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VISCOUS EFFECTS IN HIGHLY IONIZED ROTATING PLASMAS*

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Summary in English.

Rotating highly ionized plasmas may be generated in systems of cylindrical symmetry by passing a large radial current through a low density gas in an axial magnetic field. The rotation is hindered by contact with the stationary walls of the vacuum chamber. Under the simplifying assumptions of steady state laminar fluid flow and uniform viscosity, two extreme cases may be treated analytically. In the first, the flat disc approximation, the flow is considered to be controlled by axial shear alone. If zero slip is assumed to exist ~~at the~~ insulating surfaces which confine the plasma axially the current is shown to flow in a thin boundary layer, the thickness of which is of the order $\delta = (1/B)\sqrt{(\mu/\sigma)}$. Consequently, the electrical resistance of the plasma appears many times larger than in the absence of the magnetic field. It also follows that the speed of rotation of the plasma in this case should be inversely proportional to the distance from the axis. These conclusions are partially supported by experimental evidence.

In the second case treated analytically, the long cylinder approximation, only radial shear is considered important. In this case the current density is independent of the axial position. Here the apparent resistance is even higher because now all the current has to pass through regions of rapidly spinning plasma in whose frame of reference the electric field is very low. It is therefore practically impossible to construct a simple rotating plasma machine whose volume resistance is lower than the resistance along the end plates. In one obvious scheme to prevent the current from flowing near the insulators an attempt is made to keep the entire plasma from making contact with the insulator surfaces. The first of such models is briefly introduced at the end of this paper and the main results are summarized.

*Work done under the auspices of the U. S. Atomic Energy Commission.

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1. Introduction

Some experiments with highly ionized rotating plasmas have been described in the literature[1-3] and their relationship to controlled thermonuclear fusion research has been discussed[2-4]. As an example, the machine described as a hydromagnetic capacitor[1] has been sketched schematically in Fig. 1a. Very briefly, the operation is as follows. In this case a condenser bank of 50 or 100 μf is discharged through the gas between two coaxial electrodes. This radial current serves three purposes. The initial large pulse of current is used both to ionize the gas and to set it into rotating motion because it flows across an already existing axial magnetic field. When the rapidly spinning plasma develops an EMF equal and opposite to that of the driving condenser bank the current drops to a relatively low value which is just sufficient to keep the plasma spinning in the face of viscous drag at the stationary boundaries. It is a disucssion of this quasi-stationary state of rotation of the plasma which is the topic of the present paper.

As pointed out before,[1] the viscous drag is equivalent to an internal leakage resistance of the hydromagnetic capacitor and as such is easily observable. Under very drastic simplifying assumptions concerning the plasma this effective resistance may be expressed as a relatively simple function of magnetic field and temperature so that for a given magnetic field a measurement of this resistance should yield an estimate of the temperature.

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2. Simplified viscous flow models

Let us assume there exists a nearly steady state of rotation where the plasma has nearly uniform density, i.e. where the centripetal force is supplied by an azimuthal current j_θ rather than by a pressure gradient. This means we assume laminar flow in azimuthal symmetry and we can neglect all time derivatives. Let us furthermore assume that we can treat the plasma as a fluid medium, neglecting mean free paths and Larmor radii. Because of the finite conductivity σ of the gas there must, of course, be a plasma drift v_r radially outward which can be estimated from the radial component of the equation of fluid motion

$$\rho \frac{v_\theta^2}{r} = j_\theta B_z = \sigma v_r B_z^2 \qquad (1)$$

where the notation is the same as that used previously.[1] The second equation in (1) makes use of the fact that E_θ must be negligible in this symmetry because all time derivatives are negligible. Based on eq. (1) it is easily shown that in the plasmas under study we indeed have always $v_r/v_\theta \ll 1$, sothat the flow may be regarded as almost purely azimuthal.

The viscous drag must now be calculated from the azimuthal component of the equation of motion in which the shear stress tensor is retained. The relations are rather complicated because in general the viscosity coefficient in the presence of a magnetic field is not a scalar quantity. Moreover, it is not really expected

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to be uniform throughout the plasma. In two limiting cases, however, in which one component of the velocity gradient dominates over the other, analytic solutions are easily obtained if the viscosity is not a function of position

3. The flat disc approximation*

In the first case to be treated, which we shall call the flat disc approximation, the axial shear dominates over the radial one. This geometry corresponds most closely to the hydromagnetic capacitor described before.[1] Here the azimuthal component of the equation of motion takes the simple form

$$-\mu \frac{\partial^2 v_\theta}{\partial z^2} = j_r B_z = \sigma(cE_r - v_\theta B_z)B_z \qquad (2)$$

It should be pointed out that the electric field E which appears in Ohm's law for plasmas can never be directly determined from boundary conditions but must, in general, be considered as a dependent variable to be evaluated in a self-consistent analysis of the plasma dynamics. The use of Ohm's law in this case is of advantage because the assumed symmetry implies that E_r is independent of z, so that eq. (2) can be solved directly. We find for the boundary conditions $v_\theta(+h/2) = v_\theta(-h/2) = 0$, i.e. no slip at the insulators which are located symmetrically at $z = +h/2$ and $z = -h/2$.

$$v_\theta = \frac{cE_r}{B_z}\left(1 - \frac{\cosh z/\delta}{\cosh h/2\delta}\right) \qquad (3)$$

*A more generalized treatment of this case, with essentially the same results, has recently been carried out by C. C. Chang (private communication).

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where the "skin-depth" or boundary layer δ is given by $\delta = (1/B)(\sqrt{\mu/\sigma})$. The function j_r is then found by substitution of eq. (3) into eq. (2). Since $j_r \propto 1/r$, because it cannot have a finite divergence in the steady state, it follows that in this case $v \propto 1/r$ and $E \propto 1/r$, just as the vacuum field in a long coaxial capacitor. The effective resistance is now readily evaluated to be given by

$$R_{eff} = \frac{V}{I} = \frac{\int_a^b cE dr}{2\pi r \int_{-h/2}^{+h/2} j dz} = \frac{\ln \frac{b}{a}}{4\pi\sigma\delta} \coth \frac{h}{2\delta} = R_o M \coth M \qquad (4)$$

where the ratio $M = h/2\delta$ may be called the Hartman number of this particular type of channel flow[5] and R_o is the electrical resistance of the plasma disc at zero magnetic field. It is easily verified that the total power input $I^2 R_{eff}$ balances both the electrical and viscous dissipation rate.

For an evaluation of δ and R_{eff} we have to substitute the appropriate expressions for σ and μ. The conductivity σ has been discussed amply in the literature.[6] For the coefficient of viscosity in this case where the velocity gradient is parallel to the magnetic field we have to use[7] $\mu = \mu_o/[1 + (\omega \mu_o/p)^2]$ with $\mu_o = (5/8)(m_i/\pi)^{1/2}[(kT)^{5/2}/(Ze)^4 \ln \Lambda]$. Here ω is the ion cyclotron frequency and p is the ion pressure. In many cases under consideration here the quadratic term in the denominator of μ, representing

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the effect of the magnetic field, may be neglected. If numerical values are substituted, we find that in typical cases it is predicted that $\delta \approx 10^{-2}$ cm and there is some doubt whether the fluid model used is realistic since both the ion cyclotron radius and the mean free path are considerably longer. If the observed R_{eff}, which is usually between 0.5 and 1.0 ohm in the rotating plasma under study, is used to estimate the temperature one finds values in the neighborhood of 3 eV. This seems low but it is not too unreasonable since we are dealing here with the temperature in a layer very close to the insulator surface.

4. Experimental evidence

A more direct experimental investigation of the flow pattern would be provided by a measurement of j_θ, which according to eq. (1) should be proportional to ρv_θ^2. In this symmetry j_θ may be derived from a determination of B_r. It was not possible to measure B_r inside the moving plasma, however, since the probes interfered too much with the flow. Therefore B_r was measured by means of tiny coils on the boundary only, and a network analog was used to determine the current distribution consistent with the observed field at the boundary. Such a procedure of course cannot give an unambiguous result. However, with the added assumption of near uniformity in the z-direction, consistent with the results of the above analysis, a definite distribution is found as shown in Fig. 2. If $v_\theta \propto 1/r$, as suggested by eq. (3), then according to eq. (1) $j_\theta \propto 1/r^3$, provided ρ is uniform. In most of the region the fit

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is rather good. Near the inner electrode, however, a very drastic deficit is encountered, indicating a strong deficiency in ρv_θ^2 either by loss of plasma there or because the insulators have become conducting. In most of the region the time decay is almost exactly exponential with a decay constant very nearly equal to twice that of the voltage decay, in good agreement with eq. (1). It is also noteworthy, in this connection, that the distribution does not change very much in time, showing only a slight migration of j_θ towards larger radii, in agreement with quantitative estimates of v_r according to eq. (1). On the other hand, the observed signal of B_r displays some 20% or 30% of random noise superimposed on the smoothly decreasing signal so that the assumption of stationary laminar flow is probably not quite justified.

5. The long cylinder approximation

The second case which is readily analyzed may be called the long tube approximation. Here the axial dimension is assumed to be very long compared to the radial one so that the viscous drag is dominated by the radial shear. This configuration is more nearly realized in the experiment called Ixion,[3] although even there, as will become clear, the length is probably not quite sufficient. If only radial shear is considered and the current density is independent of z, the azimuthal component of the equation of motion takes the form

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$$\mu\left(\frac{v_\theta}{r^2} - \frac{1}{r}\frac{\partial v_\theta}{\partial r} - \frac{\partial^2 v_\theta}{\partial r^2}\right) = j_r B_z = \frac{IB_z}{2\pi h} \cdot \frac{1}{r} \tag{5}$$

The Solution is

$$v_\theta = \frac{IB_z}{4\pi h\mu}\left(\frac{C_1}{r} + C_2 r - r \ln r\right) \tag{6}$$

where C_1 and C_2 are to be determined from the boundary conditions on $v_\theta(r)$ which we shall not specify at this point. The coefficient of viscosity is now given by[7] $\mu = \mu_o/[1 + 4(\omega\,\mu_o/p)^2]$. In the presence of stationary boundaries, unless there is perfect slip, v_θ now displays a maximum somewhere between the electrodes, reminiscent of the distributions shown in Fig. 2. However, now we have nowhere a simple relation such as $v \propto 1/r$.

The effective resistance R_{eff} may again be calculated as indicated in eq. (4) using Ohm's law as stated in eq. (2). In this case we find

$$R_{eff} = R_o\left(1 + \frac{L^2}{\delta^2}\right) \approx R_o \frac{L^2}{\delta^2} \tag{7}$$

where R_o and δ are the same quantities as before and L^2 is an area calculated by integrating the bracket of eq. (6) between the boundaries and dividing by $2 \ln b/a$. For zero slip at stationary boundaries L is of the order of the radius of the inner electrode. We

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are now in a position to judge whether a certain uniform rotating plasma may be considered as a long tube approximation, i.e. whether the current flowing along the insulator boundary layer may be neglected. We merely have to require that the volume resistance given in eq. (7) is much smaller than the boundary resistance determined by eq. (4), or in other words that $h \gg 2L^2/\delta$. Since δ is usually very short unless B is very weak, this condition is just about impossible to realize in practice.

6. Conclusion and introduction to the new experiments

The preceding discussion, although highly idealized, serves to demonstrate that if rotating plasmas behave as fluids it is very likely that in the steady state they are controlled by currents flowing along the insulators with which they are in contact. Evaporation of material and perhaps insulator breakdown is to be expected as a general consequence and the usefulness of the scheme is severely limited. Therefore, configurations have been and are being developed in which it is attempted to keep the plasma from touching the insulators altogether. One such design, which has been in operation for some time, is shown schematically in Fig. 1b. Here the insulators are arranged to be parallel rather than normal to the axis of rotation so that full use is made of the centrifugal force driving the plasma away from the insulators. In this case the magnetic field lines, of course, will have to be curved into arcs as indicated in the figure. In the present experiments the B field is formed by means of a slowly pulsed current through a field coil

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on the inside of the inner electrode. When the gas breaks down into a power discharge under the influence of an applied radial electric field, the current is indeed observed to be concentrated in the central region at maximum distance from the insulators. Now it is found, however, that during the period of near steady rotation the current does not enter the anode in complete axial symmetry, but clearly confines itself to a set of narrow strips, perhaps 10 or 12 in number, roughly equally spaced about the perimeter of the machine, their long dimension being parallel to the magnetic field and extending half way up and down towards the insulators. The nature of this current structure, which appears to rotate with the plasma, and a more complete discussion of the entire new configuration will be the topic of a subsequent paper.

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Figure Captions

1 a. The flat disc rotating plasma.

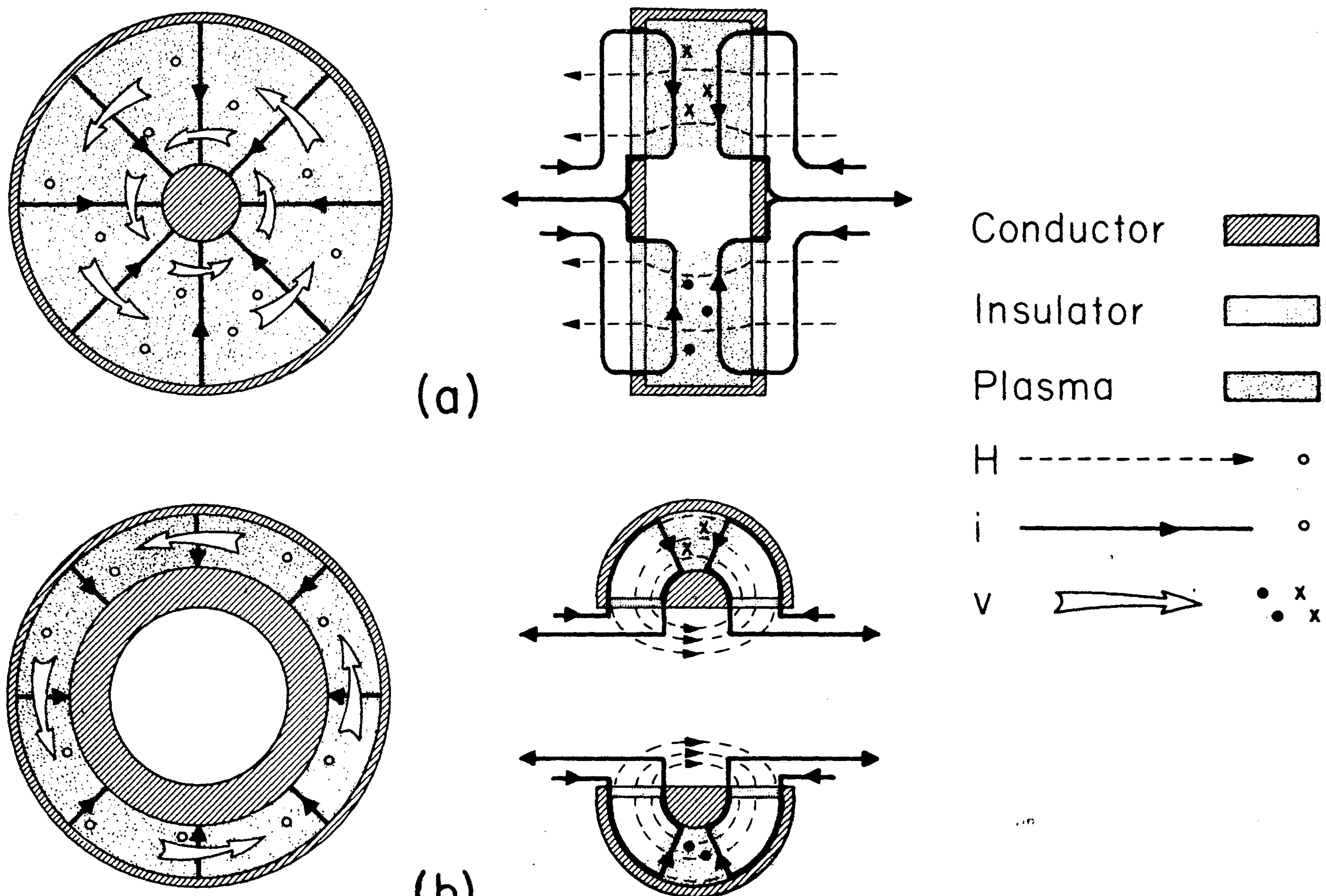
b. The "Homopolar III" configuration.

2. Azimuthal current distribution.

Identification of Figures:

Fig. 1 = Dwng. No. 55,860-2 = MU-17979

Fig. 2 = Dwng. No. 55,861-2 = MU-17980



(a)

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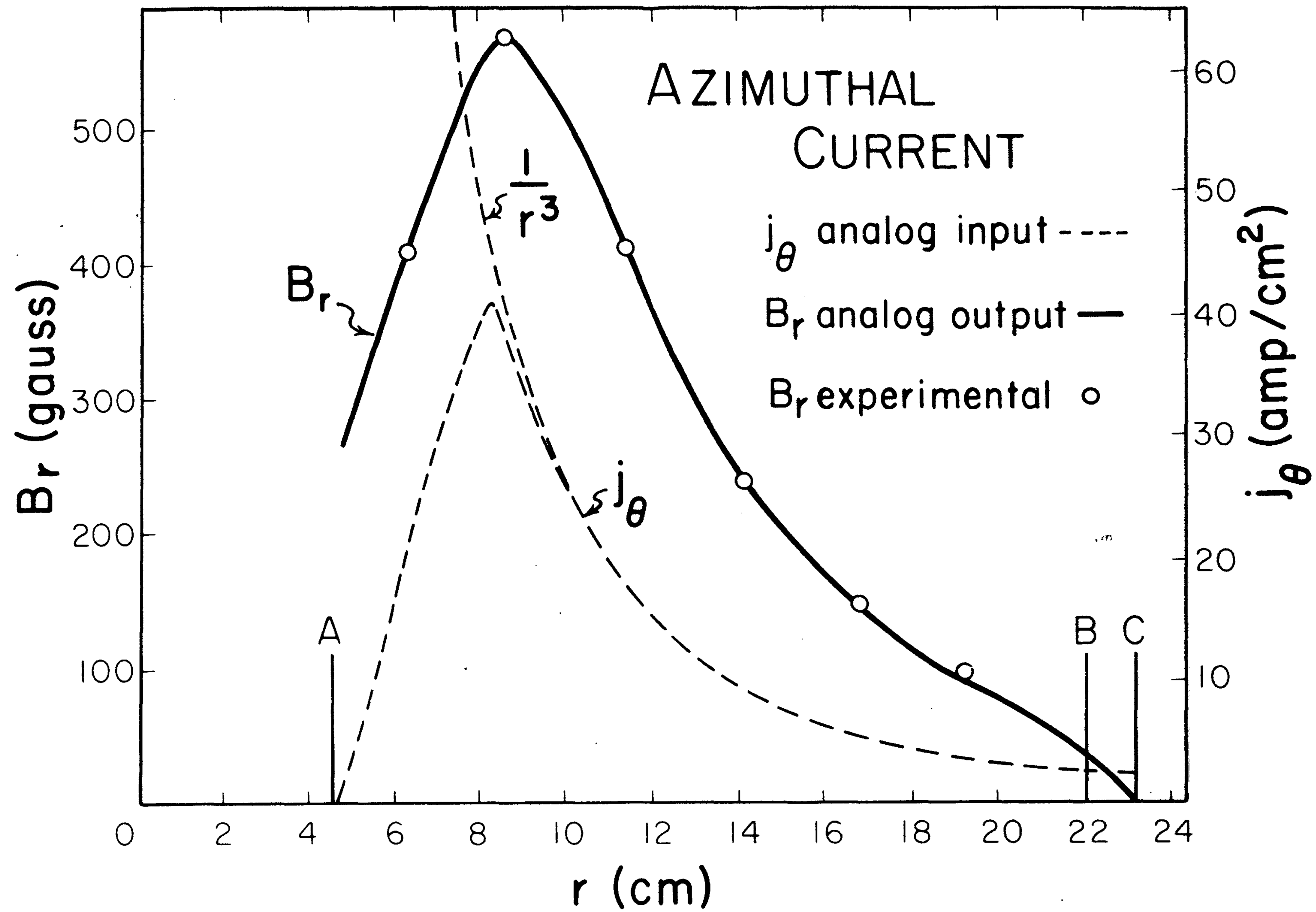
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CURRENT
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B_r experimental
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