Planck 2015 results. XIII. Cosmological parameters


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ABSTRACT

This paper presents cosmological results based on full-mission Planck observations of temperature and polarization anisotropies of the cosmic microwave background (CMB) radiation. Our results are in very good agreement with the 2013 analysis of the Planck 2013 nominal-mission temperature data, but with improved precision. The temperature and polarization power spectra are consistent with the standard spatially-flat 6-parameter ΛCDM cosmology with a power-law spectrum of adiabatic scalar perturbations (denoted “base ΛCDM”) in this paper). From the Planck temperature data combined with Planck lensing, for this Hubble constant, $H_0 = (67.8 \pm 0.9) \text{ km} \ s^{-1} \text{ Mpc}^{-1}$, a matter density parameter $\Omega_m = 0.308 \pm 0.012$, and a tilted scalar spectral index $n_s = 0.968 \pm 0.006$, consistent with the 2013 analysis. Note that in this abscissa we quote 68% limits on measured parameters and 95% upper limits on other parameters. We present the first results of polarization measurements with the Low Frequency Instrument at large angular scales. Combined with the Planck temperature and lensing data, these measurements give a reionization optical depth of $\tau = 0.066 \pm 0.016$, corresponding to a reionization redshift of $z_{\text{reio}} = 8.8^{+1.7}_{-1.4}$. These results are consistent with those from WMAP polarization measurements cleaned for dust emission using 353-GHz polarization maps from the High Frequency Instrument.

We find no evidence for any departure from base ΛCDM in the neutrino sector of the theory; for example, combining Planck observations with other astrophysical data we find $N_{\text{eff}} = 3.15 \pm 0.23$ for the effective number of relativistic degrees of freedom, consistent with the value $N_{\text{eff}} = 3.046$ of the Standard Model of particle physics. The sum of neutrino masses is constrained to $\sum m_\nu < 0.23 \text{ eV}$. The spatial curvature of our Universe is found to be very close to zero, with $\Omega_k < 0.005$. Adding a tensor component as a single-parameter extension to base ΛCDM we find an upper limit on the tensor-to-scalar ratio of $r_{\text{0.02}} = 0.11$, consistent with the Planck 2013 results and consistent with the $B$-mode polarization constraints from a joint analysis of BICEP2, Keck Array, and Planck (BKP) data. Adding the $B$-mode data to our analysis leads to a tighter constraint of $r_{\text{0.02}} < 0.09$ and disfavours inflationary models with a $V(\phi) \propto \phi^4$ potential. The addition of Planck polarization data leads to strong constraints on deviations from a purely adiabatic spectrum of fluctuations. We find no evidence for any contribution from isocurvature perturbations or from cosmic defects. Combining Planck data with other astrophysical data, including Type Ia supernovae, the equation of state of dark energy is constrained to $w = -1.006 \pm 0.045$, consistent with the expected value for a cosmological constant. The standard big bang nucleosynthesis predictions for the light elements are well matched. As shown in the 2013 analysis, the fluctuation spectrum is found to be higher than inferred from some analyses of rich cluster counts and weak gravitational lensing. We show that these tensions cannot easily be resolved in simple modifications of the base ΛCDM cosmology. Apart from these tensions, the base ΛCDM model provides an excellent description of the Planck CMB observations and many other astrophysical data sets.

Key words. Cosmology: observations – Cosmology: theory – cosmic microwave background – cosmological parameters
1. Introduction

The cosmic microwave background (CMB) radiation offers an extremely powerful way of testing the origin of fluctuations and of constraining the matter content, geometry, and late-time evolution of the Universe. Following the discovery of anisotropies in the CMB by the COBE satellite (Smoot et al. 1992), ground-based, sub-orbital experiments and notably the WMAP satellite (Bennett et al. 2003, 2013) have mapped the CMB anisotropies with increasingly high precision, providing a wealth of new information on cosmology.

Planck\(^1\) is the third-generation space mission, following COBE and WMAP, dedicated to measurements of the CMB anisotropies. The first cosmological results from Planck were reported in a series of papers (for an overview see Planck Collaboration I 2016, for further details). In addition, the Planck beams have been characterized more accurately in the 2015 data release and there have been minor modifications to the low-level data processing.

The layout of this paper is as follows. Section 2 summarizes a number of small changes to the parameter estimation methodology since PCP13. The full mission temperature and polarization power spectra are presented in Sect. 3. The first subsection (Sect. 3.1) discusses the changes in the cosmological parameters of the base ΛCDM cosmology compared to those presented in 2013. Section 3.2 presents an assessment of the impact of foreground cleaning (using the 545-GHz maps) on the cosmological parameters of the base ΛCDM model. The power spectra and associated likelihoods are presented in Sect. 3.3. This subsection also discusses the internal consistency of the Planck TT, TE, and EE spectra. The agreement of TE and EE with the TT spectra provides an important additional test of the accuracy of our foreground corrections to the TT spectra at high multipoles.

PCP13 used the WMAP polarization likelihood at low multipoles to constrain the reionization optical depth parameter \(\tau\). The 2015 analysis replaces the WMAP likelihood with polarization data from the Planck Low Frequency Instrument (LFI, Planck Collaboration II 2016). The impact of this change on \(\tau\) is discussed in Sect. 3.4, which also presents an alternative (and competitive) constraint on \(\tau\) based on combining the Planck TT spectrum with the power spectrum of the lensing potential measured by Planck. We also compare the LFI polarization constraints with the WMAP polarization data cleaned with the Planck HFI 353-GHz maps.

Section 4 compares the Planck power spectra with the power spectra from high-resolution ground-based CMB data from the Atacama Cosmology Telescope (ACT, Das et al. 2014) and the South Pole Telescope (SPT, George et al. 2015). This section applies a Gibbs sampling technique to sample over foreground and other “nuisance” parameters to recover the underlying CMB power spectrum at high multipoles (Dunkley et al. 2013; Calabrese et al. 2013). Unlike PCP13, in which we combined the likelihoods of the high-resolution experiments with the Planck temperature likelihood, in this paper we use the high-resolution experiments mainly to check the consistency of the “damping tail” in the Planck power spectrum at multipoles \(\ell \approx 2000\).

Section 5 introduces additional data, including the Planck lensing likelihood (described in detail in Planck Collaboration XV 2016) and other astrophysical data sets. As in PCP13, we are highly selective in the astrophysical

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\(^1\)Planck (http://www.esa.int/Planck) is a project of the European Space Agency (ESA) with instruments provided by two scientific consortia funded by ESA member states and led by Principal Investigators from France and Italy, telescope reflectors provided through a collaboration between ESA and a scientific consortium led and funded by Denmark, and additional contributions from NASA (USA).

\(^2\)This paper refers extensively to the earlier 2013 Planck cosmological parameters paper and CMB power spectra and likelihood paper (Planck Collaboration XVI 2014; Planck Collaboration XV 2014). To simplify the presentation, these papers will henceforth be referred to as PCP13 and PPL13, respectively.

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data sets that we combine with Planck. As mentioned above, the main purpose of this paper is to describe what the Planck data have to say about cosmology. It is not our purpose to present an exhaustive discussion of what happens when the Planck data are combined with a wide range of astrophysical data. This can be done by others, using the publicly released Planck likelihood. Nevertheless, some cosmological parameter combinations are highly degenerate using CMB power spectrum measurements alone, the most severe being the “geometrical degeneracy” that opens up when spatial curvature is allowed to vary. Baryon acoustic oscillation (BAO) measurements are a particularly important astrophysical data set. Since BAO surveys involve a simple geometrical measurement, these data are less prone to systematic errors than most other astrophysical data. As in PCP13, BAO measurements are used as a primary astrophysical data set in combination with Planck to break parameter degeneracies. It is worth mentioning explicitly our approach to interpreting tensions between Planck and other astrophysical data sets. Tensions may be indicators of new physics beyond that assumed in the base ΛCDM model. However, they may also be caused by systematic errors in the data. Our primary goal is to report whether the Planck data support any evidence for new physics. If evidence for new physics is driven primarily by astrophysical data, but not by Planck, then the emphasis must necessarily shift to establishing whether the astrophysical data are free of systematics. This type of assessment is beyond the scope of this paper, but sets a course for future research.

Extensions to the base ΛCDM cosmology are discussed in Sect. 6, which explores a large grid of possibilities. In addition to these models, we also explore constraints on big bang nucleosynthesis, dark matter annihilation, cosmic defects, and departures from the standard recombination history. As in PCP13, we find no convincing evidence for a departure from the base ΛCDM model. As far as we can tell, a simple inflationary model with a slightly tilted, purely adiabatic, scalar fluctuation spectrum fits the Planck data and most other precision astrophysical data. There are some “anomalies” in this picture, including the poor fit to the CMB temperature fluctuation spectrum at low multipoles, as reported by WMAP (Bennett et al. 2003) and in PCP13, suggestions of departures from statistical isotropy at low multipoles (as reviewed in Planck Collaboration XXIII 2014; Planck Collaboration XVI 2016), and hints of a discrepancy with the amplitude of the matter fluctuation spectrum at low redshifts (see Sect. 5.5). However, none of these anomalies are of decisive statistical significance at this stage.

One of the most interesting developments since the appearance of PCP13 was the detection by the BICEP2 team of a B-mode polarization anisotropy (BICEP2 Collaboration 2014), apparently in conflict with the 95% upper limit on the tensor-to-scalar ratio, r_{0.02} < 0.11,1 reported in PCP13. Clearly, the detection of B-mode signal from primordial gravitational waves would have profound consequences for cosmology and inflationary theory. However, a number of studies, in particular an analysis of Planck 353-GHz polarization data, suggested that polarized dust emission might contribute a significant part of the BICEP2 signal (Planck Collaboration Int. XXX 2016; Mortonson & Seljak 2014; Flauger et al. 2014). The situation is now clearer following the joint analysis of BICEP2, Keck, Array, and Planck data (BICEP2/Keck Array and Planck Collaborations 2015, hereafter BKP); this increases the signal-to-noise ratio on polarized dust emission primarily by directly cross-correlating the BICEP2 and Keck Array data at 150 GHz with the Planck polarization data at 353 GHz. The results of BKP give a 95% upper limit on the tensor-to-scalar ratio of r_{0.05} < 0.12, with no statistically significant evidence for a primordial gravitational wave signal. Section 6.2 presents a brief discussion of this result and how it fits in with the indirect constraints on r derived from the Planck 2015 data.

Our conclusions are summarized in Sect. 7.

2. Model, parameters, and methodology

The notation, definitions and methodology used in this paper largely follow those described in PCP13, and so will not be repeated here. For completeness, we list some derived parameters of interest in Sect. 2.2. We have made a small number of modifications to the methodology, as described in Sect. 2.1. We have also made some minor changes to the model of unresolved foregrounds and nuisance parameters used in the high-f likelihood. These are described in detail in Planck Collaboration XI (2016), but to make this paper more self-contained, these changes are summarized in Sect. 2.3.

2.1. Theoretical model

We adopt the same general methodology as described in PCP13, with small modifications. Our main results are now based on the lensed CMB power spectra computed with the updated January 2015 version of the camb1 Boltzmann code (Lewis et al. 2000), and parameter constraints are based on the January 2015 version of CosmoMC (Lewis & Bridle 2002; Lewis 2013). Changes in our physical modelling are as follows.

- For each model in which the fraction of baryonic mass in helium Y_{p} is not varied independently of other parameters, it is now set from the big bang nucleosynthesis (BBN) prediction by interpolation from a recent fitting formula based on results from the PArthEnOPE BBN code (Pisanti et al. 2008). We now use a fixed fiducial neutron decay constant of τ_{r} = 880.3 s, and also account for the small difference between the mass-fraction ratio Y_{p} and the nucleon-based fraction Y_{p}^{BBN}. These modifications result in changes of about 1% in the inferred value of Y_{p} compared to PCP13, giving best-fit values Y_{p} ≈ 0.2453 (Y_{p}^{BBN} ≈ 0.2467) in ΛCDM. See Sect. 6.5 for a detailed discussion of the impact of uncertainties arising from variations of τ_{r} and nuclear reaction rates; however, these uncertainties have minimal impact on our main results. Section 6.5 also corrects a small error arising from how the difference between N_{eff} = 3.046 and N_{eff} = 3 was handled in the BBN fitting formula.

- We have corrected a missing source term in the dark energy modelling for w ≠ −1. The correction of this error has very little impact on our science results, since it is only important for values of w far from −1.

- To model the small-scale matter power spectrum, we use the halofit approach (Smith et al. 2003), with the updates of Takahashi et al. (2012), as in PCP13, but with revised fitting

1http://camb.info
parameters for massive neutrino models.\textsuperscript{5} We also now include the halofit corrections when calculating the lensed CMB power spectra.

As in PCP13 we adopt a Bayesian framework for testing theoretical models. Tests using the “profile likelihood” method, described in Planck Collaboration Int. XVI (2014), show excellent agreement for the mean values of the cosmological parameters and their errors, for both the base ΛCDM model and its ND extension. Tests have also been carried out using the CClass Boltzmann code (Lesgourgues 2011) and the Monte Python MCMC code (Audren et al. 2013) in place of camb and CosmoMC, respectively. Again, for flat models we find excellent agreement with the baseline choices used in this paper.

2.2. Derived parameters

Our base parameters are defined as in PCP13, and we also calculate the same derived parameters. In addition we now compute:

- the helium nucleon fraction defined by $Y_{BBN}^P \equiv 4n_{He}/n_{B}$;
- where standard BBN is assumed, the mid-value deuterium ratio predicted by BBN, $Y_{DP} \equiv 10^3 n_{D}/n_{He}$, using a fit from the PArthEnNoPE BBN code (Pisanti et al. 2008);
- the comoving wavenumber of the perturbation mode that entered the Hubble radius at matter-radiation equality $z_{eq}$, where this redshift is calculated approximating all neutrinos as relativistic at that time, i.e., $k_{eq} \equiv a(z_{eq})H(z_{eq})$;
- the comoving angular diameter distance to last scattering, $D_A(z_c)$;
- the angular scale of the sound horizon at matter-radiation equality, $\theta_{eq} \equiv r(z_{eq})/D_A(z_c)$, where $r_c$ is the sound horizon and $z_{eq}$ is the redshift of last scattering;
- the amplitude of the CMB power spectrum $D_{l} \equiv \langle \ell(\ell + 1)C_{\ell}/2\pi \rangle$ in $\mu K^2$, for $\ell = 40, 220, 810, 1520$, and 2000;
- the primordial spectral index of the curvature perturbations at wavenumber $k = 0.02$ Mpc$^{-1}$, $n_s,0.02$ (as in PCP13, our default pivot scale is $k = 0.05$ Mpc$^{-1}$, so that $n_s \equiv n_s,0.05$);
- parameter combinations close to those probed by galaxy and CMB lensing (and other external data), specifically $\sigma_8\Omega_m^{0.5}$ and $\sigma_8\Omega_{\gamma}^{0.25}$;
- various quantities reported by BAO and redshift-space distortion measurements, as described in Sects. 5.2 and 5.5.1.

2.3. Changes to the foreground model

Unresolved foregrounds contribute to the temperature power spectrum and must be modelled to extract accurate cosmological parameters. PPL13 and PCP13 used a parametric approach to modelling foregrounds, similar to the approach adopted in the analysis of the SPT and ACT experiments (Reichardt et al. 2012; Dunkley et al. 2013). The unresolved foregrounds are described by a set of power spectrum templates together with nuisance parameters, which are sampled via MCMC along with the cosmological parameters.\textsuperscript{6} The components of the extragalactic foreground model consist of:

- the shot noise from Poisson fluctuations in the number density of point sources;
- the power due to clustering of point sources (loosely referred to as the CIB component);
- a thermal Sunyaev-Zeldovich (tSZ) component;
- a kinetic Sunyaev-Zeldovich (kSZ) component;
- the cross-correlation between tSZ and CIB.

In addition, the likelihood includes a number of other nuisance parameters, such as relative calibrations between frequencies, and beam eigenmode amplitudes. We use the same templates for the tSZ, kSZ, and tSZ/CIB cross-correlation as in the 2013 papers. However, we have made a number of changes to the CIB modelling and the priors adopted for the SZ effects, which we now describe in detail.

2.3.1. CIB

In the 2013 papers, the CIB anisotropies were modelled as a power law:

$$D_{l}^{\nu_{\nu}} = A_{\nu_{\nu}}^{\mathrm{CIB}} \left( \frac{\ell}{3000} \right)^{\gamma_{\mathrm{CIB}}} .$$

Planck data alone provide a constraint on $A_{\nu_{\nu}}^{\mathrm{CIB}}$ and very weak constraints on the CIB amplitudes at lower frequencies. PCP13 reported typical values of $A_{\nu_{\nu}}^{\mathrm{CIB}} = (29 \pm 6) \mu K^2$ and $\gamma_{\mathrm{CIB}} = 0.40 \pm 0.15$, fitted over the range $500 \leq \ell \leq 2500$. The addition of the ACT and SPT data (“highL”) led to solutions with steeper values of $\gamma_{\mathrm{CIB}}$ closer to 0.8, suggesting that the CIB component was not well fit by a power law.

Planck results on the CIB, using H I as a tracer of Galactic dust, are discussed in detail in Planck Collaboration XXX (2014). In that paper, a model with 1-halo and 2-halo contributions was developed that provides an accurate description of the Planck and IRAS CIB spectra from 217 GHz through to 3000 GHz. At high multipoles, $\ell \gtrsim 3000$, the halo-model spectra are reasonably well approximated by power laws, with a slope $\gamma_{\mathrm{CIB}} \approx 0.8$ (though see Sect. 4). At multipoles in the range $500 \leq \ell \leq 2500$, corresponding to the transition from the 2-halo term dominating the clustering power to the 1-halo term dominating, the Planck Collaboration XXX (2014) templates have a shallower slope, consistent with the results of PCP13. The amplitudes of these templates at $\ell = 3000$ are

- $A_{\nu_{\nu}}^{\mathrm{CIB}} = 63.6 \mu K^2$,
- $A_{\nu_{\nu}}^{\mathrm{CIB}} = 19.1 \mu K^2$,
- $A_{\nu_{\nu}}^{\mathrm{CIB}} = 5.9 \mu K^2$,
- $A_{\nu_{\nu}}^{\mathrm{CIB}} = 1.4 \mu K^2$.

Note that in PCP13, the CIB amplitude of the $143 \times 217$ spectrum was characterized by a correlation coefficient

$$A_{\nu_{\nu}}^{\mathrm{CIB}} = r_{\nu_{\nu}}^{\mathrm{CIB}} \sqrt{A_{\nu_{\nu}}^{\mathrm{CIB}} \times A_{\nu_{\nu}}^{\mathrm{CIB}} \times A_{\nu_{\nu}}^{\mathrm{CIB}}} .$$

The combined Planck-highL solutions in PCP13 always give a high correlation coefficient with a 95 % lower limit of $r_{\nu_{\nu}}^{\mathrm{CIB}} \gtrsim 0.85$, consistent with the model of Eq. (2), which has $r_{\nu_{\nu}}^{\mathrm{CIB}} \approx 1$. In the 2015 analysis, we use the Planck Collaboration XXX (2014) templates, fixing the relative amplitudes at $100 \times 100$, $143 \times 143$, and $143 \times 217$ to the amplitude of the $217 \times 217$ spectrum. Thus, the CIB model used in this paper is specified by only one amplitude, $A_{\nu_{\nu}}^{\mathrm{CIB}}$, which is assigned a uniform prior in the range $0 \rightarrow 200 \mu K^2$.\textsuperscript{6}
In PCP13 we solved for the CIB amplitudes at the CMB effective frequencies of 217 and 143 GHz, and so we included colour corrections in the amplitudes $A_{217}^{\text{CIB}}$ and $A_{143}^{\text{CIB}}$ (there was no CIB component in the 100 $\times$ 100 spectrum). In the 2015 Planck analysis, we do not include a colour term since we define $A_{217}^{\text{CIB}}$ to be the actual CIB amplitude measured in the Planck 217-GHz band. This is higher by a factor of about 1.33 compared to the amplitude at the CMB effective frequency of the Planck 217-GHz band. This should be borne in mind by readers comparing 2015 and 2013 CIB amplitudes measured by Planck.

### 2.3.2. Thermal and kinetic SZ amplitudes

In the 2013 papers we assumed template shapes for the thermal (tSZ) and kinetic (kSZ) spectra characterized by two amplitudes, $A^{\text{tSZ}}$ and $A^{\text{kSZ}}$, defined in equations (26) and (27) of PCP13. These amplitudes were assigned uniform priors in the range 0–10 (µK)$^2$. We used the Trac et al. (2011) KSZ template spectrum and the $\epsilon = 0.5$ tSZ template from Efstathiou & Migliaccio (2012). We adopt the same templates for the 2015 Planck analysis, since, for example, the tSZ template is actually a good match to the results from the recent numerical simulations of McCarthy et al. (2014).

In addition, we previously included a template from Addison et al. (2012) to model the cross-correlation between the CIB and tSZ emission from clusters of galaxies. The amplitude of this template was characterized by a dimensionless correlation coefficient, $\xi_{\text{tSZ}}^{\text{CIB}}$, which was assigned a uniform prior in the range 0–1. The three parameters $A^{\text{tSZ}}$, $A^{\text{kSZ}}$, and $\xi_{\text{tSZ}}^{\text{CIB}}$ are not well constrained by Planck alone. Even when combined with ACT and SPT, the three parameters are highly correlated with each other. Marginalizing over $\xi_{\text{tSZ}}^{\text{CIB}}$, Reichardt et al. (2012) find that SPT spectra constrain the linear combination

$$A^{\text{tSZ}} + 1.55 A^{\text{kSZ}} = (9.2 \pm 1.3) \mu K^2. \quad (4)$$

The slight differences in the coefficients compared to the formula given in Reichardt et al. (2012) come from the different effective frequencies used to define the Planck amplitudes $A^{\text{tSZ}}$ and $A^{\text{kSZ}}$. An investigation of the 2013 Planck+HighL solutions show a similar degeneracy direction, which is almost independent of cosmology, even for extensions to the base $\Lambda$CDM model.

$$A^{\text{tSZ}} = A^{\text{kSZ}} + 1.6 A^{\text{kSZ}} = (9.4 \pm 1.4) \mu K^2 \quad (5)$$

for Planck+WP+HighL, which is very close to the degeneracy direction (Eq. 4) measured by SPT. In the 2015 Planck analysis, we impose a conservative Gaussian prior for $A^{\text{kSZ}}$, as defined in Eq. (5), with a mean of 9.5 $\mu K^2$ and a dispersion 3 $\mu K^2$ (i.e., somewhat broader than the dispersion measured by Reichardt et al. 2012). The purpose of imposing this prior on $A^{\text{kSZ}}$ is to prevent the parameters $A^{\text{tSZ}}$ and $A^{\text{kSZ}}$ from wandering into unphysical regions of parameter space when using Planck data alone. We retain the uniform prior of $[0,1]$ for $\xi_{\text{tSZ}}^{\text{CIB}}$. As this paper was being written, results from the complete 2540 deg$^2$ SPT-SZ survey area appeared (George et al. 2015). These are consistent with Eq. (5) and in addition constrain the correlation parameter to low values, $\xi_{\text{tSZ}}^{\text{CIB}} = 0.113^{+0.057}_{-0.054}$.

### 2.3.3. Absolute Planck calibration

In PCP13, we treated the calibrations of the 100 and 217-GHz channels relative to 143 GHz as nuisance parameters. This was an approximate way of dealing with small differences in relative calibrations between different detectors at high multipoles, caused by bolometer time-transfer function corrections and intermediate and far sidelobes of the Planck beams. In other words, we approximated these effects as a purely multiplicative correction to the power spectra over the multipole range $\ell = 50–2500$. The absolute calibration of the 2013 Planck power spectra was therefore fixed, by construction, to the absolute calibration of the 143-5 bolometer. Any error in the absolute calibration of this reference bolometer was not propagated into errors on cosmological parameters. For the 2015 Planck likelihoods we use an identical relative calibration scheme between 100, 143, and 217 GHz, but we now include an absolute calibration parameter $y_0$, at the map level, for the 143-GHz reference frequency. We adopt a Gaussian prior on $y_0$ centred on unity with a (conservative) dispersion of 0.25 %. This overall calibration uncertainty is then propagated through to cosmological parameters such as $A_s$ and $\sigma_8$. A discussion of the consistency of the absolute calibrations across the nine Planck frequency bands is given in Planck Collaboration I (2016).

### 3. Constraints on the parameters of the base $\Lambda$CDM cosmology from Planck

#### 3.1. Changes in the base $\Lambda$CDM parameters compared to the 2013 data release

The principal conclusion of PCP13 was the excellent agreement of the base $\Lambda$CDM model with the temperature power spectra measured by Planck. In this subsection, we compare the parameters of the base $\Lambda$CDM model reported in PCP13 with those measured from the full-mission 2015 data. Here we restrict the comparison to the high multipole temperature ($TT$) likelihood (plus low-$\ell$ polarization), postponing a discussion of the $TE$ and $EE$ likelihood blocks to Sect. 3.2. The main differences between the 2013 and 2015 analyses are as follows.

1. There have been a number of changes to the low-level Planck data processing, as discussed in Planck Collaboration II (2016) and Planck Collaboration VII (2016). These include: changes to the filtering applied to remove “4-K” cooler lines from the time-ordered data (TOD); changes to the deglitching algorithm used to correct the TOD for cosmic ray hits; improved absolute calibration based on the spacecraft orbital dipole and more accurate models of the beams, accounting for the intermediate and far sidelobes. These revisions largely eliminate the calibration difference between Planck-2013 and WMAP reported in PCP13 and Planck Collaboration XXXI (2014), leading to upward shifts of the HFI and LFI Planck power spectra of approximately 2.0 % and 1.7 %, respectively. In addition, the mapmaking used for 2015 data processing utilizes “polarization destriping” for the polarized HFI detectors (Planck Collaboration VIII 2016).

2. The 2013 papers used WMAP polarization measurements (Bennett et al. 2013) at multipoles $\ell \leq 23$ to constrain the optical depth parameter $\tau$; this likelihood was denoted “WP” in the 2013 papers. In the 2015 analysis, the WMAP polarization likelihood is replaced by a Planck polarization likelihood constructed from
Table 1. Parameters of the base $\Lambda$CDM cosmology (as defined in PCP13) determined from the publicly released nominal-mission CamSpec DetSet likelihood [2013N(DS)] and the 2013 full-mission CamSpec DetSet and cross-yearly (Y1×Y2) likelihoods with the extended sky coverage [2013F(DS) and 2013F(CY)]. These three likelihoods are combined with the WMAP polarization likelihood to constrain $\tau$. The column labelled 2015F(CHM) lists parameters for a CamSpec cross-half-mission likelihood constructed from the 2015 maps using similar sky coverage to the 2013F(CY) likelihood (but greater sky coverage at 217 GHz and different point source masks, as discussed in the text). The column labelled 2015F(CHM) (Plik) lists parameters for the Plik cross-half-mission likelihood that uses identical sky coverage to the CamSpec likelihood. The 2015 temperature likelihoods are combined with the Planck lowP likelihood to constrain $\tau$. The last two columns list the deviations of the Plik parameters from those of the nominal-mission and the CamSpec 2015(CHM) likelihoods. To help refer to specific columns, we have numbered the first six explicitly. The high-$\ell$ likelihoods used here include only $TT$ spectra. $H_0$ is given in the usual units of km s$^{-1}$ Mpc$^{-1}$.

|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| $10^9 h \Omega_m$ | 1.04131 ± 0.00063 | 1.04162 ± 0.00047 | 1.04121 ± 0.00048 | 1.04094 ± 0.00048 | 1.04086 ± 0.00048 | 0.71 | 0.17
| $\Omega_b h^2$ | 0.02205 ± 0.00028 | 0.02234 ± 0.00023 | 0.02230 ± 0.00023 | 0.02225 ± 0.00023 | 0.02222 ± 0.00023 | −0.61 | 0.13
| $\Omega_c h^2$ | 0.1199 ± 0.0027 | 0.1189 ± 0.0022 | 0.1188 ± 0.0022 | 0.1194 ± 0.0022 | 0.1199 ± 0.0022 | 0.00 | −0.23
| $H_0$ | 67.3 ± 1.2 | 67.8 ± 1.0 | 67.8 ± 1.0 | 67.48 ± 0.98 | 67.26 ± 0.98 | 0.03 | 0.22
| $n_s$ | 0.9603 ± 0.0073 | 0.9665 ± 0.0062 | 0.9655 ± 0.0062 | 0.9682 ± 0.0062 | 0.9652 ± 0.0062 | −0.67 | 0.48
| $\tau$ | 0.315 ± 0.017 | 0.308 ± 0.013 | 0.308 ± 0.013 | 0.313 ± 0.013 | 0.316 ± 0.014 | −0.06 | −0.23
| $\sigma_8$ | 0.829 ± 0.012 | 0.831 ± 0.011 | 0.828 ± 0.012 | 0.829 ± 0.015 | 0.830 ± 0.015 | −0.08 | −0.07
| $10^9 A_s e^{-\eta_f}$ | 1.836 ± 0.013 | 1.833 ± 0.011 | 1.831 ± 0.011 | 1.875 ± 0.014 | 1.881 ± 0.014 | −3.46 | −0.42

(3) In the 2013 papers, the Planck temperature likelihood was a hybrid: over the multipole range $\ell = 2–49$, the likelihood was based on the Commander algorithm applied to 87% of the sky computed using a Blackwell-Rao estimator of the likelihood at higher multipoles ($\ell = 50–2500$) was constructed from cross-spectra over the frequency range 100–217 GHz using the CamSpec software (Planck Collaboration XV 2014), which is based on the methodology developed in Efstathiou (2004) and Efstathiou (2006). At each of the Planck HFI frequencies, the sky is observed by a number of detectors. For example, at 217 GHz the sky is observed by four unpolarized spider-web bolometers (SWBs) and eight polarization sensitive bolometers (PSBs). The TOD from the 12 bolometers can be combined to produce a single map at 217 GHz for any given period of time. Thus, we can produce 217-GHz maps for individual sky surveys (denoted S1, S2, S3, etc.), or by year (Y1, Y2), or split by half-mission (HM1, HM2). We can also produce a temperature map from each SWB and a temperature and polarization map from quadruplets of PSBs. For example, at 217 GHz we produce four temperature and two temperature-polarization maps. We refer to these maps as detectors-set maps (or “DetSets” for short); note that the DetSet maps can also be produced for any arbitrary time period. The high multipole likelihood used in the 2013 papers was computed by cross-correlating HFI DetSet maps for the "nominal" Planck mission extending over 15.5 months. For the 2015 papers we use the full-mission Planck data, extending over 29 months for the HFI and 48 months for the LFI. In the Planck 2015 analysis, we have produced cross-year and cross-half-mission likelihoods in addition to a DetSet likelihood. The baseline 2015 Planck temperature-polarization likelihood is also a hybrid, matching the high-multipole likelihood at $\ell = 30$ to the Planck pixel-based likelihood at lower multipoles.

(4) The sky coverage used in the 2013 CamSpec likelihood was intentionally conservative, retaining effectively 49% of the sky at 100 GHz and 31% of the sky at 143 and 217 GHz. This was done to ensure that on the first exposure of Planck cosmological results to the community, corrections for Galactic dust emission were demonstrably small and had negligible impact on cosmic-ray patterns.

Throughout this paper, we adopt the following labels for likelihoods: (i) Planck TT denotes the combination of the TT likelihood at multipoles $\ell \geq 30$ and a low-$\ell$ temperature-only likelihood based on the CMB map recovered with Commander; (ii) Planck TT+lowP further includes the Planck polarization data in the low-$\ell$ likelihood, as described in the main text; (iii) labels such as Planck TE+lowP denote the TE likelihood at $\ell \geq 30$ plus the polarization-only component of the map-based low-$\ell$ Planck likelihood; and (iv) Planck TT,TE,EE+lowP denotes the combination of the likelihood at $\ell \geq 30$ using TT, TE, and EE spectra and the low-$\ell$ temperature-polarization likelihood. We make occasional use of combinations of the polarization likelihoods at $\ell \geq 30$ and the temperature-polarization data at low-$\ell$, which we denote with labels such as Planck TE+lowT.P.
logical parameters. In the 2015 analysis we make more aggressive use of the sky at each of these frequencies. We have also tuned the point-source masks to each frequency, rather than using a single point-source mask constructed from the union of the point source catalogues at 100, 143, 217, and 353 GHz. This results in many fewer point source holes in the 2015 analysis compared to the 2013 analysis.

Most of the results in this paper are derived from a revised Planck likelihood, based on cross-half-mission spectra. The Planck likelihood has been modified since 2013 so that it is now similar to the CamSpec likelihood used in PCP13. Both likelihoods use similar approximations to compute the covariance matrices. The main difference is in the treatment of Galactic dust corrections in the analysis of the polarization spectra. The two likelihoods have been written independently and give similar (but not identical) results, as discussed further below. The Planck likelihood is discussed in Planck Collaboration XI (2016). The CamSpec likelihood is discussed in a separate paper (Efstathiou et al. in preparation).

We have made minor changes to the foreground modelling and to the priors on some of the foreground parameters, as discussed in Sect. 2.3 and Planck Collaboration XI (2016).

Given these changes to data processing, mission length, sky coverage, etc., it is reasonable to ask whether the base ΛCDM parameters have changed significantly compared to the 2013 numbers. In fact, the parameter shifts are relatively small. The situation is summarized in Table 1. The second column of this table lists the Planck+WP parameters, as given in Table 5 of PCP13. Since these numbers are based on the 2013 processing of the nominal mission and computed via a DetSet CamSpec likelihood, the column is labelled 2013N(DS). We now make a number of specific remarks about these comparisons.

After the submission of PCP13 we found strong evidence that a residual in the 217 × 217 DetSet spectrum at ℓ = 1800 was a systematic caused by electromagnetic interference between the Joule-Thomson 4-K cooler electronics and the bolometer readout electronics. This interference leads to a set of time-variable narrow lines in the power spectrum of the TOD. The data processing pipelines apply a filter to remove these lines; however, the filtering failed to reduce their impact on the power spectra to negligible levels. Incomplete removal of the 4-K cooler lines affects primarily the 217 × 217 PSB×PSB cross-spectrum in Survey 1. The presence of this systematic was reported in the revised versions of 2013 Planck papers. Using simulations and also comparison with the 2013 full-mission likelihood (in which the 217 × 217 power spectrum “dip” is strongly diluted by the additional sky surveys) we assessed that the 4-K line systematic was causing shifts in cosmological parameters of less than 0.5σ. Column 3 in Table 1 lists the DetSet parameters for the full-mission 2013 data. This full-mission likelihood uses more extensive sky coverage than the nominal mission likelihood (effectively 39% of sky at 217 GHz, 55% of sky at 143 GHz, and 63% of sky at 100 GHz); otherwise the methodology and foreground model are identical to the CamSpec likelihood described in PPL13. The parameter shifts are relatively small and consistent with the improvement in signal-to-noise of the full-mission spectra and the systematic shifts caused by the 217×217 dip in the nominal mission (for example, raising H0 and ns, as discussed in appendix C4 of PCP13).

In a reanalysis of the publicly released Planck maps, Spergel et al. (2015) constructed cross-survey (S1 × S2) likelihoods and found cosmological parameters for the base ΛCDM model that were close to (within approximately 1σ) the nominal mission parameters listed in Table 1. The Spergel et al. (2015) analysis differs substantially in sky coverage and foreground modelling compared to the 2013 Planck analysis and so it is encouraging that they find no major differences with the results presented by the Planck collaboration. On the other hand, they did not identify the reasons for the roughly 1σ parameter shifts. They argue that foreground modelling and the ℓ = 1800 dip in the 217 × 217 DetSet spectrum can contribute towards some of the differences but cannot produce 1σ shifts, in agreement with the conclusions of PCP13. The 2013F(DS) likelihood disfavours the Spergel et al. (2015) cosmology (with parameters listed in their table 3) by Δχ² = 11, i.e., by about 2σ, and almost all of the Δχ² is contributed by the multipole range 1000–1500, so the parameter shifts are not driven by cotemporal systematics resulting in correlated noise biases at high multipoles. However, as discussed in PPL13 and Planck Collaboration XI (2016), low-level correlated noise in the DetSet spectra affects all HFI channels at high multipoles where the spectra are noise dominated. The impact of this correlated noise on cosmological parameters is relatively small. This is illustrated by column 4 of Table 1 (labelled “2013F(CY)3”), which lists the parameters of a 2013 CamSpec cross-year likelihood using the same sky coverage and foreground model as the DetSet likelihood used for column 3. The parameters from these two likelihoods are in good agreement (better than 0.2σ), illustrating that cotemporal systematics in the DetSets are at sufficiently low levels that there is very little effect on cosmological parameters. Nevertheless, in the 2015 likelihood analysis we apply corrections for correlated noise to the DetSet cross-spectra, as discussed in Planck Collaboration XI (2016), and typically find agreement in cosmological parameters between DetSet, cross-year, and cross-half-mission likelihoods to better than 0.5σ accuracy for a fixed likelihood code (and to better than 0.2σ accuracy for base ΛCDM).

The revised version of PCP13 also reported an error in the ordering of the beam-transfer functions applied to some of the 2013 217 × 217 DetSet cross-spectra, leading to an offset of a few (μK)² in the coadded 217 × 217 spectrum. As discussed in PCP13, this offset is largely absorbed by the foreground model and has negligible impact on the 2013 cosmological parameters.

(3) 2015 versus 2013 processing. Column 5 (labelled “2015F(CHM)”) lists the parameters computed from the CamSpec cross-half-mission likelihood using the HFI 2015 data with revised absolute calibration and beam-transfer functions. We also replace the WP likelihood of the 2013 analysis with the Planck lowP likelihood. The 2015F(CHM) likelihood uses slightly more sky coverage (60%) at 217 GHz, compared to the 2013F(CY) likelihood and also uses revised point source masks. Despite these changes, the base ΛCDM parameters derived from the 2015 CamSpec likelihood are within ≈ 0.4σ of the 2013F(CY) parameters, with the exception of b84C, which is lower by 0.67σ, τ, which is lower by 1σ, and A_s e^(-i7°), which is higher by about 4σ. The change in τ simply reflects the preference for a lower value of τ from the Planck LFI polarization data compared to the WMAP polarization likelihood in the form delivered by the WMAP team (see Sect. 3.4 for further discussion).
Fig. 1. Planck 2015 temperature power spectrum. At multipoles $\ell \geq 30$ we show the maximum likelihood frequency-averaged temperature spectrum computed from the Plik cross-half-mission likelihood, with foreground and other nuisance parameters determined from the MCMC analysis of the base $\Lambda$CDM cosmology. In the multipole range $2 \leq \ell \leq 29$, we plot the power spectrum estimates from the Commander component-separation algorithm, computed over 94% of the sky. The best-fit base $\Lambda$CDM theoretical spectrum fitted to the Planck TT+lowP likelihood is plotted in the upper panel. Residuals with respect to this model are shown in the lower panel. The error bars show $\pm 1\sigma$ uncertainties.

The large upward shift in $A_s e^{-2\tau}$ reflects the change in the absolute calibration of the HFI. As noted in Sect. 2.3, the 2013 analysis did not propagate an error on the Planck absolute calibration through to cosmological parameters. Coincidentally, the changes to the absolute calibration compensate for the downward change in $\tau$ and variations in the other cosmological parameters to keep the parameter $\sigma_8$ largely unchanged from the 2013 value. This will be important when we come to discuss possible tensions between the amplitude of the matter fluctuations at low redshift estimated from various astrophysical data sets and the Planck CMB values for the base $\Lambda$CDM cosmology (see Sect. 5.6).

(4) Likelihoods. Constructing a high-multipole likelihood for Planck, particularly with $TE$ and $EE$ spectra, is complicated and difficult to check at the sub-$\sigma$ level against numerical simulations because the simulations cannot model the foregrounds, noise properties, and low-level data processing of the real Planck data to sufficiently high accuracy. Within the Planck collaboration, we have tested the sensitivity of the results to the likelihood methodology by developing several independent analysis pipelines. Some of these are described in Planck Collaboration XI (2016). The most highly developed of them are the CamSpec and revised Plik pipelines. For the 2015 Planck papers, the Plik pipeline was chosen as the baseline. Column 6 of Table 1 lists the cosmological parameters for base $\Lambda$CDM determined from the Plik cross-half-mission likelihood, together with the lowP likelihood, applied to the 2015 full-mission data. The sky coverage used in this likelihood is identical to that used for the CamSpec 2015F(CHM) likelihood. However, the two likelihoods differ in the modelling of instrumental noise, Galactic dust, treatment of relative calibrations, and multipole limits applied to each spectrum. As summarized in column 8 of Table 1, the Plik and CamSpec parameters agree to within 0.2$\sigma$, except for $n_s$, which differs by nearly 0.5$\sigma$. The difference in $n_s$ is perhaps not surprising, since this parameter is sensitive to small differences in the foreground modelling. Differences in $n_s$ between Plik and CamSpec are systematic and persist throughout the grid of extended $\Lambda$CDM models discussed in Sect. 6. We emphasize that the CamSpec and Plik likelihoods have been written independently, though they are based on the same theoretical framework. None of the conclusions in this paper (including those based on the full “TT,TE,EE” likelihoods) would differ in any substantive way had we chosen to use the CamSpec likelihood in place of Plik. The overall shifts of parameters between the Plik 2015
likelihood and the published 2013 nominal mission parameters are summarized in column 7 of Table 1. These shifts are within 0.7σ except for the parameters τ and A_s e^{−2τ}, which are sensitive to the low-multipole polarization likelihood and absolute calibration.

In summary, the Planck 2013 cosmological parameters were pulled slightly towards lower H_0 and n_s by the ℓ ≈ 1800 4-K line systematic in the 217 × 217 cross-spectrum, but the net effect of this systematic is relatively small, leading to shifts of 0.5σ or less in cosmological parameters. Changes to the low-level data processing, beams, sky coverage, etc., as well as the likelihood code also produce shifts of typically 0.5σ or less. The combined effect of these changes is to introduce parameter shifts relative to PCP13 of less than 0.7σ, with the exception of τ and A_s e^{−2τ}. The main scientific conclusions of PCP13 are therefore consistent with the 2015 Planck analysis.

Parameters for the base ΛCDM cosmology derived from full-mission DetSet, cross-year, or cross-half-mission spectra are in extremely good agreement, demonstrating that residual (i.e., uncorrected) cotemporal systematics are at low levels. This is also true for the extensions of the ΛCDM model discussed in Sect. 6. It is therefore worth explaining why we have adopted the cross-half-mission likelihood as the baseline for this and other 2015 Planck papers. The cross-half-mission likelihood has lower signal-to-noise than the full-mission DetSet likelihood; however, the errors on the cosmological parameters from the two likelihoods are almost identical, as can be seen from the entries in Table 1. This is also true for extended ΛCDM models. However, for more complicated tests, such as searches for localized features in the power spectra (Planck Collaboration XX 2016), residual 4-K line systematic effects and residual uncorrected correlated noise at high multipoles in the DetSet likelihood can produce results suggestive of new physics (though not at a high significance level). We have therefore decided to adopt the cross-half-mission likelihood as the baseline for the 2015 analysis, sacrificing some signal-to-noise in favour of reduced systematics. For almost all of the models considered in this paper, the Planck results are limited by small systematics of various types, including systematic errors in modelling foregrounds, rather than by signal-to-noise.

The foreground-subtracted, frequency-averaged, cross-half-mission spectrum is plotted in Fig. 1, together with the Commander power spectrum at multipoles ℓ ≤ 29. The high multipole spectrum plotted in this figure is an approximate maximum likelihood solution based on equations (A24) and (A25) of PPL13, with the foregrounds and nuisance parameters for each spectrum fixed to the best-fit values of the base ΛCDM solution. Note that a different way of solving for the Planck CMB spectrum, by marginalizing over foreground and nuisance parameters, is presented in Sect. 4. The best-fit base ΛCDM model is plotted in the upper panel, while residuals with respect to this model are plotted in the lower panel. In this plot, there are only four handpowers at ℓ ≥ 30 that differ from the best-fit model by more than 2σ. These are: ℓ = 434 (−2.0σ), ℓ = 465 (2.5σ); ℓ = 1214 (−2.5σ); and ℓ = 1455 (−2.1σ). The χ^2 of the coadded TT spectrum plotted in Fig. 1 relative to the best-fit base ΛCDM model is 2547 for 2479 degrees of freedom (30 ≤ ℓ ≤ 2500), which is a 0.96% fluctuation (PTE = 16.8%). These numbers confirm the extremely good fit of the base ΛCDM cosmology to the Planck TT data at high multipoles. The consistency of the Planck polarization spectra with base ΛCDM is discussed in Sect. 3.3.

PCP13 noted some mild internal tensions within the Planck data, for example, the preference of the phenomenological lensing parameter A_L (see Sect. 5.1) towards values greater than unity and a preference for a negative running of the scalar spectral index (see Sect. 6.2.2). These tensions were partly caused by the poor fit of base ΛCDM model to the temperature spectrum at multipoles below about 50. As noted by the WMAP team (Hinshaw et al. 2003), the temperature spectrum has a low quadrupole amplitude and a glitch in the multipole range 20 ≤ ℓ ≤ 30. These features can be seen in the Planck 2015 spectrum of Fig. 1. They have a similar (though slightly reduced) effect on cosmological parameters to those described in PCP13.

3.2. 545-GHz-cleaned spectra

As discussed in PCP13, unresolved extragalactic foregrounds (principally Poisson point sources and the clustered component of the CIB) contribute to the Planck TT spectra at high multipoles. The approach to modelling these foreground contributions in PCP13 is similar to that used by the ACT and SPT teams (Reichardt et al. 2012; Dunkley et al. 2013) in that the foregrounds are modelled by a set of physically motivated power spectrum template shapes with an associated set of adjustable nuisance parameters. This approach has been adopted as the baseline for the Planck 2015 analysis. The foreground model has been adjusted for this new analysis, in relatively minor ways, as summarized in Sect. 2.3 and described in further detail in Planck Collaboration XII (2016). Galactic dust emission also contributes to the temperature and polarization power spectra and must be subtracted from the spectra used to form the Planck likelihood. Unlike the extragalactic foregrounds, Galactic dust emission is anisotropic and so its impact can be reduced by appropriate masking of the sky. In PCP13, we intentionally adopted conservative masks, tuned for each of the frequencies used to form the likelihood, to keep dust emission at low levels. The results in PCP13 were therefore insensitive to the modelling of residual dust contamination.

In the 2015 analysis, we have extended the sky coverage at each of 100, 143, and 217 GHz, and so in addition to testing the accuracy of the extragalactic foreground model, it is important to test the accuracy of the Galactic dust model. As described in PPL13 and Planck Collaboration XII (2016) the Galactic dust templates used in the CmbSpec and Plik likelihoods are derived by fitting the 545-GHz mask-differenced power spectra. Mask differencing isolates the anisotropic contribution of Galactic dust from the isotropic extragalactic components. For the extended sky coverage used in the 2015 likelihoods, the Galactic dust contributions are a significant fraction of the extragalactic foreground contribution in the 217 × 217 temperature spectrum at high multipoles, as illustrated in Fig. 2. Galactic dust dominates over all other foregrounds at multipoles ℓ ≤ 500 at HFI frequencies.

A simple and direct test of the parametric foreground modelling used in the CmbSpec and Plik likelihoods is to compare results with a completely different approach in which the low-frequency maps are “cleaned” using higher frequency maps as foreground templates (see, e.g., Lueker et al. 2010). In a similar approach to Spergel et al. (2015), we can form cleaned maps at lower frequencies ν by subtracting a 545-GHz map as a template,

\[ M_{\nu, \text{clean}}(\nu) = (1 + a^{T\nu}) M_{\nu} - a^{T\nu} M_{2500}^{\nu}, \]

where ν is the frequency of the template map M_{2500}^{\nu} and a^{T\nu} is the cleaning coefficient. Since the maps have different beams,
the subtraction is actually done in the power spectrum domain:

\[
\tilde{C}_{\ell_1 \ell_2}^{T_1, T_2, \text{clean}} = (1 + a_{T_1}^T)(1 + a_{T_2}^T)\tilde{C}_{\ell_1 \ell_2}^{T_1, T_2, \text{raw}} - (1 + a_{T_1}^T)\alpha_{T_1}^T \tilde{C}_{\ell_1 \ell_2}^{T_1, T_1, \text{clean}} - (1 + a_{T_2}^T)\alpha_{T_2}^T \tilde{C}_{\ell_1 \ell_2}^{T_2, T_2, \text{clean}} + \alpha_{T_1}^T \alpha_{T_2}^T \tilde{C}_{\ell_1 \ell_2}^{T_1, T_2, \text{raw}},
\]

(7)

where \(\tilde{C}_{\ell_1 \ell_2}\) etc. are the mask-deconvolved beam-corrected power spectra. The coefficients \(a_{T_i}^T\) are determined by minimizing

\[
\sum_{\ell = \ell_{\text{min}}}^{\ell_{\text{max}}} \sum_{\ell' = \ell_{\text{min}}}^{\ell_{\text{max}}} \tilde{C}_{\ell \ell'}^{T_1, T_2, \text{clean}} \left( \hat{M}_{\ell \ell'}^{T_1, T_2} \right)^{-1} \tilde{C}_{\ell' \ell_2}^{T_1, T_2, \text{clean}},
\]

(8)

where \(\hat{M}_{\ell \ell'}^{T_1, T_2}\) is the covariance matrix of the estimates \(\tilde{C}_{\ell_1 \ell_2}^{T_1, T_2, \text{clean}}\).

We choose \(\ell_{\text{min}} = 100\) and \(\ell_{\text{max}} = 500\) and compute the spectra in Eq. (7) by cross-correlating half-mission maps on the 60% mask used to compute the 217 × 217 spectrum. The resulting cleaning coefficients are \(a_{143}^T = 0.00194\) and \(a_{217}^T = 0.00765\); note that all of the input maps are in units of thermodynamic temperature. The cleaning coefficients are therefore optimized to remove Galactic dust at low multipoles, though by using 545 GHz as a dust template we find that the cleaning coefficients are almost constant over the multipole range 50–2500. We note, however, that this is not true if the 353- and 857-GHz maps are used as dust templates, as discussed in Efstathiou et al. (in preparation).

The 545-GHz-cleaned spectra are shown by the red points in Fig. 2 and can be compared directly to the “uncleaned” spectra in the CamSpec likelihood (upper panels). As can be seen, Galactic dust emission is removed to high accuracy and the residual foreground contribution at high multipoles is strongly suppressed in the 217 × 217 and 143 × 217 spectra. Nevertheless, there remains small foreground contributions at high multipoles, which we model heuristically as power laws,

\[
\hat{D}_I = A \left( \frac{\ell}{1500} \right)^{e},
\]

(9)

with free amplitudes \(A\) and spectral indices \(e\). We construct another CamSpec cross-half-mission likelihood using exactly the same sky masks as the 2015F(CHM) likelihood, but using 545-GHz-cleaned 217 × 217, 143 × 217, and 143 × 143 spectra. We then use the simple model of Eq. (9) in the likelihood to remove residual unresolved foregrounds at high multipoles for each frequency combination. We do not clean the 100×100 spectrum and so for this spectrum we use the standard parametric foreground model in the likelihood. The lower panels in Fig. 2 show the residuals with respect to the best-fit base CDM model and foreground solution for the uncleaned CamSpec spectra (blue points) and for the 545-GHz-cleaned spectra (red points). These residuals are almost identical, despite the very different approaches to Galactic dust removal and foreground modelling. The cosmological parameters from these two likelihoods are also in very good agreement, typically to better than 0.1σ, with the exception of \(n_s\), which is lower in the cleaned likelihood by 0.26σ. It is not surprising, given the heuristic nature of the model (Eq. 9), that \(n_s\) shows the largest shift. We can also remove the 100×100 spectrum from the likelihood entirely, with very little impact on cosmological parameters.

Further tests of map-based cleaning are presented in Planck Collaboration XI (2016), which additionally describes another independently written power-spectrum analysis pipeline (MSPEC) tuned to map-cleaned cross-spectrum analysis and using a more complex model for fitting residual foregrounds than the heuristic model of Eq. (9). Planck Collaboration XI (2016) also describes power spectrum analysis and cosmological parameters derived from component-separated Planck maps. However, the simple demonstration presented in this section shows that the details of modelling residual dust contamination and other foregrounds are under control in the 2015 Planck likelihood. A further strong argument that our TT results are insensitive to foreground modelling is presented in the next section, which compares the cosmological parameters derived from the
TT, TE, and EE likelihoods. Unresolved foregrounds at high multipoles are completely negligible in the polarization spectra and so the consistency of the parameters, particularly from the TE spectrum (which has higher signal-to-noise than the EE spectrum) provides an additional cross-check of the TT results.

Finally, one can ask why we have not chosen to use a 545-GHz-cleaned likelihood as the baseline for the 2015 Planck parameter analysis. Firstly, it would not make any difference to the results of this paper had we chosen to do so. Secondly, we feel that the parametric foreground model used in the baseline likelihood has a sounder physical basis. This allows us to link the amplitudes of the unresolved foregrounds across the various Planck frequencies with the results from other ways of studying foregrounds, including the higher resolution CMB experiments described in Sect. 4.

3.3. The 2015 Planck temperature and polarization spectra and likelihood

The coadded 2015 Planck temperature spectrum was introduced in Fig. 1. In this section, we present additional details and consistency checks of the temperature likelihood and describe the full mission Planck TE and EE spectra and likelihood: preliminary Planck TE and EE spectra were presented in PCP13. We then discuss the consistency of the cosmological parameters for base ΛCDM measured separately from the TT, TE, and EE spectra. For the most part, the discussion given in this section is specific to the Planck likelihood, which is used as the baseline in this paper. A more complete discussion of the Planck and other likelihoods developed by the Planck team is given in Planck Collaboration XI (2016).

3.3.1. Temperature spectra and likelihood

(1) Temperature masks. As in the 2013 analysis, the high-multipole TT likelihood uses the 100×100, 143×143, 217×217, and 143×217 spectra. However, in contrast to the 2013 analysis, which used conservative sky masks to reduce the effects of Galactic dust emission, we make more aggressive use of sky in the 2015 analysis. The 2015 analysis retains 80 %, 70 %, and 60 % of sky at 100 GHz, 143 GHz, and 217 GHz, respectively, before apodization. We also apply apodized point source masks to remove compact sources with a signal-to-noise threshold $> 5$ at each frequency (see Planck Collaboration XXVI 2016 for a description of the Planck Catalogue of Compact Sources). Apodized masks are also applied to remove extended objects, and regions of high CO emission were masked at 100 GHz and 217 GHz (see Planck Collaboration X 2016). As an estimate of the effective sky area, we compute the following sum over pixels:

$$f_{\text{sky}}^{\text{eff}} = \frac{1}{4\pi} \sum w_i^2 \Omega_i,$$

(10)

where $w_i$ is the weight of the apodized mask and $\Omega_i$ is the area of pixel $i$. All input maps are at HEALpix (Górski et al. 2005) resolution $N_{\text{side}} = 2048$. Eq. (10) gives $f_{\text{sky}}^{\text{eff}} = 66.3 \%$ at 100 GHz, 57.4 % at 143 GHz, and 47.1 % at 217 GHz.

(2) Galactic dust templates. With the increased sky coverage used in the 2015 analysis, we take a slightly different approach to subtracting Galactic dust emission to that described in PPL13 and PCP13. The shape of the Galactic dust template is determined from mask-differenced power spectra estimated from the 545-GHz maps. The mask differencing removes the isotropic contribution from the CIB and point sources. The resulting dust template has a similar shape to the template used in the 2013 analysis, with power-law behaviour $D_{\ell}^{\text{dust}} \propto \ell^{-0.63}$ at high multipoles, but with a “bump” at $\ell \approx 200$ (as shown in Fig. 2). The absolute amplitude of the dust templates at 100, 143, and 217 GHz is determined by cross-correlating the temperature maps at these frequencies with the 545-GHz maps (with minor corrections for the CIB and point source contributions). This allows us to generate priors on the dust template amplitudes, which are treated as additional nuisance parameters when running MCMC chains (unlike the 2013 analysis, in which we fixed the amplitudes of the dust templates). The actual priors used in the Planck likelihood are Gaussians on $D_{\ell}$ with the following means and dispersions: $(7 \pm 2) \mu K^2$ for the 100×100 spectrum; $(9 \pm 2) \mu K^2$ for 143×143; $(21 \pm 8.5) \mu K^2$ for 143×217; and $(80 \pm 20) \mu K^2$ for 217×217. The MCMC solutions show small movements of the best-fit dust template amplitudes, but always within statistically acceptable ranges given the priors.

(3) Likelihood approximation and covariance matrices. The approximation to the likelihood function follows the methodology described in PPL13 and is based on a Gaussian likelihood assuming a fiducial theoretical power spectrum (a fit to Planck TT with prior $\tau = 0.07 \pm 0.02$). We have also included a number of small refinements to the covariance matrices. Foregrounds, including Galactic dust, are added to the fiducial theoretical power spectrum, so that the additional small variance associated with foregrounds is included, along with cosmic variance of the CMB, under the assumption that the foregrounds are Gaussian random fields. The 2013 analysis did not include corrections to the covariance matrices arising from leakage of low-multipole power to high multipoles via the point source holes; these can introduce errors in the covariance matrices of a few percent at $\ell \approx 300$, corresponding approximately to the first peak of the CMB spectrum. In the 2015 analysis we apply corrections to the fiducial theoretical power spectrum, based on Monte Carlo simulations, to correct for this effect. We also apply Monte Carlo based corrections to the analytic covariance matrices at multipoles $\ell \leq 50$, where the analytic approximations begin to become inaccurate even for large effective sky areas (see Efstathiou 2004). Finally, we add the uncertainties on the beam shapes to the covariance matrix following the methodology described in PPL13. The Planck beams are much more accurately characterized in the 2015 analysis, and so the beam corrections to the covariance matrices are extremely small. The refinements to the covariance matrices described in this paragraph are all relatively minor and have little impact on cosmological parameters.

(4) Binning. The baseline Planck likelihood uses binned temperature and polarization spectra. This is done because all frequency combinations of the TE and EE spectra are used in the Planck likelihood, leading to a large data vector of length 22 865 if the spectra are retained multipole-by-multipole. The baseline Planck likelihood reduces the size of the data vector by binning these spectra. The spectra are binned into bins of width $\Delta \ell = 5$ for $30 \leq \ell \leq 99$, $\Delta \ell = 9$ for $100 \leq \ell \leq 1503$, $\Delta \ell = 17$ for $1504 \leq \ell \leq 2013$ and $\Delta \ell = 33$ for $2014 \leq \ell \leq 2508$, with a weighting of $C^{\ell}_{\ell}$ proportional to $\ell(\ell+1)$ over the bin widths. The bins span an odd number of multipoles, since for approximately azimuthal masks we expect a nearly symmetrical correlation function around the central multipole. The binning does not affect the determination of cosmological parameters in ΛCDM-
Table 2. Goodness-of-fit tests for the 2015 Planck temperature and polarization spectra. $\Delta \chi^2 = \chi^2 - N_{\text{dof}}$ is the difference from the mean assuming that the best-fit base $\Lambda$CDM model (fitted to Planck TT+lowP) is correct and $N_{\text{dof}}$ is the number of degrees of freedom (set equal to the number of multipoles). The sixth column expresses $\Delta \chi^2$ in units of the expected dispersion, $\sqrt{2 N_{\text{dof}}}$, and the last column lists the probability to exceed (PTE) the tabulated value of $\chi^2$.

<table>
<thead>
<tr>
<th>Likelihood</th>
<th>Frequency</th>
<th>Multipole range</th>
<th>$\chi^2$</th>
<th>$\chi^2/N_{\text{dof}}$</th>
<th>$N_{\text{dof}}$</th>
<th>$\Delta \chi^2/ \sqrt{2 N_{\text{dof}}}$</th>
<th>PTE [%]</th>
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<tr>
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<td>1967</td>
<td>1.08</td>
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<td>1.04</td>
<td>2479</td>
<td>1.25</td>
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<td></td>
<td>217x217</td>
<td>30–2508</td>
<td>2549.66</td>
<td>1.03</td>
<td>2479</td>
<td>1.00</td>
<td>15.28</td>
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<tr>
<td></td>
<td>Combined</td>
<td>30–2508</td>
<td>2546.67</td>
<td>1.03</td>
<td>2479</td>
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<td>16.81</td>
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<td>30–999</td>
<td>1088.78</td>
<td>1.12</td>
<td>970</td>
<td>2.70</td>
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<tr>
<td></td>
<td>100x143</td>
<td>30–999</td>
<td>1032.84</td>
<td>1.06</td>
<td>970</td>
<td>1.43</td>
<td>7.90</td>
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<td></td>
<td>100x217</td>
<td>505–999</td>
<td>526.56</td>
<td>1.06</td>
<td>495</td>
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<td></td>
<td>143x143</td>
<td>30–1996</td>
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<td>1.03</td>
<td>1967</td>
<td>0.98</td>
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<td>970</td>
<td>1.31</td>
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<td>30–999</td>
<td>1048.22</td>
<td>1.08</td>
<td>970</td>
<td>1.78</td>
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<td>100x217</td>
<td>505–999</td>
<td>479.72</td>
<td>0.97</td>
<td>495</td>
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<td></td>
<td>143x143</td>
<td>30–1996</td>
<td>2000.90</td>
<td>1.02</td>
<td>1967</td>
<td>0.54</td>
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<td></td>
<td>143x217</td>
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<td>86.80</td>
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<td></td>
<td>217x217</td>
<td>505–1996</td>
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<td>1492</td>
<td>-1.51</td>
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<td>30–1996</td>
<td>1986.95</td>
<td>1.01</td>
<td>1967</td>
<td>0.32</td>
<td>37.16</td>
</tr>
</tbody>
</table>

Type models (which have smooth power spectra), but significantly reduces the size of the joint TT,TE,EE covariance matrix, speeding up the computation of the likelihood. However, for some specific purposes, e.g., searching for oscillatory features in the TT spectrum, or testing $\chi^2$ statistics, we produce blocks of the likelihood multipole-by-multipole.

(5) Goodness of fit. The first five rows of Table 2 list $\chi^2$ statistics for the TT spectra (multipole-by-multipole) relative to the Planck best-fit base $\Lambda$CDM model and foreground parameters (fitted to Planck TT+lowP). The first four entries list the statistics separately for each of the four spectra that form the TT likelihood and the fifth line (labelled “Combined”) gives the $\chi^2$ value for the maximum likelihood TT spectrum plotted in Fig. 1. Each of the individual spectra provides an acceptable fit to the base $\Lambda$CDM model, as does the frequency-averaged spectrum plotted in Fig. 1. This demonstrates the excellent consistency of the base $\Lambda$CDM model across frequencies. More detailed consistency checks of the Planck spectra are presented in Planck Collaboration XI (2016); however, as indicated by Table 2, we find no evidence for any inconsistencies between the foreground-corrected temperature power spectra computed for different frequency combinations. The temperature spectra are largely signal dominated over the multipole ranges listed in Table 2 and so the $\chi^2$ values are insensitive to small errors in the Planck noise model used in the covariance matrices. As discussed in the next subsection, this is not true for the TE and EE spectra, which are noise dominated over much of the multipole range.

(1) Masks and dust corrections. At low multipoles ($\ell \leq 300$) polarized Galactic dust emission is significant at all frequencies and is subtracted in a similar way to the dust subtraction in temperature, i.e., by including additional nuisance parameters quantifying the amplitudes of a power-law dust template with a slope constrained to $\tau_{\text{dust}} = \tau^{0.40}$ for both TE and EE (Planck Collaboration Int. XXX 2016). Polarized synchrotron emission (which has been shown to be negligible at 100 GHz and higher frequencies for Planck noise levels, Fuskeland et al. 2014) is ignored. Gaussian priors on the polarization dust amplitudes are determined by cross-correlating the lower frequency maps with the 353-GHz polarization maps (the highest frequency polarized channel of the HFI) in a similar way to the determination of temperature dust priors. We use the temperature-based apodized masks in $Q$ and $U$ at each frequency, retaining 70%, 50%, and 41% of the sky at 100, 143, and 217 GHz, respectively, after apodization (slightly smaller than the temperature masks at 143 and 217 GHz). However, we do not apply point source or CO masks to the $Q$ and $U$ maps. The construction of the full TT,TE,EE likelihood is then a straightforward extension of the TT likelihood using the analytic covariance matrices given by Efstathiou (2006) and Hamimeche & Lewis (2008).

3.3.2. Polarization spectra and likelihood

In addition to the TT spectra, the 2015 Planck likelihood includes the TE and EE spectra. As discussed in Sect. 3.1, the Planck 2015 low-multipole polarization analysis is based on the LFI 70-GHz data. Here we discuss the TE and EE spectra that are used in the high-multipole likelihood, which are computed from the HFI data at 100, 143 and 217 GHz. As summarized in Planck Collaboration XI (2016), there is no evidence for any unresolved foreground components at high multipoles in the polarization spectra. We therefore include all frequency combinations in computing the TE and EE spectra to maximize the signal-to-noise.\(^\text{11}\)

\(^{11}\)In temperature, the $100 \times 143$ and $100 \times 217$ spectra are not included in the likelihood because the temperature spectra are largely signal dominated. These spectra therefore add little new information on the CMB, but would require additional nuisance parameters to correct for unresolved foregrounds at high multipoles.
Fig. 3. Frequency-averaged $TE$ and $EE$ spectra (without fitting for temperature-to-polarization leakage). The theoretical $TE$ and $EE$ spectra plotted in the upper panel of each plot are computed from the Planck TT+lowP best-fit model of Fig. 1. Residuals with respect to this theoretical model are shown in the lower panel in each plot. The error bars show ±1σ errors. The green lines in the lower panels show the best-fit temperature-to-polarization leakage model of Eqs. (11a) and (11b), fitted separately to the $TE$ and $EE$ spectra.
(2) Polarization spectra and residual systematics. Maximum likelihood frequency coadded $T E$ and $EE$ spectra are shown in Fig. 3. The theoretical curves plotted in these figures are the $T E$ and $EE$ spectra computed from the best-fit base $\Lambda$CDM model fitted to the temperature spectra (Planck TT+lowP), as plotted in Fig. 1. The lower panels in each figure show the residuals with respect to this model. The theoretical model provides a very good fit to the $T E$ and $EE$ spectra. Table 2 lists $\chi^2$ statistics for the $T E$ and $EE$ spectra for each frequency combination (with the $T E$ and $ET$ spectra for each frequency combination coadded to form a single $T E$ spectrum). Note that since the $T E$ and $EE$ spectra are noisier than the $TT$ spectra, these values of $\chi^2$ are sensitive to the procedure used to estimate Planck noise (see Planck Collaboration XI 2016 for further details).

Some of these $\chi^2$ values are unusually high, for example the $100 \times 100$ and $143 \times 217$ $T E$ spectra and the $100 \times 143$ $EE$ spectrum all have low PTEs. The Planck $T E$ and $EE$ spectra for different frequency combinations are not as internally consistent as the Planck $TT$ spectra. Inter-comparison of the $T E$ and $EE$ spectra at different frequencies is much more straightforward than for the temperature spectra because unresolved foregrounds are unimportant in polarization. The high $\chi^2$ values listed in Table 2 therefore provide clear evidence of residual instrumental systematics in the $T E$ and $EE$ spectra.

With our present understanding of the Planck polarization data, we believe that the dominant source of systematic error in the polarization spectra is caused by beam mismatch that generates leakage from temperature to polarization (recalling that the HFI polarization maps are generated by differencing signals between quadruplets of polarization sensitive bolometers). In principle, with accurate knowledge of the beams this leakage could be described by effective polarized beam window functions. For the 2015 papers, we use the $TT$ beams rather than polarized beams, and characterize temperature-to-polarization leakage using a simplified model. The impact of beam mismatch on the polarization spectra in this model is

\[
\Delta C_\ell^{T E} = \epsilon_\ell C_\ell^{TT},
\]
\[
\Delta C_\ell^{EE} = \epsilon_\ell^2 C_\ell^{TT} + 2\epsilon_\ell C_\ell^{TE},
\]

where $\epsilon_\ell$ is a polynomial in multipole. As a consequence of the Planck scanning strategy, pixels are visited approximately every six months, with a rotation of the focal plane by 180°, leading to
a weak coupling to beam modes $b_{hm}$ with odd values of $m$. The dominant contributions are expected to come from modes with $m = 2$ and 4, describing the beam ellipticity. We therefore fit the spectra using a fourth-order polynomial,

$$
\epsilon_t = a_0 + a_2 \ell^2 + a_4 \ell^4,
$$

(12)
treating the coefficients $a_0$, $a_2$, and $a_4$ as nuisance parameters in the MCMC analysis. We have ignored the odd coefficients of the polynomial, which should be suppressed by our scanning strategy. We do however include a constant term in the polynomial to account for small deviations of the polarization efficiency from unity.

The fit is performed separately on the $TE$ and $EE$ spectra. A different polynomial is used for each cross-frequency spectrum. The coadded corrections are shown in the lower panels of Fig. 3. Empirically, we find that temperature-to-polarization leakage systematics tend to cancel in the coadded spectra. Although the best-fit leakage corrections to the coadded spectra are small, the corrections for individual frequency cross-spectra can be up to 3 times larger than those shown in Fig. 3. The model of Eqs. (11a) and (11b) is clearly crude, but gives us some idea of the impact of temperature-to-polarization leakage in the coadded spectra.

With our present empirical understanding of leakage, we find a correlation between the polarization spectra that have the highest expected temperature-to-polarization leakage and those that display high $\chi^2$ in Table 2. However, the characterization of this leakage is not yet accurate enough to reduce the $\chi^2$ values for each frequency combination to acceptable levels.

As discussed in PCP13, each Planck data release and accompanying set of papers should be viewed as a snapshot of the state of the Planck analysis at the time of the release. For the 2015 release, we have a high level of confidence in the temperature power spectra. However, we have definite evidence for low-level systematics associated with temperature-to-polarization leakage in the polarization spectra. The tests described above suggest that these are at low levels of a few $\mu K^2$ in $D_\ell$. However, temperature-to-polarization leakage can introduce correlated features in the spectra, as shown by the EE leakage model plotted in Fig. 3. Until we have a more accurate characterization of these systematics, we urge caution in the interpretation of features in the $TE$ and $EE$ spectra. For some of the 2015 papers, we use the $TE$ and $EE$ spectra, without leakage corrections. For most of the models considered in this paper, the $TT$ spectra alone provide tight constraints and so we take a conservative approach and usually quote the $TT$ results. However, as we will see, we find a high level of consistency between the Planck $TT$ and full Planck $TT$, $TE$, $EE$ likelihoods. Some models considered in Sect. 6 are, however, sensitive to the polarization blocks of the likelihood. Examples include constraints on isocurvature modes, dark matter annihilation, and non-standard recombination histories. Planck 2015 constraints on these models should be viewed as preliminary, pending a more complete analysis of polarization systematics, which will be presented in the next series of Planck papers accompanying a third data release.

(3) $TE$ and $EE$ conditionals. Given the best-fit base $\Lambda$CDM cosmology and foreground parameters determined from the temperature spectra, one can test whether the $TE$ and $EE$ spectra are consistent with the $TT$ spectra by computing conditional probabilities. Writing the data vector as

$$
\hat{C} = (\hat{C}^{TT}, \hat{C}^{TE}, \hat{C}^{EE})^\top = (\hat{X}_T, \hat{X}_E)^\top,
$$

(13)
where the quantities $\hat{C}^{TT}$, $\hat{C}^{TE}$, and $\hat{C}^{EE}$ are the maximum likelihood frequency co-added foreground-corrected spectra. The covariance matrix of this vector can be partitioned as

$$
\hat{M} = \begin{pmatrix}
M_T & M_{TP}
M_{TP}^\top & M_E
\end{pmatrix}.
$$

(14)

The expected value of the polarization vector, given the observed temperature vector $\hat{X}_T$, is

$$
\hat{X}_p^{\text{cond}} = \hat{X}_p^{\text{theory}} + M_{TP}^\top (\hat{X}_T - \hat{X}_T^{\text{theory}}),
$$

(15)
with covariance

$$
\hat{\Sigma}_p = M_p + M_{TP} M_{TP}^\top.
$$

(16)

In Eq. (15), $\hat{X}_p^{\text{theory}}$ and $\hat{X}_p^{\text{theory}}$ are the theoretical temperature and polarization spectra deduced from minimizing the Planck $TT$+lowP likelihood. Equations (15) and (16) give the expectation values and distributions of the polarization spectra conditional on the observed temperature spectra. These are shown in Fig. 4. Almost all of the data points sit within the $\pm 2\sigma$ bands and in the case of the $TE$ spectra, the data points track the fluctuations expected from the $TT$ spectra at multipoles $\ell \lesssim 1000$. Figure 4 therefore provides an important additional check of the consistency of the $TE$ and $EE$ spectra with the base $\Lambda$CDM cosmology.

(4) Likelihood implementation. Section 3.1 showed good consistency between the independently written CamSpec and PIlik codes in temperature. The methodology used for the temperature likelihoods are very similar, but the treatment of the polarization spectra in the two codes differs substantially. CamSpec uses low-resolution CMB-subtracted 353-GHz polarization maps thresholded by $P = (Q^2 + U^2)^{1/2}$ to define diffuse Galactic polarization masks. The same apodized polarization mask, with an effective sky fraction $f_{\text{sky}} = 48.8\%$ (as defined by Eq. (10)), is used for 100-, 143-, and 217-GHz $Q$ and $U$ maps. Since there are no unresolved extragalactic foregrounds detected in the $TE$ and $EE$ spectra, all of the different frequency combinations of $TE$ and $EE$ spectra are compressed into single $TE$ and $EE$ spectra (weighted by the inverse of the diagonals of the appropriate covariance matrices), after foreground cleaning using the 353-GHz maps (generalizing the map cleaning technique described in Sect. 3.2 to polarization). This allows the construction of a full $TT$, $TE$, $EE$ likelihood with no binning of the spectra and with no additional nuisance parameters in polarization. As noted in Sect. 3.1 the consistency of results from the polarization blocks of the CamSpec and PIlik likelihoods is not as good as in temperature. Cosmological parameters from fits to the $TE$ and $EE$ CamSpec and PIlik likelihoods can differ by up to about $1.5\sigma$, although no major science conclusions would change had we chosen to use the CamSpec likelihood as the baseline in this paper. We will, however, sometimes quote results from CamSpec in addition to those from PIlik to give the reader an indication of the uncertainties in polarization associated with different likelihood implementations. Figure 5 shows the CamSpec $TE$ and $EE$ residuals and error ranges conditional on the best-fit base $\Lambda$CDM and foreground model fitted to the CamSpec temperature+lowP

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12To reduce the impact of noise at 353 GHz, the map-based cleaning of the $TE$ and $EE$ spectra is applied at $\ell \lesssim 300$. At higher multipoles, the polarized dust corrections are small and are subtracted as power laws fitted to the Galactic dust spectra at lower multipoles.
likelihood. The residuals in both $TE$ and $EE$ are similar to those from Plik. The main difference can be seen at low multipoles in the $EE$ spectrum, where CamSpec shows a higher dispersion, consistent with the error model, though there are several high points at $\ell \approx 200$ corresponding to the minimum in the $EE$ spectrum, which may be caused by small errors in the subtraction of polarized Galactic emission using 353 GHz as a foreground template (and there are also differences in the covariance matrices at high multipoles caused by differences in the methods used in CamSpec and Plik to estimate noise). Generally, cosmological parameters determined from the CamSpec likelihood have smaller formal errors than those from Plik because there are no nuisance parameters describing polarized Galactic foregrounds in CamSpec.

3.3.3. Consistency of cosmological parameters from the $TT$, $TE$, and $EE$ spectra

The consistency between parameters of the base $\Lambda$CDM model determined from the Plik temperature and polarization spectra are summarized in Table 3 and in Fig. 6. As pointed out by Zaldarriaga et al. (1997) and Galli et al. (2014), precision measurements of the CMB polarization spectra have the potential to constrain cosmological parameters to higher accuracy than measurements of the $TT$ spectra because the acoustic peaks are narrower in polarization and unresolved foreground contributions at high multipoles are much lower in polarization than in temperature. The entries in Table 3 show that cosmological parameters that do not depend strongly on $\tau$ are consistent between the $TT$ and $TE$ spectra, to within typically 0.5 $\sigma$ or better. Furthermore, the cosmological parameters derived from the $TE$ spectra have comparable errors to the $TT$ parameters. None of the conclusions in this paper would change in any significant way were we to use the $TE$ parameters in place of the $TT$ parameters. The consistency of the cosmological parameters for base $\Lambda$CDM between temperature and polarization therefore gives added confidence that Planck parameters are insensitive to the specific details of the foreground model that we have used to correct the $TT$ spectra. The $EE$ parameters are also typically within about 1 $\sigma$ of the $TT$ parameters, though because the $EE$ spectra from Planck are noisier than the $TT$ spectra, the errors on the $EE$ parameters are significantly larger than those from $TT$. However, both the $TE$ and $EE$ likelihoods give lower values of $\tau$, $A_s$, and $\sigma_8$, by over 1 $\sigma$ compared to the $TT$ solutions. Notice that the $TE$ and $EE$ entries in Table 3 do not use any information from the temperature in the low-multipole likelihood. The tendency for higher values of $\sigma_8$, $A_s$, and $\tau$ in the Planck TT+lowP solution is driven, in part, by the temperature power spectrum at low multipoles.

Columns [4] and [5] of Table 3 compare the parameters of the Planck TT likelihood with the full Planck TT,TE,EE likelihood. These are in agreement, shifting by less than 0.2 $\sigma$. Although we have emphasized the presence of systematic effects in the Planck polarization spectra, which are not accounted for in the errors quoted in column [4] of Table 3, the consistency of the Planck TT and Planck TT,TE,EE parameters provides strong evidence that residual systematics in the polarization spectra have little impact on the scientific conclusions in this paper. The consistency of the base $\Lambda$CDM parameters from temperature and polarization is illustrated graphically in Fig. 6. As a rough rule-of-thumb, for base $\Lambda$CDM, or extensions to $\Lambda$CDM with spatially flat geometry, using the full Planck TT,TE,EE likelihood produces improvements in cosmological parameters of about the same size as adding BAO to the Planck TT+lowP likelihood.

3.4. Constraints on the reionization optical depth parameter $\tau$

The reionization optical depth parameter $\tau$ provides an important constraint on models of early galaxy evolution and star formation. The evolution of the inter-galactic Lyα opacity measured in the spectra of quasars can be used to set limits on the epoch of reionization (Gunn & Peterson 1965). The most recent measurements suggest that the reionization of the inter-galactic medium was largely complete by a redshift $z \approx 6$ (Fan et al. 2006). The steep decline in the space density of Lyα-emitting galaxies over the redshift range $6 \leq z \leq 8$ also implies a low redshift of reionization (Choudhury et al. 2015). As a reference, for the Planck parameters listed in Table 3, instantaneous reionization at redshift $z = 7$ results in an optical depth of $\tau = 0.048$.

The optical depth $\tau$ can also be constrained from observations of the CMB. The WMAP9 results of Bennett et al. (2013) give $\tau = 0.089 \pm 0.014$, corresponding to an instantaneous redshift of reionization $z_{\tau} = 10.6 \pm 1.1$. The WMAP constraint comes mainly from the $EE$ spectrum in the multipole range $\ell = 2−6$. It has been argued (e.g., Robertson et al. 2013, and references therein) that the high optical depth reported by WMAP cannot be produced by galaxies seen in deep redshift surveys.
Planck Collaboration: Cosmological parameters

Fig. 6. Comparison of the base $\Lambda$CDM model parameter constraints from Planck temperature and polarization data.

even assuming high escape fractions for ionizing photons, implying additional sources of photoionizing radiation from still fainter objects. Evidently, it would be useful to have an independent CMB measurement of $\tau$.

The $\tau$ measurement from CMB polarization is difficult because it is a small signal, confined to low multipoles, requiring accurate control of instrumental systematics and polarized foreground emission. As discussed by Komatsu et al. (2009), uncertainties in modelling polarized foreground emission are comparable to the statistical error in the WMAP $\tau$ measurement. In particular, at the time of the WMAP9 analysis there was very little information available on polarized dust emission. This situation has been partially rectified by the 353-GHz polarization maps from Planck (Planck Collaboration Int. XXII 2015; Planck Collaboration Int. XXX 2016). In PPL13, we used preliminary 353-GHz Planck polarization maps to clean the WMAP Ka, Q, and V maps for polarized dust emission, using WMAP K-band as a template for polarized synchrotron emission. This lowered $\tau$ by about 1 $\sigma$ to $\tau = 0.075 \pm 0.013$, compared to $\tau = 0.089 \pm 0.013$ using the WMAP dust model.\textsuperscript{13} However, given the preliminary nature of the Planck polarization analysis we decided for the Planck 2013 papers to use the WMAP polarization likelihood, as produced by the WMAP team.

\textsuperscript{13}Neither of these error estimates reflect the true uncertainty in foreground removal.
In the 2015 papers, we use Planck polarization maps based on low-resolution LFI 70-GHz maps, excluding Surveys 2 and 4. These maps are foreground-cleaned using the LFI 30-GHz and HFI 353-GHz maps as polarized synchrotron and dust templates, respectively. These cleaned maps form the polarization part (‘lowP’) of the low-multipole Planck pixel-based likelihood, as described in Planck Collaboration XI (2016). The temperature part of this likelihood is provided by the Commander component-separation algorithm. The Planck low-multipole likelihood retains 46% of the sky in polarization and is completely independent of the WMAP polarization likelihood. In combination with the Planck high multipole TT likelihood, the Planck low-multipole likelihood gives $\tau = 0.078 \pm 0.019$. This constraint is somewhat higher than the constraint $\tau = 0.067 \pm 0.022$ derived from the Planck low-multipole likelihood alone (see Planck Collaboration XI 2016 and also Sect. 5.1.2).

Following the 2013 analysis, we have used the 2015 HFI 353-GHz polarization maps as a dust template, together with the WMAP K-band data as a template for polarized synchrotron emission, to clean the low-resolution WMAP Ka, Q, and V maps (see Planck Collaboration XI 2016 for further details). For the purpose of cosmological parameter estimation, this data set is masked using the WMAP P06 mask, which retains 73% of the sky. The noise-weighted combination of the Planck 353-cleaned WMAP polarization maps yields $\tau = 0.071 \pm 0.013$ when combined with the Planck TT information in the range $2 \leq \ell \leq 2508$, consistent with the value of $\tau$ obtained from the LFI 70-GHz polarization maps. In fact, null tests described in Planck Collaboration XI (2016) demonstrate that the LFI and WMAP polarization data are statistically consistent. The HFI polarization maps have higher signal-to-noise than the LFI and could, in principle, provide a third cross-check. However, at the time of writing, we are not yet confident that systematics in the HFI maps at low multipoles ($\ell \leq 20$) are at negligible levels. A discussion of HFI polarization at low multipoles will therefore be deferred to future papers.\textsuperscript{14}

Given the difficulty of making accurate CMB polarization measurements at low multipoles, it is useful to investigate other ways of constraining $\tau$. Measurements of the temperature power spectrum provide a highly accurate measurement of the amplitude $A_s e^{-2\tau}$. However, as shown in PCP13 CMB lensing breaks the degeneracy between $\tau$ and $A_s$. The observed Planck TT spectrum is, of course, lensed, so the degeneracy between $\tau$ and $A_s$ is partially broken when we fit models to the Planck TT likelihood. However, the degeneracy breaking is much stronger if we combine the Planck TT likelihood with the Planck lensing likelihood constructed from measurements of the power spectrum of the lensing potential $C_\ell^{\phi\phi}$. The 2015 Planck TT and lensing likelihoods are statistically more powerful than their 2013 counterparts and the corresponding determination of $\tau$ is more precise. The 2015 Planck lensing likelihood (labelled “lensing”) is summarized in Sec. 5.1 and discussed in more detail in Planck Collaboration XV (2016). The constraints on $\tau$ and $z_{re}$ for various data combinations excluding low-multipole polarization data from Planck are summarized in Fig. 7 and compared with the baseline Planck TT+lowP parameters. This figure also shows the shifts of other parameters of the base ΛCDM cosmology, illustrating their sensitivity to changes in $\tau$.

The Planck constraints on $\tau$ and $z_{re}$ in the base ΛCDM model for various data combinations are:

\begin{align}
\tau &= 0.078^{+0.019}_{-0.019} \quad z_{re} = 9.9^{+1.5}_{-1.3}, \; \text{Planck TT+lowP;} \\
\tau &= 0.070^{+0.024}_{-0.024} \quad z_{re} = 9.0^{+2.5}_{-2.1}, \; \text{Planck TT+lensing;} \\
\tau &= 0.066^{+0.016}_{-0.016} \quad z_{re} = 8.8^{+1.7}_{-1.4}, \; \text{Planck TT+lowP+lensing;} \\
\tau &= 0.067^{+0.016}_{-0.016} \quad z_{re} = 8.9^{+1.7}_{-1.4}, \; \text{Planck TT+lensing+BAO;} \\
\tau &= 0.066^{+0.013}_{-0.013} \quad z_{re} = 8.8^{+1.3}_{-1.2}, \; \text{Planck TT+lowP+lensing+BAO.}
\end{align}

\textsuperscript{14}See Planck Collaboration Int. XLVI (2016), which has been submitted since this paper was written.

\textsuperscript{15}We use the same specific definition of $z_{re}$ as in the 2013 papers, where reionization is assumed to be relatively sharp, with a mid-point parameterized by a redshift $z_{re}$ and width $\Delta z_{re} = 0.5$. Unless otherwise stated we impose a flat prior on the optical depth with $\tau > 0.01$. 

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Fig. 7. Marginalized constraints on parameters of the base ΛCDM model for various data combinations, excluding low-multipole polarization, compared to the Planck TT+lowP constraints.
The constraint from \textit{Planck} TT+\textit{lensing}+BAO on \(\tau\) is completely independent of low-multipole CMB polarization data and agrees well with the result from \textit{Planck} polarization (and has comparable precision). These results all indicate a lower redshift of reionization than the value derived from WMAP9 polarization likelihood. The low values of \(\tau\) from \textit{Planck} are also consistent with the lower value of \(\tau\) derived from the WMAP \textit{Planck} 353-GHz-cleaned polarization likelihood, suggesting strongly that the WMAP9 value is biased slightly high by residual polarized dust emission.

The \textit{Planck} results of Eqs. (17a)–(17c) provide evidence for a lower optical depth and redshift of reionization than inferred from WMAP (Bennett et al. 2013), partially alleviating the difficulties in reionizing the intergalactic medium using starlight for common nuisance parameters describing the tSZ, kSZ, and CIB components, extending the templates used for \textit{Planck} to \(\ell = 13,000\) to cover the full ACT and SPT multipole range. As in \textit{PCP13}, we use five point-source amplitudes to fit for the total dust and radio Poisson power, namely \(A^{PS}_{\text{ACT}}\), \(A^{PS}_{\text{SPT}}\), \(A^{PS}_{\text{CIB}}\), and \(A^{PS}_{\text{CIB}}\). We rescale these amplitudes to cross-frequency spectra using point-source correlation coefficients, improving on the 2013 treatment by using different parameters for the ACT and SPT correlations, \(r^{PS}_{\text{ACT}}\) and \(r^{PS}_{\text{SPT}}\) (a single \(r^{PS}_{\text{SPT}}\) parameter was used in 2013). We vary \(\sigma_{\text{r}}\) and \(\sigma_{\text{CIB}}\) as in 2013, and include dust amplitudes for ACT, with Gaussian priors as in \textit{PCP13}.

As described in Sect. 2.3 we use a theoretically motivated clustered CIB model fitted to \textit{Planck}+IRAS estimates of the CIB. The model at all frequencies in the range 95–220 GHz is specified by a single amplitude \(A^{CIB}_{\text{CIB}}\). The CIB power is well constrained by \textit{Planck} data at \(\ell < 2000\). At multipoles \(\ell \gtrsim 3000\), the 1-halo component of the CIB model steepens and becomes degenerate with the Poisson power. This causes an underestimate of the Poisson levels for ACT and SPT, inconsistent with predictions from source counts. We therefore use the \textit{Planck} CIB template only in the range 2 < \(\ell < 3000\), and extrapolate to higher multipoles using a power law \(D_{\ell} \propto \ell^{0.8}\). While this may not be a completely accurate model for the clustered

4. Comparison of the \textit{Planck} power spectrum with high-resolution experiments

In \textit{PCP13} we combined \textit{Planck} with the small-scale measurements of the ground-based, high-resolution Atacama Cosmology Telescope (ACT) and South Pole Telescope (SPT). The primary role of using ACT and SPT was to set limits on foreground components that were poorly constrained by \textit{Planck} alone and to provide more accurate constraints on the damping tail of the temperature power spectrum. In this paper, with the higher signal-to-noise levels of the full mission \textit{Planck} data, we have taken a different approach, using the ACT and SPT data to impose a prior on the thermal and kinetic SZ power spectrum parameters in the \textit{Planck} foreground model as described in Sect. 2.3. In this section, we check the consistency of the temperature power spectra measured by \textit{Planck}, ACT, and SPT, and test the effects of including the ACT and SPT data on the recovered CMB power spectrum.

We use the latest ACT temperature power spectra presented in Das et al. (2014), with a revised binning described in Calabrese et al. (2013) and final beam estimates in Hasselfield et al. (2013). As in \textit{PCP13} we use ACT data in the range 1000 < \(\ell < 10,000\) at 148 GHz, and 1500 < \(\ell < 10,000\) for the 148 \(\times\) 218 and 218–GHz spectra. We use SPT measurements in the range 2000 < \(\ell < 13,000\) from the complete 2540 deg\(^2\) SPT-SZ survey at 95, 150, and 220 GHz presented in George et al. (2015).

Each of these experiments uses a foreground model to describe the multi-frequency power spectra. Here we implement a common foreground model to combine \textit{Planck} with the high-multipole data, following a similar approach to \textit{PCP13} but with some refinements. Following the 2013 analysis, we solve for common nuisance parameters describing the tSZ, kSZ, and tSZ×CIB components, extending the templates used for \textit{Planck} to \(\ell = 13,000\) to cover the full ACT and SPT multipole range. As in \textit{PCP13}, we use five point-source amplitudes to fit for the total dust and radio Poisson power, namely \(A^{PS}_{\text{ACT}}\), \(A^{PS}_{\text{SPT}}\), \(A^{PS}_{\text{CIB}}\), and \(A^{PS}_{\text{CIB}}\). We rescale these amplitudes to cross-frequency spectra using point-source correlation coefficients, improving on the 2013 treatment by using different parameters for the ACT and SPT correlations, \(r^{PS}_{\text{ACT}}\) and \(r^{PS}_{\text{SPT}}\) (a single \(r^{PS}_{\text{SPT}}\) parameter was used in 2013). We vary \(\sigma_{\text{r}}\) and \(\sigma_{\text{CIB}}\) as in 2013, and include dust amplitudes for ACT, with Gaussian priors as in \textit{PCP13}.
CIB spectrum at high multipoles (see, e.g., Viero et al. 2013; Planck Collaboration XXX 2014), this extrapolation is consistent with the CIB model used in the analysis of ACT and SPT. We then need to extrapolate the Planck 217-GHz CIB power to the ACT and SPT frequencies. This requires converting the CIB measurement in the HFI 217-GHz channel to the ACT and SPT bandpasses assuming a spectral energy distribution; we use the CIB spectral energy distribution from Béthermin et al. (2012). Combining this model with the ACT and SPT bandpasses, we find that $A_{\text{CIB}}^{\text{217}}$ has to be multiplied by 0.12 and 0.89 for ACT 148 and 218 GHz, and by 0.026, 0.14, and 0.91 for SPT 95, 150, and 220 GHz, respectively. With this model in place, the best-fit Planck, ACT, and SPT Poisson levels agree with those predicted from source counts, as discussed further in Planck Collaboration XI (2016).

The nuisance model includes seven calibration parameters as in FCP13 (four for ACT and three for SPT). The ACT spectra are internally calibrated using the WMAP 9-year maps, with 2% and 7% uncertainty at 148 and 218 GHz, while the SPT calibrates using the Planck 2013 143-GHz maps, with 1.1%, 1.2%, and 2.2% uncertainty at 95, 150, and 220 GHz. To account for the increased 2015 Planck absolute calibration (2% higher in power) we increase the mean of the SPT map-based calibrations from 1.00 to 1.01.

This common foreground and calibration model fits the data well. We first fix the cosmology to that of the best-fit Planck TT+lowP ΛCDM best-fit model for the Planck (grey), ACT south (orange), ACT equatorial (red), and SPT (green) CMB bandpowers. The ACT and SPT bandpowers are scaled by the best-fit calibration factors.

In Fig. 9 we show the residual CMB power with respect to the Planck TT+lowP ΛCDM best-fit model for the Planck (grey), ACT south (orange), ACT equatorial (red), and SPT (green) CMB bandpowers. The ACT and SPT bandpowers are scaled by the best-fit calibration factors.

Fig. 10. Planck CMB power spectrum that is marginalized over foregrounds (red), including a prior on the thermal and kinetic SZ power. The inclusion of the full higher resolution ACT and SPT data (shown in blue) does not significantly decrease the errors.

For the Planck data, we do not require the Planck CMB bandpowers to be the same as those for ACT or SPT, so that we can check for consistency between the three experiments.

In Fig. 9 we show the residual CMB power with respect to the Planck TT+lowP ΛCDM best-fit model for the three experiments. All of the data sets are consistent over the multipole range plotted in this figure. For ACT-S, we find $\chi^2 = 17.54$ (18 data points, PTE = 0.49); for ACT-E we find $\chi^2 = 23.54$ (18 data points, PTE = 0.17); and for SPT $\chi^2 = 5.13$ (6 data points, PTE = 0.53).

Figure 10 shows the effect of including ACT and SPT data on the recovered Planck CMB spectrum. We find that including the ACT and SPT data does not reduce the Planck errors significantly. This is expected because the dominant small-scale foreground contributions for Planck are the Poisson source amplitudes, which are treated independently of the Poisson amplitudes for ACT and SPT. The high-resolution experiments do help

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\[^{16}\text{This means that the other calibration factors (e.g., ACT 218 GHz) are re-defined to be relative to 148 GHz (or 150 GHz for SPT) data.}\]
tighten the CIB amplitude (which is reasonably well constrained by Planck) and the tSZ and kSZ amplitudes (which are sub-dominant foregrounds for Planck). The kSZ effect in particular is degenerate with the CMB, since both have blackbody components; imposing a prior on the allowed kSZ power (as discussed in Sect. 2.3) breaks this degeneracy. The net effect is that the errors on the recovered Planck CMB spectrum are only marginally reduced with the inclusion of the ACT and SPT data. This motivates our choice to include the information from ACT and SPT into the joint tSZ and kSZ prior applied to Planck.

The Gibbs sampling technique recovers a best-fit CMB spectrum marginalized over foregrounds and other nuisance parameters. The Gibbs samples can then be used to form a fast CMB-only likelihood that depends on only one nuisance parameter, the overall calibration \( y_p \), MCMC chains run using the CMB-only likelihood therefore converge much faster than using the full multi-frequency Planck likelihood. The CMB-only likelihood is also extremely accurate, even for extensions to the base LCDM cosmology and is discussed further in Planck Collaboration XI (2016).

5. Comparison of the Planck base LCDM model with other astrophysical data sets

5.1. CMB lensing measured by Planck

Gravitational lensing by large-scale structure leaves imprints on the CMB temperature and polarization that can be measured in high angular resolution, low-noise observations, such as those from Planck. The most relevant effects are a smoothing of the acoustic peaks and troughs in the \( TT \), \( TE \), and \( EE \) power spectra, the conversion of \( E \)-mode polarization to \( B \)-modes, and the generation of significant non-Gaussianity in the form of a non-zero connected 4-point function (see Lewis & Challinor 2006 for a review). The latter is proportional to the power spectrum \( C_\phi \) of the lensing potential \( \phi \), and so one can estimate this power spectrum from the CMB 4-point functions. In the 2013 Planck release, we reported a 10\( \sigma \) detection of the lensing effect in the \( TT \) power spectrum (see PCP13) and a 25\( \sigma \) measurement of the amplitude of \( C_\phi \) from the \( TTTT \) 4-point function (Planck Collaboration XVII 2014). The power of such lensing measurements is that they provide sensitivity to parameters that affect the late-time expansion, geometry, and matter clustering (e.g., spatial curvature and neutrino masses) from the CMB alone.

Since the 2013 Planck release, there have been significant developments in the field of CMB lensing. The SPT team have reported a 7.7\( \sigma \) detection of lens-induced \( B \)-mode polarization based on the \( EEB^{\text{CIB}} \) 3-point function, where \( \phi^{\text{CIB}} \) is a proxy for the CMB lensing potential \( \phi \) derived from CIB measurements (Hanson et al. 2013). The POLARBEAR collaboration (POLARBEAR Collaboration 2014b) and the ACT collaboration (van Engelen et al. 2015) have performed similar analyses at somewhat lower significance (POLARBEAR Collaboration 2014b). In addition, the first detections of the polarization 4-point function from lensing, at a significance of around 4\( \sigma \), have been reported by the POLARBEAR (Ade et al. 2014) and SPT (Story et al. 2015) collaborations, and the former have also made a direct measurement of the \( BB \) power spectrum due to lensing on small angular scales with a significance around 2\( \sigma \) (POLARBEAR Collaboration 2014a). Finally, the \( BB \) power spectrum from lensing has also been detected on degree angular scales, with similar significance, by the BICEP2 collaboration (BICEP2 Collaboration 2014); see also BKP.

5.1.1. The Planck lensing likelihood

Lensing results from the full-mission Planck data are discussed in Planck Collaboration XV (2016).\(^{17}\) With approximately twice the amount of temperature data, and the inclusion of polarization, the noise levels on the reconstructed \( \phi \) are a factor of about 2 better than in Planck Collaboration XVII (2014). The broadband amplitude of \( C_\phi \) is now measured to better than 2.5\% accuracy, the most significant measurement of CMB lensing to date. Moreover, lensing \( B \)-modes are detected at 10\( \sigma \), both through a correlation analysis with the CIB and via the \( TTEB \) 4-point function. Many of the results in this paper make use of the Planck measurements of \( C_\phi \). In particular, they provide an alternative route to estimate the optical depth (as already discussed in Sect. 3.4), and to tightly constrain spatial curvature (Sect. 6.2.4).

The estimation of \( C_\phi \) from the Planck full-mission data is discussed in detail in Planck Collaboration XV (2016). There are a number of significant changes from the 2013 analysis that are worth noting here.

- The lensing potential power spectrum is now estimated from lens reconstructions that use both temperature and polarization data in the multipole range \( 100 \leq \ell \leq 2048 \). The likelihood used here is based on the power spectrum of a lens reconstruction derived from the minimum-variance combination of five quadratic estimators (\( TT \), \( TE \), \( EE \), \( TB \), and \( EB \)). The power spectrum is therefore based on 15 different 4-point functions.
- The results used here are derived from foreground-cleaned maps of the CMB synthesized from all nine Planck frequency maps with the SMICA algorithm, while the baseline 2013 results used a minimum-variance combination of the 143-GHz and 217-GHz nominal-mission maps. After masking the Galaxy and point-sources, 67.3\% of the sky is retained for the lensing analysis.
- The lensing power spectrum is estimated in the multipole range \( 8 \leq \ell \leq 2048 \). Multipoles \( \ell \leq 8 \) have large mean-field corrections due to survey anisotropy and are rather unstable to analysis choices; they are therefore excluded from all lensing results. Here, we use only the range \( 40 \leq \ell \leq 400 \) (the same as used in the 2013 analysis), with eight bins each with width \( \Delta \ell = 45 \). This choice is based on the extensive suite of null tests reported in Planck Collaboration XV (2016). Nearly all tests are passed over the full multipole range \( 8 \leq \ell \leq 2048 \), with the exception of a slight excess of curl modes in the \( TT \) reconstruction around \( \ell = 500 \). Given that the range \( 40 \leq \ell \leq 400 \) contains most of the statistical power in the reconstruction, we have conservatively adopted this range for use in the Planck 2015 cosmology papers.
- To normalize \( C_\phi \) from the measured 4-point functions requires knowledge of the CMB power spectra. In practice, we normalize with fiducial spectra, but then correct for changes in the true normalization at each point in parameter space within the likelihood. The exact renormalization scheme adopted in the 2013 analysis proved to be too slow for the extension to polarization, so we now use a linearized approximation, based on pre-computed response functions, which is very efficient within an MCMC analysis. Spot-checks have confirmed the accuracy of this approach.

\(^{17}\)In that paper we are careful to highlight the 4-point function origin of the lensing power spectrum reconstruction by using the index \( L \); however, in this paper we use the notation \( \ell \).
Fig. 11. Planck measurements of the lensing power spectrum compared to the prediction for the best-fitting base ΛCDM model to the Planck TT+lowP data. Left: the conservative cut of the Planck lensing data used throughout this paper, covering the multipole range 40 ≤ ℓ ≤ 400. Right: lensing data over the range 8 ≤ ℓ ≤ 2048, demonstrating the general consistency with the ΛCDM prediction over this extended multipole range. In both cases, green points are the power from lensing reconstructions using only temperature data, while blue points combine temperature and polarization. They are offset in ℓ for clarity and error bars are ±1 σ. In the top panels the solid lines are the best-fitting base ΛCDM model to the Planck TT+lowP data with no renormalization or δN(1) correction applied (see text for explanation). The bottom panels show the difference between the data and the renormalized and δN(1)-corrected theory bandpowers, which enter the likelihood. The mild preference of the lensing measurements for lower lensing power around ℓ = 200 pulls the theoretical prediction for Cℓφφ downwards at the best-fitting parameters of a fit to the combined Planck TT+lowP+lensing data, shown by the dashed blue lines (always for the conservative cut of the lensing data, including polarization).

- The measurement of Cℓφφ can be thought of as being derived from an optimal combination of trispectrum configurations. In practice, the expectation value of this combination at any multipole ℓ has a local part proportional to Cℓφφ, but also a non-local ("N(1) bias") part that couples to a broad range of multipoles in Cℓφφ (Kesden et al. 2003); this non-local part comes from non-primary trispectrum couplings. In the Planck 2013 analysis we corrected for the N(1) bias by making a fiducial correction, but this ignores its parameter dependence. We improve on this in the 2015 analysis by correcting for errors in the fiducial N(1) bias at each point in parameter space within the lensing likelihood. As with the renormalization above, we linearize this δN(1) correction for efficiency. As a result, we no longer need to make an approximate correction in the Cℓφφ covariance matrix to account for the cosmological uncertainty in N(1).
- Beam uncertainties are no longer included in the covariance matrix of Cℓφφ, since, with the improved knowledge of the beams, the estimated uncertainties are negligible for the lensing analysis. The only inter-bandpower correlations included in the Cℓφφ bandpower covariance matrix are from the uncertainty in the correction applied for the point-source 4-point function.

As in the 2013 analysis, we approximate the lensing likelihood as Gaussian in the estimated bandpowers, with a fiducial covariance matrix. Following the arguments in Schmittfull et al. (2013), it is a good approximation to ignore correlations between the 2- and 4-point functions; so, when combining the Planck power spectra with Planck lensing, we simply multiply their respective likelihoods.

It is also worth noting that the changes in absolute calibration of the Planck power spectra (around 2% between the 2013 and 2015 releases) do not directly affect the lensing results. The CMB 4-point functions do, of course, respond to any recalibration of the data, but in estimating Cℓφφ this dependence is removed by normalizing with theory spectra fit to the observed CMB spectra. The measured Cℓφφ bandpowers from the 2013 and current Planck releases can therefore be directly compared, and are in good agreement (Planck Collaboration XV 2016). Care is needed, however, in comparing consistency of the lensing measurements across data releases with the best-fitting model predictions. Changes in calibration translate directly into changes in Aℓe−2ℓ, which, along with any change in the best-fitting optical depth, alter Ar, and hence the predicted lensing power. These changes from 2013 to the current release go in opposite directions, leading to a net decrease in Ar of 0.6%. This, combined with a small (0.15%) increase in θ0, reduces the expected Cℓφφ by approximately 1.5% for multipoles ℓ > 60.

The Planck measurements of Cℓφφ, based on the temperature and polarization 4-point functions, are plotted in Fig. 11 (with results of a temperature-only reconstruction included for comparison). The measured Cℓφφ are compared with the predicted lensing power from the best-fitting base ΛCDM model to the Planck TT+lowP data in this figure. The bandpowers that are used in the conservative lensing likelihood adopted in this paper are shown in the left-hand plot, while bandpowers over the range 8 ≤ ℓ ≤ 2048 are shown in the right-hand plot, to demonstrate the general consistency with the ΛCDM prediction over the full multipole range. The difference between the measured bandpowers and the best-fit prediction are shown in the bottom
panels. Here, the theory predictions are corrected in the same way as they are in the likelihood.

Figure 11 suggests that the *Planck* measurements of \( C_\ell^{\phi \phi} \) are mildly in tension with the prediction of the best-fitting ΛCDM model. In particular, for the conservative multipole range \( 40 \leq \ell \leq 400 \), the temperature+ polarization reconstruction has \( \chi^2 = 15.4 \) (for eight degrees of freedom), with a PTE of 5.2 %. For reference, over the full multipole range \( \chi^2 = 40.8 \) for 19 degrees of freedom (PTE of 0.3 %); the large \( \chi^2 \) is driven by a single bandpass (638 \( \leq \ell \leq 762 \)), and excluding this gives an acceptable \( \chi^2 = 26.8 \) (PTE of 8 %). We caution the reader that this multipole range is where the lensing reconstruction shows a mild excess of curl-modes (*Planck* Collaboration XV 2016), and for this reason we adopt the conservative multipole range for the lensing likelihood in this paper.

This simple \( \chi^2 \) test does not account for the uncertainty in the predicted \( C_\ell^{\phi \phi} \). In the ΛCDM model, the dominant uncertainty in the multipole range \( 40 \leq \ell \leq 400 \) comes from that in \( A_s \) (1σ uncertainty of 3.7 % for *Planck* TT+lowP), which itself derives from the uncertainty in the reionization optical depth, \( \tau \). The predicted rms lensing deflection from *Planck* TT+lowP data is \( \langle d^2 \rangle^{1/2} = (2.50 \pm 0.05) \text{arcmin} \), corresponding to a 3.6 % uncertainty (1σ) in the amplitude of \( C_\ell^{\phi \phi} \) (which improves to 3.1 % uncertainty for the combined *Planck*+WP likelihood). Note that this is larger than the uncertainty on the measured amplitude, i.e., the lensing measurement is more precise than the prediction from the CMB power spectra in even the simplest ΛCDM model. This model uncertainty is reflected in a scatter in the \( \chi^2 \) of the lensing data over the *Planck* TT+lowP chains, \( \chi^2_{\text{lens}} = 17.9 \pm 9.0 \), which is significantly larger than the expected scatter in \( \chi^2 \) at the true model, due to the uncertainties in the lensing bandpowers (\( \sqrt{2N_{\text{ dof}} = 4} \)). Following the treatment in PCP13, we can assess consistency more carefully by introducing a parameter \( A_L^{\phi \phi} \) that scales the theory lensing trispectrum at every point in parameter space in a joint analysis of the CMB spectra and the lensing spectrum. We find

\[
A_L^{\phi \phi} = 0.95 \pm 0.04 \quad (68\%, \text{Planck TT+lowP+lensing}),
\]

in good agreement with the expected value of unity. The posterior for \( A_L^{\phi \phi} \), and other lensing amplitude measures discussed below, is shown in Fig. 12.

Given the precision of the measured \( C_\ell^{\phi \phi} \) compared to the uncertainty in the predicted spectrum from fits to the *Planck* TT+lowP data, the structure in the residuals seen in Fig. 11 might be expected to pull parameters in joint fits. As discussed in *Planck* Collaboration XV (2016) and Pan et al. (2014), the primary parameter dependence of \( C_\ell^{\phi \phi} \) at multipoles \( \ell \geq 100 \) is through \( A_s \) and \( \ell_{\text{eq}} \) in ΛCDM models. Here, \( \ell_{\text{eq}} \approx 1/\theta_{\text{eq}} \) is the angular multipole corresponding to the horizon size at matter-radiation equality observed at a distance \( \chi_s \). The combination \( A_s/\ell_{\text{eq}} \) determines the mean-squared deflection \( \langle d^2 \rangle \), while \( \ell_{\text{eq}} \) controls the shape of \( C_\ell^{\phi \phi} \). For the parameter ranges of interest,

\[
\frac{\delta C_\ell^{\phi \phi}}{C_\ell^{\phi \phi}} = \frac{\delta A_s/A_s + (n_\ell + 1)\delta \ell_{\text{eq}}/\ell_{\text{eq}}}{C_\ell^{\phi \phi}}
\]

where \( n_\ell \) arises (mostly) from the strong wavenumber dependence of the transfer function for the gravitational potential, with \( n_\ell \approx 1.5 \) around \( \ell = 200 \).

In joint fits to *Planck* TT+lowP+lensing, the main parameter changes from *Planck* TT+lowP alone are a 2.6 % reduction in the best-fit \( A_s \), with an accompanying reduction in the best-fit \( \tau \), to 0.067 (around 0.6σ; see Sect. 3.4). There is also a 0.7 % reduction in \( \ell_{\text{eq}} \), achieved at fixed \( \theta_s \), by reducing \( \omega_m \). These combine to reduce \( C_\ell^{\phi \phi} \) by approximately 4 % at \( \ell = 200 \), consistent with Eq. (19). The difference between the theory lensing spectrum at the best-fit parameters in the *Planck* TT+lowP and *Planck* TT+lowP+lensing fits are shown by the dashed blue lines in Fig. 11. In the joint fit, the \( \chi^2 \) for the lensing bandpowers improves by 6, while the \( \chi^2 \) for the *Planck* TT+lowP data degrades by only 1.2 (2.8 for the high-\( \ell \) TT data and −1.6 for the low-\( \ell \) TE data).

The lower values of \( A_s \) and \( \omega_m \) in the joint fit give a 2 % reduction in \( \sigma_8 \), with

\[
\sigma_8 = 0.815 \pm 0.009 \quad (68\%, \text{Planck TT+lowP+lensing}),
\]

as shown in Fig. 19. The decrease in matter density leads to a corresponding decrease in \( \Omega_m \), and at fixed \( \theta_s \), (approximately \( \propto \Omega_m h^2 \)) a 0.5 σ increase in \( H_0 \), giving

\[
H_0 = (67.8 \pm 0.9) \text{ km s}^{-1} \text{Mpc}^{-1}
\]

Joint *Planck*+lensing constraints on other parameters of the base ΛCDM cosmology are given in Table 4.

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18 In detail, the theory spectrum is binned in the same way as the data, renormalized to account for the (very small) difference between the CMB spectra in the best-fit model and the fiducial spectra used in the lensing analysis, and corrected for the difference in \( N_{\text{101}} \), calculated for the best-fit and fiducial models (around a 4 % change in \( N_{\text{101}} \), since the fiducial-model \( C_\ell^{\phi \phi} \) is higher by this amount than in the best-fit model).
Planck Collaboration XV (2016) discusses the effect on parameters of extending the lensing multipole range in joint fits with Planck TT+lowP. In the base ΛCDM model, using the full multipole range 8 ≤ ℓ ≤ 2048, the parameter combination \( \sigma_8 \Omega_m^{1/2} \approx (A_s G_0^2)^{1/2} \) (which is well determined by the lensing measurements) is pulled around 1σ lower that its value using the conservative lensing range, with a negligible change in the uncertainty. Around half of this shift comes from the 3.6σ outstanding bandpower (638 ≤ ℓ ≤ 762). In massive neutrino models, the total mass is similarly pulled higher by around 1σ when using the full lensing multipole range.

5.1.2. Detection of lensing in the CMB power spectra

The smoothing effect of lensing on the acoustic peaks and troughs of the TT power spectrum is detected at high significance in the Planck data. Following PCP13 (see also Calabrese et al. 2008), we introduce a parameter \( A_L \), which scales the theory \( C_\ell^{\phi\phi} \) power spectrum at each point in parameter space, and which is used to lens the CMB spectra.\(^{19}\) The expected value for base ΛCDM is \( A_L = 1 \). The results of such an analysis for models with variable \( A_L \) is shown in Fig. 12. The marginalized constraint on \( A_L \) is

\[
A_L = 1.22 \pm 0.10 \quad (68\%, \text{Planck TT+lowP}).
\]

This is very similar to the result from the 2013 Planck data reported in PCP13. The persistent preference for \( A_L > 1 \) is discussed in detail there. For the 2015 data, we find that \( \Delta A_L^2 = -6.4 \) between the best-fitting ΛCDM+\( A_L \) model and the best-fitting base ΛCDM model. There is roughly equal preference for high \( A_L \) from intermediate and high multipoles (i.e., the P11k likelihood; \( \Delta A_L^2 = -2.6 \)) and from the low-\( \ell \) likelihood (\( \Delta A_L^2 = -3.1 \)), with a further small change coming from the priors.

Increases in \( A_L \) are accompanied by changes in all other parameters, with the general effect being to reduce the predicted CMB power on large scales, and in the region of the second acoustic peak, and to increase CMB power on small scales (see Fig. 13). A reduction in the high-\( \ell \) foreground power compensates the CMB increase on small scales. Specifically, \( n_s \) is increased by 1% relative to the best-fitting base model and \( A_L \) is reduced by 4%, both of which lower the large-scale power to provide a better fit to the measured spectra around \( \ell = 20 \) (see Fig. 1). The densities \( \omega_b \) and \( \omega_c \) respond to the change in \( n_s \), following the usual ΛCDM acoustic degeneracy, and \( A_L e^{-2\tau} \) falls by 1%, attempting to reduce power in the damping tail due to the increase in \( n_s \) and reduction in the diffusion angle \( \theta_D \) (which follows from the reduction in \( \omega_b \)). The changes in \( A_L \) and \( A_s e^{-2\tau} \) lead to a reduction in \( \tau \) from 0.078 to 0.060. With these cosmological parameters, the lensing power is lower than in the base model, which additionally increases the CMB power in the acoustic peaks and reduces it in the troughs. This provides a poor fit to the measured spectra around the fourth and fifth peaks, but can be mitigated by increasing \( A_L \) to give more smoothing from lensing than in the base model. However, \( A_L \) further increases power in the damping tail, but this is partly offset by reduction of the power in the high-\( \ell \) foregrounds.

\(^{19}\)We emphasize the difference between the phenomenological parameters \( A_L \) and \( A_s e^{-2\tau} \) (introduced earlier). The amplitude \( A_L \) multiplies \( C_\ell^{\phi\phi} \) when calculating both the lensed CMB theory spectra and the lensing likelihood, while \( A_s e^{-2\tau} \) affects only the lensing likelihood by scaling the theory \( C_\ell^{\phi\phi} \) when comparing with the power spectrum of the reconstructed lensing potential \( \phi \).

Fig. 13. Changes in the CMB TT spectrum and foreground spectra, between the best-fitting \( A_L \) model and the best-fitting base ΛCDM model to the Planck TT+lowP data. The solid blue line shows the difference between the \( A_L \) model and ΛCDM while the dashed line has the the same values of the other cosmological parameters, but with \( A_L \) set to unity, to highlight the changes in the spectrum arising from differences in the other parameters. Also shown are the changes in the best-fitting foreground contributions to the four frequency cross-spectra between the \( A_L \) model and the ΛCDM model. The data points (with ±1σ errors) are the differences between the high-\( \ell \) maximum-likelihood frequency-averaged CMB spectrum and the best-fitting ΛCDM model to the Planck TT+lowP data (as in Fig. 1). Note that the changes in the CMB spectrum and the foregrounds should be added when comparing to the residuals in the data points.

The trends in the TT spectrum that favour high \( A_L \) have a similar pull on parameters such as curvature (Sect. 6.2.4) and the dark energy equation of state (Sect. 6.3) in extended models. These parameters affect the late-time geometry and clustering and so alter the lensing power, but their effect on the primary CMB fluctuations is degenerate with changes in the Hubble constant (to preserve \( \theta_D \)). The same parameter changes as those in \( A_L \) models are found in these extended models, but with, for example, the increase in \( A_L \) replaced by a reduction in \( \Omega_b \). Adding external data, however, such as the Planck lensing data or BAO (Sect. 5.2), pull these extended models back to base ΛCDM.

Finally, we note that lensing is also detected at lower significance in the polarization power spectra (see Fig. 12): \[
A_L = 0.98^{+0.21}_{-0.24} \quad (68\%, \text{Planck TE+lowP}); \quad (23a)
\]
\[
A_L = 1.54^{+0.28}_{-0.33} \quad (68\%, \text{Planck EE+lowP}). \quad (23b)
\]

These results use only polarization at low multipoles, i.e., with no temperature data at multipoles \( \ell < 30 \). These are the first detections of lensing in the CMB polarization spectra, and reach almost 5σ in TE. We caution the reader that the \( A_L \) constraints from EE and low-\( \ell \) polarization are rather unstable between high-\( \ell \) likelihoods because of differences in the treatment of the polarization data (see Fig. 12, which compares constraints from the P11k and CamSpec polarization likelihoods). The result of replacing P11k with the CamSpec likelihood is \( A_L = 1.19^{+0.20}_{-0.24} \), i.e., around 1σ lower than the result from P11k reported in Eq. (23b). If we additionally include the low-\( \ell \) temperature data, \( A_L \) from TE increases:

\[
A_L = 1.13 \pm 0.2 \quad (68\%, \text{Planck TE+lowT,P}). \quad (24)
\]
The pull to higher $A_L$ in this case is due to the reduction in $TT$ power in these models on large scales (as discussed above).

5.2. Baryon acoustic oscillations

Baryon acoustic oscillation (BAO) measurements are geometric and largely unaffected by uncertainties in the nonlinear evolution of the matter density field and additional systematic errors that may affect other types of astrophysical data. As in PCP13, we therefore use BAO as a primary astrophysical data set to break parameter degeneracies from CMB measurements.

Figure 14 shows an updated version of figure 15 from PCP13. The plot shows the acoustic-scale distance ratio $D_V(z)/r_{	ext{drag}}$ measured from a large number of scale structure surveys with effective redshift $z$, divided by the mean acoustic-scale ratio in the base $\Lambda$CDM cosmology using Planck TT+lowP+lensing. Here $r_{	ext{drag}}$ is the comoving sound horizon at the end of the baryon drag epoch and $D_V$ is a combination of the angular diameter distance $D_A(z)$ and Hubble parameter $H(z)$.

$$D_V(z) \approx \left[1 + z^2 D_A^2(z) \frac{cz}{H(z)} \right]^{1/3}.$$  

The grey bands in the figure show the $\pm 1\sigma$ and $\pm 2\sigma$ ranges allowed by Planck in the base $\Lambda$CDM cosmology.

The changes to the data points compared to figure 15 of PCP13 are as follows. We have replaced the SDSS DR7 measurements of Percival et al. (2010) with the recent analysis of the SDSS Main Galaxy Sample (MGS) of Ross et al. (2015) at $z_{\text{eff}} = 0.15$, and by the Anderson et al. (2014) analysis of the Baryon Oscillation Spectroscopic Survey (BOSS) “LOWZ” sample at $z_{\text{eff}} = 0.32$. Both of these analyses use peculiar velocity field reconstructions to sharpen the BAO feature and reduce the errors on $D_V/r_{	ext{drag}}$. The blue points in Fig. 14 show a reanalysis of the WiggleZ redshift survey by Kazin et al. (2014) that applies peculiar velocity reconstructions. These reconstructions cause small shifts in $D_V/r_{	ext{drag}}$ compared to the unreconstructed WiggleZ results of Blake et al. (2011) and lead to reductions in the errors on the distance measurements at $z_{\text{eff}} = 0.44$ and $z_{\text{eff}} = 0.73$. The point labelled BOSS CMASS at $z_{\text{eff}} = 0.57$ shows $D_V/r_{	ext{drag}}$ from the analysis of Anderson et al. (2014), updating the BOSS-DR9 analysis of Anderson et al. (2012) used in PCP13.

In fact, the Anderson et al. (2014) analysis solves jointly for the positions of the BAO feature in both the line-of-sight and transverse directions (the distortion in the transverse direction caused by the background cosmology is sometimes called the Alcock-Paczynski effect, Alcock & Paczynski 1979), leading to joint constraints on the angular diameter distance $D_A(z_{\text{eff}})$ and the Hubble parameter $H(z_{\text{eff}})$. These constraints, using the tabulated likelihood included in the CosmoMC module, are plotted in Fig. 15. Samples from the Planck TT+lowP+lensing chains are shown for comparison, coloured by the value of $\Omega_m h^2$. The length of the degeneracy line is set by the allowed variation in $H_0$ (or equivalently $\Omega_m h^2$). In the Planck TT+lowP+lensing $\Lambda$CDM analysis this line is defined approximately by

$$D_A(0.57)/r_{	ext{drag}} \left( \frac{H(0.57) r_{\text{drag}}/c}{0.4582} \right) = 1.0000 \pm 0.0004.$$  

which just grazes the BOSS CMASS 68% error ellipse plotted in Fig. 15. Evidently, the Planck base $\Lambda$CDM parameters are in good agreement with both the isotropised $D_V$ BAO measurements plotted in Fig. 14, and with the anisotropic constraints plotted in Fig. 15.

In this paper, we use the 6dFGS, SDSS-MGS, and BOSS-LOWZ BAO measurements of $r_{\text{d}z}$ (Beutler et al. 2011; Ross et al. 2015; Anderson et al. 2014) and the CMASS-DR11 anisotropic BAO measurements of Anderson et al. (2014). Since the WiggleZ volume partially overlaps that of the BOSS-CMASS sample and the correlations have not been quantified, we do not use the WiggleZ results in this paper. It is clear from Fig. 14 that the combined BAO likelihood is dominated by the two BOSS measurements.

In the base $\Lambda$CDM model, the Planck data constrain the Hubble constant $H_0$ and matter density $\Omega_m$ to high precision:

$$H_0 = (67.3 \pm 1.0) \text{ km s}^{-1}\text{Mpc}^{-1}$$

$$\Omega_m = 0.315 \pm 0.013$$

Planck TT+lowP. (27)

With the addition of the BAO measurements, these constraints are strengthened significantly to

$$H_0 = (67.6 \pm 0.6) \text{ km s}^{-1}\text{Mpc}^{-1}$$

$$\Omega_m = 0.310 \pm 0.008$$

Planck TT+lowP+BAO. (28)

These numbers are consistent with the Planck+lensing constraints of Eq. (21). Section 5.4 discusses the consistency of these estimates of $H_0$ with direct measurements.

Although low-redshift BAO measurements are in good agreement with Planck for the base $\Lambda$CDM cosmology, this may not be true at high redshifts. Recently, BAO features have been measured in the flux-correlation function of the Lyα forest of BOSS quasars (Delubac et al. 2015) and in the cross-correlation of the Lyα forest with quasars (Font-Ribera et al. 2014). These observations give measurements of $c/(H(z)r_{\text{d}z})$ and $D_{\text{s}}(z)/r_{\text{d}z}$ (with somewhat lower precision) at $z = 2.34$ and $z = 2.36$, respectively. For example, from table II of Aubourg et al. (2015) the two Lyα BAO measurements combined give $c/(H(2.34)r_{\text{d}z}) = 9.14 \pm 0.20$, compared to the predictions of the base Planck $\Lambda$CDM cosmology of 8.586 ± 0.021, which are discrepant at the 2.7σ level. At present, it is not clear whether this discrepancy is caused by systematics in the Lyα BAO measurements (which are more complex and less mature than galaxy BAO measurements) or an indicator of new physics. As Aubourg et al. (2015) discuss, it is difficult to find a physical explanation for the Lyα BAO results without disrupting the consistency with the much more precise galaxy BAO measurements at lower redshifts.

5.3. Type Ia supernovae

Type Ia supernovae (SNe) are powerful probes of cosmology (Riess et al. 1998; Perlmuter et al. 1999) and particularly of the equation of state of dark energy. In PCP13, we used two samples of type Ia SNe, the “SNLS” compilation (Conley et al. 2011) and the “Union2.1” compilation (Suzuki et al. 2012). The SNLS sample was found to be in mild tension, at about the 2σ level, with the 2013 Planck base $\Lambda$CDM cosmology favouring a value of $\Omega_m \approx 0.23$ compared to the Planck value of $\Omega_m = 0.315 \pm 0.017$. Another consequence of this tension showed up in extensions to the base $\Lambda$CDM model, where the combination of Planck and the SNLS sample showed 2σ evidence for a “phantom” ($w < -1$) dark energy equation of state.

Following the submission of PCP13, Betoule et al. (2013) reported the results of an extensive campaign to improve the relative photometric calibrations between the SNLS and SDSS supernova surveys. The “Joint Light-curve Analysis” (JLA) sample, used in this paper, is constructed from the SNLS and SDSS SNe data, together with several samples of low redshift SNe.21 Cosmological constraints from the JLA sample are discussed by Betoule et al. (2014) and residual biases associated with the photometry and light curve fitting are assessed by Mosher et al. (2014). For the base $\Lambda$CDM cosmology, Betoule et al. (2014) find $\Omega_m = 0.295 \pm 0.034$, consistent with the 2013 and 2015 Planck values for base $\Lambda$CDM. This relieves the tension between the SNLS and Planck data reported in PCP13. Given the consistency between Planck and the JLA sample for base $\Lambda$CDM, one can anticipate that the combination of these two data sets will constrain the dark energy equation of state to be close to $w = -1$ (see Sect. 6.3).

Since the submission of PCP13, first results from a sample of Type Ia SNe discovered with the Pan-STARRS survey have been reported by Rest et al. (2014) and Scolnic et al. (2014). The Pan-STARRS sample is still relatively small (consisting of 146 spectroscopically confirmed Type Ia SNe) and is not used in this paper.

5.4. The Hubble constant

CMB experiments provide indirect and highly model-dependent estimates of the Hubble constant. It is therefore important to compare CMB estimates with direct estimates of $H_0$, since any significant evidence of a tension could indicate the need for new physics. In PCP13, we used the Riess et al. (2011, hereafter R11) Hubble Space Telescope (HST) Cepheid+SNe based estimate of $H_0 = (73.8 \pm 2.4) \text{ km s}^{-1}\text{Mpc}^{-1}$ as a supplementary “$H_0$ prior.” This value was in tension at about the 2.5σ level with the 2013 Planck base $\Lambda$CDM value of $H_0$.

For the base $\Lambda$CDM model, CMB and BAO experiments consistently find a value of $H_0$ lower than the R11 value. For example, the 9-year WMAP data (Bennett et al. 2013; Hinshaw et al. 2013) give:

$$H_0 = (69.7 \pm 2.1) \text{ km s}^{-1}\text{Mpc}^{-1}, \text{ WMAP9},$$

$$H_0 = (68.0 \pm 0.7) \text{ km s}^{-1}\text{Mpc}^{-1}, \text{ WMAP9+BAO}.$$ (29b)

These numbers can be compared with the Planck 2015 values given in Eqs. (27) and (28). The WMAP constraints are driven towards the Planck values by the addition of the BAO data and so there is persuasive evidence for a low $H_0$ in the base $\Lambda$CDM cosmology independently of the high-multipole CMB results from Planck. The 2015 Planck TT+lowP value is entirely consistent with the 2013 Planck value and so the tension with the R11 $H_0$ determination remains at about 2.4σ.

The tight constraint on $H_0$ in Eq. (29b) is an example of an “inverse distance ladder,” where the CMB primarily constrains the sound horizon within a given cosmology, providing an absolute calibration of the BAO acoustic-scale (e.g., Percival et al. 2010; Cuesta et al. 2015; Aubourg et al. 2015, see also PCP13). In fact, in a recent paper Aubourg et al. (2015) use the 2013 Planck constraints on $r_s$ in combination with BAO and the JLA SNe data to find $H_0 = (67.3 \pm 1.1) \text{ km s}^{-1}\text{Mpc}^{-1}$, in excellent agreement with the 2015 Planck value for base $\Lambda$CDM given in

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21A CosmMoC likelihood model for the JLA sample is available at http://supernovae.in2p3.fr/sdss_snls_jla/ReadMe.html. The latest version in CosmMoC includes numerical integration over the nuisance parameters for use when calculating joint constraints using importance sampling; this can give different $\chi^2$ values compared to parameter best fits.

22These numbers are taken from our parameter grid, which includes a neutrino mass of 0.06 eV and the same updated BAO compilation as Eq. (28) (see Sect. 5.2).
Eq. (27), which is based on the Planck temperature power spectrum. Note that by adding SNe data, the Aubourg et al. (2015) estimate of \( H_0 \) is insensitive to spatial curvature and to late time variations of the dark energy equation of state. Evidently, there are a number of lines of evidence that point to a lower value of \( H_0 \) than the direct determination of R11.

The R11 Cepheid data have been reanalysed by Efstathiou (2014, hereafter E14) using the revised geometrical distance to NGC 4258 of Humphreys et al. (2013). Using NGC 4258 as a distance anchor, E14 finds

\[
H_0 = (70.6 \pm 3.3) \text{ km s}^{-1}\text{Mpc}^{-1}, \quad \text{NGC 4258}, \tag{30}
\]

which is within 1\( \sigma \) of the Planck TT estimate given in Eq. (27). In this paper we use Eq. (30) as a “conservative” \( H_0 \) prior.

R11 also use Large Magellanic Cloud Cepheids and a small sample of Milky Way Cepheids with parallax distances as alternative distance anchors to NGC4258. The R11 \( H_0 \) prior used in PCP13 combines all three distance anchors. Combining the LMC and MW distance anchors, E14 finds

\[
H_0 = (73.9 \pm 2.7) \text{ km s}^{-1}\text{Mpc}^{-1}, \quad \text{LMC + MW}, \tag{31}
\]

under the assumption that there is no metallicity variation of the Cepheid period-luminosity relation. This is discrepant with Eq. (27) at about the 2.2 \( \sigma \) level. However, neither the central value nor the error in Eq. (31) is reliable. The MW Cepheid sample is small and dominated by short period (<10 day) objects. The MW Cepheid sample therefore has very little overlap with the period range of SNe host galaxy Cepheids observed with HST. As a result, the MW solutions for \( H_0 \) are unstable (see Appendix A of E14). The LMC solution is sensitive to the metallicity dependence of the Cepheid period-luminosity relation which is poorly constrained by the R11 data. Furthermore, the estimate in Eq. (30) is based on a differential measurement, comparing HST photometry of Cepheids in NGC 4258 with those in SNe host galaxies. It is therefore less prone to photometric systematics, such as crowding corrections, than is the LMC+MW estimate of Eq. (31). It is for these reasons that we have adopted the prior of Eq. (30) in preference to using the LMC and MW distance anchors.\(^{23}\)

Direct measurements of the Hubble constant have a long and sometimes contentious history (see, e.g., Tammann et al. 2008). The controversy continues to this day and in the literature one can find “high” values, e.g., \( H_0 = (74.3 \pm 2.6) \text{ km s}^{-1}\text{Mpc}^{-1} \) (Freedman et al. 2012), and “low” values, e.g., \( H_0 = (63.7 \pm 2.3) \text{ km s}^{-1}\text{Mpc}^{-1} \) (Tammann & Reindl 2013). The key point that we wish to make is that the Planck-only estimates of Eqs. (21) and (27), and the Planck+BAO estimate of Eq. (28) all have small errors and are consistent. If a persuasive case can be made that a direct measurement of \( H_0 \) conflicts with these estimates, then this will be strong evidence for additional physics beyond the base \( \Lambda \)CDM model.

Finally, we note that in a recent analysis Bennett et al. (2014) derive a “concordance” value of \( H_0 = (69.6 \pm 0.7) \text{ km s}^{-1}\text{Mpc}^{-1} \) for base \( \Lambda \)CDM by combining WMAP9+SPT+ACT+BAO with a slightly revised version of the R11 \( H_0 \) value, \((73.0 \pm 2.4) \text{ km s}^{-1}\text{Mpc}^{-1} \). The Bennett et al. (2014) central value for \( H_0 \) differs from the Planck value of Eq. (28) by nearly 3\% (or 2.5 \( \sigma \)). The reason for this difference is that the Planck data are in tension with the Story et al. (2013) SPT data (as discussed in Appendix B of PCP13; note that the tension is increased with the Planck full mission data) and with the revised R11 \( H_0 \) determination. Both tensions drive the Bennett et al. (2014) value of \( H_0 \) away from the Planck solution.

5.5. Additional data

5.5.1. Redshift space distortions

Transverse versus line-of-sight anisotropies in the redshift-space clustering of galaxies induced by peculiar motions can, potentially, provide a powerful way of constraining the growth rate of structure (e.g., Percival & White 2009). A number of studies of redshift-space distortions (RSD) have been conducted to measure the parameter combination \( f\sigma_8(z) \), where models with scale-independent growth

\[
f(z) = \frac{d\ln D}{d\ln a}, \tag{32}
\]

and \( D \) is the linear growth rate of matter fluctuations. Notice that the parameter combination \( f\sigma_8 \) is insensitive to differences between the clustering of galaxies and dark matter, i.e., to galaxy bias (Song & Percival 2009). In the base \( \Lambda \)CDM cosmology, the growth factor \( f(z) \) is well approximated as \( f(z) = \Omega_m(z)^{0.545} \). More directly, in linear theory the quadrupole of the redshift-space clustering anisotropy actually probes the density-velocity correlation power spectrum, and we therefore define

\[
f\sigma_8(z) \equiv \bigg( \frac{\sigma_8^{(dd)}(z)}{\sigma_8^{(id)}(z)} \bigg)^2, \tag{33}
\]

as an approximate proxy for the quantity actually being measured. Here \( \sigma_8^{(id)} \) measures the smoothed density-velocity correlation and is defined analogously to \( \sigma_8 \equiv \sigma_8^{(dd)} \), but using the correlation power spectrum \( P_N(k) \), where \( v = -\nabla \cdot r_N/H \) and \( v_N \) is the Newtonian-gauge (peculiar) velocity of the baryons and dark matter, and \( d \) is the total matter density perturbation. This definition assumes that the observed galaxies follow the flow of the cold matter, not including massive neutrino velocity effects. For models close to \( \Lambda \)CDM, where the growth is nearly scale independent, it is equivalent to defining \( f\sigma_8 \) in terms of the growth of the baryon+CDM density perturbations (excluding neutrinos).

The use of RSD as a measure of the growth of structure is still under active development and is considerably more difficult than measuring the positions of BAO features. Firstly, adopting the wrong fiducial cosmology can induce an anisotropy in the clustering of galaxies, via the Alcock-Paczynski (AP) effect, which is strongly degenerate with the anisotropy induced by peculiar motions. Secondly, much of the RSD signal currently comes from scales where nonlinear effects and galaxy bias are significant and must be accurately modelled in order to relate the density and velocity fields (see, e.g., the discussions in Bianchi et al. 2012; Okumura et al. 2012; Reid et al. 2014; White et al. 2015).
Fig. 16. Constraints on the growth rate of fluctuations from various redshift surveys in the base ΛCDM model: green star (6dFGS, Beutler et al. 2012); purple square (SDSS MGS, Howlett et al. 2015); cyan cross (SDSS LRG, Oka et al. 2014); red triangle (BOSS LOWZ survey, Gil-Marín et al. 2015); large red circle (BOSS CMASS, as analysed by Samushia et al. 2014); blue circles (WiggleZ, Blake et al. 2012); and green diamond (VIPERS, de la Torre et al. 2013). The points with dashed red error bars correspond to alternative analyses of BOSS CMASS from Beutler et al. (2014, small circle, offset for clarity) and Chuang et al. (2013, small square). Of the BOSS CMASS points, two are based on the more recent DR11 data set (Samushia et al. 2014; Beutler et al. 2014), while the third is based on the more recent DR12 (Chuang et al. 2013), and are therefore not independent. The grey bands show the range allowed by Planck TT+lowP+lensing in the base ΛCDM model. Where available (for SDSS MGS and BOSS), we have plotted conditional constraints on $f\sigma_8$ assuming a Planck ΛCDM background cosmology. The WiggleZ points are plotted conditional on the mean Planck cosmology prediction for $F_{\text{AP}}$ (evaluated using the covariance between $f\sigma_8$ and $F_{\text{AP}}$ given in Blake et al. 2012). The 6dFGS point is at sufficiently low redshift that it is insensitive to the cosmology.

Current constraints assuming a Planck base ΛCDM model, are shown in Fig. 16. Neglecting the AP effect can lead to biased measurements of $f\sigma_8$ if the assumed cosmology differs, and to significant underestimation of the errors (Howlett et al. 2015). The analyses summarized in Fig. 16 solve simultaneously for RSD and the AP effect, except for the 6dFGS point (which is insensitive to cosmology) and the VIPERS point (which has a large error). The grey bands show the range allowed by Planck TT+lowP+lensing in the base ΛCDM model, and are consistent with the RSD data. The tightest constraints on $f\sigma_8$ in this figure come from the BOSS CMASS-DR11 analyses of Beutler et al. (2014) and Samushia et al. (2014). The Beutler et al. (2014) analysis is performed in Fourier space and shows a small bias in $f\sigma_8$, compared to numerical simulations when fitting over the wavenumber range $0.01–0.20\ h\text{Mpc}^{-1}$. The Samushia et al. (2014) analysis is performed in configuration space and shows no evidence of biases when compared to numerical simulations.

The updated DR12 CMASS result from Chuang et al. (2013) is illustrated in Fig. 17, compared to the constraint from Planck TT+lowP+lensing in the base ΛCDM cosmology. The Planck results sit slightly high but overlap the 68 % contour from Samushia et al. (2014). The Planck result lies about 1.5σ higher than the Beutler et al. (2014) analysis of the BOSS CMASS sample.

RSD measurements are not used in combination with Planck in this paper. However, in the companion paper exploring dark energy and modified gravity (Planck Collaboration XIV 2016), the RSD/BAO measurements of Samushia et al. (2014) are used together with Planck. Where this is done, we exclude the Anderson et al. (2014) BOSS-CMASS results from the BAO likelihood. Since Samushia et al. (2014) do not apply a density field reconstruction in their analysis, the BAO constraints from BOSS-CMASS are then slightly weaker, though consistent, with those of Anderson et al. (2014).
5.5.2. Weak gravitational lensing

Weak gravitational lensing offers a potentially powerful technique for measuring the amplitude of the matter fluctuation spectrum at low redshifts. Currently, the largest weak lensing data set is provided by the CFHTLenS survey (Heymans et al. 2012; Erben et al. 2013). The first science results from this survey appeared shortly before the completion of PCP13 and it was not possible to do much more than offer a cursory comparison with the Planck 2013 results. As reported in PCP13, at face value the results from CFHTLenS appeared to be in tension with the Planck 2013 base ΛCDM cosmology at about the 2–3σ level. Since neither the CFHTLenS results nor the 2015 Planck results have changed significantly from those in PCP13, it is worth discussing this discrepancy in more detail in this paper.

Weak lensing data can be analysed in various ways. For example, one can compute two correlation functions from the ellipticities of pairs of images separated by angle θ, which are related to the convergence power spectrum \( P^\kappa(\ell) \) of the survey at multipole \( \ell \) via

\[
\xi_\kappa(\theta) = \frac{1}{2\ell + 1} \int \! \! d\ell P^\kappa(\ell) J_\ell(\ell \theta),
\]

where the Bessel functions in (35) are \( J_0 \equiv J_0 \) and \( J_1 \equiv J_1 \) (see, e.g., Bartelmann & Schneider 2001). Much of the information from the CFHTLenS survey correlation function analyses comes from wave numbers at which the matter power spectrum is strongly nonlinear, complicating any direct comparison with Planck.

This can be circumventing by performing a 3D spherical harmonic analysis of the shear field, allowing one to impose lower limits on the wave numbers that contribute to a weak lensing likelihood. This has been done by Kitching et al. (2014). Including only wave numbers with \( k \leq 1.5 \text{hMpc}^{-1} \), Kitching et al. (2014) find constraints in the \( \sigma_8-\Omega_m \) plane that are consistent with the results from Planck. However, by excluding modes with higher wave numbers, the lensing constraints are weakened. When they increase the wavenumber cut-off to \( k = 5 \text{hMpc}^{-1} \) some tension with Planck begins to emerge (which these authors argue may be an indication of the effects of baryonic feedback in suppressing the matter power spectrum at small scales). The large-scale properties of CFHTLenS therefore seem broadly consistent with Planck and it is only as CFHTLenS probes higher wave numbers, in particular in the 2D and tomographic correlation function analyses (Heymans et al. 2013; Kilbinger et al. 2013; Fu et al. 2014; MacCrann et al. 2015), that apparently strong discrepancies with Planck appear.

The situation is summarized in Fig. 18. The sample points show parameter values in the \( \sigma_8-\Omega_m \) plane for the ΛCDM base model, computed from the Heymans et al. (2013, hereafter H13) tomographic measurements of \( \xi_\kappa \). These data consist of correlation function measurements in six photometric redshift bins extending over the redshift range 0.2–1.3. We use the red galaxy sample, since H13 find that this sample shows no evidence for intrinsic galaxy alignments (simplifying the comparison with theory) and we apply the “conservative” cuts of H13, intended to reduce sensitivity to the nonlinear part of the power spectrum; these cuts eliminate measurements with \( \theta < 3' \) for any redshift combination that involves the lowest two redshift bins. Here we have used the halofit prescription of Takahashi et al. (2012) to model the nonlinear power spectrum, but do not include any model of baryon feedback or intrinsic alignments. For the lensing-only constraint we also impose additional priors in a similar way to the CMB lensing analysis described in Planck Collaboration XV (2016), i.e., Gaussian priors \( \Omega_m h^2 = 0.0223 \pm 0.0009 \) and \( n_s = 0.96 \pm 0.02 \), where the exact values (chosen to span reasonable ranges given CMB data) have little impact on the results. The sample range shown also restricts the Hubble parameter to \( 0.2 < h < 1 \); note that when comparing with constraint contours, the location of the contours can change significantly depending on the \( H_0 \) prior range assumed. We also use a weak prior on the primordial amplitude, \( 2 < \ln(10^{10} A_s) < 4 \), which shows up the strong correlation between \( \Omega_m-\sigma_8-H_0 \) in the region of parameter space relevant for comparison with Planck. In Fig. 18 we only show lensing contours after the samples have been projected into the space allowed by the BAO data (blue contours), or additionally restricting to the reduced space where \( \theta_{MC} \) is fixed to the Planck value, which is accurately measured. The black contours show the constraints from Planck TT+lowP.

The lensing samples just overlap with Planck, and superficially one might conclude that the two data sets are consistent. However, the weak lensing constraints approximately define a 1D degeneracy in the 3D \( \Omega_m-\sigma_8-H_0 \) space, so consistency of the Hubble parameter at each point in the projected space must also be considered (see appendix E1 of Planck Collaboration XV 2016). Comparing the contours in Fig. 18 (the regions where the weak lensing constraints are consistent with BAO observations) the CFHTLenS data favour a lower value of \( \sigma_8 \) than the Planck data (and much of the area of the blue contours also has higher \( \Omega_m \)). However, even with the conservative angular cuts applied by H13, the weak lensing constraints depend on the nonlinear model of the power spectrum and on the possible influence of baryonic feedback in reshaping the matter power spectrum at small spatial scales (Harnois-Dérap et al. 2015; MacCrann et al. 2015). The importance of these effects can be reduced by imposing even more conservative angular
cuts on $\xi_s$, but of course, this weakens the statistical power of the weak lensing data. The CFHTLenS data are not used in combination with Planck in this paper (apart from specific cases in Sects. 6.3 and 6.4.4) and, in any case, would have little impact on most of the extended $\Lambda$CDM constraints discussed in Sect. 6. Weak lensing can, however, provide important constraints on dark energy and modified gravity. The CFHTLenS data are therefore used in combination with Planck in the companion paper (Planck Collaboration XIV 2016), which explores several halo fil prescriptions and the impact of applying more conservative angular cuts to the H13 measurements.

5.5.3. Planck cluster counts

In 2013 we noted a possible tension between our primary CMB constraints and those from the Planck SZ cluster counts, with the clusters preferring lower values of $\sigma_8$ in the base $\Lambda$CDM model in some analyses (Planck Collaboration XX 2014). The comparison is interesting because the cluster counts directly measure $\sigma_8$ at low redshift; any tension could signal the need for extensions to the base model, such as non-minimal neutrino mass (though see Sect. 6.4). However, limited knowledge of the scaling relation between SZ signal and mass have hampered the interpretation of this result.

With the full mission data we have created a larger catalogue of SZ clusters with a more accurate characterization of its completeness (Planck Collaboration XXIV 2016). By fitting the counts in redshift and signal-to-noise, we are able to simultaneously constrain the slope of the SZ signal–mass scaling relation and the cosmological parameters. A major uncertainty, however, remains the overall mass calibration, which in Planck Collaboration XX (2014) we quantified with a “hydrostatic bias” parameter, $(1 - b)$, with a fiducial value of 0.8 and a range $0.7 < (1 - b) < 1$ (consistent with some other studies, e.g., Simet et al. 2015). In the base $\Lambda$CDM model, the primary CMB constraints prefer a normalization below the lower end of this range, $(1 - b) \approx 0.6$. The recent, empirical normalization of the relation by the Weighing the Giants lensing programme (WtG; von der Linden et al. 2014) gives $0.69 \pm 0.07$ for the 22 clusters in common with the Planck cluster sample. This calibration reduces the tension with the primary CMB constraints in base $\Lambda$CDM. In contrast, correlating the entire Planck 2015 SZ cosmology sample with Planck CMB lensing gives $1/(1 - b) = 1.0 \pm 0.2$ (Planck Collaboration XXIV 2016), toward the upper end of the range adopted in Planck Collaboration XX (2014), although with a large uncertainty. An alternative lensing calibration analysis by the Canadian Cluster Comparison Project, which uses 37 clusters in common with the Planck cluster sample (Hoekstra et al. 2015) finds $(1 - b) = 0.76 \pm 0.05$ (stat.) $\pm 0.06$ (syst.), which lies between the other two mass calibrations. These calibrations are not yet definitive and the situation will continue to evolve with improvements in mass measurements from larger samples of clusters.

A recent analysis of cluster counts for an X-ray-selected sample (REFLEX II) shows some tension with the Planck base $\Lambda$CDM cosmology (Böhringer et al. 2014). However, an analysis of cluster counts of X-ray-selected clusters by the WtG collaboration, incorporating the WtG weak lensing mass calibration, finds $\sigma_8(\Omega_m/0.3)^{0.37} = 0.81 \pm 0.03$, in good agreement with the Planck CMB results for base $\Lambda$CDM (Mantz et al. 2015). This raises the possibility that there may be systematic biases in the assumed scaling relations for SZ-selected clusters compared to X-ray-selected clusters (in addition to a possible mass calibration bias). Mantz et al. (2015) give a brief review of recent determinations of $\sigma_8$ from X-ray, optically-selected, and SZ-selected samples, to which we refer the reader. More detailed discussion of constraints from combining Planck cluster counts with primary CMB anisotropies and other data sets can be found in Planck Collaboration XXIV (2016).

5.6. Cosmic concordance?

Table 4 summarizes the cosmological parameters in the base $\Lambda$CDM for Planck combined with various data sets discussed in this section. Although we have seen from the survey presented above that base $\Lambda$CDM is consistent with a wide range of cosmological data, there are two areas of tension:

1. the Ly$\alpha$ BAO measurements at high redshift (Sect. 5.2);
2. the Planck CMB estimate of the amplitude of the fluctuation spectrum and the lower values inferred from weak lensing, and (possibly) cluster counts and redshift space distortions (Sect. 5.5).

The first point to note is that the astrophysical data in areas (1) and (2) are complex and more difficult to interpret than most of the astrophysical data sets discussed in this section. The interpretation of the data in area (2) depends on nonlinear modelling of the power spectrum, and in the case of clusters and weak lensing, on uncertain baryonic physics. Understanding these effects more accurately sets a direction for future research.

It is, however, worth reviewing our findings on $\sigma_8$ and $\Omega_m$ from Planck assuming base $\Lambda$CDM. These are summarized in...
Fig. 19 and the following constraints:
\[ \sigma_8 = 0.829 \pm 0.014, \quad \text{Planck TT+lowP}; \]  
\[ \sigma_8 = 0.815 \pm 0.009, \quad \text{Planck TT+lowP+lensing}; \]  
\[ \sigma_8 = 0.810 \pm 0.006, \quad \text{Planck TT+lensing+\tau_e}. \]  
The last line imposes a Gaussian prior of \( \tau_e = 7 \pm 1 \) with a limit \( \tau_e > 6.5 \) on the reionization redshift in place of the reionization constraints from the lowP likelihood. As discussed in Sect. 3.4, such a low redshift of reionization is close to the lowest plausible value allowed by astrophysical data (though such low values are not favoured by either the WMAP or LFI polarization data). The addition of Planck lensing data pulls \( \sigma_8 \) down by about 1 \( \sigma \) from the Planck TT+lowP value, so Eq. (36c) is the lowest possible range allowed by the Planck CMB data. As shown in Fig. 19, adding the \( TE \) and \( EE \) spectra at high multipoles does not change the Planck constraints. If a convincing case can be made that astrophysical data conflict with the estimate of Eq. (36c), then this will be powerful evidence for new physics beyond base \( \Lambda \)CDM with minimal-mass neutrinos.

A number of authors have interpreted the discrepancies in area (2) as evidence for new physics in the neutrino sector (e.g., Planck Collaboration XX 2014; Hamann & Hasenkamp 2013; Battye & Moss 2014; Battye et al. 2015; Wyman et al. 2014; Beutler et al. 2014). They use various data combinations together with Planck to argue for massive neutrinos with mass \( \sum m_\nu \approx 0.3 \text{eV} \) or for a single sterile neutrino with somewhat higher mass. The problem here is that any evidence for new neutrino physics is driven mainly by the additional astrophysical data, not by Planck CMB anisotropy measurements. In addition, the external data sets are not entirely consistent, so tensions remain. As discussed in PCP13 (see also Leistedt et al. 2014; Battye et al. 2015) Planck data usually favour base \( \Lambda \)CDM over extended models. Implications of the Planck 2015 data for neutrino physics are discussed in Sect. 6.4 and tensions between Planck and external data in various extended neutrino models are discussed further in Sect. 6.4.4.

As mentioned above, we do not use RSD or galaxy weak lensing measurements for combined constraints in this paper (apart from Sects. 6.3 and 6.4.4, where we use the CFHTLenS data). They are, however, used in the paper exploring constraints on dark energy and modified gravity (Planck Collaboration XIV 2016). For some models discussed in that paper, the combination of Planck, RSD, and weak lensing data does prefer extensions to the base \( \Lambda \)CDM cosmology.

6. Extensions to the base \( \Lambda \)CDM model

6.1. Grid of models

The full grid results are available online.\textsuperscript{25} Figure 20 and Table 5 summarize the constraints on one-parameter extensions to base \( \Lambda \)CDM. As in PCP13, we find no strong evidence in favour of any of these simple one-parameter extensions using Planck or Planck combined with BAO. The entire grid has been run using both the PLik and CamSpec likelihoods. As noted in Sect. 3, the parameters derived from these two \( TT \) likelihoods agree to better than 0.5 \( \sigma \) for base \( \Lambda \)CDM. This level of agreement also holds for the extended models analysed in our grid. In Sect. 3 we also pointed out that we have definite evidence, by comparing spectra computed with different frequency combinations, of residual systematics in the \( TE \) and \( EE \) spectra. These systematics average down in the coadded \( TE \) and \( EE \) spectra, but the remaining level of systematics in these coadded spectra are not yet well quantified (though they are small). Thus, we urge the reader to treat parameters computed from the \( TT,TE,EE \) likelihoods with some caution. In the case of polarization, the agreement between the PLik and CamSpec \( TE \) and \( EE \) likelihoods is less good, with shifts in parameters of up to 1.5 \( \sigma \) (though such large shifts are unusual). In general, the behaviour of the \( TT,TE,EE \) likelihoods is as shown in Fig. 20. For extended models, the addition of the Planck polarization data at high multipoles reduces the errors on extended parameters compared to the Planck temperature data and pulls the parameters towards those of base \( \Lambda \)CDM. A similar behaviour is seen if the Planck TT (or Planck TT,TE,EE) data are combined with BAO.

The rest of this section discusses the grid results in more detail and also reports results on some additional models (specifically dark matter annihilation, tests of the recombination history, and cosmic defects) that are not included in our grid.

6.2. Early-Universe physics

Arguably the most important result from 2013 Planck analysis was the finding that simple single-field inflationary models, with a tilted scalar spectrum \( n_s \approx 0.96 \), provide a very good fit to the Planck data. We found no evidence for a tensor component or running of the scalar spectral index, no strong evidence for isocurvature perturbations or features in the primordial power spectrum (Planck Collaboration XXII 2014), and no evidence for non-Gaussianity (Planck Collaboration XXIV 2014), cosmic strings or other topological defects (Planck Collaboration XXV 2014). On large angular scales, the Planck data showed some evidence for “anomalies” seen previously in the WMAP data (Bennett et al. 2011). These include a dip in the power spectrum in the multipole range \( 20 \lesssim \ell \lesssim 30 \) (see Fig. 1) and some evidence for a departure from statistical isotropy on large angular scales (Planck Collaboration XXIII 2014). However, the statistical significance of these anomalies is not high enough to provide compelling evidence for new physics beyond simple single-field inflation.

The Planck 2013 results led to renewed interest in the \( R^2 \) inflationary model, originally introduced by Starobinsky (1980), and related inflationary models that have flat effective potentials of similar form (e.g., Kallosh & Linde 2013; Ferrara et al. 2013; Buchmüller et al. 2013; Ellis et al. 2013). A characteristic of these models is that they produce a red tilted scalar spectrum and a low tensor-to-scalar ratio. For reference, the Starobinsky model predicts

\[ n_s \approx 1 - \frac{2}{N} \in (0.960, 0.967), \]  
\[ r \approx \frac{12}{N^2} \in (0.003, 0.005), \]  
\[ \frac{dn_s}{d\ln k} \approx -\frac{2}{N^2} \in (-0.0008, -0.0006), \]  

where \( N \) is the number of e-foldings between the end of inflation and the time that our present day Hubble scale crossed the inflationary horizon, and numerical values are for the range \( 50 \lesssim N \lesssim 60 \).

Although the Planck 2013 results stimulated theoretical work on inflationary models with low tensor-to-scalar ratios, the cosmological landscape became more complicated following the detection of a \( B \)-mode polarization anisotropy by the BICEP2
Table 4. Parameter 68 % confidence limits for the base $\Lambda$CDM model from Planck CMB power spectra, in combination with lensing reconstruction (“lensing”) and external data (“ext”), BAO+JLA+$H_0$. While we see no evidence that systematic effects in polarization are biasing parameters in the base $\Lambda$CDM model, a conservative choice would be to use the parameter values listed in Column 3 (i.e., for $TT+\ell^2P+$lensing). Nuance parameters are not listed here for brevity, but can be found in the extensive tables on the Planck Legacy Archive, http://pla.esac.esa.int/pla; however, the last three parameters listed here give a summary measure of the total foreground amplitude (in $\mu K^2$) at $\ell = 2000$ for the three high-\ell temperature power spectra used by the likelihood. In all cases the helium mass fraction used is predicted by BBN from the baryon abundance (posterior mean $Y_B = 0.2453$, with theoretical uncertainties in the BBN predictions dominating over the Planck error on $\Omega_B h^2$). The Hubble constant is given in units of km s$^{-1}$ Mpc$^{-1}$, while $r_s$ is in Mpc and wavenumbers are in Mpc$^{-1}$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$TT+\ell^2P+$lensing</th>
<th>$TT+\ell^2P+$lensing + ext</th>
<th>$TT+\ell^2P+$lensing + ext</th>
<th>$TT+\ell^2P+$lensing + ext</th>
<th>$TT+\ell^2P+$lensing + ext</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega_k$</td>
<td>$0.2222 \pm 0.00003$</td>
<td>$0.2276 \pm 0.00023$</td>
<td>$0.2227 \pm 0.00020$</td>
<td>$0.2225 \pm 0.00016$</td>
<td>$0.2226 \pm 0.00016$</td>
</tr>
<tr>
<td>$\Omega_m^{\ell^2P}$</td>
<td>$0.1197 \pm 0.0022$</td>
<td>$0.1186 \pm 0.0020$</td>
<td>$0.1184 \pm 0.0012$</td>
<td>$0.1198 \pm 0.0015$</td>
<td>$0.1193 \pm 0.0014$</td>
</tr>
<tr>
<td>$H_0^{\ell^2P}$</td>
<td>$1.0480 \pm 0.00047$</td>
<td>$1.0410 \pm 0.00046$</td>
<td>$1.0410 \pm 0.00041$</td>
<td>$1.0407 \pm 0.00032$</td>
<td>$1.0407 \pm 0.00032$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>$0.078 \pm 0.019$</td>
<td>$0.066 \pm 0.016$</td>
<td>$0.067 \pm 0.013$</td>
<td>$0.079 \pm 0.017$</td>
<td>$0.063 \pm 0.014$</td>
</tr>
<tr>
<td>$\ln(10^{10}A_s)$</td>
<td>$3.089 \pm 0.036$</td>
<td>$3.062 \pm 0.029$</td>
<td>$3.064 \pm 0.024$</td>
<td>$3.094 \pm 0.034$</td>
<td>$3.059 \pm 0.025$</td>
</tr>
<tr>
<td>$n_s$</td>
<td>$0.9655 \pm 0.0062$</td>
<td>$0.9677 \pm 0.0060$</td>
<td>$0.9681 \pm 0.0044$</td>
<td>$0.9645 \pm 0.0049$</td>
<td>$0.9653 \pm 0.0048$</td>
</tr>
</tbody>
</table>

Table 5. Constraints on 1-parameter extensions to the base $\Lambda$CDM model for combinations of Planck power spectra, Planck lensing, and external data (BAO+JLA+$H_0$, denoted “ext”). All limits and confidence regions quoted here are 95 %.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$TT$</th>
<th>$TT+\ell^2P$</th>
<th>$TT+\ell^2P$ + ext</th>
<th>$TT+\ell^2P$ + ext</th>
<th>$TT+\ell^2P$ + ext</th>
<th>$TT+\ell^2P$ + ext</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega_k$</td>
<td>$-0.059^{+0.049}_{-0.055}$</td>
<td>$-0.005^{+0.016}_{-0.017}$</td>
<td>$-0.0001^{+0.005}_{-0.0052}$</td>
<td>$-0.040^{+0.013}_{-0.014}$</td>
<td>$-0.004^{+0.015}_{-0.015}$</td>
<td>$0.0008^{+0.040}_{-0.039}$</td>
</tr>
<tr>
<td>$\Sigma m_{n}\text{[eV]}$</td>
<td>$&lt; 0.715$</td>
<td>$&lt; 0.675$</td>
<td>$&lt; 0.234$</td>
<td>$&lt; 0.492$</td>
<td>$&lt; 0.589$</td>
<td>$&lt; 0.194$</td>
</tr>
<tr>
<td>$N_{\text{eff}}$</td>
<td>$3.13^{+0.06}_{-0.05}$</td>
<td>$3.13^{+0.06}_{-0.05}$</td>
<td>$3.15^{+0.04}_{-0.07}$</td>
<td>$2.99^{+0.11}_{-0.07}$</td>
<td>$2.94^{+0.13}_{-0.08}$</td>
<td>$3.04^{+0.07}_{-0.06}$</td>
</tr>
<tr>
<td>$Y_P$</td>
<td>$0.252^{+0.041}_{-0.042}$</td>
<td>$0.251^{+0.040}_{-0.039}$</td>
<td>$0.251^{+0.038}_{-0.037}$</td>
<td>$0.250^{+0.026}_{-0.027}$</td>
<td>$0.247^{+0.026}_{-0.027}$</td>
<td>$0.249^{+0.026}_{-0.027}$</td>
</tr>
<tr>
<td>$d_n/d\ln k$</td>
<td>$-0.008^{+0.016}_{-0.016}$</td>
<td>$-0.003^{+0.015}_{-0.015}$</td>
<td>$-0.003^{+0.015}_{-0.015}$</td>
<td>$-0.006^{+0.014}_{-0.014}$</td>
<td>$-0.007^{+0.013}_{-0.013}$</td>
<td>$-0.002^{+0.013}_{-0.013}$</td>
</tr>
<tr>
<td>$r_0$</td>
<td>$&lt; 0.103$</td>
<td>$&lt; 0.114$</td>
<td>$&lt; 0.114$</td>
<td>$&lt; 0.0987$</td>
<td>$&lt; 0.112$</td>
<td>$&lt; 0.113$</td>
</tr>
<tr>
<td>$w$</td>
<td>$-1.54^{+0.62}_{-0.60}$</td>
<td>$-1.41^{+0.64}_{-0.6}$</td>
<td>$-1.06^{+0.85}_{-0.91}$</td>
<td>$-1.58^{+0.58}_{-0.49}$</td>
<td>$-1.42^{+0.62}_{-0.56}$</td>
<td>$-1.019^{+0.075}_{-0.080}$</td>
</tr>
</tbody>
</table>
Fig. 20. 68 % and 95 % confidence regions on 1-parameter extensions of the base ΛCDM model for Planck TT+lowP (grey), Planck TT,TE,EE+lowP (red), and Planck TT,TE,EE+lowP+BAO (blue). Horizontal dashed lines correspond to the parameter values assumed in the base ΛCDM cosmology, while vertical dashed lines show the mean posterior values in the base model for Planck TT,TE,EE+lowP+BAO.
team (BICEP2 Collaboration 2014). If the BICEP2 signal were primarily caused by primordial gravitational waves, then the inferred tensor-to-scalar ratio would have been $r_{0.01} \approx 0.2$, apparently in conflict with the 2013 Planck 95% upper limit of $r_{0.02} < 0.11$, based on fits to the temperature power spectrum. Since the Planck constraints on $r$ are highly model dependent (and fixed mainly by lower $k$) it is possible to reconcile these results by introducing additional parameters, such as large tilts or strong running of the spectral indices.

The situation has been clarified following a joint analysis of BICEP2/Keck observations and Planck polarization data reported in BKP. This analysis shows that polarized dust emission contributes a significant part of the BICEP2 signal. Correcting for polarized dust emission, BKP report a 95% upper limit of $r_{0.05} < 0.12$ on scale-invariant tensor modes, eliminating the tension between the BICEP2 and the Planck 2013 results. There is therefore no evidence for inflationary tensor modes from B-mode polarization measurements at this time (although the BKP analysis leaves open the possibility of a much higher tensor-to-scalar ratio than the prediction of Eq. 37b for Starobinsky-type models).

The layout of the rest of this subsection is as follows. In Sect. 6.2.1 we review the Planck 2015 and Planck+BKP constraints on $n_s$ and $r$. Constraints on the running of the scalar spectral index are presented in Sect. 6.2.2. Polarization data provide a powerful way of testing for isocurvature modes, as discussed in Sect. 6.2.3. Finally, Sect. 6.2.4 summarizes our results on spatial curvature. A discussion of specific inflationary models and tests for features in the primordial power spectrum can be found in Planck Collaboration XX (2016).

6.2.1. Scalar spectral index and tensor fluctuations

Primordial tensor fluctuations (gravitational waves) contribute to both the CMB temperature and polarization power spectra. Gravitational waves entering the horizon between recombination and the present day generate a tensor contribution to the large-scale CMB temperature anisotropy. In this data release, the strongest constraint on tensor modes from Planck data still comes from the CMB temperature spectrum at $\ell \lesssim 100$. The corresponding comoving wavenumbers probed by the Planck temperature spectrum have $k \lesssim 0.008 \text{Mpc}^{-1}$, with very little sensitivity to higher wavenumbers because gravitational waves decay on sub-horizon scales. The precision of the Planck constraint is limited by cosmic variance of the large-scale anisotropies (which are dominated by the scalar component), and it is also model dependent. In polarization, in addition to $B$-modes, the $EE$ and $TE$ spectra also contain a signal from tensor modes coming from the last-scattering and reionization epochs. However, in this release the addition of Planck polarization constraints at $\ell \geq 30$ do not significantly change the results from temperature and low-$\ell$ polarization (see Table 5).

Figure 21 shows the 2015 Planck constraint in the $n_s$–$r$ plane, adding $r$ as a 1-parameter extension to base ΛCDM. For base ΛCDM ($r = 0$), the value of $n_s$ is

$$n_s = 0.9655 \pm 0.0062, \quad \text{Planck TT+lowP}.$$ (38)
We highlight this number here since $n_s$, a key parameter for inflationary cosmology, shows one of the largest shifts of any parameter in base $\Lambda$CDM between the Planck 2013 and Planck 2015 analyses (about 0.7 $\sigma$). As explained in Sect. 3.1, part of this shift was caused by the $\ell \approx 1800$ systematic in the nominal-mission $217 \times 217$ spectrum used in PCP13.

The red contours in Fig. 21 show the constraints from Planck TT+lowP. These are similar to the constraints shown in Fig. 23 of PCP13, but with $n_s$ shifted to slightly higher values. The addition of BAO or the Planck lensing data to Planck TT+lowP lowers the value of $\Omega_b h^2$, which, at fixed $\theta_s$, increases the small-scale CMB power. To maintain the fit to the Planck temperature power spectrum for models with $r = 0$, these parameter shifts are compensated by a change in the amplitude $A_s$ and the tilt $n_s$ (by about 0.4 $\sigma$). The increase in $n_s$ to match the observed power on small scales leads to a decrease in the scalar power on large scales, allowing room for a slightly larger contribution from tensor modes. The constraints shown by the blue contours in Fig. 21, which combine Planck lensing, BAO, and other astrophysical data, are therefore tighter in the $n_s$ direction and shifted to slightly higher values, but marginally weaker in the $r$-direction. The 95\% limits on $r_{0.02}$ are

$$r_{0.02} < 0.10, \text{ Planck TT+lowP},$$
$$r_{0.02} < 0.11, \text{ Planck TT+lowP+lensing+ext},$$

consistent with the results reported in PCP13. Here we assume the second-order slow-roll consistency relation for the tensor spectral index. The result in Eqs. (39a) and (39b) are mildly scale dependent, with equivalent limits on $r_{0.05}$ being weaker by about 5\%.

PCP13 noted a mismatch between the best-fit base $\Lambda$CDM model and the temperature power spectrum at multipoles $\ell \leq 40$, partly driven by the dip in the multipole range $20 \leq \ell \leq 30$. If this mismatch is simply a statistical fluctuation of the $\Lambda$CDM model (and there is no compelling evidence to think otherwise), the strong Planck limit (compared to forecasts) is the result of chance low levels of scalar mode confusion. On the other hand, if the dip represents a failure of the $\Lambda$CDM model, the 95\% limits of Eqs. (39a) and (39b) may be underestimates. These issues are considered at greater length in Planck Collaboration XX (2016) and will not be discussed further in this paper.

As mentioned above, the Planck temperature constraints on $r$ are model-dependent and extensions to $\Lambda$CDM can give significantly different results. For example, extra relativistic degrees of freedom increase the small-scale damping of the CMB anisotropies at a fixed angular scale, which can be compensated by increasing $n_s$, allowing a larger tensor mode. This is illustrated by the grey contours in Fig. 21, which show the constraints for a model with $\Delta N_{\text{eff}} = 0.39$. Although this value of $\Delta N_{\text{eff}}$ is disfavoured by the Planck data (see Sect. 6.4.1) it is not excluded at a high significance level.

This example emphasizes the need for direct tests of tensor modes based on measurements of a large-scale B-mode pattern in CMB polarization. Planck B-mode constraints from the 100- and 143-GHz HFI channels, presented in Planck Collaboration XI (2016), give a 95\% upper limit of $r \leq 0.27$. However, at present the tightest B-mode constraints on $r$ come from the BKP analysis of the BICEP2/Keck field, which covers approximately 400 $\text{deg}^2$ centred on RA = 0$^\circ$, Dec = $-57.5^\circ$. These measurements probe the peak of the B-mode power spectrum at around $\ell = 100$, corresponding to gravitational waves with $k \approx 0.01 \text{Mpc}^{-1}$ that enter the horizon during recombination (i.e., somewhat smaller than the scales that contribute to the Planck temperature constraints on $r$). The results of BKP give a posterior for $r$ that peaks at $r_{0.05} \approx 0.05$, but is consistent with $r_{0.05} \approx 0$. Thus, at present there is no convincing evidence of a primordial B-mode signal. At these low values of $r$, there is no longer any tension with Planck temperature constraints.

The analysis of BKP constrains $r$ defined relative to a fixed fiducial B-mode spectrum, and on its own does not give a useful constraint on either the scalar amplitude or $n_s$. A combined analysis of the Planck CMB spectra and the BKP likelihood can, self-consistently, give constraints in the $n_s$-$r$ plane, as shown in the right-hand panel of Fig. 21. The BKP likelihood pulls the contours to slightly non-zero values of $r$, with best fits of around $r_{0.02} \approx 0.03$, but at very low levels of statistical significance. The BKP likelihood also rules out the upper tail of $r$ values allowed by Planck alone. The joint Planck+BKP likelihood analyses give the 95\% upper limits

$$r_{0.02} < 0.08, \text{ Planck TT+lowP+BKP},$$
$$r_{0.02} < 0.09, \text{ Planck TT+lowP+lensing+ext+BKP}.$$

The exact values of these upper limits are weakly dependent on the details of the foreground modelling applied in the BKP analysis (see BKP for further details). The results given here are for the baseline 2-parameter model, varying the $B$-mode dust amplitude and frequency scaling, using the lowest five $B$-mode bandpowers.

Allowing a running of the scalar spectral index as an additional free parameter weakens the Planck constraints on $r_{0.02}$, as shown in Fig. 22. The coloured samples in Fig. 22 illustrate how
a negative running allows the large-scale scalar spectral index $n_{s,0.002}$ to shift towards higher values, lowering the scalar power on large scales relative to small scales, thereby allowing a larger tensor contribution. However adding the BKP likelihood, which directly constrains the tensor amplitude on smaller scales, significantly reduces the extent of this degeneracy leading to a 95% upper limit of $n_{r,0.002} < 0.10$ even in the presence of running (i.e., similar to the results of Eqs. 40a and 40b).

The Planck+BKP joint analysis rules out a quadratic inflationary potential ($V(\phi) \propto m^2 \phi^2$, predicting $r \approx 0.16$) at over 99% confidence and reduces the allowed range of the parameter space for models with convex potentials. Starobinsky-type models are an example of a wider class of inflationary theories in which $n_s - 1 = \mathcal{O}(1/N)$ is not a coincidence, yet $r = \mathcal{O}(1/N^2)$ (Roest 2014; Creminelli et al. 2015). These models have concave potentials, and include a variety of string-inspired models with exponential potentials. Models with $r = \mathcal{O}(1/N)$ are, however, still allowed by the data, including a simple linear potential and fractional-power monomials, as well as regions of parameter space in between where $n_s - 1 = \mathcal{O}(1/N)$ is just a coincidence.

Models that have sub-Planckian field evolution, so satisfying the Lyth bound (Lyth 1997; Garcia-Bellido et al. 2014), will typically have $r \lesssim 2 \times 10^{-3}$ for $n_s \approx 0.96$, and are also consistent with the tensor constraints shown in Fig. 21. For further discussion of the implications of the Planck 2015 data for a wide range of inflationary models see Planck Collaboration XX (2016).

In summary, the Planck limits on $r$ are consistent with the BKP limits from $B$-mode measurements. Both data sets are consistent with $r = 0$; however, the combined data sets yield an upper limit to the tensor-to-scalar ratio of $r = 0.09$ at the 95% level. The Planck temperature constraints on $r$ are limited by cosmic variance. The only way of improving these limits, or potentially detecting gravitational waves with $r < 0.09$, is through direct $B$-mode detection. The Planck 353-GHz polarization maps (Planck Collaboration Int. XXX 2016) show that at frequencies of around 150 GHz, Galactic dust emission is an important contaminant at the $r = 0.05$ level even in the cleanest regions of the sky. BKP demonstrates further that on small regions of the sky covering a few hundred square degrees (typical of ground based $B$-mode experiments), the Planck 353-GHz maps are of limited use as monitors of polarized Galactic dust emission because of their low signal-to-noise level. To achieve limits substantially below $r \approx 0.05$ will require observations of comparably high sensitivity over a range of frequencies, and with increased sky coverage. The forthcoming measurements from Keck Array and BICEP3 at 95 GHz and the Keck Array receivers at 220 GHz should offer significant improvements on the current constraints.

A number of other ground-based and sub-orbital experiments should also return high precision $B$-mode data within the next few years (see Abazajian et al. 2015a, for a review).

6.2.2. Scale dependence of primordial fluctuations

In simple single-field models of inflation, the running of the spectral index is of second order in inflationary slow-roll parameters and is typically small, $|dn_s/d\ln k| \approx (n_s - 1)^2 \approx 10^{-3}$ (Kosowsky & Turner 1995). Nevertheless, it is possible to construct models that produce a large running over a wavenumber range accessible to CMB experiments, whilst simultaneously achieving enough e-folds of inflation to solve the horizon problem. Inflation with an oscillatory potential of sufficiently long period, perhaps related to axion monodromy, is an example (Silverstein & Westphal 2008; Meerburg 2014; Czerny et al. 2014; Minor & Kaplinghat 2015).

As reviewed in PCP13, previous CMB experiments, either on their own or in combination with other astrophysical data, have sometimes given hints of a non-zero running at about the 2 $\sigma$ level (Spergel et al. 2003; Hinshaw et al. 2013; Hou et al. 2014). The results of PCP13 showed a slight preference for negative running at the 1.4 $\sigma$ level, driven almost entirely by the mismatch between the CMB temperature power spectrum at high multipoles and the spectrum at multipoles $\ell \lesssim 50$.

The 2015 Planck results (Fig. 23) are similar to those in PCP13. Adding running as an additional parameter to base $\Lambda$CDM with $r = 0$, we find

$$dn_s/d\ln k = -0.0084 \pm 0.0082, \quad \text{Planck TT+lowP}, \hspace{1cm} (41a)$$

$$dn_s/d\ln k = -0.0057 \pm 0.0071, \quad \text{Planck TT, TE, EE+lowP}. \hspace{1cm} (41b)$$

There is a slight preference for negative running, which, as in PCP13, is driven by the mismatch between the high and low multipoles in the temperature power spectrum. However, in the 2015 Planck data the tension between high and low multipoles is reduced somewhat, primarily because of changes to the HFI beams at multipoles $\ell \lesssim 200$ (see Sect. 3.1). A consequence of this reduced tension can be seen in the 2015 constraints on models that include tensor fluctuations in addition to running:

$$dn_s/d\ln k = -0.012^+0.009_{-0.008}, \quad \text{Planck TT+lowP}, \hspace{1cm} (42a)$$

$$dn_s/d\ln k = -0.0085 \pm 0.0076, \quad \text{Planck TT, TE, EE+lowP} \hspace{1cm} (42b)$$

$$dn_s/d\ln k = -0.0065 \pm 0.0076, \quad \text{Planck TT+lowP+lensing} \hspace{1cm} (42c)$$

PCP13 found an approximately 2 $\sigma$ pull towards negative running for these models. This tension is reduced to about 1 $\sigma$ with the 2015 Planck data, and to lower values when we include the BKP likelihood, which reduces the range of allowed tensor amplitudes.
In summary, the Planck data are consistent with zero running of the scalar spectral index. However, as illustrated in Fig. 23, the Planck data still allow running at roughly the $10^{-2}$ level, i.e., an order of magnitude higher than expected in simple inflationary models. One way of potentially improving these constraints is to extend the wavenumber range from CMB scales to smaller scales using additional astrophysical data, for example by using measurements of the Lyα flux power spectrum of high-redshift quasars (as in the first year WMAP analysis, Spergel et al. 2003). Palanque-Delabrouille et al. (2015) have recently reported an analysis of a large sample of quasar spectra from the SDSSIII/BOSS survey. These authors find a low value of the scalar spectral index $n_s = 0.928 \pm 0.012$ (stat.) $\pm (0.02)$ (syst.) on scales of $k \approx 1$ Mpc$^{-1}$. To extract physical parameters, the Lyα power spectra need to be calibrated against numerical hydrodynamical simulations. The large systematic error in this spectral index determination is dominated by the fidelity of the hydrodynamic simulations and by the splicing used to achieve high resolution over large scales. These uncertainties need to be reduced before addressing the consistency of Lyα results with CMB measurements of the running of the spectral index.

6.2.3. Isocurvature perturbations

A key prediction of single-field inflation is that the primordial perturbations are adiabatic. More generally, the observed fluctuations will be adiabatic in any model in which the curvature perturbations were the only super-horizon perturbations left by the time that dark matter (and other matter) first decoupled, or was produced by decay. The different matter components then all have perturbations proportional to the curvature perturbation, so there are no isocurvature perturbations. However, it is possible to produce an observable amount of isocurvature modes by having additional degrees of freedom present during inflation and through reheating. For example, the curvaton model can generate correlated adiabatic and isocurvature modes from a second field (Mollerach 1990; Lyth & Wands 2002).

Isocurvature modes describe relative perturbations between the different species (Bucher et al. 2001b), with perhaps the simplest being a perturbation in the baryonic or dark matter sector (relative to the radiation). However, only one total matter isocurvature mode is observable in the linear CMB (in the accurate approximation in which the baryons are pressureless); a compensated mode (between the baryons and the cold dark matter) with $\delta \rho_b = -\delta \rho_\nu$ has no net density perturbation, and produces no CMB anisotropies (Gordon & Lewis 2003; Grin et al. 2011; Grin et al. 2014). It is possible to generate isocurvature modes in the neutrino sector; however, this requires interaction of an additional perturbed super-horizon field with neutrinos after they have decoupled, and hence is harder to achieve. Finally, neutrino velocity potential and vorticity modes are other possible consistent perturbations to the photon-neutrino fluid after neutrino decoupling. However, they are essentially impossible to excite, since they consist of photon and neutrino fluids coherently moving in opposite directions on super-horizon scales (although the relative velocity would have been zero before neutrino decoupling).

Planck Collaboration XXII (2014) presented constraints on a variety of general isocurvature models using the Planck temperature data, finding consistency with adiabaticity, though with some mild preference for isocurvature models that reduce the power at low multipoles to provide a better match to the Planck temperature spectrum at $\ell \lesssim 50$. For matter isocurvature perturbations, the photons are initially unperturbed but perturbations develop as the Universe becomes more matter dominated. As a result, the phase of the acoustic oscillations differs from adiabatic modes; this is most clearly distinctive with the addition of polarization data (Bucher et al. 2001a).

An extended analysis of isocurvature models is given in Planck Collaboration XX (2016). Here we focus on a simple illustrative case of a totally-correlated matter isocurvature mode. We define an isocurvature amplitude parameter $\alpha$, such that\(^\text{27}\)

\[ S_m = \text{sgn}(\alpha) \sqrt{\frac{|\alpha|}{1 - |\alpha|}} \zeta, \quad (43) \]

where $\zeta$ is the primordial curvature perturbation. Here $S_m$ is the total matter isocurvature mode, defined as the observable sum of the baryon and CDM isocurvature modes, i.e., $S_m = S_c + S_b(\rho_\gamma/\rho_\gamma)$, where

\[ S_i \equiv \frac{\delta \rho_i}{\rho_i} - \frac{3\delta \rho_\gamma}{4\rho_\gamma}. \quad (44) \]

All modes are assumed to have a power spectrum with the same spectral index $n_s$, so that $\alpha$ is independent of scale. For positive $\alpha$ this agrees with the definitions in Bean et al. (2006) and Larson et al. (2011) for $\alpha_+1$, but also allows for the correlation to have the opposite sign. Approximately, $\text{sgn}(\alpha) \alpha^2 \approx B_c$, where $B_c$ is the CDM version of the amplitude defined as in Amendola et al. (2002). Note that in our conventions, negative values of a lower the Sachs–Wolfe contribution to the large-scale TT power spectrum. We caution the reader that this convention differs from some others, e.g., Larson et al. (2011).

\(^{27}\)Planck Collaboration XX (2016) gives equivalent one-tailed constraints on $\beta_{iso} = |\alpha|$, where the correlated and anti-correlated cases are considered separately.
Fig. 25. Power spectra drawn from the Planck TT+lowP posterior for the correlated matter isocurvature model, colour-coded by the value of the isocurvature amplitude parameter $\alpha$, compared to the Planck data points. The left-hand figure shows how the negatively-correlated modes lower the large-scale temperature spectrum, slightly improving the fit at low multipoles. Including polarization, the negatively-correlated modes are disfavoured, as illustrated at the first acoustic peak in EE on the right-hand plot. Data points at $\ell < 30$ are not shown for polarization, as they are included with both the default temperature (i.e., TT+lowP) and polarization (i.e., TT,TE,EE+lowP) likelihood combinations.

Planck constraints on the correlated isocurvature amplitude are shown in Fig. 24, with and without high-multipole polarization. The corresponding marginalized limit from the temperature data is

$$\alpha = -0.0025^{+0.0035}_{-0.0047} \; (95\%, \text{Planck TT+lowP}),$$

which is significantly tightened around zero when Planck polarization information is included at high multipoles:

$$\alpha = 0.0003^{+0.0016}_{-0.0012} \; (95\%, \text{Planck TT,TE,EE+lowP}).$$

This strongly limits the isocurvature contribution to be less than about 3% of the adiabatic modes. Figure 25 shows how models with negative correlation parameter, $\alpha$, fit the temperature data at low multipoles slightly better than models with $\alpha = 0$; however, these models are disfavoured from the corresponding change in the polarization acoustic peaks.

In this model most of the gain in sensitivity comes from relatively large scales, $\ell \lesssim 300$, where the correlated isocurvature modes with delayed phase change the first polarization acoustic peak ($\ell \approx 140$) significantly more than in temperature (Bucher et al. 2001a). The polarization data are not entirely robust to systematics on these scales, but in this case the result appears to be quite stable between the different likelihood codes. However, it should be noted that a particularly low point in the TE spectrum at $\ell \approx 160$ (see Fig. 3) pulls in the direction of positive $\alpha$, and could be giving an artificially strong constraint if this were caused by an unidentified systematic.

6.2.4. Curvature

The simplifying assumptions of large-scale homogeneity and isotropy lead to the familiar Friedman-Lemaître-Robertson-Walker (FLRW) metric that appears to be an accurate description of our Universe. The base $\Lambda$CDM cosmology assumes an FLRW metric with a flat 3-space. This is a very restrictive assumption that needs to be tested empirically. In this subsection, we investigate constraints on the parameter $\Omega_K$, where for $\Lambda$CDM models $\Omega_K \equiv 1 - \Omega_m - \Omega_\Lambda$. For FLRW models $\Omega_K > 0$ corresponds to negatively-curved 3-geometries while $\Omega_K < 0$ corresponds to positively-curved 3-geometries. Even with perfect data within our past lightcone, our inference of the curvature $\Omega_K$ is limited by the cosmic variance of curvature perturbations that are still super-horizon at the present, since these cannot be distinguished from background curvature within our observable volume.

Fig. 26. Constraints in the $\Omega_m$-$\Omega_\Lambda$ plane from the Planck TT+lowP data (samples; colour-coded by the value of $H_0$) and Planck TT,TE,EE+lowP (solid contours). The geometric degeneracy between $\Omega_m$ and $\Omega_\Lambda$ is partially broken because of the effect of lensing on the temperature and polarization power spectra. These limits are improved significantly by the inclusion of the Planck lensing reconstruction (blue contours) and BAO (solid red contours). The red contours tightly constrain the geometry of our Universe to be nearly flat.
The parameter $\Omega_K$ decreases exponentially with time during inflation, but grows only as a power law during the radiation and matter-dominated phases, so the standard inflationary prediction has been that curvature should be unobservably small today. Nevertheless, by fine-tuning parameters it is possible to devise inflationary models that generate open (e.g., Bucher et al. 1995; Linde 1999) or closed universes (e.g., Linde 2003). Even more speculatively, there has been interest recently in multiverse models, in which topologically-open “pocket universes” form by bubble nucleation (e.g., Coleman & De Luccia 1980; Gott 1982) between different vacua of a “string landscape” (e.g., Freivogel et al. 2006; Bouso & et al. 2015). Clearly, the detection of a significant deviation from $\Omega_K = 0$ would have profound consequences for inflation theory and fundamental physics.

The Planck power spectra give the constraint

$$\Omega_K = -0.052^{+0.049}_{-0.055} \quad (95\%, \text{Planck TT+lowP}).$$

(47)

The well-known geometric degeneracy (Bond et al. 1997; Zaldarriaga et al. 1997) allows for the small-scale linear CMB spectrum to remain almost unchanged if changes in $\Omega_K$ are compensated by changes in $H_0$ to obtain the same angular diameter distance to last scattering. The Planck constraint is therefore mainly determined by the (wide) priors on $H_0$, and the effect of lensing smoothing on the power spectra. As discussed in Sect. 5.1, the Planck temperature power spectra show a slight preference for more lensing than expected in the base $\Lambda$CDM cosmology, and since positive curvature increases the amplitude of the lensing signal, this preference also drives $\Omega_K$ towards negative values.

Taken at face value, Eq. (47) represents a detection of positive curvature at just over 2 $\sigma$, largely via the impact of lensing on the power spectra. One might wonder whether this is mainly a parameter volume effect, but that is not the case, since the best fit closed model has $\Delta\chi^2 \approx 6$ relative to base $\Lambda$CDM, and the fit is improved over almost all the posterior volume, with the mean improvement being $\langle \Delta\chi^2 \rangle \approx 5$ (very similar to the phenomenological case of $\Lambda$CDM+$k$). Addition of the Planck polarization spectra shifts $\Omega_K$ towards zero by $\Delta\Omega_K \approx 0.015$:

$$\Omega_K = -0.040^{+0.038}_{-0.041} \quad (95\%, \text{Planck TT,TE,EE+lowP}),$$

(48)

but $\Omega_K$ remains negative at just over 2 $\sigma$.

What’s more, the lensing reconstruction from Planck measures the amplitude directly and, as discussed in Sect. 5.1, this does not prefer more lensing than base $\Lambda$CDM. The combined constraint shows impressive consistency with a flat universe:

$$\Omega_K = -0.005^{+0.016}_{-0.017} \quad (95\%, \text{Planck TT+lowP+lensing}).$$

(49)

The dramatic improvement in the error bar is another illustration of the power of the lensing reconstruction. The Planck constraint can be sharpened further by adding external data that break the main geometric degeneracy. Combining the Planck data with BAO, we find

$$\Omega_K = 0.000 \pm 0.005 \quad (95\%, \text{Planck TT+lowP+lensing+BAO}).$$

(50)

This constraint is unchanged at the quoted precision if we add the JLA supernovae data and the $H_0$ prior of Eq. (30).

Figure 26 illustrates these results in the $\Omega_m$-$\Omega_k$ plane. We adopt Eq. (50) as our most reliable constraint on spatial curvature. Our Universe appears to be spatially flat to a 1 $\sigma$ accuracy of 0.25 $\%$.

6.3. Dark energy

The physical explanation for the observed accelerated expansion of the Universe is currently not known. In standard $\Lambda$CDM the acceleration is provided by a cosmological constant, i.e., an additional fluid satisfying an equation of state $w \equiv p/\rho = -1$. However, there are many possible alternatives, typically described either in terms of extra degrees of freedom associated with scalar fields or modifications of general relativity on cosmological scales (for reviews see, e.g.,Copeland et al. 2006; Tsujikawa 2010). A detailed study of these models and the constraints imposed by Planck and other data are presented in a separate paper, Planck Collaboration XIV (2016).

Here we will limit ourselves to the most basic extensions of $\Lambda$CDM, which can be phenomenologically described in terms of the equation of state parameter $w$ alone. Specifically we will use the camb implementation of the “parameterized post-Friedmann” (PPF) framework of Hu & Sawicki (2007) andFang et al. (2008) to test whether there is any evidence that $w$ varies with time. This framework aims to recover the behaviour of canonical (i.e., those with a standard kinetic term) scalar field cosmologies minimally coupled to gravity when $w \leq -1$, and accurately approximates them for values $w \approx -1$. In these models the speed of sound is equal to the speed of light, so that the clustering of the dark energy inside the horizon is strongly suppressed. The advantage of using the PPF formalism is that it is possible to study the phantom domain, $w < -1$, including transitions across the “phantom barrier,” $w = -1$, which is not possible for canonical scalar fields.

The CMB temperature data alone do not tightly constrain $w$, because of a strong geometrical degeneracy, even for spatially-flat models. From Planck we find

$$w = -1.54^{+0.62}_{-0.50} \quad (95\%, \text{Planck TT+lowP}),$$

(51)

i.e., almost a 2 $\sigma$ shift into the phantom domain. This is partly, but not entirely, a parameter volume effect, with the average effective $\chi^2$ improving by $\langle \Delta\chi^2 \rangle \approx 2$ compared to base $\Lambda$CDM.
This is consistent with the preference for a higher lensing amplitude discussed in Sect. 5.1.2, improving the fit in the $w < -1$ region, where the lensing smoothing amplitude becomes slightly larger. However, the lower limit in Eq. (51) is largely determined by the (arbitrary) prior $H_0 < 100 \text{ km s}^{-1}\text{Mpc}^{-1}$, chosen for the Hubble parameter. Much of the posterior volume in the phantom region is associated with extreme values for cosmological parameters, which are excluded by other astrophysical data. The mild tension with base ΛCDM disappears as we add more data that break the geometrical degeneracy. Adding Planck lensing and BAO, JLA and $H_0$ ("ext") gives the 95 % constraints

$$w = -1.023^{+0.091}_{-0.096} \quad \text{(Planck TT+lowP+ext)},$$

$$w = -1.006^{+0.085}_{-0.091} \quad \text{(Planck TT+lowP+lensing+ext)},$$

$$w = -1.019^{+0.075}_{-0.080} \quad \text{(Planck TT, TE, EE+lowP+lensing+ext)}.$$  

The addition of Planck lensing, or using the full Planck temperature+polarization likelihood together with the BAO, JLA, and $H_0$ data does not substantially improve the constraint of Eq. (52a). All of these data set combinations are compatible with the base ΛCDM value of $w = -1$. In PCP13, we conservatively quoted $w = -1.13^{+0.24}_{-0.25}$, based on combining Planck with BAO, as our most reliable limit on $w$. The errors in Eqs. (52a)–(52c) are substantially smaller, mainly because of the addition of the JLA SNe data, which offer a sensitive probe of the dark energy equation of state at $z \leq 1$. In PCP13, the addition of the SNLS SNe data pulled $w$ into the phantom domain at the 2σ level, reflecting the tension between the SNLS sample and the Planck 2013 base ΛCDM parameters. As noted in Sect. 5.3, this discrepancy is no longer present, following improved photometric calibrations of the SNe data in the JLA sample. One consequence of this is the tightening of the errors in Eqs. (52a)–(52c) around the ΛCDM value $w = -1$ when we combine the JLA sample with Planck.

If $w$ differs from $-1$, it is likely to change with time. We consider here the case of a Taylor expansion of $w$ at first order in the scale factor, parameterized by

$$w = w_0 + (1 - a)w_a.$$  

More complex models of dynamical dark energy are discussed in Planck Collaboration XIV (2016). Figure 27 shows the 2D marginalized posterior distribution for $w_0$ and $w_a$ for the combination Planck+BAO+JLA. The JLA SNe data are again crucial in breaking the geometrical degeneracy at low redshift and with these data we find no evidence for a departure from the base ΛCDM cosmology. The points in Fig. 27 show samples from these chains colour-coded by the value of $H_0$. From these MCMC chains, we find $H_0 = (68.2 \pm 1.1) \text{ km s}^{-1}\text{Mpc}^{-1}$. Much higher values of $H_0$ would favour the phantom regime, $w < -1$.

As pointed out in Sects. 5.5.2 and 5.6 the CFHTLenS weak lensing data are in tension with the Planck base ΛCDM parameters. Examples of this tension can be seen in investigations of dark energy and modified gravity, since some of these models can modify the growth rate of fluctuations from the base ΛCDM predictions. This tension can be seen even in the simple model of Eq. (53). The green regions in Fig. 28 show 68 % and 95 % contours in the $w_0$–$w_a$ plane for Planck TT+lowP combined with the CFHTLenS H13 data. In this example, we have applied ultra-conservative cuts, excluding $\xi_\ell$ entirely and excluding measurements with $\theta < 17^\prime$ in $\xi_\ell$ for all tomographic redshift bins. As discussed in Planck Collaboration XIV (2016), with these cuts the CFHTLenS data are insensitive to modelling the nonlinear evolution of the power spectrum, but this reduction in sensitivity comes at the expense of reducing the statistical power of the weak lensing data. Nevertheless, Fig. 28 shows that the combination of Planck+CFHTLenS pulls the contours into the phantom domain and is discrepant with base ΛCDM at the 2σ level. The Planck+CFHTLenS data also favour a high value of $H_0$. If we add the (relatively weak) $H_0$ prior of Eq. (30), the contours (shown in cyan) in Fig. 28 shift towards $w = -1$. It therefore seems unlikely that the tension between Planck and CFHTLenS can be resolved by allowing a time-variable equation of state for dark energy.

A much more extensive investigation of models of dark energy and also models of modified gravity can be found in Planck Collaboration XIV (2016). The main conclusions of that analysis are as follows:

- an investigation of more general time-variations of the equation of state shows a high degree of consistency with $w = -1$;
- a study of several dark energy and modified gravity models either finds compatibility with base ΛCDM, or mild tensions, which are driven mainly by external data sets.

6.4. Neutrino physics and constraints on relativistic components

In the following subsections, we update Planck constraints on the mass of standard (active) neutrinos, additional relativistic degrees of freedom, models with a combination of the two, and models with massive sterile neutrinos. In each subsection we emphasize the Planck-only constraint, and the implications of the Planck result for late-time cosmological parameters measured from other observations. We then give a brief discussion of tensions between Planck and some discordant external data, and assess whether any of these model extensions can help to resolve them. Finally we provide constraints on neutrino interactions.
6.4.1. Constraints on the total mass of active neutrinos

Detection of neutrino oscillations has proved that neutrinos have mass (see, e.g., Lesgourgues & Pastor 2006 and Nakamura & Petcov 2014 for reviews). The Planck base ΛCDM model assumes a normal mass hierarchy with $\sum m_\nu \approx 0.06$ eV (dominated by the heaviest neutrino mass eigenstate) but there are other possibilities, including a degenerate hierarchy with $\sum m_\nu \approx 0.1$ eV. At this time there are no compelling theoretical reasons to strongly prefer any of these possibilities, so allowing for larger neutrino masses is perhaps one of the most well-motivated extensions to base ΛCDM considered in this paper. There has also been significant interest recently in larger neutrino masses as a possible way to lower Λ motivated extensions to base CDM considered in this paper.

Here we give constraints assuming three species of degenerate massive neutrinos, neglecting the small differences in mass expected from the observed mass splittings. At the level of sensitivity of Planck this is an accurate approximation, but note that it does not quite match continuously on to the base ΛCDM model (which assumes two massless and one massive neutrino with $\sum m_\nu = 0.06$ eV). We assume that the neutrino mass is constant, and that the distribution function is Fermi-Dirac with zero chemical potential.

Masses well below 1 eV have only a mild effect on the shape of the CMB power spectra, since they became non-relativistic after recombination. The effect on the background cosmology can be compensated by changes in $H_0$, to ensure the same observed acoustic peak scale $\theta_*$, There is, however, some sensitivity of the CMB anisotropies to neutrino masses as the neutrinos start to become less relativistic at recombination (modifying the early ISW effect), and from the late-time effect of lensing on the power spectrum. The Planck power spectrum (95% constraints are

$$\sum m_\nu < 0.72 \text{ eV} \quad \text{Planck TT+lowP},$$

$$\sum m_\nu < 0.21 \text{ eV} \quad \text{Planck TT+lowP+BAO},$$

$$\sum m_\nu < 0.49 \text{ eV} \quad \text{Planck TT, TE, EE+lowP},$$

$$\sum m_\nu < 0.17 \text{ eV} \quad \text{Planck TT, TE, EE+lowP+BAO}.$$ (54d)

The Planck TT+lowP constraint has a broad tail to high masses, as shown in Fig. 29, which also illustrates the acoustic scale degeneracy with $H_0$. Larger masses imply a lower $\sigma_8$ through the effects of neutrino free-streaming on structure formation, but the larger masses also require a lower Hubble constant, leading to possible tensions with direct measurements of $H_0$. Masses below about 0.4 eV can provide an acceptable fit to the direct $H_0$ measurements, and adding the BAO data helps to break the acoustic scale degeneracy and tightens the constraint on $\sum m_\nu$ substantially. Adding Planck polarization data at high multipoles produces a relatively small improvement to the Planck TT+lowP+BAO constraint (and the improvement is even smaller with the alternative CamSpec likelihood), so we consider the TT results to be our most reliable constraints.

The constraint of Eq. (54b) is consistent with the 95% limit of $\sum m_\nu < 0.23$ eV reported in PCP13 for Planck+BAO. The limits are similar because the linear CMB is insensitive to the mass of neutrinos that are relativistic at recombination. There is little to be gained from improved measurement of the CMB temperature power spectra, though improved external data can help to break the geometric degeneracy to higher precision. CMB lensing can also provide additional information at lower redshifts, and future high-resolution CMB polarization measurements that accurately reconstruct the lensing potential can probe much smaller masses (see, e.g., Abazajian et al. 2015b).

As discussed in detail in PCP13 and Sect. 5.1, the Planck CMB power spectra prefer somewhat more lensing smoothing than predicted in ΛCDM (allowing the lensing amplitude to vary gives $A_\ell > 1$ at just over 2σ). The neutrino mass constraint from the power spectra is therefore quite tight, since increasing the neutrino mass lowers the predicted smoothing even further compared to base ΛCDM. On the other hand the lensing reconstruction data, which directly probes the lensing power, prefers lensing amplitudes slightly below (but consistent with) the base ΛCDM prediction (Eq. 18). The Planck+lensing constraint therefore pulls the constraints slightly away from zero towards higher neutrino masses, as shown in Fig. 30. Although the posterior has less weight at zero, the lensing data are incompatible with very large neutrino masses so the Planck+lensing 95% limit is actually tighter than the Planck TT+lowP result:

$$\sum m_\nu < 0.68 \text{ eV} \quad (95\%, \text{Planck TT+lowP+lensing}).$$ (55)

Adding the polarization spectra improves this constraint slightly to

$$\sum m_\nu < 0.59 \text{ eV} \quad (95\%, \text{Planck TT,TE,EE+lowP+lensing}).$$ (56)

We take the combined constraint that further includes BAO, JLA, and $H_0$ (“ext”) as our best limit:

$$\sum m_\nu < 0.23 \text{ eV} \quad \Omega_\Lambda h^2 < 0.0025 \quad (95\%, \text{Planck TT+lowP+lensing+ext}).$$ (57)
Planck Collaboration: Cosmological parameters

Fig. 30. Constraints on the sum of the neutrino masses for various data combinations.

This is slightly weaker than the constraint from Planck TT,TE,EE+lowP+lensing+BAO (which is tighter in both the CamSpec and Plik likelihoods), but is immune to low level systems that might affect the constraints from the Planck polarization spectra. Equation (57) is therefore a conservative limit. Marginalizing over the range of neutrino masses, the Planck constraints on the late-time parameters are\(^{28}\)

\[
\begin{align*}
H_0 &= 67.7 \pm 0.6 \\
\sigma_8 &= 0.810^{+0.015}_{-0.012}
\end{align*}
\]

Planck TT+lowP+lensing+ext. (58)

For this restricted range of neutrino masses, the impact on the other cosmological parameters is small and, in particular, low values of \(\sigma_8\) will remain in tension with the parameter space preferred by Planck.

The constraint of Eq. (57) is weaker than the constraint of Eq. (54b) excluding lensing, but there is no good reason to disregard the Planck lensing information while retaining other astrophysical data. The CMB lensing signal probes very-nearly linear scales and passes many consistency checks over the multipole range used in the Planck lensing likelihood (see Sect. 5.1 and Planck Collaboration XV 2016). The situation with galaxy weak lensing is rather different, as discussed in Sect. 5.5.2. In addition to possible observational systematics, the weak lensing data probe lower redshifts than CMB lensing, and smaller spatial scales, where uncertainties in modelling nonlinearities in the matter power spectrum and baryonic feedback become important (Harnois-Dérap et al. 2015).

A larger range of neutrino masses was found by Beutler et al. (2014) using a combination of RSD, BAO, and weak lensing information. The tension between the RSD results and base \(\Lambda\)CDM was subsequently reduced following the analysis of Samushia et al. (2014), as shown in Fig. 17. Galaxy weak lensing and some cluster constraints remain in tension with base \(\Lambda\)CDM, and we discuss possible neutrino resolutions of these problems in Sect. 6.4.4.

---

\(^{28}\)To simplify the displayed equations, \(H_0\) is given in units of \(\text{km} \, \text{s}^{-1} \, \text{Mpc}^{-1}\) in this section.

Fig. 31. Samples from Planck TT+lowP chains in the \(N_{\text{eff}}-H_0\) plane, colour-coded by \(\sigma_8\). The grey bands show the constraint \(H_0 = (70.6 \pm 3.3) \, \text{km} \, \text{s}^{-1} \, \text{Mpc}^{-1}\) of Eq. (30). Notice that higher \(N_{\text{eff}}\) brings \(H_0\) into better consistency with direct measurements, but increases \(\sigma_8\). Solid black contours show the constraints from Planck TT,TE,EE+lowP+BAO. Models with \(N_{\text{eff}} < 3.046\) (left of the solid vertical line) require photon heating after neutrino decoupling or incomplete thermalization. Dashed vertical lines correspond to specific fully-thermalized particle models, for example one additional massless boson that decoupled around the same time as the neutrinos \((\Delta N_{\text{eff}} \approx 0.57)\), or before muon annihilation \((\Delta N_{\text{eff}} \approx 0.39)\), or an additional sterile neutrino that decoupled around the same time as the active neutrinos \((\Delta N_{\text{eff}} \approx 1)\).

Another way of potentially improving neutrino mass constraints is to use measurements of the Ly\(\alpha\) flux power spectrum of high-redshift quasars. Palanque-Delabrouille et al. (2015) have recently reported an analysis of a large sample of quasar spectra from the SDSSIII/BOSS survey. When combining their results with 2013 Planck data, these authors find a bound \(\sum m_\nu < 0.15 \, \text{eV}\) (95% CL), compatible with the results presented in this section.

An exciting future prospect is the possible direct detection of non-relativistic cosmic neutrinos by capture on tritium, for example with the PTOLEMY experiment (Cocco et al. 2007; Betts et al. 2013; Long et al. 2014). Unfortunately, for the mass range \(\sum m_\nu < 0.23 \, \text{eV}\) preferred by Planck, detection with the first generation experiment will be extremely difficult.

6.4.2. Constraints on \(N_{\text{eff}}\)

Dark radiation density in the early Universe is usually parameterized by \(N_{\text{eff}}\), defined so that the total relativistic energy density in neutrinos and any other dark radiation is given in terms of the photon density \(\rho_T\) at \(T \ll 1\, \text{MeV}\) by

\[
\rho = N_{\text{eff}} \left( \frac{4}{11} \right)^{4/3} \rho_T.
\]

The numerical factors in this equation are included so that \(N_{\text{eff}} = 3\) for three standard model neutrinos that were thermalized in the early Universe and decoupled well before electron-positron annihilation. The standard cosmological prediction is
actually \( N_{\text{eff}} = 3.046 \), since neutrinos are not completely decoupled at electron-positron annihilation and are subsequently slightly heated (Mangano et al. 2002).

In this section we focus on additional energy density from massless particles. In addition to massless sterile neutrinos, a variety of other particles could contribute to \( N_{\text{eff}} \). We assume that the additional massless particles are produced well before recombination, and neither interact nor decay, so that their energy density scales with the expansion exactly like massless neutrinos. An additional \( \Delta N_{\text{eff}} = 1 \) could correspond to a fully thermalized sterile neutrino that decoupled at \( T \lesssim 100 \text{ MeV} \); for example, any sterile neutrino with mixing angles large enough to provide a potential resolution to short-baseline reactor neutrino oscillation anomalies would most likely thermalize rapidly in the early Universe. However, this solution to the neutrino oscillation anomalies requires approximately 1-eV sterile neutrinos, rather than the massless case considered in this section; exploration of anomalies requires approximately 1-eV sterile neutrinos, rather than the massless case considered in this section; exploration of the two parameters \( N_{\text{eff}} \) and \( \sum m_\nu \) is reported in Sect. 6.4.3. For a review of sterile neutrinos see Abazajian et al. (2012).

More generally the additional radiation does not need to be fully thermalized, for example there are many possible models of non-thermal radiation production via particle decays (see, e.g., Hasenkamp & Kensten 2013; Conlon & Marsh 2013). The radiation could also be produced at temperatures \( T > 100 \text{ MeV} \), in which case typically \( \Delta N_{\text{eff}} < 1 \) for each additional species, since heating by photon production at muon annihilation (corresponding to \( T \approx 100 \text{ MeV} \)) decreases the fractional importance of the additional component at the later times relevant for the CMB. For particles produced at \( T \gg 100 \text{ MeV} \) the density would be diluted even more by numerous phase transitions and particle annihilations, and give \( \Delta N_{\text{eff}} \ll 1 \). Furthermore, if the particle is not fermionic, the factors entering the entropy conservation equation are different, and even thermalized particles could give specific fractional values of \( \Delta N_{\text{eff}} \). For example Weinberg (2013) considers the case of a thermalized massless boson, which contributes \( \Delta N_{\text{eff}} = 4/7 \approx 0.57 \) if it decouples in the range \( 0.5 \text{ MeV} < T < 100 \text{ MeV} \) like the neutrinos, or \( \Delta N_{\text{eff}} = 0.39 \) if it decouples at \( T > 100 \text{ MeV} \) (before the photon production at muon annihilation, hence undergoing fractional dilution).

In this paper we follow the usual phenomenological approach, where one constrains \( N_{\text{eff}} \) as a free parameter with a wide flat prior, although we comment on a few discrete cases separately below. Values of \( N_{\text{eff}} < 3.046 \) are less well motivated, since they would require the standard neutrinos to be incompletely thermalized or additional photon production after neutrino decoupling, but we include this range for completeness.

Figure 31 shows that \textit{Planck} is entirely consistent with the standard value \( N_{\text{eff}} = 3.046 \). However, a significant density of additional radiation is still allowed, with the (68\%) constraints

\[
\begin{align*}
N_{\text{eff}} &= 3.13 \pm 0.32 \quad \text{Planck TT+lowP,} & (60a) \\
N_{\text{eff}} &= 3.15 \pm 0.23 \quad \text{Planck TT+lowP+BAO,} & (60b) \\
N_{\text{eff}} &= 2.99 \pm 0.20 \quad \text{Planck TT, TE, EE+lowP,} & (60c) \\
N_{\text{eff}} &= 3.04 \pm 0.18 \quad \text{Planck TT, TE, EE+lowP+BAO.} & (60d)
\end{align*}
\]

Notice the significantly tighter constraint with the inclusion of \textit{Planck} high-\( l \) polarization, with \( \Delta N_{\text{eff}} < 1 \) at over 4\( \sigma \) from \textit{Planck} alone. This constraint is not very stable between likelihoods, with the CamSpec likelihood giving a roughly 0.8\( \sigma \) lower value of \( N_{\text{eff}} \). However, the strong limit from polarization is also consistent with the joint \textit{Planck} TT+lowP+BAO result, so Eq. (60b) leads to the robust conclusion that \( \Delta N_{\text{eff}} < 1 \) at over 3\( \sigma \). The addition of \textit{Planck} lensing has very little effect on this constraint.

For \( N_{\text{eff}} > 3 \), the \textit{Planck} data favour higher values of the Hubble parameter than the \textit{Planck} base \( \Lambda \)CDM value, which as discussed in Sect. 5.4 may be in better agreement with some direct measurements of \( H_0 \). This is because \textit{Planck} accurately measures the acoustic scale \( r_s/\Delta\chi \); increasing \( N_{\text{eff}} \) means (via the Friedmann equation) that the early Universe expands faster, so the sound horizon at recombination, \( r_s \), is smaller and hence recombination has to be closer (larger \( H_0 \) and hence smaller \( \Delta\chi \)) for it to subdue the same angular size observed by \textit{Planck}. However, models with \( N_{\text{eff}} > 3 \) and a higher Hubble constant also have higher values of the fluctuation amplitude \( \sigma_8 \), as shown by the coloured samples in Fig. 31. As a result, these models increase the tensions between the CMB measurements and astrophysical measurements of \( \sigma_8 \) discussed in Sect. 5.6. It therefore seems unlikely that additional radiation alone can help to resolve tensions with large-scale structure data.

The energy density in the early Universe can also be probed by the predictions of BBN. In particular \( \Delta N_{\text{eff}} > 0 \) increases the primordial expansion rate, leading to earlier freeze-out, with a higher neutron density and hence a greater abundance of helium and deuterium after BBN has completed. A detailed discussion of the implications of \textit{Planck} for BBN is given in Sect. 6.5. Observations of both the primordial helium and deuterium abundance are compatible with the predictions of standard BBN for the \textit{Planck} base \( \Lambda \)CDM value of the baryon density. The \textit{Planck}+BBN constraints on \( N_{\text{eff}} \) (Eqs. 75 and 76) are compatible, and slightly tighter than Eq. (60b).

Although there is a large continuous range of plausible \( N_{\text{eff}} \) values, it is worth mentioning briefly a few of the discrete values from fully thermalized models. This serves as an indication of how strongly \textit{Planck} prefers base \( \Lambda \)CDM, and also how the inferred values of other cosmological parameters might be affected by this particular extension to base \( \Lambda \)CDM. As discussed above, one fully thermalized neutrino \( (\Delta N_{\text{eff}} \approx 1) \) is ruled out at over 3\( \sigma \), and is disfavoured by \( \Delta\chi^2 \approx 8 \) compared to base \( \Lambda \)CDM by \textit{Planck} TT+lowP, and much more strongly in combination with \textit{Planck} high-\( l \) polarization or BAO data. The thermalized boson models that give \( \Delta N_{\text{eff}} = 0.39 \) or \( \Delta N_{\text{eff}} = 0.57 \) are disfavoured by \( \Delta\chi^2 \approx 1.5 \) and \( \Delta\chi^2 \approx 3 \), respectively, and are therefore not strongly excluded. We focus on the former, since it is also consistent with the \textit{Planck} TT+lowP+BAO constraint at 2\( \sigma \). As shown in Fig. 31, larger \( N_{\text{eff}} \) corresponds to a region of parameter space with significantly higher Hubble parameter,

\[ H_0 = 70.6 \pm 1.0 \quad (68\%, \text{Planck TT+lowP}; \Delta N_{\text{eff}} = 0.39). \quad (61) \]

This can be compared to the direct measurements of \( H_0 \) discussed in Sect. 5.4. Evidently, Eq. (61) is consistent with the \( H_0 \) prior adopted in this paper (Eq. 30), but this example shows that an accurate direct measurement of \( H_0 \) can potentially provide evidence for new physics beyond that probed by \textit{Planck}. As shown in Fig. 31, the \( \Delta N_{\text{eff}} = 0.39 \) cosmology also has a significantly higher small-scale fluctuation amplitude and the spectral index \( n_s \) is also bluer, with

\[
\begin{align*}
\sigma_8 &= 0.850 \pm 0.015 & (62a) \\
n_s &= 0.983 \pm 0.006 & (62b)
\end{align*}
\]

The \( \sigma_8 \) range in this model is higher than preferred by the \textit{Planck} lensing likelihood in base \( \Lambda \)CDM. However, the fit to the \textit{Planck} lensing likelihood is model dependent and the lensing degeneracy direction also associates high \( H_0 \) and low \( \Omega_m \) values with higher \( \sigma_8 \). The joint \textit{Planck} TT+lowP+lensing constraint does
Fig. 32. Samples from Planck TT+lowP in the $N_{\text{eff}}$-$m_{\nu, \text{sterile}}$ plane, colour-coded by $\sigma_8$, for models with one massive sterile neutrino family, with effective mass $m_{\nu, \text{sterile}}^\text{eff}$, and the three active neutrinos as in the base $\Lambda$CDM model. The physical mass of the sterile neutrino in the thermal scenario, $m_{\nu, \text{sterile}}$, is constant along the grey dashed lines, with the indicated mass in eV; the grey shading shows the region excluded by our prior $m_{\nu, \text{sterile}} < 10$ eV, which cuts out most of the area where the neutrinos behave nearly like dark matter. The physical mass in the Dodelson-Widrow scenario, $m_{\nu, \text{sterile}}^\text{DW}$, is constant along the dotted lines (with the value indicated on the adjacent dashed lines).

Pull $\sigma_8$ down slightly to $\sigma_8 = 0.84 \pm 0.01$ and provides an acceptable fit to the Planck data. For Planck TT+lowP+lensing, the difference in $\chi^2$ between the best fit base $\Lambda$CDM model and the extension with $\Delta N_{\text{eff}} = 0.39$ is only $\Delta \chi^2_{\text{CMB}} \approx 2$. The higher spectral index with $\Delta N_{\text{eff}} = 0.39$ gives a decrease in large-scale power, fitting the low $\ell < 30$ Planck TT spectrum better by $\Delta \chi^2 \approx 1$, but at the same time the high-$\ell$ data prefer $\Delta N_{\text{eff}} \approx 0$. Correlations with other cosmological parameters can be seen in Fig. 20. Clearly, a very effective way of testing these models would be to obtain reliable, accurate, astrophysical measurements of $H_0$ and $\sigma_8$.

In summary, models with $\Delta N_{\text{eff}} = 1$ are disfavoured by Planck combined with BAO data at about the $3 \sigma$ level. Models with fractional changes of $\Delta N_{\text{eff}} \approx 0.39$ are mildly disfavoured by Planck, but require higher $H_0$ and $\sigma_8$ compared to base $\Lambda$CDM.

### 6.4.3. Simultaneous constraints on $N_{\text{eff}}$ and neutrino mass

As discussed in the previous sections, neither a higher neutrino mass nor additional radiation density alone can resolve all of the tensions between Planck and other cosmophysical data. However, the presence of additional massive particles, such as massive sterile neutrinos, could potentially improve the situation by introducing enough freedom to allow higher values of the Hubble constant and lower values of $\sigma_8$. As mentioned in Sect. 6.4.2, massive sterile neutrinos offer a possible solution to reactor neutrino oscillation anomalies (Kopp et al. 2013; Giunti et al. 2013) and this has led to significant recent interest in this class of models (Hamann & Hasenkamp 2013; Wyman et al. 2014; Battye & Moss 2014; Leistert et al. 2014; Bergström et al. 2014; MacCrann et al. 2015). Alternatively, active neutrinos could have significant degenerate masses above the minimal baseline value together with additional massless particles contributing to $N_{\text{eff}}$. Many more complicated scenarios could also be envisaged.

In the case of massless radiation density, the cosmological predictions are independent of the actual form of the distribution function, since all particles travel at the speed of light. However, for massive particles the results are more model dependent. To formulate a well-defined model, we follow PCP13 and consider the case of one massive sterile neutrino parameterized by $m_{\nu, \text{sterile}}^\text{eff} = (94.1 \Delta \Omega_{\nu, \text{sterile}} h^2)$ eV, in addition to the two approximately massless and one massive neutrino of the base-line model. For thermally-distributed sterile neutrinos, $m_{\nu, \text{sterile}}^\text{eff}$ is related to the true mass via

$$m_{\nu, \text{sterile}}^\text{eff} = (T_v/T_r)^3 m_{\nu, \text{sterile}}^\text{thermal} = (\Delta N_{\text{eff}})^{3/4} m_{\nu, \text{sterile}}^\text{thermal}^\text{eff},$$

and for the cosmologically-equivalent Dodelson-Widrow (DW) case (Dodelson & Widrow 1994) the relation is given by

$$m_{\nu, \text{sterile}}^\text{eff} = \chi_s m_{\nu, \text{sterile}}^\text{DW},$$

with $\Delta N_{\text{eff}} = \chi_s$. We impose a prior on the physical thermal mass, $m_{\nu, \text{sterile}}^\text{thermal} < 10$ eV, when generating parameter chains, to exclude regions of parameter space in which the particles are so massive that their effect on the CMB spectra is identical to that of cold dark matter. Although we consider only the specific case of one massive sterile neutrino with a thermal (or DW) distribution, our constraints will be reasonably accurate for other models, for example eV-mass particles produced as non-thermal decay products (Hasenkamp 2014).

Figure 32 shows that although Planck is perfectly consistent with no massive sterile neutrinos, a significant region of parameter space with fractional $\Delta N_{\text{eff}}$ is allowed, where $\sigma_8$ is lower than in the base $\Lambda$CDM model. This is also the case for massless sterile neutrinos combined with massive active neutrinos. In the single massive sterile model, the combined constraints are

$$\begin{align*}
N_{\text{eff}} < 3.7 & \quad m_{\nu, \text{sterile}}^\text{eff} < 0.52 \text{ eV} \\
& \quad 95\%, \text{ Planck TT+lowP+lensing+BAO.}
\end{align*}$$

The upper tail of $m_{\nu, \text{sterile}}^\text{eff}$ is largely associated with high physical masses near to the prior cutoff; if instead we restrict to the region where $m_{\nu, \text{sterile}}^\text{thermal} < 2$ eV the constraint is

$$\begin{align*}
N_{\text{eff}} < 3.7 & \quad m_{\nu, \text{sterile}}^\text{eff} < 0.38 \text{ eV} \\
& \quad 95\%, \text{ Planck TT+lowP+lensing+BAO.}
\end{align*}$$

Massive sterile neutrinos with mixing angles large enough to help resolve the reactor anomalies would typically imply full thermalization in the early Universe, and hence give $\Delta N_{\text{eff}} = 1$ for each additional species. Such a high value of $N_{\text{eff}}$, especially combined with $m_{\nu, \text{sterile}} = 1$ eV, as required by reactor anomaly solutions, were virtually ruled out by previous cosmological data (Mirizzi et al. 2013; Archidiacono et al. 2013a; Gariazzo et al. 2013). This conclusion is strengthened by the analysis presented here, since $N_{\text{eff}} = 4$ is excluded at greater than 99% confidence. Unfortunately, there does not appear to be a consistent resolution to the reactor anomalies, unless thermalization of the massive neutrinos can be suppressed, for example, by large lepton asymmetry, new interactions, or particle decay (see Gariazzo et al. 2014; Bergström et al. 2014, and references therein).
6.4.4. Neutrino models and tension with external data

The extended models discussed in this section allow \textit{Planck} to be consistent with a wider range of late-Universe parameters than in base ΛCDM. Figure 33 summarizes the constraints on \( \Omega_m, \sigma_8, \) and \( H_0 \) for the various models that we have considered. The inferred Hubble parameter can increase or decrease, as required to maintain the observed acoustic scale, depending on the relative contribution of additional radiation (changing the sound horizon) and neutrino mass (changing mainly the angular diameter distance). However, all of the models follow similar degeneracy directions in the \( \Omega_m-\sigma_8 \) and \( H_0-\sigma_8 \) planes, so these models remain predictive: \textit{large common areas of the parameter space are excluded in all of these models.} The two-parameter extensions are required to fit substantially lower values of \( \sigma_8 \) without also decreasing \( H_0 \) below the values determined from direct measurements, but the scope for doing this is clearly limited.

External data sets need to be reanalysed consistently in extended models, since the extensions change the growth of structure, angular distances, and the matter-radiation equality scale. For example, the dashed lines in Fig. 33 show how different models affect the CFHTLenS galaxy weak lensing constraints from Heymans et al. (2013) (see Sect. 5.5.2), when restricted to the region of parameter space consistent with the \textit{Planck} acoustic scale \( \delta_{\text{ac}} \) (fixed to the \textit{Planck} TT+lowP ΛCDM best fit) combined with BAO and the Hubble constant measurement of Eq. (30). These additional constraints break large parameter degeneracies in the weak lensing likelihood that would otherwise obscure the comparison with the \textit{Planck} contours. Here priors on other parameters applied to the CFHTLenS analysis are as described in Sect. 5.5.2.

We have also considered the case of additional radiation and degenerate massive active neutrinos, with the combined constraint

\[
N_{\text{eff}} = 3.2 \pm 0.5 \\
\sum m_\nu < 0.32 \text{ eV}
\]

95%, \textit{Planck} TT+lowP+lensing+BAO. \hfill (67)

Again \textit{Planck} shows no evidence for a deviation from the base ΛCDM model.

\[\text{Fig. 33. 68\% and 95\% constraints from \textit{Planck} TT+lowP (green), \textit{Planck} TT+lowP+lensing (grey), and \textit{Planck} TT+lowP+lensing+BAO (red) on the late-Universe parameters \( H_0, \sigma_8, \) and \( \Omega_m \) in various neutrino extensions of the base ΛCDM model. The blue contours show the base ΛCDM constraints from \textit{Planck} TT+lowP+lensing+BAO. The dashed cyan contours show joint constraints from the H13 CFHTLenS galaxy weak lensing likelihood (with angular cuts as in Fig. 18) at constant CMB acoustic scale measurements and the local Hubble parameter. The filled green, grey, and red contours in Fig. 33 show the CMB constraints on these models for various data combinations. The tightest of these constraints comes from the \textit{Planck} TT+lowP+lensing+BAO combination. The blue contours show the constraints in the base ΛCDM cosmology. The red contours are broader than the blue contours and there is greater overlap with the CFHTLenS contours, but this offers only a marginal improvement compared to base ΛCDM (compare with Fig. 18; see also the discussions in Leistedt et al. 2014 and Battye et al. 2015). For each of these models, the CFHTLenS results prefer lower values of \( \sigma_8 \). Allowing for a higher neutrino mass lowers \( \sigma_8 \) from \textit{Planck}, but does not help alleviate the discrepancy with the CFHTLenS data, since the \textit{Planck} data prefer a lower value of \( H_0 \). A joint analysis of the CFHTLenS likelihood with \textit{Planck} TT+lowP shows a \( \Delta \chi^2 < 1 \) preference for the extended neutrino models compared to base ΛCDM, and the fits to \textit{Planck} TT+lowP are worse in all cases. In base ΛCDM the CFHTLenS data prefer a region of parameter space \( \Delta \chi^2 \approx 4 \) away from the \textit{Planck} TT+lowP+CFHTLenS joint fit, indicative of the tension between the data sets. This is only slightly relieved to \( \Delta \chi^2 \approx 3 \) in the extended models.

In summary, modifications to the neutrino sector alone cannot easily explain the discrepancies between \textit{Planck} and other astrophysical data described in Sect. 5.5, including the inclusion of a low value of \( \sigma_8 \) from rich cluster counts.\]
6.4.5. Testing perturbations in the neutrino background

As shown in the previous sections, the Planck data provide evidence for a cosmic neutrino background at a very high significance level. Neutrinos affect the CMB anisotropies at the background level, by changing the expansion rate before recombination and hence relevant quantities such as the sound horizon and the damping scales. Neutrinos also affect the CMB anisotropies via their perturbations. Perturbations in the neutrino background are coupled through gravity to the perturbations in the photon background, and can be described (for massless neutrinos) by the following set of equations (Hu 1998; Hu et al. 1999; Trotta & Melchiorri 2005; Archidiacono et al. 2011):

\[
\delta \nu = \frac{\dot{a}}{a} \left( 1 - 3 \epsilon_{\text{eff}} \right) \left[ \delta \nu + 3 \frac{\dot{\delta}}{a} + \frac{2}{3k} \frac{\dot{a}}{a} \right] - k \frac{q}{q_\nu} + \frac{2}{3k} h ;
\]

\[
\dot{q}_\nu = k c_{\text{eff}}^2 \left( \delta \nu + 3 \frac{\dot{\delta}}{a} + \frac{2}{3} \frac{\dot{a}}{a} \right) - \frac{a}{a} q_\nu - \frac{2}{3} k \pi_\nu ;
\]

\[
\pi_\nu = 3k c_{\text{vis}}^2 \left( \frac{2}{5} q_\nu + \frac{4}{15k} (h + 6\eta) \right) - \frac{3}{5} k F_{\nu,3} ;
\]

\[
F_{\nu,\ell} = \frac{k}{2\ell + 1} \left( 3F_{\nu,\ell-1} - (\ell + 1) F_{\nu,\ell+1} \right), \quad (\ell \geq 3).
\]

Here dots denote derivatives with respect to conformal time, \(\delta \nu\) is the neutrino density contrast, \(q_\nu\) is the neutrino velocity perturbation, \(\pi_\nu\) the anisotropic stress, \(F_{\nu,\ell}\) are higher-order moments of the neutrino distribution function, and \(h\) and \(\eta\) are the scalar metric perturbations in the synchronous gauge. In these equations, \(c_{\text{eff}}^2\) is the neutrino sound speed in its own reference frame and \(c_{\text{vis}}^2\) parameterizes the anisotropic stress. For standard non-interacting massless neutrinos \(c_{\text{eff}}^2 = c_{\text{vis}}^2 = 1/3\). Any deviation from the expected values could provide a hint of non-standard physics in the neutrino sector.

A greater (lower) neutrino sound speed would increase (decrease) the neutrino pressure, leading to a lower (higher) perturbation amplitude. On the other hand, changing \(c_{\text{vis}}^2\) alters the viscosity of the neutrino fluid. For \(c_{\text{vis}}^2 = 0\), the neutrinos act as a perfect fluid, supporting undamped acoustic oscillations.

Several previous studies have used this approach to constrain \(c_{\text{eff}}^2\) and \(c_{\text{vis}}^2\) using cosmological data (see, e.g., Trotta & Melchiorri 2005; Smith et al. 2012; Archidiacono et al. 2013b; Gerbino et al. 2013; Audren et al. 2015), with the motivation that deviations from the expected values could be a hint of non-standard physics in the neutrino sector. Non-standard interactions could involve, for example, neutrino coupling with light scalar particles (Hannestad 2005; Beacom et al. 2004; Bell 2005; Sawyer 2006). If neutrinos are strongly coupled at recombination, this would result in a lower value for \(c_{\text{vis}}^2\) than in the standard model. Alternatively, the presence of early dark energy that mimics a relativistic component at recombination could possibly lead to a value for \(c_{\text{eff}}^2\) that differs from 1/3 (see, e.g., Calabrese et al. 2011).

In this analysis, for simplicity, we assume \(N_\nu = 3.046\) and massless neutrinos. By using an equivalent parameterization for massive neutrinos (Audren et al. 2015) we have checked that assuming one massive neutrino with \(M_{\nu} \approx 0.06\,eV\), as in the base model used throughout this paper, has no impact on the constraints on \(c_{\text{eff}}^2\) and \(c_{\text{vis}}^2\) reported in this section.\(^{29}\) We adopt a flat prior between zero and unity for both \(c_{\text{vis}}^2\) and \(c_{\text{eff}}^2\).

\(^{29}\)We also do not explore extended cosmologies in this section, since no significant degeneracies are expected between \((\sum m_\nu, N_{\text{eff}}, w, dn_\nu/d\ln k)\) and \((c_{\text{eff}}, c_{\text{vis}}^2)\) (Audren et al. 2015).

The top and bottom panels of Fig. 34 show the posterior distributions of \(c_{\text{eff}}^2\) and \(c_{\text{vis}}^2\) from Planck TT+lowP, Planck TT+lowP+BAO, Planck TT,TE,EE+lowP, and Planck TT,TE,EE+lowP+BAO. The mean values and 68 % errors on \(c_{\text{eff}}^2\) and \(c_{\text{vis}}^2\) are

\[
\begin{align*}
\langle c_{\text{eff}}^2 \rangle &= 0.312 \pm 0.011, & \text{Planck TT+lowP,} \\
\langle c_{\text{vis}}^2 \rangle &= 0.47 \pm 0.26 - 0.12. & \text{(69a)}
\end{align*}
\]

\[
\begin{align*}
\langle c_{\text{eff}}^2 \rangle &= 0.316 \pm 0.010, & \text{Planck TT+lowP+BAO,} \\
\langle c_{\text{vis}}^2 \rangle &= 0.44 \pm 0.15 - 0.10. & \text{(69b)}
\end{align*}
\]

\[
\begin{align*}
\langle c_{\text{eff}}^2 \rangle &= 0.3240 \pm 0.0060, & \text{Planck TT,TE,EE+lowP,} \\
\langle c_{\text{vis}}^2 \rangle &= 0.327 \pm 0.037. & \text{(69c)}
\end{align*}
\]

\[
\begin{align*}
\langle c_{\text{eff}}^2 \rangle &= 0.3242 \pm 0.0059, & \text{Planck TT,TE,EE+lowP+BAO,} \\
\langle c_{\text{vis}}^2 \rangle &= 0.331 \pm 0.037. & \text{(69d)}
\end{align*}
\]
Constraints on these parameters are consistent with the conventional values $c_{vis}^2 = c_{vis}^3 = 1/3$. A vanishing value of $c_{vis}^2$, which might imply a strong interaction between neutrinos and other species, is excluded at more than the 95% level arising from the Planck temperature data. This conclusion is greatly strengthened (to about 9σ) when Planck polarization data are included. As discussed in Bashinsky & Seljak (2004), neutrino anisotropic stresses introduce a phase shift in the CMB angular power spectra, which is more visible in polarization than temperature because of the sharper acoustic peaks. This explains why we see such a dramatic reduction in the error on $c_{vis}^2$ when including polarization data.

The precision of our results is consistent with the forecasts discussed in Smith et al. (2012), and we find strong evidence, purely from CMB observations, for neutrino anisotropies with the standard values $c_{vis}^2 = 1/3$ and $c_{eff}^2 = 1/3$.

6.5. Primordial nucleosynthesis

6.5.1. Details of analysis approach

Standard big bang nucleosynthesis (BBN) predicts light element abundances as a function of parameters relevant to the CMB, such as the baryon-to-photon density ratio $n_b \equiv m_b/n_b$, the radiation density parameterized by $N_{eff}$, and the chemical potential of the electron neutrinos. In PCP13, we presented consistency checks between the Planck 2013 results, light element abundance data, and standard BBN. The goal of Sect. 6.5.2 below is to update these results and to provide improved tests of the standard BBN model. In Sect. 6.5.3 we show how Planck data can be used to constrain nuclear reaction rates, and in Sect. 6.5.4 we will present the most stringent CMB bounds to date on the primordial helium fraction.

For simplicity, our analysis assumes a negligible leptonic asymmetry in the electron neutrino sector. For a fixed photon temperature today (which we take to be $T_0 = 2.7255 K$), $n_b$ can be related to $\omega_b \equiv \Omega_b h^2$, up to a small (and negligible) uncertainty associated with the primordial helium fraction. Standard BBN then predicts the abundance of each light element as a function of only two parameters, $\omega_b$ and $N_{eff} \equiv N_{eff} - 3.046$, with a theoretical error coming mainly from uncertainties in the neutron lifetime and a few nuclear reaction rates.

We will confine our discussion to BBN predictions for the primordial abundances of $^4$He and deuterium, expressed respectively as

$$\frac{Y_P}{Y_{PBBN}} = 4n_{He}/n_H$$

and

$$\frac{Y_{DP}}{Y_{DPBBN}} = Y_{DP}(\omega_b, \Delta N_{eff})$$

and $Y_{DP}(\omega_b, \Delta N_{eff})$ are given by Taylor expansions obtained with the $ParthEliope$ code (Pisanti et al. 2008), similar to the ones presented in Iocco et al. (2009), but updated by the $ParthEliope$ team with the latest observational data on nuclear rates and on the neutron lifetime:

$$Y_{PBBN}^0 = 0.2311 + 0.9502\omega_b - 11.27\omega_b^2$$

+ $\Delta N_{eff}(0.01356 + 0.00858\omega_b - 0.1810\omega_b^2)$

+ $\Delta N_{eff}(-0.0009795 - 0.001370\omega_b + 0.017462\omega_b^2)$ ;

(70)

$$Y_{DP} = 18.754 - 1534.40\omega_b + 4865.6\omega_b^2 - 552670\omega_b^3$$

+ $\Delta N_{eff}(2.4671 - 208.11\omega_b + 6760.9\omega_b^2 - 78007\omega_b^3)$

+ $\Delta N_{eff}(0.012907 - 1.3653\omega_b + 37.388\omega_b^2 - 267.78\omega_b^3)$.

(71)

By averaging over several measurements, the Particle Data Group 2014 (Olive et al. 2014) estimates the neutron life-time to be $\tau_n = (880.3 \pm 1.1) s$ at 68% CL. The expansions in Eqs. (70) and (71) are based on this central value, and we assume that Eq. (70) predicts the correct helium fraction up to a standard error $\sigma_{Y_{PBBN}} = 0.0003$, obtained by propagating the error on $\tau_n$.

The uncertainty on the deuterium fraction is dominated by that on the rate of the reaction $d(p, \gamma)^4He$. For that rate, in PCP13 we relied on the result of Serpico et al. (2004), obtained by fitting several experiments. The expansions of Eqs. (70) and (71) now adopt the latest experimental determination by Adelberger et al. (2011) and use the best-fit expression in their

---

However, the most recent individual measurement by Yue et al. (2013) gives $\tau_n = [887.8 \pm 2.2 (stat.) \pm 1.9 (syst.)] s$, which is discrepant at 3.5σ with the previous average (including only statistical errors). Hence one should bear in mind that systematic effects could be underestimated in the Particle Data Group result. Adopting the central value of Yue et al. (2013) would shift our results by a small amount (by a factor of 1.0062 for $Y_P$ and 1.0036 for $Y_{DP}$).

---

\*\*BBN calculations usually refer to nucleon number density fractions rather than mass fractions. To avoid any ambiguity with the helium mass fraction $Y_P$, normally used in CMB physics, we use superscripts to distinguish between the two definitions $Y_P^{CMB}$ and $Y_P^{BBN}$. Typically, $Y_P^{BBN}$ is about 0.5% higher than $Y_P^{CMB}$.
equation (29). We also rely on the uncertainty quoted in Adelberger et al. (2011) and propagate it to the deuterium fraction. This gives a standard error \( \sigma_{\text{DP}} = 0.06 \), which is more conservative than the error adopted in PCP13.

6.5.2. Primordial abundances from Planck data and standard BBN

We first investigate the consistency of standard BBN and the CMB by fixing the radiation density to its standard value, i.e., \( N_{\text{eff}} = 3.046 \), based on the assumption of standard neutrino decoupling and no extra light relics. We can then use Planck data to measure \( \omega_b \), assuming base \( \Lambda \)CDM, and test for consistency with experimental abundance measurements. The 95\% CL bounds obtained for the base \( \Lambda \)CDM model for various data combinations are

\[
\omega_b = \begin{cases} 
0.02226^{+0.00210}_{-0.00210} & \text{Planck TT,TE,EE+lowP,} \\
0.02226^{+0.00209}_{-0.00209} & \text{Planck TT+lowP,} \\
0.02225^{+0.00132}_{-0.00132} & \text{Planck TT+lowP+BAO,} \\
0.02225^{+0.00130}_{-0.00130} & \text{Planck TT+TE,EE+lowP,} \\
0.02225^{+0.00129}_{-0.00129} & \text{Planck TT+TE,EE+lowP+BAO,} \\
\end{cases}
\]

(72)

corresponding to a predicted primordial \(^4\)He number density fraction (95\% CL) of

\[
\begin{align*}
\Y_{\text{BBN}} &= 0.2466^{+0.00200}_{-0.00199} \times 0.0063 \quad \text{Planck TT+lowP,} \\
&= 0.2466^{+0.00198}_{-0.00198} \times 0.0063 \quad \text{Planck TT+lowP+BAO,} \\
&= 0.2466^{+0.00194}_{-0.00194} \times 0.0062 \quad \text{Planck TT+TE,EE+lowP,} \\
&= 0.2466^{+0.00193}_{-0.00193} \times 0.0061 \quad \text{Planck TT+TE,EE+lowP+BAO,}
\end{align*}
\]

(73)

and deuterium fraction (95\% CL)

\[
\begin{align*}
\Y_{\text{DP}} &= 2.626^{+0.083}_{-0.085} \times 0.15 \quad \text{Planck TT+lowP,} \\
&= 2.614^{+0.075}_{-0.074} \times 0.14 \quad \text{Planck TT+lowP+BAO,} \\
&= 2.614^{+0.057}_{-0.060} \times 0.13 \quad \text{Planck TT+TE,EE+lowP,} \\
&= 2.606^{+0.051}_{-0.054} \times 0.13 \quad \text{Planck TT+TE,EE+lowP+BAO.}
\end{align*}
\]

(74)

The first set of error bars (in parentheses) in Eqs. (73) and (74) reflect only the uncertainty on \( \omega_b \). The second set includes the theoretical uncertainty on the BBN predictions, added in quadrature to the errors from \( \omega_b \). The total errors in the predicted helium abundances are dominated by the BBN uncertainty, as in PCP13. For deuterium, the Planck 2015 results improve the determination of \( \omega_b \) to the point where the theoretical errors are comparable or larger than the errors from the CMB. In other words, for base \( \Lambda \)CDM the predicted abundances cannot be improved substantially by further measurements of the CMB. This also means that Planck results can, in principle, be used to investigate nuclear reaction rates that dominate the theoretical uncertainty (see Sect. 6.5.3).

The results of Eqs. (73) and (74) are well within the ranges indicated by the latest measurements of primordial abundances, as illustrated in Fig. 35. The helium data compilation of Aver et al. (2013) gives \( \Y_{\text{BBN}} = 0.2465 \pm 0.0097 \) (68\% CL), and the Planck prediction is near the middle of this range.\(^{32}\) As summarized by Aver et al. (2013) and Peimbert (2008), helium abundance measurements derived from emission lines in low-metallicity H\( \alpha \) regions are notoriously difficult and prone to systematic errors. As a result, many discrepant helium abundance measurements can be found in the literature. Izotov et al. (2014) have reported \( \Y_{\text{BBN}} = 0.2551 \pm 0.0022 \), which is discrepant with the base \( \Lambda \)CDM predictions by 3.4\( \sigma \). Such a high helium fraction could be accommodated by increasing \( N_{\text{eff}} \) (see Fig. 36 and Sect. 6.5.4); however, at present it is not clear whether the error quoted by Izotov et al. (2014) accurately reflects systematic uncertainties, including in particular the error in extrapolating to zero metallicity.

Historically, deuterium abundance measurements have shown excess scatter over that expected from statistical errors, indicating the presence of systematic uncertainties in the observations. Figure 35 shows the data compilation of Iocco et al. (2009), \( \Y_{\text{DP}} = 2.87 \pm 0.22 \) (68\% CL), which includes measurements based on damped Ly\( \alpha \) and Lyman limit systems. We also show the more recent results by Cooke et al. (2014) (see also Pettini & Cooke 2012) based on their observations of low-metallicity damped Ly\( \alpha \) absorption systems in two quasars (SDSS J1358+6522, \( z_{\text{abs}} = 3.0672 \) and SDSS J1419+0829, \( z_{\text{abs}} = 3.04973 \)) and a reanalysis of archival spectra of damped Ly\( \alpha \) systems in three further quasars that satisfy strict selection criteria. The Cooke et al. (2014) analysis gives \( \Y_{\text{DP}} = 2.53 \pm 0.04 \) (68\% CL), somewhat lower than the central Iocco et al. (2009) value, and with a much smaller error. The Cooke et al. (2014) value is almost certainly the more reliable measurement, as evidenced by the consistency of the deuterium abundances of the five systems in their analysis. The Planck base \( \Lambda \)CDM predictions of Eq. (74) lie within 1\( \sigma \) of the Cooke et al. (2014) result. This is a remarkable success for the standard theory of BBN.

\(^{32}\)A substantial part of this error comes from the regression to zero metallicity. The mean of the 17 measurements analysed by Aver et al. (2013) is \( \Y_{\text{BBN}} = 0.2535 \pm 0.0036 \), i.e., about 1.7\( \sigma \) higher than the Planck predictions of Eq. (73).

Fig. 36. Constraints in the \( \omega_b-N_{\text{eff}} \) plane from Planck and Planck+BAO data (68\% and 95\% contours) compared to the predictions of BBN, given primordial element abundance measurements. We show the 68\% and 95\% confidence regions derived from \(^4\)He bounds compiled by Aver et al. (2013) and from deuterium bounds compiled by Cooke et al. (2014). In the CMB analysis, \( N_{\text{eff}} \) is allowed to vary as an additional parameter to base \( \Lambda \)CDM, with \( \Y_P \) fixed as a function of \( \omega_b \) and \( N_{\text{eff}} \), according to BBN predictions. These constraints assume no significant lepton asymmetry.
It is worth noting that the Planck data are so accurate that $\omega_b$ is insensitive to the underlying cosmological model. In our grid of extensions to base $\Lambda$CDM the largest degradation of the error in $\omega_b$ is in models that allow $N_{\text{eff}}$ to vary. In these models, the mean value of $\omega_b$ is almost identical to that for base $\Lambda$CDM, but the error on $\omega_b$ increases by about 30%. The value of $\omega_b$ is stable to even more radical changes to the cosmology, for example, adding general isocurvature modes (Planck Collaboration XX 2016).

If we relax the assumption that $N_{\text{eff}} = 3.046$ (but adhere to the hypothesis that electron neutrinos have a standard distribution, with a negligible chemical potential), BBN predictions depend on both parameters ($\omega_b$ and $N_{\text{eff}}$). Following the same methodology as in Sect. 6.4.4 of PCP13, we can identify the region of the $\omega_b$--$N_{\text{eff}}$ parameter space that is compatible with direct measurements of the primordial helium and deuterium abundances, including the BBN theoretical errors. This is illustrated in Fig. 36 for the $N_{\text{eff}}$ extension to base $\Lambda$CDM. The region preferred by CMB observations lies at the intersection between the mean value of $\omega_b$ and $\Omega_{\text{CDM}}$.

Figure 36 also shows that the Planck polarization data help in reducing the degeneracy between $\omega_b$ and $N_{\text{eff}}$.

We can actually make a more precise statement by combining the posterior distribution on $\omega_b$ and $N_{\text{eff}}$, obtained for Planck with that inferred from helium and deuterium abundance, including observational and theoretical errors. This provides joint CMB+BBN predictions on these parameters. After marginalizing over $\omega_b$, the 95% CL preferred ranges for $N_{\text{eff}}$ are

\[
N_{\text{eff}} = \begin{cases} 
3.11^{+0.59}_{-0.57} & \text{He+Planck TT+lowP}, \\
3.14^{+0.44}_{-0.43} & \text{He+Planck TT+lowP+BAO}, \\
2.99^{+0.39}_{-0.39} & \text{He+Planck TT, TE, EE+lowP},
\end{cases}
\]

when combining Planck with the helium abundance estimated by Aver et al. (2013), or

\[
N_{\text{eff}} = \begin{cases} 
2.95^{+0.52}_{-0.52} & \text{D+Planck TT+lowP}, \\
3.01^{+0.38}_{-0.37} & \text{D+Planck TT+lowP+BAO}, \\
2.91^{+0.37}_{-0.37} & \text{D+Planck TT, TE, EE+lowP},
\end{cases}
\]

when combining with the deuterium abundance measured by Cooke et al. (2014). These bounds represent the best currently-available estimates of $N_{\text{eff}}$ and are remarkably consistent with the standard model prediction.

The allowed region in $\omega_b$--$N_{\text{eff}}$ space does not increase significantly when other parameters are allowed to vary at the same time. From our grid of extended models, we have checked that this conclusion holds in models with neutrino masses, tensor fluctuations, or running of the scalar spectral index, for example.

6.5.3. Constraints from Planck and deuterium observations on nuclear reaction rates

We have seen that primordial element abundances estimated from direct observations are consistent with those inferred from Planck data under the assumption of standard BBN. However, the Planck determination of $\omega_b$ is so precise that the theoretical errors in the BBN predictions are now a dominant source of uncertainty. As noted by Cooke et al. (2014), one can begin to think about using CMB measurements together with accurate deuterium abundance measurements to learn about the underlying BBN physics.

While for helium the theoretical error comes mainly from the uncertainties in the neutron lifetime, for deuterium it is dominated by uncertainties in the radiative capture process $d(p, \gamma)^3$He, converting deuterium into helium. The present experimental uncertainty for the $S$-factor at low energy (relevant for BBN), is in the range 6–10% (Ma et al. 1997). However, as noted by several authors (see, e.g., Nollett & Holder 2011; Di Valentino et al. 2014) the best-fit value of $S(E)$ inferred from experimental data in the range 30 keV $\leq E \leq 300$ keV is about 5–10% lower than theoretical expectations (Viviani et al. 2000; Maručič et al. 2005). The PArthENOPE BBN code assumes the lower experimental value for $S(p, \gamma)^3$He, and this might explain why the deuterium abundance determined by Cooke et al. (2014) is slightly lower than the value inferred by Planck. To investigate this further, following the methodology of Di Valentino et al. (2014), we perform a combined analysis of Planck and deuterium observations, to constrain the value of the $d(p, \gamma)^3$He reaction rate. As in Di Valentino et al. (2014), we parameterize the thermal rate $R(T)$ of the $d(p, \gamma)^3$He process in the PArthENOPE code by introducing a rescaling factor $A_2$ of the experimental rate $R_{\text{ex}}(T)$, i.e., $R(T) = A_2 R_{\text{ex}}(T)$, and solve for $A_2$ using various Planck+BAO data combinations, given the Cooke et al. (2014) deuterium abundance measurements.

Assuming the base $\Lambda$CDM model we find (68% CL)

\[
A_2 = 1.106 \pm 0.071 \text{ Planck TT+lowP}, \quad (77a)
\]

\[
A_2 = 1.098 \pm 0.067 \text{ Planck TT+lowP+BAO}, \quad (77b)
\]

\[
A_2 = 1.110 \pm 0.062 \text{ Planck TT, TE, EE+lowP}, \quad (77c)
\]

\[
A_2 = 1.109 \pm 0.058 \text{ Planck TT, TE, EE+lowP+BAO}. \quad (77d)
\]

The posteriors for $A_2$ are shown in Fig. 37. These results suggest that the $d(p, \gamma)^3$He reaction rate may have been underestimated by about 10%. Evidently, tests of the standard BBN picture appear to have reached the point where they are limited by uncertainties in nuclear reaction rates. There is therefore a strong case to improve the precision of experimental measurements (e.g., Anders et al. 2014) and theoretical computations of key nuclear reaction rates relevant for BBN.

6.5.4. Model-independent bounds on the helium fraction from Planck

Instead of inferring the primordial helium abundance from BBN codes using ($\omega_b$, $N_{\text{eff}}$) constraints from Planck, we can measure it directly, since variations in $Y^\text{BBN}_p$ modify the density of free electrons between helium and hydrogen recombination and therefore affect the damping tail of the CMB anisotropies.

If we allow $Y^\text{BBN}_P$ to vary as an additional parameter to base $\Lambda$CDM, we find the following constraints (at 95% CL):

\[
y^\text{BBN}_p = \begin{cases} 
0.253^{+0.041}_{-0.042} & \text{Planck TT+lowP}, \\
0.254^{+0.036}_{-0.038} & \text{Planck TT+lowP+BAO}, \\
0.251^{+0.026}_{-0.027} & \text{Planck TT, TE, EE+lowP}, \\
0.253^{+0.025}_{-0.026} & \text{Planck TT, TE, EE+lowP+BAO}.
\end{cases}
\]

Joint constraints on $Y^\text{BBN}_p$ and $\omega_b$ are shown in Fig. 38. The addition of Planck polarization measurements results in a substantial reduction in the uncertainty on the helium fraction. In fact, the standard deviation on $Y^\text{BBN}_p$ in the case of Planck TT, TE, EE+lowP is only 30% larger than the observational error quoted by Aver et al. (2013). As emphasized throughout this
Fig. 37. Posteriori for the $A_2$ reaction rate parameter for various data combinations. The vertical dashed line shows the value $A_2 = 1$ that corresponds to the current experimental estimate of the $d(p,\gamma)^4$He rate used in the PAReThEnoPE BBN code.

Fig. 38. Constraints in the $\omega_b - Y_{BBN}$ plane from Planck and Planck+BAO, compared to helium abundance measurements. Here 68 % and 95 % contours are plotted for the CMB(+BAO) data combinations when $Y_{BBN}$ is allowed to vary as an additional parameter to base $\Lambda$CDM. The horizontal band shows observational bounds on $^4$He compiled by Aver et al. (2013) with 68 % and 95 % errors, while the dashed line at the top of the figure delineates the conservative 95 % upper bound inferred from the Solar helium abundance by Serenelli & Basu (2010). The green stripe shows the predictions of standard BBN for the primordial abundance of $^4$He as a function of the baryon density. Both BBN predictions and CMB results assume $N_{\text{eff}} = 3.046$ and no significant lepton asymmetry.

Fig. 39. As in Fig. 38, but now allowing $Y_P^{BBN}$ and $N_{\text{eff}}$ to vary as parameter extensions to base $\Lambda$CDM.

abundance measurements and with the predictions of standard BBN.

There is a well-known parameter degeneracy between $Y_p$ and the radiation density (see the discussion in PCP13). Helium abundance predictions from the CMB are therefore particularly sensitive to the addition of the parameter $N_{\text{eff}}$ to base $\Lambda$CDM. Allowing both $Y_P^{BBN}$ and $N_{\text{eff}}$ to vary we find the following constraints (at 95 % CL):

$$Y_P^{BBN} = \begin{cases} 
0.252^{+0.058}_{-0.065} & \text{Planck TT+lowP;} \\
0.251^{+0.058}_{-0.064} & \text{Planck TT+lowP+BAO;} \\
0.263^{+0.034}_{-0.037} & \text{Planck TT,TE,EE+lowP;} \\
0.262^{+0.035}_{-0.037} & \text{Planck TT,TE,EE+lowP+BAO.}
\end{cases}$$

(79)

Contours in the $Y_P^{BBN}$-$N_{\text{eff}}$ plane are shown in Fig. 39. Here again, the impact of Planck polarization data is important, and helps to substantially reduce the degeneracy between these two parameters. The Planck TT,TE,EE+lowP contours are in very good agreement with standard BBN and $N_{\text{eff}} = 3.046$. However, even if we relax the assumption of standard BBN, the CMB does not allow high values of $N_{\text{eff}}$. It is therefore difficult to accommodate an extra thermalized relativistic species, even if the standard BBN prior on the helium fraction is relaxed.

6.6. Dark matter annihilation

Energy injection from dark matter (DM) annihilation can alter the recombination history, leading to changes in the temperature and polarization power spectra of the CMB (e.g., Chen & Kamionkowski 2004; Padmanabhan & Finkbeiner 2005). As demonstrated in several papers (e.g., Galli et al. 2009; Slatyer et al. 2009; Finkbeiner et al. 2012), CMB anisotropies offer an opportunity to constrain the nature of DM. Furthermore, CMB experiments such as Planck can achieve limits on the annihilation cross-section that are relevant for the interpretation of the rise in the cosmic-ray positron fraction at energies $\geq 10$ GeV observed by PAMELA, Fermi, and AMS (Adriani et al. 2009; Ackermann et al. 2012; Aguilar et al. 2014). The CMB constraints are complementary to those determined from other astro-
physical probes, such as the γ-ray observations of dwarf galaxies by the Fermi satellite (Ackermann et al. 2014).

The way in which DM annihilations heat and ionize the gaseous background depends on the nature of the cascade of particles produced following annihilation and, in particular, on the production of e± pairs and photons that couple to the gas. The fraction of the rest mass energy that is injected into the gas can be modelled by an “efficiency factor,” \( f(z) \), which is typically in the range 0.01–1 and depends on redshift. Computations of \( f(z) \) for various annihilation channels can be found in Slatyer et al. (2009), Hutsi et al. (2009), and Evoli et al. (2013). The rate of energy release per unit volume by annihilating DM can therefore be written as

\[
dE \frac{dt}{dV}(z) = 2g \rho_\text{crit}^2 \Omega_\text{m}^2(1 + z)^3 p_{\text{ann}}(z),
\]

where \( p_{\text{ann}} \) is defined as

\[
p_{\text{ann}}(z) \equiv f(z) \frac{\langle rv \rangle}{m_r}.
\]

Here \( \rho_\text{crit} \) the critical density of the Universe today, \( m_r \) is the mass of the DM particle, and \( \langle rv \rangle \) is the thermally-averaged annihilation cross-section times the velocity (explicitly the so-called Møller velocity); we will refer to this quantity loosely as the “cross-section” hereafter. In Eq. (80), \( g \) is a degeneracy factor that is equal to 1/2 for Majorana particles and 1/4 for Dirac particles. In this paper, the constraints will refer to Majorana particles. Note that to produce the observed dark matter density from thermal DM relics requires an annihilation cross-section of \( \langle rv \rangle \approx 3 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1} \) (assuming s-wave annihilation) at the time of freeze-out (see, e.g., the review by Profumo 2013).

Both the amplitude and redshift dependence of the efficiency factor \( f(z) \) depend on the details of the annihilation process (e.g., Slatyer et al. 2009). The functional shape of \( f(z) \) can be taken into account using generalized parameterizations or principal components (Finkbeiner et al. 2012; Hutsi et al. 2011), similar to the analysis of the recombination history presented in Sect. 6.7.4. However, as shown in Galli et al. (2011), Giesen et al. (2012), and Finkbeiner et al. (2012), to a first approximation the redshift dependence of \( f(z) \) can be ignored, since current CMB data (including Planck) are sensitive to energy injection over a relatively narrow range of redshift, typically \( z \approx 1000–600 \). The effects of DM annihilation can therefore be reasonably well parameterized by a single constant parameter, \( p_{\text{ann}} \) (with \( f(z) \) set to a constant \( f_{\text{crit}} \)), which encodes the dependence on the properties of the DM particles. In the following, we calculate constraints on the \( p_{\text{ann}} \) parameter, assuming that it is constant, and then project these constraints on to a particular dark matter model assuming \( f_{\text{crit}} \equiv f(z = 600) \), since the effect of dark matter annihilation peaks at \( z \approx 600 \) (see Finkbeiner et al. 2012). The \( f(z) \) functions used here are those calculated in Slatyer et al. (2009), with the updates described in Galli et al. (2013) and Madhavacheril et al. (2014). Finally, we estimate the fractions of injected energy that affect the gaseous background, from heating, ionizations, or LyC excitations, using the updated calculations described in Galli et al. (2013) and Valdes et al. (2010), following Shull & van Steenberg (1985).

\[\text{Fig. 40.} \text{ 2D marginal distributions in the } p_{\text{ann}}-n_\gamma \text{ plane for Planck TT+lowP (red), Planck EE+lowP (yellow), Planck TE+lowP (green), and Planck TT,TE,EE+lowP (blue) data combinations. We also show the constraints obtained using WMAP9 data (light blue).}\]

We compute the theoretical angular power spectrum in the presence of DM annihilations by modifying the recfast routine (Seager et al. 1999) in the camb code as in Galli et al. (2011).\(^{34}\) We then add \( p_{\text{ann}} \) as an additional parameter to those of the base ΛCDM cosmology. Table 6 shows the constraints for various data combinations.

<table>
<thead>
<tr>
<th>Data combinations</th>
<th>( p_{\text{ann}} ) (95 % upper limits)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TT+lowP</td>
<td>( &lt; 5.7 \times 10^{-27} )</td>
</tr>
<tr>
<td>EE+lowP</td>
<td>( &lt; 1.4 \times 10^{-27} )</td>
</tr>
<tr>
<td>TE+lowP</td>
<td>( &lt; 5.9 \times 10^{-28} )</td>
</tr>
<tr>
<td>TT+lowP+lensing</td>
<td>( &lt; 4.4 \times 10^{-27} )</td>
</tr>
<tr>
<td>TT,TE,EE+lowP</td>
<td>( &lt; 4.1 \times 10^{-28} )</td>
</tr>
<tr>
<td>TT,TE,EE+lowP+lensing</td>
<td>( &lt; 3.4 \times 10^{-28} )</td>
</tr>
<tr>
<td>TT,TE,EE+lowP+ext</td>
<td>( &lt; 3.5 \times 10^{-28} )</td>
</tr>
</tbody>
</table>

The constraints on \( p_{\text{ann}} \) from the Planck TT+lowP spectra are about 3 times weaker than the 95 % limit of \( p_{\text{ann}} < 2.1 \times 10^{-27} \text{ cm}^3 \text{ s}^{-1} \text{ GeV}^{-1} \) derived from WMAP9, which includes WMAP polarization data at low multipoles. On the other hand, the Planck TE or EE spectra improve the constraints on \( p_{\text{ann}} \) by about an order of magnitude compared to those from Planck TT alone. This is because the main effect of dark matter annihilation is to increase the width of last scattering, leading to a suppression of the amplitude of the peaks, both in temperature and polarization. As a result, the effects of DM annihilation on the power spectra at high multipole are degenerate with other parameters of base ΛCDM, such as \( n_s \) and \( A_s \) (Chen & Kamionkowski 2004; Padmanabhan & Finkbeiner 2005). At large angular scales (\( \ell < 200 \)), however, dark matter annihilation can produce an enhancement in polarization, caused

\[\text{33} \text{To maintain consistency with other papers on dark matter annihilation, we retain the notation } f(z) \text{ for the efficiency factor in this section; it should not be confused with the growth rate factor introduced in Eq. (32).}\]

\[\text{34} \text{We checked that we obtain similar results using either the HyRec code (Ali-Haimoud & Hirata 2011), as detailed in Giesen et al. (2012), or CosmoRec (Chluba & Thomas 2011), instead of recfast.}\]
by the increased ionization fraction in the freeze-out tail following recombination. As a result, large-angle polarization information is crucial for breaking the degeneracies between parameters, as illustrated in Fig. 40. The strongest constraints on $p_{\text{ann}}$ therefore come from the full Planck temperature and polarization likelihood and there is little improvement if other astrophysical data, or Planck lensing, are added.\(^{35}\)

We verified the robustness of the Planck TT,TE,EE+lowP constraint by also allowing other parameter extensions of base $\Lambda$CDM ($N_{\text{eff}},dn_{s}/d\ln k$, or $Y_P$) to vary together with $p_{\text{ann}}$. We found that the constraint is weakened by up to 20%. Furthermore, we have verified that we obtain consistent results when relaxing the priors on the amplitudes of the Galactic dust templates or if we use the CamSpec likelihood instead of the baseline P11k likelihood.

Figure 41 shows the constraints from WMAP9, Planck TT,TE,EE+lowP, and a forecast for a cosmic-variance-limited experiment with similar angular resolution to Planck.\(^{36}\) The horizontal red band includes the values of the thermal-relic cross-section multiplied by the appropriate $f_{\text{eff}}$ for different DM annihilation channels. For example, the upper red line corresponds to $f_{\text{eff}} = 0.67$, which is appropriate for a DM particle of mass $m_\chi = 10\,\text{GeV}$ annihilating into $e^+e^-$, while the lower red line corresponds to $f_{\text{eff}} = 0.13$, for a DM particle annihilating into $2\pi^+\pi^-$ through an intermediate mediator (see, e.g., Arkani-Hamed et al. 2009). The Planck data exclude at 95% confidence level a thermal relic cross-section for DM particles of mass $m_\chi \lesssim 44\,\text{GeV}$ annihilating into $e^+e^-$ ($f_{\text{eff}} \approx 0.6$), $m_\chi \lesssim 16\,\text{GeV}$ annihilating into $\mu^+\mu^-$ or $b\bar{b}$ ($f_{\text{eff}} \approx 0.2$), and $m_\chi \lesssim 11\,\text{GeV}$ annihilating into $\tau^+\tau^-$ ($f_{\text{eff}} \approx 0.15$).

The dark grey shaded area in Fig. 41 shows the approximate allowed region of parameter space, as calculated by Cholis & Hooper (2013) on the assumption that the PAMELA, AMS, and Fermi cosmic-ray excesses are caused by DM annihilation; the dark grey dots indicate the best-fit dark matter models described in that paper (for a recent discussion on best-fitting models, see also Boudaud et al. 2015). The favoured value of the cross-section is about two orders of magnitude higher than the thermal relic cross-section ($\approx 3 \times 10^{-26}\,\text{cm}^3\,\text{s}^{-1}$). Attempts to reconcile such a high cross-section with the relic abundance of DM include a Sommerfeld enhanced cross-section (that may saturate at $(\sigma v) \approx 10^{-25}\,\text{cm}^3\,\text{s}^{-1}$) or non-thermal production of DM (see, e.g., the discussion by Madhavacheril et al. 2014). Both of these possibilities are strongly disfavoured by the Planck data. We cannot, however, exclude more exotic possibilities, such as DM annihilation through a p-wave channel with a cross-section that scales as $s^2$ (Diamanti et al. 2014). Since the relative velocity of DM particles at recombination is many orders of magnitude smaller than in the Galactic halo, such a model cannot be constrained using CMB data.

Observations from the Fermi Large Area Telescope of extended $\gamma$-ray emission towards the centre of the Milky Way, peaking at energies of around 1–3 GeV, have been interpreted as evidence for annihilating DM (e.g., Goodenough & Hooper 2009; Gordon & Macias 2013; Daylan et al. 2016; Abazajian et al. 2014; Lacroix et al. 2014). The light grey stars in Fig. 41 show specific models of DM annihilation designed to fit the Fermi $\gamma$-ray excess (Calore et al. 2015), while the light grey box shows the uncertainties of the best-fit cross-sections due to imprecise knowledge of the Galactic DM halo profile. Although the interpretation of the Fermi excess remains controversial (because of uncertainties in the astrophysical backgrounds), DM annihilation remains a possible explanation. The best-fit models of Calore et al. (2015) are consistent with the Planck constraints on DM annihilation.

6.7. Testing recombination physics with Planck

The cosmological recombination process determines how CMB photons decoupled from baryons around redshift $z \approx 10^3$, when the Universe was about 400,000 years old. The importance of this transition on the CMB anisotropies has long been recognized (Sunyaev & Zeldovich 1970; Peebles & Yu 1970). The most advanced computations of the ionization history (e.g., Ali-Haimoud & Hirata 2010; Chluba & Thomas 2011; Ali-Haimoud & Hirata 2011; Chluba et al. 2012) account for many subtle atomic physics and radiative transfer effects that were not included in the earliest calculations (Zeldovich et al. 1968; Peebles 1968).

With precision data from Planck, we are sensitive to sub-percent variations of the free electron fraction around last-scattering (e.g., Hu et al. 1995; Seager et al. 2000; Seljak et al. 2003). Quantifying the impact of uncertainties in the ionization history around the maximum of the Thomson visibility function

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\(^{35}\)It is interesting to note that the constraint derived from Planck TT,TE,EE+lowP is consistent with the forecast given in Galli et al. (2009), $p_{\text{ann}} \lesssim 3 \times 10^{-26}\,\text{cm}^3\,\text{s}^{-1}\,\text{GeV}^{-1}$.

\(^{36}\)We assumed here that the cosmic-variance-limited experiment would measure the angular power spectra up to a maximum multipole of $\ell_{\max} = 2500$, observing a sky fraction $f_{\text{sky}} = 0.65$.  

**Fig. 41.** Constraints on the self-annihilation cross-section at recombination, $(\sigma v)_\chi$, times the efficiency parameter, $f_{\text{eff}}$ (Eq. 81). The blue area shows the parameter space excluded by the Planck TT,TE,EE+lowP data at 95% CL. The yellow line indicates the constraint using WMAP9 data. The dashed green line delineates the region ultimately accessible to a cosmic-variance-limited experiment with angular resolution comparable to that of Planck. The horizontal red band includes the values of the thermal-relic cross-section multiplied by the appropriate $f_{\text{eff}}$ for different DM annihilation channels. The dark grey circles show the best-fit DM models for the PAMELA/AMS-02/Fermi cosmic-ray excesses, as calculated in Cholis & Hooper (2013) (caption of their figure 6). The light grey stars show the best-fit DM models for the Fermi Galactic centre $\gamma$-ray excess, as calculated by Calore et al. (2015) (their tables I, II, and III), with the light grey area indicating the astrophysical uncertainties on the best-fit cross-sections.
on predictions of the CMB power spectra is thus crucial for the scientific interpretation of data from Planck. In particular, for tests of models of inflation and extensions to ΛCDM, the interpretation of the CMB data can be significantly compromised by inaccuracies in the recombination calculation (e.g., Wong et al. 2008; Rubio-Martín et al. 2010; Shaw & Chluba 2011). This problem can be approached in two ways, either by using modified recombination models with a specific physical process (or parameter) in mind, or in a semi-blind, model-independent way. Both approaches provide useful insights in assessing the robustness of the results from Planck.

Model-dependent limits on varying fundamental constants (Kaplinghat et al. 1999; Scóccola et al. 2009; Galli et al. 2009), annihilating or decaying particles (e.g., Chen & Kamionkowski 2004; Padmanabhan & Finkbeiner 2005; Zhang et al. 2006, and Sect. 6.6), or more general sources of extra ionization and excitation photons (Peebles et al. 2000; Doroshkevich et al. 2003; Galli et al. 2008), have been discussed extensively in the literature.

As already discussed in PCP13, the choice for Planck has been to use the rapid calculations of the recfast code, modified using corrections calculated with the more precise codes. To start this sub-section we quantify the effects caused by using corrections calculated with the more precise codes. Although this is about 5 times larger than the differences on the CMB power spectra is a bias in ln(10^10A_s) at the level of 0.012 for Planck TT,TE,EE+lowP+BAO. Although this is about 5 times larger than the difference in n_s between CosmoRec and HyRec, this bias is nevertheless unimportant at the current level of precision (and smaller than the differences seen from different likelihoods, see Sect. 3.1).

Finally we compare CosmoRec with recfast in its original form (i.e., before recalibrating the fitting-functions on refined recombination calculations). For base ΛCDM, we expect to see biases of ΔΩ_bh^2 ∼ −2.1σ ∼ −0.00028 and Δn_s ∼ −3.3σ ∼ −0.012 (Shaw & Chluba 2011). Using the actual data (Planck TT,TE,EE+lowP+BAO) we find biases of ΔΩ_bh^2 ∼ −1.8σ ∼ −0.00024 and Δn_s ∼ −2.6σ ∼ −0.010, very close to the expected values. This illustrates explicitly the importance of the improvements of CosmoRec and HyRec over the original version of recfast for the interpretation of Planck data. However, CosmoRec and HyRec themselves are much more computationally intensive than the modified recfast, which is why we use recfast in most Planck cosmological analyses.

6.7.1. Comparison of different recombination codes

Even for pre-Planck data, it was realized that the early recombination calculations of Zeldovich et al. (1968) and Peebles (1968) had to be improved. This led to the development of the widely-used and computationally quick recfast code (Seager et al. 1999, 2000). However, for Planck, the recombination model of recfast in its original form is not accurate enough. Percent-level corrections, due to detailed radiative transfer and atomic physics effects have to be taken into account. Ignoring these effects can bias the inferred cosmological parameters, some by as much as a few standard deviations.

The recombination problem was solved as a common effort of several groups (Dubrovich & Grachev 2005; Khlopinenko et al. 2007; Chluba & Sunyaev 2006b; Rubio-Martín et al. 2006; Karshenboim & Ivanov 2008; Wong & Scott 2007; Switzer & Hirata 2008; Grin & Hirata 2010; Ali-Haimoud & Hirata 2010). This work was undertaken, to a large extent, in preparation for the precision data from Planck. Both CosmoRec (Chluba & Thomas 2011) and HyRec (Ali-Haimoud & Hirata 2011) allow fast and precise computations of the ionization history, explicitly capturing the physics of the recombination problem. For the standard cosmology, the ionization histories obtained from these two codes in their default settings agree to within 0.05 % for hydrogen recombination (600 ≤ z ≤ 1600) and 0.35 % during helium recombination (1600 ≤ z ≤ 3000). The effect of these small differences on the CMB power spectra is ≤ 0.1 % at ℓ < 4000 and so has a negligible impact on the interpretation of precision CMB data; for the standard six parameters of base ΛCDM, we find that the largest effect is a bias in ln(10^10A_s) at the level of 0.04σ ≈ 0.0012 for Planck TT,TE,EE+lowP+BAO.

For Planck analyses, the recombination model of recfast is used by default. In recfast, the precise dynamics of recombination is not modelled physically, but approximated with fitting-functions calibrated against the full recombination calculations assuming a reference cosmology (Seager et al. 1999, 2000; Wong et al. 2008). At the level of precision required for Planck, the recfast approach is sufficiently accurate, provided that the cosmologies are close to base ΛCDM (Rubino-Martín et al. 2010; Shaw & Chluba 2011). Comparing the latest version of recfast (camb version) with CosmoRec, we find agreement to within 0.2 % for hydrogen recombination (600 ≤ z ≤ 1600) and 0.2 % during helium recombination for the standard ionization history. The effect on the CMB power spectra is ≤ 0.15 % at ℓ < 4000, although with slightly more pronounced shifts in the peak positions than when comparing CosmoRec and HyRec. For the base ΛCDM model, we find that the largest bias is on n_s at the level of 0.15σ (≈ 0.0006) for Planck TT,TE,EE+lowP+BAO. Although this is about 5 times larger than the difference in n_s between CosmoRec and HyRec, this bias is nevertheless unimportant at the current level of precision (and smaller than the differences seen from different likelihoods, see Sect. 3.1).

6.7.2. Measuring A_{2s−1s} with Planck

The crucial role of the 2s−1s two-photon decay channel for the dynamics of hydrogen recombination has been appreciated since the early days of CMB research (Zeldovich et al. 1968; Peebles 1968). Recombination is an out-of-equilibrium process and energetic photons emitted in the far Wien tail of the CMB by Lyman continuum and series transitions keep the primordial plasma ionized for a much longer period than expected from simple equilibrium recombination physics. Direct recombinations to the ground state of hydrogen are prohibited, causing a modification of the free electron number density, N_e, by only ΔN_e/N_e ≈ 10^{-6} around z ≈ 10^3 (Chluba & Sunyaev 2007). Similarly, the slow escape of photons from the Lyα resonance reduces the effective Ly-α transition rate to A_{2s−1s} ≈ 1−10 s^{-1} (by more than seven orders of magnitude), making it comparable to the vacuum 2s−1s two-photon decay rate of A_{2s−1s} ≈ 8.2 s^{-1}. About 57 % of all hydrogen atoms in the Universe became neutral through the 2s−1s channel (e.g., Wong et al. 2006; Chluba & Sunyaev...37Helium recombination is treated in more detail by CosmoRec (e.g., Rubino-Martín et al. 2008; Chluba et al. 2012), which explains most of the difference.
The high sensitivity of the recombination process to the 2s–1s two-photon transition rate also implies that instead of simply adopting a value for $A_{2s-1s}$ from theoretical computations (Breit & Teller 1940; Spitzer & Greenstein 1951; Goldman 1989) one can directly determine it with CMB data. From the theoretical point of view it would be surprising to find a value that deviates significantly from $A_{2s-1s} = 8.2206\, s^{-1}$, derived from the most detailed computation (Labzowsky et al. 2005).

However, laboratory measurements of this transition rate are extremely challenging (O’Connell et al. 1975; Krüger & Oed 1975; Cesar et al. 1996). The most stringent limit is for the differential decay rate, $A_{2s-1s}(\lambda)\, d\lambda = (1.5\pm0.65)\, s^{-1}$ (a 43 % error) at wavelengths $\lambda = 255.4$–$232.0\, \text{nm}$, consistent with the theoretical value of $A_{2s-1s}(\lambda)\, d\lambda = 1.02\, s^{-1}$ in the same wavelength range (Krüger & Oed 1975). With precision data from *Planck* we are in a position to perform the best measurement to date, using cosmological data to inform us about atomic transition rates at last scattering (as also emphasized by Mukhanov et al. 2012).

The 2s–1s two-photon rate affects the CMB anisotropies only through its effect on the recombination history. A larger value of $A_{2s-1s}$, accelerates recombination, allowing photons and baryons to decouple earlier, an effect that shifts the acoustic peaks towards smaller scales. In addition, slightly less damping occurs, as in the case of the stimulated 2s–1s two-photon decays (Chluba & Sunyaev 2006b). This implies that for flat cosmologies, variations of $A_{2s-1s}$ correlate with $\Omega_b h^2$ and $H_0$ (which affect the distance to the last scattering surface), while $A_{2s-1s}$ anti-correlates with $\Omega_b h^2$ and $n_s$ (which modify the slope of the damping tail). Despite these degeneracies, one expects that *Planck* will provide a measurement of $A_{2s-1s}$ to within $\pm0.5\, s^{-1}$, corresponding to an approximately 6 % uncertainty (Mukhanov et al. 2012).

In Fig. 42, we show the marginalized posterior for $A_{2s-1s}$ from *Planck* and for *Planck* combined with COBE. Using *CosmoRec* to compute the recombination history, we find

\[
\begin{align*}
A_{2s-1s} &= 7.70 \pm 1.01\, s^{-1} \quad \text{*Planck* TT+lowP, } (82a) \\
A_{2s-1s} &= 7.72 \pm 0.60\, s^{-1} \quad \text{*Planck* TT, TE, EE+lowP, } (82b) \\
A_{2s-1s} &= 7.71 \pm 0.99\, s^{-1} \quad \text{*Planck* TT+lowP+BAO, } (82c) \\
A_{2s-1s} &= 7.75 \pm 0.61\, s^{-1} \quad \text{*Planck* TT, TE, EE+lowP, } +\text{BAO. } (82d)
\end{align*}
\]

These results are in very good agreement with the theoretical value, $A_{2s-1s} = 8.2206\, s^{-1}$. For *Planck* TT, TE, EE+lowP+BAO, approximately 8 % precision is reached using cosmological data. These constraints are not sensitive to the addition of BAO, or other external data (JLA+H0). The slight shift away from the theoretical value is accompanied by small fractions of a or $x$ shifts in $n_s$, $\Omega_b h^2$, and $H_0$, to compensate for the effects of $A_{2s-1s}$ on the distance to the last scattering surface and damping tail. This indicates that additional constraints on the acoustic scale are required to fully break degeneracies between these parameters and their effects on the CMB power spectrum, a task that could be achieved in the future using large-scale structure surveys and next generation CMB experiments.

The values for $A_{2s-1s}$ quoted above were obtained using *CosmoRec*. When varying $A_{2s-1s}$, the range of cosmologies becomes large enough to potentially introduce a mismatch of the *recfast* fitting-functions that could affect the posterior. In particular, with *recfast* the 2s–1s two-photon and Lyα channels are not treated separately, so that changes specific to the 2s–1s decay channel propagate inconsistently.\footnote{One effect is that by increasing $A_{2s-1s}$, fewer Lyα photons are produced. This reduces the Lyα feedback correction to the 2s–1s channel, which further accelerates recombination, an effect that is not captured with *recfast* in the current implementation.} However, repeating the analysis with *recfast*, we find $A_{2s-1s} = 7.78 \pm 0.58\, s^{-1}$ (see Fig. 42), for *Planck* TT, TE, EE+lowP+BAO, which is in excellent agreement with *CosmoRec*, showing that these effects can be neglected.

6.7.3. Measuring $T_0$ at last-scattering with *Planck*

Our best constraint on the CMB monopole temperature comes from the measurements of the CMB spectrum with COBE/FIRAS, giving a 0.02 % determination of $T_0$ (Fixsen et al. 1996; Fixsen 2009). Other constraints from molecular lines typically reach 1 % precision (see table 2 in Fixsen 2009, for an overview), while independent BBN constraints provide 5–10 % limits (Simha & Steigman 2008; Jeong et al. 2014).

The CMB anisotropies provide additional ways of determining the value of $T_0$ (for fixed values of $N_{\text{eff}}$ and $Y_p$). One is through the energy distribution of the CMB anisotropies (Fixsen et al. 1996; Fixsen 2003; Chluba 2014) and another through their power spectra (Opher & Pelinson 2004, 2005; Chluba 2014). Even small changes in $T_0$, compatible with the COBE/FIRAS error, affect the ionization history at the 0.5 % level around last-scattering, propagating to a roughly 0.1 % uncertainty in the CMB power spectrum (Chluba & Sunyaev 2008). Overall, the effect of this uncertainty on the parameters of ΛCDM models is small (Hamann & Wong 2008); however,
Fig. 43. Marginalized posterior for $T_0$. We find excellent agreement with the COBE/FIRAS measurement. For comparison, we show the result for Planck TT, TE, EE + lowP + BAO obtained using both CosmoRec and recfast, emphasizing the consistency of different treatments.

without prior knowledge of $T_0$ from the COBE/FIRAS measurement, the situation would change significantly.

The CMB monopole affects the CMB anisotropies in several ways. Most importantly, for larger $T_0$, photons decouple from baryons at lower redshift, since more ionizing photons are present in the Wien-tail of the CMB. This effect is amplified because of the exponential dependence of the atomic level populations on the ratio of the ionization potentials and CMB temperature. In addition, increasing $T_0$ lowers the expansion timescale of the Universe and the redshift of matter-radiation equality, while increasing the photon sound speed. Some of these effects are also produced by varying $N_{\text{eff}}$; however, the effects of $T_0$ on the ionization history and photon sound speed are distinct.

With CMB data alone, the determination of $T_0$ is degenerate with other parameters, but the addition of other data sets breaks this degeneracy. Marginalized posterior distributions for $T_0$ are shown in Fig. 43. Using CosmoRec, we find

$$ T_0 = 2.722 \pm 0.027 \text{ K} \quad \text{Planck TT + lowP + BAO}, $$

$$ T_0 = 2.718 \pm 0.021 \text{ K} \quad \text{Planck TT, TE, EE + lowP + BAO}, $$

and similar results are obtained with recfast. This is in excellent agreement with the COBE/FIRAS measurement. $T_0 = 2.7255 \pm 0.0006$ K (Fixsen et al. 1996; Fixsen 2009). These measurements of $T_0$ reach a precision that is comparable to the accuracy obtained with interstellar molecules. Since the systematics of these independent methods are very different, this result demonstrates the consistency of all these data. Allowing $T_0$ to vary causes the errors of the other cosmological parameters to increase. The strongest effect is on $\theta_{SC}$, which is highly degenerate with $T_0$. The error on $\theta_{SC}$ increases by a factor of roughly 25 if $T_0$ is allowed to vary. The error on $\Omega_b h^2$ increases by a factor of about 4, while the errors on $n_s$ and $\Omega_c h^2$ increase by factors of 1.5–2, and the other cosmological parameters are largely unaffected by variations in $T_0$. Because of the strong degeneracy with $\theta_{SC}$, no constraint on $T_0$ can be obtained using Planck data alone. External data, such as BAO, are therefore required to break this geometric degeneracy.

It is important to emphasize that the CMB measures the temperature at a redshift of $z \approx 1100$, so the comparison with measurements of $T_0$ at the present day is effectively a test of the constancy of $aT_{\text{CMB}}$, where $a \approx 1/1100$ is the scale-factor at the time of last-scattering. It is remarkable that we are able to test the constancy of $aT_{\text{CMB}} \equiv T_0$ over such a large dynamic range in redshift. Of course, if we did find that $aT_{\text{CMB}}$ around recombination were discrepant with $T_0$ now, then we would need to invent a finely-tuned late-time photon injection mechanism to explain the anomaly. Fortunately, the data are consistent with the standard $T_{\text{CMB}} \approx (1 + z)$ scaling of the CMB temperature.

Another approach to measuring $aT_{\text{CMB}}$ is through the thermal Sunyaev-Zeldovich effect in rich clusters of galaxies at various redshifts (Fabbri et al. 1978; Rephaeli 1980), although it is unclear how one would interpret a failure of this test without an explicit model. In practice this approach is consistent with a scaling $aT_{\text{CMB}} = \text{constant}$, but with lower precision than obtained here from Planck (e.g., Battistelli et al. 2002; Luzzi et al. 2009; Saro et al. 2014; Hurier et al. 2014). A simple $T_{\text{CMB}} = T_0 (1 + z)^{−\beta}$ modification to the standard temperature redshift relation is frequently discussed in the literature (though this case is not justified by any physical model and is difficult to realize without creating a CMB spectral distortion, see Chluba 2014).

For this parameterization we find

$$ \beta = (0.2 \pm 1.4) \times 10^{-3} \quad \text{Planck TT + lowP + BAO}, $$

$$ \beta = (0.4 \pm 1.1) \times 10^{-3} \quad \text{Planck TT, TE, EE + lowP + BAO}, $$

where we have adopted a recombination redshift of $z_r = 1100$. Because of the long lever-arm in redshift afforded by the CMB, this is an improvement over earlier constraints by more than an order of magnitude (e.g., Hurier et al. 2014).

In a self-consistent picture, changes of $T_0$ would also affect the BBN era. We might therefore consider a simultaneous variation of $N_{\text{eff}}$ and $Y_P$ to reflect the variation of the neutrino energy density accompanying a putative variation in the photon energy density. Since we find $aT_{\text{CMB}}$ at recombination to be highly consistent with the observed CMB temperature from COBE/FIRAS, considering this extra variation seems unnecessary. Instead, we may view the $aT_{\text{CMB}}$ variation investigated here as further support for the limits discussed in Sects. 6.4 and 6.5.

6.7.4. Semi-blind perturbed recombination analysis

The high sensitivity of small-scale CMB anisotropies to the ionization history of the Universe around the epoch of recombination allows us to constrain possible deviations from the standard recombination scenario in a model-independent way (Farhang et al. 2012, 2013). The method relies on an eigen-analysis (often referred to as a principle component analysis) of perturbations in the free electron fraction, $X_e(z) = N_e / N_H$, where $N_H$ denotes the number density of hydrogen nuclei. The eigenmodes selected are specific to the data used in the analysis. Similar approaches have been used to constrain deviations of the reionization history from the simplest models (Mortonson & Hu 2009). Pure energy release in the form of heating of ordinary matter would leave a Compton $y$-distortion (Zeldovich & Sunyaev 1969) at these late times (Burgina et al. 1991; Hu & Silk 1993; Chluba & Sunyaev 2012). The test depends on the logarithm of the redshift and so is insensitive to the precise value adopted for $z_r$. 
Table 7. Standard parameters and the first three $X_e$-modes, as determined for Planck TT,TE,EE+lowP+BAO.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$+1$ mode</th>
<th>$+2$ modes</th>
<th>$+3$ modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega_m^2$</td>
<td>$0.02229 \pm 0.00017$</td>
<td>$0.02237 \pm 0.00018$</td>
<td>$0.02237 \pm 0.00019$</td>
</tr>
<tr>
<td>$\Omega_b^2$</td>
<td>$0.1190 \pm 0.0010$</td>
<td>$0.1186 \pm 0.0011$</td>
<td>$0.1187 \pm 0.0012$</td>
</tr>
<tr>
<td>$H_0$</td>
<td>$67.64 \pm 0.48$</td>
<td>$67.80 \pm 0.51$</td>
<td>$67.80 \pm 0.56$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>$0.065 \pm 0.012$</td>
<td>$0.068 \pm 0.013$</td>
<td>$0.068 \pm 0.013$</td>
</tr>
<tr>
<td>$n_s$</td>
<td>$0.9667 \pm 0.0053$</td>
<td>$0.9677 \pm 0.0055$</td>
<td>$0.9678 \pm 0.0067$</td>
</tr>
<tr>
<td>$\ln(10^{10} A_s)$</td>
<td>$3.062 \pm 0.023$</td>
<td>$3.066 \pm 0.024$</td>
<td>$3.066 \pm 0.024$</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>$-0.03 \pm 0.12$</td>
<td>$0.03 \pm 0.14$</td>
<td>$0.02 \pm 0.15$</td>
</tr>
<tr>
<td>$\mu_2$</td>
<td>$-0.17 \pm 0.18$</td>
<td>$-0.18 \pm 0.19$</td>
<td>$-0.19 \pm 0.19$</td>
</tr>
<tr>
<td>$\mu_3$</td>
<td>$-0.02 \pm 0.88$</td>
<td>$-0.02 \pm 0.88$</td>
<td>$-0.02 \pm 0.88$</td>
</tr>
</tbody>
</table>

2008) and annihilating dark matter scenarios (Finkbeiner et al. 2012), both with the prior assumption that the standard recombination physics is fully understood, as well as for constraining trajectories in inflation Planck Collaboration XX (2016) and dark energy Planck Collaboration XIV (2016) parameterizations.

Here, we use Planck data to find preferred ionization fraction trajectories $X_e(z)$ composed of low-order perturbation eigenmodes to the standard history ($X_e$-modes). The $X_e$-modes are constructed through the eigen-decomposition of the inverse of the Fisher information matrix for base $\Lambda$CDM (the six cosmological parameters and the nuisance parameters) and recombination perturbation parameters (see Farhang et al. 2012, for details). This procedure allows us to estimate the errors on the eigenmode amplitudes, $\mu_i$, providing a rank ordering of the $X_e$-modes and their information content.

The first three $X_e$-modes for Planck TT,TE,EE+lowP are illustrated in Fig. 44, together with their impact on the differential visibility function. Figure 45 shows the response of the CMB temperature and polarization power spectra to these eigenmodes. The first mode mainly leads to a change in the width and height of the Thomson visibility function (bottom panel of Fig. 44). This implies less diffusion damping, which is also reflected in the modifications to the CMB power spectra (as shown in Fig. 45). The second mode causes the visibility maximum to shift towards higher redshifts for $\mu_2 > 0$, which leads to a shift of the CMB extrema to smaller scales; however, for roughly constant width of the visibility function it also introduces less damping at small scales. The third mode causes a combination of changes in both the position and width of the visibility function, with a pronounced effect on the location of the acoustic peaks. For the analysis of Planck data combinations, we only use $X_e$-modes that are optimized for Planck TT,TE,EE+lowP.

We modified CosmoMC to estimate the mode amplitudes. The results for Planck TT,TE,EE+lowP+BAO are presented in Table 7. Although all mode amplitudes are consistent with standard recombination, adding the second $X_e$-mode causes mild shifts in $H_0$ and $\tau$. For Planck TT,TE,EE+lowP, we find $\mu_1 = -0.11 \pm 0.51$ and $\mu_2 = -0.23 \pm 0.50$, using the Planck TT,TE,EE+lowP eigenmodes, again consistent with the standard recombination scenario. Adding the polarization data improves the errors by more than a factor of 2. However, the mode amplitudes are insensitive to the addition of external data.

With pre-Planck data, only the amplitude, $\mu_1$, of the first eigenmode could be constrained. The corresponding change in the ionization history translates mainly into a change in the slope of the CMB damping tail, with this mode resembling the first mode determined using Planck data (Fig. 44). The WMAP9+SPT data gave a non-zero value for the first eigenmode at about 2 $\sigma$, $\mu_1^{SPT} = -0.80 \pm 0.37$. However, the WMAP9+ACT data gave $\mu_1^{ACT} = 0.14 \pm 0.45$ and the combined pre-Planck data (WMAP+ACT+SPT) gave $\mu_1^{pre} = -0.44 \pm 0.33$, both consistent with the standard recombination scenario (Calabrese et al. 2013). The variation among these results is another manifestation of the tensions between different pre-Planck CMB data, as discussed in PCP13.

Although not optimal for Planck data, we also compute the amplitudes of the first three $X_e$-modes constructed for the WMAP9+SPT data set. This provides a more direct comparison with the pre-Planck constraints. For Planck TT,TE,EE+lowP+BAO we obtain $\mu_1^{SPT} = -0.10 \pm 0.13$ and $\mu_2^{SPT} = -0.13 \pm 0.18$. The mild tension of the pre-Planck data with the standard recombination scenario disappears when using Planck data. This is especially impressive, since the er-
errors have improved by more than a factor of 2. By projecting onto the Planck modes, we find that the first two SPT modes can be expressed as $\mu_{1}^{\text{SPT}} \approx 0.69\mu_1 + 0.66\mu_2 \approx -0.09$ and $\mu_{2}^{\text{SPT}} \approx -0.70\mu_1 + 0.64\mu_2 \approx -0.13$, which emphasizes the consistency of the results. Adding the first three SPT modes, we obtain $\mu_{1}^{\text{SPT}} = -0.09 \pm 0.13$, $\mu_{2}^{\text{SPT}} = -0.14 \pm 0.21$, and $\mu_{3}^{\text{SPT}} = -0.12 \pm 0.86$, which again is consistent with the standard model of recombination. The small changes in the mode amplitudes when adding the third mode arise because the SPT modes are not optimal for Planck and so are correlated.

### 6.8. Cosmic defects

Topological defects are a generic by-product of symmetry-breaking phase transitions and a common phenomenon in condensed matter systems. Cosmic defects of various types can be formed in phase transitions in the early Universe (Kibble 1976). In particular, cosmic strings can be produced in some supersymmetric and grand-unified theories at the end of inflation (Jeannerot et al. 2003), as well as in higher-dimensional theories (e.g., Polchinski 2005). Constraints on the abundance of cosmic strings and other defects therefore place limits on a range of models of the early Universe. More discussion on the formation, evolution, and cosmological role of topological defects can be found, for example, in the reviews by Vilenkin & Shellard (2000), Hindmarsh & Kibble (1995), and Copeland & Kibble (2010).

In this section we revisit the power spectrum-based constraints on the abundance of cosmic strings and other topological defects using the 2015 Planck data, including Planck polarization measurements. The general approach follows that described in the Planck 2013 analysis of cosmic defects (Planck Collaboration XXV 2014), so here we focus on the updated constraints rather than on details of the methodology.
Topological defects are non-perturbative excitations of the underlying field theory and their study requires numerical simulations. Unfortunately, since the Hubble scale, $c/H_0$, is over 50 orders of magnitude greater than the thickness of a GUT-scale string, approximately $(h/\mu c)^{1/2}$ with $\mu$ the mass per unit length of the string, it is impractical to simulate the string dynamics exactly in the late Universe. For this reason one needs to make approximations. One approach considers the limit of an infinitely thin string, which corresponds to using the Nambu-Goto ("NG") action for the string dynamics. In an alternative approach, the actual field dynamics for a given model are solved on a lattice. In this case it is necessary to resolve the string core, which generally requires more computationally intensive simulations than in the NG approach. Lattice simulations, however, can include additional physics, such as field radiation that is not present in NG simulations. Here we will use field-theory simulations of the Abelian-Higgs action ("AH"); details of these simulations are discussed in Bevis et al. (2007, 2010).

The field-theory approach also allows one to simulate theories in which the defects are not cosmic strings and so cannot be described by the NG action. Examples include semi-local strings ("SL", Urrestilla et al. 2008) and global defects. Here we will specifically consider the breaking of a global $O(4)$ symmetry resulting in texture defects ("TX").

For the field-theory defects, we measure the energy-momentum tensor from the simulations and insert it as an additional constituent into a modified version of the CMBEASY Boltzmann code (Doran 2005) to predict the defect contribution to the CMB temperature and polarization power spectra (see, e.g., Durrer et al. 2002). The same approach can be applied to NG strings, but rather than using simulations directly, we model the strings using the unconnected segment model ("USM", Albrecht et al. 1999; Pogosian & Vachaspati 1999). In this model, strings are represented by a set of uncorrelated straight segments, with scaling properties chosen to match those determined from numerical simulations. In this case, the string energy-momentum tensor can be computed analytically and used as an active source in a modified Boltzmann code. For this analysis we use CMBACT version 4,\(^{41}\) whereas Planck Collaboration XXV (2014) used version 3. There have been several improvements to the code since the 2013 analysis, including a correction to the normalization of vector mode spectra. However, the largest change comes from an improved treatment of the scaling properties. The string correlation length and velocity are described by an updated velocity-dependent one-scale model (Martins & Shellard 2002), which provides better agreement with numerical simulations. Small-scale structure of the string, which was previously a free parameter, is accounted for by the one-scale model.

The CMBEASY spectra from defects are proportional to $(G\mu/c^2)^2$. We scale the computed template CMBSpectra, and add these to the inflationary and foreground power spectra, to form the theory spectra that enter the likelihood. In practice, we parameterize the defects with their relative contribution to the TT spectrum at multipole $\ell = 10$, $f_{10} \equiv C_{10}^{TT(\text{defect})}/C_{10}^{TT(\text{total})}$.

We vary $f_{10}$ and the standard six parameters of the base $\Lambda$CDM model, using CosmoMC. We also report our results in terms of the derived parameter $G\mu/c^2$.

The constraints on $f_{10}$ and the inferred limits on $G\mu/c^2$ are summarized in Table 8. The marginalized 1D posterior distribution functions are shown in Fig. 46. For Planck TT+lowP we find that the results are similar to the Planck+WP constraints reported in Planck Collaboration XXV (2014), for the AH model, or somewhat better for SL and TX. However, the addition of the Planck high-$\ell$ TT and EE polarization data leads to a significant improvement compared to the 2013 constraints.

For the NG string model, the results based on Planck TT+lowP are slightly weaker than the 2013 Planck+WP constraints. This is caused by a difference in the updated defect spectrum from the USM model, which has a less pronounced peak and shifts towards the AH spectrum. With the inclusion of polarization, Planck TT,TE,EE+lowP improves the upper limit on $f_{10}$ by a factor of 2, as for the AH model. The differences between the AH and NG results quoted here can be regarded as a rough indication of the uncertainty in the theoretical string power spectra.

In summary, we find no evidence for cosmic defects from the Planck 2015 data, with tighter limits than before.

### 7. Conclusions

1. The six-parameter base $\Lambda$CDM model continues to provide a very good match to the more extensive 2015 Planck data, including polarization. This is the most important conclusion of this paper.

2. The 2015 Planck TT, TE, EE, and lensing spectra are consistent with each other under the assumption of the base $\Lambda$CDM cosmology. However, when comparing the TE and EE spectra computed for different frequency combinations, we find evidence for systematic effects caused by temperature-to-polarization leakage. These effects are at low levels and have little impact on the science conclusions of this paper.

3. We have presented the first results on polarization from the LFI at low multipoles. The LFI polarization data, together with Planck lensing and high-multipe spectrum temperature data, gives a reionization optical depth of $\tau = 0.066 \pm 0.016$ and a reionization redshift of $z_{\text{re}} = 8.8_{-1.2}^{+1.1}$. These numbers are in good agreement with those inferred from the WMAP9 polarization data cleaned for polarized dust emission using HFI 353-GHz maps. They are also in good agreement with results from Planck temperature and lensing data, i.e., excluding any information from polarization at low multipoles.

4. The absolute calibration of the Planck 2015 HFI spectra is higher by 2% (in power) compared to 2013, largely resolving

\(^{41}\)http://www.sfu.ca/~levon/cmbact.html

**Table 8.** 95% upper limits on the parameter $f_{10}$ and on the derived parameter $G\mu/c^2$ for the defect models discussed in the text. We show results for Planck TT+lowP data as well as for Planck TT,TE,EE+lowP.

<table>
<thead>
<tr>
<th>Defect type</th>
<th>TT+lowP $f_{10}$</th>
<th>TT,TE,EE+lowP $f_{10}$</th>
<th>TT,TE,EE+lowP $G\mu/c^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NG . . . . .</td>
<td>&lt;0.020 &lt;1.8x10^{-7}</td>
<td>&lt;0.011 &lt;1.3x10^{-7}</td>
<td></td>
</tr>
<tr>
<td>AH . . . . .</td>
<td>&lt;0.030 &lt;3.3x10^{-7}</td>
<td>&lt;0.015 &lt;2.4x10^{-7}</td>
<td></td>
</tr>
<tr>
<td>SL . . . . .</td>
<td>&lt;0.039 &lt;10.6x10^{-7}</td>
<td>&lt;0.024 &lt;8.5x10^{-7}</td>
<td></td>
</tr>
<tr>
<td>TX . . . . .</td>
<td>&lt;0.047 &lt;9.8x10^{-7}</td>
<td>&lt;0.036 &lt;8.6x10^{-7}</td>
<td></td>
</tr>
</tbody>
</table>

\(^{42}\)As in the abstract, here we quote 68% confidence limits on measured parameters and 95% upper limits on other parameters.
we find an even tighter constraint, favouring inflationary models with a $V_{\text{on}}$ term with the $B$ we derive a 95% upper limit of $\sigma_B$ for Planck. The $H$ is consistent with Type Ia SNe. The Hubble constant in this cosmology of $\Omega_K = 0$. The $H$ is higher than inferred from weak lensing measurements from this paper. The $H$ is in good agreement with those reported in PCP13.

(5) The Planck $TT$, $TE$, and $EE$ spectra are accurately described by a purely adiabatic spectrum of fluctuations with a spectral tilt $n_s = 0.968 \pm 0.006$, consistent with the predictions of single-field inflationary models. Combining Planck data with BAO, we find tight limits on the spatial curvature of the Universe, $|\Omega_K| < 0.005$, again consistent with the inflationary prediction of a spatially-flat Universe.

(6) The Planck data show no evidence for tensor modes. Adding a tensor amplitude as a one-parameter extension to base $\Lambda$CDM, we derive a 95% upper limit of $r_{\text{002}} < 0.11$. This is consistent with the B-mode polarization analysis reported in BKP, resolving the apparent discrepancy between the Planck constraints on $r$ and the BICEP2 results reported by BICEP2 Collaboration (2014). In fact, by combining the Planck and BKP likelihoods, we find an even tighter constraint, $r_{\text{002}} < 0.09$, strongly disfavouring inflationary models with a $V(\phi) \propto \phi^2$ potential.

(7) The Planck data show no evidence for any significant running of the spectral index. We also set strong limits on a possible departure from a purely adiabatic spectrum, either through an ad-mixture of fully-correlated isocurvature modes or from cosmic defects.

(8) The Planck best-fit base $\Lambda$CDM cosmology (we quote numbers for Planck $TT+lowP+lensing$ here) is in good agreement with results from BAO surveys, and with the recent JLA sample of Type Ia SNe. The Hubble constant in this cosmology is $H_0 = (67.8 \pm 0.9)$ km s$^{-1}$Mpc$^{-1}$, consistent with the direct measurement of $H_0$ of Eq. (30) used as an $H_0$ prior in this paper. The $Planck$ base $\Lambda$CDM cosmology is also consistent with the recent analysis of redshift-space distortions of the BOSS CMASS-DR11 data by Samushia et al. (2014) and Beutler et al. (2014). The amplitude of the present-day fluctuation spectrum, $\sigma_8$, of the Planck base $\Lambda$CDM cosmology is higher than inferred from weak lensing measurements from the CFHTLenS survey (Heymans et al. 2012; Erben et al. 2013) and, possibly, from counts of rich clusters of galaxies (including Planck cluster counts reported in Planck Collaboration XXIV 2016). The Planck base $\Lambda$CDM cosmology is also discordant with Ly$\alpha$ BAO measurements at $z \approx 2.35$ (Delubac et al. 2015; Font-Ribera et al. 2014). At present, the reasons for these tensions are unclear.

(9) By combining the Planck $TT+lowP+lensing$ data with other astrophysical data, including the JLA supernovae, the equation of state for dark energy is constrained to $w = -1.006 \pm 0.045$ and is therefore compatible with a cosmological constant, as assumed in the base $\Lambda$CDM cosmology.

(10) We have presented a detailed analysis of possible extensions to the neutrino sector of the base $\Lambda$CDM model. Combining Planck $TT+lowP+lensing$ with BAO we find $N_{\text{eff}} = 3.15 \pm 0.23$ for the effective number of relativistic degrees of freedom, consistent with the value $N_{\text{eff}} = 3.046$ of the standard model. The sum of neutrino masses is constrained to $\sum m_\nu < 0.23$ eV. The $Planck$ data strongly disfavour fully thermalized sterile neutrinos with $m_{\text{sterile}} = 1$ eV that have been proposed as a solution to reactor neutrino oscillation anomalies. From Planck, we find no evidence for new neutrino physics. Standard neutrinos with masses larger than those in the minimal mass hierarchy are still allowed, and could be detectable in combination with future astrophysical and CMB lensing data.

(11) The standard theory of big bang nucleosynthesis, with $N_{\text{eff}} = 3.046$ and negligible leptonic asymmetry in the electron neutrino sector, is in excellent agreement with Planck data and observations of primordial light element abundances. This agreement is particularly striking for deuterium, for which accurate primordial abundance measurements have been reported recently (Cooke et al. 2014). The BBN theoretical predictions for deuterium are now dominated by uncertainties in nuclear reaction rates (principally the $d(p, \gamma)^3$He radiative capture process), rather than from Planck uncertainties in the physical baryon density $\omega_B \equiv \Omega_B h^2$.

(12) We have investigated the temperature and polarization signatures associated with annihilating dark matter and possible deviations from the standard recombination history. Again, we find no evidence for new physics from the Planck data.

In summary, the Planck temperature and polarization spectra presented in Figs. 1 and 3 are more precise (and accurate) than those from any previous CMB experiment, and improve on the 2013 spectra presented in PCP13. Yet we find no signs for any significant deviation from the base $\Lambda$CDM cosmology. Similarly, the analysis of 2015 Planck data reported in Planck Collaboration XVII (2016) sets unprecedentedly tight limits on primordial non-Gaussianity. The Planck results offer powerful evidence in favour of simple inflationary models, which provide an attractive mechanism for generating the slightly tilted spectrum of (nearly) Gaussian adiabatic perturbations that match our data to such high precision. In addition, the Planck data show that the neutrino sector of the theory is consistent with the assumptions of the base $\Lambda$CDM model and that the dark energy is compatible with a cosmological constant. If there is new physics beyond base $\Lambda$CDM, then the corresponding observational signatures in the CMB are weak and difficult to detect. This is the legacy of the Planck mission for cosmology.

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Planck Collaboration: Cosmological parameters

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