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Mechanism for the Difference in Lifetimes of Charged and Neutral $D$ Mesons

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The reaction $D^0 \rightarrow s + d + \text{gluon}$ is proposed as a source for the difference in the lifetimes of the charged and neutral $D$ mesons. In a nonrelativistic bound-state model the rate for the reaction is found to depend on the ratio $f_D/m_s$. For reasonable values of this ratio the observed difference in the lifetimes may be accounted for.

A number of experiments\(^1\) have recently reported a significant difference in the lifetimes of the charged and neutral $D$ mesons, with $\tau_{D^+}$ perhaps as much as six times as large as $\tau_{D^0}$. It has been argued that mesons containing a heavy quark $c$, $b$, or $t$ will decay through a mechanism where the light quark acts as a spectator\(^2\) [Fig. 1(a)]. The process depicted in Fig. 1(b) can contribute only to the decay of the $D^0$.\(^3\) However, by the usual helicity arguments the contribution of Fig. 1(b) is suppressed by the square of the ratio of light-\(~\)to heavy-quark masses and by $f_D/m_c^2$, $f_D$ being the pure leptonic decay constant of the $D$ defined by

$$\langle D(p) | J_{\mu}^A | 0 \rangle = \frac{i}{(2\pi)^{3/2}} \frac{f_D b_{\mu}}{2\omega_D} ,$$

where $J_{\mu}^A$ is the weak hadronic axial-vector current. The spectator graph leads to equal charged and neutral decay rates given by\(^4\)

$$\Gamma_{\mu} = \Gamma_{\mu}(m_c/m_D)^2 [2 + 3a_s] ,$$

where $\Gamma_{\mu} = G_F^2 m_D^3 / 192\pi^2$ is the rate for muon decay $\mu \rightarrow e\nu\nu$. The factor of 2 is for leptons, and 3 for colors, and $a_s = (2f_s^2 + f_m^2)/3$. The coefficients $f_c$ and $f_m$ incorporate renormalization effects due to gluon exchange on the terms in the weak Lagrangian transforming as the 20 and 84 of $SU(4)$, respectively.\(^5\) Using $a_s(m_c^2) = 0.6$, we obtain $f_c \sim 2$ and $f_m \sim 0.7$, leading to $a_s = 1.7$.

In this note, we propose a mechanism that may account for the observed difference in lifetimes. It is the one depicted in Fig. 2, namely

$$D^0 \rightarrow s + d + \gamma_s(\text{gluon}).$$

We have calculated the contribution of this process by considering the $D^0$ meson (mass = 1.86 GeV) as a nonrelativistic bound state of $c$ and $u$ quarks with "constituent" quark masses of $m_c = 1.55$ GeV and $m_u = 0.3$ GeV. The momentum variation of the bound-state wave function is faster than that

\(\text{FIG. 1. Graphs contributing to } D \text{-meson decays.}
\)

\(\text{(a) The "spectator" graph that contributes to the nonleptonic and semileptonic decays of both the charged and the neutral } D \text{ mesons. (b) This contributes to the decays of the } D^0(\bar{D}^0) \text{ only. See Ref. 3.}
\)
of the amplitude multiplying it and thus the total amplitude is proportional to the wave function at the origin and, in turn, to $f_D$.\textsuperscript{5}

The gauge-invariant amplitude for the contribution of Figs. 2(a) and 2(b) can be written as (color indices are suppressed)

$$A = \frac{e}{\sqrt{2}} \left( F_A q \bar{p} q + i F_V \bar{q} \gamma_\mu p \gamma^\mu q \right)$$

$$= \frac{\epsilon^\mu(q) l^\mu}{[2w_s(2\pi^3)]^{1/2}},$$

where $\epsilon^\mu$ is the polarization of the gluon and $l^\mu$ the weak current of the light quarks:

$$l^\mu = \bar{u}_s(q) \gamma^\mu (1 - \gamma_5) v_d(q).$$

Since we are dealing with gluon emission from a color-neutral state the gauge-invariant amplitude (4) is infrared finite. Note that the contribution for gluon emission from final-state light-quark lines will be suppressed by powers of $m_s^2$ and/or $m_u^2$ and is therefore neglected.

In the nonrelativistic model that we have adopted we find

$$F_V = \frac{\psi(0)}{m_s m_c} (2m_D)^{1/2} = \frac{f_D}{\sqrt{6}} \frac{m_s}{m_u m_c},$$

$$F_A = \frac{\psi(0)}{m_s} \frac{m_u - m_s}{m_u m_c m_D} (2m_D)^{1/2} = \frac{f_D}{\sqrt{6}} \frac{m_u - m_s}{m_u m_c}.$$ \hspace{0.5cm} (7)

The decay rate, $\Gamma_s$, for the process (3) is then found to be

$$\Gamma_s = G_F^2 a_0^+ \alpha_s \left| F_V \right|^2 + \left| F_A \right|^2 m_D^5 / 108 \pi^2$$

$$\approx G_F^2 a_0^+ \alpha_s f_D m_D^5 / 324 \pi^2 m_u^2,$$

where $a_0^+ = (f_+ + f_-)^2 / 4$. This leads to a ratio of lifetimes:

$$R = \frac{\tau_{\pi^+}}{\tau_{\pi^-}} = 1 + \frac{16 \alpha_s \pi}{27} \frac{f_D^2}{m_u^2} \frac{a_0^+}{2 + 3 a_3}.$$ \hspace{0.5cm} (10)

With $\alpha_s = 4\pi / [9 \ln(m_D^2 / \Lambda^2)]$, and $\Lambda = 0.5$ GeV, we obtain

$$R = 1 + 0.7 (f_D^2 / m_u^2).$$ \hspace{0.5cm} (11)

Both $f_D$ and $m_u$ are not accurately known. In the literature estimates for $f_D$ range from about 150 to 800 MeV. In fact, a nonrelativistic-potential model calculation based on the potential $V(r) = -4\alpha_s / 3r + \alpha_s / r^2$, with $a = 1.95$ GeV$^{-1}$, yields\textsuperscript{8}

$$1 \approx f_D / m_u \approx 2.$$ \hspace{0.5cm} (12)

Using $m_u = 300$ MeV and the values of $f_D$ from the literature quoted above, we find that $R$ varies from 1.2 to 7. The larger values of $f_D / m_u$ could therefore account for a significant difference in the lifetimes of neutral and charged $D$ mesons.

Our method of calculating $F_V$ and $F_A$ of Eqs. (6) and (7) based on a nonrelativistic bound-state model are expected to work, at best, for heavy-quark systems. They are totally unreliable for $\pi$ or $K$ mesons. Analogous form factors exist\textsuperscript{9} for $\pi (K) - \nu \bar{\nu}$ (also $\bar{\pi} - \gamma \gamma$), but they are smaller by a factor of 10 for the $\pi$ case and a factor of 10 for the case of $K$ mesons than a model as ours would suggest. For light mesons these form factors can be understood on the basis of partial conservation of axial-vector current (PCAC) arguments. We do not expect soft $-D$-meson limits to work. On the other hand, nonrelativistic bound-state models have had considerable success in the heavier systems.\textsuperscript{10}

We expect an analogous mechanism to be important in other heavy-meson decays. Some consequences are as follows:

(1) The contribution of the gluon mechanism of Fig. 2 to the width of the charmed $F$ meson can be obtained by replacing $a_0^+$ with $a_0^- = (f_+ - f_-)^2 / 4$, $m_D$ with $m_F$, $m_u$ with $m_s$, and $f_D$ with $f_F$ in Eq. (9). Note that since the W carries no color, the renormalization of the weak four-fermion vertex via gluon exchange is crucial to this contribution and it vanishes in the limit of $f_+ = f_- = 1$. We thus obtain

$$\frac{\tau_{\pi^+}}{\tau_{\pi^-}} \approx 1 + \left[ \frac{m_F^2}{m_c^2} \frac{16 \alpha_s \pi}{27} \frac{f_F^2}{m_s^2} \frac{a_0^-}{2 + 3 a_3} \right]$$

$$\approx \left[ \frac{24 \pi^2}{2 + 3 a_3} \frac{m_F m_s^2}{m_c^2} \left( 1 - \frac{m_s^2}{m_c^2} \right) \frac{f_F^2}{m_s^2} \right]$$

$$\approx 1 + 0.2 (f_F^2 / m_s^2) + 2.4 (f_F^2 / m_c^2).$$ \hspace{0.5cm} (13)

FIG. 2. Dominant contribution to the decay $D^0 \to s + d + \gamma$. Graphs with emission of the gluons off the final quark lines are suppressed by the helicity factor and are being ignored.
In Eqs. (13) and (14) the last factors are for the pure leptonic mode $F - \tau + \nu_\tau$. Thus the lifetime of the $F$ meson and its semileptonic branching ratio would be somewhat smaller than that of the charged $D$ meson.\footnote{J. Kirkby, in Proceedings of the International Symposium on Lepton and Photon Interactions at High Energies, Fermilab, Batavia, 23–29 August 1979 (to be published); V. Lüth, \textit{ibid.}; L. Veyvodice, \textit{ibid.}; H. Harari, Rapporteur's talk, \textit{ibid.}}

(2) The lifetimes of the neutral mesons containing $b$ and $t$ quarks and their semileptonic branching ratios will also be smaller than those of their charged isospin counterparts.

(3) Of course, the strongest prediction of our model is the existence of a gluon jet in the decays of heavy mesons. Anticipating an ability to distinguish gluon from quark jets (for instance by $(p_\perp)$ or multiplicities), we give the energy ($\omega$) distribution of the gluon as

$$\Gamma^{-1} d\Gamma/d\omega = 6\pi (1 - \omega),$$


(4) Similar considerations should apply to radiative leptonic decays of $D$ (Cabibbo suppressed) and $F$ (Cabibbo allowed) decays. The rate for $D^+ (F^+) - e^+ \nu$ should be $10^4$ times that for $D^- (F^-) - e^- \bar{\nu}$.\footnote{A crossed-channel version of Fig. 1(b) exists for charged-$D$-meson decays but it is Cabibbo suppressed.

Throughout this paper we will assume the "standard" Weinberg-Salam and Glashow-Weinberg-Model for weak interactions and set the Cabibbo angle in charm decays to zero.}

(5) As the gluon carries no isospin our mechanism indicates that isospin-$\frac{1}{2}$ final states may dominate Cabibbo-allowed $D^0$ decays. It is not clear whether this dominance would extend to the exclusive two-body channels. If it does, then it is worth pointing out that the mechanism of Fig. 2 yields

$$\Gamma(D^0 - K^0 \pi^0)/\Gamma(D^0 - K^- \pi^+) = \frac{1}{2}.\footnote{See, for example, Cabibbo and Maiani, Ref. 2.}$$

Recall that the contribution to this ratio from the spectator graph [Fig. 1(a)] is highly suppressed and amounts to $\frac{1}{30}$.\footnote{R. Van Royen and V. F. Weisskopf, Nuovo Cimento 3, 617 (1967).} Experimentally this ratio is $0.7 \pm 0.35$.\footnote{N. G. Deshpande and D. Iskandar, Phys. Rev. D 19, 3457 (1979), and references therein; V. A. Novikov, L. B. Okun, M. A. Shifman, A. I. Vainshtein, M. B. Voloshin, and V. I. Zakharov, Phys. Rev. Lett. 38, 626 (1977); S. S. Gerstein and M. Y. Kholopov, Pisma Zh. Eksp. Teor. Fiz. 23, 374 (1976) [JETP Lett. 23, 338 (1976)].}

If our mechanism is important for the above two-body modes, then it will also be important to Cabibbo-suppressed decays such as $D^0 - K^- K^+$ and $D^0 - \pi^- \pi^-$.\footnote{These parameters are taken from the charmonium model of E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane, and T.-M. Yan, Phys. Rev. D 17, 3090 (1978).}

In short, even a large difference in the lifetimes of charged and neutral $D$ mesons can be explained without requiring a revision of the underlying gauge model and/or invoking exotic new interactions, provided $f_D/m_D \approx 2$. The critical point in our calculation is the observation that in the rate for the reaction $D^0 - s + \bar{d} + $-gluon, the dependence on $f_D^2$ is compensated for by the appearance of $m_D^2$ in the denominator.

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