Title
ADDITIONAL TABLES FOR THE EVALUATION Of Soo xBe-Xf(x)dx BY GAUSS-LAGUERRE QUADRATURE

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Tables of abscissae and weight coefficients to fifteen places are presented for the Gauss-Laguerre quadrature formula $\int_0^\infty x^\beta e^{-x} f(x)dx \sim \sum_{k=1}^{N} H_k f(a_k)$ for $\beta = -2/3$ and $-1/3$ and $N = 1(1)15$. The tables supplement those presented previously in [1] for $\beta = -1/4$, $-1/2$, and $-3/4$. The same computer program was used, and the accuracy of the tables is the same. The values of $\Gamma(1/3)$ and $\Gamma(2/3)$ used in the calculations were taken from [2] and [3].

This work was carried out under the auspices of the U. S. Atomic Energy Commission.

REFERENCES

1. P. Concus, D. Cassatt, G. Jaehnig, and E. Melby, "Tables for the evaluation of $\int_0^\infty x^\beta e^{-x} f(x)\,dx$ by Gauss-Laguerre Quadrature," Math. Comp. v. 17, 1963, p. 245-256.

2. B. Zondek, "The values of $\Gamma(1/3)$ and $\Gamma(2/3)$ and their logarithms accurate to 28 decimals," MTAC v. 9, 1955, p. 24-25.

3. M. E. Sherry and S. Fulda, "Calculation of gamma functions to high accuracy," MTAC v. 13, 1959, p. 314-315.

Table I. Abscissae and weight coefficients for the Gauss-Laguerre quadrature formula,

$$\int_0^\infty x^{-2/3} e^{-x} f(x)\,dx \sim \sum_{k=1}^{N} H_k f(a_k).$$

a_k	H_k	$H_k \cdot \exp(a_k)$
	N=1	
0.33333 33333 33333	2.67893 85347 07748	3.73875 99050 80055
	N=2	
0.17863 27949 54082	2.49948 36804 70859	2.98833 67977 03566
2.48803 38717 12585	0.17945 48542 36889	2.16020 31335 74152
	N=3	
0.12209 68169 45682	2.32145 53653 79710	2.62292 76526 89097
1.61334 90056 81775	0.34850 20018 91653	1.74933 84732 65220
5.26455 41773 72543	0.(2)89811 67436 38545	1.73659 93013 00220
	N=4	
0.(1)92758 17522 83156	2.17838 28683 12138	2.39011 37841 24326
1.20439 96301 60831	0.46624 94584 48529	1.55482 83607 98918
3.72129 63905 01274	0.(1)33935 59339 05516	1.40214 82539 18999
8.31487 91374 42913	0.(3)37061 45565 29098	1.51366 09259 25455
	N=5	
0.(1)74791 34285 81872	2.06295 68539 57088	2.22316 45692 75146
0.96333 70637 95945	0.54578 89626 23030	1.43019 98224 66154
2.91502 11893 17740	0.(1)67680 33373 62046	1.24864 82381 78496
6.18840 21619 37290	0.(2)24988 53673 06475	1.21710 97356 21169
11.52511 49087 57504	0.(4)13530 71836 03497	1.36966 59602 26564
	N=6	
0.(1)62656 68620 61241	1.96778 06602 95184	2.09501 98442 45480
0.80351 97800 50894	0.59986 39639 15075	1.33972 90657 35762
2.40603 76993 79709	0.10385 48643 90971	1.15174 34181 56171
5.00182 61236 75312	0.(2)72865 26890 63232	1.08339 30791 64780
8.88395 04252 69825	0.(3)15206 55152 20666	1.09718 88485 50533
14.84200 92854 18137	0.(6)45370 06641 92254	1.26640 36339 42300
	N=7	
0.(1)53910 56289 48034	1.88762 35247 29122	1.99217 93822 92348
0.68953 36065 25160	0.63699 81561 83309	1.26940 09402 31050
2.05218 56851 08167	0.13896 51438 20625	1.08182 94507 55712
4.21907 51067 26376	0.(1)14712 44697 01646	1.00001 36960 11105
7.34319 66930 89941	0.(3)63118 70707 16657	0.97559 12522 55892
11.73951 90976 21715	0.(5)80616 49496 47417	1.01118 88596 66047
18.23591 25813 67171	0.(7)14284 31440 42804	1.18745 07529 32144
	N=8	
0.(1)47307 35376 73493	1.81888 64860 09543	1.90700 09918 78877
0.60403 61659 97319	0.66264 58853 58518	1.21230 27194 08334
1.79078 83091 20698	0.17142 86358 11587	1.02757 33918 48145
3.65704 99530 09369	0.(1)24268 43793 20532	0.94032 59858 34089
6.29469 00373 44909	0.(2)16622 74053 39440	0.90043 37700 37319
9.86999 21193 05589	0.(4)46430 24940 98539	0.89801 61154 53170
14.71435 65010 15497	0.(6)38486 49395 15074	0.94552 31851 15890
21.68844 62271 05938	0.(9)42830 29443 30813	1.12440 70951 93164

Table I (continued).

a_k	H_k	$H_k \exp(a_k)$
	N=9	
0.(1)42145 37239 52375	1.75904 92200 60616	1.83476 94248 91784
0.53748 97739 81778	0.68033 07638 73045	1.16452 53654 44683
1.58933 21171 28713	0.20069 43256 14273	0.98349 75036 13614
3.23139 82625 40138	0.(1)35323 74980 99306	0.89422 17611 58128
5.52383 87307 37347	0.(2)33799 04390 33494	0.84698 76832 67099
8.56764 43801 55673	0.(3)15754 73958 61560	0.82849 88429 24810
12.53876 15496 37149	0.(5)30066 35156 48875	0.83867 88330 83233
17.78203 90365 89241	0.(7)16916 17998 75387	0.89318 97445 94946
25.18735 07768 34723	0.(10)12349 65839 61333	1.07246 32048 13696
	N=10	
0.(1)37999 16082 97694	1.70630 09190 22129	1.77238 65699 42899
0.48420 09002 69970	0.69237 95775 88061	1.12364 73414 11566
1.42910 30695 99249	0.22672 14818 34479	0.94655 15012 59328
2.89675 76520 43951	0.(1)47300 60548 75355	0.85686 53157 80554
4.92896 47179 95256	0.(2)58298 75094 16652	0.80590 05062 07236
7.59257 21985 69778	0.(3)39292 01022 13102	0.77932 11408 89289
10.99512 97186 49276	0.(4)12979 14306 41960	0.77333 94837 27197
15.32024 38240 26925	0.(6)17574 04643 54222	0.79135 09422 03390
20.92436 35583 62038	0.(9)69529 01304 74543	0.85016 20120 98616
28.72399 85329 87120	0.(12)34481 66051 58073	1.02863 72155 84583
	N=11	
0.(1)34595 73297 18913	1.65930 71147 20939	1.71771 65943 89995
0.44055 54614 97032	0.70036 58979 37819	1.08806 73970 48753
1.29850 93142 41873	0.24970 22417 57856	0.91486 68076 70198
2.62622 22517 26409	0.(1)59732 53574 03607	0.82559 06841 65244
4.45404 41220 65282	0.(2)89878 27679 53684	0.77271 88765 06399
6.82890 53754 19783	0.(3)80251 97794 33279	0.74167 16302 68702
9.82246 89540 51510	0.(4)39434 09488 81933	0.72730 39437 97399
13.54739 05648 23864	0.(6)95352 83902 78314	0.72927 37721 25854
18.19370 48672 40577	0.(8)94414 85025 45465	0.75242 97141 06519
24.12820 59427 32845	0.(10)27029 89193 34113	0.81393 74074 67420
32.29206 40798 95600	0.(14)93711 73949 44720	0.99096 71054 24927
	N=12	
0.(1)31751 88504 51077	1.61706 14034 16468	1.66922 99938 72122
0.40414 66951 98345	0.70537 83064 11777	1.05667 34089 88548
1.18996 38622 24016	0.26992 01710 37720	0.88721 74590 66608
2.40269 68135 67196	0.(1)72268 47053 13811	0.79877 93543 53391
4.06513 93596 22009	0.(1)12784 66101 32197	0.74500 09202 39689
6.21168 94430 01020	0.(2)14268 84487 49885	0.71136 29874 69343
8.89298 71270 21559	0.(4)95070 23013 05451	0.69218 11723 11777
12.18441 31208 35406	0.(5)35034 36026 14408	0.68567 38316 09196
16.20282 56879 21650	0.(7)63669 95355 26442	0.69299 85414 40771
21.14386 65559 01885	0.(9)47257 04650 30065	0.71966 62674 40484
27.38376 18231 14808	0.(11)10024 70156 56743	0.78286 72151 83144
35.88675 76265 46998	0.(15)24888 02465 71807	0.95810 08798 17937

Table I (continued).

a_k	H_k	$H_k \cdot \exp(a_k)$
	N=13	
0.(1)29340 08751 67801	1.57878 90828 80581	1.62579 71307 06276
0.37330 88560 20493	0.70818 55342 81667	1.02866 27836 62236
1.09828 21077 57930	0.28768 04626 77323	0.86275 64752 79910
2.21474 57562 23859	0.(1)84656 71343 45083	0.77537 76258 19947
3.74028 65422 50479	0.(1)17128 27220 83108	0.72127 24804 00863
5.70096 41062 17142	0.(2)22934 73670 26269	0.68610 56749 99242
8.13412 87674 57322	0.(3)19478 34511 79359	0.66398 65698 86201
11.09327 94632 60340	0.(5)99273 30361 16095	0.65250 30283 82450
14.65682 04475 20963	0.(6)28083 06880 38128	0.65135 95823 91913
18.94521 04183 09559	0.(8)39205 63695 92509	0.66244 35215 86128
24.15911 58002 46050	0.(10)22267 32968 66359	0.69157 42572 37391
30.68349 49996 09917	0.(13)35706 13401 04430	0.75581 48046 13943
39.50435 59809 42499	0.(17)64792 61153 80598	0.92907 18438 40821
	N=14	
0.(1)27268 82877 04741	1.54388 29397 33886	1.58656 20797 27195
0.34685 19081 44841	0.70934 05000 34417	1.00343 81914 04170
1.01979 56701 79682	0.30327 71171 93785	0.84087 46801 04957
2.05440 54961 18154	0.(1)96724 31138 26186	0.75466 22342 90282
3.46455 32629 10342	0.(1)21919 00222 67659	0.70057 90484 07075
5.27050 70014 09558	0.(2)34163 10102 81989	0.66452 20448 71674
7.50073 18661 86253	0.(3)35401 59688 06054	0.64054 45083 57575
10.19485 80676 55877	0.(4)23386 33827 15056	0.62594 01790 98641
13.40869 49332 51236	0.(6)93088 68956 78354	0.61975 37106 70982
17.22320 36891 13055	0.(7)20613 32423 42376	0.62242 97716 46072
21.76201 94942 60384	0.(9)22516 10224 72324	0.63623 59333 85036
27.23039 92668 54783	0.(12)99585 49110 30801	0.66712 65287 78568
34.02147 55051 18916	0.(14)12278 70956 77136	0.73196 75111 71761
43.14190 16766 93111	0.(18)16575 95272 79500	0.90316 75731 68542
	N=15	
0.(1)25470 73765 09968	1.51185 93971 76593	1.55086 21774 46671
0.32390 27607 42317	0.70924 74123 68479	0.98054 36728 17963
0.95183 32997 39326	0.31697 89812 48985	0.82111 95929 07569
1.91594 74727 50201	0.10835 79275 85128	0.73611 57419 93559
3.22739 04717 02161	0.(1)27059 36565 95438	0.68226 87570 73234
4.90226 73378 06081	0.(2)47975 41448 96299	0.64572 31983 10489
6.96284 14474 97537	0.(3)58728 91781 94435	0.62054 83387 47174
9.43944 36134 56714	0.(4)48024 94998 49877	0.60390 02297 10192
12.37354 21882 40066	0.(5)25141 32471 86987	0.59449 64303 77271
15.82288 38244 98723	0.(7)79545 36865 86074	0.59211 49571 29327
19.87065 75221 65249	0.(8)14018 24563 22847	0.59759 99868 21143
24.64336 25910 87589	0.(10)12169 83784 66314	0.61342 45888 45275
30.35050 93912 56378	0.(13)42549 67757 96976	0.64558 73695 51794
37.39294 50367 90201	0.(16)40940 47891 25074	0.71072 66660 51776
46.79700 23046 16461	0.(20)41756 93177 88322	0.87984 96194 45458

Table II. Abscissae and weight coefficients for the Gauss-Laguerre quadrature formula,

$$\int_0^\infty x^{-1/3} e^{-x} f(x)dx \sim \sum_{k=1}^{N} H_k f(a_k).$$

a_k	H_k	$H_k \cdot \exp(a_k)$
	N=1	
0.66666 66666 66667	1.35411 79394 26400	2.63746 16062 23614
	N=2	
0.37567 22179 30861	1.20150 65925 33115	1.74935 73309 49625
2.95766 11154 02472	0.15261 13468 93285	2.93820 92866 39453
	N=3	
0.26208 52606 66292	1.05156 60135 89033	1.36665 44626 78457
1.95493 25013 75445	0.29323 21025 71361	2.07122 80179 52962
5.78298 22379 58263	0.(2)93198 23266 00662	3.02639 04071 81615
	N=4	
0.20133 92018 56116	0.93478 72957 93188	1.14328 18417 80921
1.47419 79346 41924	0.38384 16942 34837	1.67644 06223 15675
4.13132 51919 59209	0.(1)35053 09226 73989	2.18241 86778 47587
8.85980 43382 09417	0.(3)43585 71309 76715	3.06978 73394 08706
	N=5	
0.16348 24973 14192	0.84387 34511 46022	0.99374 93792 03699
1.18664 67489 32585	0.43813 92106 15197	1.43537 79040 09759
3.25643 95006 43721	0.(1)69158 74524 89632	1.79515 02794 61967
6.63967 97553 91305	0.(2)29290 96149 50739	2.24031 92330 14211
12.08708 48310 51530	0.(4)17436 26671 03234	3.09604 84470 52807
	N=6	
0.13761 93190 61857	0.77148 15585 97697	0.88530 48839 07387
0.99414 01486 44880	0.46932 17982 01660	1.26829 50772 60388
2.69920 64156 71707	0.10463 20896 95678	1.55566 23852 06413
5.38978 06821 93424	0.(2)84865 06544 56794	1.85986 30202 83691
9.36322 11351 59500	0.(3)19535 80411 62902	2.27628 46189 30619
15.41603 22992 68631	0.(6)62834 56337 99592	3.11384 04933 95291
	N=7	
0.11882 62404 17822	0.71248 54729 94972	0.80238 27701 90461
0.85587 18337 35907	0.48601 38312 02244	1.14379 72377 42467
2.30933 54229 33882	0.13781 69246 67347	1.38750 38140 60685
4.56022 60611 97541	0.(1)16983 22218 95258	1.62368 24857 24287
7.76418 74767 47606	0.(3)80733 75958 29542	1.90107 37040 17769
12.23922 58894 76617	0.(4)11129 78160 79360	2.30099 09424 20269
18.81899 37421 57292	0.(7)20994 87403 64955	3.12678 78126 15295
	N=8	
0.10455 14821 36296	0.66341 97017 63645	0.73653 68778 25913
0.75160 89226 95132	0.49355 02248 35070	1.04652 82634 36930
2.01993 67576 43745	0.16724 86333 58232	1.26069 48111 33682
3.96188 27396 45473	0.(1)27721 72353 09577	1.45694 79602 12167
6.67114 85966 22730	0.(2)21132 05295 48468	1.66795 64756 37209
10.31583 13659 55020	0.(4)63886 17647 96298	1.92981 43671 15440
15.22974 28971 60944	0.(6)56380 43379 03193	2.31911 19515 86911
22.27863 05714 73993	0.(9)66219 32688 64781	3.13668 62736 25755

Table II (continued).

a_k	H_k	$H_k \cdot \exp(a_k)$
	N=9	
0.(1)93339 83979 40810	0.62190 39621 41154	0.68274 77869 07662
0.67011 78735 20849	0.49523 90400 36961	0.96792 87011 16060
1.79605 10753 01967	0.19259 74065 33472	1.16055 44093 48033
3.50710 90543 86493	0.(1)39892 35190 67410	1.33047 81639 54644
5.86479 31321 17533	0.(2)42649 55010 38095	1.50300 95811 89710
8.97133 50481 77276	0.(3)21580 55055 19200	1.69927 56208 12020
13.00407 28516 73079	0.(5)43922 08719 43820	1.95110 23478 26202
18.30989 79271 52198	0.(7)26063 49997 31779	2.33303 02458 26026
25.78328 31978 76526	0.(10)19953 47042 49209	3.14453 22982 37908
	N=10	
0.(1)84300 61248 82692	0.58626 16128 53416	0.63782 67793 48929
0.60464 22226 23999	0.49316 56158 65161	0.90278 75686 44797
1.61743 38033 43365	0.21405 46452 24968	1.07886 53190 25413
3.14854 18461 74402	0.(1)52786 28478 43986	1.23002 92705 42650
5.24080 33974 52569	0.(2)72953 72122 58754	1.37752 48609 79256
7.96198 07047 30965	0.(3)53524 33422 13233	1.53601 53612 82337
11.42055 90176 29076	0.(4)18894 01906 31765	1.72270 18131 34140
15.80128 88723 27903	0.(6)27009 46676 09465	1.96756 26705 81606
21.46241 97718 71361	0.(8)11193 46027 60859	2.34409 29978 36740
29.32469 64180 24758	0.(12)57927 43518 95306	3.15092 55211 87019
	N=11	
0.(1)76858 00539 27447	0.55528 10484 96007	0.59964 17446 59646
0.55086 68988 17950	0.48866 97541 24379	0.84772 28976 08883
1.47148 16027 93306	0.23201 77010 67849	1.01059 57316 36361
2.85797 09629 57677	0.(1)65857 35300 17655	1.14763 89774 07342
4.74150 44317 40556	0.(1)11147 14380 07979	1.27753 50210 86366
7.16971 76822 49429	0.(2)10863 03553 34605	1.41162 72271 39307
10.21495 82517 70330	0.(4)57158 90864 41214	1.56093 18047 59215
13.99063 58413 90188	0.(5)14612 63952 56867	1.74094 32526 42159
18.68776 91662 98648	0.(7)15164 40272 49596	1.98070 74613 82573
24.67478 10330 24956	0.(10)45240 05609 51428	2.35312 16498 18609
32.89678 94568 97549	0.(13)16303 36339 81931	3.15624 96146 83743
	N=12	
0.(1)70623 23349 12133	0.52806 58072 36857	0.56670 79798 57435
0.50590 37324 06639	0.48262 77375 90317	0.80043 02209 19685
1.34991 02566 12651	0.24693 76995 77046	0.95245 83061 69766
2.61740 79814 43835	0.(1)78716 61603 60781	1.07843 07410 48909
4.33185 31483 20145	0.(1)15709 37435 41090	1.19525 01525 11202
6.52820 48415 40092	0.(2)19181 32479 07767	1.31232 66205 15438
9.25763 08452 07617	0.(3)13711 89380 82870	1.43759 12898 08004
12.59605 73212 52730	0.(5)53504 11989 47923	1.58046 69298 64750
16.66097 89581 99512	0.(6)10201 19338 65808	1.75558 63898 44533
21.64891 07741 63243	0.(9)78917 19295 07022	1.99147 16010 59824
27.93757 84724 47858	0.(11)17373 80097 91134	2.36064 67879 94156
36.49494 04349 14465	0.(15)44693 10897 20129	3.16076 21399 62358

Table II (continued).

a_k	H_k	$H_k \cdot \exp(a_k)$
	N=13	
0.(1)65324 28667 26846	0.50393 84322 21635	0.53795 68696 39432
0.46774 52334 82929	0.47562 15217 98530	0.75927 77474 79817
1.24703 80470 01365	0.25925 06692 12252	0.90219 75193 79990
2.41478 22742 32756	0.(1)91104 33901 75796	1.01921 46987 21640
3.98910 97498 27019	0.(1)20847 48851 28927	1.12590 59008 46467
5.99653 68963 23493	0.(2)30605 41430 58622	1.23044 20016 32546
8.47483 52245 34307	0.(3)27939 17888 18456	1.33902 23047 49679
11.47792 18864 48622	0.(4)15100 21715 66940	1.45807 99295 37048
15.08465 55887 52354	0.(6)44865 54706 42586	1.59623 06706 07439
19.41605 04632 26589	0.(8)65329 07774 29949	1.76762 48718 37371
24.67356 51806 44890	0.(10)38506 83586 67700	2.00046 53834 77357
31.24355 62492 37133	0.(13)63870 32900 75507	2.36702 70846 97316
40.11554 55862 82525	0.(16)11977 45676 85612	3.16464 28093 15029
	N=14	
0.(1)60765 14197 70151	0.48237 72745 69923	0.51259 78795 06265
0.43495 19690 91790	0.46804 23544 03705	0.72307 34168 71353
1.15883 21678 00457	0.26934 98268 47637	0.85820 51505 17097
2.24166 50319 52486	0.10285 89080 66357	0.96779 78729 29923
3.69777 77154 67489	0.(1)26423 52049 90244	1.06638 77237 88853
5.54781 66824 93765	0.(2)45243 29176 47568	1.16128 91466 70299
7.82059 16861 85185	0.(3)50479 56399 04542	1.25763 73002 70822
10.55606 62821 62901	0.(4)35413 30957 94956	1.36021 59181 59127
13.81039 54452 39036	0.(5)14822 59324 05586	1.47469 67587 01564
17.66493 19286 73231	0.(7)34264 09373 44922	1.60924 32986 79887
22.24380 76991 97140	0.(9)38859 65717 06297	1.77771 40511 03925
27.75300 99558 96515	0.(11)17774 90523 54986	2.00810 47992 17162
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