International Debt Crisis and the Prices of Options on Bank Stocks

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Abstract

Modelling the stock price processes of banks with large exposures to Latin American debt as a combination of diffusion and jump processes leads to a no-arbitrage pricing restriction which can be used to infer the implied market prices of Latin American debt.

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1 Introduction

The breakthrough in the pricing of options came through the brilliant observation by Black and Scholes (1973) that if the stochastic process followed by the stock price can be described by a lognormal diffusion process, the payoff of an option on the stock can be replicated by the payoff of a portfolio consisting of the riskless bond and underlying stock. Thus, the option can be priced by arbitrage arguments alone. However, if the stock price, in addition to local movements, has a possibility of *jumps*, modelling its return process as diffusion may not be appropriate. For instance, stocks of banks that have a significant amount of debt to the Latin American countries may exhibit large price changes if one or more of the countries were to declare default on their loans. *The Economist* (1984) reports that at the end of 1983, nine leading American banks had outstanding loans to Argentina, Brazil, Mexico and Venezuela equal to *more than 100 percent* of their equity. Given that the probabilities of default by the Latin American countries are non-trivial, the possibility of jumps in the stock prices of these banks must be explicitly modelled.

Merton (1976) models the stock price dynamics as a mixture of diffusion and a poisson process. However, he shows that if the jump amplitude associated with the poisson process is stochastic with positive dispersion, then a riskless hedge cannot be formed using the underlying stock and the riskless bond. The reason is that portfolio mixing is a linear operation and the stock price is a non-linear function of the stock price. If the jump amplitude is constant, it is still not possible to form a perfect hedge for the option using only the riskless bond and the underlying stock since we have more than one risk term – the diffusion term and the poisson term – but only one asset containing these risks – the underlying stock. Jones (1984) argues that we can use *other* options on the same stock to form a perfect hedge since the options on the stock also contain the same risk terms. Thus, he is able to price an option on the stock in terms the price of the underlying stock and the prices of other options on the same stock.

The key observation we make in this paper is the following. All international loan contracts contain what is known as *cross-default* and *sharing* clauses. The effect of this clause is that if a borrower decides to repudiate its debt, it must repudiate its debt to *all* the banks [see Chowdhry (1991)]. The implication of this is that the Poisson processes describing the jump components of loans to a given country are perfectly correlated across all the stock prices of banks that have outstanding loans to the same country. Therefore, we can obtain a perfect hedge for an option on a stock of one bank using the riskless bond, the underlying stock, stocks of *other* banks and options on the stocks of these banks. We can therefore obtain testable restrictions on the prices of stocks and prices of options on stocks of all banks that have a significant Latin American debt exposure.

2 The Model

We start by making the following assumptions.

Assumption 1 *Perfect Capital Markets:*

- *No transactions costs.*
- *No taxes.*
- *No restriction on short sales.*

Assumption 2 *The riskless rate is known and constant.*

Assumption 3 *The market operates continuously.*

Assumption 4 *The stock prices of banks S_i's, follow stochastic processes given by the following system of differential equations:*

$$\frac{dS_i}{S_i} = \mu_i dt + \sigma_i dz_i + \sum_{j=1}^{n} k_{ij} d\pi_j \qquad \forall i. \tag{1}$$

Here dz_i's denote standard Wiener processes and $d\pi_j$'s denote independent Poisson processes with

$$Pr[d\pi_j = 1] = \lambda_j dt,$$

$$Pr[d\pi_j = 0] = 1 - \lambda_j dt.$$

This assumption allows each stock price to have its own drift and diffusion movements. In addition, the stock price may jump if one of the Poisson event was to occur. Each Poisson process here models the event of default by a particular country. Notice that the Poisson processes $d\pi_j$'s are identical for the stock price processes of all the banks capturing the notion that default by a given country implies default against *all* the banks. k_{ij}'s measure proportional price jumps which would be different for various banks depending on the size of the loans to different countries relative to the equity.

Assumption 5 *The dynamics of the option prices C_i's contain the same risks as the underlying stocks:*

$$\frac{dC_i}{C_i} = \mu_i^C dt + \sigma_i^C dz_i + \sum_{j=1}^{n} k_{ij}^C d\pi_j \qquad \forall i. \tag{2}$$

Let us now form a portfolio consisting of stocks of $(n+1)$ banks and an option on each of these stocks. There are $(2n+1)$ risk terms – $(n+1)$ diffusion terms and n Poisson terms – and $(2n+2)$ assets containing the risk terms. Any $(2n+1)$ assets would span all the risk terms. Therefore, the payoff of an option on any one bank can be replicated by forming a perfect hedge using the riskless bond, the underlying stock, stocks of n other banks and an option on each of these n stocks. In other words, there is a restriction on the prices of these $(2n+2)$ assets. Let the portfolio consist of

- N_i units of the stock i and
- N_i^C units of the option on stock i.

The value of the portfolio V, therefore, would be

$$V = \sum_{i=1}^{n+1} (N_i S_i + N_i^C C_i).$$

If the portfolio is self financing, then

$$\begin{aligned} dV &= \sum_{i=1}^{n+1} (N_i dS_i + N_i^C dC_i) \\ &= \left[\sum_{i=1}^{n+1} (N_i \mu_i S_i + N_i^C \mu_i^C C_i)\right] dt + \sum_{i=1}^{n+1} (N_i \sigma_i S_i + N_i^C \sigma_i^C C_i) dz_i \\ &\quad + \sum_{j=1}^{n} \left[\sum_{i=1}^{n+1} (N_i k_{ij} S_i + N_i^C k_{ij}^C C_i)\right] d\pi_j. \end{aligned}$$

If N_i and N_i^C are chosen such that

$$N_i \sigma_i S_i + N_i^C \sigma_i^C C_i = 0 \qquad \forall i, \tag{3}$$

$$\sum_{i=1}^{n+1}(N_i k_{ij} S_i + N_i^C k_{ij}^C C_i) = 0 \qquad \forall j \tag{4}$$

then we have obtained a perfect hedge. The change in the value of the portfolio is deterministic and by arbitrage must earn the risk free rate of interest r. Thus, we have

$$dV = \left[\sum_{i=1}^{n+1}(N_i \mu_i S_i + N_i^C \mu_i^C C_i)\right] dt = rV\,dt$$

$$\Rightarrow \sum_{i=1}^{n+1}(N_i \mu_i S_i + N_i^C \mu_i^C C_i) = r\sum_{i=1}^{n+1}(N_i S_i + N_i^C C_i)$$

$$\Rightarrow \sum_{i=1}^{n+1}\left[N_i S_i(\mu_i - r) + N_i^C C_i(\mu_i^C - r)\right] = 0. \tag{5}$$

Once we have shown that we can obtain a perfect hedge, we can use the risk-neutral argument of Cox and Ross (1976) to obtain the pricing formula. Since we need an arbitrage argument only, the value of the claim does not depend on investors preferences directly. So, in particular, we can assume a world in which investors are risk neutral. In such a world equilibrium would require that unconditional expected returns on *all* assets equal the riskless rate. Therefore the stock price dynamics can now be represented by the following system of equations:

$$\frac{dS_i}{S_i} = (r - \sum_{j=1}^{n}\theta_j k_{ij})dt + \sigma_i dz_i + \sum_{j=1}^{n} k_{ij}d\pi_j \tag{6}$$

with

$$Pr[d\pi_j = 1] = \theta_j dt,$$

$$Pr[d\pi_j = 0] = 1 - \theta_j dt.$$

Here, θ_j's denote the pseudoprobabilties associated with the Poisson events.

The term $(-\sum_{j=1}^{n}\theta_j k_{ij})$ – which would be positive for negative k_{ij}'s representing downward jumps – could be thought of as a continuous rate of dividend the stock must be earning in equilibrium to compensate for the possibility of downward jumps.

The price of an option on a stock, therefore, is given by

$$C_i = e^{-rT}E^* f(S_i(T))$$

where E^* denotes expectation with respect to the risk neutral system in (6), and $f(S_i(T))$ denotes the payoff at maturity. If the claim in question is a call option, the price is given by

$$C_i = e^{-rT}E^* Max[0, S_i - X_i]$$

where X_i denotes the exercise price.

We now make the following assumption.

Assumption 6 *The downward jumps are large enough that if any one jump was to occur, the at the money call options would become worthless.*

This assumption is reasonable for banks with Latin American debt since the amount of loans to a given Latin American countries as a percentage of equity is quite large for these banks.

The pricing formula for at the money call options can therefore be written as

$$C_i = e^{-(\sum_{j=1}^{n} \theta_j)T} \left[S_i(0) e^{-(\sum_{j=1}^{n} \theta_j k_{ij})T} N(d_{1i}) - X_i e^{-rT} N(d_{2i}) \right] \tag{7}$$

where

$$d_{1i} = \frac{ln \frac{S_i(0)}{X_i} + (r - \sum_{j=1}^{n} \theta_j k_{ij} + \frac{1}{2}\sigma_i^2)T}{\sigma_i \sqrt{T}},$$

$$d_{2i} = d_{1i} - \sigma_i \sqrt{T}.$$

The pricing formula above has a simple risk neutral interpretation. The first term $e^{-(\sum_{j=1}^{n} \theta_j)T}$ represents the probability that no jump occurs. The term in the square brackets represents the price of the call option *conditional* on the fact that no jump occurs.

The pricing result in (7) requires θ_j's as inputs. If we know the call option prices on any n stocks, then we can invert these valuation functions to obtain the values of these parameters. Therefore, if we know the prices of options on stocks of *any n* banks, the prices of options on *all* other banks can be obtained. In other words, this valuation provides testable restrictions on the prices of stocks and the prices of options on stocks of all banks that have a significant Latin American debt exposure.

The approach can be generalized to price any claim whose price process can be represented by the Poisson process representing the Latin American debt. An important application of this approach therefore would be to infer the market prices of Latin American debt that are implicit in the prices of stocks and the prices of options on stocks of banks with Latin American debt.

3 Conclusion

The key insight of this paper is that because of the presence of ***cross-default*** and ***sharing*** clauses in international debt contracts, the jump components of loans to any given country by several different banks can be modeled as perfectly correlated Poisson processes. This allows us to obtain no-arbitrage pricing restrictions which can be used to infer the implied market prices of Latin American debt.

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