Training for Insight: The Case of the Nine-Dot Problem

Trina C. Kershaw (tkersh1@uic.edu) and Stellan Ohlsson (stellan@uic.edu)
Department of Psychology
University of Illinois at Chicago
1007 W. Harrison St., Chicago, IL 60607

Abstract

Three sources of difficulty for the nine-dot problem were hypothesized: 1) *turning on a non-dot point*, i.e., ending one line and beginning a new line in a space between dots; 2) *crossing lines*, i.e., drawing lines that intersect and cross; and 3) *picking up interior dots*, i.e., drawing lines that cross dots that are in the interior of the nine-dot and its variants. Training was designed to either facilitate or hinder participants in overcoming these difficulties. Participants were then tested on variants of the nine-dot problem. Results showed that participants in the facilitating training condition performed significantly better than the hindering or control group.

Constraints and Insights

Prior knowledge is the main resource that a problem solver can bring to bear on a problem. Prior knowledge produces unconscious biases that might influence perception and/or encoding of a problem. In general, prior knowledge can be helpful and productive when reasoning or solving a problem. However, when a problem solver faces a very unfamiliar or novel type of problem, there is no guarantee that prior knowledge will be relevant or helpful. The defining characteristic of so-called insight problems is that they activate seemingly relevant prior knowledge which is not, in fact, relevant or helpful (Ohlsson, 1984b, 1992; Wiley, 1998). To succeed, the problem solver must de-activate or relax the constraints imposed by the more or less automatically activated but unhelpful knowledge. To understand human performance on an insight problem, we should therefore try to identify the particular prior concepts, principles, skills or dispositions that constrain performance on that problem. Knoblich, Ohlsson, Haider, and Rhenius (1999) and Knoblich, Ohlsson, and Raney (1999) applied this perspective with considerable success to a class of match stick problems. In this paper, we apply it to the nine-dot problem and other connect-the-dots (CD) problems.

The nine-dot problem (Maier, 1930) requires that nine dots arranged in a square be connected by four straight lines drawn without lifting the pen from the paper and without retracing any lines (Figure 1). This task is ridiculously simple in the formal sense that there are only a few possible solutions to try, but ridiculously difficult in the psychological sense that the solution rate among college undergraduates who are given a few minutes to think about it is less than 5% (Lung & Dominowski, 1985; MacGregor, Ormerod, & Chronicle, 2001). The problem is surely of an unfamiliar type – when in everyday life do we ever draw lines to connect dots under certain constraints? – but what, exactly, are the sources of difficulty? Interestingly, seventy years of research (Maier, 1930) have not sufficed to answer this question.

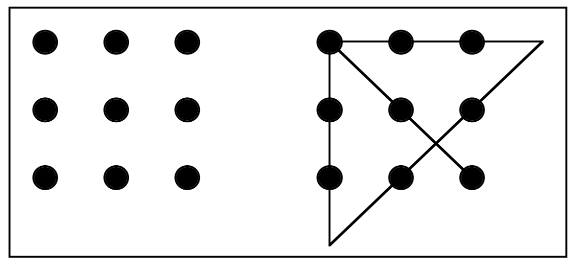

Figure 1: The nine-dot problem and its solution

The Gestalt psychologists introduced insight problems into cognitive psychology and explained their difficulty in terms of Gestalts, schemas that supposedly organize perceptual information (Ohlsson, 1984a). Consequently, they hypothesized that the nine-dot problem is difficult because people are so dominated by the perception of a square that the do not 'see' the possibility of extending lines outside the square formed by the dots (Scheerer, 1963). This hypothesis predicts that telling participants that they can draw lines outside the figure should facilitate the solution. Burnham and Davis (1969) and Weisberg and Alba (1981) tested this hypothesis, and found that the instruction only worked if combined with other hints that gave away part of the solution, e.g., telling the participants at which point to start or giving them the first line of the solution. A second prediction from the Gestalt hypothesis is that altering the shape of the problem and thus breaking up the square should also help. Both Burnham and Davis (1969) and Weisberg and Alba (1981) found facilitating effects of this manipulation. A third prediction from the Gestalt hypothesis is that giving people experience in extending lines outside the figure should help. Weisberg and Alba (1981) and Lung and Dominowski (1985) indeed found facilitating effects of such training.

Recently, MacGregor et al. (2001) and Chronicle, Ormerod, and MacGregor (in press) have proposed a theory that attempts to predict quantitative differences in solution rates for different CD problems. Their explanation is based on four principles: (a) People always draw their next line so as to go through as many dots as possible. (b) People judge the value of a line as a function of how many dots it picks up in relation to how many dots are left and how many lines they have left to draw. (c) People look ahead 1, 2, 3 or at most 4 steps when deciding which line to draw next. (d) When lookahead indicates that every possible line from the current dot will end in a situation where the next line does not provide sufficient progress, they consider, with some probability, lines that go outside the figure formed by the dots.

This theory successfully predicts the differences in solution rates between different several different CD problems. It provides a more detailed description of why people get stuck than any previous theory – their lookahead is not deep enough to reveal that the solution path they are trying will dead end eventually – but the basic explanation for the difficulty is similar to that of previous theories: people consider lines within the shape formed by the dots before they consider lines that go outside the figure.

However, if this is true of variants of the nine-dot problems that do not form squares or any other 'good figure', then the Gestalt explanation for why people do not go outside the figure no longer holds. So what is the difficulty?

By analyzing pilot data and inspecting MacGregor et al.'s (2001) solution rates for the different nine-dot variants, we hypothesize that "hesitating to go outside the figure formed by the dots" is the wrong formulation of the constraint operating in this type of problem. Instead, we propose that *people are disposed to turn on a dot*, as opposed to turn on a point on the paper where there is no dot (a non-dot point). This constraint overlaps in meaning with the stay-within-the-figure constraint, so it explains the success of the training provided by Lung and Dominowski (1985). At the same time, this formulation is different enough to explain why telling people that they can go outside the figure does not help; they do not hesitate to extend lines outside the figure, but they do not want to turn on a non-dot point. As a secondary constraint, we hypothesize that people hesitate *to cross lines*, having a strong disposition towards thinking of the four lines they are supposed to draw as forming a closed outline. As a consequence, they do not see how *to pick up the dots in the interior of the figure*, an operation that requires crossing lines in many CD problems.

In the present study, we tested this hypothesis by both comparing problems that did and did not require turns on non-dot points and by attempting to facilitate the solution via training. As a novel methodological feature, we also tried to *hinder* the solution with training intended to *strengthen* the inappropriate constraints.

Method

Participants

Participants were 90 undergraduates (30 in each training group: facilitating, hindering, and no training) from UIC's Participant Pool. No demographic data were collected about the participants.

Materials

The training exercises were designed by the first author.

Facilitating Training The facilitating training was designed to eliminate the difficulties that participants were thought to face when solving the nine-dot problem. Twelve training exercises were designed, each with similar instructions to the nine-dot problem (Connect all of these dots using ___ straight lines without lifting your pen from the page and without retracing any lines). Each exercise required a different number of lines to connect the dots.

Six of the training exercises required participants to cross lines and pick up interior dots (Figure 2), and the other six could only be solved by turning on a non-dot point (Figure 3). Each training exercise was presented on its own page. The dots were filled circles that were .5 cm in diameter, and the centers of each dot were approximately 3.75 cm apart.

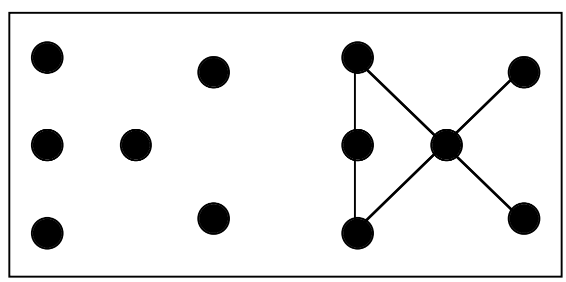

Figure 2: Facilitating Training Exercise and its Solution: Crossing Lines and Picking up Interior Dots

Hindering Training The hindering training consisted of 12 exercises that were solved by drawing lines that always turned on a dot and never crossed another line (Figure 4). As in the facilitating training, participants were instructed with similar directions to the nine-dot problem (Connect all of these dots using ___ straight lines without lifting your pen from the page and without retracing any lines). Again, each exercise required a different number of lines to connect the dots. The hindering training was constructed just as the facilitating training, with the dots, or filled circles, being .5 cm in diameter and the centers of each dot being approximately 3.75 cm apart.

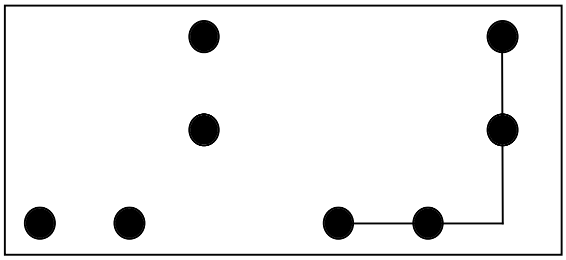

Figure 3: Facilitating Training Exercise and its Solution: Turning on a Non-Dot Point

Nine-Dot Variants The three insight and three non-insight versions of the nine-dot problem that were used had been designed by MacGregor et al. (2001). The insight problems required participants to turn on a non-dot point (Figure 5), while the non-insight problems were the insight problems with an added dot, which excused participants from having to turn on a non-dot point (Figure 6). Each problem was presented on its own page. The dots were filled circles that were 1 cm in diameter. The center of each dot was approximately 3.75 cm apart.

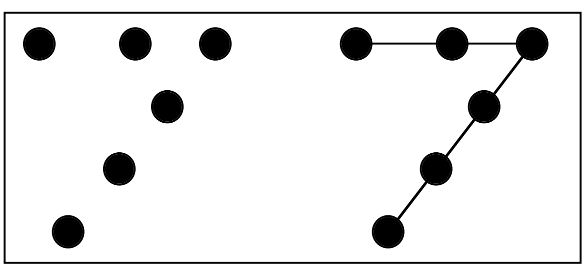

Figure 4: Hindering Training Exercise and its Solution

Procedure

Participants were seen in groups of 2-10. Each session lasted from 40 minutes to an hour. All test materials were contained in a booklet.

Training Phase Participants in both the facilitating and hindering training conditions were given the same directions for the training exercises. The instructions explained that they would be connecting dots using the number of lines specified on each page without lifting their pens from the page or retracing any lines. They were also told to start at the dot marked with a star for each group. The purpose of giving participants a set starting point was to make sure that there was a single solution for each training exercise.

Participants had one minute to work on each training exercise. Time was kept by the experimenter. Participants in the no training (control) group did not complete the training exercises and instead began with the problems.

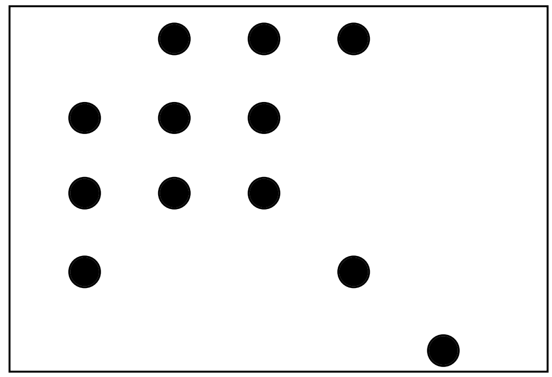

Figure 5: Nine-Dot Variant: Insight Version

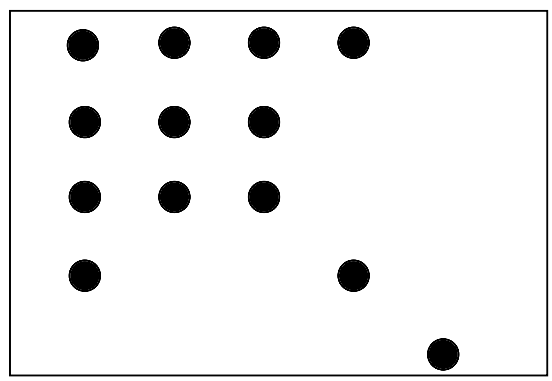

Figure 6: Nine-Dot Variant – Non-Insight Version

Problem-Solving Phase. After completing the training, participants began the problem-solving section of the booklet. Participants were instructed that they would have four minutes to connect all the dots in the figure using four straight lines without lifting their pens from the page and without retracing any lines. For each problem, there was a practice sheet followed by a second identical sheet with the problem on it. Participants were instructed to try the problem as many times as they wanted to on the practice sheet. When they thought they had come up with a solution, they were to record the time out of the four minutes that had passed, using a large clock at the front of the room.

The participants then were to turn the page and redraw their final solutions on the clean page.

The order of problems was the same for all participants. The first problem was an insight problem, the second was a non-insight problem, the third was an insight problem, and so on. Each insight problem was followed by its non-insight version. The first insight problem was of principal interest in comparing the effect of the training manipulations. The other problems were included to obtain baseline solving rates for various problem types, and to compare solving rates in the UIC participant population to the solving rates obtained by MacGregor et al. (2001).

Results

The results focus on the first insight problem (see Figure 5).

Group Analysis

A 3 x 2 chi-square analysis was conducted for the first insight problem. The independent variable was the type of training: facilitating, hindering, or no training. The dependent variable was whether or not a participant had solved the first insight problem. Nineteen participants (63%) in the facilitating group solved the first insight problem, eight (27%) in the hindering group solved, and 11 (37%) in the no training group solved. The chi-square was significant, $\chi^2(2, \underline{N} = 90) = 8.836$, $\underline{p} < .05$. Post-hoc comparisons showed significant differences between the facilitating group and the hindering group, $\chi^2(1, \underline{N} = 60) = 8.148$, $\underline{p} < .05$, and between the facilitating group and the no training group, $\chi^2(1, \underline{N} = 60) = 4.267$, $\underline{p} < .05$, but not between the hindering group and the no training group, $\chi^2(1, \underline{N} = 60) = .693$, $\underline{p} > .05$.

A between-groups analysis of variance (ANOVA) was conducted for the average amount of time it took participants in each group to solve the first insight problem. Participants in the facilitating group averaged 116 seconds to solve the first insight problem, the hindering group averaged 188 seconds, and the no training group averaged 185 seconds to solve. The ANOVA was significant, $\underline{F}(2,87) = 5.873$, $\underline{p} < .05$. Post-hoc Tukey tests revealed significant differences between the facilitating group and the hindering group ($\underline{p} < .05$) and the facilitating group and the no training group ($\underline{p} < .05$), but not between the hindering group and the no-training group ($\underline{p} > .05$).

Individual Differences Analysis

Although the facilitating group did better on the first insight problem than the other two groups, there was a large amount of variation in solving rate within the facilitating training group. Specifically, not all participants in the facilitating training group completed the training correctly. Participants in the facilitating group were split into two sub-groups based on whether or not they had completed the training correctly. In order to be classified as having completed the training correctly, a participant had to correctly complete over half (six) of the training exercises. If a participant did not correctly complete at least six of the training exercises, then he or she was put into the "did not complete training" group. In the "completed training" group, no participant got more than four training exercises incorrect. In the "did not complete training" group, one participant got six exercises wrong, and the others got seven or more exercises wrong.

There were a total of 19 participants in the "completed training" group, 17 (89%) of which solved the first insight problem, and 11 participants in the "did not complete training" group, two (18%) of which solved the first insight problem. A chi-square analysis comparing the performance of the two sub-groups on the first insight problem was significant, $\chi^2(1, \underline{N} = 30) = 15.248$, $\underline{p} < .05$.

Within each group, there were large differences in the amount of time needed to solve the first insight problem. Participants in the facilitating group who solved the first insight problem needed between 8 and 180 seconds to solve, with the majority of solvers requiring between eight and 109 seconds. The amount of time needed to solve ranged between 15 and 235 seconds in the hindering group, and between 20 and 195 seconds in the no training group.

Discussion

The results show that the problems that did not require turns on non-dot points were easier than those that did, and that the facilitating training improved performance on our CD problems, supporting the idea that the difficulties of the nine-dot problem and of CD problems generally might be some combination of a disposition towards turning on a dot and a disposition to think of the four lines they are supposed to draw as forming an outline and hence not crossing each other.

Contrary to expectation, the hindering training did not suppress the solution rate below that of a control group. There are several possible explanations. First, the solution rate was low enough so that attempting to suppress it further encountered a floor effect. Second, it is possible that the constraining dispositions were entrenched enough already so that attempting to entrench them yet further with a brief intervention did not succeed.

An interesting finding was that for the facilitating group, the degree to which participants completed the training determined their success in solving the first insight problem. It is likely that only the participants in the facilitating group who fully completed the training were able to successfully transfer what they had learned during training to solving the insight problems. This finding shows that despite common difficulties for participants in solving CD problems, there exists individual variation in the degree to which participants can be guided to overcome these difficulties.

Where would people acquire the two central dispositions to want to turn on a dot and to draw outlines? The first disposition might stem from yet another Gestalt concept, the difference between figure and ground. The dots on the paper is the figure and the paper is the background and hence not part of what they are working on. The disposition to draw outlines might be grounded in how people draw when they try to make representational drawings; they trace the outline of the object they are trying to represent. Even if plausible sources can be identified for these two constraints, it remains to prove that they are operating in the nine-dot problem itself as well as in the altered versions we used in this study.

Another reason that people find the nine-dot and related CD problems difficult is that their prior experience in solving CD problems is based in children's connect-the-dot puzzles. This experience is irrelevant to the knowledge that is needed to solve the nine-dot problem. Prior knowledge creates unconscious biases that are not always helpful (cf. Ohlsson, 1984b, 1992; Wiley, 1998). The presentation of a problem can interact with prior knowledge, thus resulting in an incorrect and unhelpful encoding of the problem.

What is of most interest in studies on CD problems is to comprehend how people can get stuck on such trivial problems. To understand how the human mind works, we must understand unhelpful interactions between problems and prior knowledge, the impasses that result, and how people overcome those impasses by relaxing the inappropriate constraints. Insight problems are tools with which to study these processes.

References

Burnham, C.A., & Davis, K.G. (1969). The nine-dot problem: Beyond perceptual organization. Psychonomic Science, 17(6), 321-323.

Chronicle, E.P., Ormerod, T.C., & MacGregor, J.N. (in press). When insight just won't come: The failure of visual cues in the nine-dot problem. Quarterly Journal of Experimental Psychology.

Knoblich, G., Ohlsson, S., Haider, H., & Rhenius, D. (1999). Constraint relaxation and chunk decomposition in insight problem solving. Journal of Experimental Psychology: Learning, Memory, and Cognition, 25, 1534-1555.

Knoblich, G., Ohlsson, S., & Raney, G. (1999). Resolving impasses in problem solving: An eye movement study. In M. Hahn and S. Stoness (Eds.), Proceedings of the Twenty-First Annual Meeting of the Cognitive Science Society (pp. 276-281). Mahwah, NJ: Erlbaum.

Lung, C.T., & Dominowski, R.L. (1985). Effects of strategy instructions and practice on nine-dot problem solving. Journal of Experimental Psychology: Learning, Memory, and Cognition, 11(4), 804-811.

MacGregor, J.N., Ormerod, T.C., & Chronicle, E.P. (2001). Information-processing and insight: A process model of performance on the nine-dot and related problems. Journal of Experimental Psychology: Learning, Memory, and Cognition,27(1), 176-201.

Maier, N.R.F. (1930). Reasoning in humans: I. On direction. Journal of Comparative Psychology, 10, 115-143.

Ohlsson, S. (1984a). Restructuring revisited I: Summary and critique of the Gestalt theory of problem solving. Scandinavian Journal of Psychology, 25, 65-78.

Ohlsson, S. (1984b). Restructuring revisited II: An information processing theory of restructuring and insight. Scandinavian Journal of Psychology, 25, 117-129.

Ohlsson, S. (1992). Information processing explanations of insight and related phenomena. In M. Keane and K. Gilhooly (Eds.), Advances in the Psychology of Thinking (Vol.1, pp. 1-44). London: Harvester-Wheatsheaf.

Scheerer, M. (1963) Problem solving. Scientific American, 208(4), 118-128.

Weisberg, R.W., & Alba, J.W. (1981). An examination of the alleged role of "fixation" in the solution of several "insight" problems. Journal of Experimental Psychology: General, 110(2), 169-192.

Wiley, J. (1998). Expertise as mental set: The effects of domain knowledge on creative problem solving. Memory & Cognition, 26(4), 16-730.