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Propagation of Load Shed in Cascading Line Outages Simulated by OPA

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Abstract

We estimate with a branching process model the propagation of load shed and the probability distribution of load shed in simulated blackouts of an electric power system. The average propagation of the simulated load shed data is estimated and then the initial load shed is discretized and propagated with a Galton-Watson branching process model of cascading failure to estimate the probability distribution of total load shed. We initially test the estimated distribution of total load shed using load shed data generated by the OPA simulation of cascading transmission line outages on the 300 bus IEEE test system. We discuss the effectiveness of the estimator in terms of how many cascades need to be simulated to predict the distribution of load shed accurately.

1 Introduction

Large blackouts are rarer than small blackouts, but are costly to society when they do occur and have substantial risk [12]. Large blackouts generally become widespread by a cascading process of successive failures [18, 22, 23]. It is useful to study mechanisms of cascading failure so that blackout risk may be better quantified and mitigated. The electric power infrastructure is vital in maintaining our society, and maintaining high reliability is especially important as the electric power infrastructure is being transformed in response to changes in new energy sources, new loads, technological advances, sustainability, markets and climate change.

There are many and diverse mechanisms in power systems by which components tripping or failures cause further components tripping [12, 15, 18, 22, 23]. These include line overloads, failures in protection, communication, maintenance or software, various types of instability, and errors in coordination, situational awareness, planning or operations. It is infeasible to analyze a full range of these mechanisms with one simulation, so cascading failure simulations model and analyze a selected subset of these mechanisms [15]. In this paper we analyze load shed data produced by the OPA simulation of cascading line overloads. Each simulated cascade has successive generations in which transmission lines are tripped and load is shed, and the total number of lines tripped and the total amount of load shed are measures of the size of the blackout.

In the OPA simulation model [2], the power system is represented with a standard DC load flow approximation. Starting from a solved base case, blackouts are initiated by random line outages. Whenever a line is outaged, the generation and load is redispatched using standard linear programming methods. The cost function is weighted to ensure that load shedding is avoided where possible. If any lines were overloaded during the optimization, then these lines are outaged with a specified probability. The process of redispatch and testing for outages is iterated until there are no more outages. Then the total load shed is the power lost in the blackout. The OPA model neglects many of the cascading processes in blackouts and the timing of events. However, the OPA model does represent in a simplified way a dynamical process of cascading overloads and outages that is consistent with some basic network and operational constraints. This paper considers a restricted form of the OPA model in which the power grid is fixed and does not evolve or upgrade; in other work the OPA model also represents the complex dynamics of an evolving grid [3, 12, 19].

Branching processes have long been used in a variety of applications to model cascading processes [1, 14], but their application to the risk of cascading failure is recent [7, 8]. In particular, Galton-Watson branching processes give a high-level and tractable probabilistic model of cascading failure. There is some initial evidence that Galton-Watson branching processes can capture some general features of simulated and observed cascading line trips [8, 9, 20] and can approximate other probabilistic models of cascading failure [7, 11, 17]. The branching process gives a simple probabilistic description of the cascading process as an initial disturbance followed by an average tendency for the cascade to propagate in stages until the cascade dies out or all the components fail.

In previous work [9, 10], we obtained cascading failure data from the OPA simulation with 118 and 300 bus IEEE standard test systems, estimated the initial number of lines tripped and average propagation of line trips from this data, and then used the branching process to predict the probability distribution of the total number of lines tripped. This predicted distribution was then shown to match well with the empirical distribution produced by exhaustively running the OPA simulation in most of the cases tested. It is useful to predict the distribution of total number of lines tripped via the branching process because this can be done with significantly fewer simulated cascades. The total number of lines tripped is a measure of blackout size of interest to utilities, whereas load shed is a measure of blackout size and impact of much more direct interest to all users of electricity. Therefore in this paper we test estimating the propagation and probability distribution of load shed.

In contrast with the case of number of lines tripped, which are nonnegative integers, the amounts of load shed are nonnegative real numbers. We estimate the initial distribution of load shed and the average propagation λ from the simulated load shed data. Then we discretize the continuous initial distribution of load shed and use this discrete distribution as the initial distribution of a Galton-Watson branching process with average propagation λ to estimate a discretized distribution of the total load shed.

Our previous work [10, 24] also estimated the initial distribution of load shed and the average propagation λ from simulated load shed data, but then took a different approach using continuous state branching processes [13, 16, 21] to estimate the distribution of the total load shed. The offspring distribution was assumed to be a gamma distribution, with mean λ and variance estimated from the data. Then computer algebra was used to manipulate cumulant generating functions to compute the distribution of total load shed. In this approach, it is not yet known what form of offspring distribution fits power system cascading data well (the gamma distribution was chosen in [10, 24] because it is easy to compute with). Also, there remain challenges in estimating a second parameter of the offspring distribution such as variance and in improving the methods that compute the distribution of load shed for general offspring distributions. These challenges for the approach based directly on continuous state branching processes may be met in the future, but here we are able to suggest an alternative approach that seems simpler.

We assume some background explanations in previous papers. The OPA model is explained in detail in [2] and references to a variety of cascading failure methods and simulations are in [12, 15]. The branching process model and parameter estimation are explained in more detail in [9] and general background on branching processes is in [1, 13, 14].

2 Estimating propagation and distribution of load shed with a branching process

This section describes the procedure for estimating the propagation and probability distribution of load shed with a branching process.

For each simulated cascade the total load shed as well as the load shed at each intermediate generation of the cascade is recorded. The first step is to round very small load shed amounts that are considered negligible (less than 0.5% of total load) to zero. Then the data is modified so that each cascade starts with a nonzero amount of shed. In particular, cascades with no load shed are discarded. The remaining K cascades are those with some non-negligible load shed. Therefore the computed statistics, such as the probability distributions of initial and total load shed, are conditioned on the cascade starting with some non-negligible amount of load shed. Moreover, for the cascades with no load shed in initial generations and non-negligible load shed in subsequent generations, we discard the initial generations with no load shed so that generation zero always starts with a positive amount of load shed.

Now the data has K cascades with non-negligible load shed. Letting X_n^i denote the load shed at generation n of cascade i, the data looks like this:

$$\begin{array}{lcccc} & \text{gen. } 0 & \text{gen. } 1 & \text{gen. } 2 & \cdots \\ \text{cascade } 1 & X_0^{(1)} & X_1^{(1)} & X_2^{(1)} & \cdots \\ \text{cascade } 2 & X_0^{(2)} & X_1^{(2)} & X_2^{(2)} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \text{cascade } K & X_0^{(K)} & X_1^{(K)} & X_2^{(K)} & \cdots \end{array}$$

The total load shed in cascade i is

$$Y^{(i)} = X_0^{(i)} + X_1^{(i)} + \dots$$

The estimator for the average propagation λ is the standard Harris estimator [6, 13, 14, 25]:

$$\widehat{\lambda} \quad = \quad \frac{\displaystyle\sum_{k=1}^{K} \left(X_1^{(k)} + X_2^{(k)} + ... \right)}{\displaystyle\sum_{k=1}^{K} \left(X_0^{(k)} + X_1^{(k)} + ... \right)} \tag{1}$$

The Harris estimator (1) is an asymptotically unbiased maximum likelihood estimator [14, 25]. Our cascading process is assumed to be subcritical ($\lambda < 1$) and saturation effects are neglected. (In supercritical or saturating cases, other estimators for λ are appropriate as discussed in [10, 17].)

The load shed amounts $X_0^{(1)}, X_0^{(2)}, \cdots, X_0^{(K)}$ are samples from the probability distribution of initial load shed, assuming that some non-negligible load is shed. The average initial load shed θ is estimated as

$$\theta = \frac{1}{K}\sum_{k=0}^{K} X_0^{(k)} \tag{2}$$

To estimate the probability distribution of total load shed from the initial load shed and the estimated propagation $\widehat{\lambda}$, we discretize the samples of the initial load distribution and assume they are propagated by a Galton-Watson branching process with a Poisson offspring distribution of mean $\widehat{\lambda}$.

There are general arguments suggesting that the choice of a Poisson offspring distribution is appropriate [4, 5]. The Poisson distribution is a good approximation when each load shed increment is related to stress on the supply to a large number of other loads so that each load shed increment can be associated with a small, fairly uniform probability of independently leading to other load shed increments in a large number of locations.

We choose a discrete amount of load shed Δ. Then each initial load shed sample X_0^k is discretized to an integer multiple of Δ:

$$Z_0^k = \text{int}\left[\frac{X_0^k}{\Delta} + 0.5\right], \tag{3}$$

where $\text{int}[x]$ = integer part of x. Write Z_0 for the initial load shed expressed in integer multiples of Δ. Then the empirical probability distribution of Z_0 is

$$P[Z_0 = z_0] = \frac{1}{K}\sum_{k=1}^{K} I[Z_0^k = z_0] \tag{4}$$

Now, given the probability distribution (4) of the initial distribution Z_0 and the average propagation estimated from (1), branching process theory implies that the discretized total load shed is distributed according to a mixture of Borel-Tanner distributions:

$$P[Y = r\Delta] = \sum_{z_0=1}^{r} P[Z_0 = z_0] z_0 \lambda (r\lambda)^{r-z_0-1} \frac{e^{-r\lambda}}{(r-z_0)!} \tag{5}$$

3 Results

The cascading failure data is produced by the OPA simulation on the IEEE 300 bus standard test system [26]. Three load levels are considered: 1.0, 1.05 and 1.1 times the base case load. 20 000 cascades were simulated for each load level. The number of cascades K with non-negligible load shed is shown in Table 1 for each load level. The probability of a cascade with non-negligible load shed (that is, a significant blackout) is $K/(20\,000)$.

For the IEEE 300 bus system the load shed discretization Δ is chosen to be 952 MW, which is 4% of the base case load of 23 800 MW. This value of Δ is chosen by experimenting with a range of values. (As a possible point of reference, the power system contains 409 lines as discrete elements and each line comprises 0.24% of the total number of lines.) Too small a value of Δ does not allow sufficient samples within each discretization bin to get a good estimate of the frequency of blackouts in that discretization bin. Too large a value of Δ gives insufficient resolution in the load shed. In the cases tested we find that varying Δ by a factor of 2 has not much effect on the results. The choice of Δ does affect the way that the branching process models the cascading load, and we hope that future work will establish more systematic methods for the choice of discretization.

The average propagation λ is estimated using (1) for each load level and is shown in Table 1. The average initial load shed θ estimated using (2) for each load level is also shown in Table 1.

Table 1. Average propagation λ and average initial load shed θ in IEEE 300 bus test system

load level	λ	θ(GW)	K
1.0	0.09	3.72	4137
1.05	0.21	3.57	8568
1.1	0.42	3.29	9381

For the base case load level 1.0, the probability distribution of total load shed estimated via the branching process is compared to the empirical distribution of total load shed in Figure 1. Although both probability distributions are discretized in load, the distribution of total load shed estimated via the branching process has its points joined by a line so it can be clearly distinguished. The match is good, but this is expected in this case since the average propagation $\lambda = 0.09$ is small and the cascading effect is small, so that the distribution of total load is close to the initial distribution of load.

For the higher load level 1.05, the probability distribution of total load shed estimated via the branching process is compared to the empirical distribution of total load shed in Figure 3. The average propagation $\lambda = 0.21$ and the match is good. The empirical initial load shed distribution is shown in Figure 2. The cascading has the effect of changing the initial distribution of load shed into a distribution of total load shed with larger blackouts.

For the higher load level 1.1, the probability distribution of total load shed estimated via the branching process is compared to the empirical distribution of total load shed in Figure 5. The average propagation $\lambda = 0.42$ and the match is good except for the sharply dropping portion of the tail. The empirical initial load shed distribution is shown in Figure 4. The cascading has a larger effect of changing the

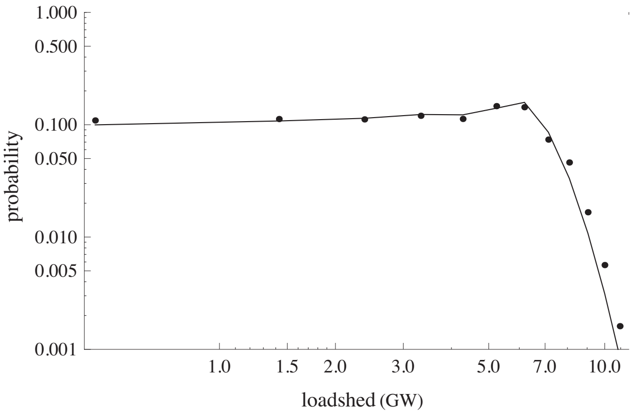

Figure 1. Probability distributions of total load shed for IEEE 300 bus system at load level 1.0. Dots are the empirical distribution; line is estimated with the branching process.

initial load shed distribution into the total load shed distribution.

4 Number of cascades for accurate estimates

This section roughly estimates how many fewer cascades are needed to estimate propagation and then estimate the probability distribution of load shed with the branching process compared to direct empirical estimation of the probability distribution of load shed.

In our case of a Poisson offspring distribution, the asymptotic standard deviation of the Harris estimator can be worked out using the methods of [25] to be

$$\sigma(\widehat{\lambda}) \sim \frac{\sqrt{\lambda(1-\lambda)}}{\sqrt{K\theta/\Delta}} \tag{6}$$

Note that θ/Δ estimates $EX_0/\Delta = EZ_0$, which is the mean number of discretized amounts of initial load shed.

Let p_{branch} be the probability of shedding total load S, computed via estimating λ from K_{branch} simulated cascades with non-negligible load shed and then using the branching process model. p_{branch} is conditioned on a non-negligible amount of load shed. Assume that the initial distribution of load shed is known with high accuracy. Then the standard deviation of p_{branch} is

$$\begin{aligned}\sigma(p_{\mathrm{branch}}) &= \left|\frac{dp_{\mathrm{branch}}}{d\lambda}\right| \sigma(\widehat{\lambda}) \\ &= \left|\frac{dp_{\mathrm{branch}}}{d\lambda}\right| \sqrt{\frac{\lambda(1-\lambda)\Delta}{K_{\mathrm{branch}}\theta}} \end{aligned} \tag{7}$$

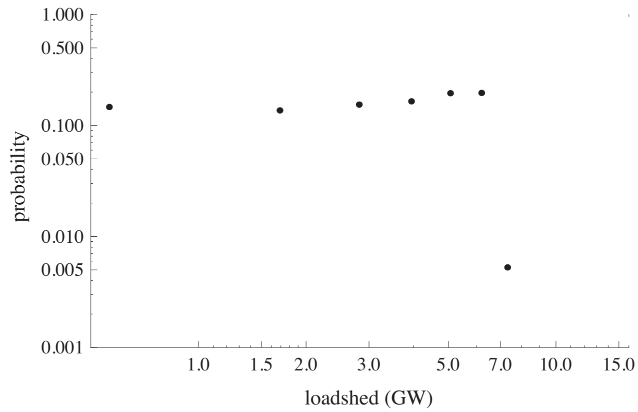

Figure 2. Probability distribution of initial load shed at load level 1.05.

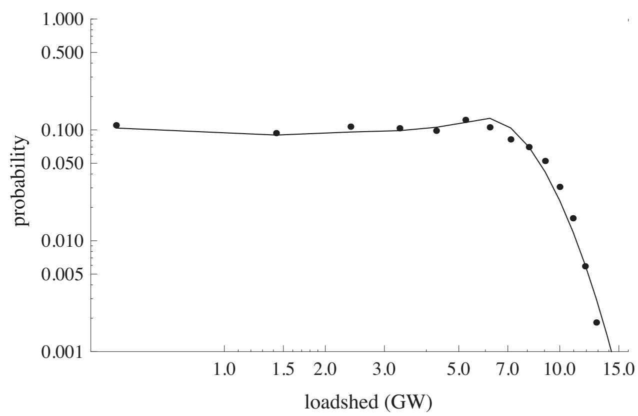

Figure 3. Probability distributions of total load shed at load level 1.05. Dots are the empirical distribution; line is predicted with the branching process.

Let p_{empiric} be the probability of shedding total load S, computed empirically by simulating K_{empiric} cascades with non-negligible load shed. Then the standard deviation of p_{empiric} is

$$\sigma(p_{\mathrm{empiric}}) = \sqrt{\frac{p_{\mathrm{empiric}}(1-p_{\mathrm{empiric}})}{K_{\mathrm{empiric}}}} \tag{8}$$

If we require the same standard deviation for both methods, then we can equate (7) and (8) to approximate the ratio

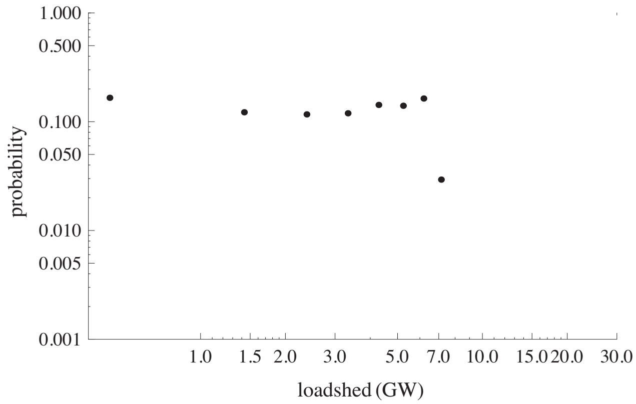

Figure 4. Probability distribution of initial load shed at load level 1.1.

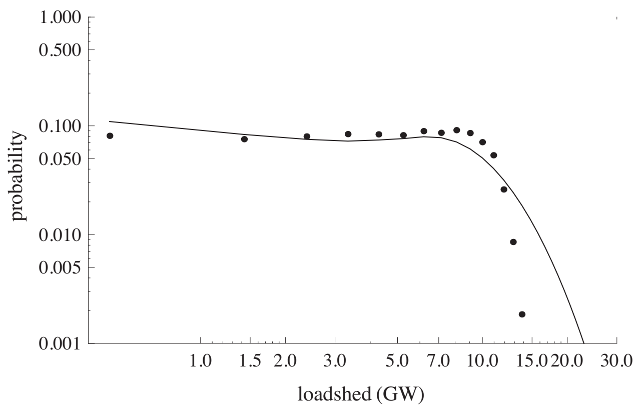

Figure 5. Probability distributions of total load shed at load level 1.1. Dots are the empirical distribution; line is predicted with the branching process.

of the required number of simulated cascades as

$$\frac{K_{\text{empiric}}}{K_{\text{branch}}} = \frac{p_{\text{empiric}}(1 - p_{\text{empiric}})\theta}{\lambda(1-\lambda)\Delta} \left(\frac{dp_{\text{branch}}}{d\lambda} \right)^{-2} \quad (9)$$

To obtain a rough estimate of the ratio, we evaluate (9) for total load shed $S = 9.52$ GW for each of the three load levels. $dp_{\text{branch}}/d\lambda$ is estimated by numerical differencing. We find that K_{empiric} exceeds K_{branch} by an order of magnitude or more.

5 Conclusion

In this paper, we suggest approximating the cascading process of load shed in blackouts by discretizing the load shed and then using a Galton-Watson branching process. The average propagation of failures λ is estimated using the standard Harris estimator from cascading load shed data that records the load shed in each cascade generation. Then the branching process model estimates the probability distribution of load shed from the discretized distribution of initial load shed and the estimate of λ. We test this estimation on cascading failure data from the OPA simulation of cascading transmission line outages in the 300 bus IEEE electric power test system. The estimated distribution is close to the empirical distribution in most of the cases tested, suggesting that the branching process model with an averaged propagation can capture some aspects of the cascading of load shed, at least for the purpose of estimating the probability distribution of total load shed.

The approach via propagation and the branching process opens opportunities for estimation of the probability distribution of load shed from fewer observed or simulated cascades. We assume that the probability distribution of initial load shed is known accurately. These initial load shed statistics can be estimated by methods of conventional reliability or by observations, since some load is shed much more frequently than there is a large cascading blackout. Given that the probability distribution of initial load shed is known accurately, our initial testing of the estimation via the branching process of the probability distribution of total load shed suggests that an order of magnitude or more fewer cascades are needed for this estimation in the tail of the distribution than is needed for direct empirical estimation of the probability distribution of load shed. This is not only helpful in reducing simulation times, which are always burdensome and often prohibitive for cascading failure simulations of large power system models, but also will be a crucial attribute in designing practical methods of estimating the probability distribution of load shed from cascades observed in the power system. Empirical methods of accumulating blackout statistics that simply wait for enough cascades to occur take too long to be practical when estimating the rare but important large blackouts in the tail of the distribution. Model based approaches to cascading failure such as the method presented here are needed to estimate the probability of large blackouts from observations over a time scale of about a year rather than over decades.

The approach seems to be easier than a previous method [10, 24] that estimates the offspring distribution of a continuous state branching process and then uses computer algebra to compute cumulant generating functions of the distribution of total load shed.

This paper estimates average propagation and the distribution of load shed using a branching process. These first results are sufficiently promising that further testing with other power system models or more detailed cascading failure simulations is warranted.

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