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CURRENT ALGEBRA SUM RULES FOR
CROSS SECTIONS OF INCLUSIVE EXPERIMENT*

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May 26, 1971

ABSTRACT

On the basis of equal-time commutation relations in current algebras we derive sum rules for differential cross sections of inclusive experiment. The sum rules hold good at high incident energies and they relate integrated cross sections for different charge states of the secondary to be detected.

Inclusive measurement of hadron collisions has recently interested both experimentalists and theorists. Although not enough experimental data have been accumulated, the problem has been investigated from various theoretical viewpoints, such as parton model, multiperipheral model, limiting fragmentation theory, and phenomenological $O(2,1)$ expansion. In this Letter we present another theoretical approach to inclusive processes through current algebras.

To be specific let us consider an equal-time commutator

$$[F_+^5(t), F_-^5(t)] = 2F_3(t), \quad (1)$$

where $F_i(t)$ and $F_j^5(t)$ [$i, j = 1, 2, 3$; $(\pm) \equiv (1) \pm i(2)$] are generators of the chiral $SU(2) \times SU(2)$. We sandwich (1) with incoming states $\langle p_1 p_2^{in} |$ and $| p_1 p_2^{in} \rangle$, and insert a complete set of intermediate states between $F_+^5(t)$ and $F_-^5(t)$. The right-hand side is

equal to

$$2(2\pi)^6 \delta(p_1 - p_1') \delta(p_2 - p_2') (I_3^{(1)} + I_3^{(2)}), \quad (2)$$

where $I_3^{(i)}$ is the z-component of the isospin. One must be careful in dealing with the disconnected and the partially connected contributions on the left-hand side:¹

$$\begin{aligned} & \langle p_1' p_2^{in} | [F_+^5(t), F_-^5(t)] | p_1 p_2^{in} \rangle \\ &= \sum_n \langle p_1' p_2^{in} | F_+^5(t) | n^{out} \rangle_c \langle n^{out} | F_-^5(t) | p_1 p_2^{in} \rangle_c \\ &+ \sum_{n'} \langle p_2' | F_+^5(t) | n'^{out} \rangle \langle n'^{out} | F_-^5(t) | p_1 p_2^{in} \rangle_c \\ &+ \sum_{n'} \langle p_1' | F_+^5(t) | n'^{out} \rangle \langle n'^{out} | F_-^5(t) | p_1 p_2^{in} \rangle_c \\ &+ \sum_{n'} \langle p_1' p_2^{in} | F_+^5(t) | n' p_1^{out} \rangle_c \langle n' p_1^{out} | F_-^5(t) | p_2 \rangle \\ &+ \sum_{n'} \langle p_1' p_2^{in} | F_+^5(t) | n' p_2^{out} \rangle_c \langle n' p_2^{out} | F_-^5(t) | p_1 \rangle \\ &+ (2\pi)^3 \delta(p_2 - p_2') \langle p_1' | F_+^5(t) F_-^5(t) | p_1 \rangle \\ &+ (2\pi)^3 \delta(p_1 - p_1') \langle p_2' | F_+^5(t) F_-^5(t) | p_2 \rangle \\ &- \text{terms with } F_+^5 \leftrightarrow F_-^5, \end{aligned} \quad (3)$$

where the subscript c stands for a connected diagram. The disconnected diagrams (the sixth, seventh, and the corresponding terms with $F_+^5 \leftrightarrow F_-^5$) are equal to

$$2(2\pi)^6 \delta(p_1 - p_1') \delta(p_2 - p_2') (I_3^{(1)} + I_3^{(2)}) \quad (4)$$

which cancels the expectation value of $2F_3$.

To derive a covariant form of sum rule we use the technique of infinite momentum. Since the "target" has degrees of freedom due to

internal motion, the method of boost is not unique. Go to the frame in which the time-component of $(p_1 - \kappa p_2)_\mu$ is equal to zero, and boost along the direction perpendicular to both p_1 and p_2 . For the three independent variables, $s = (p_1 \cdot p_2)$, $s_1 = (p_1 \cdot q)$, and $s_2 = (p_2 \cdot q)$, the variables s and $s_1 - \kappa s_2 (= 0)$ are kept fixed under this boost. By the standard technique of infinite momentum limit² we obtain

$$\int_{-\infty}^{v_0} dv v^{-2} [A_+^c(v, \kappa, s) - A_-^c(v, \kappa, s)] + \int_{-\infty}^0 dv v^{-2} [A_+^{p.c.}(v, \kappa, s) - A_-^{p.c.}(v, \kappa, s)] = 0, \quad (5)$$

where $v = (p_1 + p_2) \cdot q$, the upper limit of the integral is determined by

$$\begin{aligned} M_n^2 &= (p_1 + p_2 - q)^2 \\ &= 2s - 2v + q^2 + m_1^2 + m_2^2 \\ &= \text{squared mass of the lowest intermediate state in the} \\ &\quad (p_1, p_2, -q) \text{ channel,} \end{aligned}$$

leading to $v_0 \simeq s$, and $A_\pm^c$ is defined as

$$A_\pm^c(v, \kappa, s) = 8q_0 p_{10} p_{20} \sum_n |\langle n^{\text{out}} | j_\pm(0) | p_1 p_2^{\text{in}} \rangle_c|^2 \delta(p_1 + p_2 - q - p_n), \quad (6)$$

with $j_\pm(0)$ being the pion source function, while $A_\pm^{p.c.}$ stands for the sum of the partially connected diagrams, one of which is, for instance,

$$8q_0 p_{10} p_{20} \sum_n \langle p_2 | j_+(0) | n^{\text{out}} \rangle \langle n p_1 | j_+(0) | p_1 p_2^{\text{in}} \rangle_c \delta(p_2 - q - p_n). \quad (7)$$

It is worthwhile to note that the upper limit in the integral for

$A_\pm^{p.c.}$ is zero (no emission of the pion from a stable particle because

of energy conservation). The sum rule contains contributions from the three-body scattering region which is hardly accessible experimentally. However, we argue in the following that the integral over v is in fact saturated with the contribution above $v \simeq (2s)^{\frac{1}{2}} m_\pi$ (the pion emission region) if s is sufficiently high. First we note that convergence of (5) is assured since $A_+(v, \kappa, s) - A_-(v, \kappa, s) \sim (-v)^{\alpha_\rho(0)}$ as $v \rightarrow \infty$ with s fixed³ (s corresponding to a "target" mass). We are now interested in the behavior of $A_+ - A_-$ in the region of v which is much larger than hadron masses, but not larger than s . This is not the same asymptotic limit as Mueller considered in his $O(2,1)$ expansion.⁴ We expect that at lower M_n^2 there are peaks and bumps due to resonances, and that $A_+ - A_-$ becomes smooth as M_n^2 increases. Duality implies that intermediate states of higher mass are dual to a smooth asymptotic region of crossed channel Regge trajectories. We therefore deduce that the asymptotic behavior at large M_n^2 with s and κ fixed is given as $\sim (M_n^2)^{\alpha_1(0)}$ with $\alpha_1(0)$ being intercepts at $t = 0$ of relevant Regge trajectories.^{5,6} To obtain an asymptotic behavior of the amplitude for pion emission, we compare it with the axial vector amplitude

$$\begin{aligned} &4p_{10} p_{20} \int d^4x e^{-iqx} \langle p_1 p_2^{\text{in}} | A_{\mu+}(x) A_{\nu-}(0) | p_1 p_2^{\text{in}} \rangle \\ &= (p_1 + p_2)_\mu (p_1 + p_2)_\nu A_1 + (p_1 - \kappa p_2)_\mu (p_1 - \kappa p_2)_\nu A_2 \\ &\quad + q_\mu q_\nu A_3 + \delta_{\mu\nu} A_4 + \dots \end{aligned} \quad (8)$$

Because of helicity flip in the crossed channel, the preceding argument in the asymptotic behavior leads us to

$$A_1, A_2 \sim r_{1,2}(\kappa, s) (M_n^2)^{\alpha_\rho(0)-2}, \quad A_3, A_4 \sim r_{3,4}(\kappa, s) (M_n^2)^{\alpha_\rho(0)}, \dots \quad (9)$$

with κ and s fixed. Taking divergence in μ and ν in (8), we find that the pion amplitude $(\phi_i = F_\pi^{-1} \partial^\mu A_{\mu,i})$ for the massless pion ($q^2 = 0$) behaves as⁷

$$A_+ - A_- \sim \gamma(\kappa, s) v^2 (M_\pi^2)^{\alpha_\rho(0)-2}. \quad (10)$$

Because of the rapidly decreasing behavior of $A_+ - A_-$ as v decreases from v_0 , it is a good approximation for a large s to retain only the pion emission region in our sum rule (5). The second integral of the partially connected graphs may, therefore, be discarded. We finally obtain a two-particle version of the Adler-Weisberger sum rule

$$\int_{(2s)^{\frac{1}{2}} m_\pi}^{v_0} dv v^{-1} \left[\frac{d\sigma_+}{d^3q} - \frac{d\sigma_-}{d^3q} \right] (v, \kappa, s) \simeq 0, \quad (11)$$

where $d\sigma_\pm/d^3q$ is the differential cross section in the c.m. frame for (massless) $\pi^\pm$ emission. The foregoing argument indicates that near the lower bound of the integral the integrand is probably very small and negligible.⁸ One remarkable feature is not only that one can choose any two-particle state as an initial state, but also that (11) must hold for arbitrary values of the parameters s and κ as long as they meet some restriction to be mentioned later. It imposes, therefore, a very strong restriction on the production cross section of inclusive experiments.

The same technique applies to the isospin-two part of

$$[dF_i^5(t)/dt, F_j^5(t)] = \delta_{ij} \sigma(t).^9 \quad (12)$$

The resulting sum rule is

$$\int_{(2s)^{\frac{1}{2}} m_\pi}^{v_0} dv \left[\frac{d\sigma_+}{d^3q} + \frac{d\sigma_-}{d^3q} - 2 \frac{d\sigma_0}{d^3q} \right] (v, \kappa, s) \simeq 0. \quad (13)$$

This sum rule will be tested when the π^0 inclusive experiment is done.

Finally let us briefly mention how our sum rules should be tested with experiment. The physical region of inclusive experiment is restricted as

$$s_1 > 0, \quad s_2 > 0, \quad s_1 + s_2 \lesssim s \quad (s \rightarrow \infty) \quad (14)$$

and

$$(\beta_2/\beta_1)(1 - \beta_1\beta_\pi)(1 + \beta_2\beta_\pi)^{-1} \leq \kappa \leq (\beta_2/\beta_1)(1 + \beta_1\beta_\pi)(1 - \beta_2\beta_\pi)^{-1}, \quad (15)$$

where β_1, β_2 , and β_π are the velocities in the c.m. frame of incident hadrons and emitted pion, respectively. The restriction (15) comes from $|\cos \theta_{c.m.}| \leq 1$. It is trivially satisfied in the limit of zero pion mass at high energies ($s \rightarrow \infty$). For a finite pion mass and a finite incident energy, the parameter κ must be chosen in such a way that an important region of integral in sum rules falls in the physical region of emission. This is always possible for large s . From various arguments like Regge theory, limiting fragmentation and so on, it is known that in the π^-p collision, for instance, the π^- can be emitted energetically near the forward direction while the π^0 is emitted less forwardly and the π^+ has even a flatter angular distribution. The sum rule (9) tells us that the $\pi^\pm$ emission is independent of the charge when it is integrated over energy of the secondary with an appropriate weight.

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FOOTNOTES AND REFERENCES

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- 1. We call the second to fifth terms in (3) as partially connected, and the sixth and seventh as disconnected. The same is true for the terms with $F_+^5 \leftrightarrow F_-^5$.
- 2. The infinite momentum method does not give rise to an anomalous threshold while it looks as if the covariant derivation [see S. Fubini, Nuovo Cimento 43A, 475 (1966)] introduces it. In simple perturbation diagrams an anomalous singularity disappears from the region of $v \geq 0$ when $q^2 \leq 0$ (space-like). Since the infinite momentum method works for $q^2 \leq 0$, the same sum rule will be derived in the covariant way if the pion is treated as massless.
- 3. This is the high energy limit of three-body scattering, $|v/s| \rightarrow \infty$.
- 4. A. H. Mueller, Phys. Rev. 2D, 2963 (1970).
- 5. This part of the argument has already been discussed in detail in A. I. Sanda and M. Suzuki, Phys. Rev. 3D (in press). The Regge expansion $(M_n^2)^{\alpha_1(0)} (s/M_n^2)^{2\alpha_2(t)}$, where t is the momentum transfer to the pion, is not fruitful since either M_n^2 or $|t|$ is large in our region.
- 6. In the finite-energy sum rule for an inelastic electron proton scattering amplitude it has been claimed that $(m^2 + 2m\nu)/q^2 (= M_n^2/q^2 - 1)$ is an appropriate variable in which the amplitude is extrapolated down near to the threshold ($m > \nu$). This may be considered as a support for our underlying idea. E. D. Bloom and F. J. Gilman, Phys. Rev. Letters 25, 1140 (1970).
- 7. The factor ν^2 must exist in front since, going back to (3), there is no singular contribution at $\nu = 0$. This also assures the PCAC condition.

8. In various sum rules of current algebras so far derived, the contribution from a smooth Regge asymptotic region is very small numerically. See, for instance, S. L. Adler and F. J. Gilman, Phys. Rev. 156, 1598 (1968).
9. This is true in the σ -model, quark model and many other models.

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