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UNIVERSITY OF CALIFORNIA
SANTA CRUZ

**COSMOLOGICAL APPLICATIONS OF STRONG
GRAVITATIONAL LENSING**

A dissertation submitted in partial satisfaction of the
requirements for the degree of

DOCTOR OF PHILOSOPHY

in

PHYSICS

by

Jackson H. O'Donnell

August 2026

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Abstract

Cosmological Applications of Strong Gravitational Lensing

by

Jackson H. O'Donnell

Strong gravitational lensing is poised to become one of the strongest probes of cosmology. Though rare, strong lensing is highly sensitive to the geometry of the universe and the distribution of mass in foreground galaxies or clusters. While strong lensing has long been used to study distant magnified objects, the past decade has seen rapid progress in applying SL to novel studies of dark matter and dark energy. I provide a survey of these possibilities, with a focus on studies of dark matter in lensing galaxy clusters. I present the most rigorous constraint to date on Self-Interacting Dark Matter (SIDM) at cluster scales, combining strong lensing with resolved stellar kinematics in lens system MACS J0138-2155. Additionally, I discuss a powerful possibility for constraining the dark energy equation of state with strong lensing, a technique known as *compound lens cosmography*, and encouraging results with a remarkable lens system known as the Carousel Lens.

The discovery of a new dish confers more happiness on humanity than the discovery of a new star.

Physiologie du goût
JEAN-ANTHELME BRILLAT-SAVARIN

It seems, then, that nature has pointed out a mixed kind of life as most suitable to the human race... Indulge your passion for science, says she, but let your science be human, and such as may have a direct reference to action and society.

An Enquiry Concerning Human Understanding
DAVID HUME

The trouble is just that we have a bad habit, encouraged by pedants and sophisticates, of considering happiness as something rather stupid.

The Dawn of Everything
DAVID GRAEBER

If a man's imagination were not so weak, so easily tired, if his capacity for wonder not so limited, he would abandon forever such fantasies of the supernal. He would learn to perceive in water, leaves, and silence more than sufficient of the absolute and marvelous, more than enough to console him for the loss of the ancient dreams.

Desert Solitaire
EDWARD ABBEY

It was the end of something – if not his innocence, at least of his faith that things would always happen gradually enough to afford time to do something about it in.

Against the Day
THOMAS PYNCHON

What do such machines really do? They increase the number of things we can do without thinking. Things we do without thinking, there's the real danger.

FRANK HERBERT

“Oh, dear,” Pips making with an eyebrow, “do you really not know? ‘Normal’? Things will never go back to the way they were, it’ll all just keep getting more, what the Chinese call, ‘interesting.’”

Shadow Ticket
THOMAS PYNCHON

But a certain dullness of mind, it seems, is almost a necessary quality, if not of every active man, at least of every serious maker of money.

The Idiot
FYODOR DOSTOEVSKY

There is no steady unretracing progress in this life; we do not advance through fixed gradations, and at the last one pause:—through infancy's unconscious spell, boyhood's thoughtless faith, adolescence' doubt (the common doom), then scepticism, then disbelief, resting at last in manhood's pondering repose of If. But once gone through, we trace the round again; and are infants, boys, and men, and Ifs eternally.

Moby Dick
HERMAN MELVILLE

Slowly is the fastest way to get to where you want to be.

ANDRÉ DE SHIELDS

I maintain that truth is a pathless land, and you cannot approach it by any path whatsoever...

JIDDU KRISHNAMURTI

The word is the most imprecise of signs. Only a science-obsessed age could fail to comprehend that this is its great virtue, not its defect.

JOHN FOWLES

There is no royal road to science, and only those who do not dread the fatiguing climb of its steep paths have a chance of gaining its luminous summits.

KARL MARX

From the age of six I had a penchant for copying the form of things, and from about fifty, my pictures were frequently published; but until the age of seventy, nothing that I drew was worthy of notice. At seventy-three years, I was somewhat able to fathom the growth of plants and trees, and the structure of birds, animals, insects, and fish. Thus, when I reach eighty years, I hope to have made increasing progress, and at ninety to see further into the underlying principles of things, so that at one hundred years I will have achieved a divine state in my art, and at one hundred and ten, every dot and every stroke will be as though alive.

KATSUSHIKA HOKUSAI

*We shall not cease from exploration
And the end of all our exploring
Will be to arrive where we started
And know the place for the first time.*

T.S. ELIOT

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Chapter 1

Introduction

The contents and expansion history of the universe are the core of physical cosmology. On both accounts, the universe has surprised us. Over the past century overwhelming evidence has accumulated that visible matter cannot account for all the mass in the universe (early evidence includes [414, 416, 323]), and must be outnumbered by an unknown **dark matter** by roughly 5:1 [87]. Although cosmologists have had great success in both measuring and explaining the distribution of dark matter in the universe, its precise nature remains unknown. Perhaps more strangely, it has been conclusively determined that the expansion of the universe is *accelerating*. This acceleration implies another quantity filling the universe, termed **dark energy**, which currently exceeds the matter density of the universe by roughly 2:1 [87, 113, 175]. All told, our best understanding of physical cosmology is that 95% of our universe is made up of a mysterious, unknown ‘dark sector’.¹ This fact remains one of the greatest mysteries, or embarrassments, of the physical sciences.

Intriguingly, further cracks have begun to show. For the past several decades,

¹While the term ‘dark sector’ refers collectively to dark matter and dark energy, it remains unknown whether dark matter and dark energy have anything to do with each other.

the standard model of cosmology has been comprised of a collisionless, cold dark matter (CDM), in a universe whose expansion is being driven by a cosmological constant (or Λ). This model is known as Λ CDM. As cosmological experiments have grown and improved, a number of discrepancies have emerged between probes, including the current expansion rate of the universe H_0 [185, 136] and the amplitude of structure S_8 [106, 164, 280]. No consensus has yet emerged if these tensions are due to systematic errors, unknown physics, or mere statistical flukes. While CDM remains highly successful at explaining the large scale structure of matter in the universe, a wide array of alternative models could preserve CDM's success at large scales while predicting concrete deviations on small scales. Observed deviations on small scales could therefore determine nontrivial properties of the dark matter through astrophysical observations, a highly enticing possibility. Elsewhere in the dark sector, evidence is mounting that dark energy is evolving in time, rather than a cosmological constant Λ . We are beginning to answer remarkable questions: Are there hidden forces in the dark sector? How did the dark matter form? Is the dark energy growing, or shrinking with time? Has dark energy's evolution changed over cosmic time?

Strong gravitational lensing has remarkable potential to weigh in on many of these questions, independently of other probes. Though a rare phenomenon, strong lensing is highly sensitive to both extremes of the mass distribution of the universe, and the geometry of the universe itself. The aim of this thesis is twofold. The first is to provide an overview of strong lensing, including an (incomplete) survey of its promising applications. The second is to present novel strong lensing results, including the most rigorous constraint to date on Self-Interacting Dark Matter (SIDM) in clusters of galaxies, and promising constraints on dark energy from a system known as the Carousel lens.

Here, we briefly provide essential background on modern physical cosmology. First, an overview of the expansion history of the universe is provided in Section 1.2, along with key quantities used throughout this work. Next, we introduce the theory of Cold Dark Matter (CDM), along with intriguing alternatives such as Self-Interacting Dark Matter (SIDM), in Section 1.3. Finally, an introduction to the phenomenology of dark energy and is given in Section 1.4.

1.1 Conventions

Throughout this thesis, scalar quantities are given as x , and vectors are given in bold $\boldsymbol{x}$. Where spacetime metrics are needed, we use the ‘mostly positive’ sign convention $(-+++)$. Comoving distances are given as χ_i for an object at redshift z_i . Distances given simply as D_i are angular diameter distances, and D_{ij} is the angular diameter distance between z_i and z_j . Unless otherwise noted, cosmology is assumed to be flat Λ CDM, $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_m = 0.3$. Key terms are rendered in **bold**. The term **physical cosmology** is used to distinguish this field from philosophical, religious, or mythological cosmology. The term **cosmography** describes the empirical study of the expansion of the universe through geometric measurements.

1.2 Cosmological Background

The foundation of modern physical cosmology is Einstein’s general theory of relativity. General Relativity describes the geometry of spacetime by a metric $g_{\mu\nu}$, which can evolve according to the contents of spacetime. This relationship is governed by the Einstein field equations:

$$R_{\mu\nu} - \frac{R}{2}g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi GT_{\mu\nu} \quad (1.1)$$

Where $R_{\mu\nu}$ is the Ricci curvature tensor, Λ is a cosmological constant, and $T_{\mu\nu}$ describes the energy and momentum of matter. The Einstein field equations have no general solution, and describe a wide array of complex phenomena. Fortunately for physical cosmologists, we do not require a universal solution, and we can usefully rely on particular limits with simple solutions. First, to describe a non-static universe, we introduce a **scale factor** $a(t)$ which allows the universe to expand (or contract) over time. For simplicity, we can declare that the scale factor today is unity, $a_0 = a(t_0) = 1$. Next, we assume the universe is *homogeneous*, meaning that it has a uniform energy density everywhere, and *isotropic*, meaning the universe looks the same in all directions. While this is obviously false – a perfectly homogeneous universe would have no planets, stars, or galaxies – on large scales our universe is ‘close enough’ to homogeneous that these assumptions hold. These assumptions give us the **Friedmann-Lemaître-Robertson-Walker (FLRW)** metric [113, 175], given here in radial coordinates:

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - \kappa r^2} + r^2 d\Omega^2 \right] \quad (1.2)$$

Where $d\Omega$ is an infinitesimal solid angle. For completeness, we have also introduced a curvature $\kappa \in \{-1, 0, 1\}$, describing a hyperbolic, flat (Euclidean), and elliptical universe, respectively. While it is an open question whether the universe is *perfectly* flat, as far as we can tell, our universe is indeed very *close* to flat. Throughout this work, we will generally assume a flat universe ($\kappa = 0$) except where otherwise noted. Combined with the Einstein equations, the FLRW metric implies that the universe expands or contracts according to the **Friedmann**

equation:²

$$\begin{aligned}
\left(\frac{\dot{a}}{a}\right)^2 &= H^2(t) = \frac{8\pi G\rho}{3} - \frac{k}{a^2} + \frac{\Lambda}{3} \\
&= \frac{8\pi G\rho_{\text{crit}}}{3} \left(\frac{\rho_m}{\rho_{\text{crit}}} + \frac{\rho_r}{\rho_{\text{crit}}} + \frac{\rho_k}{\rho_{\text{crit}}} + \frac{\rho_\Lambda}{\rho_{\text{crit}}} \right) \\
&= H_0^2 (\Omega_{m,0} a^{-3} + \Omega_{r,0} a^{-4} + \Omega_{k,0} a^{-2} + \Omega_{\Lambda,0})
\end{aligned} \tag{1.3}$$

Where the Hubble constant H_0 is the current expansion rate, and the fractional contribution of each species s today is given by $\Omega_{s,0} \equiv \rho_s(a=1)/\rho_{\text{crit}}(a=1)$. The contributions of matter (m), radiation (r), curvature (k),³ and a cosmological constant (Λ) are shown with their appropriate scalings. The radiation density $\Omega_{r,0}$ is negligible today, and the curvature contribution $\Omega_{k,0}$ is known to be very close to zero. As such, radiation and curvature will generally be ignored in this work. Armed with this Friedmann equation, we can now compute various cosmological distance metrics.

Distance metrics in cosmology are essential: cosmography essentially consists of studying the size and expansion of the universe through distances. Cosmological distance metrics are surprisingly subtle. Consider an observer at the center of a sphere of radius R , regarding an object of size L on the surface of the sphere. In our intuitive Euclidean geometry, the quantity R tells us several things: 1) the object is a distance R from the observer, 2) its apparent angular size is $\theta = L/R$, and 3) its luminosity is proportional to R^{-2} . In an expanding universe, these three equations no longer hold: each requires a *different* distance metric! For an excellent practical overview of this topic, see [167]; here we will simply define the distance metrics.

²This equation is sometimes called the First Friedmann Equation, to distinguish it from a Second Friedmann Equation governing $\ddot{a}/a$.

³While the curvature of spacetime does not ‘truly’ have an energy density, it is conventionally assigned an ‘effective energy density’ $\rho_k \equiv \Omega_k \rho_{\text{crit}}$.

The **comoving distance** is the line-of-sight distance which remains constant with time, essentially ‘ignoring’ the expansion of the universe. The comoving distance is fixed to the current scale of the universe, $a(z = 0) = 1$. It is given by:

$$\chi_i = \frac{c}{H_0} \int_0^{z_i} \frac{dz'}{E(z')} \quad (1.4)$$

The **angular diameter distance** relates the angular size of an object to its physical size in its own frame. An object of angular size θ has a physical size of $D_A \theta$. In a flat universe, the angular diameter distance to an object at redshift $z = (1 - a)/a$ is simply:

$$D_A = a\chi = \frac{\chi}{1 + z} \quad (1.5)$$

We will also find it useful to consider the angular diameter distance *between* two redshifts z_1 and z_2 . If an observer were transported back in time to redshift z_1 , they would observe objects at z_2 with angular diameter distance $D_{A,12}$. In a flat universe, this is:

$$D_{A,12} = a_2(\chi_2 - \chi_1) = \frac{1}{1 + z_2} \frac{c}{H_0} \int_{z_1}^{z_2} \frac{dz'}{E(z')} \quad (1.6)$$

For the correct expressions in a curved spacetime, see [167]. The angular diameter distance is the most relevant to this work, and in later chapters we will drop the subscript A , so that $D_{A,i} = D_i$ and $D_{A,ij} = D_{ij}$.

Lastly, the **luminosity distance** describes how the apparent flux of an object relates to its total luminosity. If standard candles of uniform brightness were scattered throughout the universe, their observed flux would scale as D_L^{-2} . The luminosity distance is:

$$D_L = (1 + z)^2 D_A \quad (1.7)$$

Though it will not be used in this work, the luminosity distance is an essential cosmological concept. The relation between redshift and D_L can be directly constrained by samples of standard candles. Samples of type Ia Supernovae have long been used for such measurements, and provided the first widely accepted evidence of dark energy [309, 285].⁴

1.3 Dark Matter

The standard model of cosmology attributes most of the mass in the universe to Cold Dark Matter (CDM), characterized as an ideal gas interacting only through gravity. In this picture, dark matter is caught between three forces: the pull of gravity, accreting matter into massive structures; the gas pressure of dark matter, counteracting structure formation at small scales; and the expansion of the universe, suppressing structure formation on large scales. This basic theory has been enormously successful at explaining the distribution of matter in the universe, as measured by galaxy clustering [12, 116, 100, 127], weak lensing [116, 100, 127], the number counts of galaxy clusters [143, 48, 212], and combinations thereof [5]. These are generally referred to as probes of **large scale structure** (LSS), measuring the distribution of matter and its evolution over cosmic time. However, there is room for discrepancies at small scales, which could inform us of non-trivial properties of the dark matter.

One such property is the temperature of the dark matter. A dynamically hot dark matter would have significant pressure at early times, suppressing the

⁴Technically, type Ia SNe are *standardizable* candles, rather than standard candles. This distinction emphasizes the challenge of SNe Ia cosmology, but the basic concept is the same.

formation of visible structure such as galaxies [175]. The standard CDM model is *cold* for precisely this reason: if all the dark matter were hot, our universe would not exhibit the rich structure that we take for granted.⁵ While the dark matter cannot all be hot, there is still room for deviation from CDM, allowing for a **Warm Dark Matter** (WDM). WDM is conceptually simple, and predicts a suppression of low-mass dark matter halos below galaxy scales [279, 56].

The dark sector could generically harbor additional forces, the simplest of which might manifest as elastic scattering between particles of dark matter. This is known as **Self-Interacting Dark Matter** (SIDM), and provides a wide array of intriguing predictions.⁶ The strongest effects of SIDM are expected at opposite ends of the halo mass function: in groups and clusters of galaxies [186, 9, 317, 381]; as well as dwarf galaxies, satellites, and subhalos of dark matter [186, 381, 9, 251, 314]. In the most massive halos, SIDM is expected to modify the dark matter in the center of a halo, forming a ‘cored’ density profile with a flat central $\rho(r)$ rather than a ‘cusp’ with $\rho(r) \propto r^{-1}$ [319, 391, 317]. It has long been known that an isothermal matter halo with a heat transport mechanism eventually undergoes a runaway collapse process known as the **gravothermal catastrophe**; this once posed a theoretical problem for globular clusters, where pairwise gravitational scattering between stars was expected to cause this process [37]. While this problem is resolved for globular clusters,⁷ SIDM is expected to trigger this process in halos of dark matter, causing a runaway collapse process that can potentially

⁵We know of *a* hot dark matter. The cosmic neutrino background (CνB) is expected to consist of thermal relic neutrinos which decoupled from baryonic matter early in the universe. The CνB has a small effect on structure formation, and observed structure provides an upper limit on neutrino masses ($\sum_i m_{\nu_i}$).

⁶SIDM should not be confused with *annihilating* or *decaying dark matter*, which predict inelastic scattering or coupling to the standard model. Neither should it be confused with *interacting dark energy*, which refers to hypothetical interactions between dark matter and dark energy.

⁷It is thought that interactions with hard binary star systems provide a reheating mechanism, interrupting gravothermal collapse; see Ch. 8.2 of [37].

form a black hole [278, 314]. Thus SIDM is expected to produce opposite effects in high- and low-mass halos: cluster halos have not evolved long enough to trigger gravothermal collapse, and are expected to exhibit cores; whereas in low-mass halos SIDM can potentially trigger collapse. Furthermore, different halo masses differ in the depth of their gravitational potentials, and therefore the velocity of their dark matter. SIDM constraints at different masses therefore probe a potential velocity-dependence of the SIDM interaction strength [314, 317, 326]. A primary result of this work is a constraint on SIDM at cluster scales, presented in Chapter 4. We briefly note that other alternatives to CDM exist, such as axion-like or **fuzzy dark matter**, which would similarly modify the structure of dark matter on small scales [173, 118, 149].

1.4 Dark Energy

The expansion of the universe is accelerating. There is little room to dispute this basic fact; it has been confirmed by a long list of cosmological probes including the cosmic microwave background [29, 87, 299], baryon acoustic oscillation (BAO) measurements [84, 12, 236], samples of type Ia supernovae [293, 310, 322], and tracers of large scale structure such as galaxy clustering and cosmic shear correlations [101, 164]. The simplest explanation of this fact would be a cosmological constant Λ , an intrinsic vacuum energy which is constant in space and time [113]. Other possibilities exist, which are most often described by an equation-of-state parameter w relating the relativistic pressure to the energy density, as well as the time-evolution of dark energy. Replacing Ω_Λ with a more general dark energy density Ω_{de} , this yields:

$$w(a) = P_{\text{de}}/\rho_{\text{de}} \quad (1.8)$$

$$\frac{d \ln \Omega_{\text{de}}}{d \ln a} = -3(1 + w(a)) \quad (1.9)$$

In fact, these equations hold for each species, where matter has constant $w_m = 0$, and radiation has $w_r = 1/3$.⁸ A cosmological constant Λ corresponds to $w_\Lambda = -1$. Constraints on cosmology beyond Λ CDM often allow for a $w \neq -1$ which is constant in time, known as w CDM. A time-varying $w(a)$ with two degrees of freedom is most commonly given as $w(a) = w_0 + w_a(1 - a)$, known as w_0w_a CDM cosmology, where Λ CDM is $w_0 = -1$, $w_a = 0$.

Evidence is accumulating for an evolving dark energy, rather than Λ . Most notably, the second data release of the Dark Energy Spectroscopic Instrument (DESI) alone gave a 1.7σ preference for w_0w_a CDM over Λ CDM from baryon acoustic oscillation measurements [84]. When combined with CMB measurements and samples of Type Ia SNe, the significance rises to $2.8 - 4.2\sigma$. While this DESI result is widely-discussed, other independent probes likewise are beginning to disfavor Λ CDM, including the full results of the Dark Energy Survey [83].

Moreover, the w_0w_a CDM parameters favored by DESI are a surprise to theorists, as they imply a transition from $w < -1$ at early times to $w > -1$ at late times. A value of $w < -1$ implies a growing dark energy density, sometimes termed **phantom dark energy**. A transition of $w(z)$ across $w = -1$ is likewise called a **phantom crossing**. Phantom dark energy was widely considered unphysical, as is a phantom crossing [389]. No consensus has yet emerged on how to

⁸We just learned that most of the matter in the universe can be described by an ideal gas, which must have some positive pressure. But an equation of state $w_m = 0$ implies that matter has zero pressure. How can this be? It's true that CDM does have a nonzero positive pressure, but since it is non-relativistic, this pressure rounds to zero when compared to the mass energy of matter. Thus for the Friedmann equations, $w_m = 0$.

interpret this evidence, despite ongoing theoretical work (see e.g. [225]). While there is room to quibble with analysis details, and so far evidence for evolving dark energy has limited significance, [226] has shown that DESI results do seem to robustly favor a phantom crossing around $z = 0.5$, as shown by both binned analyses and non-parametric Gaussian process regression. Future cosmology results may provide more surprises.

Chapter 2

Gravitational Lensing

This chapter provides an introduction to the phenomenon of gravitational lensing and its applications. First, we provide a derivation of gravitational lensing from General Relativity, with solutions to the lensing equation in several regimes. Except where otherwise noted, the formalisms introduced here describe all gravitational lensing, spanning both ‘weak’ and ‘strong’ scales. Next, we provide an overview of methods for modeling strong lensing systems. Finally, we provide a survey of cosmological applications of strong lensing, focusing on probes of dark matter and dark energy.

2.1 Origins of Lensing

In Section 1.2, we described a homogeneous and isotropic universe with the FLRW metric. Now it is time to consider the *inhomogeneous* universe. An inhomogeneous matter distribution perturbs the curvature of spacetime, contributing **scalar perturbations** to the FLRW metric. The metric of a (flat) expanding universe with scalar perturbations can be given as:

$$g_{00} = -1 - 2\Psi(\mathbf{x}, t) \quad (2.1)$$

$$g_{0i} = 0 \quad (2.2)$$

$$g_{ij} = a^2(t)\delta_{ij}(1 + 2\Phi(\mathbf{x}, t)) \quad (2.3)$$

Where Ψ, Φ are our scalar perturbations, varying in space and time. The first, $\Psi(\mathbf{x}, t)$, is simply the Newtonian gravitational potential in natural units. In the absence of anisotropic stress from radiation or relativistic species, $\Phi = -\Psi$ [113]. These perturbations are typically $\lesssim \mathcal{O}(10^{-4})$, and thus are usually treated at linear order. A particle in such a universe follows equations of motion given by the geodesic equation. The magnitude and direction of its momentum (p and $\hat{p}$ respectively) evolve according to the particle's energy E , the Hubble parameter $H(a)$, and the scalar perturbations given above, yielding ([113] §3.3.2, Eqs. 3.71,3.72):

$$\frac{dp}{dt} = -[H + \dot{\Phi}]p - \frac{E}{a}\hat{p}^i\partial_i\Psi \quad (2.4)$$

$$\frac{d\hat{p}^i}{dt} = \frac{E}{ap}\left[\delta^{ik} - \hat{p}^i\hat{p}^k\right]\partial_k\left(\frac{p^2}{E^2}\Phi - \Psi\right) \quad (2.5)$$

These two equations are remarkably powerful. In the first, the term $-[H + \dot{\Phi}]p$ describes cosmological redshift via the Hubble parameter H , and implies a time-varying potential induces similar red- or blue-shifting. This describes an important correction to the observed CMB, known as the integrated Sachs-Wolfe effect. This same term also implies a massive object moving transverse to the line-of-sight can induce a dipole in background light, known as the moving lens effect, which was recently detected for the first time in the Cosmic Microwave Background [170].

For a non-relativistic particle, $p/E \ll 1$, and deflections $d\hat{p}^i/dt$ are governed by the Newtonian potential as $-\nabla\Psi$. For massless or highly relativistic particles, $p/E = 1$, and deflections are governed by $\nabla(\Phi-\Psi)$, or in the absence of anisotropic stress, $-2\nabla\Psi$. Famously, this factor-of-two discrepancy in the predicted lensing of light was among the first observed predictions of General Relativity, confirmed by measurements of stars near the limb of the sun during a solar eclipse in 1919 [117].

Let's take a step back and consider what a gravitational lensing formalism might look like. We wish to study the observed lensing of objects at cosmological distance. An object observed at an angular position $\boldsymbol{\theta}$ has been deflected along its path by a cumulative angle $\boldsymbol{\alpha}$:

$$\boldsymbol{\beta} = \boldsymbol{\theta} - \boldsymbol{\alpha}(\boldsymbol{\theta}) \quad (2.6)$$

Such that $\boldsymbol{\beta}$ represents the intrinsic, or ‘unlensed’, source position. (In the absence of lensing, we would observe this object at position $\boldsymbol{\beta}$). We can expect $\boldsymbol{\alpha}(\boldsymbol{\theta})$ to depend on the gravitational potential along the photon's entire path, from emission to observation. This simple formula is the fundamental equation of gravitational lensing. For now, we will derive $\boldsymbol{\alpha}(\boldsymbol{\theta})$ from the geodesic of a photon.

In the comoving spatial coordinates of our metric g_{ij} , a luminous object at $\boldsymbol{x}_\beta$ is deflected along its path to the observer, so it appears to be at a position $\boldsymbol{x}$.

$$\boldsymbol{x}(\boldsymbol{\theta}, \chi) = (\boldsymbol{\theta}, 1)\chi \quad (2.7)$$

$$\boldsymbol{x}_\beta = (\boldsymbol{\beta}, 1)\chi_\beta \quad (2.8)$$

Where χ is the comoving distance, and we have aligned the z -axis with the

observer's line-of-sight. When deflections are small, they can be approximated along a single line-of-sight $\mathbf{x}(\chi) = \mathbf{x}(\boldsymbol{\theta}, \chi)$. Using $p_{\perp}^i = -dx_{\perp}^i/d\chi$, we have:

$$\hat{p}_{\perp}^i(\chi'') = -2 \int_0^{\chi''} d\chi' \partial_i \Phi(\mathbf{x}(\boldsymbol{\theta}, \chi'), t) + (\hat{p}_{\perp}^i)_{\text{initial}} \quad (2.9)$$

$$\begin{aligned} \boldsymbol{\beta} &= -(\hat{\mathbf{p}}_{\perp})_{\text{init}} + \frac{2}{\chi_{\beta}} \int_0^{\chi_{\beta}} d\chi'' \int_0^{\chi''} d\chi' \nabla_{\mathbf{x}_{\perp}} \Phi(\mathbf{x}(\boldsymbol{\theta}, \chi')) \\ &= \boldsymbol{\theta} - \nabla_{\boldsymbol{\theta}} \left(-2 \int_0^{\chi_{\beta}} d\chi' \Phi(\mathbf{x}(\boldsymbol{\theta}, \chi')) \left(\frac{1}{\chi'} - \frac{1}{\chi_{\beta}} \right) \right) \end{aligned} \quad (2.10)$$

$$\boldsymbol{\beta} = \boldsymbol{\theta} - \nabla_{\boldsymbol{\theta}} \Psi_L(\boldsymbol{\theta}) \quad (2.11)$$

The deflection angles $\boldsymbol{\alpha}(\boldsymbol{\theta})$ can therefore be expressed as the gradient of a **lensing potential** $\Psi_L(\boldsymbol{\theta})$, which is closely related to the line-of-sight integral of the Newtonian potential. This is a very convenient fact! The equations above describe the cumulative lensing effect of all mass near the line-of-sight, which is quite useful for relating the large-scale structure of the universe to weak lensing across large fields. Here, however, we are interested in strong lensing, where the lensing potential is often dominated by a single massive object much smaller than χ . Considering a single lensing mass at comoving distance χ_L yields the **thin lens approximation**:

$$\begin{aligned} \Psi_L(\boldsymbol{\theta}) &= -2 \frac{\chi_{\beta} - \chi_L}{\chi_L \chi_{\beta}} \int_{-\infty}^{+\infty} \frac{dz}{a_L} \Phi(\mathbf{x}(\boldsymbol{\theta}, \frac{z}{a_L})) \\ &= -2 \frac{D_{LS}}{D_L D_S} \int dz \int d^3x' \frac{-G\rho(\mathbf{x}')}{c^2 |\mathbf{x}' - \mathbf{x}(\boldsymbol{\theta}, z)|} \\ &= \frac{4GD_{LS}}{c^2 D_L D_S} \int d^2x_{\perp} \Sigma(\mathbf{x}_{\perp}) \ln |D_L \boldsymbol{\theta} - \mathbf{x}_{\perp}| + (\text{infy.}) \end{aligned} \quad (2.12)$$

Where D_L, D_S, D_{LS} are the angular diameter distances to the lens, source, and *between* lens and source, and $\Sigma(\mathbf{x}_{\perp})$ is the projected mass density of the lens. While the integral diverges, the divergent term does not depend on $\boldsymbol{\theta}$ and can

safely be ignored. A note on units: as given in Eq. 2.12, $\Psi_L(\boldsymbol{\theta})$ has units of steradian, yielding deflections $\boldsymbol{\alpha} = \nabla_{\boldsymbol{\theta}}\Psi_L(\boldsymbol{\theta})$ in units of radians. Strong lensing deflections are much smaller than a radian, the most massive galaxy clusters produce deflections on the order of 1 arcmin. In practice, it is more common to represent all angles in units of arcsecond, with $\Psi_L(\boldsymbol{\theta})$ in units of square arcsec.

2.1.1 Gravitational Lensing Formalism

We now have the fundamental equations of gravitational lensing, which govern the correspondence between observed **image plane** positions $\boldsymbol{\theta}$ and unlensed **source plane** positions $\boldsymbol{\beta}$. These equations are:

$$\boldsymbol{\beta} = \boldsymbol{\theta} - \boldsymbol{\alpha}(\boldsymbol{\theta}) \quad (2.13)$$

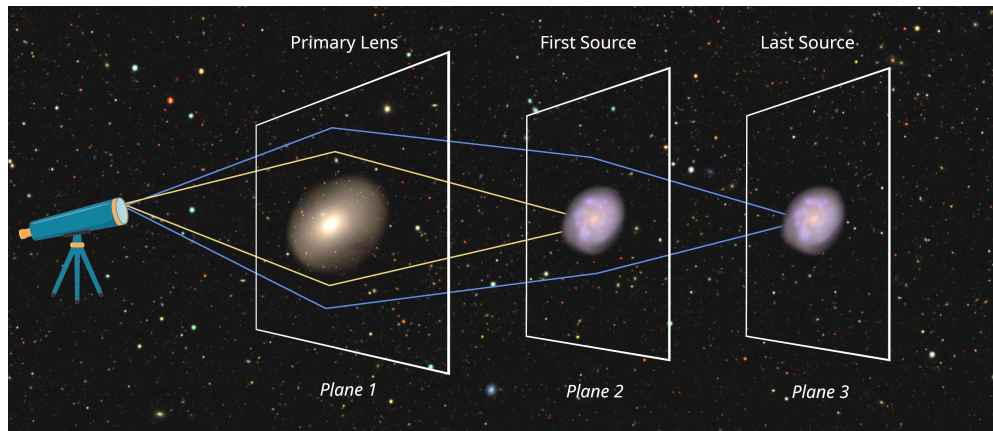
$$\boldsymbol{\alpha}(\boldsymbol{\theta}) = \nabla_{\boldsymbol{\theta}}\Psi_L(\boldsymbol{\theta}) \quad (2.14)$$

$$\Psi_L(\boldsymbol{\theta}) = \frac{1}{\pi} \int d^2\theta' \kappa(\boldsymbol{\theta}') \ln |\boldsymbol{\theta} - \boldsymbol{\theta}'| \quad (2.15)$$

$$\kappa(\boldsymbol{\theta}) = \frac{\Sigma(D_L\boldsymbol{\theta})}{\Sigma_{\text{crit}}} = \frac{4\pi G D_{LS} D_L}{c^2 D_S} \Sigma(D_L\boldsymbol{\theta}) \quad (2.16)$$

Where $\kappa(\boldsymbol{\theta})$ is the **convergence**, the dimensionless field which sources the lens potential, and the **critical surface density** Σ_{crit} , which converts between the physical surface mass density $\Sigma(\boldsymbol{x}_{\perp})$ and κ . The critical surface density Σ_{crit} is a geometric quantity sensitive to angular diameter distances between observer, lens, and source; it therefore encodes quite a bit of information on the size and expansion history of the universe.

These equations make no distinction between ‘weak’ and ‘strong’ lensing. What, then, is the distinction? Note that the first equation allows multiple solutions: a source can appear at multiple positions $\boldsymbol{\theta}_i$ which satisfy $\boldsymbol{\beta} = \boldsymbol{\theta}_i - \boldsymbol{\alpha}(\boldsymbol{\theta}_i)$.



(a)



(b)

Figure 2.1: Example strong lens consisting of three planes. **Top:** Illustration of light paths taken between source and observer, with each plane deflecting light rays traversing it. **Bottom:** Observer's view of such a system, with both background sources appearing in four images surrounding the primary lens.

The appearance of multiple lensed images defines the **strong lensing regime**. Strong lensing is a rare effect, requiring α to vary significantly over a small field. In practice, strong lensing requires a source aligned almost directly behind a massive lens. The source plane area yielding multiple images, sometimes referred to as the ‘lensing cross-section’ of a given lens, only reaches $> \mathcal{O}(10)$ arcsec² for massive galaxy clusters. While strong lensing can be adequately defined by the production multiple images of a given source, the converse definition of weak lensing, as ‘singly-imaged objects’, is not particularly useful. We will revisit this below.

A quick note on terminology: in the context of strong lensing, the term ‘lens’ is ambiguous. Does it refer to the massive lens alone, or the combined system of a foreground lens galaxy and background multiply-imaged source? To ease this ambiguity, the foreground object which causes lensing is sometimes termed the **deflector**. In all cases, **lensed source**, or simply source, refers to the distant multiply-imaged object.

If α is constant across a field, the observed image of a lensed source will be translated across the sky, but remain unchanged in size or shape. Moreover, we would have no way of knowing it is lensed at all, since we could not possibly know its intrinsic position. Lensing therefore becomes interesting when $\alpha(\theta)$ changes in θ . Changing deflections are described by the lensing Jacobian A , conventionally decomposed as follows:

$$\begin{aligned}
A_{ij} &= \frac{d\beta^i}{d\theta^j} = \delta_{ij} - \frac{d\alpha^i}{d\theta^j} \\
&= \delta_{ij} - \frac{d^2\Psi_L}{d\theta^i d\theta^j} \\
&= \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & 1 - \kappa + \gamma_1 \end{pmatrix}
\end{aligned} \tag{2.17}$$

In addition to the convergence κ we are already familiar with, the **lensing**

shear $\gamma_1^2 + \gamma_2^2 = \gamma^2$ has appeared. The shear ‘stretches’ a source’s appearance, inducing an apparent ellipticity, or even ‘canceling out’ the intrinsic ellipticity of a source. Together the convergence and shear change the observed size of an object S_θ relative to its intrinsic size S_β , inducing a magnification μ as follows:

$$\begin{aligned}\mu &= \frac{S_\theta}{S_\beta} = \left(\frac{S_\beta}{S_\theta}\right)^{-1} = \left(\det A\right)^{-1} \\ &= \left((1 - \kappa)^2 - \gamma^2\right)^{-1}\end{aligned}\tag{2.18}$$

In the **weak lensing regime**, where $|\kappa| \ll 1$ and $|\gamma| \ll 1$, the magnification is $\mu \approx 1 + 2\kappa$ at linear order.¹ Strong lenses, by contrast, exhibit extreme values of μ , including negative parity (mirrored) images with $\mu < 0$, and demagnified central images with $\mu < 1$. A common visual diagnostic for strong lens models is the contour of infinite magnification, or **critical curve**, where $(1 - \kappa)^2 = \gamma^2$. In lenses described by a single (possibly elliptical) mass distribution, there exist two concentric critical curves, which correspond to $1 - \kappa + \gamma = 0$ and $1 - \kappa - \gamma = 0$. The former describes the inner, or **radial critical curve**, while the latter describes an outer **tangential critical curve**. The names derive from the direction of infinite magnification: at the tangential critical curve, a lensed source is stretched tangentially, producing magnified arcs or an Einstein ring. Lensed images near the radial critical curve are stretched outward, and may appear to radiate out from the lens mass. Radial images are generally smaller and fainter than tangential arcs, and are rarely seen in galaxy-scale lenses, where they are often blended with the lens galaxy’s light. At cluster scales, radial and central images are fairly common, and we will see several examples in this work.

In the previous paragraph, we introduced a definition of the weak lensing

¹As a projected mass quantity, one would expect the convergence to be strictly non-negative. However, underdensities such as cosmic voids can be usefully described with $\kappa < 0$, hence $|\kappa| \neq \kappa$ in some cases.

regime as $|\kappa| \ll 1$ and $|\gamma| \ll 1$. We noted earlier that our implied definition of ‘not strongly lensed’, where a lensed source is singly-imaged, is not sufficient to define weak lensing. There exists an intermediate regime, where a source is singly-imaged, but perturbed by κ and γ beyond linear order. This regime of ‘more-than-weak’ lensing may be called the **moderate lensing regime**, where higher-order derivatives of Ψ_L become relevant. The next order perturbations beyond $\kappa, \gamma_1, \gamma_2$ are known as **flexion**, and are conventionally decomposed as follows [23, 38]:

$$\begin{aligned}
D_{ijk} &= \partial_k A_{ij} \\
D_{ij1} &= \begin{pmatrix} -2\gamma_{1,1} - \gamma_{2,2} & -\gamma_{2,1} \\ -\gamma_{2,1} & -\gamma_{2,2} \end{pmatrix} \\
D_{ij2} &= \begin{pmatrix} -\gamma_{2,1} & -\gamma_{2,2} \\ -\gamma_{2,2} & 2\gamma_{1,2} - \gamma_{2,1} \end{pmatrix}
\end{aligned} \tag{2.19}$$

With four degrees of freedom represented by $\gamma_{i,j}$.² Flexion is described here for completeness, it will not be directly used in this work. For visualizations of the effects of flexion, see Figure 1 of [23].

2.1.2 Compound Strong Lensing

In Section 2.1, we found solutions to the lens equation in two regimes: small deflections sourced by full line-of-sight structure $\Phi(\mathbf{x}, t)$ (Eq. 2.10), and arbitrary deflections sourced by a single lens mass (Eq. 2.12). What about *strong* deflections sourced by *multiple* lens planes, known as **compound lensing**? While this regime is rarely important, it is quite interesting, for reasons we will see in Sec. 2.3.5. Consider a set of lensing masses $\Sigma_i(\boldsymbol{\theta})$ each at comoving distance χ_i , and ordered

²[23] introduces another decomposition of the flexion into ‘spin-1’ and ‘spin-3’ fields.

by distance from the observer $\chi_i < \chi_{i+1}$. A light ray propagates to the observer at $\hat{\mathbf{p}}_\perp^0 = -\boldsymbol{\theta}_1$, after intersecting each lens plane at an apparent angle $\boldsymbol{\theta}_i$. Tracing its path back to the source, it is deflected as follows:

$$\chi_{i+1}\boldsymbol{\theta}_{i+1} = \chi_i\boldsymbol{\theta}_i - (\chi_{i+1} - \chi_i)\hat{\mathbf{p}}_\perp^i \quad (2.20)$$

$$\begin{aligned} \hat{\mathbf{p}}_\perp^i &= \hat{\mathbf{p}}_\perp^{i-1} + \frac{4G}{c^2 a_i} \nabla_{\mathbf{x}_\perp^i} \int d^2 x'_\perp \Sigma_i(\mathbf{x}'_\perp) \ln |\mathbf{x}_\perp^i - \mathbf{x}'_\perp| \\ &= \hat{\mathbf{p}}_\perp^{i-1} + \frac{D_S}{D_{iS}} \nabla_{\boldsymbol{\theta}_i} \Psi_L^i(\boldsymbol{\theta}_i) \end{aligned} \quad (2.21)$$

Where Ψ_L^i is the lensing potential between χ_i and the final source plane, in the thin lens approximation. It is generally more convenient to work purely in angle, which yields:

$$\boldsymbol{\theta}_i = \boldsymbol{\theta}_1 - \sum_{j=1}^{i-1} \frac{D_{ji} D_S}{D_i D_{jS}} \nabla_{\boldsymbol{\theta}_j} \Psi_L^j(\boldsymbol{\theta}_j) \quad (2.22)$$

Note that the deflection angles $\boldsymbol{\alpha}$ are no longer simply the derivative of a single lensing potential. Therefore the deflection field $\boldsymbol{\alpha}(\boldsymbol{\theta} = \boldsymbol{\theta}_1)$ is no longer sourced by an effective convergence κ , and can exhibit a nonzero curl! In the weak lensing regime, or where the lensing is dominated by a single lens plane, this effect is negligible, but in general multi-plane lensing can be quite a bit more complex than single-plane lensing. We will revisit the multi-plane lensing equation later.

2.1.3 Time Delays

We have shown that gravitational lensing follows from the geodesic equation of General Relativity. This encourages an intuitive conception of ‘why’ lensing occurs: photons strive to take the “shortest path” through a curved spacetime

to reach us.³ So far, we have only considered transverse deviations of a photon's path from a straight line-of-sight. But the perturbations caused by lensing also change the *length* of a photon's path through spacetime, and therefore the time taken between emission and observation.

While we have no way of precisely knowing how long light has taken to reach us, strong lensing affords an interesting possibility. Strong lensing creates multiple separated images of a lensed source. If a transient event occurs in the source, such as a supernova or AGN activity, it will appear at different times in each source image. This relative time between observed images, or **time delay**, is sensitive to cosmological distances. For a single lens plane, with all angles in radians, the time delay induced by lensing is [365]:

$$t(\boldsymbol{\theta}, \boldsymbol{\beta}) = \frac{D_{\Delta t}}{c} \left[\frac{(\boldsymbol{\theta} - \boldsymbol{\beta})^2}{2} - \Psi_L(\boldsymbol{\theta}) \right] \quad (2.23)$$

$$D_{\Delta t} = (1 + z_L) \frac{D_L D_S}{D_{LS}} \quad (2.24)$$

Where $t(\boldsymbol{\theta}, \boldsymbol{\beta})$, sometimes known as the **Fermat potential**, is the arrival time of light emitted at $\boldsymbol{\beta}$ and observed at position $\boldsymbol{\theta}$, and $D_{\Delta t}$ is the **time-delay distance**.⁴ The time-delay distance is sensitive to the Hubble constant, $D_{\Delta t} \propto (H_0)^{-1}$, with smaller dependence on the universe's expansion history. Therefore, if the time delay between images $\Delta t_{ij} = t(\boldsymbol{\theta}_i, \boldsymbol{\beta}) - t(\boldsymbol{\theta}_j, \boldsymbol{\beta})$ is known, and the lens potential $\Psi_L(\boldsymbol{\theta})$ is constrained precisely, competitive constraints on H_0 can be produced. This method was originally proposed by [305], and has come to be

³This also implies a useful analogy to optical lensing. Light rays traveling through a medium such as glass similarly 'take the shortest path', and optical lenses magnify or distort light accordingly. Unlike optical lensing, where the effect can vary with the wavelength of light, gravitational lensing is fundamentally achromatic.

⁴The Fermat potential implies the lensing equation. Lensed images appear at stationary points of the arrival time surface, where $\nabla_{\boldsymbol{\theta}} t(\boldsymbol{\theta}, \boldsymbol{\beta}) = 0$.

known as **time-delay cosmography**. Fortunately, the time delays Δt_{ij} induced by strong lenses are generally accessible on human timescales. Galaxy-scale strong lenses exhibit typical time delays of $\sim$ weeks to months. Strong lensing galaxy clusters, with deeper gravitational potentials and higher deflection angles, exhibit time delays of $\sim$ months to years, with some predicted time delays of $\mathcal{O}(\text{decades})$. Time-delay cosmography is discussed further in Section 2.3.4.

2.1.4 The Mass Sheet Degeneracy

We have seen that in a single-plane lens, the observed lensing is governed by the convergence field $\kappa(\boldsymbol{\theta})$. But is κ uniquely determined from the observed lensing? Consider the following transformation of κ :

$$\kappa(\boldsymbol{\theta}) \mapsto (1 - \lambda) + \lambda\kappa(\boldsymbol{\theta}) \quad (2.25)$$

The convergence becomes a perturbation $\lambda\kappa$ atop an infinite sheet of convergence $1 - \lambda$. This is known as the **mass-sheet transform** (MST).⁵ The other lensing quantities become:

$$\begin{aligned} \Psi_L(\boldsymbol{\theta}) &\mapsto (1 - \lambda)\frac{\theta^2}{2} + \lambda\Psi_L(\boldsymbol{\theta}) & \boldsymbol{\beta} &\mapsto \lambda\boldsymbol{\beta} & A &\mapsto \lambda A \\ \boldsymbol{\alpha}(\boldsymbol{\theta}) &\mapsto (1 - \lambda)\boldsymbol{\theta} + \lambda\boldsymbol{\alpha}(\boldsymbol{\theta}) & \mu &\mapsto \mu/\lambda^2 & \Delta t_{ij} &\mapsto \lambda\Delta t_{ij} \end{aligned} \quad (2.26)$$

Under the MST, the source plane is stretched by a factor of λ in each dimension, changing the source size and magnification. All other properties of the observed lensing would remain unchanged. Therefore, for any convergence field $\kappa(\boldsymbol{\theta})$

⁵The MST is a special case of the ‘Source-Position Transformation’ (SPT) [336]. The MST is of particular interest since it is perfectly degenerate with time-delays. Other forms of the SPT leave time-delays unchanged, or modify them in a spatially-dependent way.

which correctly explains observed lensing, a convergence field $\kappa' = (1 - \lambda) + \lambda\kappa(\boldsymbol{\theta})$ would fit observations equally well, with the inferred source size changed by a factor λ . This is known as the **mass-sheet degeneracy** (MSD), and represents a fundamental problem for modeling strong lenses with one lens and one source. Critically, since the MST also modifies time delays, it represents one of the primary systematic concerns for time-delay cosmography.

The MSD can be broken in several ways. If the intrinsic size or luminosity of a source is known, the magnification μ/λ^2 can be inferred, constraining λ . However, if strongly lensed standard candles or standard rulers were common, cosmographic measurements with strong lensing would be considerably easier. We are not so lucky. In practice, the best avenue to break the MSD is to measure the projected lens mass $\Sigma(\boldsymbol{\theta})$ through independent probes. This is typically accomplished with resolved measurements of the stellar kinematics of lens galaxies, probing the combined dark and luminous mass in the center of the lens halo. The TDCOSMO collaboration has made considerable progress in refining the precision of such measurements [e.g. 194, 193, 343, 375].

Compound lenses offer another opportunity to break the MSD. The MST holds for a single source plane: a MST which preserves the lensing of one source will modify the lensing of other sources. Simultaneously modeling multiple source planes therefore breaks the MSD. There is one notable caveat: [335] has shown that there exists an extended MST for systems of three planes, simultaneously modifying the mass of the first two planes to preserve the observed lensing. Curiously, this extended MST preserves the magnification of the final source plane. This does not appear to generalize to beyond three planes, so compound lenses with more than two sources unambiguously break the MSD.

2.2 Introduction to Strong Lens Modeling

The gravitational lensing formalism introduced in the previous section dealt purely in angles, where lensing maps each observed angle $\boldsymbol{\theta}$ to a source position $\boldsymbol{\beta}$. Observationally, we measure not simply angles, but a spectral flux density $I_\lambda(\boldsymbol{\theta}, \lambda)$ which varies in both position and wavelength. The observed spectral flux density of a lensed source at $\boldsymbol{\theta}$ is simply that of the unlensed source at $\boldsymbol{\beta}$:

$$I_\lambda(\boldsymbol{\theta}, \lambda) = I_\lambda^S(\boldsymbol{\beta} = \boldsymbol{\theta} - \boldsymbol{\alpha}(\boldsymbol{\theta}), \lambda) \quad (2.27)$$

This is equivalent to the statement that **surface brightness is conserved**. Lensed objects are observed with the same I_λ as unlensed sources, and only appear brighter because they ‘take up’ a larger solid angle $d\Omega_\theta = |\mu|d\Omega_\beta$.⁶ In addition to strong lensing distortions, observations of lensed sources are affected by the atmosphere, telescope, and instrument. These effects can be described by the **point spread function** (PSF), and wavelength response $B(\lambda)$. The observed flux in a pixel P_i is therefore:

$$P_i = \int d^2\theta W_i(\boldsymbol{\theta}) \int d\lambda B(\lambda) \int d^2\theta' \text{PSF}(\boldsymbol{\theta}' - \boldsymbol{\theta}, \lambda) I_\lambda^S(\boldsymbol{\theta}' - \boldsymbol{\alpha}(\boldsymbol{\theta}'), \lambda) \quad (2.28)$$

Where $W_i(\boldsymbol{\theta})$ is a window function describing the spatial response of pixel i . These combined operations can be expressed more succinctly in operator notation, with a mass- and cosmology-dependent lensing $L(M, C)$ acting on source light S as:

⁶For extended lensed arcs, where the surface brightness can be resolved, this is a useful fact: the critical curves correspond to stationary points of the observed surface brightness.

$$P = (W \circ B \circ \text{PSF} \circ L(M, C))S \quad (2.29)$$

The task of strong lens modeling is to infer the lensing mass M , source reconstruction S , and possibly cosmology C , from one or more images $\{P\}$. A bewildering array of lens modeling tools exist; it has been said that “*there are as many lens modeling codes as lenses*”. For galaxy-scale lenses, the most widely-used tools include `lenstronomy` [39, 43], `PyAutoLens` [262, 263], and `GLEE` [363, 364]. The standard tool for cluster-scale lens modeling is `lenstool` [182]. In this section we will summarize the variety of approaches and strategies to lens modeling.

2.2.1 Mass Models

The mass model governs the lens potential $\Psi_L(\boldsymbol{\theta})$ and deflections $\boldsymbol{\alpha}(\boldsymbol{\theta})$, most often using parametric models corresponding to the matter distributions of individual objects. There are two general approaches. One is to model the convergence field $\kappa(\boldsymbol{\theta})$, agnostic of the physical mass of the lens objects. This has several advantages: no cosmology needs to be assumed, and lenses without known redshifts can likewise be modeled. Additionally, parameters such as the ‘Einstein radius’ θ_E can be estimated quickly by a user, allowing simple model configuration. However, for lenses with multiple source planes, or when a mass model needs to be related to dynamical measurements, a cosmology and $\Sigma_{\text{crit}}(z_L, z_S)$ must be assumed, negating these advantages. The other approach is to parameterize a mass model in terms of *mass* itself, with physically motivated quantities such as the mass and concentration of an NFW halo (M_{200m}, c_{200m}), or the stellar velocity dispersion of a lens galaxy. In this case, redshifts of lens and source must be known, and a cosmology must be assumed to convert a mass model into $\kappa(\boldsymbol{\theta}) = \Sigma(\boldsymbol{\theta})/\Sigma_{\text{crit}}$ and compute predicted lensing. While this is generally more complex, it likewise

has advantages: lens model parameters are now physically interpretable, and a modeler can impose physically motivated priors based on scaling relations or other observations. However, where strong lensing is used to constrain cosmology, care must be taken, as priors on the physical mass of the lens can implicitly bias the inferred cosmology.

The former dimensionless approach based on κ is frequently used at galaxy-scales, and the latter approach based on physical mass is generally used for cluster-scale lensing. We will review these in turn, then briefly touch on alternative non-parametric approaches.

Parametric Models: Galaxy-Scale

Galaxy-scale strong lenses are dominated by the mass of a single deflector galaxy. As such, they are generally modeled as a single mass component. The most common choice is an **elliptical power law** (EPL) (used in e.g. [372, 352, 328]), corresponding to a three dimensional density profile $\rho(r) \propto r^{-\gamma_{\text{pl}}}$ [39, 43].⁷ (Synonyms exist: the EPL is also sometimes called a ‘Power-law Elliptical Mass Distribution’, or PEMD.) In two dimensions, with a projected axis ratio q , the convergence field of an EPL can be written as:⁸

$$\kappa(\boldsymbol{\theta}) = \frac{3 - \gamma_{\text{pl}}}{2} \left(\frac{\theta_E}{\sqrt{q\theta_x^2 + \theta_y^2/q}} \right)^{\gamma_{\text{pl}}-1} \quad (2.30)$$

Galaxy-scale lenses are typically well-described by $\gamma_{\text{pl}} \sim 2$, which is a possible solution for an isothermal halo of dark matter. Special cases of the EPL include the **singular isothermal sphere** (SIS, $\gamma_{\text{pl}} = 2, q = 1$), and **singular**

⁷Be advised that both the negative power-law slope and the lensing shear are conventionally called γ . Take care not to confuse them.

⁸The EPL *can* be written in this form, which preserves the total mass with changing q . However, it is not always implemented as such! Take care to understand your lens modeling code’s implementation choices.

isothermal ellipse (SIE, $\gamma_{\text{pl}} = 2, q \in [0, 1]$). It is perhaps surprising that galaxy scale lenses would be well described by a single power-law γ_{pl} , especially where $\gamma_{\text{pl}} > 2$ implies a divergent central mass. Two factors are at play here. First, γ_{pl} is only constrained by lensing at the radii where images are observed: a shallower slope yields more diffuse, ‘fuzzier’ observed images, and a steeper slope conversely yields sharper, thinner arcs. A measured $\gamma_{\text{pl}} > 2$ from lensing therefore does not actually require a divergent central mass. Second, the observed lensing is a combined effect from the dark and baryonic mass of the lens, which differ in their spatial distributions. In practice, the combination of the concentrated baryonic mass (the ‘bulge’) and the more diffuse dark ‘halo’, appears to closely resemble a single power-law at the radii probed by strong lensing [219, 216]. This is known as the **bulge-halo conspiracy** [115]. Kinematic analyses of samples of galaxies are beginning to show that this ‘conspiracy’ is far from perfect [217, 99], and therefore future lens models may require separate baryonic and dark components, or broken power-laws. For now, however, the bulge-halo conspiracy reigns in strong lensing analyses.

The spatial distribution of lens mass is not necessarily elliptical, and state-of-the-art models often require further flexibility to precisely model observed arcs [276, 283]. An increasingly common approach is to perturb a mass model with higher-order **multipole moments**, describing ‘diskiness’ and ‘boxyness’ beyond simple ellipticity. Multipole moments are essential to reach the precision needed for dark matter probes, such as gravitational imaging [276].

Foreground or environmental perturbations additionally contribute to the observed strong lensing, perturbing the images in addition to the lens mass. It is common to include an **external shear** (γ_{ext}) in lens models to describe the contribution of large scale structure near the line-of-sight. The external shear is

described by a constant shear across the lensing region:

$$\Psi_L^{\gamma_{\text{ext}}}(\boldsymbol{\theta}) = \frac{\gamma_{\text{ext},1}}{2}(\theta_x^2 - \theta_y^2) + \gamma_{\text{ext},2}\theta_x\theta_y \quad (2.31)$$

$$\boldsymbol{\alpha}^{\gamma_{\text{ext}}}(\boldsymbol{\theta}) = \hat{\theta}_x(\gamma_{\text{ext},1}\theta_x + \gamma_{\text{ext},2}\theta_y) + \hat{\theta}_y(-\gamma_{\text{ext},1}\theta_y + \gamma_{\text{ext},2}\theta_x) \quad (2.32)$$

Including γ_{ext} is often necessary to produce a high-quality lens model. However, caution should be exercised in interpreting γ_{ext} : it has been shown that γ_{ext} does not correlate with the ‘true’ shear source by large scale structure as inferred from weak lensing [125]. It has been suggested that γ_{ext} is merely capturing unmodeled features of the lens mass, implying that more complex mass models are needed to fully describe observed lensing [168, 169, 14].

Parametric Models: Cluster Scales

Galaxy clusters are massive halos of dark matter, typically $10^{14} - 10^{15} M_\odot$, which contain numerous smaller galaxy-scale ‘subhalos’. The subhalos associated with member galaxies meaningfully affect the observed lensing, and cannot be neglected [253, 196, 94]. The mass models of cluster strong lenses are therefore quite complex. The established approach is to describe the lensing as a combination of smooth, group- or cluster-scale masses c_i and galaxy-scale perturbers p_j [182]:

$$\Psi_{L,\text{tot}}(\boldsymbol{\theta}) = \sum_i \Psi_{L,c_i}(\boldsymbol{\theta}) + \sum_j \Psi_{L,p_j}(\boldsymbol{\theta}) \quad (2.33)$$

Relaxed lensing clusters can often be modeled with a single cluster-scale halo c_1 , while merging clusters often require multiple cluster-scale halos $\{c_i\}$. There are typically dozens of cluster member galaxies $\{p_j\}$ included in a model, and modeling each galaxy independently would yield a prohibitively large parameter

space, where most parameters for individual galaxies are entirely unconstrained. Fortunately, there is a known empirical relation between the luminosity and velocity dispersion of elliptical galaxies, known as the **Faber-Jackson relation** [126], according to which $\sigma \propto L^\alpha$ with $\alpha \approx 0.2 - 0.3$ [30, 154, 155]. Given a parametric mass model for each galaxy which can be related to σ , the lensing mass of each cluster member can be linked by this relation. The standard parameterization is the **dual Pseudo-Isothermal Ellipse** (dPIE, [119]), which has an analytic solution for the velocity dispersion σ given a mass, core radius r_a , and scale radius r_s .⁹ With a negligible core radius $r_a \approx 0$, the parameters of each member galaxy i scale as follows:

$$\frac{\sigma_i}{\sigma_{\text{ref}}} = \left(\frac{L_i}{L_{\text{ref}}} \right)^\alpha \quad (2.34)$$

$$\frac{r_{s,i}}{r_{s,\text{ref}}} = \left(\frac{L_i}{L_{\text{ref}}} \right)^\beta \quad (2.35)$$

It can be shown that if the total mass-to-light ratio of member galaxies Υ_i^{tot} scales as L_i^γ , then $\beta = \gamma - 2\alpha + 1$; γ is generally fixed, and lens models float the parameters α , σ_{ref} , and $r_{s,\text{ref}}$. Frequently, the smooth cluster-scale halo is also modeled as a dPIE. The widespread use of the dPIE profile is more of a practical convention than a theoretically-motivated model, as the simple analytical relation between dPIE parameters and central velocity dispersion presented in [119] conveniently relates lens model parameters to the Faber-Jackson relation. Nevertheless, the dPIE seems to describe cluster member galaxies reasonably well, and [36] has recently shown that resolved dynamical measurements of cluster members are described better by dPIE profiles than NFW profiles. This result could help explain a well-studied discrepancy between the simulated and observed

⁹The dPIE has a number of synonyms, including the ‘pseudo-Jaffe profile’ [39, 43]

‘lensing strength’ of cluster member galaxies [239, 240, 241].

Non-Parametric Mass Models

We note that non-parametric methods are also used to reconstruct the lensing mass field, though they are generally less common than parametric models. The parametric models discussed above can only represent a limited range of lensing mass models, implicitly asserting a prior on the lens mass distribution. Non-parametric or ‘free-form’ models allow for highly complex lensing mass distributions, enabling data-driven constraints without ascribing a strong prior on what the lens mass ‘should’ look like. Naturally, non-parametric models have trade-offs; they can significantly increase computational complexity, introduce additional hyperparameters to be tuned, and ignore other observable handles on the lens mass. Non-parametric models have primarily been applied to cluster-scale strong lenses, with tools such as **GRALE** [338], **RELENSING** [378], and **MARS** [78].

2.2.2 Model Constraints

Pixel-Level Modeling

The ‘gold standard’ method for modeling strong lenses is to fit fully model observed imaging data as described by Eq. 2.28, forward modeling the mass model, the observed light from lensed sources, and the luminous contribution of the lens galaxy (or galaxies) [as in e.g. 8, 407, 25, 344, 372, 352]. (Though we have neglected the light of the deflector up to this point, it is frequently blended with the lensed source and must also be modeled in most cases.) This is known as **pixel-level modeling** or **extended surface brightness modeling**. Astronomers have long studied the observed light distributions of distant galaxies, and strong lens modelers naturally draw on conventional models of the spatially varying lu-

minosity of galaxies. Typically, this involves modeling the light of both lens and source as a sum of one or more **Sérsic profiles** [340], or where more flexibility is needed, a Gaussian expansion [69]. In order to use multiple observed imaging bands, it is essential to accurately model the variation of the PSF between each band [as in e.g. 300], and it is often necessary to model a relative astrometric offset between each image during modeling. Frequently, the position and shape parameters of the lens and source light are enforced to be identical in each band, with the relative luminosity in each band allowed to float.

Where simple parametric light models are not sufficient to capture all observed details, several strategies exist. The simplest approach is to additionally include **shapelet** perturbations [303, 304] on top of the parametric light model [41]. Shapelets are sets of spatially varying functions which can be used as a basis for complex distributions. While a given (infinite) shapelet basis can represent any arbitrary distribution, in practice the N lowest order shapelets are used. Including more shapelets (higher N) helps describe more complex source morphology, at the cost of increased computational complexity and reduced sampling efficiency. Shapelets can improve the fidelity of a source reconstruction, but *not all shapelets are alike*, and different choices of shapelet bases can yield very different results. While **lenstronomy** [39, 43] and **GIGA-Lens** [159] use the shapelet basis originally proposed in [303, 304], other shapelet bases may be better suited to reconstructing lensed sources.

In the absence of a shapelet basis that is perfectly suited to describe lensed sources, fully pixelated models can be used to represent arbitrary source morphology [91, 92]. Such models represent the source light as a field of pixels, introducing hundreds or thousands of additional model parameters. This approach is computationally expensive, and generally requires leveraging highly parallel computing

hardware such as GPUs or TPUs. In order to manage this complexity, a regularization scheme is needed to suppress overfitting to small scale features. Some modelers have found success in representing source light in Fourier space, using a Matérn power spectrum prior to generalize regularization choices [219, 121]. An alternative cutting-edge approach is to use ML techniques to ‘learn’ the prior on lens and source light, first training a model on a catalog of imaging data, then using that model to condition pixelated reconstructions of lens and source light. This approach was taken in [209], which trained a neural network on DESI Legacy Survey galaxy imaging to construct a prior on source light, and applied that model in a strong lensing analysis to produce remarkable reconstructions of lensed sources. However, such an approach requires a selection of imaging data to train on, and a large number of model choices, both of which could potentially bias results. The authors of [209] trained their source prior model on low-redshift galaxies imaged with the Dark Energy Camera (DECam), yet applied their model to reconstruct higher-redshift lensed sources imaged with HST. Such a mismatch between training data and applied data could introduce subtle biases in the inferred source light and lens model.

Conjugate Point Constraints

A lensed source that appears at positions $\boldsymbol{\theta}_i$ requires a lens model that aligns, or ‘focuses’, all images to the same source position $\boldsymbol{\beta} = \boldsymbol{\theta}_i - \boldsymbol{\alpha}(\boldsymbol{\theta}_i)$. This naturally implies the simplest way to constrain a lens model – the model must explain each measured position $\boldsymbol{\theta}_i^f$ by a single source position $\boldsymbol{\beta}^f$, for each feature f . A family of position-based methods for strong lens modeling are known as **conjugate point methods**.¹⁰ These methods necessarily ignore the overwhelming majority of the

¹⁰While this family of methods has been used since at least the 1990s ([271] lists early implementations including [187, 198, 199]), the term ‘conjugate point’ or ‘conjugate image’ seems to

data: while pixel-level models make use of the full imaging information, conjugate point methods reduce observations to a small data vector of observed positions. Nevertheless, conjugate point methods have practical advantages, and remain the standard for modeling cluster lenses.

Conjugate point constraints can be computed in either the image-plane or the source-plane. The image-plane construction requires finding a source-plane position β^f whose predicted image-plane positions $\theta_{i,\text{pred}}^f$ most closely match observed positions θ_i^f with corresponding covariance matrices Σ_i^f . This requires maximizing the following likelihood:

$$\log \mathcal{L} = - \sum_{f,i} \frac{1}{2} (\theta_i^f - \theta_{i,\text{pred}}^f)^T (\Sigma_i^f)^{-1} (\theta_i^f - \theta_{i,\text{pred}}^f) \quad (2.36)$$

The image-plane construction further allows penalizing models which predict extra spurious images (i.e., if 3 images of a source are seen, a model predicting 5 images is disallowed). However, identifying all predicted image positions $\theta_{i,\text{pred}}^f$ is a complex problem, requiring nonlinear solvers to explore the observed field. The image-plane construction is therefore computationally expensive, and source-plane methods are sometimes used instead.

The source plane method involves computing the model-predicted source plane images $\beta_i^f = \theta_i^f - \alpha(\theta_i^f)$, and quantifying the consistency of each β_i^f for each feature. A perfect lens model would yield $\beta_i^f = \beta_j^f, \forall i, j$. The most commonly used source-plane likelihood is the one implemented in `lenstool` [182], and used in other work [e.g. 384]:

$$\log \mathcal{L} = - \sum_{f,i} \frac{(\beta_i^f - \bar{\beta}^f)^2}{2(\mu(\theta_i^f))^{-2}(\sigma_i^f)^2} \quad (2.37)$$

have entered common usage later [380, 141].

Where $\bar{\boldsymbol{\beta}}^f$ is the mean source-plane position for observed feature f , and σ_i^f is the (circular) astrometric uncertainty on $\boldsymbol{\theta}_i^f$. **This likelihood is incorrect on several counts.** First, the magnification μ is the ratio of sizes between the image- and source-planes, not the ratio of 1D distances; therefore the equation above should have μ^{-1} rather than μ^{-2} . Second, it assumes position uncertainties in the source plane are circular. This is usually false: even where image-plane uncertainties are circular, the lensing effect creates elliptical uncertainties in the source plane. Finally, allowing the source-plane uncertainty $\sigma_i^f/\mu(\boldsymbol{\theta}_i^f)$ to vary as a function of model-predicted magnification makes the *normalization* of the likelihood vary with model parameters. The model-dependent normalization must be properly handled in the likelihood, which Eq. 2.37 fails to do.

In [268], we introduced an improved source plane likelihood which corrects the faults listed above. Given $\boldsymbol{\theta}_i^f$ with corresponding astrometric uncertainty matrices Σ_i^f , then the corresponding source-plane uncertainty matrices B_i^f , best fit source-plane feature position $\hat{\boldsymbol{\beta}}^f$, and log-likelihood for feature f are:

$$B_i^f = A(\boldsymbol{\theta}_i^f) \Sigma_i^f A^T(\boldsymbol{\theta}_i^f) \quad (2.38)$$

$$(\hat{B}^f)^{-1} = \sum_i (B_i^f)^{-1} \quad (2.39)$$

$$\hat{\boldsymbol{\beta}}^f = \hat{B}^f \left(\sum_i (B_i^f)^{-1} \boldsymbol{\beta}_i^f \right) \quad (2.40)$$

$$\log \mathcal{L}^f = -\log \sqrt{\frac{|\hat{B}^f|}{2\pi}} - \frac{1}{2} \sum_i (\boldsymbol{\beta}_i^f - \hat{\boldsymbol{\beta}}^f)^T (\hat{B}^f)^{-1} (\boldsymbol{\beta}_i^f - \hat{\boldsymbol{\beta}}^f) \quad (2.41)$$

While this formalism assumes $A(\boldsymbol{\theta})$ varies little between $\boldsymbol{\theta}_i^f$ and $\boldsymbol{\theta}_{i,\text{pred}}^f$, this is generally true for accurate lens models where $\boldsymbol{\theta}_i^f \approx \boldsymbol{\theta}_{i,\text{pred}}^f$. This formalism is also an improvement when observed astrometric uncertainties are circular, i.e.

$(\Sigma_i^f)_{jk} = \delta_{jk}(\sigma_i^f)^2$, in which case lensing distortions induce ellipticity in the source-plane uncertainties B_i^f . This elliptical source-plane likelihood is a computationally efficient way to quickly obtain accurate lens models, and can even improve pixel-level analyses which use conjugate-point methods as a ‘first pass’ constraint (e.g. [92]).

Lens Dynamics

Precision strong lens models increasingly employ dynamical constraints on lens mass, most often derived from spectroscopic observations. While lensing is sensitive to the projected mass quantity $\kappa(\boldsymbol{\theta}) = \Sigma(\boldsymbol{\theta})/\Sigma_{\text{crit}}$, dynamical constraints derive from measured velocity dispersions of luminous counterparts. Assuming a static, spherically symmetric mass distribution $\rho(r)$, the virial theorem implies a simple relation between the RMS velocity $v_{\text{rms}} = \sqrt{\langle v^2 \rangle}$ and the Newtonian gravitational potential $\Psi(r)$:

$$v_{\text{rms}}^2 = -\Psi(r) = -\frac{GM_{\text{enc}}(< r)}{r} \quad (2.42)$$

$$(v_{\text{rms}}^{\text{obs}})^2 = -\frac{\int \nu \Psi(r) dz}{\int \nu dz} = \frac{\int \nu GM_{\text{enc}} r^{-1} dz}{\int \nu dz} \quad (2.43)$$

Where ν is the 3D distribution of the luminous tracer with measured kinematics, and z represents the line-of-sight direction. Thus the measured LOS velocity $v_{\text{los}} = \langle v \rangle$ and LOS velocity dispersion $\sigma_v = \sqrt{v_{\text{rms}}^2 - v_{\text{los}}^2}$ are sensitive to the enclosed 3D mass $M_{\text{enc}}(< r)$. This provides a lever on the physical lens mass distribution, both improving the mass model precision and constraining the mass-sheet parameter λ [343, 193, 375]. Dynamical constraints are essential for time-delay cosmography, as discussed in Section 2.3.4. Dynamical constraints are

also used in cluster-scale lenses to calibrate the lensing contribution of galaxy substructures with the Faber-Jackson relation. The Faber-Jackson relation can even be calibrated for individual lensing clusters with sufficient spectroscopic data (e.g. [30, 154, 132]).

The simple discussion above made strong assumptions of a spherically symmetric lens mass, and a luminous tracer with an isotropic velocity dispersion. In practice, more flexibility is needed to accurately model observed kinematics. One common choice is the ‘Jeans Anisotropic MGE’ (JAM) framework, which allows for an axisymmetric mass distribution (prolate or oblate, but not fully triaxial), in either spherical or cylindrical coordinates [70, 71, 74]. JAM allows for velocity anisotropy, describing luminous tracers with motions that are preferentially radial rather than tangential, or vice versa. An application of JAM to cluster strong lens modeling using cutting-edge methodology can be found in Ch. 4. More detailed dynamical modeling possibilities include Schwarzschild orbit reconstruction methods [337], which make use of higher-order velocity moments (e.g. $\langle v^3 \rangle$, $\langle v^4 \rangle$). For an application of Schwarzschild dynamical modeling to strong lensing, see [396].

2.3 Applications of Strong Lensing

Gravitational lensing combines three elements: a) the gravitational potentials of massive objects; b) the geometry of the universe, encoded in Σ_{crit} and $D_{\Delta t}$; and c) the light of distant objects. While these elements can never be fully separated, the science applications of strong lensing are often divided into three corresponding categories. These are:

- **Lens science:** Studies of lens objects, particularly their mass, and the spatial distribution of their mass.

- **Cosmography:** Constraints on the size, shape, and expansion rate of the universe.
- **Source science:** Studies of distant magnified objects. The magnification of strong lensing enables measurements of smaller features and fainter objects than otherwise possible.

In this section we summarize interesting probes of cosmology with strong lensing, particularly probes of dark matter or the expansion history of the universe. While source science is not a topic of this work, we note that it has a rich history of astrophysical studies, particularly in galaxy cluster lenses. Key observational programs include [294, 82], selected studies include [311, 57, 312, 58, 180, 313, 313, 59].

2.3.1 Dark Matter: Halo Cores

Strong lensing has an unparalleled ability to measure the amount and distribution of matter in massive objects. In lensing configurations with observed arcs surrounding the deflector, the observed lensing constrains both the total projected mass in the lensing region and the slope of the density profile at the arcs. In galaxy-scale lenses, constraints on the power-law density slope γ have implications for galaxy evolution, especially where a sample of lenses can probe its evolution with redshift [e.g. 328]. In cluster-scale lenses, which are vastly more spatially extended, strong lensing generally probes only the innermost region of the halo. While this gives cluster lensing only weak constraining power on the overall mass of clusters, it is uniquely powerful at probing the inner slope of cluster halos, and even demonstrating dark matter cores. Seminal work along these lines included [256, 257], which combined strong lensing and slit kinematics in a sample of clusters. Their results implied that while the total (baryonic + dark) matter profile closely matched an NFW, the dark matter profile alone appeared to

be cored. These results inspired considerable follow-up work, including dedicated constraints on SIDM from samples of lensing clusters [e.g. 18, 326]. This line of work will be revisited and expanded on in Chapter 4. Additionally, demagnified central images are regularly seen in lensing clusters, which provide further constraints on the inner density profile and often indicate a cored profile. For a recent study on lensing clusters with central images, see [77].

2.3.2 Dark Matter: Flux-Ratio Anomalies

Any strongly lensed point source, such as a quasar, will not exhibit any morphology; the only constraints it provides on a mass model are the observed positions θ_i and relative flux between images, $r_{ij} = L_i/L_j = \mu_i/\mu_j$. However, these flux ratios are perturbed by all low-mass halos along the line-of-sight, and differ significantly from the flux ratios predicted by a smooth parametric mass model [409, 188]. These differences between observed flux ratios and those predicted by a smooth mass model (the **macromodel**) are known as **flux-ratio anomalies** [409], and are sensitive to the aggregate population of low-mass matter halos (the perturbing **micromodel**) [187].

Flux-ratio anomalies therefore probe the small-scale distribution of dark matter and constrain any model which modifies the population and morphology of low-mass halos, such as WDM [279, 56] or SIDM [146, 9, 251]. However, samples of lenses are needed, as an individual quadruply imaged point source only provides three flux ratios, and doubly imaged sources only provide a single ratio. Recent work has used populations of lensed quasars to constrain the temperature of dark matter [147], the free streaming length of dark matter [145], and probe the minimum dark matter halo mass [261], testing a key prediction of Λ CDM theory. Upcoming results will use the same data to constrain SIDM (D. Gilman, private

communication).

2.3.3 Dark Matter: Gravitational Imaging

Any massive object along the line-of-sight will perturb strongly lensed images, changing their position or morphology. This effect scales inversely with the projected distance from an image, and matter halos lying directly on an observed image will have the strongest effect. With a sufficiently detailed model of the lens macromodel and surface brightness of a source, it is possible to infer the presence of discrete low-mass halos perturbing arcs [385]. This powerful method is known as **gravitational imaging** [386]. Conceptually, this is quite similar to the flux-ratio anomaly method in that it uses deviations between observed lensing and a macromodel to constrain small-scale structure. Gravitational imaging, however, has the unique ability to precisely detect individual objects, and potentially demonstrate the presence of dark matter-only halos devoid of baryons.

Gravitational imaging is an active subfield of strong lensing, and has successfully detected a number of intriguing low-mass objects. The most well-known is likely in SDSSJ0946+1006, also known as the “Jackpot” lens, which is incidentally the only known galaxy-scale lens to host three sources at distinct redshift [355]. In this system, a perturber of roughly $\sim 10^9 M_\odot$ has been independently reported by several groups using different modeling tools [25, 244, 218]. More recent gravitational imaging detections include JVAS B1938+666, where deep radio observations with the Very Long Baseline Array (VLBI) allowed sufficient angular resolution to detect a $\sim 10^6 M_\odot$ perturber at high significance [234], and PJ011646, where ALMA dust-continuum maps enabled the detection of a $\sim 3 \times 10^{10} M_\odot$ object [16]. The ‘Jackpot’ and JVAS B1938+666 detections notably appear to be very highly concentrated objects in tension with Λ CDM [218, 387]. In some analyses,

the perturbers are indistinguishable from point masses; for instance [25] found a slight preference for a point mass over a modified NFW. Such objects have strong implications for beyond- Λ CDM physics such as SIDM or fuzzy DM [121, 218, 387]. Should these detections stand up to further scrutiny, and especially if future detections likewise appear to defy Λ CDM, gravitational imaging could provide strong evidence for non-trivial dark sector physics.

2.3.4 Cosmography: Time-delay H_0 measurements

As we have already seen in Section 2.1.3, the time-delays observed between strongly lensed images are sensitive to the universe’s expansion rate H_0 . Time-delay constraints on H_0 are perhaps the most mature and widely-known strong lensing probe of cosmology. The Time-Delay COSMOgraphy collaboration (TDCOSMO) has steadily refined measurement and modeling techniques, with the most recent TDCOSMO constraints [375] yielding $H_0 = 71.6^{+3.9}_{-3.3}$ km s^{−1} Mpc^{−1} (Fig. 2.2) from a sample of eight lensed quasars with time-delay measurements, incorporating resolved stellar kinematics measurements and a prior on Ω_m from a sample of supernovae. The TDCOSMO 2025 result constitutes a $\sim 5\%$ measurement of H_0 , and ongoing work is expected to improve this precision to $\sim 3\%$ [195] within the next year.

Despite this progress, strong lenses with measured time delays remain rare, with multiply-imaged AGN as the largest sample by far. Strongly lensed supernovae are preferable to AGN for a variety of reasons, including more precise time delay measurements and easier lens modeling once the supernova images fade [330]. While few ($\mathcal{O}(10)$) strongly lensed SNe are known, including SN Requiem [320], SN Encore [289], SN H0pe [282], SN Refsdal [189], and SN Winny [373], sample sizes are expected to grow rapidly in the next few years. LSST in particular

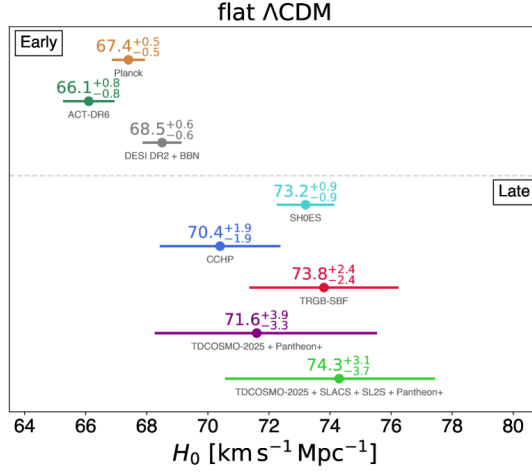


Figure 2.2: TDCOSMO 2025 results: Time-delay measurement of H_0 from TDCOSMO 2025 [375], taken from Fig. 14 of that paper. For comparison, higher-precision measurements are included from CMB experiments (ACT, Planck), BAO (DESI), and local distance ladder (SH0ES, CCHP, TRGB).

is expected to expand current samples by at least an order of magnitude, finding approximately 50 lensed Type Ia SNe per year [19, 150], with other estimates in the range 20 – 100 per year [274, 405]. Even excluding lensed SNe, the complete sample of lensed AGN from LSST will have competitive constraining power on both H_0 and the dark energy equation of state [122].

Lensed SNe and AGN are not the only candidates for time-delay cosmography. Any other lensed transient event can be used for these measurements, including ‘exotic’ transients such as gamma ray bursts [324], fast radio bursts [272], kilonovae [220], or gravitational waves [368, 220, 272]. While no such ‘exotic’ lensed transients have yet been observed, they offer distinct possibilities, such as < 1 second time-delay precision [220, 324]. Strongly lensed gravitational waves, in particular, would represent a novel test of General Relativity. The gravitational lensing formalism above was derived from the geodesic equation of a massless particle propagating through a perturbed universe. As tensor perturbations to the spacetime metric, the geodesic equations for gravitational waves are somewhat

more complicated. Fortunately, lensing of gravitational waves does follow the same equations at leading order, but is expected to exhibit additional wave-optics effects [368] which may probe dark matter structure on small scales [184].

2.3.5 Cosmography: Compound Lensing

We have seen that Σ_{crit} is sensitive to the geometry and expansion history of the universe. If Σ_{crit} could be measured precisely through strong lensing, it would enable precise measurements of the expansion history of the universe. Unfortunately, we are generally unable to measure Σ_{crit} , as it is directly degenerate with the projected lens mass $\Sigma(\boldsymbol{\theta})$. Indeed, lens models are often parameterized by the convergence $\kappa(\boldsymbol{\theta}) = \Sigma(\boldsymbol{\theta})/\Sigma_{\text{crit}}$ to remove this degeneracy.

Consider, however, a compound lens, where each background source is lensed by the same foreground lens mass. If each source occurs at a distinct cosmological redshift, the strength of the lensing effect on each source is governed by a different Σ_{crit} . In this case, the relative lensing strength between each source contains information on how Σ_{crit} varies with source redshift [148]. For a lens at z_i and source at z_j , the relative lensing strength to a source at z_S can be described as follows:

$$\eta_{ij} = \frac{\Sigma_{\text{crit},iS}}{\Sigma_{\text{crit},ij}} = \frac{D_{ij}D_S}{D_jD_{iS}} \quad (2.44)$$

For a source at redshift $z_j < z_S$, $\eta_{ij} < 1$, and the lensing strength converges to zero at $z_i = z_j$.¹¹ Treating z_j as a continuous variable yields the **lensing Hubble diagram**, seen in Fig. 2.3. Compound strong lenses sample the lensing Hubble

¹¹This construction of η_{ij} privileges a source at z_S , and defines all lensing in reference to it. A more natural approach is to reference the lensing strength against an infinitely distant source, in which case $\eta_{ij} = D_{ij}/D_j$. Nevertheless, the definition with a reference redshift z_S is standard throughout the literature, and will be used in this work.

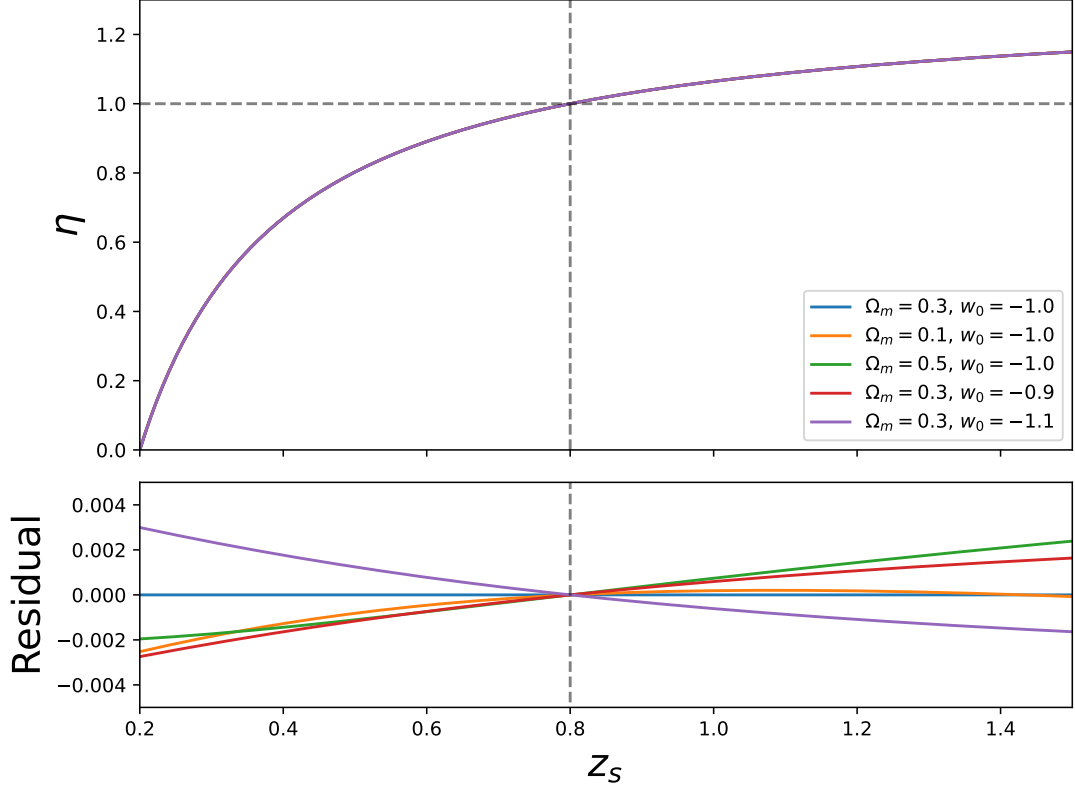


Figure 2.3: Lensing Hubble Diagram: An illustration of how the strength of lensing varies as a function of redshift and cosmology, as described in Equation 2.44. The lensing strength η is shown for a lens at $z_l = 0.2$ and a reference source $z_{\text{ref}} = 0.8$, in five different w CDM cosmologies. The variations are subtle: differences in the top panel are not visible by eye, and a corresponding residual panel on the bottom shows $(\Delta\eta)/\eta$ relative to $(\Omega_m, w_0) = (0.3, -1)$.

diagram at multiple redshifts, directly constraining the evolution of the angular diameter distance between each source, the lens, and the observer. Note that the relative lensing strength is often erroneously described as “ratios of Einstein radii”. While this is true for a point mass lens, where η_{ij} uniquely determines the ratio of Einstein radii for each source, this is *not true* in other cases. The ratios of Einstein radii are a function of both η_{ij} and the slope of the lens density profile $\rho \propto r^{-\gamma}$, creating a degeneracy between the power-law slope γ and η_{ij} . For an excellent illustration of this degeneracy, see Fig. 19 of [384]. Compound strong lensing is similar to weak lensing shear-ratios, which likewise measure the variation of lensing strength with source redshift [374, 120].¹² While in weak lensing, shear ratios are more often used as a calibration tool or consistency check, they can in principle be used to constrain cosmology just like compound strong lensing. The emerging technique of kinematic lensing promises to provide much higher signal-to-noise than traditional weak lensing, and could enable cosmological shear-ratio constraints with future data sets [214].

In practice, compound lensing has limitations. Recall the compound lens formalism introduced in Section 2.1.2. Equation 2.22 can be rewritten using our newly defined η_{ij} :

$$\boldsymbol{\theta}_i = \boldsymbol{\theta}_1 - \sum_{j=1}^{i-1} \eta_{ji} \nabla_{\boldsymbol{\theta}_j} \Psi_L^j(\boldsymbol{\theta}_j) \quad (2.45)$$

If only one lens plane contributes ($\Psi_L^1 \neq 0$ and $\Psi_L^{j>1} \approx 0$) then η_{ij} can be precisely constrained from the observed lensing. Of course, all lensed sources will have associated mass, and in general $\Psi_L^j \neq 0$. Correct modeling of compound

¹²The lensing Hubble diagram introduced here is identical to the shear ratio $\gamma_{t,i}/\gamma_{t,j}$, though it applies to all lensing effects, not merely shear. Compare Eq. 2.44 to Eq. 11 of [374].

lensing therefore requires the mass of each plane, save the last. The Zigzag lens [334] is the first system demonstrated to have a source strongly lensed by two mass sheets, and is named for to the circuitous light path produced by two deflector planes.

In such cases where multiple mass planes contribute to the observed lensing, the lensing Jacobian A becomes somewhat more complicated. The new Jacobian to source plane k is A'_k :

$$\begin{aligned}
(A'_k)_{ij} &= \frac{\partial(\beta_k)_i}{\partial(\theta_1)_j} = \delta_{ij} - \frac{\partial}{\partial(\theta_1)_j} \sum_{l=1}^{k-1} \eta_{lk} \frac{\partial}{\partial(\theta_l)_i} \Psi_L^l(\boldsymbol{\theta}_l(\boldsymbol{\theta}_1)) \\
&= \delta_{ij} - \sum_{l=1}^{k-1} \eta_{lk} (1 - A_l|_{\boldsymbol{\theta}_l})_{ij} - \sum_{l=1}^{k-1} \sum_{m=1}^{l-1} \eta_{lk} \eta_{ml} [(1 - A_l|_{\boldsymbol{\theta}_l})(1 - A_m|_{\boldsymbol{\theta}_m})]_{ij} \\
&\quad - \sum_{l=1}^{k-1} \sum_{m=1}^{l-1} \sum_{n=1}^{m-1} \eta_{lk} \eta_{ml} \eta_{nm} [(1 - A_l|_{\boldsymbol{\theta}_l})(1 - A_m|_{\boldsymbol{\theta}_m})(1 - A_n|_{\boldsymbol{\theta}_n})]_{ij} \\
&\quad - \dots
\end{aligned} \tag{2.46}$$

Where unprimed A_l represents the lensing Jacobian produced by mass plane l alone, in the absence of other mass planes. The new lensing Jacobian involves not only the contributions of individual lens planes, but the mixing between pairs of planes, *triplets* of planes, and so on. At linear order in κ, γ , the first two terms dominate, suppressing mixing between planes and recovering the weak lensing limit (Eq. 2.10). Here we are interested in the strong lensing regime, where such mixing cannot be neglected. Inter-plane mixing produces a new term r in the lensing Jacobian:

$$A'_{ij} = \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 - r \\ -\gamma_2 + r & 1 - \kappa + \gamma_1 \end{pmatrix} \quad (2.47)$$

$$\mu^{-1} = (1 - \kappa)^2 - \gamma_1^2 - \gamma_2^2 + r^2 \quad (2.48)$$

The new term r induces rotations in lensed images, and can be understood as a curl of the effective deflection field $\boldsymbol{\alpha}_i(\boldsymbol{\theta}_1) = \boldsymbol{\theta}_1 - \boldsymbol{\theta}_i(\boldsymbol{\theta}_1)$. This curl contribution from compound lensing is not widely appreciated. In the limit of one dominant mass plane with many smaller halos along the line-of-sight, the deflections can be decomposed into the sum of an ordinary convergence field κ_{div} and a field κ_{curl} sourcing rotations [103]. This formalism can potentially measure the contribution of line-of-sight halos, similar to flux-ratio anomalies [103, 104].

We will present an ideal lens system for compound lens cosmography in Chapter 5, along with preliminary cosmography constraints. Compound lensing has considerable potential for the future of cosmology, and will be discussed further in the conclusion (Section 6.2).

Chapter 3

Searching for Lenses: The DES Bright Arcs Survey

Here we present a search for strong gravitational lenses in the first three years of Dark Energy Survey (DES) imaging data. This chapter includes a reprint of the following published material: O’Donnell et al. [267]. This heterogeneous search combined the results of eight distinct lens searches, producing a diverse catalog of 247 candidate lenses. Of these 247 systems, 81 were identified for the first time in this work. Two strong lens systems discovered in this work, MACSJ0138 and the “Carousel Lens”, are studied in detail in Chapters 4 and 5 respectively.

3.1 Introduction

Gravitational lensing occurs when the gravitational field of a massive foreground object, the “lens”, deflects the light of a more distant object, the “source”. If the lens is massive enough, and the source, lens, and observer sufficiently collinear and distant from one another, the observed image of the source object can be distorted into rings, arcs, or multiple images. This effect is known as

strong gravitational lensing (SL), and has become a powerful and popular tool for studying a wide variety of extragalactic physics and cosmology.

Though strong gravitational lensing of very distant objects by foreground galaxies was predicted [415] four decades prior, the first discovery of a strongly lensed object was that of QSO 0957+561 [392], a double image of a single quasar at redshift $z = 1.4$. Not long after, it was realized that the long, narrow, curved features bent around the centers of galaxy clusters were actually SL systems as well [359, 229, 158]. In the subsequent 40-year interval, the count of candidate strongly lensed galaxies has expanded to more than 2000. At the same time, so has their scientific interest.

Strong gravitational lensing provides new opportunities to study astrophysics and the physics of dark matter. Because the surface brightness of a source galaxy is unchanged by lensing, magnification of the source provides amplification of the image flux. Colloquially named “Einstein’s Telescopes” [140], strong-lens systems enable studies of details of distant galaxies that would otherwise be unresolved or too faint. For example, faint lensed sources with relatively large redshifts enable studies of the initial mass-to-light ratio, star formation and metallicity in young galaxies [e.g. 26, 210]. Studies of the lens systems, whether galaxies, galaxy groups, or galaxy clusters, provide information on their total mass and mass profiles, including baryon and dark matter [e.g. 200, 403, 379, 258], and explore the connection between these characteristics and the occurrence of strong lensing [316, 135, 356]. Strong lens systems are now being used to study galaxy and cluster substructures and to, in turn, constrain the interaction physics of dark matter [239, 146].

Strong gravitational lensing enables studies of cosmology. Studies of characteristics of ensembles of strong lensing systems can be used to constrain Ω_M in

wCDM and Λ CDM [208], albeit loosely, with samples of $O(100)$ confirmed lensing systems. Studies of gravitationally lensed time-varying systems [305, 44, 42, 272], such as lensed quasars [333, 365] or supernovae [157], are used to constrain the expansion history between source, lens, and observer.¹ Individual measurement of strongly-lens quad quasars systems provide [54, 342] constraints on H_0 with about 4% precision, and combinations [366, 406] are now in mild tension, or not [40], with early Universe probes, depending [40] on assumptions about the lens' radial mass profiles. Lens systems with multiple sources at differing redshifts can provide [224, 141, 88] complementary information about the expansion history, independent of the Hubble constant. Constraints on Ω_M and the dark energy equation of state, w , are at about the 20% level from individual lensing systems [183, 91]. In these systems it can be critical to accurately model massive lens substructures to obtain unbiased cosmological constraints [94].

The count of candidate SL systems has increased rapidly with the advent of deep wide-field surveys with dedicated SL searches. In the CFHTLS Strong Lensing Legacy Survey [64], 54 systems with promising lenses were identified by the ARCFINDER algorithm [246] in 150 sq. deg., 49 confirmed strong lens systems identified with RINGFINDER [142], and 29 promising and 59 total [247] from a crowdsourcing [231] effort. CFHTLS imaging yielded 16 more probable to definite SL systems using neural network-based searches [176]. The Survey of Gravitationally-lensed Objects in HSC Imaging (SuGoHI, [357]) used three different methods, including YATTALENS, to search the Hyper Suprime-Cam Subaru Strategic Program (HSC SSP) images. The program yielded 333 galaxy-galaxy SL candidates from an area of 442 deg². The Kilo Degree Survey images were searched using machine-learning (ML) classifiers that discovered 227 high grade SL candi-

¹Note that our SL searches could identify lensed quasars, but it is unlikely that we would have identified short duration transients, such as SN, in our wide-field images.

dates, plus an additional 200 candidates discovered serendipitously [286, 287, 215]. [68] discovered 330 new SL systems in the 30,000 sq. deg. Pan-Starrs 3π Survey [79]. The 1.64 deg² Hubble Space Telescope (HST) COSMOS survey field yielded 67 galaxy-galaxy lens candidates [128]. HST was also used to confirm a total of 110 SL systems identified [51, 353] in the Sloan Digital Sky Survey (SDSS) spectroscopic data. SDSS also yielded 19 confirmed systems to the Sloan Bright Arcs Survey [15, 111, 205, 203, 222], and more than 30 confirmed and 50 additional candidate lenses to the CASSOWARY survey [28, 288, 361]. Searches of galaxy clusters identified in SDSS yielded 16 strong lens systems with $> 10''$ radius and 21 additional SL candidates [163], 68 giant arcs in [399], ten SL systems in [138], and 37 more in [350]. More recently, a search of galaxy clusters in HSC imaging [179] revealed more than 600 candidate strong lens systems, of which 8 are confirmed spectroscopically. The eBOSS spectroscopic data produced [369] 838 likely galaxy-galaxy SL systems with Einstein radius $\lesssim 1''$. The DESI Legacy Imaging Surveys team used 14,000 sq. degs. of images from several instruments, including the Dark Energy Camera [131], in combination with a neural network trained on real SL systems, to identify 1,545 candidate systems [171, 172].

Galaxy clusters, among the most massive objects in the universe, are ideal candidates for strong lenses, and optical observations of galaxy cluster catalogs have yielded many SL systems. Galaxy cluster catalogs selected via optical observations, the Sunyaev-Zel'dovich effect [362], or as extended high-flux x-ray sources have been used. Follow-up of South Pole Telescope (SPT) [306] detections yielded 34 new and 10 previously known SL systems in [46], and 44 new SL systems in [47]. Follow-up of Atacama Cosmology Telescope galaxy cluster identifications [165] yielded 67 new candidates including many in the DES footprint that are characterized here for the first time.

The DES data have proven to be a productive source of SL candidate systems. Searches for lensed quasars have yielded four spectroscopically confirmed systems [10, 277, 221], two new quad SL systems and 96 more quasar lens candidates ripe for spectroscopic follow-up [11]. Spectroscopic observations by the STRIDES collaboration confirmed 10 new lensed quasars and 10 quasar pairs [211]. Visual scanning of 400,000 color cutout images at coordinates selected, using techniques similar to those of this paper, from the ~ 250 sq. deg. Science Verification (SV) and $\sim 1,800$ sq. deg. Y1 catalogs yielded 348 new SL candidates in [109], as well as one serendipitous SL discovery [35]. The convolutional neural network-based search technique described in [176] contributed [178, 177] 485 ranked candidates in the full $\sim 5,000$ sq. deg. DES footprint, principally of the galaxy-galaxy lens configuration. The latter paper also included 26 additional SL candidates identified by visual scanning of $\sim 54,000$ potential lensing targets identified using procedures similar to those of [109]. Spectroscopic confirmation of 17 DES systems is described in [265, 221, 92, 266].

The recent competitive success of automated and machine learning methods for identifying strong lens candidates is evident from a simple comparison of the numbers of candidates in these references. Neural network and ML algorithm searches for SL systems have advanced because of increased attention to the technique’s power and capabilities. Advances in computer vision have given rise to automated searches for strong lens candidates, and for classification, deblending, and modelling of strong lens candidates [13, 246, 203, 138, 181, 281, 213, 408, 52, 207, 243, 291]. Nonetheless, these ML searches continue to benefit from visual scanning and ranking of candidate systems. Furthermore, continued efforts to attain large samples of real lenses with a variety of morphologies is important for training and validation of new automated lens-finding algorithms, particularly

for identification of group and cluster-scale SL systems, where ML has been less successful.

In this paper we report 247 strong gravitational lens candidates from searches of the full 5,000 sq. deg. DES footprint, including 81 previously unreported candidates. These lenses constitute a diverse set of primarily galaxy group- and cluster-scale systems, with both red and blue sources. We provide details of the locations (RA & DEC), and g,r,i,z,Y band magnitudes of source and lens objects, as well as Einstein radii and a rank, quantifying the confidence level of the selection, for each SL candidate system. This paper is organized as follows. In §2 we describe the Dark Energy Survey observations and catalogs. In §3 we describe our strong gravitational lens search procedures. In §4 we describe the results from the searches and provide the properties of the candidate lens systems. We highlight some of the systems that have notable properties. Finally, in §5 we summarize and discuss our results.

3.2 Dark Energy Survey Imaging Data

The Dark Energy Survey has completed a six season program that imaged 5,100 sq. deg. of the southern galactic cap using the Dark Energy Camera (DECam) [131], which is operated on the 4m Victor M. Blanco Telescope at Cerro Tololo Interamerican Observatory (CTIO) near La Serena, Chile. Following a “Science Verification” period [95], DES data collection typically occurred from mid-to-late August to mid-February, starting on August 31, 2013 and ending on January 9, 2019. Over the six year survey, we accumulated ten high-quality, 90-second duration observations of the full survey field in each of the four filters g , r , i , and z -bands, and six to ten high-quality observations of 45 or 90 seconds length for the Y -band (totalling 450 seconds). In addition to the wide-field survey, DES

performed a time-domain supernova survey during the same time period, visiting ten fields in the g , r , i , and z -band filters with an approximately weekly cadence and at much greater depth [190] than the wide-field survey. Details of the observation strategy, survey operations, and the survey progress for each of the seasons are available [254, 108, 110, 107]. Most of the searches described in this paper used as a starting point the object catalogs and images from the first three seasons of observations. Much of the information that we provide about the candidate strong lens systems comes from the full six-season data set. When we refer to “Y1” [114], “Y3” [1], and “Y6” [3] data sets, we mean to include all the preceding seasons of observations. For instance, Y3 refers to the data from the first three seasons.

The observations were processed by the Dark Energy Survey Data Management (DESDM) system [245, 248] in three pipelined stages: single epoch “detrending”, photometric calibration, and coaddition. The detrending operation removes the instrumental signature, subtracts the sky background, and removes artifacts such as cosmic rays from the individual exposures. The resulting images are in FITS-formatted files with an inverse-variance weight (WGT), and a mask of bad pixels (MSK). Single epoch catalogs were produced using PSFEX [34] and SEXTRACTOR [32]. Astrometric calibration is performed by matching bright stars on each exposure to reference stellar catalogs (2MASS [354] for Y3, GAIA DR2 [85] for Y6) using SCAMP [31]. Magnitude zeropoints for each CCD on each exposure are determined using the “Forward Global Calibration Module” [63]. Finally, exposures in each filter were coadded using SWARP [33] in 10,000 by 10,000 pixel “tiles” 0.72 degrees on a side. SEXTRACTOR was then run on a weighted combination of coadded $r + i + z$ “detection tiles” to form catalogs of objects, and then rerun on the individual filter bands at the location of the detected objects.

The separation or “deblending” of closely positioned (or even overlapping) objects is a challenge where the goal is to balance completeness against the spurious separation of features within a single galaxy. The deblending was performed using the detection images. For Y6, the SourceExtractor source detection threshold was lowered from 10σ (in Y3) to 5σ to detect fainter sources; this was accompanied by a reoptimization of the deblending parameters. As a result, objects are sometimes detected and/or deblended better in one data release than they are in the other. The absolute photometric calibration for both Y3 and Y6 is tied to the spectrophotometric Hubble CALSPEC standard star C26202 [49]. Other differences between the Y3 and Y6 processing are described in 3. All of the previous seasons’ observations were reprocessed to produce the new data sets. Each time, object catalogs containing the list of objects, their shapes, and their astrometric and photometric properties were calculated for each coadd tile. Unless noted otherwise, the SEXTRACTOR MAG_AUTO magnitudes are the primary measures of coadd flux used in further analysis.

Photometric redshifts (photo- z ’s) are derived by the DNF algorithm [98]. The lens photo- z ’s are reasonably well-estimated, given that our lens samples consist predominantly of red galaxies, which have strong 4000 Å break features that yield better photo- z measurements. However, we caution that our sources, which are typically fainter blue objects, will have photo- z ’s that are subject to larger uncertainties and systematic errors.

The DES Y1 and Y3/Y6 observing footprints are described in [95] and shown in Figure 3.1. Table 3.1 characterizes these data sets for comparative purposes. Additional information about the “Science Verification” (SVA1) data can be found in [109].

Table 3.1: Comparison between Y1, Y3, and Y6 catalogs. Magnitude limits are given for detections with $S/N \geq 10$.

Parameter	Y1	Y3	Y6
Sky Coverage, grizY intersection (deg ²)	1793	5186	4913
Objects (Galaxies/Stars)	137M Total	310M/80M	543M/145M
i-band Uniformity (typical # of exp.)	3 to 4	4 to 6	8 to 10
i-band Single-epoch Median PSF FWHM (")	0.97	0.88	0.88
Coadd Median Astrometric Relative Precision (ang. dist., mas)	25	30	27
i-band Coadd Mag. Lim. (MAG_AUTO)	22.5	22.5	23.1
i-band Coadd Mag. Lim. (MAG_APER_4, 1.95" diam.)	22.9	23.4	23.8

3.3 Gravitational Lens Candidate Search Procedures

We completed eight different searches, described below, to identify candidate strong lens systems using the Y3 and Y6 data. One of the searches was a by-product of the Y6 data quality effort. The seven other searches involved identifying and visually scanning a list of potential SL systems. While the origin of the lists was different for each search (those details are described below), the rest of the procedure is in common. The lists were loaded into the “DES Science Portal” [3], which includes “Target Viewer”, a tool for visualizing the DES fields that can also provide catalog information about the objects. The Target Viewer produced 5-color (g , r , i , z , and Y-band) cutout images, roughly $2'$ across, with one system centered on each page. Cutouts were scanned by either scientists with experience identifying strong lens systems or students trained to do so. Each page

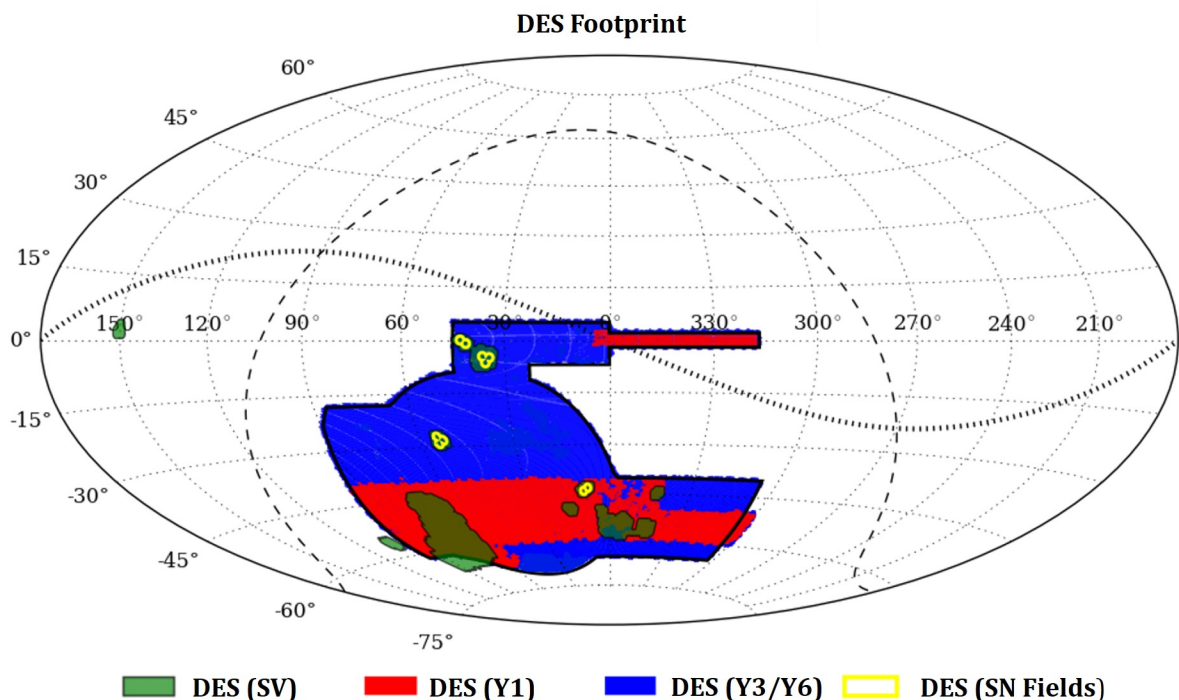


Figure 3.1: The footprint of the Dark Energy Survey. This result is based on searches of the DES Y3/Y6 footprint, outlined in black. Note that the DES Y3 footprint included additional observations of the Y1 and SV fields, and the the Y6 footprint included additional observations in all of those.

required only a few seconds to scan. Figure 3.2 shows a sample page as seen on the Target Viewer. Potential SL candidates were identified by the occurrence of an apparent arc, or a pattern of arc-like knots or objects suggestive of an instance of strong lensing. It was not required that the potential sources or lenses that we identified were part of the selection that caused the cutout to be made in the first place. Interesting candidates were flagged for further evaluation. Some bright or particularly interesting candidates were immediately designated for further study.

Each search discovered new SL candidate systems. Some new systems were found by more than one of our searches, as well as some that had previously been discovered. After each search was completed, a short list of systems identified

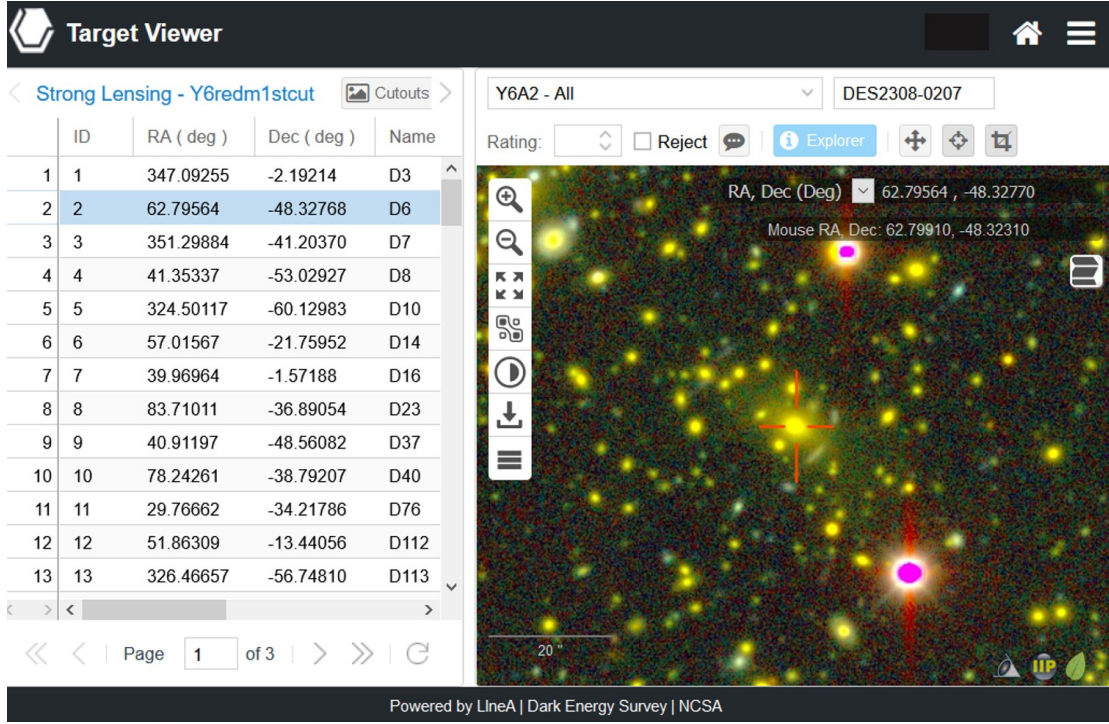


Figure 3.2: An example of an SL candidate shown in the NCSA Target Viewer. The left hand panel is a list of cutouts and characteristics, such as the RA and DEC. On the right side of the viewer is the optical image as well as some of the controls for manipulating the viewer. One cutout is displayed at a time, with an adjustable size of about $2'$ on a side. In addition, the Target Viewer can access DES object catalogs and encircle the identified objects (not shown). Each circle can display useful information about the object, including the OBJECTID, which provides reference to the object in the respective data release.

as candidates was compiled and all re-ranked by a team of five scientists. Each person assigned a score of 0 to 2 to each system - 0 points if the system was thought to not be a SL candidate, 1 point if it might be, and 2 points if the system was expected to be an instance of strong lensing. Scores from all judges were summed to produce a “rank” between 0 and 10. Systems with a rank of at least 3 were taken as the final list for this paper. The candidate rankings of 3 to 10 span the range from “possible” to “probable” to “definite” strong lens systems with rankings consistent with those used in the Master Lens Database [249] and other graded samples of strong lens candidates.

3.3.1 Blue Near Anything “Knot” Searches

We searched the DES Y3 catalogs for SL candidates using a “Blue Near Anything” (BNA) algorithm, originally motivated by [202] and used extensively by [109]. This algorithm is aimed at identifying star-forming Lyman break galaxies and Lyman- α emitting galaxies strongly lensed by massive luminous red galaxies (LRGS). There were two such searches.

The first was performed on the single coadd tile object catalogs and is illustrated in Figure 3.3. First, a list of candidate lens galaxies is created. The criteria for a galaxy to be in the list of possible lenses are that at least one of the r -band, i -band, or z -band magnitudes is less than 21.5. Selection criteria on SExtractor outputs removed galaxies that were faint, objects that were not well deblended, objects that are likely to be stars, and artifacts left over from objects with saturated pixels. In each $\frac{1}{2}$ sq. deg. tile, this search typically identified 5,000 to 7,000 candidate lens galaxies. Next, we formed a list of source candidates. The criteria for an object to be in the list of possible sources are that at least one of the magnitudes for the g -band, r -band, or i -band must also be less than 21.5 and that

the object was not poorly deblended or contained saturated pixels. We did not apply star-galaxy separation in order to preserve strongly lensed quasars which appear star-like [302]. Blue-colored source candidates were selected by requiring that $g - r < 1.0$ and $r - i < 1.0$. There were typically 2,000 to 3,000 candidate source objects per tile. Next, for each object in the lens list we identified the objects in the source list that were within $8''$ of the lens. Then, we identified the largest set of those sources, associated with a given lens candidate object, that each had a similar color, where the “similar” requirement was that $|\Delta(g - r)|$ and $|\Delta(r - i)|$ both be less than 0.25 magnitudes. This algorithm predominantly finds blue-colored source galaxies lensed by red galaxies or galaxy clusters, and we refer to it as the BNA 2+ selection. After we identified the list of candidates on each tile, we removed those candidates that were within $10''$ of a position that we had previously scanned in the Y1 BNA search [109]. There were 247,076 systems that had 2 or more matched source objects in the 10345 tile catalogs. We performed a visual scan of the 30,319 systems that had 3 or more matched source objects. This yielded a short list of 95 SL candidates.

The second BNA search was similar, but it was aimed at identifying those bright SL candidates that we might have missed by scanning only those systems with three or more blue knots. For this “Bright Blue Near Anything” (BBNA) selection, we raised the lens galaxy magnitude threshold to 20.5 and the source galaxy threshold to 20.0 and reran the algorithm. We performed a visual scan of the 18,985 resulting systems with two or more matched source objects. This yielded a short list of 8 additional BBNA SL candidates, all of which received a rank of 3 or more.

3.3.2 Red Near Anything 3+ “Knot” Search

The motivation for this search, refereed to as “Red Near Anything” (RNA), was to discover SL systems with red-colored sources. It was similar to the RNA search used extensively for Y1 in [109], and similar to the BNA search described above except as noted. The lens candidate selection criteria required that any of $r, i, z < 22.0$. In both iterations, there were two selection criteria for the source lists. The first was that any of $r, i, z < 22.0$, that $g > 23$, and that $g - r > 0$ and $r - i > 0$. The second was that any of $r, i, z < 22.0$, that $g > 23$ and $r > 23$, and that $r - i > 0$ and $i - z > 0$. Again, a list of source candidates was matched against a list of lens candidates with an $8''$ maximum radius. Next, for each system we found the largest set of matching sources for which $|\Delta(g - r)| < 0.25$ and $|\Delta(r - i)| < 0.25$. After eliminating candidates within $10''$ of a location scanned in [109], there were 439,077 candidates. We visually scanned 30,994 targets with 3 or more source “knots”, short-listing 110 for further study.

We carried out the second iteration of this campaign on the Y3 data after we realized that the color-matching selection criterion $|\Delta(g - r)| < 0.25$ would eliminate source objects that were g and r -band dropouts. We reran the algorithm, this time requiring that $|\Delta(r - i)| < 0.25$ and $|\Delta(i - z)| < 0.25$. A total of 501,184 candidate systems remained after removing locations we had checked in [109]. There were 20,203 targets with 3 or more source knots. We visually scanned 5,100 of them, finding four systems for the short list. Due to its low productivity, this search (RNA2) was abandoned before we scanned all the targets. All four of these had a grade of 3 or more.

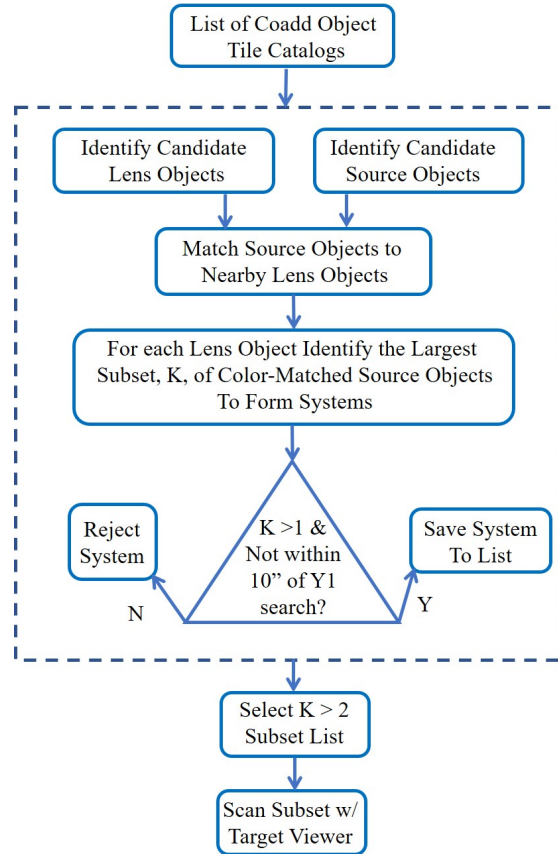


Figure 3.3: Typical flowchart for the “Blue Near Anything” and “Red Near Anything” search algorithms. The box with the dotted outline is a single computer program. At the last step within the program, ‘Y’ and ‘N’ stand for ‘Yes’ and ‘No’. The “ $K > 2$ ” subset selection was done on 3 of 4 of these searches.

3.3.3 An Undirected Search of DES Y6 Tiles

As part of the Y6A1 DES data release, we performed a data quality check comprising a visual scan of a subset of the 10,167 $1/2$ -sq. deg. coadd tiles. The scanning took place in two stages. Initially 2,377 tiles were scanned in RA-order starting with RA's close to 0 hrs. Later, we repeated the process on the 1,708 tiles that had not had any targets in the BNA or RNA SL searches. We were looking for obvious quality defects such as the inclusion of problematic single exposures into the coadds. We identified coadds that included exposures that had been taken during an earthquake, some that had a trail from the reflections off the International Space Station, and others that had subtle astrometric failures that resulted in a “doubling-up” of the objects in one filter band. All of these bad exposures were removed from the data set, and coadds were remade without them. This visual scan procedure was as follows. First, we displayed the full coadd tile at a scale that permitted us to notice macroscopic problems, but not with a resolution to notice SL systems. Then we “zoomed-in” at a dozen or more different places on the coadd tile, looking for problems that could only be seen with finer resolution. See Figure 3.4. In this process we viewed only a fraction of the area of the tile, perhaps one-tenth, with sufficient resolution to spot a candidate SL system. We recorded 44 candidates for the short list. Seventeen of these were graded as rank 3 or more.

3.3.4 Scans of Galaxy Cluster Catalogs

Galaxy clusters are among the most massive structures in the Universe, and form high-magnification regions where SL systems are likely to appear. We searched for SL systems in three catalogs of galaxy clusters: the optically-selected DES Y3 redMaPPer catalog, and two cluster catalogs selected via the Sunyaev-

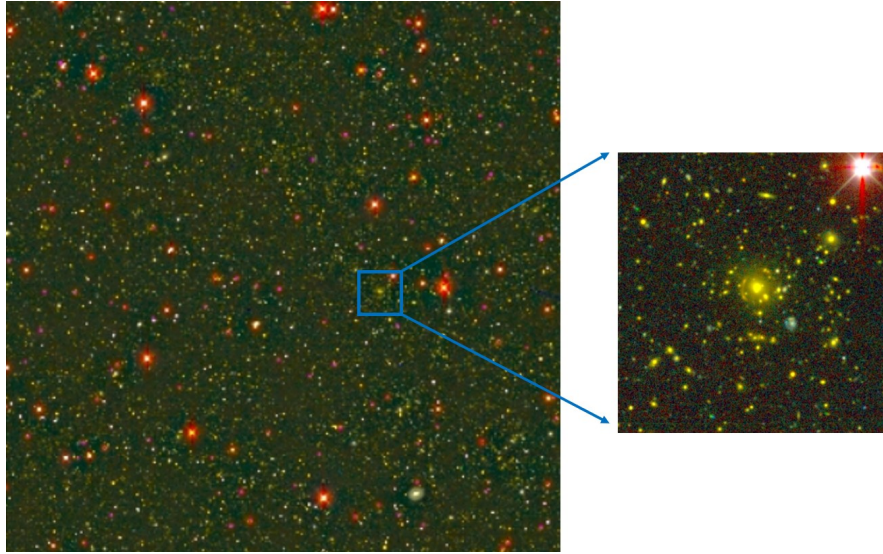


Figure 3.4: In the visual data quality test, we first displayed the full tile of DES data, 0.72 deg on each side (left). Then we zoomed-in a factor of 50 to 100 (in area) at roughly a dozen places on each tile, looking for problems noticeable in a cutout roughly 5' on a side. If we noticed a potential SL candidate we recorded its position. The right-side is about 1/4 of our typical zoomed-in area, but at the scale that we viewed it.

Zel’dovich effect [362]: the Atacama Cosmology Telescope (ACT) cluster catalog [165], and the SPTPol-Extended Cluster Survey catalog [47]. The overlap of all cluster catalog footprints is shown in Fig 3.5.² A Venn diagram of the how many SL systems were discovered in each cluster catalog can be found in Figure 3.6. Each catalog and their resulting SL systems are described in this section.

The red sequence Matched-filter Probabilistic Percolation cluster finder algorithm (redMaPPer) [325, 233, 2] identifies galaxy clusters as overdensities of red-sequence galaxies in a catalog. The algorithm counts the excess number of red-sequence galaxies of similar colors that are brighter than a specified luminosity threshold within a circle of radius $R_\lambda = 1.0 \text{ h}^{-1} \text{ Mpc} (\lambda/100)^{0.2}$, where λ , the “richness”, is the number of galaxies in the cluster [321]. The galaxy cluster catalog used in this work was obtained from redMaPPer version 6.4.22+2 on DES Y3A2 Gold data [341]. It is a volume-limited catalog with redshifts of the clusters ranging from $0.04 < z < 0.95$ and richness $\lambda > 5$. We visually scanned the 21,092 cluster sample with $\lambda > 20$ in this search. Though we have previously reported searches of the locations of $\lambda \geq 20$ redMaPPer clusters in the DES SV and Y1A1 fields [265, 109], respectively, we didn’t exclude those positions from this search. Of the 152 systems shortlisted for final grading, 143 were ranked ≥ 3 .

The ACT cluster catalog identified more than 4,000 galaxy clusters using the Sunyaev-Zel’dovich effect [165]. The full ACT data set [252] comprises 18,000 sq. degs. and overlaps the DES Y3 footprint almost completely. We visually scanned the 1857 ACT galaxy clusters within the DES footprint, identifying 93 for further evaluation and final grading. Of these, 77 were ranked ≥ 3 . Some of these were already identified in [165].

The SPTPol Extended Cluster Survey [47] (SPTPol-ECS) identified 266 galaxy

²The dust map was obtained from Planck at http://irsa.ipac.caltech.edu/data/Planck/release_2/all-sky-maps/maps/component-maps/foregrounds/COM_CompMap_DustPol-commander_1024_R2.00.fits

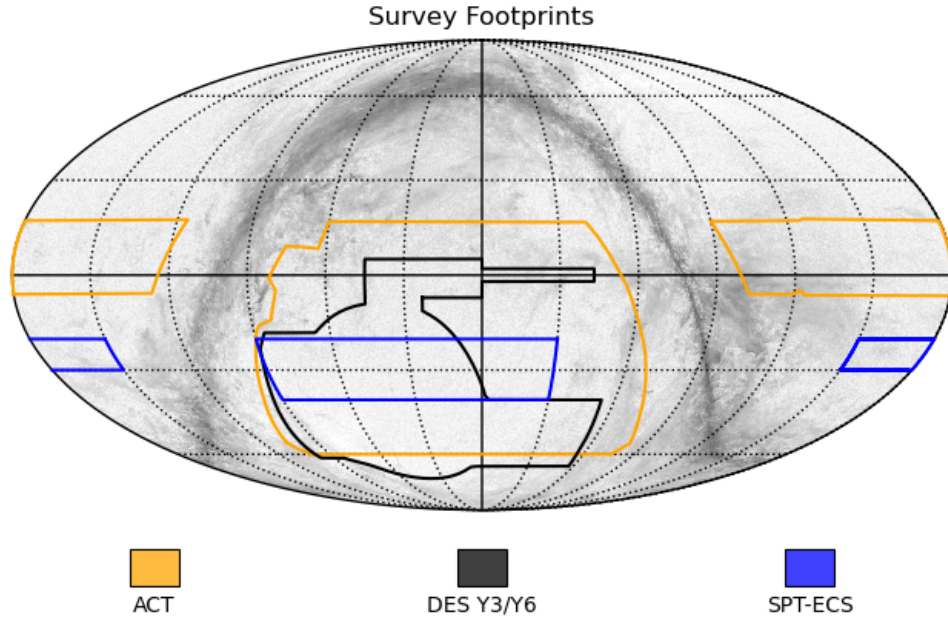


Figure 3.5: The footprints of the Dark Energy Survey and other cluster catalogs referenced in this work. The DES redMaPPer catalogs span the DES footprint, outlined in black in the center. Searches were also conducted based on galaxy cluster catalogs from the SPTPol-ECS and ACT, whose footprints are shown in blue and orange, respectively. Background dust map obtained from Planck Data Release 2. The overlap between the DES and ACT cluster footprints is approximately 4770 sq. degs., and the overlap between the SPT and DES footprints is approximately 1510 sq. degs. The intersection of all three footprints is approximately 1470 sq. degs.

clusters with detection significance $\chi > 5$, and 204 with $4 < \chi < 5$. The full DES footprint partially covered that of SPTPol-ECS. From the 470 total SPTPol-ECS galaxy cluster candidates, we scanned 325 locations in the DES Y6 data. Of these, 17 SL candidate systems were identified for final grading, and all 17 were ranked ≥ 3 .

3.4 Search Results

The ranked lists from all searches were combined, yielding 247 SL systems of rank 3 or greater. Of these systems, 81 are presented for the first time. Figure 3.6 shows Venn diagrams illustrating the overlap between each search algorithm or catalog, for all 247 SL candidates and separately for the newly discovered systems. Table 3.2 shows the number of candidates searched and systems found for each sub-search. Figure 3.7 shows the distribution of the system ranks. Figures 3.8 and 3.9 show the lens and source photometric redshifts, and Einstein radii for the 247 SL systems. For strong lensing to occur, the source must be strictly more distant than the lens, yet in Figure 3.8 it is clear that some sources have a lower measured redshift than their respective lenses. For some systems, this discrepancy is likely due to catastrophic redshift errors in blue source objects, as mentioned in Section 3.2.

Table ?? provides an overview of each candidate lens system, including: the system name, algorithms or catalogs which identified the candidates for scanning, system rank, Einstein radius estimate (described below), and references to previous discoveries of the system. Figures 10 through 17 of the published version of this work [267] show a 3-color cutout of each system. The cutouts are made from the DES Y6A1 coadded tiles using the i, r, and g-bands for the RGB color channels respectively. Where SL systems fell across tile edges, multiple coadd tiles

Table 3.2: Summary of the number of objects visually scanned, the number ranked, and the count of those with rank ≥ 3 for the various searches. We kept track of overlaps between searches for the columns “# Rank ≥ 3 ” and “# New”, but not for the columns “# Scanned” and “# Ranked”. Where there are empty fields, we have not kept track of the distinct counts. Note that a single “New” system can be discovered by multiple searches. Therefore the subtotals and “Totals” can be less than the sums of the respective columns.

Search	# Scanned	# Ranked	# Rank ≥ 3	# New
BNA 3+	30319	95	21	6
BBNA 2+	18985	8	7	3
BNA Combined		103	28	9
RNA 3+ No. 1	30994	110	91	25
RNA 3+ No. 2	5100	4	3	1
RNA Combined		114	93	25
Tile Inspection	4085	44	17	12
RedMapper	21092	152	142	35
ACT	1857	93	85	21
SPTPOL	325	17	16	2
Clusters Combined		202	189	50
Total			247	81

were stitched together to form one cutout. Each cutout was dynamically scaled to fit the system well within the cutout based on the radial distance from the lens center to the furthest lensed source image.

Each image is centered on the most likely lens galaxy, labeled by the letter “A”. For systems where the primary lens was uncertain, or where sources were clearly perturbed by multiple galaxies, additional lens galaxies are marked with roman letters “B”, “C”, etc. All lenses are found in the DESDM Y3 or Y6 catalogs. Source objects which were identified in DESDM are labeled by number, e.g. “1”, “2”, etc. Source objects which were not identified in DESDM had their positions determined manually, and are shown by an arrow.

To identify positions and photometry for lens and source objects, the DES

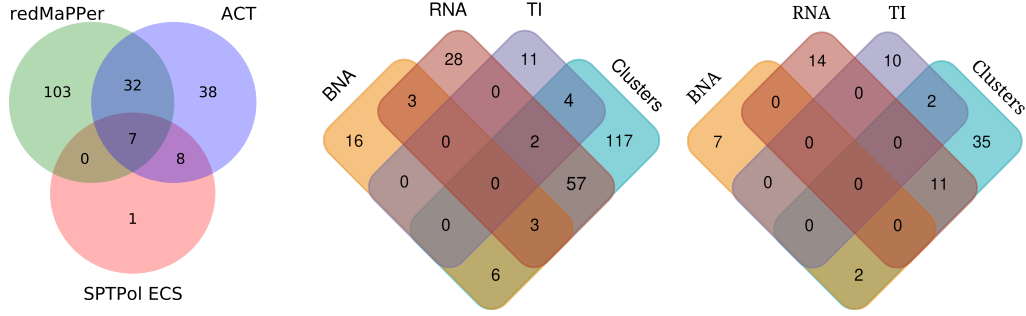


Figure 3.6: The systems presented in this work, sorted by the algorithms which identified them. **Left:** a breakdown of the galaxy cluster subset, showing how many systems were identified in redMaPPer, ACT, and SPTPol ECS searches. **Center:** all 247 systems presented in this work. RNA (BNA) includes both Red (Blue)-Near-Anything searches, TI are systems found serendipitously in tile inspections, and Clusters includes redMaPPer, ACT, and SPTPol ECS catalogs. **Right:** all 81 newly identified systems presented in this work.

Y3 and Y6 catalogs were overlaid on images of candidates in the NCSA target viewer. Lens and source images were matched to catalog object IDs. Y6 object identifications were preferred over Y3; however, in cases where an object was not detected in Y6, or was blended with another object, the object was matched to the Y3 catalog. Within each system, catalogs were kept consistent, such that no system mixes Y3 data for some objects with Y6 data for others. Some source images could not be accurately associated with a known DES object, and their position was estimated manually. Table 4 of the published version of this work [267] shows the positions and photometry for all systems presented in this paper.

We identified those systems presented here that were previously known, as well as those that are new. To do so, we checked the candidates against the Master Lens Database [249], the list of objects labeled as strong lens candidates or known systems in SIMBAD [400], and other SL search papers. Previous discoveries of all systems are given in Table 3 of the published version of this work [267]. Note that references include both confirmed and candidate strong lens systems, i.e. systems

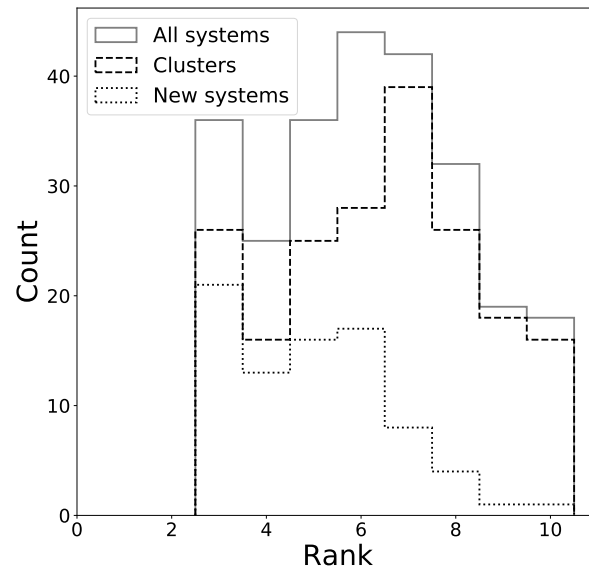


Figure 3.7: The rank of all systems presented in this work. The solid black line shows all systems, the dashed line shows the galaxy clusters subset, and the dotted line shows newly discovered systems. The mean (median) rank of all systems presented is 6.12 (6).

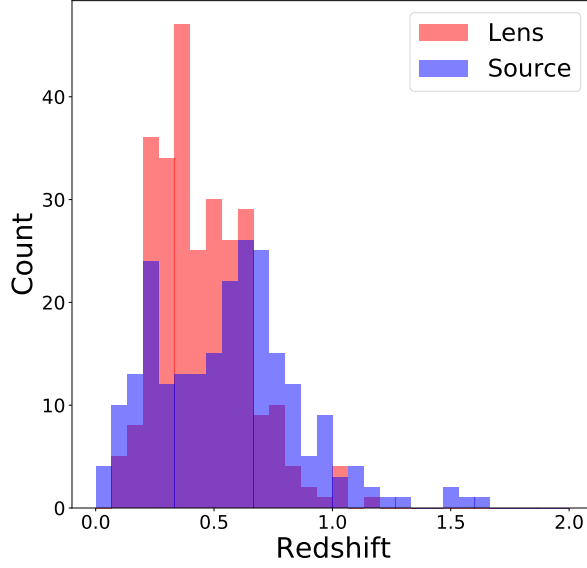


Figure 3.8: DESDM-calculated photometric redshifts for the sources and lenses. As explained in §2, the redshifts of the sources are subject to larger uncertainties and systematic errors.

with external references are not guaranteed to be true lenses. We include these previously discovered systems in this work because we provide positions, photometry, and photometric redshifts of system components (see Table 4 of [267]), as well as an estimate of the Einstein radius (see Table 3 of [267]), forming a standardized dataset which may be of use in further studies.

The average radius of source images is an approximation for the Einstein radius, and is identical to that when the true source position is directly behind the lens. We estimate the Einstein radius from the mean separation between the primary lens and each source image. For systems with multiple source images, the uncertainty is taken as the standard deviation of these separations. For systems with only one source image, the uncertainty is taken as 10% of the source-lens separation, typical for the scatter of individual sources in systems with multiple

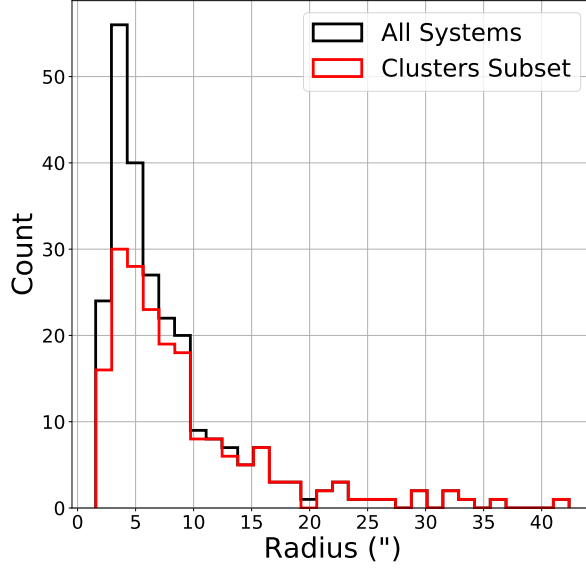


Figure 3.9: The binned distribution of Einstein radius estimates for the lens candidates. The clusters subset, as defined in Figure 3.6, is shown separately, and accounts for nearly all systems with an Einstein radius above $10''$.

images. Especially for these systems with a single lensed image, this tends to provide an overestimate of the Einstein radius. [196]. In all cases, the uncertainty is summed in quadrature with the DES plate scale of $0.263''$ per pixel. The image separation distribution is sensitive to a number of inputs such as the halo mass, the lens mass distribution, and the source redshift. It therefore contains information about the cosmological parameters and various scaling relations between galaxy properties and halo mass, and can be measured from galaxy to cluster scales [270, 247]. A similar metric has been found to be an effective measure for the inner halo mass by Remolina González et al. [307].

3.4.1 Notable Systems

Here we highlight some notable systems. The newly identified system DESJ2138-6007 is particularly spectacular. It contains a long blue arc with radius nearly $30''$ and several possible counterimages. The system is centered on a redMaPPer Y3 galaxy cluster of richness 152 ± 5 . Four SL systems appear to contain two sources, one red and one blue. In each case the system is listed twice in Tables 3 and 4 of [267], with suffixes A and B for the two distinct sources. These systems are DESJ0151-3544, DESJ0342-5355, DESJ0610-5559, and DESJ0611-5514, which have Y3 redMaPPer richnesses of 73 ± 3 , 109 ± 4 , 48 ± 4 , and 44 ± 4 , respectively. System DESJ0151-3544 was previously discovered in SPT follow up [46], and systems DESJ0342-5355 and DESJ0610-5559 were identified and noted as double source SL systems in [109]. The final one, DESJ0611-5514, is new.

System DESJ0043-2037 is a large, unrelaxed, potential merging galaxy cluster which appears to have two red arcs near its center. In this work, it is presented as two systems with suffixes A and B, centered on two separate luminous galaxies. It is a complex system which merits further study.

Higher resolution imaging of other systems presented here might expose additional remarkable features, such as central images, radial arcs, or substructure lensing in some of these systems. These features would provide significant information about the matter distribution in the lens systems.

3.5 Summary and Conclusion

We report the results of eight searches for strong gravitational lens candidates in the full 5,000 sq. deg. footprint of the DES imaging data. We searched the DES catalogs for spatial matches of potential lens and source candidates using

the “Blue Near Anything” and “Red Near Anything” algorithms. We searched the positions of known ACT, SPTPol, and DES redMaPPer galaxy clusters. The scanning of wide-area tiles for the DES data quality effort led to serendipitous discovery of several SL systems. For all of the searches we produced a short list of candidates and then evaluated cutouts to identify the most promising systems based on color and morphology. We then assigned those a rank that quantifies our confidence that the system is a potential strong gravitational lens. Of the 247 systems that we found, 81 are presented for the first time. Of the newly discovered systems, at least one was uniquely found by each of our searches. For each system we provide the position, the magnitudes, and photometric properties of the lens and source objects, and the distance (radius) of the source(s) from the lens center. Some of these are striking systems with giant arcs. Some have red-colored sources. Four have both blue and red candidate sources at differing distances from the candidate lens.

Each category of search (RNA, BNA, galaxy cluster, and tile inspection) provided unique newly-discovered systems, as shown in Figure 3.6. These varied search strategies serve as effective complements to each other, and each contributed new systems to the final catalog.

These SL systems tend to be galaxy group and cluster scale candidates, particularly those from the searches of galaxy clusters described in Section 3.3.4. Therefore, we expect it will be a useful and valuable training and validation set for future automated searches of galaxy clusters in new fields and at lower richness range. At the same time, this catalog also highlights the need for, and importance of, crowdsourced or automated lens modeling techniques [41, 206] being developed.³

While a majority of the systems discovered in these searches were previously

³<https://github.com/DES-SL/EasyLens> and <http://linan7788626.github.io/pages/Hoopla/index.html>

identified (166 of 247), it is clear that many undiscovered systems exist in current data sets. Predictions for the number of lenses in a survey depend on depth and area, but estimates for the full Dark Energy Survey range around a few thousand [274]. In the near future, they are expected to expand to more than a hundred thousand in LSST data [90]. Fortunately, there will be no shortage of high quality strong lens systems for follow up and detailed study.

Chapter 4

Dark Matter: SIDM Constraints in Lensing Clusters

This chapter presents constraints on the SIDM cross section at cluster scales from the galaxy cluster MACSJ0138, host to the strongly lensed supernovae Requiem and Encore. This chapter includes a reprint of the following published material: O'Donnell et al. [268]. Our analysis combines strong gravitational lensing with spatially resolved stellar kinematics of the central galaxy, employing several methodological advances over previous cluster-scale SIDM studies. The result is a self-consistent measurement of the density profile of MACS J0138-2155 across two orders of magnitude in radius. Our lensing and kinematics analyses individually yield highly consistent results, and from their combination we report a 95% confidence upper limit on the SIDM cross section of $\sigma/m < 0.613 \text{ cm}^2/\text{g}$, at an interaction velocity of $\langle v_{\text{pair}} \rangle < 2090 \text{ km/s}$. This constraint, derived from the most detailed single-system analysis to date, is competitive with previous cluster-scale limits while demonstrating the power of combining complementary gravitational probes. The methodology developed here advances precision cluster lens modeling and will inform future studies of dark matter physics, as well as

time-delay cosmography in this unique strong lensing system. Additionally, our results imply SN Requiem will reappear sooner than previously reported, with a 1σ CL between January 2027 and November 2028 at $H_0 = 67.7 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

4.1 Introduction

The nature of dark matter remains one of the most significant outstanding problems in physics. An unambiguous, non-gravitational signature of dark matter has yet to be found in direct or indirect dark matter searches, nor have signs of beyond the Standard Model theories, like supersymmetry, containing dark matter particle candidates been seen. However, the growth and properties of cosmic structure place strong constraints on the nature of dark matter over a large range of scales. The model of cold dark matter (CDM), in which the dark matter particles are collisionless, has been highly successful at modeling large-scale structure and its evolution, but faces potential tensions at smaller scales like dwarf galaxies. This and a lack of detection of CDM particle candidates motivate thinking more broadly about the properties of dark matter.

Fortunately, non-CDM models for dark matter predict potentially measurable differences in the properties of dark matter structures, including their densities, shapes, and substructure abundances. In particular, several models including self-interacting dark matter (SIDM), warm dark matter (WDM), and ultralight or “fuzzy” dark matter predict lower central densities in dark matter halos compared to CDM. All these alternatives to CDM reproduce its success at explaining large scale structures. WDM suppresses the linear matter power spectrum at small scales, accordingly limiting the growth of small scale dark matter halos. Axion-like or “fuzzy” dark matter can form soliton cores that in principle can affect the inner slope of the dark matter density profile, but on cluster scales the soliton core

would be negligible compared to the size of the central galaxy and the main dark matter halo. On the other hand, SIDM can drive core-evolution and substructure on larger scales, which could impact lensing observables. Therefore, we limit our analysis to the case of SIDM.

In this paper, we focus on using the combination of strong gravitational lensing (SL) and spatially resolved stellar kinematics to constrain the dark matter density to small radii in the galaxy cluster MACS J0138.0-2155, and we motivate these choices below. In addition, the modeling presented here represents several improvements to the methodology and consistency of strong lens cluster modeling.

4.1.1 Self-Interacting Dark Matter

Self-Interacting Dark Matter has received intense interest in recent years, becoming one of the most well-studied alternatives to CDM. SIDM entails a simple phenomenological change to collisionless DM, in which DM particles are allowed to interact through a new force with a (possibly velocity-dependent) elastic scattering cross section. Such self-interactions are well-motivated theoretically, and arise naturally from many particle models of DM [381, 9]. A common example is a massive DM particle coupled to a light mediator through a Yukawa interaction [381, 129, 191].¹

SIDM can produce a range of different effects at different mass scales. In particular, scattering and associated heat transfer leads to the reduction of “cuspy” DM density profiles into flattened “cored” DM density profiles [97, 275]. On longer timescales, the gravothermal evolution caused by SIDM potentially leads to runaway gravothermal collapse. This evolution from core formation to core collapse naturally explains the diversity seen in dwarf galaxies; SIDM-induced

¹We stress that our results do not assume a particular form of this particle interaction.

core collapse could potentially even lead to the formation of supermassive black hole seeds in the early universe [278, 292, 156, 81, 130]. At intermediate mass scales, the effects of SIDM on halo morphology are expected to be “washed out” by baryonic effects [301, 232]. Accordingly, most constraints on SIDM come from the extreme ends of the halo mass scale: dwarf galaxies, low-surface brightness (LSB) galaxies, and galaxy clusters [9]. Observable signals could also exist at smaller scales, including detections of core collapsed low-mass haloes with gravitational imaging [218, 234, 387].

Currently, the most stringent absolute constraints on the SIDM cross section come from strong lensing of galaxy clusters, which suggest $\sigma/m \lesssim 0.1 \text{ cm}^2/\text{g}$ [326, 18]. Constraints on dwarf and LSB mass scales from fitting rotation curves suggest a higher value, around $O(1 - 10) \text{ cm}^2/\text{g}$ [308]. However, Roberts et al. [314] suggests that σ/m could be as high as $20 - 40 \text{ cm}^2/\text{g}$ at this mass scale, when SIDM-induced core collapse is accounted for. These higher cross-sections alleviate tensions associated with the diversity of dwarf rotation curves by predicting that these low-mass galaxies are in different phases of their gravothermal evolution; most will be mildly in the core-collapsed phase, while others may only just be entering this phase, and some may be more deeply core collapsed.

The discrepancy in the size of the cross-section implied by cluster and dwarf galaxy observations suggests that the SIDM cross-section must necessarily be a velocity-dependent interaction if we wish to explain dwarf rotation curves through this mechanism. It is clear that clusters can provide stringent constraints on the SIDM interaction strength at high mass scales. Therefore, to further probe the SIDM cross-section at these scales, in this paper we focus on strong lensing in galaxy clusters.

4.1.2 Cluster SIDM constraints

We aim to improve on previous SIDM constraints at cluster scales by constructing the most detailed model to date of an individual system. This work constrains the dark matter density across a wide range of radii by combining stellar kinematics of the central galaxy at small radii and observed SL features at intermediate radii. In this section we summarize previous related studies, and contrast the approach taken in this paper.

Early work along these lines aimed to differentiate between the luminous and dark matter distributions in clusters, independently using SL and stellar kinematics measured along a 1D slit [257, 256], and found the DM profile alone to be shallower than expected in a CDM cosmology. Recently, Cerny et al. [77] took a similar approach, combining 2D IFU kinematics with SL to model six clusters, and likewise found a DM density profile shallower than a Navarro-Frenk-White (NFW) profile. However, these studies did not translate their measurements to SIDM constraints or any other alternate cosmological model.

More recent studies explicitly constrained SIDM using similar approaches, albeit with less detailed models of larger samples. Sagunski et al. [326] studied a sample of 8 galaxy groups and 7 galaxy clusters, constraining their density profile by combining stellar kinematics and strong gravitational lensing. Their results show a clear preference for non-zero cross-sections, with $\sigma/m = 0.19 \pm 0.09 \text{ cm}^2/\text{g}$ at cluster scales, and $\sigma/m = 0.5 \pm 0.2 \text{ cm}^2/\text{g}$ at group scales. For both kinematics and lensing, however, they limit their analysis to simplified approximations. For stellar kinematics, they use stellar velocity dispersions in bins along a 1D slit computed previously in Newman et al. [257], which yielded between 3 and 8 bins per cluster. In this work, we measure stellar velocity dispersions of a central cluster galaxy in 2D, using an IFU data cube to obtain 25 spatial bins, as described in

Section 4.3.3. Furthermore, their strong lens models are limited to reconstructing an average convergence in a certain aperture, derived from previous strong lens models of each system.

Andrade et al. [18] presented an SIDM constraint from 8 lensing clusters, constructing a detailed SL mass model of each system, and relating those results to the SIDM cross-section σ/m . However, they use *only* SL, without stellar kinematics to provide constraints on the innermost region. In addition, they choose a parametric cored-NFW density profile for the cluster DM halo, and relate their result to an SIDM cross-section after-the-fact. We improve on this latter choice by computing the SIDM halo density profile via the formalism described in Appendix 4.7.1, and directly predicting strong lensing observables from this profile at every step.

We note that other probes exist to constrain SIDM in clusters, which do not explicitly reconstruct the DM density profile in the innermost region. These include offsets between the galaxies and dark matter in merging clusters [404], offsets between the central galaxy and the center of mass [93, 160], the radial acceleration relation of members in the cluster outskirts [371], and potentially the ellipticity of dark matter haloes [318, 151]. Due to a combination of observational limitations and theoretical uncertainties, however, these constraints are not yet competitive with those from SL, such as Sagunski et al. [326] and Andrade et al. [18] described above.

4.1.3 Lensing Cluster MACS J0138.0-2155

In this work, we focus on the strong lensing galaxy cluster MACS J0138.0-2155 (hereafter MACSJ0138), whose primary source is a quiescent galaxy at $z=1.95$ known as MRG-0138 Newman et al. [259]. We leverage its deep, publicly-available

data to constrain the inner density profile of its dark matter halo. This lensing cluster is of particular interest to cosmology: one of the first known strongly lensed supernovae, known as “SN Requiem”, was discovered in archival HST imaging of this system [320]. More recently, JWST observations of this system discovered a *second* strongly lensed supernova, named “SN Encore” [289], making this the first known lens system to host multiple strongly-lensed SNe. As a unique, highly interesting system, MACSJ0138 has been the subject of extensive astronomical observations. We describe the data used in this analysis in Section 4.2.

Furthermore, MACSJ0138 is a very “clean” system, ideal for constraining its halo’s inner density profile. It is known to be a relaxed, non-merging cluster, whose primary lensed source is a bright, quiescent galaxy at redshift 1.95. This source appears in at least 5 separate images, surrounding the cluster’s center; furthermore, X-ray data show it to be a relaxed system, exhibiting close to spherical symmetry [132].

Here we briefly summarize previous studies on MACSJ0138. Multiple spectroscopic studies have been done on the dynamical state of the cluster and its member galaxies [132, 154], and the quiescent source hosting SNe Requiem and Encore [289, 105, 259, 260]. The strong lensing observed in MACSJ0138 has been modeled previously by Newman et al. [259], which reported spectroscopic redshifts of the lens and source, and Rodney et al. [320], which presented the discovery of SN Requiem. We confirm the redshifts presented in both papers using the IFU data described in Section 4.2.2. More recent mass models leveraging deeper data on this system were published in Ertl et al. [123] and Acebron et al. [7]. We compare the results presented in this work with the latter two papers in Section 4.5.1.

4.2 Data

The data used in this work includes space-telescope imaging and ground-based spectroscopy which are described below.

4.2.1 High-resolution Imaging

MACSJ0138 has received extensive high-resolution imaging from both HST and JWST. MACSJ0138 was first observed by HST in 2016 (Proposal ID 14496, PI A. Newman). These observations included three instrument/filter combinations: ACS/WFC band F555W, and WFC3/IR bands F105W and F160W. SN Requiem was later identified in these observations [320]. In 2019, MACSJ0138 was observed again by HST as part of the REQUIEM program (Proposal ID 15663, PI M. Akhshik). These observations utilized WFC3 exclusively, using the UVIS channel with filters F814W and F390W, and the IR channel with filters F110W, F125W, and F140W.

Following the discovery of SN Encore in November 2023, a JWST director’s discretionary time program (JWST-GO-6549, PI J. Pierel) was approved to monitor SN Encore. This program observed MACSJ0138 in three epochs between December 5 2023, and January 8 2024. During each epoch, MACSJ0138 was observed by NIRCcam using six filters: F115W, F150W, F200W, F277W, F356W, and F444W. We utilize the WFC3/IR imaging from Program 14496 to measure the observed positions of SN Requiem. To measure observed positions of other features in the lensed source, as well as SN Encore, we utilize JWST NIRCcam imaging in bands F115W, F150W, and F200W from Program JWST-GO-6549.

4.2.2 MUSE Spectroscopy

The Multi Unit Spectrographic Explorer (MUSE) is a second-generation integral field spectrograph on the VLT [24]. MACSJ0138 received three MUSE pointings of 970s each during Period 103, through Program 0103.A-0777(A). These observations used MUSE in wide-field mode, which has a field-of-view of approximately $1' \times 1'$, without adaptive optics. In this work, we utilize the reduced data cube of these observations obtained from the ESO data archive, processed by ESO pipeline version `muse/2.8` [398].

4.3 Modeling Methods

In this work, we self-consistently model and constrain the density profile of the dark matter halo of MACSJ0138, using both stellar kinematics and strong lensing, combining complementary constraints of the DM density profile at $\sim 1 - 20$ kpc, and ~ 100 kpc, respectively. Central to this work is providing a coherent model for the 3D density of the overall DM halo, modified by the effects of SIDM. This shared mass model used by both the strong lensing and stellar kinematics probes is described in Section 4.3.1. Next, Section 4.3.2 describes how we construct a strong lensing mass model, and fit it to observed strong lensing features, including handling of the contribution of cluster member galaxies, and how strong lensing observables are computed for the SIDM-modified halo. Finally, the measurement and modeling of the central galaxy’s kinematics are described in Section 4.3.3. In addition, a high-level overview of the steps in this work is provided by the diagram in Figure 4.1.

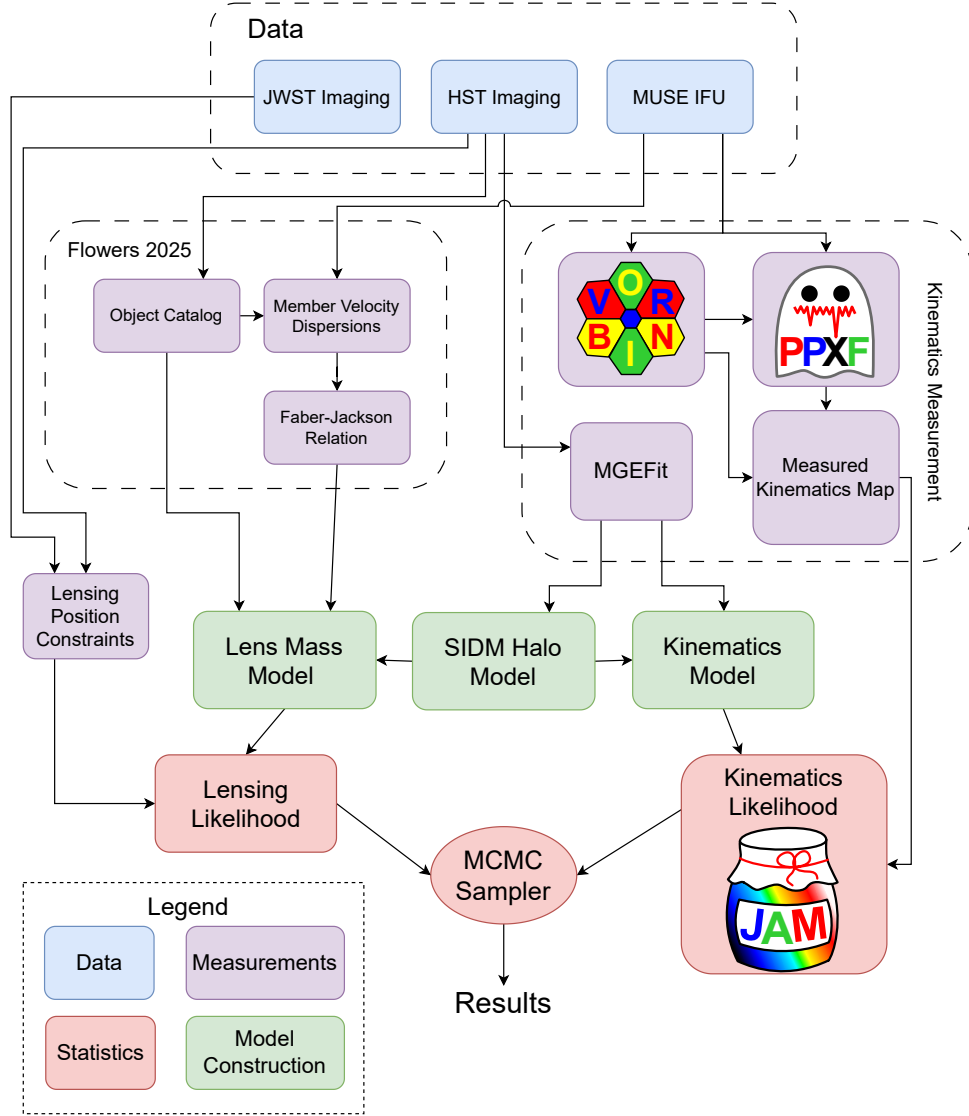


Figure 4.1: Analysis Outline: High-level overview of the analysis done in this work. Pieces of this analysis were presented in Flowers et al. [132]. Lens modeling is accomplished with `PyAutoLens` [262, 263]. Kinematics modeling in this work uses `vorbin` [75] to spatially segment the MUSE observations into regions, `ppxf` [76, 73] to model the spectrum of each region and extract stellar kinematics, `MGEfit` [69] to produce a Gaussian expansion approximating the stellar luminosity of the BCG, and `JAM` [70, 71] to forward model the observed kinematics.

4.3.1 Shared SIDM Mass Model

We construct a self-consistent mass model of the dark and baryonic matter of MACSJ0138 across the regions probed by stellar kinematics and strong lensing. To represent the effect of self-interactions on the overall DM halo of MACSJ0138, we utilize the isothermal Jeans method (see Appendix 4.7.1 for details), as used by previous constraints on SIDM in clusters [18, 326]². In contrast to previous work such as Andrade et al. [18], which constrained a parametric density profile and only related this constraint to SIDM after-the-fact, we compute the SIDM-predicted density profile at every step and consistently model both stellar kinematics and strong lensing observables directly from this profile. The SIDM-modified profile is predicted using the well-known isothermal Jeans formalism introduced in [391], as previous studies of SIDM in clusters have done [326, 18]. This semi-analytic method consists of a simple modification of an NFW DM density profile, where a boundary is drawn at a spherical radius r_1 , and all DM within r_1 is assumed to be isothermal. This boundary is conventionally defined as the radius at which DM particles have experienced, on average, one scattering over the lifetime of the halo:

$$1 = \left\langle \frac{\sigma v}{m} \right\rangle \rho_{\text{dm}}(r_1) t_{\text{halo}} \quad (4.1)$$

Where $\left\langle \frac{\sigma v}{m} \right\rangle$ represents the velocity-weighted SIDM cross section per particle mass, and t_{halo} is the age of the halo. We take the age of the halo to be the age of the universe at the redshift of the halo, which is 9.9 Gyr given our fiducial cos-

²We note that while this method only describes the ‘core-expansion’ phase of SIDM, and fails to describe ‘core-collapse’ behavior, current constraints already rule out SIDM-induced ‘core-collapse’ at cluster scales. A simple estimate based on the relation in Outmezguine et al. [278] predicts that a cluster like MACSJ0138 would experience ‘peak core’ (i.e., the core-expansion/core-collapse phase transition) and begin collapsing after 216 Gyr of evolution at an SIDM cross section of 0.1 cm²/g or 22 Gyr at a cross section of 1.0 cm²/g.

mology³. This simple formalism has been shown to effectively capture the ‘core formation’ behavior of SIDM on cluster scales; see [317] for a detailed study. Our implementation of the isothermal Jeans method is described in Appendix 4.7.1, including the handling of priors on r_1 and σ/m , and the effect stellar density profile. Alongside this paper, we present our open-source code for computing the SIDM density profile with this method, available at github.com/jhod0/sidm_halos.

The baryon contribution in the innermost part of the halo is essential to both accurately computing the predicted SIDM profile, and predicting strong lensing and kinematics observables. In this work we consider the impact of the stellar mass of the central galaxy of the cluster (typically the Brightest Cluster Galaxy, or BCG). The light profile of the BCG is approximated as a Multi-Gaussian Expansion (MGE), as described in Section 4.3.3, and the baryonic mass is derived with a stellar mass-to-light ratio. We use **kcorrect**[45] version 5.1.3 to compute the V-band absolute magnitude of the BCG using Legacy Survey photometry in the g, r, i, and z bands [102]; in addition, **kcorrect** provides an estimate of the stellar mass-to-light ratio Υ_* . This yields a V-band absolute luminosity of $4.94 \times 10^{11} L_\odot$ with a corresponding $\Upsilon_{*,V}$ of 2.05. We note this is similar to the result in Newman et al. [256], which similarly studied BCGs of lensing clusters and found $\Upsilon_{*,V}$ between 1.8 and 2.32 across seven systems. It is well known that there is considerable uncertainty on the true mass-to-light ratio, which is highly dependent on the assumed initial mass function (IMF) [326, 256]. As such, and following Sagunski et al. [326], we use a wide prior on $\log \Upsilon_*$ with a scatter of 0.3 dex.

Additionally, we allow for a prolate overall DM halo in three dimensions, rather than simply ellipticity in the plane of the sky. This treatment of the ellipticity

³This approach to t_{halo} is the same as Andrade et al. [18]. See Section 5.4 of [317] for further discussion of how to treat t_{halo} .

introduces three parameters: the prolate axis ratio q_{3d} , the angle between the halo’s axis of symmetry and the line of sight $\cos i$, and a position angle ϕ of the apparent ellipticity in the plane of the sky. We introduce ellipticity to the overall halo while ensuring that the mass and concentration of the halo are preserved, and analytically project this prolate halo into an elliptical surface density in the plane of the sky. A detailed explanation can be found in Appendix 4.7.2.

In order to compute lensing and kinematic observables from this prolate, SIDM-modified density profile, the density profile is first approximated by a basis sum appropriate for each observable. The lensing observables are computed with Cored Steep Ellipsoids (CSEs), as described in Sec. 4.3.2, and the kinematic observables are computed with a Multi-Gaussian Expansion (MGE), as described in Sec. 4.3.3. A visual summary of this process is presented in Figure 4.2.

Shared Model Parameters

There are thus seven parameters common to both the strong lensing and kinematics models. Together, they describe the overall dark matter halo and inner baryon density profile. They are:

- $\log_{10} M_{200m}$: The overall mass of the cluster’s DM halo, defined as the mass within radius r_{200m} such that the mean density within r_{200m} is 200 times the mean matter density of the universe;
- c_{200m} : The corresponding concentration of the cluster’s DM halo;
- $\log_{10} r_1$: The radius within which DM particle self-interactions modify the DM density profile;⁴
- Υ_* : The stellar mass-to-light ratio of the central galaxy;

⁴Though we sample r_1 rather than σ/m , likelihoods are weighted to enforce a flat prior on $\ln(\sigma/m)$. See Appendix 4.7.1.

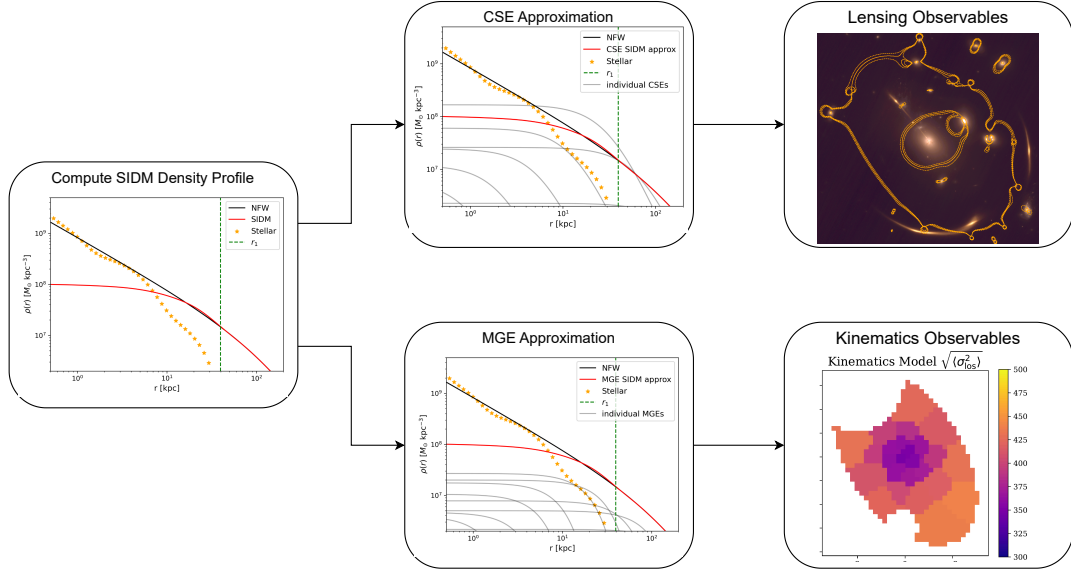


Figure 4.2: SIDM Analysis: Illustration of how the SIDM-modified dark density profile is translated into lensing and kinematics observables. In all figures, an unmodified NFW profile is shown for reference in black, and the SIDM-predicted dark density profile is shown in red. **Left:** First, the isothermal Jeans equation is solved to compute the predicted SIDM-modified density profile, as described in Section 4.3.1 and Appendix 4.7.1. **Middle:** The SIDM-modified density profile is approximated by a sum in a particular basis suitable for each observable. For lensing, we use CSEs, and for kinematics we use an MGE. In both cases, the approximation is accurate to within 1.5% across the radii shown. **Right:** The model is fit to data. See Sections 4.3.2 and 4.3.3 for details.

- q_{3d} : The 3D axis ratio of the cluster halo;
- $\cos(i)$: The *cosine* of the inclination angle relative to LOS;
- ϕ : The position angle of the projected, 2D ellipticity on the sky.

4.3.2 Strong Lens Modeling

Lens modeling in this work is done with the flexible open-source package `PyAutoLens` [262, 263]. In addition to the built-in functionality of `PyAutoLens`, we implement several extensions to describe our lens model and constraints. The mass model used in this work, and the functionality implemented to enable it, are described in Section 4.3.2; the strong lensing constraints, and the likelihood implemented in this work, is described in Section 4.3.2.

Mass Model

Our strong lensing mass model takes a similar approach to the widely-used `lenstool` software [182, 197]. It contains two primary components: one dark matter halo for the cluster as a whole, and a set of dPIE[119] profiles representing all member galaxies, save for the central galaxy.

To represent the former, we approximate our SIDM-modified DM density profile via a sum of Cored Steep Ellipsoids (CSEs), as proposed by Oguri [273]. This CSE decomposition is computed by the `scipy` method `least_squares`, using the NFW approximation in Table 1 of Oguri [273] as an initial value. The position of the overall halo is allowed to vary slightly, with a Gaussian prior centered on the known BCG center.

In order to model the member galaxies, we implement a dPIE density profile in `PyAutoLens`. The mass of these member galaxies are all scaled by their

photometry using the Faber-Jackson relation, described below; we additionally leverage a direct measurement of the Faber-Jackson relation in MACSJ0138, as advocated by Bergamini et al. [30]. This measurement was previously reported in Flowers et al. [132], yielding a slope of $\alpha \sim 0.26 \pm 0.06$ at a reference velocity dispersion of ~ 220 km/s. We note that while this measurement was done using the older, shallower MUSE data, an independent team reported a similar measurement of the Faber-Jackson relation in MACSJ0138 using the newer, deeper MUSE observations. These results, published in Granata et al. [154], used different methodology on different data, yet yielded the highly consistent measurement of $\alpha \sim 0.25 \pm 0.05$. Both results suggest a slightly shallower slope than those reported for three clusters in Bergamini et al. [30].

The dPIE profiles of the member galaxies are parameterized by their velocity dispersion σ_i , core radius $r_{a,i}$, and scale (or ‘cut’) radius $r_{s,i}$. Except where otherwise noted, all member galaxies receive a fixed core radius of 0.15 kpc. Their velocity dispersions and cut radii are scaled in the conventional way, as in [182] and [30]:

$$\sigma_{\text{LT},i} = \sigma_{\text{LT,ref}} \left(\frac{L_i}{L_0} \right)^\alpha \quad (4.2)$$

$$r_{s,\text{LT},i} = r_{s,\text{LT,ref}} \left(\frac{L_i}{L_0} \right)^\beta \quad (4.3)$$

Here, LT denotes the fiducial parameterization used in `lenstool`, rather than observed quantities. If the total mass-to-light ratio Υ_i^{tot} for these galaxies scales as L_i^γ , then $\beta = \gamma - 2\alpha + 1$. Following both Bergamini et al. [30] and Acebron et al. [7], we fix $\gamma = 0.2$. The fiducial dPIE velocity dispersion, σ_{LT} , is known to differ from observed aperture-averaged velocity dispersions σ_{ap} ; in the limit

$r_a \rightarrow 0, r_s \rightarrow \infty$, the dPIE becomes an SIS, and $c_p = \sigma_{\text{ap}}/\sigma_{\text{LT}} = \sqrt{3/2}$ [119]. In practice, c_p is generally closer to unity. Bergamini et al. [30] provides a practical treatment of c_p in three lensing clusters with measured Faber-Jackson relations, they find that for all three clusters, given physical values of r_a and r_s , $c_p \approx 1.12$. For our priors on the substructure scaling relations in MACSJ0138, we therefore adjust the measured Faber-Jackson relation from [132] by $c_p = 1.12$.

Red-sequence cluster members included in the mass model are chosen from the catalog introduced in Flowers et al. [132], all of which are spectroscopically confirmed to lie at or near the cluster’s redshift.⁵ In addition, several emission-line galaxies near the cluster redshift lie close to the observed lensing features, and their mass contributions are necessary to construct a precision mass model. We include three of these in our mass model, as shown in Figure 4.3; note that we label these following the convention in Gibson et al. [144], which differs from that in Rodney et al. [320] and Acebron et al. [7]. The fourth galaxy studied in [144], J2, is not included in our mass model: its projected position is so close to the cluster center that it’s contribution to the observed lensing is dwarfed by the BCG and cluster DM halo.

Our lensing model includes one further component, a constant ‘external shear’ γ_{ext} across the MACSJ0138 field. This shear is parameterized by the two conventional components (γ_1, γ_2) , where γ_1 shears in the vertical and horizontal directions, and γ_2 shears along the diagonals. Acebron et al. [7] likewise includes a γ_{ext} component to improve their mass model, however [123] models MACSJ0138 using two halo components without γ_{ext} . A discussion of our γ_{ext} result and its interpretation can be found in Section 4.4.4.

⁵There exists a small subset of red-sequence galaxies at higher redshift, with a velocity offset of $\sim 8,000$ km/s relative to MACSJ0138 [132, 154].

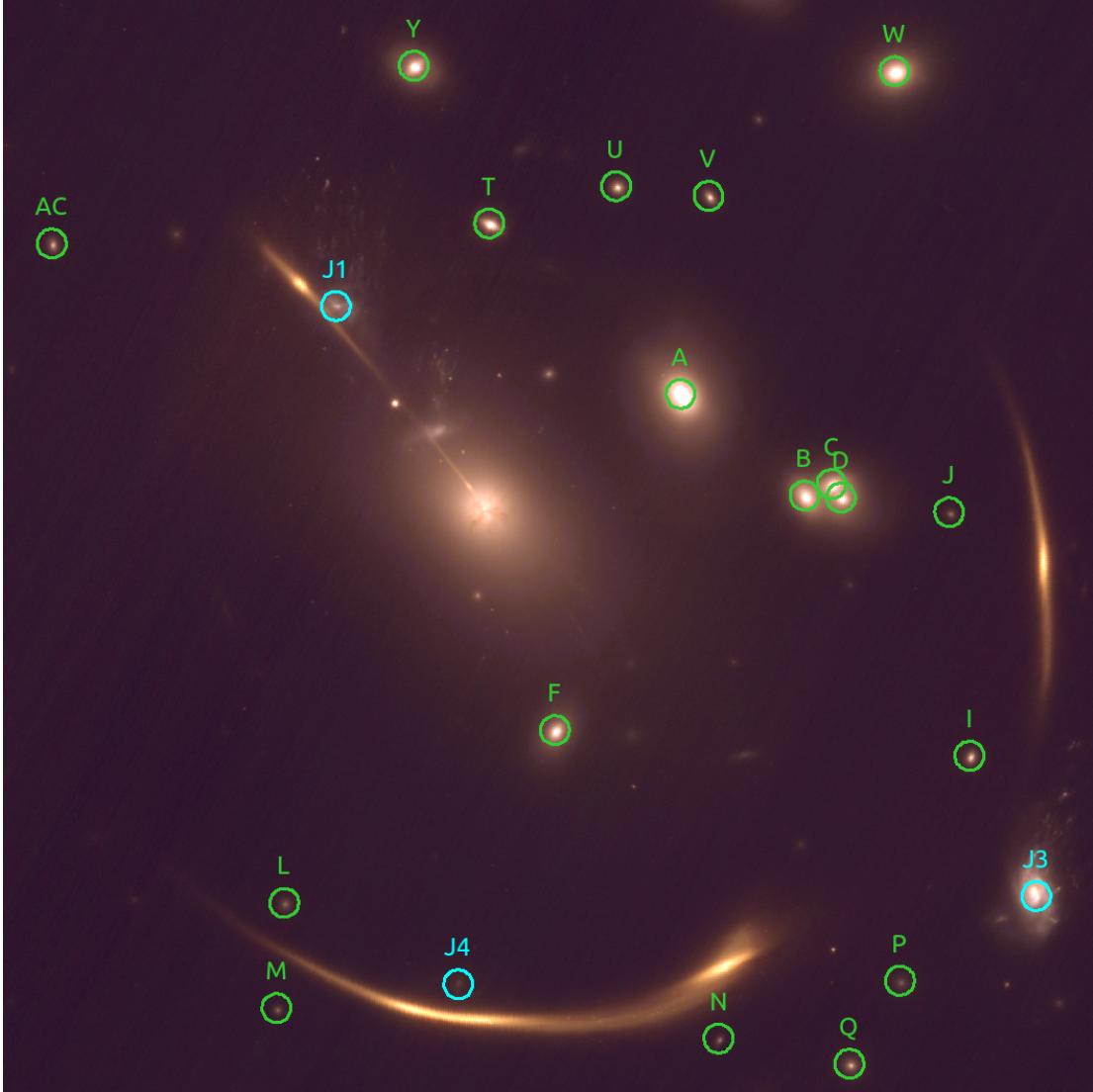


Figure 4.3: JWST NIRCам imaging of MACSJ0138. All galaxies labeled here included as components of the strong lensing mass model. Red-sequence galaxies are shown in green, emission-line galaxies are in cyan. All circles have radius $0.5''$. Several cluster members outside this FOV are also included in the mass model. In this RGB image, the blue, green, red channels show the F115W, F150W, and F200W NIRCам filters, respectively.

Lensing Constraints

Our lensing mass model is constrained by a variation of the conjugate point method, computed in the source plane. For each observed source feature F , observed at multiple positions θ_i^F , the image-plane uncertainties are ellipsoids described by the covariance matrix Σ_i^F . For a given mass model, the corresponding source-plane positions and covariance are:

$$\beta_i^F = \theta_i^F - \alpha(\theta_i^F) \quad (4.4)$$

$$B_i^F = A(\theta_i^F) \Sigma_i^F A^T(\theta_i^F) \quad (4.5)$$

Where $A(\theta)$ is the lensing Jacobian at image plane position θ . Given these predicted source plane positions, we analytically marginalize over the unknown intrinsic source plane position for each feature. In practice, to avoid overfitting to critical curves of the lens model, we average B_i^F across all image-plane positions within the uncertainty ellipsoid described by (θ_i^F, Σ_i^F) . For each set of observed features F , this yields a best-fit source plane position $\hat{\beta}^F$, with corresponding uncertainty ellipsoid $\hat{B}^F$, and a likelihood $\mathcal{L}^F$:

$$\hat{\beta}^F = \hat{B}^F \left(\sum_i (B_i^F)^{-1} \beta_i^F \right) \quad (4.6)$$

$$(\hat{B}^F)^{-1} = \sum_i (B_i^F)^{-1} \quad (4.7)$$

$$\begin{aligned} \log \mathcal{L}^F &= -\log \sqrt{\frac{|\hat{B}^F|}{2\pi}} \\ &\quad - \frac{1}{2} \sum_i (\beta_i^F - \hat{\beta}^F)^T (\hat{B}^F)^{-1} (\beta_i^F - \hat{\beta}^F) \end{aligned} \quad (4.8)$$

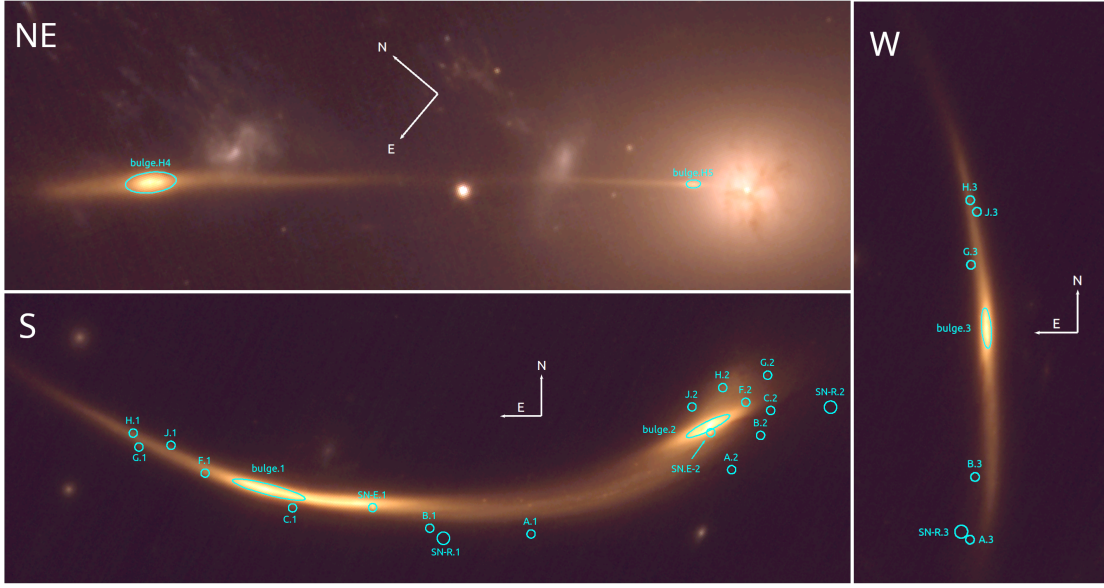


Figure 4.4: Lensing position constraints: The positions of lensed features used to constrain the strong lens model, using the likelihood described in Section 4.3.2. The RGB color bands and scaling are the same as Figure 4.3. Three fields are shown here, showing lensed features of the quiescent source in the South, West, and North-East. In each field, a compass is displayed showing North and East; both segments of the compass are 1" long for scale.

This likelihood is implemented as an extension of `PyAutoLens`. This method was later found to be equivalent to that introduced in Section 5.1 of Lombardi [227].

Our constraints include features in all five arcs of the primary quiescent source. Although recent work has reported additional sources from faint emission lines in deep MUSE data[154], those sources are not used in this work. All position constraints are measured from JWST NIRCам imaging, save for three observed images of SN Requiem, which was only seen at an earlier epoch. The positions of the three SN Requiem images are taken from [320], and assigned a higher position uncertainty due to the lower resolution of HST imaging. The position constraints used here are shown in Figure 4.4, and listed in Table 4.1.

Table 4.1: Position constraints: The constraints used for the strong lensing mass model. Each group of observed features is given a separate label (e.g. ‘SN-E’, ‘A’), and each separately observed position is given a number (‘SN-E.1’, ‘A.2’). Position angles are given in degrees east-of-north.

Feature Label	RA	Declination	Semi-Major Axis	Semi-Minor Axis	Position Angle
bulge.1	01:38:03.93	-21:55:49.23	0.91"	0.13"	75.69
bulge.2	01:38:03.17	-21:55:47.69	0.58"	0.11"	-63.68
bulge.3	01:38:02.38	-21:55:33.68	0.47"	0.11"	4.55
bulge.4	01:38:04.23	-21:55:24.07	0.43"	0.17"	46.12
bulge.5	01:38:03.80	-21:55:31.09	0.12"	0.07"	41.44
SN-E.1	01:38:03.75	-21:55:49.66	0.1"	—	—
SN-E.2	01:38:03.17	-21:55:47.85	0.1"	—	—
SN-R.1	01:38:03.63	-21:55:50.40	0.15"	—	—
SN-R.2	01:38:02.96	-21:55:47.23	0.15"	—	—
SN-R.3	01:38:02.42	-21:55:38.43	0.15"	—	—
A.1	01:38:03.48	-21:55:50.29	0.1"	—	—
A.2	01:38:03.13	-21:55:48.74	0.1"	—	—

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Table 4.1 — continued from previous page —

Feature Label	RA	Declination	Semi-Major Axis	Semi-Minor Axis	Position Angle
A.3	01:38:02.40	-21:55:38.62	0.1"	—	—
B.1	01:38:03.65	-21:55:50.16	0.1"	—	—
B.2	01:38:03.08	-21:55:47.92	0.1"	—	—
B.3	01:38:02.39	-21:55:37.15	0.1"	—	—
C.1	01:38:03.89	-21:55:49.67	0.1"	—	—
C.2	01:38:03.06	-21:55:47.31	0.1"	—	—
F.1	01:38:04.04	-21:55:48.83	0.1"	—	—
F.2	01:38:03.10	-21:55:47.12	0.1"	—	—
G.1	01:38:04.16	-21:55:48.20	0.1"	—	—
G.2	01:38:03.07	-21:55:46.46	0.1"	—	—
G.3	01:38:02.40	-21:55:32.20	0.1"	—	—
H.1	01:38:04.17	-21:55:47.86	0.1"	—	—
H.2	01:38:03.14	-21:55:46.76	0.1"	—	—
H.3	01:38:02.40	-21:55:30.69	0.1"	—	—

— continued on next page —

Table 4.1 — continued from previous page —

Feature Label	RA	Declination	Semi-Major Axis	Semi-Minor Axis	Position Angle
J.1	01:38:04.10	-21:55:48.16	0.1 "	—	—
J.2	01:38:03.20	-21:55:47.23	0.1 "	—	—
J.3	01:38:02.39	-21:55:30.96	0.1 "	—	—

4.3.3 Stellar Kinematics of the BCG

Stellar kinematics of SL deflectors provides a powerful, independent probe of the lensing mass distribution, especially if it can be resolved spatially. Early work along these lines used slit spectra of central cluster galaxies to measure variations in the stellar velocity dispersions in one dimension [332, 257, 256]. Much recent work has leveraged IFU spectra to study the dynamics of lensed systems, including lensed quasar systems [343, 194, 193], time delay constraints on the Hubble constant [375, 345], galaxy-scale multi-source-plane lenses [382], and even a group-scale strong lens [393, 394]. In the majority of these studies, the primary contribution of the stellar dynamics is to refine the strong lensing model, yielding a more accurate result and potentially breaking the mass-sheet degeneracy. In our case, however, the inner density profile constrained by central galaxy’s dynamics is of direct scientific interest: SIDM affects the halo’s dark matter density in precisely this region.

Here we describe the procedure for measuring spatially resolved stellar kinematics, and how it is used to constrain our mass model. Sections 4.3.3 and 4.3.3 describe how the kinematics map was measured and validated; Section 4.3.3 describes the light profile of the central galaxy, essential to modeling the kinematics; and Section 4.3.3 describes how our mass model is constrained by this data.

Measuring the Kinematics Map

To robustly measure the stellar kinematics in numerous spatial bins across the central galaxy, we take a similar approach to [343]. We consider all IFU ‘spaxels’ along the central galaxy, binning them spatially using `vorbin`[75] such that each bin attains a target spectral signal-to-noise. We use all pixels within an ellipse tracing the BCG, with a major axis of 8.5" and a minor axis of 5". Two circular

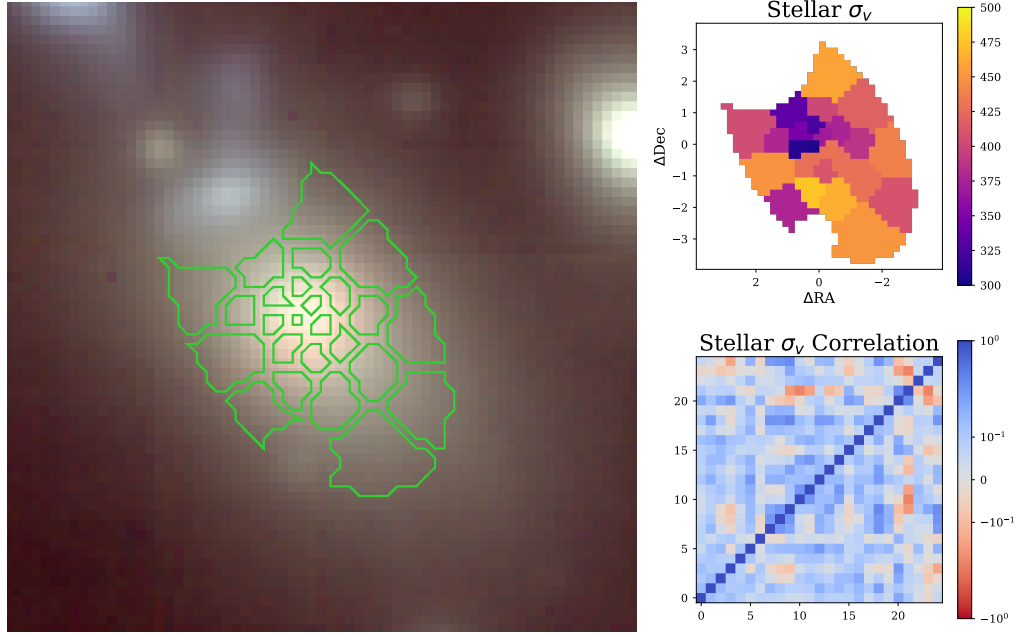


Figure 4.5: The central galaxy of MACSJ0138, and the LOS stellar velocity dispersions measured in this work. **Left:** MUSE color imaging of the central galaxy, with the Voronoi bins used in this work overlaid in green. **Upper Right:** The LOS stellar velocity dispersion in each bin, in km/s. **Lower Right:** The correlation matrix of the LOS velocity dispersion in all 25 spatial bins, displayed in a `symlog` scaling with a linear threshold of 0.2.

regions are excluded due to contamination by other galaxies along the line of sight. These spectra are spatially binned to a target signal-to-noise of $23/\text{\AA}$ between the rest-frame wavelengths of 5035 and 5160 $\text{\AA}$, just below the Mg b absorption feature. This yields 25 spatial bins, as shown in Figure 4.5.

Spectral fitting is done with the widely used software package `ppxf` [76]. This tool requires a set of input templates, and an optimal combination of templates is found which closely represents the user’s spectrum. In this work, we use the X-Shooter Library (XSL) Data Release 3 [388], a set of 830 spectra of 683 diverse stars, as our template library. All template spectra are normalized by their mean flux between 4000 $\text{\AA}$ and 4200 $\text{\AA}$. To exclude low-quality spectra, the mean SNR per pixel in this region is computed, and template spectra with SNR less than one are excluded. This is similar in spirit to the “cleaning” advocated by [193].

We compute maps of the stellar kinematics in each Voronoi bin as follows. First, the spectrum of the entire BCG is extracted (i.e., all IFU ‘spaxels’ in all Voronoi bins). Then `ppxf` is run once on this total spectrum, and provided as templates the XSL stellar spectra described above. An example of this result is shown in Fig 4.17. This fit determines a linear combination of XSL template spectra which most closely matches our observed spectrum; we record this combination as the *optimal template*. Next, for each Voronoi region, `ppxf` is applied to the region’s spectrum, this time using the single optimal template for the stellar component rather than providing a library of templates. For each spectral fit, we allow `ppxf` to fit velocity moments to the 4th order, meaning we allow for nonzero values of the third- and fourth-order moments h_3 and h_4 ⁶.

We note that this central galaxy exhibits strong emission lines, potentially associated with an AGN and star formation. As such, we provide a list of emission line templates to `ppxf` in order to obtain the highest quality spectral fit. All

⁶See Cappellari and Emsellem [76] for an explanation of these moments.

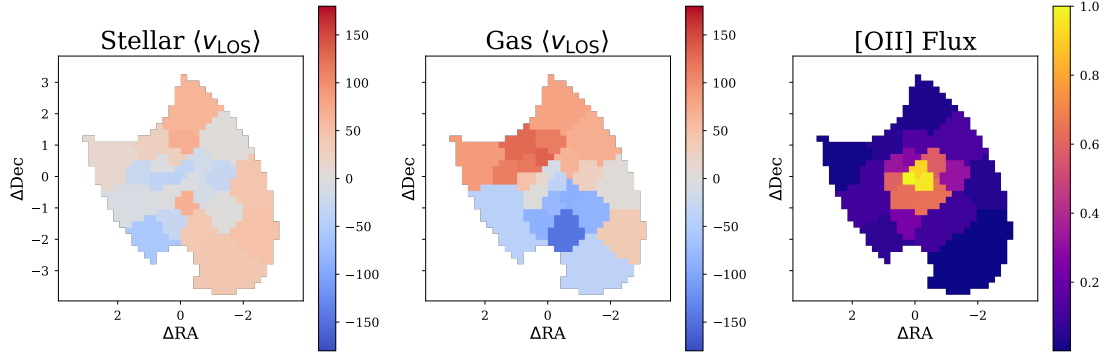


Figure 4.6: Additional features of the central galaxy of MACSJ0138 measured with MUSE IFU spectra. Axes show offsets from the BCG center in arcseconds. **Left:** The measured LOS velocity of the stellar component in km/s, showing negligible rotation. In all Voronoi bins, $(\langle v \rangle / \sigma_v)^2 < 0.04$. **Center:** The measured LOS velocity of the emission lines in km/s, showing clear evidence of rotation. **Right:** Observed flux-per-spaxel of [O II] 3729Å, showing the spatial extent of gas emission. Flux is normalized by the brightest Voronoi bin.

emission lines are associated with a single kinematic component, separate from the stellar component. We include all emission lines in the relevant wavelength range known by `ppxf`, and manually add three lines not known by `ppxf`: [SII] 4069Å, FII 5198Å, and [NI] 5201Å. While the kinematics of this gas component are not used to constrain our mass model, and have no bearing on the primary results of this paper, we note that this gas component is clearly kinematically decoupled from the stellar component, and shows clear evidence of rotation about the galactic center (see Fig. 4.6). The stellar component shows negligible rotation, as expected in a BCG.

Systematics Test and Covariance

Measurements of stellar velocity dispersions can be highly sensitive to the wavelength range used, the choice of template library, and even the degree of the polynomial used to model background. As such, we conduct the following tests to verify the robustness of our kinematics map. Our approach follows Shajib et al.

[343]. We aim to bracket a range of reasonable settings, all of which compute high-quality kinematics maps, and we compute a full kinematics map at each setting as described in the section above. We choose four different ‘axes’ along which we can reasonably change settings, and along each ‘axis’, we choose between 2 and 6 settings. These four axes, and their various settings, are:

- **Wavelength Range:** We extract and fit six different rest-frame wavelength ranges: 3700-4450Å, 3750-4500Å, 3800-4550Å, 3650-4550Å, 3700-4600Å, 3750-4650Å.
- **Background Polynomial Order:** The degree of the additive polynomial used to represent the background. We use three separate values, 4 through 6.
- **Input Template Library:** We use the XSL stellar template library as described in Section 4.3.3. As in Shajib et al. [343], we bisect the XSL library into two subsets, and compute kinematics maps with each subset. This results in three possible settings: ‘Full’ XSL library, bisected half-‘A’, and bisected half-‘B’.
- **Dust extinction:** The central galaxy of MACSJ0138 exhibits clear evidence of dust extinction, most evident in JWST NIRCам imaging (c.f. Fig 4.4, NE field). We utilize two possible settings. The first ignores any effect of dust on the spectral fits. The second uses the one-parameter attenuation curve presented in Calzetti et al. [65], applying the same attenuation to both the stellar and gas components. The dust extinction is allowed to vary between 0 and 4 magnitudes, and is allowed a different value in each voronoi bin.

These constitute 108 possible settings, and produce 108 maps of the BCG’s stellar kinematics. For each of these measured kinematics maps, we draw 1000 random realizations of the velocity dispersion map using the statistical errors produced by `ppxf`. From these 108,000 samples of the velocity dispersion in each voronoi bin, a mean and covariance are computed. The resulting best-fit kinematics map and correlation matrix are shown in Fig. 4.5. This covariance Σ is used in a conventional multivariate-Gaussian likelihood, as described in Section 4.3.3.

We find our resulting kinematics map to be consistent across all settings, with systematic uncertainty subdominant to statistical uncertainty in all but one spatial bin. Note that while Knabel et al. [193] have demonstrated the leading systematic uncertainty comes from the choice of template library, and advocate comparing a wide array of such libraries, they demonstrate this uncertainty is at most $\mathcal{O}(3\text{--}4\%)$ in fits to MUSE spectra. This is well below the $\approx 10\%$ uncertainty per bin measured in this work. See Appendix 4.7.3 for further discussion.

BCG Light Profile

The central galaxy’s light profile is represented as a Multi-Gaussian Expansion (MGE), measured using `MGEfit`[69] on the ACS/WFC F555W imaging. This band corresponds closely to the spectral wavelength range used to measure stellar kinematics. We apply `MGEfit` in an ellipse aligned with and centered on the BCG, with a semi-major axis of 7" and a semi-minor axis of 5.5", excluding several regions contaminated by member galaxies and other LOS structure, as shown in Figure 4.7. Using an initial `ngauss` of 30, the resulting best-fit MGE contains 6 Gaussian components.

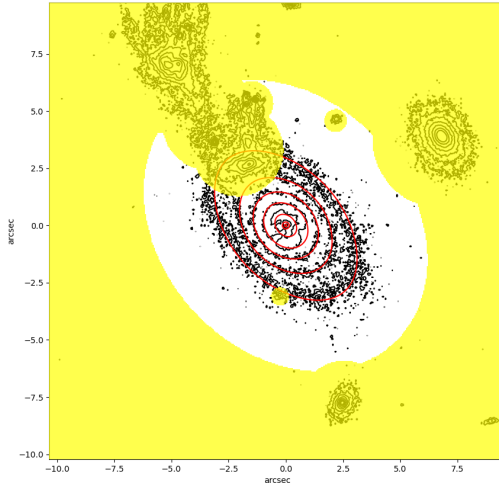


Figure 4.7: The MGE approximation of the central galaxy’s light profile, as provided by `MGEfit`. Regions in yellow are excluded from the fit. Black contours represent ACS F555W imaging data, and red contours represent the MGE profile. See Section 4.3.3 for details.

JAM Model and Likelihood

We compute the predicted stellar kinematics map using the widely used JAM package [70, 71, 74]. JAM requires two primary inputs, each represented as an MGE: (1) the luminous tracer used to measure the kinematics (in this case, the light profile of the central galaxy), and (2) the mass profile in which that tracer is embedded. The former is described in Section 4.3.3. For the latter, we combine the BCG stellar profile scaled by a mass-to-light ratio with the SIDM-modified dark matter density profile. The dark density profile is decomposed into an MGE via the `mge_fit_1d` function provided with `MGEfit`.

In addition, JAM requires several further parameters: a point spread function (PSF) model; the inclination angle relative to the line-of-sight; and the velocity anisotropy. The inclination angle, i , is also relevant to the triaxiality of the lensing halo, and has been described in Appendix 4.7.2. The meaning of the velocity anisotropy β depends on whether the cylindrical [70] or spherical [74] alignment

of the velocity ellipsoid is chosen, and whether the halo mass model is prolate or oblate. In this work, we use a spherical alignment of the velocity ellipsoid, where $\beta_{\text{sph}} = 1 - \sigma_\theta^2/\sigma_r^2$, and $\beta_{\text{sph}} = 0$ indicates isotropic velocity of the stellar population. As noted in Tdcosmo Collaboration et al. [375], JAM models with spherical and cylindrical alignment have been found to generally agree [74, 413], and both Shajib et al. [343], Tdcosmo Collaboration et al. [375] use the spherical alignment in joint models of lensing and dynamics. Considerable uncertainty, however, exists on the value of the velocity anisotropy β ; in this work, we adopt a flat prior on $\beta_{\text{sph}} \sim \mathcal{U}(-0.4, 0.4)$, slightly wider than the allowed values in [343, 375].

We use a simple Gaussian model of the PSF of our IFU data, from which the kinematics are measured. Metadata provided with the MUSE cube suggest the seeing varied between FWHM 0.66" and 0.8" over the course of the observations. In addition, we perform a simple test to compare our the relative seeing between the HST F555W imaging and the MUSE IFU data. Given the MGE model computed in Section 4.3.3, we convolve by a single-Gaussian PSF model to compute a predicted, PSF-smoothed image, and compare to the IFU cube summed between 5300 and 5700Å. Residuals between these suggest the true PSF at the MUSE slicer could be higher, up to around $\sim 1.15''$ FWHM. We thus assume a single-component Gaussian PSF, and adopt a prior on its size $\sigma_{\text{PSF}} \sim \mathcal{U}(0.35'', 0.5'')$, spanning FWHM values of approximately 0.5" to 1.2".

Given all the above inputs, for each spatial bin i , the predicted luminosity-weighted velocity dispersion is:

$$\langle \sigma_{\text{los},i}^2 \rangle = \frac{\int_{\Theta_i} I \langle v_{\text{los}}^2 \rangle \otimes \text{PSF } d^2\theta}{\int_{\Theta_i} I \otimes \text{PSF } d^2\theta} \quad (4.9)$$

Where Θ_i denotes the aperture of the i th spatial bin, I represents the projected

surface brightness of the tracer population, $\otimes$ PSF denotes convolution by the PSF, and $\langle v_{\text{los}}^2 \rangle$ is the predicted LOS velocity dispersion at position θ computed by JAM. As in Shajib et al. [343], we take the first velocity moment to be negligible, leaving $\langle \sigma_{\text{los}}^2 \rangle = \langle v_{\text{los}}^2 \rangle$. (See also Figure 4.6.) With our measured kinematics map $\boldsymbol{\sigma}_{\text{los}}^{\text{obs}} = \{\sigma_{\text{los},i}^{\text{obs}}\}$ and covariance matrix $\boldsymbol{\Sigma}$, the likelihood is therefore:

$$\Delta\boldsymbol{\sigma} = \boldsymbol{\sigma}_{\text{los}}^{\text{obs}} - \langle \boldsymbol{\sigma}_{\text{los}} \rangle \quad (4.10)$$

$$\mathcal{L}(\boldsymbol{\sigma}_{\text{los}}^{\text{obs}}|\mathcal{M}) = \frac{1}{\sqrt{(2\pi)^k |\boldsymbol{\Sigma}|}} \exp\left(-\frac{1}{2}(\Delta\boldsymbol{\sigma})^T \boldsymbol{\Sigma}^{-1}(\Delta\boldsymbol{\sigma})\right) \quad (4.11)$$

Where $\mathcal{M}$ denotes our model parameters.

4.4 Results

Table 4.2: Results: Priors and posteriors for all three models fit in this work. In the priors column, $\mathcal{U}(a, b)$ denotes a uniform prior on the interval $[a, b]$, and $\mathcal{N}(\mu, \sigma)$ denotes a Gaussian prior of mean μ and variance σ^2 . The ‘Result’ columns list the median and 1σ bounds on each parameter. The shared, lensing-only, and kinematics-only parameters and priors are explained in detail in Sections 4.3.1, 4.3.2, and 4.3.3 respectively; for convenience, brief descriptions of each parameter are included here.

Parameter	Description	Prior	Lensing Result	Kinematics Result	Joint Result
Shared Parameters					
$\log_{10} \Upsilon_*$	BCG stellar mass-to-light ratio	$\mathcal{N}(0.31, 0.3)$	$0.50^{+0.12}_{-0.21}$	$0.40^{+0.21}_{-0.29}$	$0.47^{+0.19}_{-0.35}$
$\log_{10}(\frac{M_{200m}}{M_\odot})$	Overall halo NFW mass	$\mathcal{U}(13.5, 15.5)$	$14.90^{+0.20}_{-0.17}$	$14.52^{+0.61}_{-0.59}$	$14.81^{+0.14}_{-0.12}$
c_{200m}	Overall halo NFW concentration	$\mathcal{U}(5, 15)$	$10.14^{+2.11}_{-3.03}$	$10.73^{+2.93}_{-3.41}$	$9.92^{+2.19}_{-1.88}$
$\log_{10}(r_1/\text{kpc})$	Isothermal Jeans radius, defines SIDM behavior	$\mathcal{U}(0.0, 3)$	$2.28^{+0.28}_{-0.92}$	$1.69^{+0.60}_{-0.95}$	$1.58^{+0.69}_{-0.99}$

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Table 4.2 — continued from previous page —

Parameter	Description	Prior	Lensing Result	Kinematics Result	Joint Result
q	3D halo prolate axis ratio	$\mathcal{U}(0.3, 1)$	$0.64^{+0.05}_{-0.10}$	$0.67^{+0.22}_{-0.24}$	$0.61^{+0.05}_{-0.13}$
$\cos(i)$	Halo inclination relative to LOS	$\mathcal{U}(0, 1)$	0.44 ± 0.29	$0.73^{+0.18}_{-0.40}$	$0.51^{+0.28}_{-0.32}$
ϕ	Position angle of projected ellipticity in the plane of the sky. East-of-North	$\mathcal{U}(-48 32)$	$44.3^{+1.8}_{-1.7}$	$42.0^{+43.1}_{-43.3}$	$43.6^{+1.6}_{-1.5}$
Lensing Parameters					
x_{halo}	Horizontal offset between halo and BCG center, arcsec	$\mathcal{N}(0.0, 0.3)$	$-0.10^{+0.22}_{-0.26}$	—	$0.02^{+0.16}_{-0.18}$

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Table 4.2 — continued from previous page —

Parameter	Description	Prior	Lensing Result	Kinematics Result	Joint Result
y_{halo}	Vertical offset between halo and BCG center, arcsec	$\mathcal{N}(0.0, 0.3)$	$-0.33^{+0.23}_{-0.26}$	—	$-0.25^{+0.19}_{-0.21}$
γ_1	External shear component 1	$\mathcal{N}(0.0, 0.05)$	0.02 ± 0.01	—	0.02 ± 0.01
γ_2	External shear component 2	$\mathcal{N}(0.0, 0.05)$	$-0.08^{+0.03}_{-0.02}$	—	-0.06 ± 0.02
α	Slope of the Faber-Jackson relation	$\mathcal{N}(0.26, 0.06)$	0.26 ± 0.06	—	0.26 ± 0.06
$\sigma_{\text{LT}}^{\text{piv}}$	dPIE velocity dispersion scale of Faber-Jackson relation, km/s	$\mathcal{N}(199.1, 22.3)$	$184.8^{+19.3}_{-20.4}$	—	$188.8^{+19.4}_{-19.5}$

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Table 4.2 — continued from previous page —

Parameter	Description	Prior	Lensing Result	Kinematics Result	Joint Result
$\sigma_{\text{LT}}^{\text{J1}}$	dPIE velocity	$\mathcal{N}(100, 50)$	$66.5^{+31.0}_{-33.0}$	—	$65.1^{+35.7}_{-33.7}$
	dispersion of J1, km/s				
$\sigma_{\text{LT}}^{\text{J3}}$	dPIE velocity	$\mathcal{N}(100, 50)$	$51.0^{+24.9}_{-27.0}$	—	$49.6^{+25.3}_{-26.8}$
	dispersion of J3, km/s				
r_s^{J4}	dPIE velocity	$\mathcal{N}(100, 50)$	19.6 ± 5.2	—	$19.5^{+5.1}_{-5.2}$
	dispersion of J4, km/s				
$\sigma_{\text{LT}}^{\text{J4}}$	dPIE scale radius of	$\mathcal{N}(20, 5)$	$48.3^{+14.7}_{-16.8}$	—	$56.6^{+12.0}_{-14.6}$
	J4, kpc				
Kinematics					
Parameters					
β_{sph}	Velocity anisotropy	$\mathcal{U}(-0.4, 0.4)$	—	$-0.09^{+0.21}_{-0.19}$	$-0.15^{+0.15}_{-0.14}$
σ_{PSF}	Gaussian PSF size,	$\mathcal{U}(-0.35, 0.5)$	—	0.43 ± 0.05	0.43 ± 0.05
	arcsec				

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Table 4.2 — continued from previous page —

Parameter	Description	Prior	Lensing Result	Kinematics Result	Joint Result
Derived Parameters					
σ/m	SIDM Cross Section, cm ² /g	—	$0.16^{+0.50}_{-0.15}$	$0.04^{+0.35}_{-0.03}$	$0.02^{+0.18}_{-0.02}$
$\log_{10}(\sigma/m)$	Logarithm of σ/m in cm ² /g	—	$-0.8^{+0.6}_{-1.1}$	$-1.4^{+1.0}_{-0.8}$	$-1.7^{+1.0}_{-0.8}$
σ_0	Thermal velocity of isothermal region, km/s	—	919^{+88}_{-333}	565^{+375}_{-250}	691^{+200}_{-406}
$V_{\max}$	Maximum circular velocity of cluster DM halo	—	1518^{+161}_{-137}	1144^{+640}_{-410}	1422^{+95}_{-82}

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Table 4.2 — continued from previous page —

Parameter	Description	Prior	Lensing Result	Kinematics Result	Joint Result
q_{2D}	Projected apparent axis ratio of cluster DM	—	0.68 ± 0.03	$0.82^{+0.14}_{-0.28}$	0.67 ± 0.02
$\log_{10} M_{20}^{\text{enc}}$	Enclosed mass within 20kpc	—	$12.22^{+0.14}_{-0.16}$	$12.24^{+0.13}_{-0.19}$	$12.34^{+0.04}_{-0.05}$
$\log_{10} M_{100}^{\text{proj}}$	Projected mass within 100kpc	—	13.79 ± 0.03	$13.61^{+0.29}_{-0.33}$	$13.78^{+0.02}_{-0.04}$

Given the model, constraints, and likelihoods described in Section 4.3, we compute three sets of posteriors. First, we present ‘lensing-only’ results, using only the strong lensing constraints; second, we present the ‘kinematics-only’ constraints, derived only from the measured kinematics map of the BCG; lastly, we present a joint model of the combined lensing and kinematics constraints.

All cosmology-dependent quantities use the best-fit parameters from Planck 2015 (P15) [86]. The following posteriors were obtained with the **emcee** ensemble MCMC sampler [134]. For the kinematics-only constraints, 80 walkers were used, whereas the lensing-only and joint results utilized 48 walkers each. While the kinematics-only result used a single sampler run, the lensing-only and joint posteriors were each estimated with two independent sampler runs in order to check the consistency of results. All samplers used a combination of ‘moves’, primarily the original **StretchMove** presented in Goodman and Weare [153], but with some additional weight given to **DEMove** and **DESnookerMove** [255, 376] to better explore multimodal posteriors.

All corner plots in this work were made with **corner** [133]. In all corner plots of posteriors, 2D contours shade the 1- and 2- σ posteriors, and 1D posteriors overlay the -1σ , median, and $+1\sigma$ confidence intervals with dotted vertical lines. The priors and posteriors of all parameters for all three models are shown in Table 4.2.

4.4.1 Lensing-Only Results

The lensing-only model consists of seventeen parameters, constrained by 29 measured positions of 10 source features listed in Table 4.1. As stated above, two independent **emcee** samplers of 48 walkers each were run for this model, in order to verify the robustness of posteriors. Both samplers ran for over 1,220,000 iterations, and yielded consistent estimates of the mean autocorrelation time of

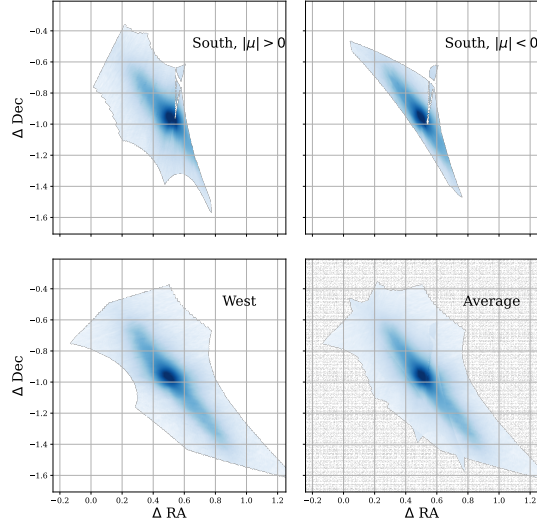


Figure 4.8: Lensing-Only Fit: Source plane projection of the observed lensing arcs of the primary source at $z \approx 1.95$. **Top:** Luminosity of the southern arc, separated into the positive and negative magnification images. **Bottom Left:** The luminosity of the western arc. **Bottom Right:** The average of the former three images, demonstrating their consistency.

$\sim 52,450$ and $\sim 52,270$ iterations. This extremely long autocorrelation time suggests that `emcee` struggled to efficiently sample this parameter space, likely due to a complex likelihood surface. Nevertheless, we consider these results to be robust: the two independent samplers produced posteriors which are quantitatively and qualitatively consistent.

The predicted source-plane light derived from the southern and western arcs is shown in Figure 4.8, using the best fit mass model and NIRCcam F150W imaging. The imaging was cropped to regions tightly bracketing the observed arcs, and the positions of each observed pixel were projected to the source plane given the best-fitting lens model. They were then interpolated to a grid of source-plane pixels with the `scipy` method `griddata`.

The lensing favors an overall DM halo with ellipticity closely aligned to the axis of the central galaxy, yielding $\phi = 44.3^{+1.8}_{-1.7}$ degrees East-of-North, compared

to the BCG’s position angle of 42° . The position of the overall DM halo is also consistent with the center of the BCG, although the lensing alone favors a DM halo centered slightly to the south.

The lensing only model gives constraints on the mass-to-light ratio, mass, and concentration of $\log_{10} \Upsilon_* = 0.5^{+0.1}_{-0.2}$, $\log_{10}(M_{200\text{m}}/M_\odot) = 14.9 \pm 0.2$, $c_{200\text{m}} = 10^{+2}_{-3}$. The isothermal radius is constrained to be $\log_{10}(r_1/\text{kpc}) = 2.3^{+0.3}_{-0.9}$.

4.4.2 Kinematics-Only Results

The kinematics model has nine parameters (the seven shared parameters describing the BCG and cluster DM halo, and β_{sph} and σ_{PSF}), and is constrained by the measured velocity dispersion in 25 Voronoi bins. The 80-walker `emcee` chain for this model ran for 192,648 iterations, with a mean autocorrelation time of 910 iterations across all nine parameters. Having run for over 200 autocorrelation times, we consider these samples to be exceedingly stable and well converged.

This model provides constraints of $\log_{10} \Upsilon_* = 0.4^{+0.2}_{-0.3}$, $\log_{10}(M_{200\text{m}}/M_\odot) = 14.5 \pm 0.6$, $c_{200\text{m}} = 11 \pm 3$, and $\log_{10}(r_1/\text{kpc}) = 1.7^{+0.6}_{-1.0}$. These are fully consistent with the lensing-only model.

The predicted kinematics map of the maximum likelihood sample is shown in Figure 4.9. The best-fit model yields a reduced χ^2 of 2.76. As expected, the stellar kinematics is most sensitive to the total mass within $\sim 20\text{kpc}$, and has only weak constraining power on the mass and concentration of the overall halo. It is reassuring, however, that the kinematics model alone yields parameters that are highly consistent with the lensing-only model: the posteriors on Υ_* are similar, and in $M_{200\text{m}}\text{-}c_{200\text{m}}$ space, the 1σ posteriors of the lensing-only model lie almost entirely within those of the kinematics-only model. A larger difference can be seen in the SIDM cross section σ/m , where the lensing alone prefers markedly higher

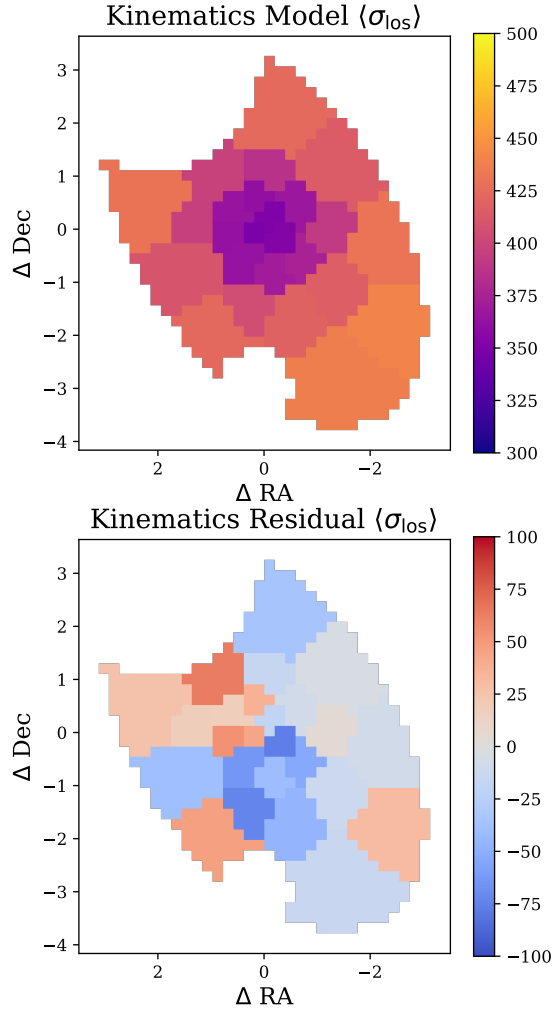


Figure 4.9: Kinematics Fit: The best-fit predicted kinematics map and residual (predicted - data). Axes units are arcsec relative to BCG center, color map units are km/s.

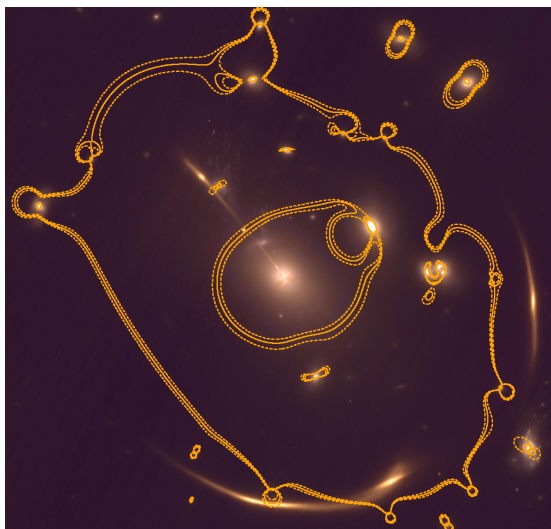


Figure 4.10: Critical Curves: Posteriors on the lensing critical curves of the joint model. Median critical curves are displayed as a solid line, $\pm 1\sigma$ critical curves are dashed lines. Color bands and scaling are identical to Figure 4.3.

values. Nevertheless, they are consistent, justifying a combination of both lensing and kinematics constraints.

Our kinematics model prefers an anisotropy $\beta_{\text{sph}} < 0$, implying $\sigma_{\theta}/\sigma_r > 1$. This preference only strengthens when lensing constraints are included, see Table 4.2. [345] similarly found $\sigma_{\theta}/\sigma_r > 1$ in both ground- and space-based IFU measurements of a galaxy-scale lens. While the PSF size σ_{PSF} is entirely unconstrained, it fortunately demonstrates almost no degeneracy with any other parameters.

4.4.3 Combined Strong Lensing and Kinematics Results

The results of the joint model, combining strong lensing and stellar kinematics, are the primary result of this work. As in Section 4.4.1, we run two separate `emcee` samplers of 48 walkers each to verify the robustness of these results. Reassuringly, `emcee` was able to sample the joint model more efficiently than the lensing-only model, despite having slightly higher dimensionality and a more expensive like-

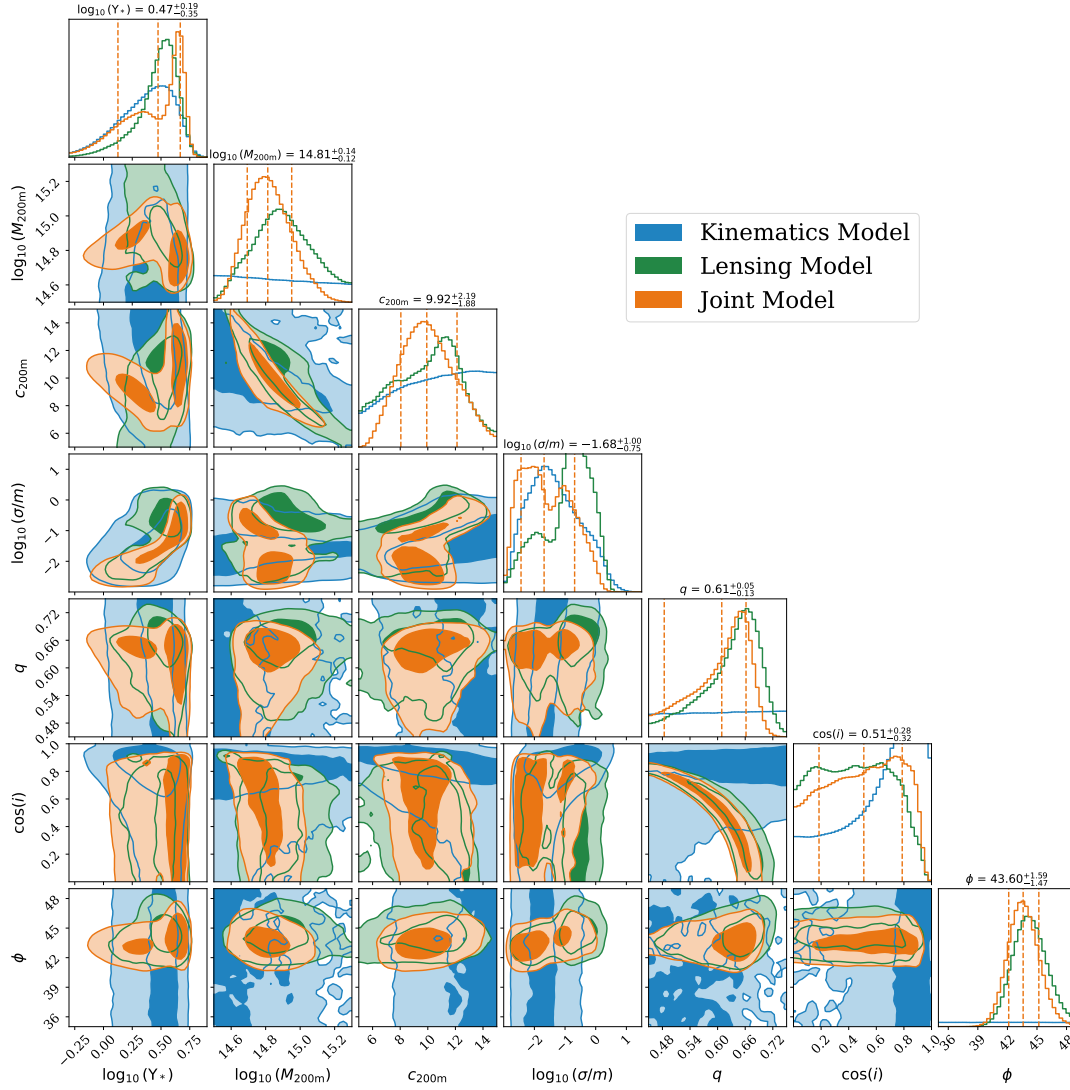


Figure 4.11: Posteriors of shared parameters across all models. For every pair of parameters, colored contours show the 1- and 2- σ posteriors. At the top of each column, the median and $\pm 1\sigma$ posteriors are listed for the joint model, and marked with dashed vertical lines.

likelihood. This suggests the kinematics constraints contribute to a ‘smoothing-out’ of the likelihood surface. The two joint samplers both ran for over 660,000 iterations, yielding a mean autocorrelation time across all parameters of $\sim 21,400$ and $\sim 22,900$ iterations, substantially shorter than for the lensing-only model. Both independent sampling runs yielded nearly identical posteriors, and we consider these posteriors to be robust.

The predicted critical curves (positions of infinite magnification) from this model are displayed in Figure 4.10. To create this plot, the magnification field $\mu(\vec{\theta})$ was computed from 10,000 randomly drawn samples of the posterior. The median ($\pm 1\sigma$) critical curves are the contours where the median (15.9%, 84.1% quantile) of μ^{-1} is zero. As expected, the tangential critical curve divides the southern arc neatly into two images of positive and negative magnification, and the radial critical curve divides the central and North-East images near a point where their light is dimmest. These curves are very consistent with those derived from the lensing-only model, which is not overlaid to improve the clarity of the plot. The predicted magnification at the conjugate points, however, differs between the joint and lensing-only models; see Section 4.4.5 for further discussion. The best-fit kinematics map from this joint model is nearly identical to that of the kinematics-only model, with a (non-reduced) χ^2 of 45.5, compared to 44.3 for the kinematics model shown in Figure 4.9.

This joint model yields constraints of $\log_{10} \Upsilon_* = 0.5^{+0.2}_{-0.4}$, $\log_{10}(M_{200\text{m}}/M_\odot) = 14.8 \pm 0.1$, $c_{200\text{m}} = 10 \pm 2$, and $\log_{10}(r_1/\text{kpc}) = 1.6^{+0.7}_{-1.0}$. For numerical constraints on other parameters, see Table 4.2. Curiously, the joint posteriors exhibit a strong bimodality absent from either the kinematics- or lensing-only models. This is most evident in the Υ_* - $M_{200\text{m}}$ space, see Fig. 4.11. In addition, the stellar mass-to-light ratio Υ_* is strongly degenerate with the SIDM cross section σ/m , with higher

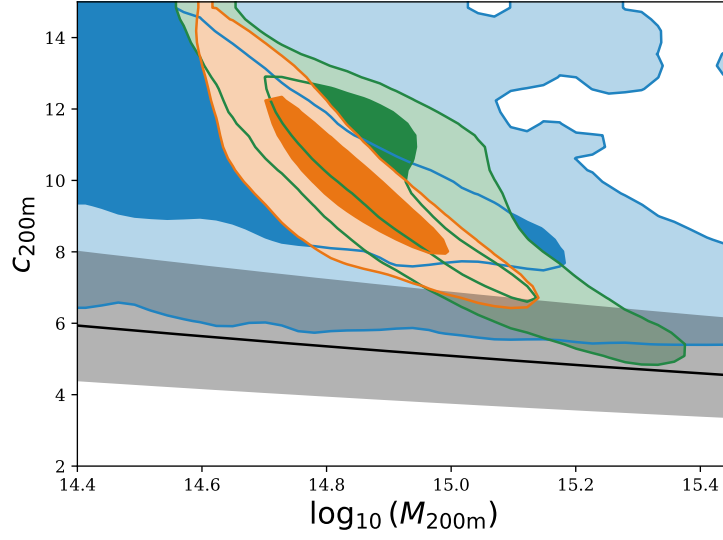


Figure 4.12: Mass-Concentration: A comparison of the posteriors on $M_{200\text{m}}$ and $c_{200\text{m}}$ in this work to theoretical predictions. The mass-concentration relation of L16 is displayed in black, with a shaded band showing 0.13 dex scatter. The L16 relation was computed at the cluster redshift, and translated to $M_{200\text{m}}, c_{200\text{m}}$ using Colossus [112].

cross sections occurring exclusively at high Υ_* .

While the kinematics and lensing models alone both prefer a cluster halo with relatively high concentration, the significance of this preference increases in the joint model. The posteriors of all models are reproduced in Figure 4.12, and compared to the expected concentration from [228] (L16) with a scatter of 0.13 dex. These choices are identical that used in [317]. The joint model prefers a halo concentration around $\sim 2\sigma$ higher than the L16 relation. Strong lensing has a strong selection function for halos that are massive, elongated along the line-of-sight, or, indeed, highly concentrated, so this result is not surprising. See Section 4.5 for further discussion.

4.4.4 External Shear

The lensing-only and joint models above yielded external shear with magnitude $|\gamma_{\text{ext}}|$ of 0.09 ± 0.02 and 0.07 ± 0.01 , respectively. The external shear term is intended to represent the contribution of cosmic shear sourced by large scale structure, and other mass near the line of sight, which are fundamental systematic uncertainties in strong gravitational lensing. While the motivation of this approach is to generically capture and account for any such perturbations, [125] has demonstrated in galaxy-galaxy lenses that the magnitude and direction of SL-derived γ_{ext} shows little correlation with known weak lensing shear, and is instead highly correlated with the elliptical orientation of the lens itself. They suggest that, in practice, the external shear accounts for structure of the lens mass which parametric models fail to properly describe.

To investigate whether the measured γ_{ext} may be explained physically, we conduct the following test. In the MACSJ0138 field, there is a luminous red galaxy $\sim 2.5'$ to the south, which is likely associated with the MACSJ0138 cluster and may host a significant DM halo of its own. We conduct another **emcee** sampling of the joint likelihood, including a further NFW halo of fixed $c_{200m} = 6$ and varied $M_{200m} = M_c$ at the position of this bright LRG, (RA, Dec) =(01:38:06, -21:57:51). To simplify the parameter space, we identify six parameters with negligible degeneracy, and fix them to the median values obtained by the joint sampling in Section 4.4.3. These parameters are α , r_s^{J4} , $\sigma_{\text{LT}}^{\text{J1,J3,J4}}$, σ_{PSF} .

The results of this test are illustrated in Figure 4.13, showing posteriors on the position angle and magnitude of γ_{ext} , as well as the mass of the nearby NFW halo, M_c . Acebron et al. [7] independently conducted a similar test, sampling separate models with a mass component associated with the same luminous galaxy to the south. The γ_{ext} measured by their **reference** model is overlaid for comparison.

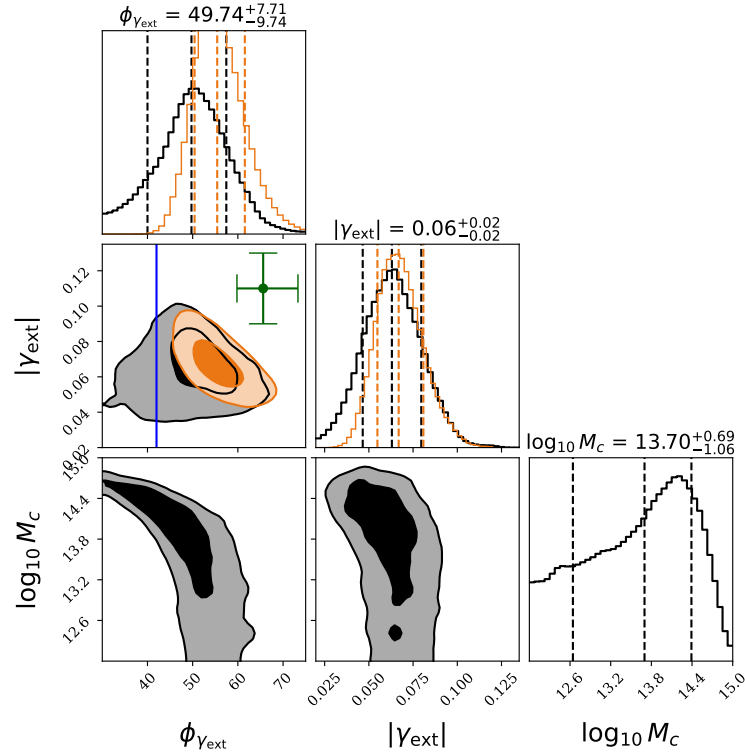


Figure 4.13: External Shear: The direction and magnitude of the external shear γ_{ext} . Black contours show the results of the test described in Section 4.4.4, and orange contours denote the results of the joint model. In $\phi_{\gamma_{\text{ext}}}$ - $|\gamma_{\text{ext}}|$ space, a blue line marks the orientation of the central galaxy of MACSJ0138, and a green point marks the result of [7].

Allowing for the nearby mass slightly decreases the derived shear magnitude, suggesting this mass can only partly explain the observed shear. The data clearly prefer a substantial mass at the location of the LRG, with a mass up to $\sim 10^{14.4} M_{\odot}$. While the effect on the shear magnitude $|\gamma_{\text{ext}}|$ is slight, the nearby mass has a significant effect on the position angle of the shear, extending the posteriors to include the orientation of the central galaxy of MACSJ0138. Similarly, Etherington et al. [125] found that γ_{ext} frequently aligned with the lens orientation. Our lensing constraints additionally show a clear preference for the presence of this other mass: this test yielded a minimum lensing χ^2 of 3.5, significantly lower than the best lensing χ^2 of 4.08 produced by the fiducial joint model. Nevertheless, the presence of the extra mass results in little change to our parameters of interest compared to the joint model; the posteriors on Υ_* , $M_{200\text{m}}$, $c_{200\text{m}}$, and σ/m are all consistent within 1σ .

4.4.5 Supernovae Requiem and Encore

Here we present model-predicted time delays and magnifications of both lensed SNe. The predicted magnifications of previously observed SNe images are shown in Figure 4.14. For clarity, we use the same labels as Acebron et al. [7], Ertl et al. [123], and mark images which are not used as model constraints with an asterisk (*). In this nomenclature, images (2a, 2b, 2c) and (1a, 1b) correspond respectively to (SN-R.2, SN-R.1, SN-R.3) and (SN-E.2, SN-E.1) in our Table 4.1. We additionally include predicted images (d*) and (e*), with predicted positions in the central and North-East radial arcs where neither supernova has yet been observed. While the lensing-only model provides very poor constraints on the magnifications, our joint model including kinematics yields clear predictions.

Time delays predicted by the joint model are listed in Table 4.3. These are

Feature Group	Requiem (2)		Encore (1)	
	Delay (d)	Median	Delay (d)	Median
a	55 ± 6	Jun 2016	41 ± 4	Sept 2023
b	0	Apr 2016	0	July 2023
c	-31 ± 5	Mar 2016	-300^{+31}_{-34}	Sept 2022
d	4245^{+343}_{-322}	Nov 2027	3507^{+291}_{-280}	Feb 2033
e	4372^{+339}_{-323}	Apr 2028	3819^{+307}_{-283}	Jan 2034

Table 4.3: Time Delays: The predicted time delays between all five images of both lensed SNe, given in days. Delays are normalized relative to the southernmost image (b). The ‘Median’ columns give the median date of predicted peak SN brightness, rounded to the month. Image groups (d) and (e) are anticipated in the next decade; their dates are shown in bold.

computed using the fixed P15 cosmology, with $H_0 = 67.7 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and a time-delay distance $D_{\Delta t}$ of $\sim 1860 \text{ Mpc}$ for this lens and source. As in Rodney et al. [320], time delays are normalized relative to the southernmost image of both SNe, labeled (b) here. Dates of peak brightness are estimated from the observed SN ages reported by Rodney et al. [320] and Dhawan et al. [105] for Requiem and Encore, respectively. These results imply the next observed image will be feature (d) of SN Requiem in late 2027 with ~ 1 year uncertainty, approximately two years from the date of this publication.

4.5 Discussion

We have presented independent and joint constraints from strong lensing and stellar kinematics models. The constraints on parameters of interest (chiefly the stellar mass-to-light ratio Υ_* , halo mass and concentration, and SIDM cross-section) are remarkably consistent between the individual probes. The joint model provides notably tighter constraints on the mass and concentration, and a more stringent upper limit on the SIDM interaction cross-section. As a means of com-

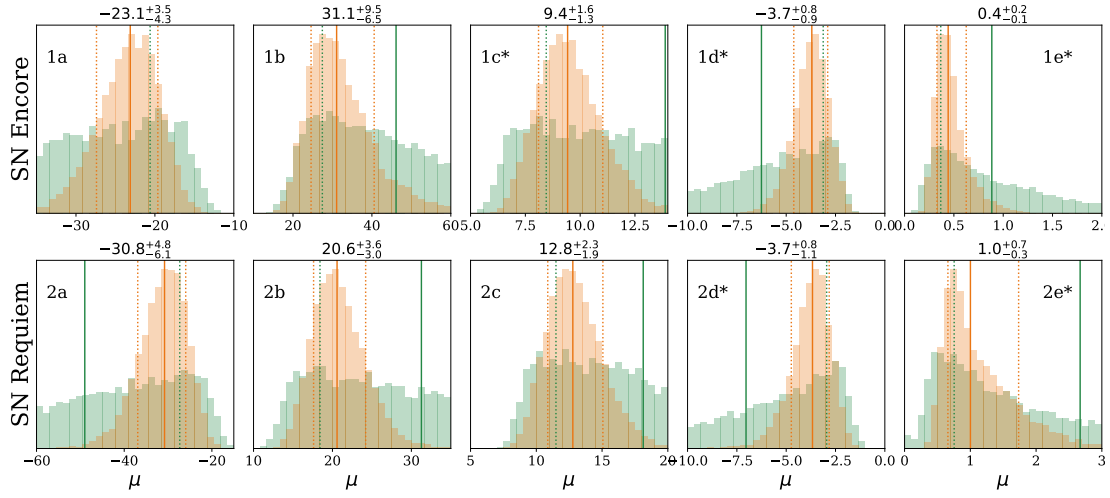


Figure 4.14: Model SNe Magnifications: The magnification of the observed and predicted images of SNe Requiem and Encore. Each histogram is generated from 10,000 randomly chosen samples of the `emcee` chains. The posteriors of the lensing-only and joint model are shown in green and orange respectively, and their median (1σ) limits are overlaid as solid (dotted) vertical lines. The title of each subplot lists the result of the joint model. Images which are not used to constrain the model are marked with an asterisk (*), including image groups (d*) and (e*) which have not been observed.

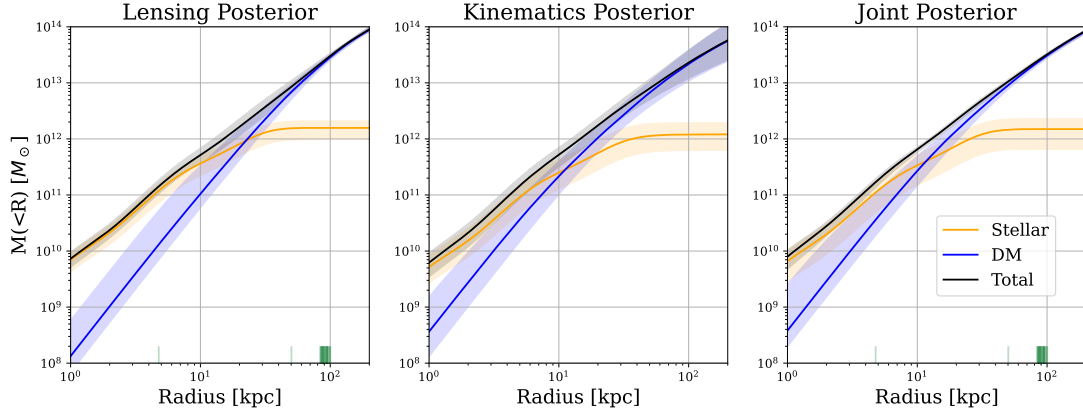


Figure 4.15: Enclosed Mass: Posteriors on the enclosed 3D mass for all three models presented here. At the bottom of each plot, a shaded blue region represents the radii constrained by stellar kinematics, and vertical green lines represent the projected radii of strong lensing constraints.

parison between all three models in this work, the enclosed 3D mass relation $M(< r)$ for the DM, BCG stellar, and total mass are displayed in Figure 4.15.

Notably, our inferred NFW concentration on the cluster dark matter halo is $\sim 2\sigma$ higher than the predicted cosmological mass-concentration relation (see Fig. 4.12). However, this does not represent a significant tension with conventional cosmological theory. It is widely known that strong lensing-selected halos are biased to high concentrations, see for example Meneghetti et al. [237]. Additionally, dynamically relaxed halos are expected to have higher concentrations [237, 80, 96]; our result on c_{200m} may be further evidence of MACSJ0138’s relaxed state.

Our results exhibit a strong bimodality in the stellar mass-to-light ratio Υ_* , and less so in the SIDM cross section. These modes correspond to a strong degeneracy between Υ_* and σ/m . This degeneracy is expected. Both probes used here (lensing and kinematics) are indifferent to any distinction between baryonic and dark mass, and measure the total mass present. A high stellar mass atop a cored DM density may resemble a low stellar mass in a steep, cuspy DM density profile, yielding this degeneracy. Therefore, our method’s ability to distinguish baryonic and dark mass is limited. Better knowledge of Υ_* in central cluster galaxies may therefore help constrain the inner DM slope.

In this work, the lensing and kinematics data vectors provided similar data degrees of freedom (d.o.f.), at 25 and 38 respectively. Pixel-level modeling of strong lensing features, by contrast, typically uses data vectors of hundreds of points or more. Wang et al. [393], which also studied a strong lensing cluster, notes this as a potential difficulty in combining strong lensing and kinematics measurements. Combining lensing data with hundreds of data d.o.f., with kinematics likelihoods of typically a few dozen d.o.f., can therefore yield minimal change over a lensing-

only model, as the χ^2 of the lensing data will typically dwarf the χ^2 of the much smaller kinematics data vector. Wang et al. [393] therefore do not combine stellar kinematics with pixel-level image modeling, and only present joint posteriors for a conjugate point likelihood with kinematics, similar to what was done here. Pixel-level SL modeling is beyond the scope of the present work, but we note this as a consideration for future work combining kinematics and strong lensing.

As an additional test, we sampled a variation on our model which explicitly represented the mass of a nearby bright galaxy in Section 4.4.4. While this choice yielded minimal change to our key parameters (chiefly Υ_* , $M_{200\text{m}}$, $c_{200\text{m}}$, and σ/m), implying our results are robust, our likelihood nonetheless strongly preferred the presence of another halo. Future precision models of MACSJ0138 may benefit from explicitly including this halo.

4.5.1 Consistency with Other Mass Models

Shortly after the completion of this work, Suyu et al. [367] and Pierel et al. [290] reported constraints on H_0 from both lensed SNe in this system, combining seven independent models using six different codes. Those models included both recent analyses referenced in this work [123, 7]. Additionally, Pierel et al. [290] reported a measurement of the time delay between SN Encore images 1a and 1b of $\Delta t_{1b,1a} = -39.8^{+3.9}_{-3.3}$ days, consistent with our model predicted value of $\Delta t_{1a,1b} = -\Delta t_{1b,1a} = 41 \pm 4$ days (see Table 4.3). Note that our prediction was computed with a fixed $H_0 = 67.7$ km/s/Mpc; a higher expansion rate would lower $|\Delta t|$.

Here we compare our results with other published mass models. Common diagnostics used in cluster-scale strong lensing include the critical curves (the contours of infinite magnification), and the root-mean-square scatter between predicted and

Feature	Lensing Best-Fit	Lensing Posterior	Joint Best-Fit	Joint Posterior
SN-E	0.12	$0.11^{+0.08}_{-0.07}$	0.09	$0.11^{+0.07}_{-0.06}$
SN-R	0.12	$0.14^{+0.13}_{-0.07}$	0.05	$0.12^{+0.10}_{-0.06}$
A	0.20	$0.21^{+0.16}_{-0.10}$	0.24	$0.21^{+0.14}_{-0.10}$
B	0.17	$0.17^{+0.15}_{-0.09}$	0.17	$0.17^{+0.13}_{-0.09}$
C	0.01	$0.05^{+0.03}_{-0.02}$	0.03	$0.05^{+0.03}_{-0.02}$
F	0.04	$0.06^{+0.04}_{-0.03}$	0.06	$0.05^{+0.04}_{-0.03}$
G	0.08	$0.11^{+0.07}_{-0.04}$	0.08	$0.10^{+0.05}_{-0.04}$
H	0.08	$0.10^{+0.07}_{-0.04}$	0.05	$0.09^{+0.05}_{-0.03}$
J	0.06	$0.09^{+0.07}_{-0.04}$	0.03	$0.08^{+0.05}_{-0.03}$
bulge	0.10	$0.20^{+0.10}_{-0.07}$	0.08	$0.20^{+0.10}_{-0.06}$
all	0.11	$0.16^{+0.06}_{-0.04}$	0.11	$0.16^{+0.05}_{-0.03}$

Table 4.4: Image-plane RMS: The RMS distance between observed and predicted positions of each feature group. The last row gives the combined RMS for all features. All entries are in arcseconds. The ‘Best-Fit’ columns give the result of the maximum likelihood sample.

observed images, commonly called the RMS. The critical curves predicted by our joint model were presented above in Figure 4.10. While the critical curves can only enable a qualitative comparison, we note that ours appear very consistent with the newest mass models, see Fig. 1 of [7] and Fig. 8 of [123]. Additionally, we present the image-plane RMS of all features used to constrain our model in Table 4.4. We caution against direct comparisons of RMS between our work and other mass models, as we use an entirely different set of position constraints and sample a different likelihood. Nevertheless, both our lensing-only and joint models yield a small, best-fit RMS of 0.11 arcsec.

Reliable estimates of the lensing magnification are essential to any study of lensed sources, such as [260], and for accurate photometric classification of the observed SNe [320]. Our joint model magnifications are in clear agreement with those reported in Suyu et al. [367], including [7, 123], with the magnifications of

all observed sources within the 1σ errors of those in [7], and slightly below the absolute magnifications predicted by [123]. See Fig. 6 of [7] and Table 9 of [123] for comparison. The absolute magnifications $|\mu|$ for predicted images 1d* and 1e* of SN Encore are nearly identical to [7] as well⁷. Compared to [123], we predict $|\mu|$ approximately 20% lower for the observed image groups (a,b,c). Our predictions for (d,e) are more discrepant, where our predictions are 60% higher and $\sim 2\text{-}5\times$ lower respectively.

Our results, however, disagree significantly with the earlier model in Rodney et al. [320], which gave $|\mu|$ of observed images roughly 3-5x lower than ours, and a significantly longer time-delay. Acebron et al. [7] similarly noted that their predicted magnifications strongly disagree with previously reported lens models in [320, 259]. We do not consider this problematic, as we believe our mass model to be higher quality than that in [320], which predicted an unreasonably large ellipticity for the overall halo and a BCG more massive than the overall halo. Intriguingly, our results in this work also imply a very different time-delay baseline from [320], which predicted a roughly 20-year delay between the observed images of SN Requiem and the next predicted image, placing the recurrence in the mid-2030s. Our model yields a much lower delay of roughly ~ 12 years, in which case SN Requiem would be seen again in the next few years, likely before 2030 (see Table 4.3). Our time-delay prediction is broadly consistent with the meta-analysis in Suyu et al. [367], which predicts SN Requiem will appear in $\sim$ April-December 2026 for $H_0 = 73 \text{ km s}^{-1} \text{ Mpc}^{-1}$, or $\sim$ March-November 2027 for $H_0 = 67 \text{ km s}^{-1} \text{ kpc}^{-1}$.

We stress that our group has independently measured the Faber-Jackson relation of the members of MACSJ0138 [132], chosen cluster members to include in

⁷Curiously, [7] lists the demagnified central image 1e* as having opposite sign, with a negative magnification. We consider this to likely be a mistake, demagnified central images always have positive parity.

the SL mass model, identified multiply imaged features to use as constraints, and uses an entirely different modeling code and parametrized mass model from other work on MACSJ0138. While agreement between our predicted magnifications and those in [7, 123] does not exclude systematic effects distorting the true magnifications, it is encouraging that these models agree with ours despite substantial differences in approach. Acebron et al. [7] suggests that the discrepancy between their reported magnifications and earlier work could be partly explained by the inclusion of newly identified sources at different redshift. However, this is unlikely to be true, as our model agrees with theirs despite not using the newly discovered sources. Lens modeling choices still have a large effect on predicted magnifications near the critical curves: our model agrees more closely with Acebron et al. [7] (which similarly used one cluster halo and an external shear) than it does with Ertl et al. [123] (which used two halos to describe the cluster DM, and no external shear).

4.6 Conclusions

We have presented a detailed mass model of MACSJ0138, combining strong gravitational lensing and stellar kinematics measurements. With the aim of refining previous SIDM constraints on cluster scales, we have improved on the methodology of previous studies [18, 326]. From our combined model, we report a 95% CL upper limit on the SIDM cross section of $\sigma/m < 0.613 \text{ cm}^2/\text{g}$, at an interaction velocity $\langle v_{\text{pair}} \rangle < 2090 \text{ km/s}$. Though in this work, we studied a single system, we aim to apply this methodology to a sample of systems in the future. Further, these results and methods are of significant interest beyond SIDM. Effectively combining strong lensing and stellar kinematics is widely acknowledged as key to advancing the state-of-the-art of SL modeling. The inclusion of kinematic

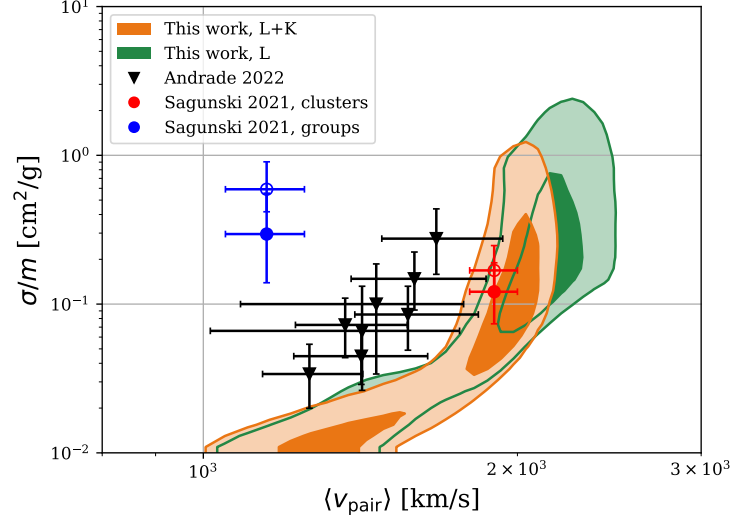


Figure 4.16: SIDM Interaction: Constraints on the magnitude and velocity scale of SIDM interactions, compared to previous work. Black triangles show the eight clusters studied in [18], while the blue and red points show the full constraints from eight groups and seven clusters in [326]. Compare to Figure 3 of [326].

measurements has the potential to break the well-known mass sheet degeneracy, improving SL-derived cosmological constraints, particularly for time delay cosmography. The MACSJ0138 system is of intense interest, as the first known SL system to host multiple strongly lensed SNe; these results inform future constraints on the Hubble constant from this system.

Our combined kinematics and lensing model places considerably tighter constraints on SIDM interactions than either measurement alone. Our posteriors on the magnitude and velocity scale of self-interaction are shown in Figure 4.16, with results from [326, 18] overlaid for comparison. For a detailed list of posteriors on measured parameters, along with derived quantities of interest, the reader is referred to Table 4.2. The lensing results alone exhibit a bimodality in σ/m , with a marked preference for the higher mode near $\sigma/m = 1 \text{ cm}^2/\text{g}$. The inclusion of kinematics constraints suppresses this mode. Notably, our joint results still

exhibit a clear bimodality, with one mode at high values of Υ_* , σ/m , and $\langle v_{\text{pair}} \rangle$, and one at lower values of those three parameters. This degeneracy resembles that of the aggregate sample of clusters in Andrade et al. [18], though offset in $\langle v_{\text{pair}} \rangle - \sigma/m$ space. Improved knowledge of the true stellar mass-to-light ratio of BCGs could therefore break this degeneracy, providing tighter constraints on the inner dark matter halo.

We also note that there is good agreement between our results and those of [326, 18]. While there is some velocity offset, each constraint is assumed to be velocity-independent locally, but all constraints are consistent with the typical $1/v^4$ global velocity-dependent behavior of SIDM theory curves in this large velocity limit [381].

Extending the methodology in this work to a sample of clusters would greatly improve constraints on SIDM. However, challenges remain. The selection effects inherent in such a sample remain a concern, as strong lensing halos are preferentially concentrated and elongated along the line-of-sight. Fortunately, other probes of the cluster density profile exist with very different systematics, such as X-ray observations of the hot intracluster medium (ICM) and weak lensing measurements. The density profile inferred from X-ray observations of the ICM could be particularly powerful for SIDM studies, as it constrains the cluster halo at similar radii as SL. Weak lensing alone is unlikely to provide meaningful constraints on SIDM [318], but is highly useful in combination with the other probes here to ensure the inferred cluster mass is accurate on larger scales.

We additionally highlight our forecast for the next predicted images of SNe Requiem and Encore. The study which discovered SN Requiem [320] predicted a ~ 20 year time delay, with the next images appearing in the mid to late 2030s. Our results differ significantly, implying a baseline of roughly 12 years, with SNe

Requiem and Encore likely reappearing in 2027-2028 and 2033-2034, respectively. These predictions use a fixed cosmology with $H_0 = 67.7 \text{ km s}^{-1} \text{ Mpc}^{-1}$; a larger expansion rate would shorten these time delays. Observations of these images would enable a time-delay measurement of the Hubble constant of unprecedented precision from a single system. Monitoring for the reappearance of SN Requiem could therefore provide enormous benefit to cosmology sooner than previously expected.

This work is the first to leverage spatially resolved stellar kinematics to model the MACSJ0138 cluster. While the kinematics map measured in this work was computed from MUSE observations, there exist newer, deeper MUSE observations on this system. Future work leveraging these data has enormous potential, especially given recent progress in rigorously characterizing the use of stellar kinematics for time-delay lenses [343, 193, 345]. Our model contributes to the growing body of published mass models on MACSJ0138 [320, 259, 7, 123], a remarkable lens system enabling an enormous variety of science.

4.7 Supplementary

4.7.1 Isothermal Jeans modeling

To describe the effect of SIDM on the cluster density profile, we utilize the isothermal Jeans formalism briefly described above in Section 4.3.1. This results in a piecewise DM density profile:

$$\rho_{\text{SIDM}}(r) = \begin{cases} \rho_{\text{iso}}(r) & r < r_1 \\ \rho_{\text{NFW}}(r) & r > r_1 \end{cases} \quad (4.12)$$

Where ρ_{NFW} denotes the usual Navarro-Frenk-White density profile, and ρ_{iso}

describes the thermalized region, obeying the isothermal Jeans equation with an isothermal velocity of σ_0 . We make the same assumption as in [317] and [326] that the expected pairwise particle velocity is $\langle v_{\text{pair}} \rangle = \frac{4\sigma_0}{\sqrt{\pi}}$. The SIDM cross sections in this work are reported as $\sigma/m = \langle \frac{\sigma v}{m} \rangle \langle v_{\text{pair}} \rangle^{-1}$ (c.f. Sagunski et al. [326] eq. 6). To ensure consistency with the outer NFW halo, boundary conditions are enforced at r_1 : the enclosed mass $M(< r_1)$ and density $\rho(r)$ are identical to an NFW halo.

We use a similar change of variables as in Section 2.2 of Robertson et al. [317], modified to account for the contribution of a baryonic density profile $\rho_b(r)$. In this work, we use the stellar density profile of the BCG as $\rho_b(r)$, inferred from the the MGE approximation of the BCG and Υ_* . From the Jeans equation for a thermalized DM profile ρ_{iso} ,

$$\frac{d}{dr} \left(r^2 \frac{d \ln \rho_{\text{iso}}}{dr} \right) = - \frac{4\pi G r^2 (\rho_{\text{iso}} + \rho_b)}{\sigma_0^2} \quad (4.13)$$

we introduce the change of variables $x = r/r_1$, $y = \ln(\rho_{\text{iso}}/\rho_0)$, $N_0 = \rho_0/\rho_1$, yielding:

$$\frac{d^2 y}{dx^2} = - \frac{2}{x} \frac{dy}{dx} - C \left(e^y + \frac{\rho_b(xr_1)}{\rho_0} \right) \quad (4.14)$$

$$C = \frac{4\pi G \rho_1 r_1^2 N_0}{\sigma_0^2} \quad (4.15)$$

To numerically solve the equation above for $\rho_{\text{iso}}(r)$, we use the `scipy` function `solve_bvp`. This solver simultaneously fits the above ODE and the free parameters N_0 and σ_0 , subject to the following boundary conditions:

$$\begin{aligned}
y|_{x=0} &= 0 & M|_{x=0} &= 0 \\
\frac{dy}{dx}|_{x=0} &= 0 & y|_{x=1} &= -\ln(N_0) \\
M|_{x=1} &= M_{\text{NFW}}(< r_1)
\end{aligned}$$

As may be apparent from the discussion above, the SIDM cross section σ/m is not directly sampled. Rather, the SIDM parameters are computed from Υ_* , M_{200m} , c_{200m} , and the radius r_1 . (This sampling via r_1 is termed the ‘outside-in’ method in Robertson et al. [317], as opposed to the ‘inside-out’ method computing directly from σ/m .) This poses a problem: since σ/m is a derived parameter, we can only place a prior on r_1 , rather than σ/m . We desire a flat prior on $\ln(\sigma/m)$, which would imply the following prior on $\ln r_1$:

$$P(\ln r_1) = \frac{d(\sigma/m)}{dr_1} \frac{r_1}{\sigma/m} \quad (4.16)$$

Though we sample from a flat prior $P(\ln r_1) = \text{const}$, we enforce our desired prior by adding a weighting $\ln \left(\frac{d(\sigma/m)}{dr_1} \frac{r_1}{\sigma/m} \right)$ to the log likelihood.

4.7.2 Triaxiality and Concentration Bias

While it is well known that strong lensing selected halos are biased to high concentrations, they additionally exhibit a severe triaxial bias: halos elongated along the line-of-sight are more likely to exhibit strong lensing. In addition, this triaxial bias is likely to artificially inflate the apparent concentration of lens halos,

potentially as much as +60% (Andrade et al. [18], Appendix C).

We allow for an elliptical halo in three dimensions, rather than limiting ellipticity to the plane of the sky. We modify both our strong lensing and stellar kinematics models accordingly. The majority of cluster halos in the universe are expected to be roughly prolate [53]; as such, we allow for a prolate, rather than oblate or fully triaxial ellipticity. (In addition, the axisymmetric solutions of **JAM**, used for kinematics modeling, only allow for oblate or prolate halos, not fully triaxial.)

The introduction of a 3D ellipticity to matter halos is rarely treated with the subtlety it deserves. [18] marginalizes over LOS ellipticity by inflating the lensing convergence $\kappa_e(\theta) = \kappa(\theta)/s$, for an axis ratio $s < 1$. While this correctly inflates the observed lensing, the mass and concentration of the halo are no longer meaningful: the apparent lensing now corresponds to a halo of mass M/s . Likewise, a common approach takes a spherical density profile $\rho_{\text{sph}}(r)$, and introduces an elliptical radius $r_e^2 = \frac{x^2}{q^2} + \frac{y^2}{q^2} + z^2$ such that $\rho_e(\vec{x}) = \rho_{\text{sph}}(r_e)$.⁸ However, this similarly fails to preserve important properties of the halo: the total halo mass is now $M_e = q^2 M_{\text{sph}}$.

In order to preserve the spherical overdensity (SO) halo definition for elliptical halos, and therefore the meaning of the mass and concentration (M_Δ, c_Δ) , we introduce ellipticity as follows. A SO halo of volume $V_\Delta = \frac{4\pi}{3} R_\Delta^3$ has a mass of $M_\Delta = \Delta V_\Delta \rho_b(z)$, where $\rho_b(z)$ is a background density, commonly taken to be either the mean matter density $\rho_m(z)$ or the critical density of the universe $\rho_c(z)$, and Δ is the overdensity of the halo within R_Δ compared to this reference. We wish to preserve $V_{\Delta,e} = V_{\Delta,\text{sph}}$ and $M_{\Delta,e} = M_{\Delta,\text{sph}}$. For a prolate halo with axis ratio $q_{3d} < 1$, this requires:

⁸For $q < 1$, this corresponds to a prolate (‘pickle’-shaped) halo, and for $q > 1$ this represents an oblate (‘pancake’-shaped) halo.

$$r_e^2 = \frac{x^2}{q^{2/3}} + \frac{y^2}{q^{2/3}} + q^{4/3} z^2 \quad (4.17)$$

$$\rho_e(\vec{x}) = \rho_{\text{sph}}(r_e) \quad (4.18)$$

Where the major axis is aligned with the z -axis. We introduce an angle i between z and the LOS, such that z' becomes the true LOS and the observed coordinates on the sky are (x', y') . Strong lensing, however, is dependent on the projected mass density $\Sigma(x', y') = \int \rho(x', y', z') dz'$. We will use lowercase r to represent spherical 3D radii, and distinguish projected 2D radii with a capital R . The projected mass density of our prolate halo is therefore:

$$\Sigma_e(R_e) = \frac{q_{3d}^{1/3}}{\sqrt{\sin^2 i + q_{3d}^2 \cos^2 i}} \Sigma_{\text{sph}}(R_e) \quad (4.19)$$

$$R_e^2 = \frac{x'^2}{q_{3d}^{2/3}} + \frac{q_{3d}^{4/3} y'^2}{\sin^2 i + q_{3d}^2 \cos^2 i} \quad (4.20)$$

$$q_{2d} = \frac{q_{3d}}{\sqrt{\sin^2 i + q_{3d}^2 \cos^2 i}} \quad (4.21)$$

Where Σ_{sph} is the projected density for a spherical halo, and q_{2d} is the apparent projected ellipticity.⁹ For any basis function for a spherically symmetric, 3D density $\rho(r; s)$ with a scale radius s , and corresponding projected density $\Sigma_{\text{sph}}(R; s)$, 3D ellipticity can thus be trivially introduced by Eqs. 4.19-4.21. We take advantage of this property to approximate our 3D, elliptical DM halo as a sum of Cored Steep Ellipsoids (CSEs) or Gaussians, in order to model the observed lensing and dynamics, respectively. See Sections 4.3.2 and 4.3.3.

⁹Compare Eq. 4.21 to Cappellari [74] Eq. 36.

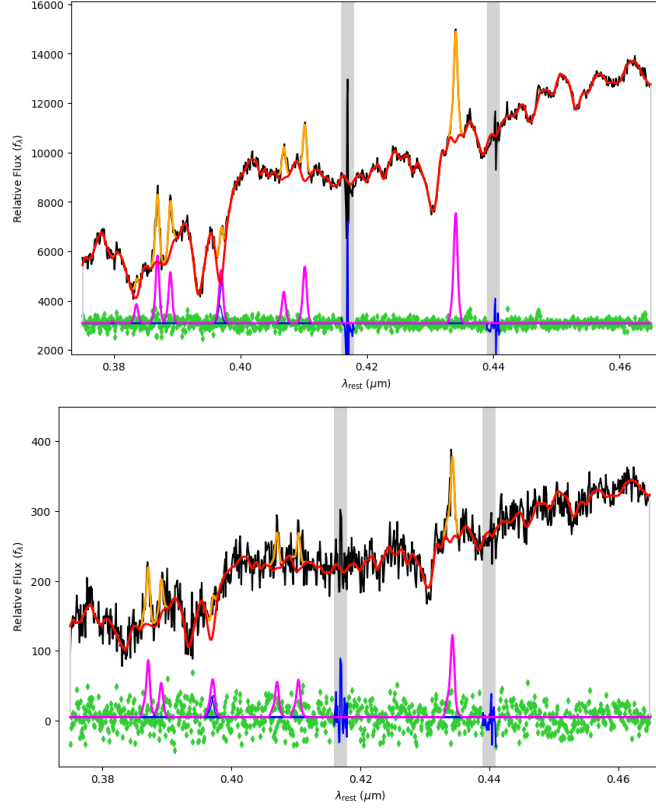


Figure 4.17: Example `ppxf` fit results. Black represents data, red represents stellar model fit, orange represents stellar + gas model fit. **Top:** An example ‘total’ fit to all Voronoi bins combined, used to determine the optimal stellar template. Fit yielded a reduced χ^2 of 1.62. **Bottom:** An example fit to an individual Voronoi bin. This fit yielded a reduced χ^2 of 1.01.

4.7.3 Robustness of the Measured Stellar Kinematics

Measurements of stellar kinematics can be highly sensitive to analysis choices, and we have drawn on recent work, particularly [193, 343], to ensure our measurements are as robust as possible. To illustrate the quality of our spectral fits, two sample `ppxf` fits in this work are shown in Figure 4.17. These demonstrate both the overall fit to the entire BCG used to determine an optimal spectral template, and a characteristic fit to an individual Voronoi bin.

The relative uncertainty of the stellar velocity dispersion measured in each

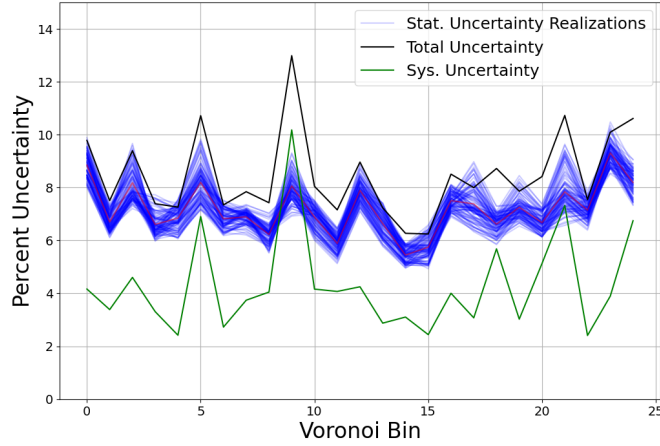


Figure 4.18: Relative uncertain of the velocity dispersions measured in this work, for each voronoi bin. The total (systematic + statistical) uncertainty is shown in black; the statistical uncertainties of all 108 realizations described in Section 4.3.3 are shown in blue; and the systematic uncertainty is shown in green.

Voronoi bin is shown in Figure 4.18. Total uncertainty is of order $\sim 10\%$ in each bin, and systematic uncertainty is subdominant to statistical uncertainty in all bins save for bin 9, the northernmost bin. Knabel et al. [193] has rigorously explored systematic uncertainty in similar measurements with the goal of sub-percent precision σ_v measurements, and demonstrated that the choice of stellar template library is often the dominant source of systematic uncertainty. In this work we exclusively used the XSL template library, limiting our ability to characterize this uncertainty. However, the same paper demonstrated such an effect on measurements using MUSE spectra was typically of order $\approx 3 - 4\%$, well below the statistical uncertainty in this work, and of similar scale to the systematic uncertainty we observe. However, our statistical uncertainty is large compared to [343, 193] in part because we use a less stringent SNR target inspired by [257]. A higher target spectral SNR threshold would yield fewer Voronoi bins, with lower statistical uncertainties. We conducted several informal tests varying the spatial mask used to Voronoi bin the BCG, and increasing the spectral SNR target: in

all cases, the kinematics maps produced were qualitatively consistent with the fiducial map used in this work.

Chapter 5

Dark Energy: Compound Lensing in Clusters

This chapter presents a spectroscopic survey of field galaxies and lensed sources in the vicinity of the strong lensing galaxy cluster known as the Carousel lens at $z=0.49$. Using both Gemini/GMOS slitmask spectra and deep VLT/MUSE observations, we bring the total number of lensed sources up to 13, including three which were not previously known from imaging observations but are apparent in the MUSE data as emission-line sources. Of these sources, 10 have confident redshifts, and an additional 2 have tentative redshifts from likely Ly α emission (including seven new redshifts determined here adding to those presented previously in [351]). The lensed sources span a redshift range from $z=0.96$ to 4.09 with most of them showing 3-5 images, including four sources displaying central or radial images. In total, we identify 43 images of these 13 sources. This lens system is remarkably symmetric and well-modeled by a simpler lens model than typical cluster lenses, and the large number of sources and their large range of redshifts make this cluster ideal for constraining cosmological parameters such as w and Ω_m as well as the cluster density profile. Additionally, we present a catalog

of 57 unlensed field galaxies with confident redshifts, of which 49 are associated with the cluster. We measure a cluster velocity dispersion of about 1100 km s^{-1} from which we estimate a halo mass $M_{200c} \approx 1.2 \times 10^{15} M_{\odot}$. This section includes a reprint of the following published material: O’Donnell et al. [269].

5.1 Introduction

Strong gravitational lenses with multiple source planes are potentially powerful probes of cosmology. The ratio of lensing strength between each source plane constrains the dark energy equation of state and matter density independent of the Hubble constant [e.g. 148, 88]. Strong lensing galaxy clusters are particularly powerful cosmic telescopes that can feature upward of tens of lensed sources, and cosmological constraints have been derived from a few multi-source plane lensing clusters [e.g. 183, 66, 67] and a group-scale system [50]. In particular, [67] use five strong lensing clusters and show that combined with cosmic microwave background (CMB) data these improve constraints on Ω_M and w . Galaxy-scale double-source plane lenses can enable similar measurements [91, 329, 55], but require much larger samples to attain similar constraining power. Cluster lenses have the advantage of a large number of sources, which give several cosmic baselines and break degeneracies in the mass model, but at the expense of more complicated mass models that e.g. must include cluster substructure.

We have identified an ideal, powerful, multi-source plane lensing cluster hosting more than 10 sources while having a very simple mass model [351]: the Carousel lens. This lensing cluster was independently identified in Dark Energy Spectroscopic Instrument (DESI) Legacy Imaging [172] and in the Dark Energy Survey [267]. Hubble Space Telescope (HST) imaging (HST proposal #16773; K. Glazebrook) revealed 10 probable lensed sources of which redshifts for five were

determined from shallow MUSE data (Prog. ID 0111.B-0400(H)) by [351]. [351] (Paper 0) also presented an initial lens modeling of this system, showing it to be well modeled by a simple, two-component mass model. A companion paper, Paper II [384], presents an updated lens modeling and cosmological constraints using four source planes. Even considering systematic uncertainties in the lens model and this limited number of sources, the Carousel lens gives strong constraints on cosmology.

In this paper, we present the results of new spectroscopic follow-up of this cluster, including deeper VLT MUSE data and Gemini GMOS targeted spectroscopy. These data allow us to identify new lensed sources, bringing the total to 13, and to determine the redshifts for additional sources, giving firm redshifts for 10 and probable redshifts for 2 more. We also determine the redshifts for 57 unlensed galaxies in the cluster field, of which 49 are cluster members.

The rest of the paper is structured as follows. In Section 2, we discuss the spectroscopic data and catalogs used. Section 3.1 presents analysis results for the lensed sources, while 3.2 presents redshift determinations for unlensed field and cluster galaxies. The cluster member catalog is used to model the cluster velocity dispersion in 3.3.

5.2 Data

We summarize spectroscopic observations used in this work below. A comparison of the FOV of both VLT/MUSE and Gemini/GMOS observations can be found in Figure 5.1. This work additionally makes use of HST WFC3 imaging in the F200LP and F140W bands (Prog. ID. 16773, PI Glazebrook), and Dark Energy Camera Legacy Survey (DECaLS) Data Release 10 object catalogs [102].

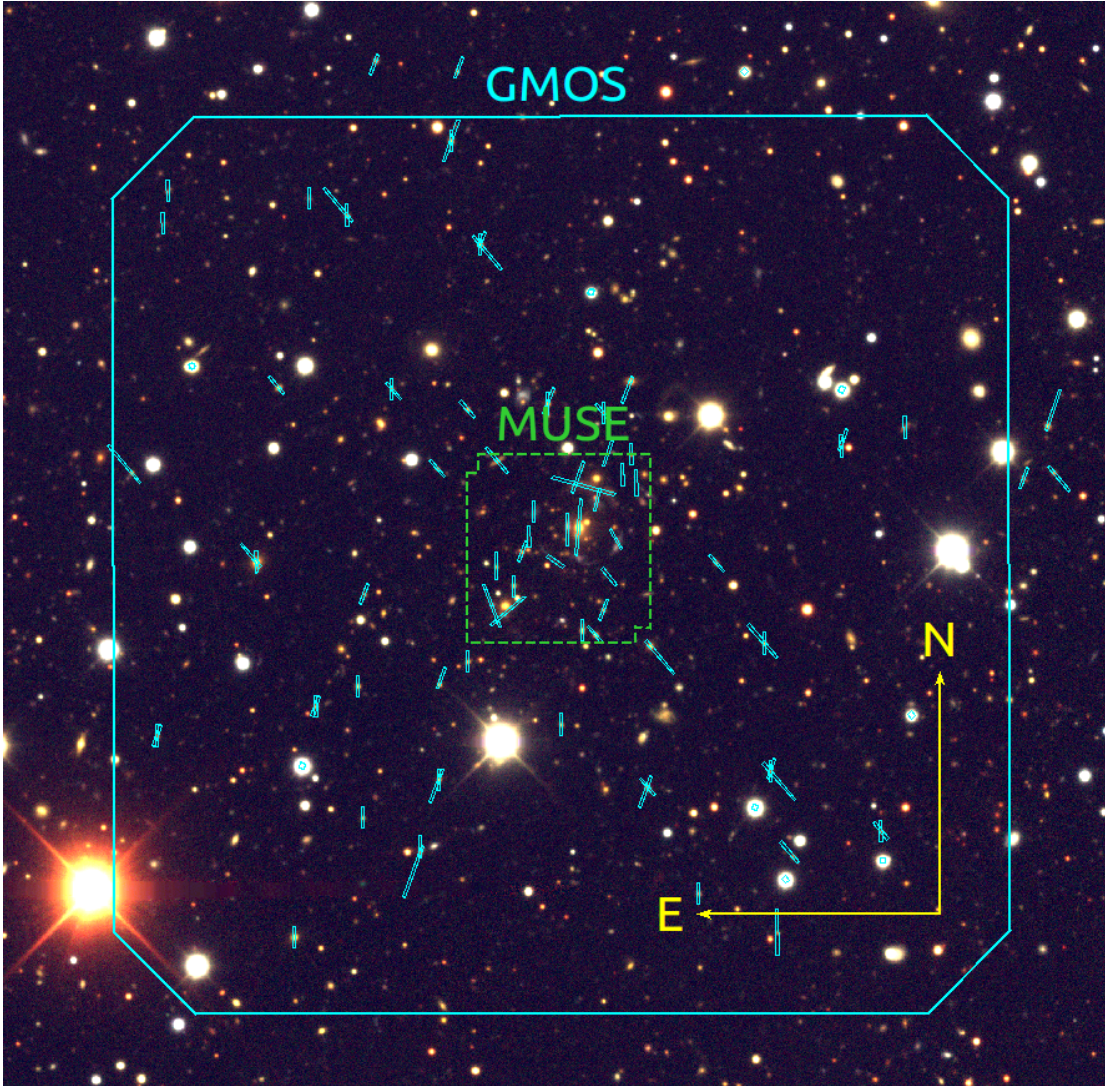


Figure 5.1: Spectral FOV Comparison: A comparison of the fields-of-view of GMOS slitmasks and MUSE observations used in this work. The slits of all four GMOS slitmasks are shown in cyan. The broader cyan region shows the size of the GMOS slitmask for scale, not the actual orientation used in our four masks. The compass arrows in the bottom-right are each 1.5' long.

5.2.1 VLT/MUSE Spectroscopy

The Carousel lens system was observed with deep VLT/MUSE [24] observations in December 2024 and January 2025 (Prog. ID 0114.A-2018(A), PI Barone). These observations were attained in MUSE Wide-Field Mode (WFM) with adap-

tive optics (AO), with two overlapping pointings covering a combined area of approximately 1.28 square arcmin, covering a wavelength range of approximately 470-935nm with a spectral resolution ranging between $\sim 1600 - 3500$. Both pointings received eight exposures of 620s each, for a total exposure of 2.76 hours. These observations were reduced and coadded into a single data cube using the MUSE pipeline version 2.10.10 [398]. Additional post processing improved sky subtraction residuals with ZAP [358], and corrected astrometry to align with two stars in the GAIA DR3 catalog [139] in the South-East of the field.

5.2.2 Gemini/GMOS Spectroscopy

The Carousel system was the target of Gemini/GMOS observations with several goals: (i) identifying the redshifts of strongly lensed sources, (ii) measuring redshifts of likely cluster member galaxies, and (iii) measuring stellar velocity dispersions of the cluster member galaxies. As part of Gemini program GS-2023B-Q-103, these data were taken in January 2024.

Gemini/GMOS observations consisted of four slit masks, one targeting the central galaxy and faintest arcs, and three targeting other lensed sources. Candidate member galaxies were selected from DES catalogs with a simple color-magnitude cut, and were included in all masks as secondary targets. All masks used the R400 grating, which provided an ideal balance of wavelength coverage and spectral resolution for both identifying redshifts and measuring stellar kinematics. These observations span a wavelength range of approximately 550-1050nm, at a spectral resolution of roughly $R \sim 2000$. A summary of these observations, including exposure times, the number of candidate cluster members observed, and a list of lensed sources targeted by each mask, is given in Table 5.1. Note that some candidate member galaxies were observed on multiple masks.

Mask	Grating and Filter	Total Exposure	λ_{cen} (nm)	Field Galaxies	Lensed Sources
bcg-01	R400, GG455	10800s	710-730	19*	3a,8b,9a
arcs-11	R400, OG515	3600s	780-800	19	3a-c,4c,5b,7c
arcs-12	R400, OG515	3600s	810-830	16	4a,7a,9c,10
arcs-13	R400, OG515	3600s	790-810	14	1a,4b,8a

Table 5.1: GMOS observations: Summary of Gemini/GMOS observations.
*Excluding BCG

Gemini/GMOS data were reduced using `PypeIt` [297, 298], a Python package for processing slit spectra from a wide array of spectrographs. As part of this work, `PypeIt` was extended to correctly handle Gemini/GMOS data with tilted slits, functionality which is now available since `PypeIt` version 1.18. These data were reduced using a development version of `PypeIt` 1.17.4.

5.3 Results

Here we present the results of the spectroscopic modeling of both lensed and unlensed objects in the Carousel field. Strongly lensed sources are described in Section 5.3.1. Spectra and redshifts of field galaxies, including cluster members, are presented in Section 5.3.2. This catalog is then used to study the dynamical state of the Carousel cluster in Section 5.3.3. All redshifts were obtained using `MARZ` [166] unless otherwise noted.

5.3.1 Strongly Lensed Sources

Previously, [351] identified sources 1 through 7 behind the Carousel cluster, and reported confident redshifts for all except Sources 6 and 7 using MUSE observations with a total integration time of ≈ 47 minutes (Prog. ID 0111.B-0400(H),

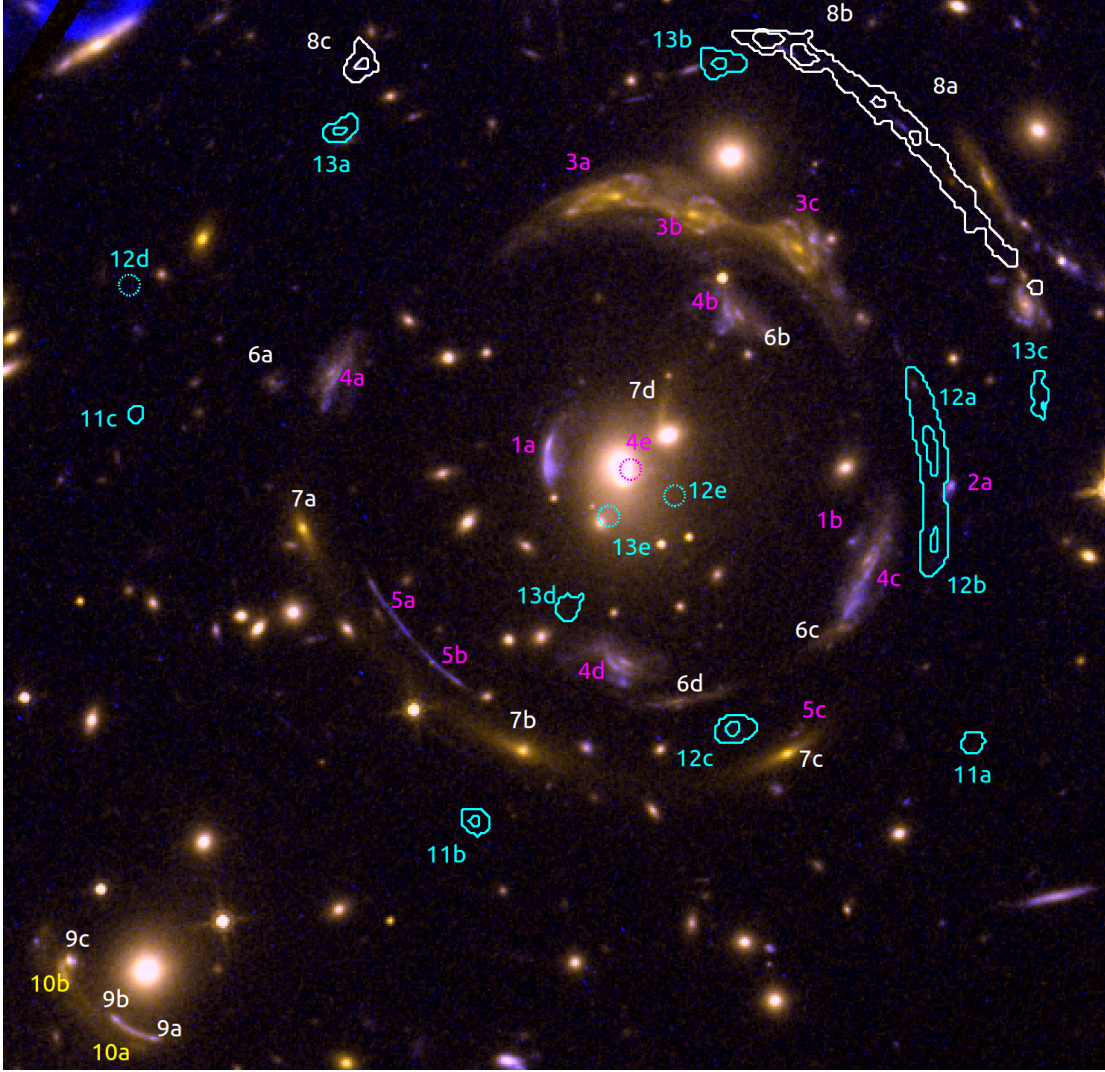


Figure 5.2: All Sources: All known lensed sources in the Carousel system, overlaid on HST imaging with color coding as follows. Magenta: Known sources with redshifts presented in Paper 0 [351]; White: Previously known sources with redshifts presented in this work; Cyan: Newly discovered sources, with extended emission observed in MUSE data; Yellow: Sources which do not yet have confident redshifts. Sources 8, 11, 12, and 13 display contours of $\text{Ly}\alpha$ emission.

PI Bian). Combining HST imaging and newer spectroscopic follow-up, including deeper MUSE observations, we report 6 further objects strongly lensed by the Carousel, bringing the sample of known multiply imaged sources up to 13¹. We

¹Source 2 is only singly imaged, and we therefore do not consider it to be a ‘true’ strongly lensed source. It is included here for consistency with Paper 0.

continue the labeling scheme introduced in [351], identifying these new sources as 8 through 13. Figure 5.2 displays all lensed sources known to date, color coded to denote previously known or newly discovered sources, with and without confident redshifts.

In addition to extracting spectra of known objects, we perform a search for other emission line sources in the MUSE data described in Section 5.2.1. This included stepping through each wavelength slice of the data cube using a number of settings, looking at both individual slices and binning by 10 spectral pixels, as well as continuum subtracting from nearby wavelength ranges to explicitly look for emission. As a result of these searches, we identify clear emission associated with newly identified strongly lensed sources 11, 12, and 13, all of which exhibit Lyman α emission at redshift $z > 3$.

In this section we describe the search for redshifts of strongly lensed sources. The MUSE and GMOS spectra of sources previously identified in imaging data are described in Section 5.3.1. Four Lyman α emitting sources are described in Section 5.3.1. A full list of all sources, including those previously presented in Paper 0, can be found in the appendix, along with spectra of all sources with newly reported redshifts.

We present a full catalog of lensed sources in the field, including those previously reported in Paper 0 [351], in Table 5.2. Table 5.2 gives the name, position, and redshift of each strongly-lensed source, as well as spectral features from which the redshifts were derived. Note that coordinates of each source are provided for clarity, and are not intended as precision measurements.

Source Label	R.A.	Dec.	Redshift	Spectral Features
<i>1a</i>	06:03:56.86	-35:58:05.05	<i>0.962</i>	See Paper 0
<i>1b</i>	06:03:55.42	-35:58:09.87	—	—
<i>2a</i>	06:03:54.95	-35:58:06.65	<i>0.962</i>	See Paper 0
<i>3a</i>	06:03:56.53	-35:57:49.95	<i>1.166</i>	See Paper 0
<i>3b</i>	06:03:56.17	-35:57:51.00	—	—
<i>3c</i>	06:03:55.66	-35:57:53.13	—	—
<i>4a</i>	06:03:57.88	-35:58:00.16	<i>1.432</i>	See Paper 0
<i>4b</i>	06:03:55.96	-35:57:56.76	—	—
<i>4c</i>	06:03:55.34	-35:58:10.92	—	—
<i>4d</i>	06:03:56.50	-35:58:16.77	—	—
4e	06:03:56.47	-35:58:05.72	1.432	[O II]
<i>5c</i>	06:03:55.68	-35:58:20.92	<i>1.432</i>	See Paper 0
<i>5b</i>	06:03:57.29	-35:58:17.88	—	—
<i>5a</i>	06:03:57.69	-35:58:12.40	—	—
<i>6d*</i>	06:03:56.24	-35:58:19.04	1.656	Fe II, Mg II
<i>6c*</i>	06:03:55.46	-35:58:14.74	—	—
6b	06:03:55.82	-35:57:57.76	—	—
6a	06:03:58.17	-35:58:00.65	—	—
<i>7a</i>	06:03:58.02	-35:58:09.07	1.627	C III], C II], MARZ template fit
<i>7b</i>	06:03:56.98	-35:58:21.89	—	—
<i>7c</i>	06:03:55.72	-35:58:22.05	—	—
<i>7d</i>	06:03:56.32	-35:58:02.53	n.a.	Not firmly detected
8a	06:03:55.17	-35:57:46.09	3.549	Ly α , O III], C III]

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Table 5.2 – continued from previous page –

Source Label	R.A.	Dec.	Redshift	Spectral Features
8b	06:03:55.75	-35:57:41.46	—	—
8c	06:03:57.76	-35:57:42.51	—	—
9a	06:03:58.73	-35:58:38.46	1.507	Fe II, Mg II, [O II]
9b	06:03:58.91	-35:58:37.33	—	—
9c	06:03:59.11	-35:58:34.02	—	—
10a	06:03:58.83	-35:58:38.87	n.a.	n.a.
10b	06:03:59.15	-35:58:34.87	n.a.	n.a.
11a	06:03:54.84	-35:58:21.43	4.090[†]	Ly α , He II
11b	06:03:57.20	-35:58:26.02	—	—
11c	06:03:58.81	-35:58:02.53	—	—
12a	06:03:55.06	-35:58:04.28	3.086	Ly α , Si II, C IV, O III], C III]
12b	06:03:55.04	-35:58:09.46	—	—
12c	06:03:55.98	-35:58:20.76	—	—
12d	06:03:58.84	-35:57:55.08	—	—
12e	06:03:56.26	-35:58:07.23	—	—
13b	06:03:56.08	-35:57:42.16	3.086[†]	Ly α
13a	06:03:57.83	-35:57:45.90	—	—
13c	06:03:54.52	-35:58:01.43	—	—
13d	06:03:56.76	-35:58:13.56	—	—
13e	06:03:56.57	-35:58:08.38	—	—

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Table 5.2 – continued from previous page –

Source Label	R.A.	Dec.	Redshift	Spectral Features
Table 5.2: Full Source List: Full list of strongly-lensed sources. Source positions and redshifts previously reported in Paper 0 [351] are shown in italics, and those newly reported in this work are shown in bold. A dash (—) indicates the same value as the previous row. Sources or images without confirmed redshifts are listed as “n.a.”. *Sources 6d and 6c were reported in Paper 0 with different labels. We have renamed them for consistency with source group 4. †Redshifts of sources 11 and 13 are considered tentative, as they are based primarily on a single spectral feature.				

Known Sources

Four images of Source 6 are apparent, in a configuration close to Source 4. Paper 0 reported two images of Source 6 (6c and 6d), and we confirm two additional images (6a and 6b). While Source 6 exhibits no emission lines, all four images have clear absorption features apparent in MUSE data. These absorption features match two distinct redshifts: Fe II 2587,2600Å and Mg II 2796,2803Å at both $z=1.656$ and $z=1.432$. We conclude that the true redshift is $z_6 = 1.656$, and each image of Source 6 additionally exhibits absorption from the CGM of Source 4, at $z_4 = 1.432$. This interpretation is consistent with a precise strong lens model (Paper II, [384]). Features for both redshifts are labeled for all four images of Source 6 in Figure 5.3.

While Paper 0 reported four images of Source 4, the newly available MUSE data demonstrates an additional image, a faint central image which we label 4e. Image 4e exhibits the same [O II] emission doublet as 4a-d. This [O II] emission doublet is also seen in the GMOS slit spectra of the central cluster galaxy, further confirming image 4e.

Source 7 similarly exhibits no emission features, but clear continuum in the

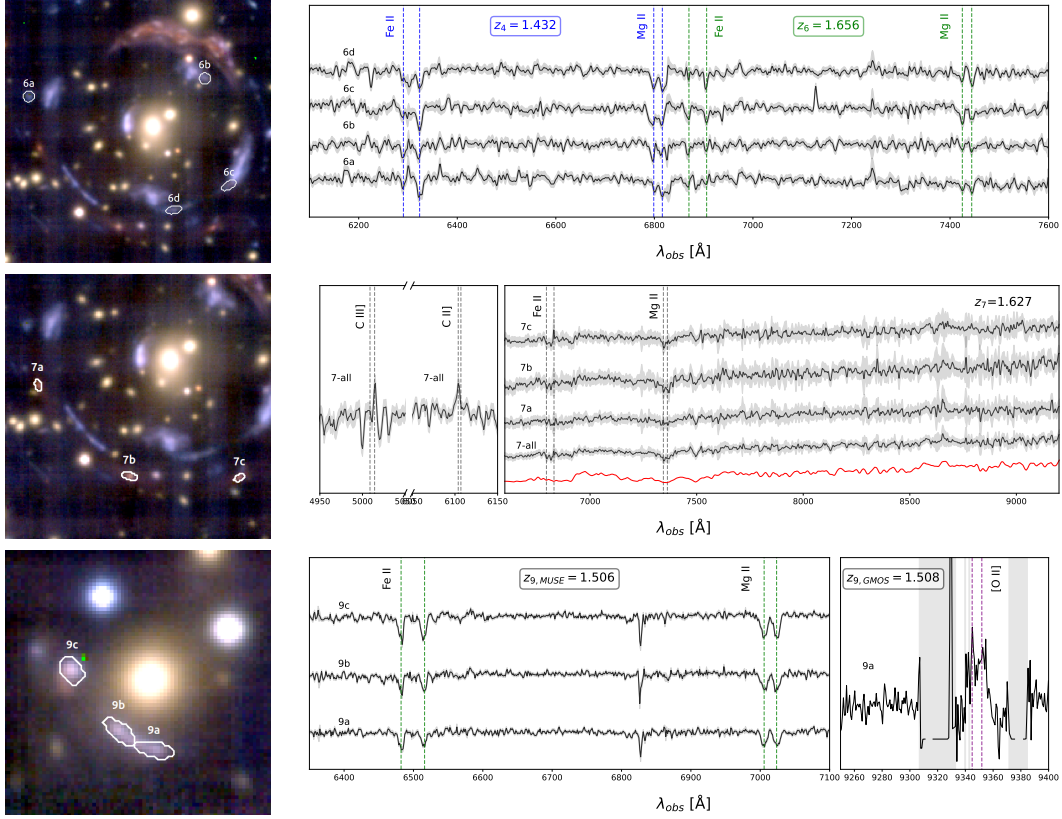


Figure 5.3: Known sources: Spectra of sources previously identified in imaging data. **Left:** MUSE data, with regions of extracted spectra overlaid. **Right:** Extracted spectra of each lensed source, identifying spectral features and redshift. For clarity, the spectrum of each lensed image is offset vertically. For Source 7, we include the Early Type Galaxy **MARZ** template match in red. For source 9, we additionally show a GMOS spectrum of image 9a exhibiting a clear [O II] doublet. Wavelengths contaminated by poor sky subtraction are indicated in vertical gray bands.

near infrared. Template matching using **MARZ** provided an excellent match to both the continuum shape and faint absorption features in 7a-c, from which two experts (TEJ and KG) independently inferred a redshift of $z=1.627$ with an ‘Early Type Absorption Galaxy’ template. The candidate radial image 7d, however, is both demagnified and blended with the light of a cluster galaxy, and therefore cannot be confirmed as the same object from spectroscopy alone. Ongoing modeling work suggests it is highly likely that 7d is in fact a counter image of Source 7 (Paper

II, [384]).

Sources 9 and 10 were identified in HST imaging, and explicitly targeted with GMOS slit spectra. Source 9 was assigned a tentative redshift from GMOS slit observations, which featured a candidate [O II] emission doublet. This redshift was confirmed by matching absorption features seen in MUSE, in all three images 9a,b,c. Source 10 is a faint red object appearing in several images just outside of Source 9; it could not be confidently distinguished from Source 9 in the MUSE cube. Source 10 is the only lensed source we identify without a redshift estimate.

Lyman α Emitters

We identify four distinct strongly lensed Ly α emitters (LAEs) apparent in MUSE data, labeled sources 8 and 11-13. Only one of these, source 8, was suspected from previous HST imaging data. All four LAEs exhibit the characteristic asymmetric double-peaked line profile of Lyman α^2 . We additionally search for faint rest-UV emission lines expected in weak Lyman break galaxies [346]. Contours of Ly α emission for all four LAEs are overlaid in Figure 5.2, and MUSE spectra of all four LAEs can be found in the appendix.

Source 8 was initially identified as a pair of faint blue features in HST imaging, corresponding to images 8a and 8b. These images were targeted by GMOS observations, but could not be detected in GMOS data. In MUSE, these images were found to lie within an extended arc of emission, and a third image 8c was also identified. Source 8 additionally exhibits fainter rest-UV emission lines O III] 1661,1666Å and C III] 1907,1909Å in stacked spectra.

Source 11, in contrast to the other LAEs here, appears as a triplet of point sources. The only confident spectral feature of Source 11 is Ly α emission consistent with $z_{11} = 4.09$, however, the stacked spectrum of all three images further

²See [161] and [162] for a detailed empirical overview of Ly α line profiles.

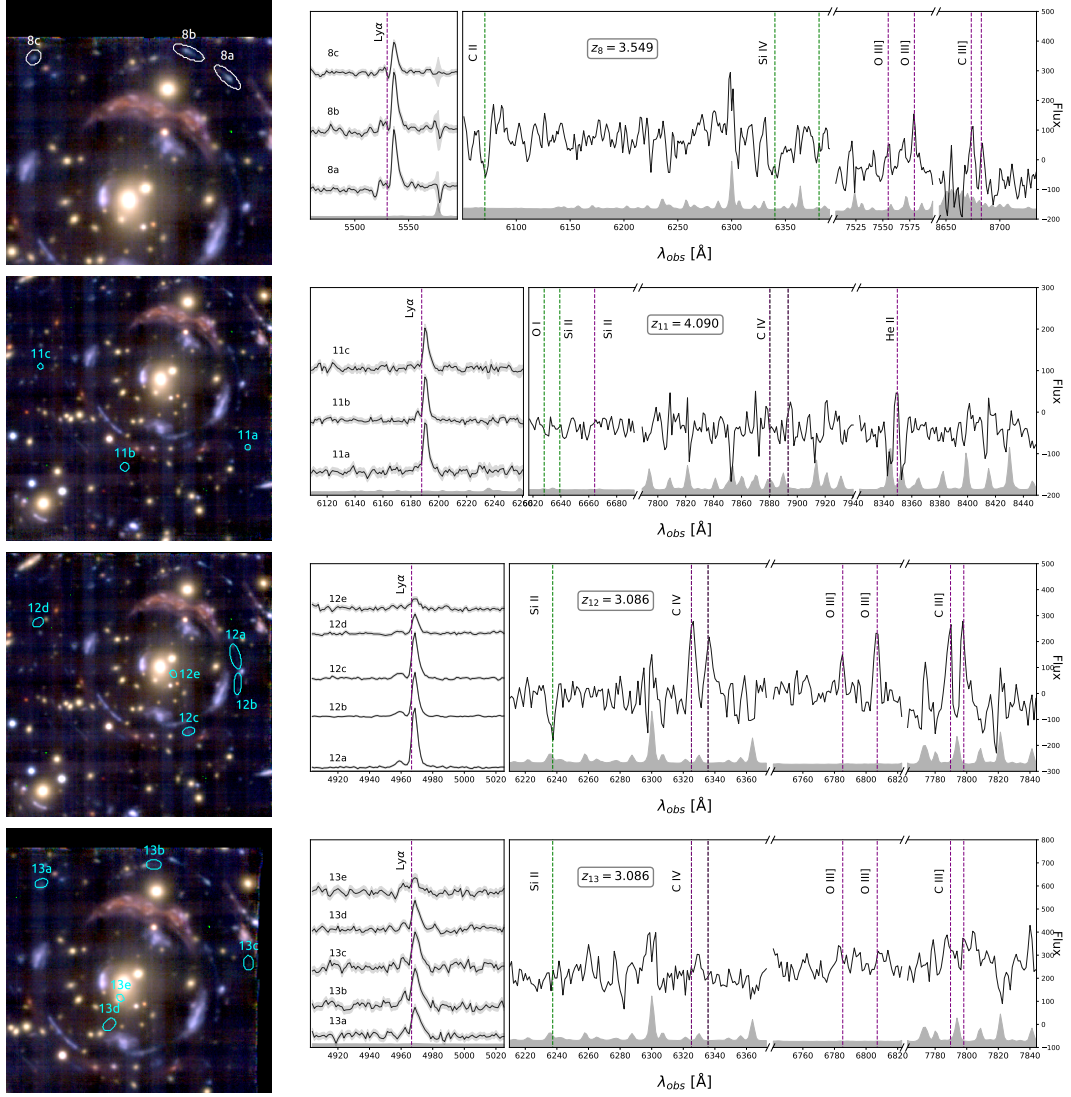


Figure 5.4: Lyman α Emitters: Spectra and newly reported redshifts of strongly-lensed LAEs discovered in MUSE data. **Left Column:** RGB image of the MUSE data, with regions used for each object overlaid. **Right Column:** Extracted spectra of each object. In each plot, the left panel shows Ly α emission visible in each image, offset and scaled for clarity. The right panels show the stacked spectrum of all images, demonstrating faint features. Grey shaded regions at the bottom show the uncertainty. Features labeled in green (purple) are expected in absorption (emission).

exhibits tentative He II 1640Å emission consistent with this redshift.

Sources 12 and 13 appear at nearly identical redshift $z_{12} = z_{13} = 3.086$, form-

ing 10 distinct images in total (see Figure 5.2). The spatial distribution of Ly α flux at this redshift is shown in Figure 5.5, with each observed position labeled. This image was produced by summing the continuum-subtracted flux between 4962 and 4972 Å, and spatially convolving this flux with a Gaussian of $\sigma = 1.5\text{px}$ (approximately $0.3''$). Initially, it was suspected that this emission originates from two different sources, and the labels 12a-e and 13a-e were assigned based on the observed lensing configuration. To verify this interpretation and confirm that 12 and 13 are distinct objects, we additionally measured the rest-frame kinematic offsets between the two modes of Ly α emission red- and blue-shifted relative to the systemic redshift, similar to [162]. These offsets were computed using **ppxf** [76, 73], with the red- and blue-shifted peaks of Ly α emission modeled as two kinematically separate emission lines atop a 4th order polynomial background. The results of this analysis are shown in Figure 5.6. In both stacked and individual spectra, Sources 12 and 13 appear kinematically distinct, with the blue Ly α emission in Source 12 offset significantly further from the system redshift. In all four LAEs, red- and blue-shifted offsets range from ~ 100 to ~ 400 km/s, consistent with offsets at similar redshift [235], as well as a larger study at lower redshift [162].

5.3.2 Cluster Members and Field Galaxies

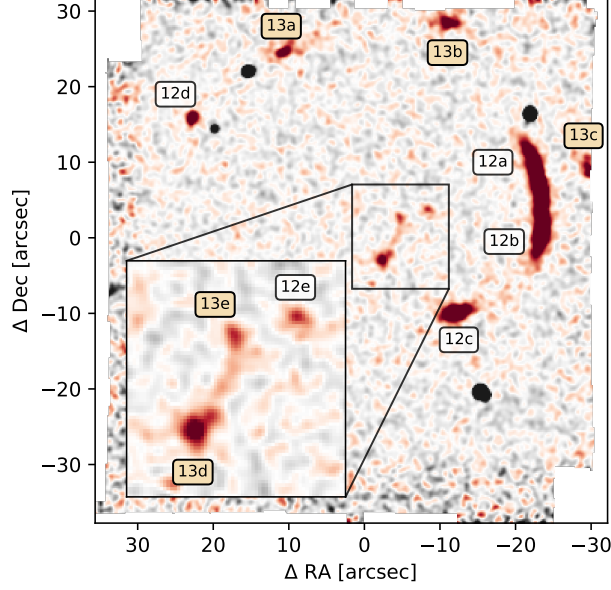


Figure 5.5: Sources 12 and 13: The spatial distribution of Ly α emission from sources 12 and 13, with each image labeled.

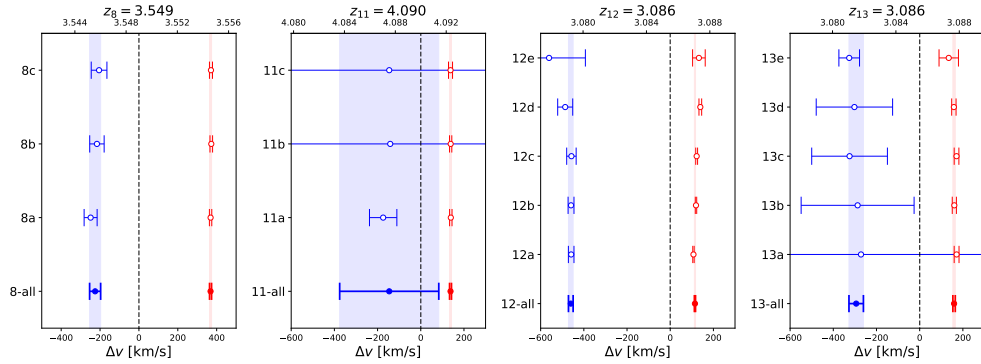


Figure 5.6: LAE Kinematics: Kinematics offsets of the red- and blue-shifted Ly α emission of all four Lyman α emitters, for each observed image. Kinematic offsets and uncertainties were computed with `ppxf` as described in Section 5.3.1, using the same spatial regions of MUSE data as in Figure 5.4.

Object ID	Redshift	RA (h:m:s)	Dec (d:m:s)	QOP	Instrument
770	0.49817	06:03:46.61	-35:57:29.08	4	GMOS
886	0.48663	06:03:47.36	-35:59:57.74	3	GMOS
1451	0.48663	06:03:50.89	-35:58:48.56	3	GMOS
2045	0.48715	06:03:54.45	-35:59:41.31	3	GMOS
2046	0.27137	06:03:54.45	-35:58:30.23	4	MUSE
2058	0.495	06:03:54.53	-35:57:46.15	4	MUSE
2067	0.95841	06:03:54.59	-35:57:56.26	4	MUSE
2136	0.49265	06:03:54.91	-35:57:38.99	3	GMOS
2188	0.4981	06:03:55.19	-35:58:26.59	4	MUSE
2234	0.49584	06:03:55.44	-35:58:05.55	4	MUSE
FGD	0.08607	06:03:55.56	-35:58:15.60	4	MUSE
2292	0.38454	06:03:55.77	-35:58:24.13	4	MUSE
2296	0.48824	06:03:55.78	-35:58:36.19	4	MUSE
2333	0.49399	06:03:55.92	-35:58:32.88	4	MUSE
2350	0.49046	06:03:55.99	-35:57:47.65	4	MUSE
2357	0.00006	06:03:56.03	-35:57:54.64	6	MUSE
2373	0.48785	06:03:56.13	-35:58:03.22	3	MUSE
2381	0.49971	06:03:56.17	-35:58:31.75	3	MUSE
2399	0.48731	06:03:56.28	-35:58:03.70	4	MUSE
2406	0.48507	06:03:56.32	-35:58:21.78	3	MUSE
2430	0.49656	06:03:56.42	-35:58:43.50	3	GMOS
2444	0.48829	06:03:56.51	-35:58:05.68	4	MUSE
2465	0.48444	06:03:56.60	-35:58:08.70	4	MUSE
2473	0.89784	06:03:56.67	-35:58:21.72	4	MUSE

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Table 5.3 – continued from previous page –

Object ID	Redshift	RA (h:m:s)	Dec (d:m:s)	QOP	Instrument
2480	0.487	06:03:56.73	-35:58:34.03	4	MUSE
2521	0.48261	06:03:56.89	-35:58:15.30	4	MUSE
2533	0.49773	06:03:56.96	-35:58:10.10	3	MUSE
2542	1.4341	06:03:57.04	-35:58:39.78	4	MUSE
2543	0.48473	06:03:57.04	-35:58:15.43	4	MUSE
2546	0.48799	06:03:57.07	-35:59:18.28	4	GMOS
2573	0.49004	06:03:57.24	-35:58:08.69	4	MUSE
2588	0.48144	06:03:57.33	-35:57:59.22	4	MUSE
2593	0.49361	06:03:57.35	-35:58:05.90	3	MUSE
2605	0.48489	06:03:57.45	-35:57:20.17	3	GMOS
2617	0.49503	06:03:57.52	-35:57:57.06	3	MUSE
2667	0.49497	06:03:57.81	-35:58:14.85	3	MUSE
2677	0.47784	06:03:57.85	-35:58:30.98	4	MUSE
2710	0.48828	06:03:58.06	-35:58:13.89	3	MUSE
2747	0.48022	06:03:58.26	-35:58:14.54	4	MUSE
2788	0.47716	06:03:58.49	-35:58:27.06	4	MUSE
2823	0.4882	06:03:58.73	-35:58:13.34	3	MUSE
2830	0.49383	06:03:58.76	-35:58:34.47	4	MUSE
2873	0.48703	06:03:58.99	-35:57:41.15	4	MUSE
2881	0.49229	06:03:59.02	-35:58:20.06	4	MUSE
2946	0.95997	06:03:59.41	-35:57:58.03	3	MUSE
2951	0.49689	06:03:59.44	-35:57:59.96	3	MUSE
2959	0.48503	06:03:59.48	-35:56:21.61	4	GMOS

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Table 5.3 – continued from previous page –

Object ID	Redshift	RA (h:m:s)	Dec (d:m:s)	QOP	Instrument
3025	0.49324	06:03:59.89	-35:57:22.13	3	GMOS
3111	0.4909	06:04:00.37	-35:55:43.37	3	GMOS
3196	0.49266	06:04:00.80	-35:57:43.98	3	GMOS
3424	0.49301	06:04:02.17	-35:57:14.62	3	GMOS
3582	0.49822	06:04:03.02	-35:58:30.08	3	GMOS
3617	0.491	06:04:03.24	-35:59:03.98	3	GMOS
3676	0.48584	06:04:03.52	-35:56:10.67	3	GMOS
3828	0.48656	06:04:04.51	-35:59:11.43	3	GMOS
4022	0.47817	06:04:05.62	-35:57:13.98	4	GMOS
4132	0.49144	06:04:06.31	-35:58:18.15	3	GMOS
4568	0.86463	06:04:08.96	-35:56:01.32	3	GMOS

Table 5.3: Redshifts for all non-lensed objects in the MUSE and Gemini datasets with confident redshifts (Q.O.P. > 3). Object IDs correspond to Legacy Survey DR10 Brick 0910m360. Spectroscopic analysis of object FGD suggests that it is a late-type emission galaxy.

We attempted to determine redshifts for all galaxies targeted by either MUSE or GMOS. All GMOS targets were matched to the DECaLS DR 10 object catalog [102], brick 0910m360. For all DECaLS objects within the MUSE field, we extract a spectrum by combining all spaxels within 1” of the object position. We exclude objects of type “PSF” as likely stars. All spectra were then examined using MARZ [166]; MARZ performs an automated template fitting for a set of galaxy and stellar templates of different types, and provides the top five redshift determinations. These were examined independently by DYW and TEJ; in some cases, the redshifts were adjusted by hand to match prominent spectral features,

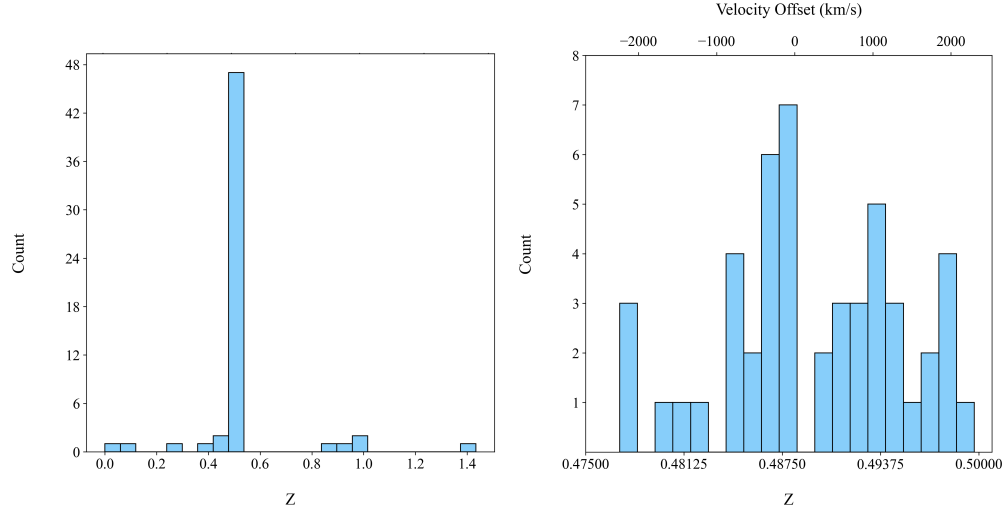


Figure 5.7: Histogram of confident redshifts for galaxies in the Carousel field. **Left:** All objects. **Right:** Likely cluster members, centered on the redshift of the Carousel cluster.

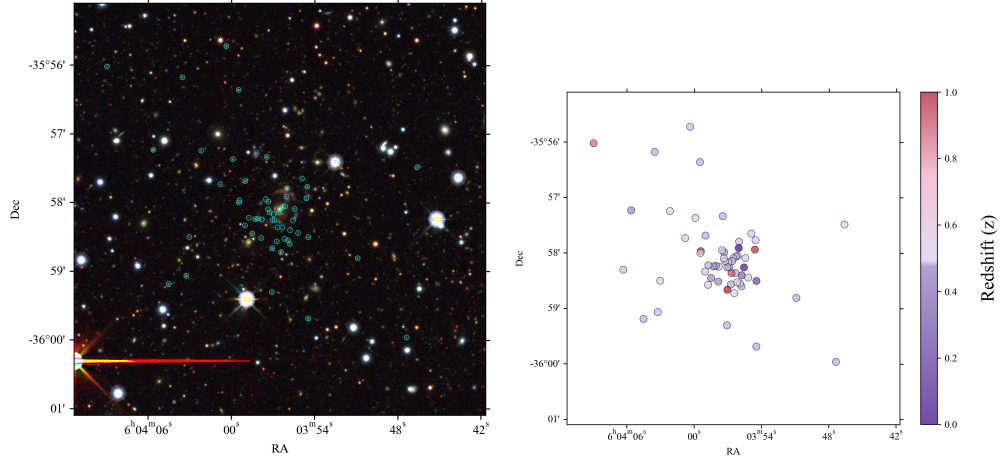


Figure 5.8: Sky map of all non-lensed galaxies in the MUSE and Gemini datasets with confident redshifts. **Left:** Catalog objects marked in cyan, overlaid on DECaLS imaging. **Right:** The same catalog and FOV, color coded by redshift.

particularly for galaxies at higher redshifts ($z \gtrsim 1$) where the template fitting became inaccurate. Redshifts were given a quality flag from 1-4, and only redshifts with quality 3 (good) and 4 (great) agreed on by both observers are reported here in Table 5.3. As the lensed sources are separately analyzed, DECaLS objects

associated to these were removed from this table. Galaxies which were observed by both MUSE and Gemini were found to have consistent redshifts between the two, independent determinations (average offset of 0.0004), and where different the higher quality redshift is reported in Table 5.3.

In total, we determine redshifts for 57 DECaLS objects, of which one is a star and 56 are galaxies. 49 of these galaxies have redshifts consistent with the cluster redshift. In addition, our analysis revealed a foreground dwarf galaxy at $z=0.086$ described below and listed as FGD in Table 5.3. The redshift distribution of galaxies in our catalog is shown in Figure 5.7.

Foreground Dwarf Galaxy

As mentioned above, we identify a previously unknown foreground galaxy at $z=0.086$, blended with lensed images 4c and 6c. This galaxy was discovered when examining the spectrum of a DECaLS object associated to 4c. In addition to the spectral features at the redshift of source 4, a set of additional emission features were found consistent with $H\alpha$, [OIII], and $H\beta$ at a redshift of 0.086. Examination revealed that the $H\alpha$ emission was prominent in a region between 4c and 6c where the color in the Hubble imaging is quite blue, and extraction of the spectrum in this region confirmed the spectral features at $z=0.086$. Understanding the presence of this galaxy is essential to properly modeling the strong lensing of sources 4 and 6.

Additionally, in order to separate the spatial distribution of source images 1b, 4c, and the foreground galaxy, we applied an Independent Component Analysis (ICA) algorithm. A small rectangular region of the IFU data centered on this region was extracted, and the spectrum of each spaxel provided to the **FastICA** routine of `scikit-learn` [284], allowing for 30 independent components. The

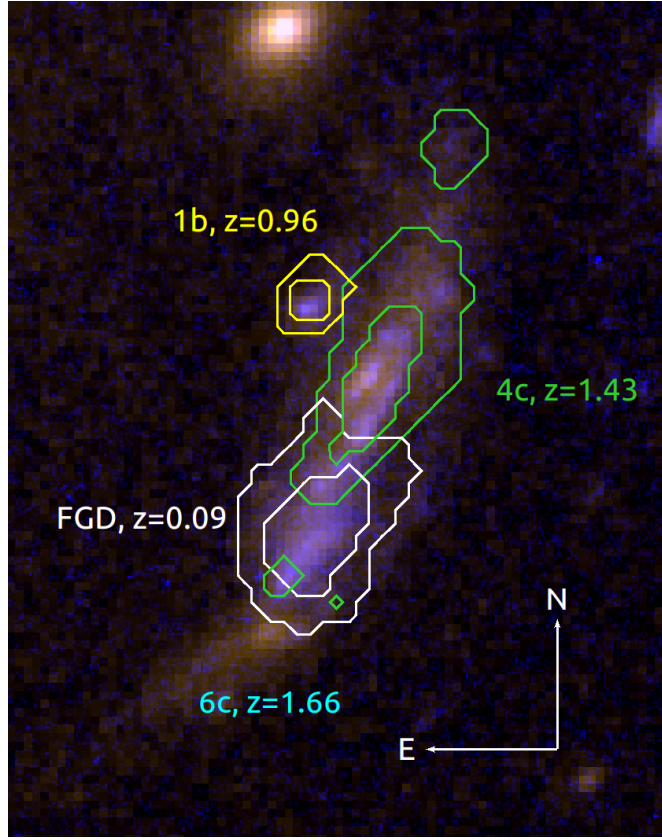


Figure 5.9: Foreground Galaxy: A region in the West of the Carousel field featuring three lensed sources in close proximity, highlighting the foreground emission-line galaxy blended with 4c and 6c. Contours show three sources as separated by ICA according to Section 5.3.2, overlaid on HST imaging. The compass in the lower-right has arms of length 2".

results effectively separated the overlapping spatial distributions of sources 1b, 4c, and the foreground galaxy, contours of which are shown in Figure 5.9. Source 6c was not detected by this process.

5.3.3 Cluster Velocity Dispersion

We estimate the velocity dispersion of member galaxies within the cluster using both the biweight scale and gapper statistics from [27] and following the methodology in [402, 132]. First, the biweight location is used to determine the

central redshift of the cluster, which was found to be $z_{\text{cl}} = 0.4895 \pm 0.0010$. The 49 galaxies within 3000 km s^{-1} of the central redshift were then used to calculate the velocity dispersion; our procedure would call for removing galaxies more than three times the velocity dispersion from the central redshift and then recalculating the dispersion, but in practice this did not remove any additional galaxies.

Uncertainties on the velocity dispersion were estimated using a bootstrap procedure wherein the cluster member galaxies were resampled with replacement and the velocity dispersion was recalculated on this resampled data. This procedure was repeated 1,000 times, and the median and 68% spread were taken as the velocity dispersion and its uncertainty.

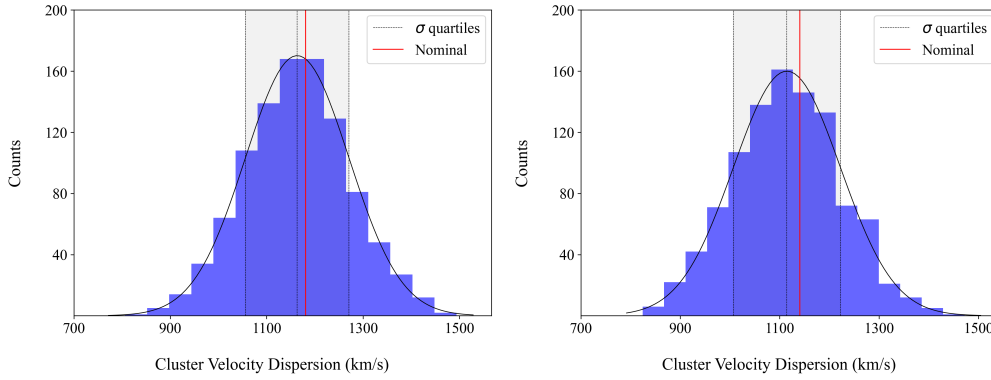


Figure 5.10: Left: Histogram of bootstrap results for the biweight method of determining the cluster velocity dispersion. Gray vertical lines and shaded region indicate the mean of the bootstrap results and the 68% confidence interval; the red line shows the nominal result for the galaxy sample. Right: Same as the top panel, but for the gapper method of determining the cluster velocity dispersion.

The results for both the biweight and gapper statistics are shown in Table 5.4, and Figure 5.10 shows the distribution of velocity dispersions found in the bootstrap trials. Both the biweight and gapper statistics indicate a velocity dispersion of around 1100 km s^{-1} with fully consistent results between methods and for the nominal sample compared to the bootstrap median. The distributions of velocity dispersions for the bootstrap trials shown in Figure 5.10 are Gaussian with no

Table 5.4: Results for cluster velocity dispersion from the biweight and gapper methods. Column 2 gives the results for the nominal galaxy sample, while column 3 gives the median of the bootstrap trials and its respective uncertainty.

Dispersion Method	Nominal Result (km s ⁻¹)	Bootstrap Median (km s ⁻¹)
Biweight	1181	1164 ⁺¹⁰⁵ ₋₁₀₆
Gapper	1141	1114 ⁺¹⁰⁸ ₋₁₀₂

indication of biases due to sample selection.

We use our estimate for the cluster velocity dispersion to estimate the M_{200c} mass using the relation from [60]. This relation is given by:

$$b_v^{1/\alpha} M_{200c} = 10^{15} M_{\odot} \frac{1}{h(z)} \left(\frac{\sigma_{gal}}{\sigma_{15}} \right)^{1/\alpha} \quad (5.1)$$

where $b_v^{1/\alpha}$ is the unknown bias in the galaxy member velocities relative to the dark matter, σ_{gal} is our estimated cluster velocity dispersion, the reference velocity dispersion $\sigma_{15} = 1082.9 \pm 4$ km s⁻¹, and $\alpha = 0.3361 \pm 0.0026$ is a dimensionless constant. We assume a flat Λ CDM cosmology with $\Omega_m = 0.3$ and $h = 0.69$. Using the gapper median and uncertainties from our bootstrap trials for the velocity dispersion, we find $b_v^{1/\alpha} M_{200c} = 1.2^{+0.4}_{-0.3} \times 10^{15} M_{\odot}$.

5.4 Discussion

The Carousel lens is a uniquely powerful lens for constraints on cosmology, as demonstrated in Paper II in this series [384]. Not only is it a relatively massive cluster at this redshift, but it is also a highly symmetric lens system leading to a relatively simple lens model compared to typical cluster lenses [351]. It also displays a large number of lensed sources over a large redshift baseline with several

different lens configurations, allowing for tight constraints on the mass model.

Our measured velocity cluster dispersion implies a very massive halo, around $10^{15} M_{\odot}$. This cluster was also detected in X-ray by the eROSITA All-Sky Survey (eRASS) [61], and through the Sunyaev-Zel’dovich effect by the Atacama Cosmology Telescope (ACT) DR5 [192]. Both eRASS and ACT-DR5 catalogs provide estimates of the cluster mass M_{500c} : the eRASS catalog estimate is $4.65^{+0.95}_{-0.73} \times 10^{14} M_{\odot}$, and the ACT-DR5 catalog provides an estimate of $5.59^{+1.09}_{-0.91} \times 10^{14} M_{\odot}$. Assuming the cosmological mass-concentration relation from [228], with a scatter on $\langle c_{500c} | M_{500c}, z \rangle$ of 0.13 dex, the eRASS and ACT-DR5 mass estimates correspond to M_{200c} of $6.87^{+1.51}_{-1.24} \times 10^{14} M_{\odot}$ and $8.28^{+1.84}_{-1.49} \times 10^{14} M_{\odot}$ respectively. Both X-ray and SZ mass estimates are somewhat lower than our kinematic estimate $b_v^{1/\alpha} M_{200c}$, though consistent.

The lens model derived in Paper 0 of this series gave an Einstein radius of $\theta_E = 13.03'' \pm 0.02''$ for Source 4, at $z_4 = 1.432$. That work modeled the cluster halo mass as an elliptical power law, rather than an NFW, complicating comparison to the NFW mass estimates above. We therefore consider the projected aperture mass within θ_E , which is $4.6 \times 10^{13} M_{\odot}$ for Source 4 using the same cosmology as above. A spherical halo of $M_{200c} = 1.2 \times 10^{15}$ would reproduce this aperture mass with a concentration $c_{200c} \sim 10$, and lower mass halos would require even higher concentrations. While this is far above the expected cosmological mass-concentration relation, the selection effect of strong lensing is expected to preference highly concentrated halos. Additionally, baryonic mass in the inner halo, or elongation along the line-of-sight, would increase the projected mass in the inner halo and cause a higher apparent concentration.

Overall, the dynamical, lensing, X-ray, and SZ mass estimates are all consistent and indicate that the Carousel is a massive cluster.

5.5 Conclusion

We have conducted a spectroscopic survey of the field surrounding the Carousel strong lensing cluster, using both Gemini/GMOS and VLT/MUSE, targeting both field galaxies and strongly lensed sources. We confirm the redshifts of the remaining sources identified in Paper 0 [351], and identify 6 new lensed sources, bringing the total to 13. Of these 13, only one (Source 10) remains without redshift. The sources with known redshifts range from $z = 0.96$ to $z = 4.1$ and nearly all of them display at least three images. Sources 4, 12, and 13 have five identified images including a central image, and sources 6 and 7 are quads. The full list of sources and their redshifts are given in the appendix, along with spectra of sources with newly reported redshifts.

We identify four strongly lensed Lyman α emitters, three of which are not apparent in imaging data and are only detected in MUSE IFU data. These sources span the redshift range $3.09 < z < 4.09$, and two of them exhibit clear central images near the BCG of the Carousel cluster. These sources could be of considerable interest for cosmology constraints in the Carousel: their central images could strongly constrain the dark matter density profile of the cluster (as in e.g. [230, 77]), and their high redshift will allow multi-source-plane cosmography studies to constrain the universe’s expansion history at much earlier times. Future deep near infrared imaging may precisely reveal these sources’ positions and morphology, enabling higher precision strong lens models.

In addition, we identify 49 cluster members as well as additional field galaxies. We find a cluster velocity dispersion of $1100 \pm 100 \text{ km s}^{-1}$ giving an estimated mass of $b_v^{1/a} M_{200c} = 1.2_{-0.3}^{+0.4} \times 10^{15} M_\odot$. This is consistent with, though somewhat higher than, mass estimates from both the eRASS X-ray and ACT-DR5 SZ catalogs, which give $M_{200c} = 6.9_{-1.2}^{+1.5} \times 10^{14} M_\odot$ and $8.3_{-1.5}^{+1.8} \times 10^{14} M_\odot$ respectively.

Spectroscopic follow-up of the Carousel system has so far spanned the wavelengths $\sim 470\text{nm}$ to $\sim 1\mu\text{m}$. This wavelength range is sensitive to the common emission lines [O II] 3729Å, [O III] 5007Å, or H α at $z \lesssim 1.7$, and Ly α at $2.9 \lesssim z \lesssim 6.7$. Further spectroscopic observations at other wavelengths will potentially identify additional lensed sources in the ‘redshift desert’ $1.7 < z < 2.9$, or at very high redshift $z \gtrsim 6.7$.

5.6 Supplemental: Preliminary cosmography with the Carousel

Following the work presented above, a detailed pixel-level modeling of the Carousel lens were published in [384], including corresponding compound lensing constraints on cosmology. That work used the **GIGA-Lens** framework [159] to forward model the light of Sources 1 and 3-7 as observed by HST, using state-of-the-art GPU-accelerated modeling methods and gradient-informed Hamiltonian Monte Carlo sampling. The sources studied span redshift $z \in [0.96, 1.656]$, and thus sample the lensing Hubble diagram for the Carousel across this redshift range. The **GIGA-Lens** model in that work consisted of four mass components, including one for the overall cluster halo, and three for the contributions of member galaxies. In order to quantify systematic tests of these measurements, the authors conducted systematic tests to quantify the potential impact of unmodeled cluster member mass, and the compound lensing contributions of lower-redshift lensed sources. The estimated contribution of those systematics is included in the results. Those results are displayed for a w CDM cosmology in Figure 5.11 in grey. For comparison, the results of earlier compound lens work combining five clusters [67] is overlaid in the middle panel, and known constraints from CMB, samples of

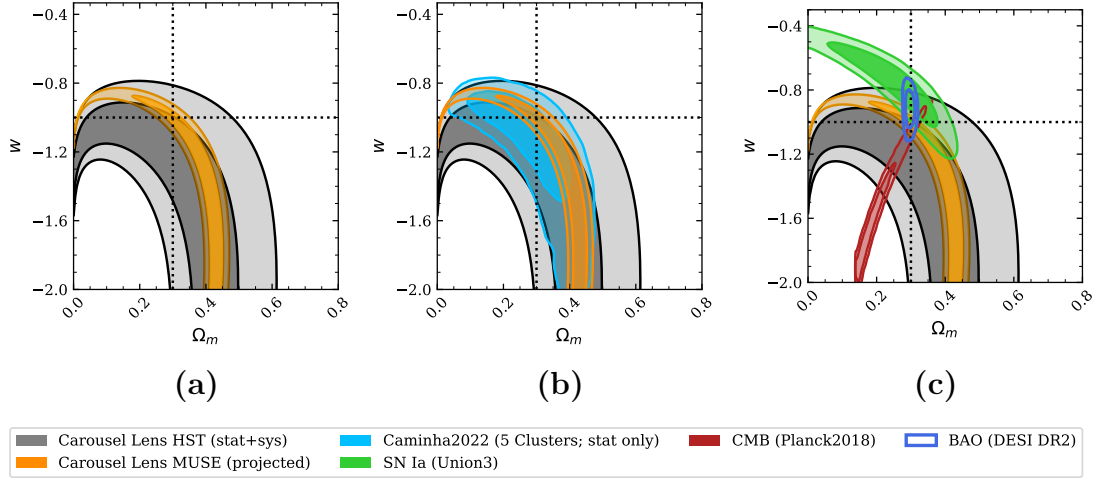


Figure 5.11: Current and predicted future constraints on $w - \Omega_m$ with the Carousel lens alone, reproduced from Fig. 7 of Urcelay et al. [384]. Constraints from the Carousel alone as measured in [384] are shown in grey, and forecast future constraints including known high-redshift sources are shown in orange. Additional contours representing other results are overlaid for reference. For further details, see Section 5.6, as well as [384].

SNe Ia, and BAO measurements are displayed in the right-most panel. Should the higher-redshift sources reported in this work (Sources 8, 11-13) receive imaging of sufficient quality that they can be modeled with similar precision, they would contribute dramatically to these constraints. A forecast of such a scenario is displayed in all panels in orange; this would certainly require near-infrared imaging, likely with JWST. We note that this forecast not only predicts similar precision to leading cosmological probes, but is *highly complementary* to them, and exhibits a degeneracy almost perfectly orthogonal to that of Planck CMB results. For further details on the extensive modeling process which produced these results, see Paper II [384].

Chapter 6

Conclusion

In this dissertation, we have presented an overview of current and future cosmological probes afforded by strong gravitational lensing. The key results of this work include a search for strong lenses in the Dark Energy Survey (Chapter 3), the most robust constraint to date on Self-Interacting Dark Matter in clusters of galaxies (Chapter 4), and a spectroscopic study of a lensing cluster ideal for compound lens cosmography, the Carousel Lens (Chapter 5).

The number of known strong lenses already exceeds our ability to model them. This gap presents a key challenge to strong lensing science, which will only continue to grow as Stage IV imaging surveys such as Euclid, LSST, and Roman delve deeper into the cosmos. To date, strong lensing studies have generally relied on detailed analysis of individual systems, or small samples of systems, with extensive human involvement in the modeling process. This is especially true with galaxy cluster lenses, which typically require laborious expert involvement to identify conjugate point constraints, produce a catalog of cluster member galaxies, choose whether to add additional cluster-scale halos for merging systems, etc. The largest samples of uniformly modeled clusters to date, by far, is [350], which modeled 37 lenses. Fully leveraging lens samples of the future will require automated analysis

pipelines, and will require developing pixel-level modeling methods suitable for cluster lenses. Notably, attempts have been made to develop automated modeling tools for galaxy-scale lenses, including **dolphin** [344], an automated modeling pipeline which has been used on samples of lenses with HST imaging.

Identifying redshifts of strongly lensed sources presents an additional bottleneck to strong lensing science. Most probes discussed in this work require redshifts of both deflector and source. Photometric redshifts of deflectors may be fairly reliable (especially in clusters, where the red sequence can be simultaneously fit to all member galaxies). Estimating redshifts of lensed sources is particularly challenging, and generally requires spectroscopic confirmation. We note that Integral Field Unit (IFU) observations of strong lenses are exceptionally powerful, simultaneously providing spectral and morphological information on both lens and source, and in some cases enabling kinematic measurements.¹ IFUs can also identify faint sources that are not apparent in imaging data. Examples include the Carousel Lens (LyA emitters at $z \in [3, 4.1]$, see Ch. 5) and MACSJ0138 [154], as well as the ‘Jackpot lens’, the only known galaxy-scale lens with more than two sources [355]. In all three cases, VLT/MUSE observations revealed unsuspected faint sources. Nevertheless, dedicated follow-up of individual lenses is resource-intensive; a complementary approach is to integrate with highly multiplexed spectroscopic surveys, which can efficiently observe many targets in the same field for disparate science cases. The planned 4MOST Strong Lensing Spectroscopic Legacy Survey (4SLSL) program takes this approach, using the 4MOST spectroscopic survey facility to target lens candidates found in Euclid with single-aperture spectra [89]. The planned FOBOS instrument on Keck II will have deployable IFUs, an exceptionally powerful design for strong lensing studies [62].

¹Though they are not the topic of this work, IFU observations of strong lens systems also enable remarkable astrophysical studies of lens deflectors and sources.

6.1 Future Directions: Cluster Lens Modeling

Strong lensing galaxy clusters are exceptional probes of cosmology, yet the tools for modeling cluster lenses are stagnant, rooted in decades-old methods which use a tiny fraction of available imaging data. Applying pixel-level modeling to lensing clusters, as is commonly done with galaxy-scale lenses, will not only increase the precision of cluster lensing models, but unlock novel probes at cluster scales such as gravitational imaging searches for low-mass dark halos. It is widely acknowledged that applying pixel-level modeling to cluster lenses is a natural direction for strong lensing. One paper on the topic is simply titled “The Next Step in Galaxy Cluster Strong Lensing” [8], and [407] has demonstrated that pixel-level modeling of a system like MACSJ0138 could improve constraints on H_0 by nearly an order of magnitude. Pixel-level modeling has been applied to individual arcs in cluster lenses a number of times [8, 259, 393], but fully realizing its potential will require modeling more lensed images simultaneously. Larger imaging fields have been successfully modeled with GIGA-Lens [159], including in the Carousel lens [384] and a smaller group-scale system [383]. However, the GIGA-Lens models utilized simpler mass models, without explicitly including all member galaxies as is typical in cluster lensing.

In addition to the computational complexity and lack of appropriate tooling, mass model choices present a significant obstacle to full surface brightness modeling of cluster lenses. Even in galaxy-scale lenses, mass model flexibility and choices of parameterizations can induce significant biases. For instance [276] has shown that multipole perturbations beyond simple ellipticity are necessary for robust gravitational imaging detections, and it is suspected that the ‘external shear’ γ_{ext} likely captures unmodeled features of the lens mass rather than a true ‘external shear’ from nearby LSS [125, 168, 169]. Problems of mass model flexibility are very

acute at cluster scales, where the lensing mass distribution is intrinsically more complex. Nevertheless, cluster lenses have a number of other observational levers which can probe their mass and dynamical state, including measured velocity dispersions, weak lensing signals, X-ray observations of the hot intracluster medium (ICM), and measurements of the Sunyaev-Zel’dovich effect in CMB data. While the availability of these additional probes provides an advantage over galaxy-scale lenses, combining them with strong lens models is challenging and work along those lines so far has been limited. Given the remarkable potential of galaxy cluster strong lensing, advances in cluster lens modeling methods are likely to enable cutting-edge cosmological constraints and new discoveries.

6.2 Future Directions: Compound Lens Cosmography

Compound strong lensing presents a powerful, somewhat novel probe of cosmology with strong lensing, as described in Section 2.1.2 and demonstrated in Chapter 5. To date, most compound lensing forecasts have considered galaxy-scale lenses with two sources, known as Double Source Plane Lenses (DSPLs). One recent forecast involved 87 DSPLs in an LSST-like data set, representing a conservative lower limit on the number of DSPLs with sufficient follow-up data for detailed modeling [347]. In a w CDM cosmology, they predicted constraints of similar precision to the five clusters studied in [67], with slightly higher constraining power; a w_0w_a CDM cosmology however yielded dramatically less constraining power, again similar to [67]. In both cases, compound lensing shines in combination with other probes, and has nearly orthogonal degeneracies to CMB cosmology surveys. A variety of related probes are under development; we choose to men-

tion “pseudo-double-source-plane lensing”, where matching pairs of single-source lenses with identical deflectors in principle yields the same constraint [348, 349]. While pseudo-DSPLs could dramatically expand sample sizes, the challenge lies in robustly matching pairs of lenses while managing systematic uncertainties.

As noted in Section 2.1.2, the lensing Hubble diagram sampled by compound strong lensing is in fact identical to shear ratios commonly measured in weak lensing surveys. While to date, weak lensing shear ratios have almost exclusively been used as calibration or consistency checks, [120] has shown that shear ratios from existing weak lensing data sets can modestly contribute to cosmological constraints. The emerging technique of kinematic lensing [295, 410, 296] has the potential to massively boost the signal of weak lensing shear measurements, and could enable competitive constraints on the dark energy equation of state with upcoming surveys such as Roman [214].

Galaxy clusters are potentially the most powerful venue for compound strong lensing. This is evidenced by the facts that 1) the 5 clusters studied in [67] had the same constraining power as a forecast for eighty seven galaxy-scale DSPLs [347], and 2) pixel-level modeling of the Carousel lens [384] again yielded nearly the same constraining power as all five studied in [67]. As a simple forecast, we provide mock cosmology constraints from 10 hypothetical compound lenses with between 2 and 6 sources out to $z_s = 3$, with deflection ratios η measured at similar precision to the Carousel [384]. With 30 total deflection ratios, this mock sample provides remarkable constraining power in a w CDM cosmology, especially when combined with CMB results from [87] (see Fig. 6.1a). In a w_0w_a CDM cosmology it provides more modest constraints, but still meaningfully contributes to real CMB + BAO measurements (Fig. 6.1b).

Strong gravitational lensing has remarkable potential, which we have only be-

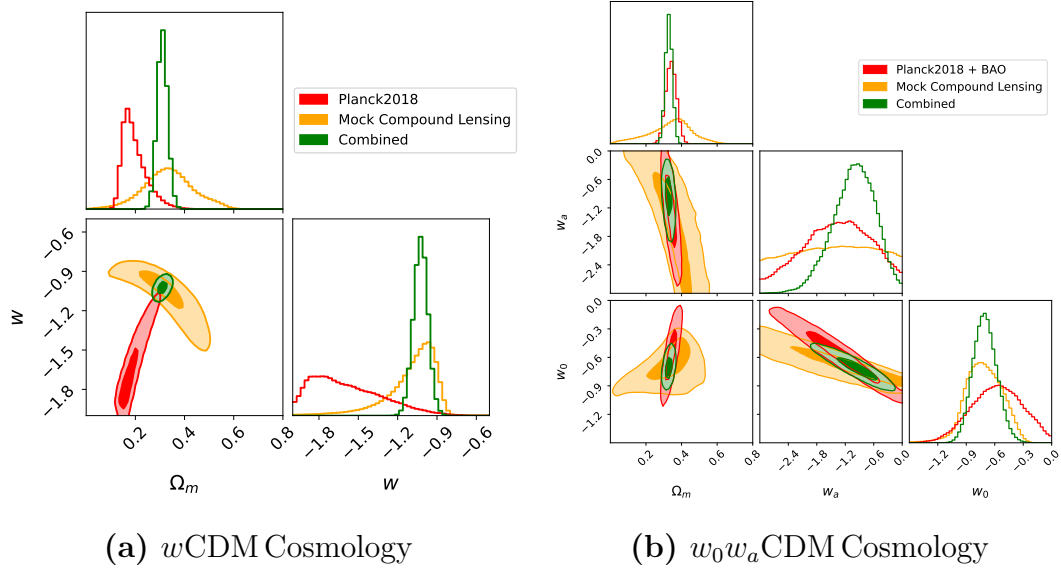


Figure 6.1: Compound lens forecast: Forecast compound lens cosmography constraints from a mock sample of 10 strong lenses with 30 deflection ratios, modeled at similar precision to the Carousel lens in [384].

gun to explore. This dissertation necessarily only scratches the surface, focusing on its applications for SIDM (Ch. 4) and dark energy (Ch. 5). Whatever discoveries are in store in the next decade of cosmology, strong lensing may well be the key to advancing our understanding of the universe.

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