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ODORICO ZEROS AND LOW ENERGY $\pi\pi$ SCATTERING DATA*

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ABSTRACT

Recent $\pi^+\pi^-$ scattering results show an anomalous behavior in the $\langle Y_1^0 \rangle$ moment of the angular distribution at 980 MeV. Odorico has suggested that this effect is unrelated to the opening of $K\bar{K}$ threshold but is explained by the straight line propagation of zeros which enter the physical region near 980 MeV. A similar explanation was also given for an "anomaly" in $\pi^-\pi^0$ scattering at roughly the same energy. We show that the zeros of both the $\pi^+\pi^-$ and $\pi^-\pi^0$ elastic scattering amplitudes, as determined from the data, contradict these suggestions of Odorico and, in fact, illustrate the effect of K^+K^- threshold in $\pi^+\pi^-$ scattering.

Odorico [1] has provided a possible explanation of recent meson-meson scattering data in terms of an hypothesis that the zeros of the scattering amplitude propagate along straight lines in the Mandelstam

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plane.* We shall refer to this as the straight-line zero (SLZ) hypothesis. The purpose of this paper is to discuss the behavior of these amplitude-zeros as given by the data and to contrast their behavior with that suggested by Odorico.

Let us begin by briefly reviewing the SLZ hypothesis [2], which is based on narrow resonance models. A single term Veneziano amplitude, for example, not only has infinitely many poles in both the s and t channels but, as an essential feature, also has an infinity of zeros at fixed values of u. These zeros are required to avoid unacceptable infinite angular momentum components in the unphysical regions where s- and t-channel poles cross each other. Indeed in reactions like KN Cex Odorico [2] has found that these straight lines of zeros, as given by the Veneziano model, are a very good approximation

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* An s-channel $\pi\pi$ amplitude $F(s,t)$ will vanish for real s at complex values of $t = t_0(s)$ or $z = 1 + 2t/(s - 4m_\pi^2) = z_0(s)$. When we talk of the path of such zeros in the Mandelstam plane we mean $t = \text{Re}\, t_0(s)$. If $\text{Im}\, t_0$ is not too large, so that the zero is 'nearby' in the complex plane, such paths are just lines of minima in the differential cross section, i.e. if a zero is nearby it will produce a minimum in the angular distribution at $t = \text{Re}\, t_0(s)$ and the smaller $\text{Im}\, t_0(s)$ is the closer this minimum will be to zero.

to the behavior of the data. This led Odorico to generalize this feature of the Veneziano model into an hypothesis concerning the global structure of hadronic amplitudes. This idea has most recently been applied [1] to the reaction $\pi^+\pi^- \to \pi^+\pi^-$ for which there are now experimental results with high statistics [3]. The data show an anomalous behavior of the $\pi^+\pi^-$ elastic amplitude close to $K\bar{K}$ threshold where, for example, the $\langle Y_1^0 \rangle$ moment shows a rapid, almost discontinuous, drop from its maximum value to zero (fig. 1). This effect has been explained by a pole in the $I = 0$, s-wave amplitude just below $K\bar{K}$ threshold which is a resonance in the $\pi\pi$ channel but a bound state of $K\bar{K}$ [4]. However Odorico remarks that a different mechanism is at work. Since we have ρ poles in the s- and t-channels we must have a zero close to $s = t = m_\rho^2$. Away from this point the SLZ hypothesis implies that the zero propagates along a line at fixed $u = 4m_\pi^2 - 2m_\rho^2$. Such a zero enters the s-channel physical region through the forward direction at $s = 2m_\rho^2 = 1.17\ (\mathrm{GeV/c})^2$ (fig. 3). Since $\langle Y_1^0 \rangle$ is a measure of the forward-backward asymmetry of the angular distribution we should expect such a nearby zero to produce a drop in $\langle Y_1^0 \rangle$ at an energy which is accidentally close to $4m_K^2 = 0.98\ (\mathrm{GeV/c})^2$. Such a drop is of course seen in fig. 1.

Odorico now applies the same considerations to $\pi^-\pi^0$ elastic scattering which has ρ poles in the s and u channels (fig. 4). The appropriate double pole killing zero in this case is at fixed t and enters the s-channel physical region through the backward direction at $s = 2m_\rho^2$. The entry of this zero is then presumed to be correlated with a rise in the $\langle Y_1^0 \rangle$ moment in this reaction which has been observed by Baton et al. [5] (fig. 2). Thus the SLZ hypothesis provides a similar explanation for the behavior of $\langle Y_1^0 \rangle$ in the two reactions $\pi^+\pi^-$ and $\pi^-\pi^0$ elastic scattering in terms of double pole killing zeros and their straight-line propagation. However, as mentioned above, Flatté et al.[4] explain the data in $\pi^+\pi^-$ scattering by means of an S^* resonance just below K^+K^- threshold. If this is correct a different explanation is required for $\pi^-\pi^0$ scattering, since that reaction involves only isospin 1 and 2. Clearly no such $I = 0$ s-wave effect could then be responsible.

It is clear that this disagreement can be resolved by looking at the actual behavior, as given by the data, of the zeros in these two reactions. We will then see if similar or different explanations are required for the "anomalies" in the $\langle Y_1^0 \rangle$ moments of figs. 1 and 2. We shall in fact show that the SLZ hypothesis cannot be realized in practice. We determined the zeros by using the phase shifts [5,8] to reconstruct the angular distributions (figs. 5, 6, 7, 8).

We begin by considering the $\pi^-\pi^0$ scattering data and discuss where the lines of zeros are. In the very low energy region there is just one zero. As discussed in ref. 6 this zero leaves the region of the Mandelstam triangle and smoothly enters the s- and u-channel physical regions where it passes close to the Legendre zero of the ρ-resonance. (Note $\langle Y_1^0 \rangle = 0$ where the zero crosses the $\mathrm{Re}\, z = 0$ line, which is close to $\sqrt{s} = m_\rho$; fig. 2,6.) This zero which is a nearby zero, having $\left| \frac{\mathrm{Im}\, t}{\mathrm{Re}\, t} \right| < \frac{1}{10}$, continues smoothly into the backward hemisphere beyond the rho (see fig. 6) and is primarily responsible for the rise in $\langle Y_1^0 \rangle$ shown in fig. 2. Beyond 1 GeV the effect of the second nearby zero becomes increasingly apparent as it approaches the physical region from outside. The approach of this double-pole killing zero, which is related to the decrease in the $I = 2$ d-wave phase shift, causes the first zero to move back into the forward hemisphere as the second zero comes closer to the physical region. How fast this occurs depends sensitively on the exact behavior of the $I = 2$ d-wave, which is not experimentally well determined.

We see that it is the movement of the first zero into the backward hemisphere between 750 and 1000 MeV that produces the rise in the $\langle Y_1^0 \rangle$ moment. It has been speculated by Odorico [1] that if the $\pi^-\pi^0$ data had better resolution the rise observed in the $\langle Y_1^0 \rangle$ moment would be as steep as the drop in the $\pi^+\pi^-$ data (fig. 1). We note that the rise will only be steeper if the first zero continues to move into the backward hemisphere as the second zero closes in on the backward direction. Then the increase in forward-backward asymmetry would be greatest. However in practice the first zero seems to be rather sensitive to the approaching second zero and before this latter zero can enter the physical region the first zero moves back across the $\mathrm{Re}\, z = 0$ line. This movement, in fact, produces the slow fall in $\langle Y_1^0 \rangle$ seen beyond 1 GeV (fig. 2); so that instead of a steeper rise being the signal of the second zero's approach it is a slow fall in this moment that heralds its arrival near the physical region.

Let us conclude our discussion of $\pi^-\pi^0$ elastic scattering by recalling that the rise in the $\langle Y_1^0 \rangle$ moment is correlated with the continued smooth behavior of a zero which acts as the Legendre zero of the rho and cannot be due to the entry of a double pole killing zero through the backward direction. In fact, the entry of this zero appears to require the first zero to move back into the forward hemisphere resulting in a slow fall in $\langle Y_1^0 \rangle$.

Whilst just comparing figs. 4 and 6 is enough to indicate that the SLZ hypothesis cannot be true locally it is also interesting to consider $\pi^+\pi^-$ scattering from the point of view of amplitude-zeros and see how they are influenced by the opening of K^+K^- threshold. In the Lovelace-Veneziano model [7] in which the zeros follow fixed u lines, the first zero is at $u = 2m_\pi^2$. In the neighborhood of the Mandelstam triangle this zero is then just the appearance of the Adler zero on-mass-shell. However such a line of zeros (at $u = 2m_\pi^2$) violates unitarity in the region $s \simeq m_\rho^2$, where it must be related to the Legendre zero of the rho. This has been implicitly recognized by Odorico who shifts this zero from $u = 2m_\pi^2$ to $u = 2m_\pi^2 - \frac{1}{2} m_\rho^2$. However whilst this is then the rho's Legendre zero it passes far from the Mandelstam triangle making a complete nonsense of PCAC. Of course the situation in Nature, as indicated by the data, is not either that this zero is the on-shell appearance of the Adler zero or the Legendre zero of the rho. It is, just as in $\pi^-\pi^0$ elastic scattering, both. This is an important fact in relating p-wave dominant and current algebra based models [6]. The path of this zero is shown in fig. 5. As can be seen the zero contour is perfectly smooth steadily moving into the backward hemisphere until it approaches $K\bar{K}$ threshold. Then within just 30 MeV from an energy of 970 MeV to 1000 MeV the $\mathrm{Re}\, z(s)$

of the zero contour changes from -0.42 to -0.21 (fig. 5,7). It is this movement of the first zero which is primarily responsible for the "anomaly" in $\langle Y_1^0 \rangle$ at 980 MeV (fig. 1). However the second zero also plays a role, though it is less crucial. This zero, which kills the double pole at $s = t = m_\rho^{\ 2}$, must enter the s-channel physical region through the forward direction before $s = m_f^{\ 2}$. Its entry is related to the growth in the $I = 0$ d-wave and it, of course, becomes one of the f_0's Legendre zeros. However its entrance into the physical region, i.e. $|\text{Re } z| \leq 1$, is not marked by a pronounced dip in the forward differential cross section since it appears far away in the complex z-plane having a large imaginary parts ($|\text{Im } z| \simeq \text{Re } z$). This is unlike the other zero which is always nearby. However as the Re z(s) decreases $|\text{Im } z(s)|$ also steadily decreases and so the effect of the zero becomes more noticeable with increasing energy until it produces a well pronounced dip in the angular distribution (see fig. 7 for behavior from $\sqrt{s} = 0.97 - 1.05$ GeV.). Again the data fails to support the SLZ explanation which requires the nearby entry of this second zero to be responsible for the rapid drop in $\langle Y_1^0 \rangle$. In fact if this zero alone had produced this effect it would imply that the anomaly was primarily due to the d-wave (instead of the s-wave effect it is) and would have been correlated with an 'anomaly' in $\langle Y_3^0 \rangle$ also, which of course does not occur.

Let us conclude with the remark that whilst zeros play a fundamental role in fulfilling and relating various features of the scattering amplitude: for example, satisfying the Adler condition, double pole killing, assigning spin to resonances and giving Regge residue's their zeros, care must be taken not to oversimplify the paths they follow. Odorico's hypothesis of zeros following roughly straight lines may be a reasonable approximation to reality in certain reactions when viewing the Mandelstam plane from afar. However in any local region zero contours are far from straight and can in fact be greatly affected by many influences, for example, the presence of an s-wave resonance just below strong inelastic channels such as $\pi\pi \rightarrow K\bar{K}$.

ACKNOWLEDGMENT

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FIGURE CAPTIONS

Fig. 1: Data for $\langle Y_1^0 \rangle$ for $\pi^+\pi^- \to \pi^+\pi^-$ from ref. 3, 4, 8.

Fig. 2: Data for $\langle Y_1^0 \rangle$ for $\pi^-\pi^0 \to \pi^-\pi^0$ from ref. 5.

Fig. 3: Structure of Odorico zeros for $\pi^+\pi^- \to \pi^+\pi^-$ scattering [1]. The dashed lines denote the Odorico zeros. The small black dots mark the Legendre zeros of the resonances shown and where zeros must occur to kill double poles. The larger black spot marks the point where the 'second' Odorico zero enters the physical region.

Fig. 4: As fig. 3 but for $\pi^-\pi^0 \to \pi^-\pi^0$ scattering.

Fig. 5: The zero contours $t = \text{Re}\, t_0(s)$ for $\pi^+\pi^-$ elastic scattering as determined from the data of ref. 8. The dotted line is the expected continuation of the first zero contour through the Mandelstam triangle.

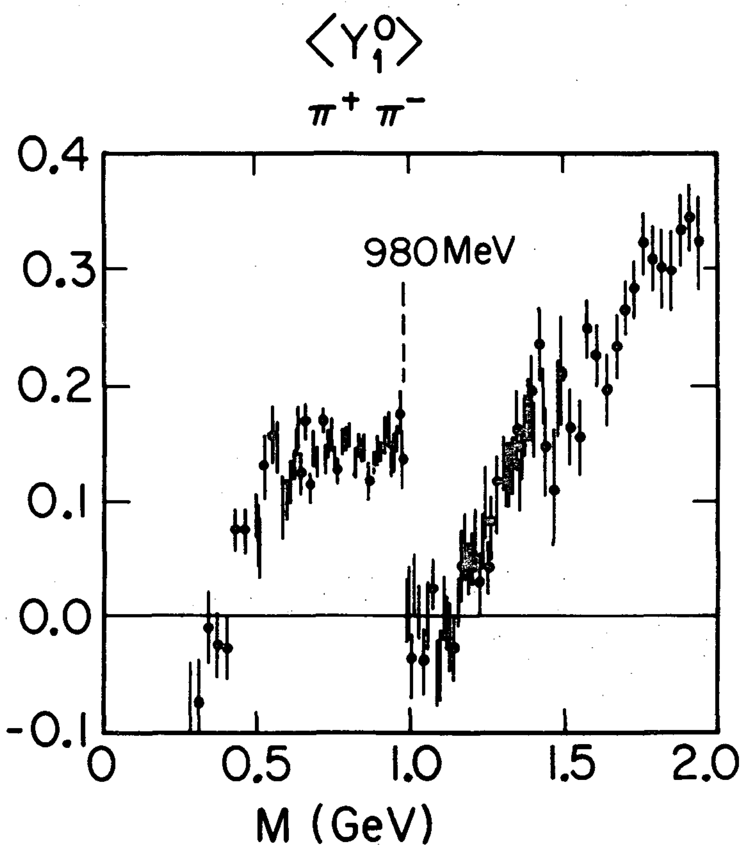
Fig. 6: The zero contours $t = \text{Re}\, t_0(s)$ for $\pi^-\pi^0$ elastic scattering as determined from the data of ref. 5. The dotted line is the expected continuation of the first zero contour through the Mandelstam triangle. Since the $I = 2$ d-wave is not well determined we have a range of positions for the zero contours marked by the shaded region.

We have used a scattering length approximation for this d-wave with the phase shift fixed to be between -3° to -6°, at 1 GeV. This defines the region shown.

Far outside the physical region the zero contour shown should not be considered very reliable since partial waves higher than d-waves become important there.

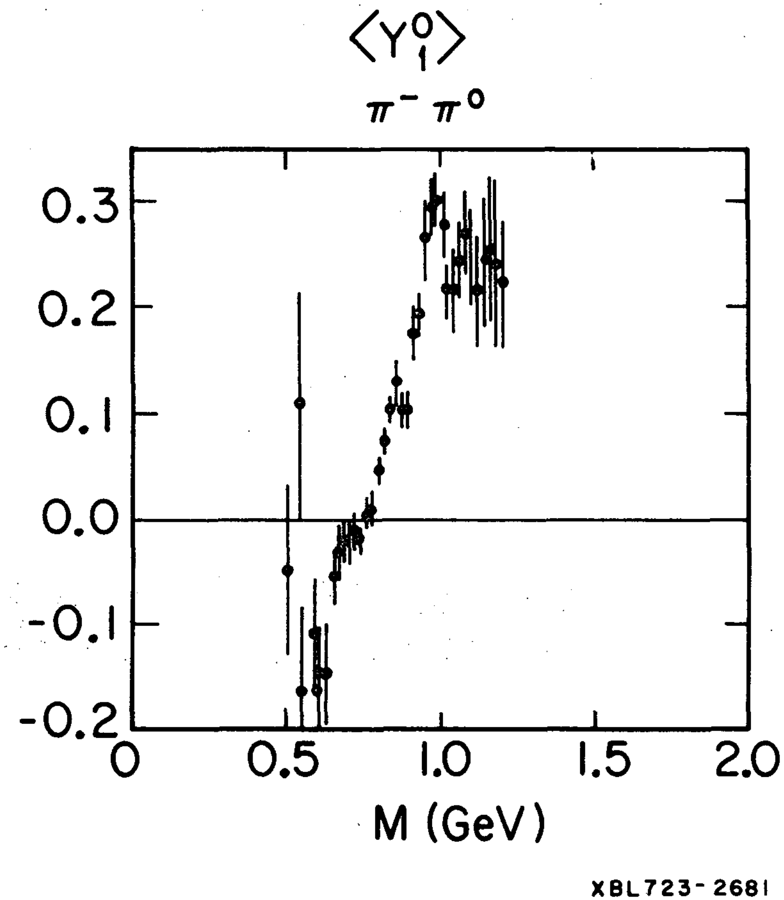
Fig. 7: The angular distribution for $\pi^+\pi^- \rightarrow \pi^+\pi^-$ scattering at 970, 990 and 1050 MeV.

Fig. 8: Same as fig. 7 but for $\pi^-\pi^o \rightarrow \pi^-\pi^o$ scattering.



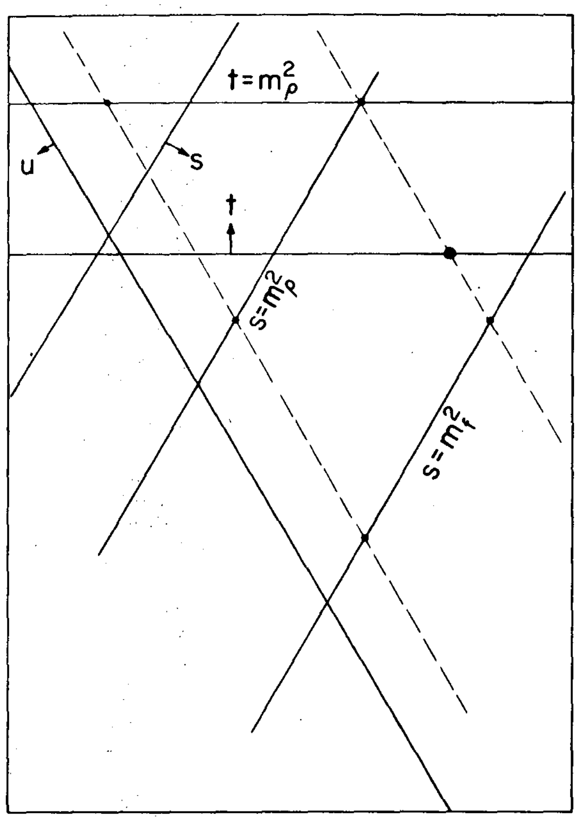
⟨Y_1^0⟩
$\pi^+\pi^-$
980MeV
0.4
0.3
0.2
0.1
0.0
-0.1
0
0.5
1.0
1.5
2.0
M (GeV)

Fig. 1



⟨Y_1^0⟩
$\pi^-\pi^0$
0.3
0.2
0.1
0.0
-0.1
-0.2
0
0.5
1.0
1.5
2.0
M (GeV)
XBL723-2681

Fig. 2



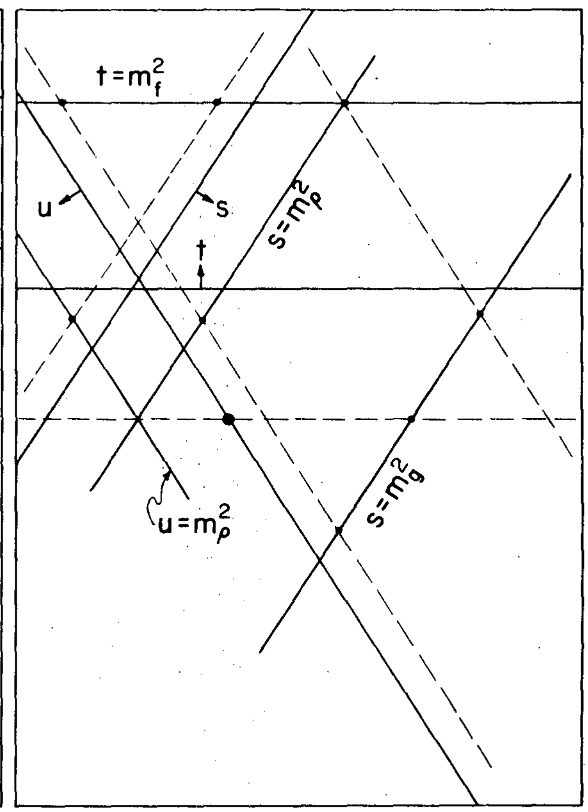


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Fig. 3

Fig. 4

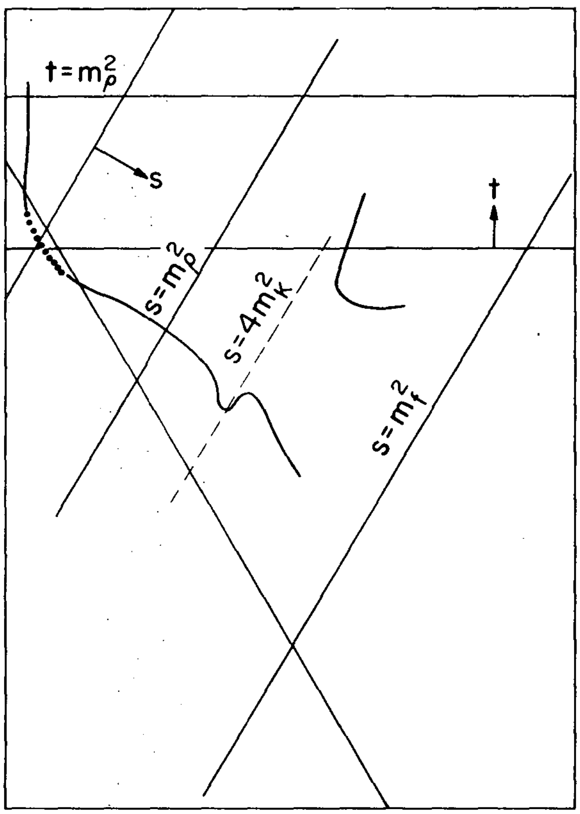

Fig. 5

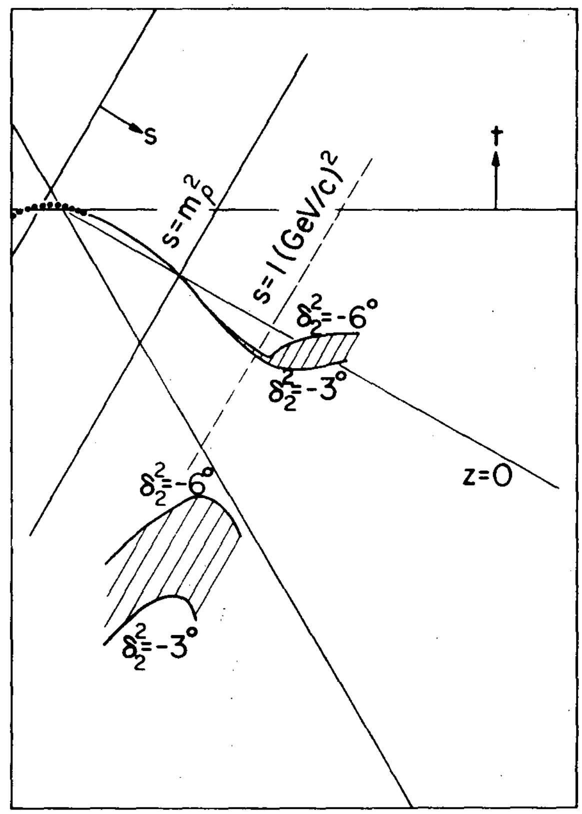

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Fig. 6

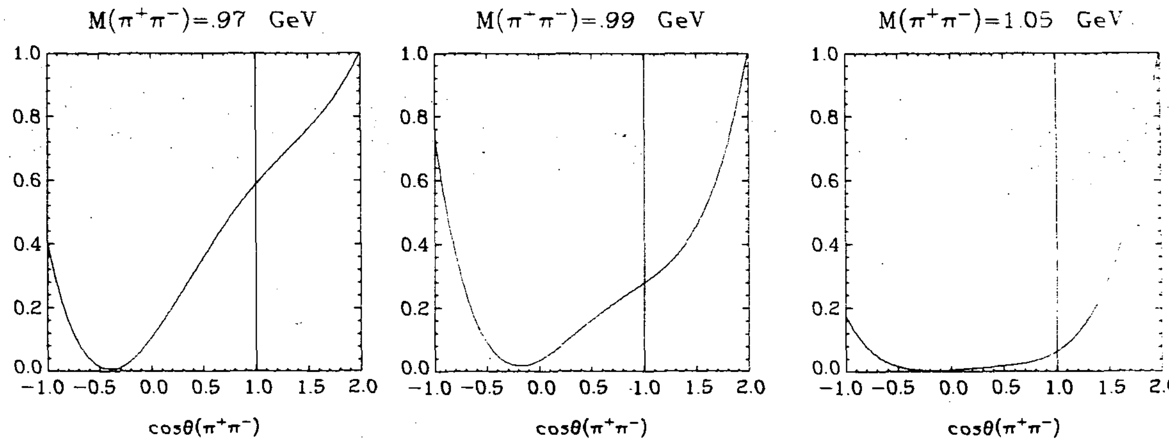
M(π⁺π⁻)=.97 GeV
M(π⁺π⁻)=.99 GeV
M(π⁺π⁻)=1.05 GeV
cosθ(π⁺π⁻)
cosθ(π⁺π⁻)
cosθ(π⁺π⁻)

Fig. 7

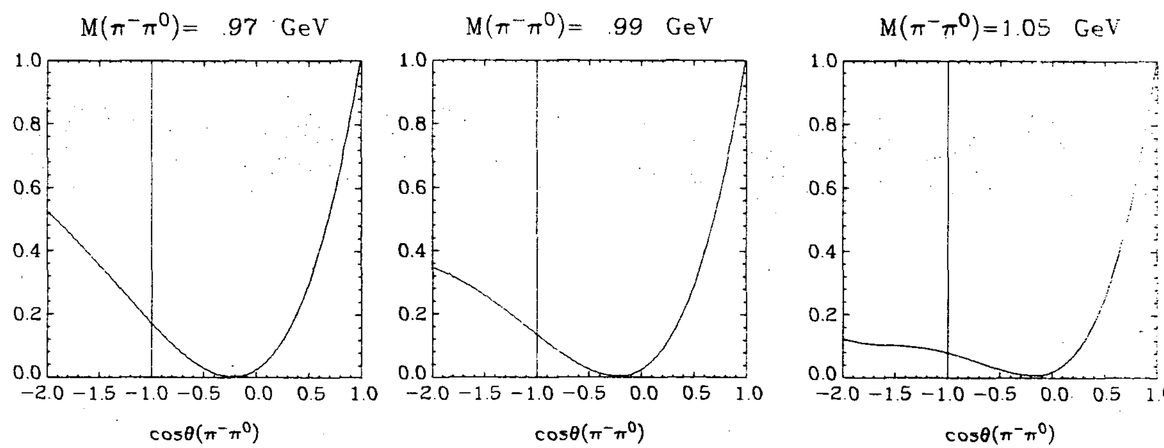
M(π⁻π⁰)= .97 GeV
M(π⁻π⁰)= .99 GeV
M(π⁻π⁰)=1.05 GeV
cosθ(π⁻π⁰)
cosθ(π⁻π⁰)
cosθ(π⁻π⁰)
XBL723-2684

Fig. 8

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