 2 where countries have been arranged in order of increasing N. Its median value is 480 people, while the extreme values range from 160 to 1700. The median for characteristic transport distance is 58 miles, with extremes ranging from around 10 miles (Haiti, Mauritius, Malta) to around 300 miles (Canada, Botswana, Algeria). All countries with L larger than 200 miles have large uninhibited areas which do not absorb products: arctic tundra (Canada), jungles (Guyana, Angola, Zambia), or deserts (Australia, Saudi Arabia, Libya, Algeria, Botswana).

While N represents mathematically a population-standardized rescaling of the Imp/GNP ratio, there is fairly little correlation between the two, because population size already accounts for 69% of the observed variation in Imp/GNP, leaving only 31% for N. In regression analysis terms, N is related to the residual of the Imp/GNP ratio when population size is controlled in

TABLE 2

Characteristic Absorption Number (*N*) and Transport Distance (*L*) around 1965[a]

Country (listed in increasing order of *N*)	*N*	*L* (miles)	Country (listed in increasing order of *N*)	*N*	*L* (miles)	Country (listed in increasing order of *N*)	*N*	*L* (miles)
Uruguay	160	26	Israel	410	23	Guyana	620	220
Paraguay	170	47	Colombia	410	65	Mozambique	620	130
Kuwait	180	21	Argentina	420	93	Denmark	630	37
Western Samoa	190	20	Ethiopia	430	61	Indonesia	650	48
Cambodia	200	21	Jamaica	450	22	Ireland	660	64
Haiti	200	10	Jordan	460	63	Malta	660	13
Central African Republic	200	86	Finland	460	77	Saudi Arabia	670	240
Iceland	230	110	Uganda	460	50	Switzerland	670	35
Dominican Republic	300	22	Ivory Coast	460	83	Brazil	670	130
Dahomey	300	41	Liberia	460	92	Morocco	680	76
Hungary	310	18	Ghana	460	50	Zambia	700	200
Togo	310	36	Australia	460	240	Canada	700	310
Romania	320	22	Peru	470	97	Sri Lanka	710	34
Turkey	320	32	Poland	470	29	Pakistan	720	45
East Germany	320	16	Congo (B)	470	190	Spain	720	56
Volta	320	47	Tunisia	470	57	Yugoslavia	740	53
New Zealand	320	65	Botswana	470	300	Portugal	740	46
Cyprus	330	26	Czechoslovakia	480	28	China	750	54
Costa Rica	330	39	Cameroon	480	92	France	750	49
Ecuador	330	48	Rhodesia	490	91	Philippines	780	47
Chad	340	130	Senegal	500	73	Kenya	790	121
Libya	340	220	Malagasy Republic	510	96	Iran	800	130
Nicaragua	360	64	Syria	520	60	South Africa	820	130
Honduras	360	49	Lesotho	530	62	Angola	880	270
El Salvador	360	19	Mexico	530	70	South Korea	930	34
Gambia	360	42	United States	530	73	Barbados	950	24
Panama	370	55	Egypt	530	60	Japan	960	37
Guatemala	370	36	Venzuela	550	110	Trinidad	960	43
Bolivia	370	130	Iraq	560	81	Italy	1060	50
Gabon	380	180	Greece	570	44	Cuba	1060	81
Afghanistan	380	49	Norway	570	100	Algeria	1070	300
Mauritius	380	12	Austria	580	38	Thailand	1080	86
Laos	390	84	Bulgaria	580	42	India	1220	61
Burma	400	41	Sweden	590	89	West Germany	1230	46
Chile	400	75	Lebanon	600	24	United Kindom	1300	54
USSR	410	79	Sudan	610	160	Netherlands	1500	48
			Somalia	610	190	Belgium	1700	60

[a]Sources: Taylor and Hudson (1972) for population and area, Taagepera and Hayes (1977) for import/GNP ratio which is subsequently converted to P/N^2, using curve in Fig. 4.

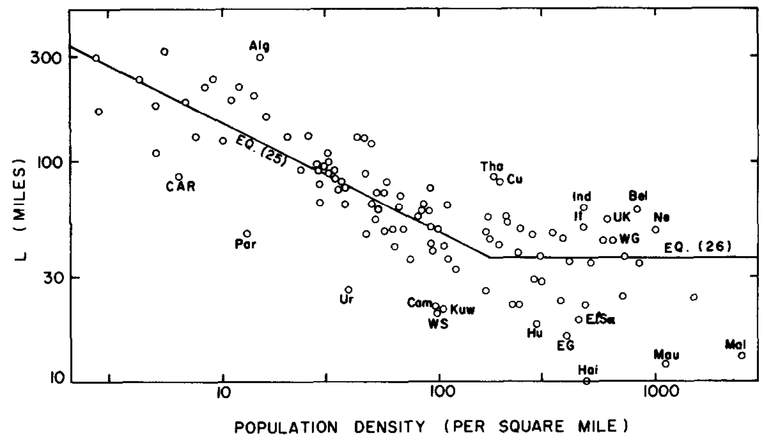

Fig. 5. Dependence of characteristic transport distance L on the country's population density D. Data: L from Table 2; D from Taylor and Hudson (1972).

accordance with the absorption model. The actual residual is $\log(N/480)$. But N is not just an abstract statistical coefficient; it has a very tangible meaning within the model: the characteristic number of *people* who absorb a flow of goods. Insofar as social science is about people, such an interpretation should be of more interest than more abstract ones.

A graph of characteristic transport distance versus population density on log-log scale (Fig. 5) shows a different pattern at low and at high densities. With less than 170 inhabitants per square mile, the average trend is close to

$$L = 480/(D)^{1/2} \qquad [D < 170]\,, \tag{25}$$

which implies, in view of the definition (24), that the median value of N tends to be around 480 irrespective of population density range considered. At population densities of more than 170 per square mile, the median L tends to remain around 37 miles regardless of the density range considered, so that definition (24) leads to N increasing with density:

$$N = 37(D)^{1/2} \qquad [D > 170]\,. \tag{26}$$

The absorption model was previously tested in terms of population rather than area, because it was assumed that goods are absorbed by people rather than by space. This assumption seems to work for low population density countries where (25) applies, but not in the high density countries where (26) applies. In the first case, 63% of goods tend to be absorbed by the closest $\pi(480)^2$ = 730,000 people, irrespective of how far they live. In the latter case, 63% of the goods seem to be absorbed by space within about 37 miles, irrespective of the number of people living within this radius, as long as the number exceeds 730,000. Maybe such a saturation effect should be expected. A sparse population would tend to consume the goods made by their relatively closest neighbors because long-distance transport costs add to prices. But if population density becomes so high that the same product is produced by numerous centers within, say, 60 miles, then those at 1 mile from a given customer have no significant transport cost advantage over those at 60 miles, and average transport distance stops decreasing when further people (and hence production centers) are packed into the same area.

The minimum L actually seems to be around 35 miles for large densely populated countries (like India) and for smaller countries surrounded by other densely populated areas (like Belgium). But if a small densely populated country is surrounded by much more sparsely populated countries (such is the case for El Salvador) or by nonpopulated areas (such is the case for all islands), then the country size itself tends to limit the transport distance. If we define "small" as having an area of less than $\pi(60)^2 = 11{,}000$ square miles, then this subset of densely populated countries includes El Salvador, Israel, Lebanon, and the island countries of Malta, Barbados, Mauritius, Jamaica, Haiti, Cyprus, and Trinidad. Except Trinidad, they all have an L of less than 30 miles. The only other high density countries with so low L are the Dominican Republic (another island country) and a number of East European countries (East Germany, Hungary, Romania, Poland, and Czechoslovakia). Cores of former empires (United Kingdom, Belgium, Netherlands) tend to have a relatively large average transport distance.

CONCLUSIONS

There is a large economic, sociological and political science literature on the reasons and conditions of foreign trade. A variety of mechanisms can and have been proposed to explain the volume of trade: supply and demand, the type of product, the position of the country in the world division of labor, political constraints, and differential wage rates and prices. However, to my best knowledge no other single factor comes even close to the explanatory power that population size has regarding the import/GNP ratio: 69% of the observed variation. Clearly this ratio should be normalized with respect to population size before a successful evaluation of other factors' impact on the relatively small residuals can emerge. India, with a low import/GNP ratio of 6%, is actually unusually active for a country of its size. Kuwait, with a much higher 28% is unusually passive for a country that small. It would be pointless to try to find politico-economic reasons for India's low and Kuwait's high Imp/GNP ratios when population-normalized residuals present the reverse picture.

This paper has a definite pµrpose: to propose a rational model for the empirically observed population-dependence. Establishing the empirical fact itself is outside the intended scope; this has been done earlier, and Taagepera and Hayes (1977) give a full review of the relevant literature. Analysis of residuals (after normalizing with respect to population) is also beyond the intended scope; that is where the various aforementioned economic and political factors would come into play. This paper only offers raw materials for such residual analysis, in the form of characteristic absorption numbers and transport distances. Within the intended scope, the following conclusions can be drawn.

The two-dimensional absorption model for homogeneous media seems to account rather well for the observed pattern of dependence of the import/GNP ratio on population size. The agreement becomes even better when the saturation effect at high population densities is taken into account; all points above the N = 960 curve in Fig. 1 belong into this category. On the average, world imports operate as if 63% of a product flow were absorbed by the nearest 730,000 people or by people within 37 miles of the production center, whichever number of people is higher. For all countries with data available, the values of the characteristic linear population and transport distance are listed in Table 2. Deviations from the world averages can be expected in the case of countries which deviate from homogeneity most (uneven geography and population distribution, irregular contours, sea borders) or least (landlocked compact countries of uniformly populated plains). Underdeveloped and developed countries may be expected to have different transportation means and hence different characteristic transport distances. Free trade countires may be expected to have larger transport distances than countries with governmental trade restrictions. The study of this detailed structure is outside the scope of the present paper; Table 2 offers a convenient starting point for further analysis. Hopefully, such studies will lead to a clearer understanding of why countries import as much as they do, and whether a country is importing too little or too much, given its geographic, demographic, and economic situation. The present paper has only shown why and how sheer area and population size should be rationally expected to affect imports, and how these expectations are largely confirmed by actual data.

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