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Research Article

Verifying the Equivalence of Representations of Joint Moment Vectors with
Application to Biomechanical Motion Capture Data from a Drop Vertical
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(November 2015)

Biomechanics software programs, such as Visual3D, Nexus, Cortex, and OpenSim, have the capability of generating several distinct component representations for joint moments and forces from motion capture data. These representations include those for orthonormal proximal and distal coordinate systems and a non-orthogonal joint coordinate system. In this article, a method is presented to address the challenging problem of evaluating and verifying the equivalence of these representations. The method accommodates the difficulty that there are two possible sets of non-orthogonal basis vectors that can be used to express a vector in the joint coordinate system and is illuminated using motion capture data from a drop vertical jump task.

Keywords: joint coordinate system; joint rotations; joint moments; joint forces; dual Euler basis

1. Introduction

The drop vertical jump task, Figure 1(a), has been championed by [Hewett, Myer, et al. \(2005\)](#) as a method to assess susceptibility to anterior cruciate ligament (ACL) injury. They have identified knee abduction moment (KAM) during the stance phase, Stages II-IV, of this maneuver as a key predictor in ACL injury risk. The KAM, which is a specific component of the knee joint moment vector $\mathbf{M}$, is featured in numerous other studies, including gait analysis research on degenerative knee joint diseases such as osteoarthritis. The moment $\mathbf{M}$ is not measured directly, rather estimates of certain components of $\mathbf{M}$ can be computed using kinematic and force plate data with the help of software packages such as Visual3D, Nexus, Cortex, and OpenSim.

Motivated by the desire to seek clinical relevant results, one can examine multiple representations for $\mathbf{M}$ and its components (see, e.g., [Schache and Baker \(2007\)](#), [Brandon and Deluzio \(2011\)](#), [Yamaguchi, Kitamura, et al. \(2015\)](#) and references therein). For instance, one can describe this vector using a basis fixed to the tibia, or a basis fixed to the femur, a laboratory fixed basis, or, dating to the seminal paper by [Grood and Suntay \(1983\)](#) on the knee joint, it is also common to describe the components of $\mathbf{M}$ in terms of a joint coordinate system. If a set of Euler angles is used to describe the relative rotation provided by the joint, then each of the three joint coordinate axes can be placed in one-to-one correspondence with the axes used to define the Euler angles. This set of axes forms a set of non-orthogonal unit vectors $\{\mathbf{g}_1, \mathbf{g}_2, \mathbf{g}_3\}$ which is known as the Euler basis. In addition, associated with every set of Euler angles, a dual Euler basis $\{\mathbf{g}^1, \mathbf{g}^2, \mathbf{g}^3\}$ is also defined. Thus multiple sets of distinct basis vectors can be associated with a given joint including

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Figure 1. (a) Schematic of the five stages of the drop vertical jump task. (b) Variation in the right knee joint angles during a drop vertical jump task. The stance phase starts at initial contact, Stage II, and ends at take-off, Stage IV. (c) Schematic of the right knee joint showing the proximal $\{p_1, p_2, p_3\}$ and distal $\{d_1, d_2, d_3\}$ bases which corotate with the femur and tibia, respectively. The Euler $\{g^1, g^2, g^3\}$ and dual Euler $\{g^1, g^2, g^3\}$ basis vectors associated with the rotation of this joint when it is parameterized using a set of 1-2-3 Euler angles ($1 = \alpha, 2 = \beta, 3 = \gamma$ as in Grood and Suntay (1983)) and the condyles C and D are also shown.

the orthonormal basis $\{p_1, p_2, p_3\}$ that corotates with the proximal body, the orthonormal basis $\{d_1, d_2, d_3\}$ that corotates with the distal body, the Euler basis, and the dual Euler basis. By way of illustration, a representative example of the progression of the joint rotation (Euler) angles α (flexion-extension), β (adduction-abduction), and γ (internal-external rotation) through Stages II-IV of the jump maneuver is shown in Figure 1(b) and, following O'Reilly, Sena, et al. (2013), the four aforementioned sets of basis vectors axes are sketched in Figure 1(c).

Software packages enable the output of multiple sets of components for $\mathbf{M}$. What is not apparent is how to check if the different sets of components are equivalent. This situation is particularly the case when the user has limited access to the algorithms producing the representations. An additional complication, that has only been recently appreciated, arises because the joint coordinate system purports, not one, but two sets of non-orthogonal basis vectors: the Euler $\{g^1, g^2, g^3\}$ and dual Euler $\{g^1, g^2, g^3\}$ bases vectors. **Our intent is to empower the research community with the ability to check output data.** In this article, we present a straightforward method to check the equivalence of the various component representations and use experimental data from a drop vertical jump task as a demonstration. **This method consists of understanding how the various components are related, how to convert between them, how to reconstruct the vector, and how to evaluate the magnitude of the vector.** We also note that the results shown in this paper are the first to explicitly demonstrate that knowledge of the dual Euler basis $\{g^1, g^2, g^3\}$ is needed to reconstruct $\mathbf{M}$ from its joint coordinate system components provided by popular biomechanics software packages.

To make this work as widely accessible as possible, a detailed discussion of the transformations between the various set of basis vectors and components can be found in the supplementary electronic resources that accompany this article. We also note that the work by Grood and Suntay (1983) prompted a series of related investigations and standardizations for other anatomical joints (see Baker (2003) and Wu, van der Helm, et al. (2005)). Consequently, many of the methods dis-

cussed in this paper can be used with other anatomical joints such as the ankle, shoulder, and elbow joints.

2. Methods

2.1 Acquisition of Sets of Kinematic and Force Data

Participant movement data was used from a larger motion analysis study to monitor return-to-play criteria after ACL injury. The study was approved by University of California, San Francisco, Committee on Human Research (IRB 12-10253). Forty eight retro-reflective spherical markers, 14 mm in diameter, were positioned on the participant on key bony landmarks and laterally on limb segments in rigid clusters of 4 markers. Three-dimensional kinematic data were collected at 250 Hz by a 10-camera system (Vicon, Oxford Metrics, UK). Kinetic data were collected at 1000 Hz by two embedded force platforms (AMTI, Watertown, MA USA).

In order to define the reference posture, a static trial was first conducted. Next, data were collected from a participant performing a drop vertical jump as described by [Hewett, Myer, et al. \(2005\)](#), sketched in Figure 1(a). The participant initially stands up-right at rest on a box. They are instructed to drop off the box onto a horizontal surface and immediately perform a maximum vertical jump, as if they were executing a basketball rebound. The stance/impact phase was defined as the participant's initial contact with the force platform following the drop, to the take-off for the maximum vertical jump. Kinematics for six-degree-of-freedom models of the thigh, shank, and foot along with force data from one of the force platforms were imported into Visual3D (C-Motion, Germantown, MD, USA). Marker position data was filtered using a 4th-order low-pass Butterworth filter with a cutoff frequency of 15 Hz. For this analysis, variables, including right knee joint angles and internal knee moment, calculated using the standard inverse dynamics in Visual3D, with components in the proximal, distal, and joint coordinate system (see (4)_{1,2,4} below), were exported from Visual3D to MATLAB (MathWorks, Natick, MA, USA). The moment in question is an estimate of the moment $\mathbf{M}$ at the proximal end of the shank (cf. [Robertson, Caldwell, et al. \(2013, pp.161–164\)](#)). MATLAB, in conjunction with the methods discussed below, was used to check the equivalence of representations of the moment in different coordinate systems.

2.2 Definition of the Segment Coordinate Systems

Segment coordinate systems for the thigh and shank are shown in Figure 1(c) and we take this opportunity to clearly define the associated axes. Segment frontal planes and joint centers are defined using a model within the motion analysis software in conjunction with marker data.

First, for the thigh, the superior-inferior axis, $\mathbf{p}_3$, is defined as passing between the hip and knee centers on the femoral frontal plane, and points towards the superior direction. The anterior-posterior axis, $\mathbf{p}_2$, is chosen to be normal to the femoral frontal plane and points towards the anterior direction. The medial-lateral axis, $\mathbf{p}_1$, is mutually perpendicular to $\mathbf{p}_2$ and $\mathbf{p}_3$, and is positive in (i.e., points towards) the lateral direction for the right side.

The corotational basis for the shank is specified by first defining the superior-inferior axis, $\mathbf{d}_3$, to pass between the knee and ankle centers on the tibial frontal plane and is positive in the superior direction. The anterior-posterior axis, $\mathbf{d}_2$, is normal to the tibial frontal plane and is positive in the anterior direction. Paralleling the specification of $\mathbf{p}_1$, the medial-lateral axis, $\mathbf{d}_1$, is mutually perpendicular to $\mathbf{d}_2$ and $\mathbf{d}_3$ and is positive in the lateral direction for the right side.

The bases $\{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3\}$ and $\{\mathbf{d}_1, \mathbf{d}_2, \mathbf{d}_3\}$ that have just been defined are right-handed. In the sequel, counterclockwise rotations are considered positive following the right-hand rule. Additional details on the specification of the distal and proximal bases sets can be found in the supplemental material.

2.3 Representations of Moments

We follow [Grood and Suntay \(1983\)](#) and assume that a 1-2-3 set of Euler angles ($1 = \alpha$, $2 = \beta$, $3 = \gamma$) is being used to parameterize the rotation of the knee joint. As mentioned previously, α denotes the flexion-extension angle, β denotes the adduction-abduction angle, and γ denote the internal-external rotation. In addition, we have the following Euler and dual Euler basis sets, respectively:¹

$$\mathbf{g}_1 = \mathbf{p}_1, \quad \mathbf{g}_2 = \frac{\mathbf{d}_3 \times \mathbf{p}_1}{\|\mathbf{d}_3 \times \mathbf{p}_1\|}, \quad \mathbf{g}_3 = \mathbf{d}_3, \quad (1)$$

and

$$\mathbf{g}^1 = \sec^2(\beta) \mathbf{g}_1 - \sec(\beta) \tan(\beta) \mathbf{g}_3, \quad \mathbf{g}^2 = \mathbf{g}_2, \quad \mathbf{g}^3 = -\sec(\beta) \tan(\beta) \mathbf{g}_1 + \sec^2(\beta) \mathbf{g}_3. \quad (2)$$

It should be noted that $\mathbf{g}^1$ and $\mathbf{g}^3$ have no apparent clinical interpretations unlike $\mathbf{g}_1$ and $\mathbf{g}_3$ which are chosen to align with the flexion-extension axis and internal-external rotation axis, respectively (cf. Figure 1(c)). Additional details on the Euler and dual Euler basis and their application to biomechanics can be found in [Desroches, Chèze and Dumas \(2010\)](#); [Dumas and Chèze \(2014\)](#); [O'Reilly \(2007, 2014\)](#) and [O'Reilly, Sena, et al. \(2013\)](#).

The moment vector $\mathbf{M}$ has a variety of representations:

$$\begin{aligned} \mathbf{M} &= M_1 \mathbf{p}_1 + M_2 \mathbf{p}_2 + M_3 \mathbf{p}_3 && \text{Proximal basis} \\ &= m_1 \mathbf{d}_1 + m_2 \mathbf{d}_2 + m_3 \mathbf{d}_3 && \text{Distal basis} \\ &= M_E^1 \mathbf{g}_1 + M_E^2 \mathbf{g}_2 + M_E^3 \mathbf{g}_3 && \text{Euler basis} \\ &= M_{E_1} \mathbf{g}^1 + M_{E_2} \mathbf{g}^2 + M_{E_3} \mathbf{g}^3. && \text{Dual Euler basis} \end{aligned} \quad (3)$$

Explicit relationships between the four sets of basis vectors appearing in these representations can be found in the supplemental material accompanying this article. The interested reader is also referred to [O'Reilly, Sena, et al. \(2013, Fig. 1\(c\)\)](#) for a graphical representation of (3)_{3,4}. The four sets of components in (3) are determined by projecting $\mathbf{M}$ onto a specific basis vector:

$$M_k = \mathbf{M} \cdot \mathbf{p}_k, \quad m_k = \mathbf{M} \cdot \mathbf{d}_k, \quad M_E^k = \mathbf{M} \cdot \mathbf{g}^k, \quad M_{E_k} = \mathbf{M} \cdot \mathbf{g}_k, \quad (k = 1, 2, 3). \quad (4)$$

As we shall see below, the moment components in a joint coordinate system are typically reported in the literature are M_{E_1} , M_{E_2} and M_{E_3} . That is,

$$\begin{aligned} M_{E_1} &= \mathbf{M} \cdot \mathbf{g}_1 = \mathbf{M} \cdot \mathbf{p}_1 = M_1, \\ M_{E_2} &= \mathbf{M} \cdot \mathbf{g}_2 = \mathbf{M} \cdot \left(\frac{\mathbf{d}_3 \times \mathbf{p}_1}{\|\mathbf{d}_3 \times \mathbf{p}_1\|} \right) = \mathbf{M} \cdot \mathbf{g}^2 = M_E^2, \\ M_{E_3} &= \mathbf{M} \cdot \mathbf{g}_3 = \mathbf{M} \cdot \mathbf{d}_3 = m_3. \end{aligned} \quad (5)$$

However, in order to show that the resulting vector is equivalent to its counterpart expressed in the proximal and distal bases, we need to compute $M_{E_1} \mathbf{g}^1 + M_{E_2} \mathbf{g}^2 + M_{E_3} \mathbf{g}^3$. That is, knowledge of the Euler basis is sufficient to compute the components M_{E_k} of $\mathbf{M}$ in the joint coordinate system, but, in order to construct the moment vector from these components, the dual Euler basis is required.

¹We tacitly assume that $\beta \neq \pm \frac{\pi}{2}$.

It is straightforward to show that $\mathbf{M} \neq M_{E_1}\mathbf{g}_1 + M_{E_2}\mathbf{g}_2 + M_{E_3}\mathbf{g}_3$. Indeed, using the identity (S.16) from the supplemental material accompanying this article, we find that

$$M_{E_1} = M_E^1 + \sin(\beta)M_E^3, \quad M_{E_3} = \sin(\beta)M_E^1 + M_E^3. \quad (6)$$

Thus, the difference in the components can be entirely attributed to the second Euler angle β . **Some models for the knee joint assume that β is small. Despite this assumption, we find data where the moment components differ substantially. In addition, this angle suffers from the ‘‘crosstalk’’ effect (cf. Dumas and Chèze (2014)) which can lead to challenges in comparing representations of $\mathbf{M}$.**

Concomitant with the representations for $\mathbf{M}$, related expressions for the magnitude $\|\mathbf{M}\|$ of $\mathbf{M}$ can be found by taking the inner (dot) product of this vector with itself and then taking the square root. After some rearranging, the resulting representations for $\|\mathbf{M}\|^2$ are

$$\begin{aligned} \|\mathbf{M}\|^2 &= \mathbf{M} \cdot \mathbf{M} \\ &= M_1^2 + M_2^2 + M_3^2 \\ &= m_1^2 + m_2^2 + m_3^2 \\ &= (M_E^1)^2 + (M_E^2)^2 + (M_E^3)^2 + \underbrace{2(M_E^1 M_E^3) \sin(\beta)} \\ &= (M_{E_1})^2 + (M_{E_2})^2 + (M_{E_3})^2 + \underbrace{\left((M_{E_1})^2 + (M_{E_3})^2 \right) \tan^2(\beta) - 2(M_{E_1} M_{E_3}) \sec(\beta) \tan(\beta)} \end{aligned} \quad (7)$$

The underbraced terms in these representations arise because of the non-orthogonality of the Euler and dual Euler basis vectors.

2.4 Signal Comparisons

In order to assess the equivalency of representations, software reported moment components M_k and m_k were used to provide estimates of M_{E_k} . The estimates for the components M_{E_k} obtained using M_k are denoted by B_{E_k} while the estimates for the components M_{E_k} obtained using m_k are denoted by b_{E_k} . The appropriate transformations are obtained using (S.15) in the supplemental data:

$$\begin{aligned} \begin{bmatrix} B_{E_1} \\ B_{E_2} \\ B_{E_3} \end{bmatrix} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & \sin(\alpha) \\ \sin(\beta) - \sin(\alpha)\cos(\beta) & \cos(\alpha)\cos(\beta) & \sin(\alpha)\sin(\beta) \end{bmatrix} \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix}, \\ \begin{bmatrix} b_{E_1} \\ b_{E_2} \\ b_{E_3} \end{bmatrix} &= \begin{bmatrix} \cos(\beta)\cos(\gamma) - \cos(\beta)\sin(\gamma)\sin(\beta) \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix}. \end{aligned} \quad (8)$$

An RMS (evaluated at 1% of stance intervals) was calculated for each of the three components (B_{E_k} and b_{E_k}) and compared to M_{E_k} . In addition, the moment magnitude $\|\mathbf{M}\|$ was calculated using M_k and m_k using (7)_{2,3} and compared to corresponding expression computed using the components M_{E_k} and (7)₅. Finally, moment vectors, $\mathbf{M}$ constructed from (3)_{1,2,4}, at 0% to 100% of stance in increments of 10% were all rotated into the proximal coordinate system and overlaid for qualitative comparison.

Figure 2. The moment vector $\mathbf{M}$ and four different representations of its components as a function of the percentage of the stance. (a) The components in the proximal basis $M_k = \mathbf{M} \cdot \mathbf{p}_k$, (b) the components in the distal basis $m_k = \mathbf{M} \cdot \mathbf{d}_k$, (c) the components $M_E^k = \mathbf{M} \cdot \mathbf{g}^k$ computed using $(S.16)_1$, and (d) the components in the joint coordinate system $M_{E_k} = \mathbf{M} \cdot \mathbf{g}_k$. As anticipated $M_{E_1} = M_1$ and $M_{E_3} = m_3$. The data shown in (a), (b), and (d) are imported directly from Visual3D.

3. Results and Discussion

Knee joint angle components from one repetition of a drop vertical jump are presented in Figure 1(b) and used in the calculations (7, 8) of the estimates $\|\mathbf{M}\|$, B_{E_k} , and b_{E_k} . Software produced moment components M_k , m_k and M_{E_k} for the drop vertical jump are shown in Figure 2(a,b,d). We find that the RMS differences between B_{E_k} and M_{E_k} are 0.0 Nm, 8.2×10^{-1} Nm, and 3.1×10^{-5} Nm, and between b_{E_k} and M_{E_k} are 4.4×10^{-6} Nm, 8.2×10^{-1} Nm, and 0.0 Nm, for the 1st, 2nd, and 3rd components, respectively. Moment magnitude from the software produced moment components M_k , m_k and M_{E_k} are displayed in Figure 3(a). The RMS difference between the $\|\mathbf{M}\|$ computed using M_k and M_{E_k} is 1.9×10^{-1} Nm, using m_k and M_{E_k} is 1.9×10^{-1} Nm, and using M_k and m_k is 4.4×10^{-6} Nm. Moment vectors, $\mathbf{M}$ constructed from the different bases, at increments of 10% were all rotated into the proximal coordinate system and overlaid in Figure 3(b). At each increment, the moment vectors visually align.

As mentioned earlier, KAM has been identified as a key indicator in ACL injury risk screening and knee osteoarthritis. This component of $\mathbf{M}$ is equivalent in the Euler and dual Euler basis, $M_{E_2} = M_E^2$. It is difficult to conclude if M_{E_2} is correct from the exported signal data (Figure 2(a,b,d)). However, using rotated M_k and m_k signals allows us to compare moment vector components. The small RMS values for M_{E_2} confirm the similarity of the signals. Both exported signal data and RMS values demonstrate M_1 aligns with M_{E_1} and m_3 aligns with M_{E_3} , as expected since $\mathbf{p}_1 = \mathbf{g}_1$ and $\mathbf{d}_3 = \mathbf{g}_3$. Reconstructing these components in the proximal coordinate system in Figure 3(b) also aids in confirming the equivalence of these signals.

Further comparison of overall magnitude gives confidence that we have obtained complete signals.

Figure 3. (a) Overlaid computations of the magnitude of the moment vector $\mathbf{M}$ using three representations (7)_{2,3,5} (i.e., the components in proximal, distal, and joint coordinate system representations). (b) The moment vectors $\mathbf{M}$, at each 10% of stance, obtained by reconstructing $\mathbf{M}$ from its components using (3)_{1,2,4}, rotated into the proximal basis, and overlaid. The axes in this figure are not to scale.

All magnitude comparisons yield small values of RMS difference, indicating equivalent signals. In this example, erroneously omitting the underbraced terms in the determination of $\|\mathbf{M}\|$ (cf. (7)₅) increases the RMS error value by a factor of 22. This magnitude calculation is dependent on β due to the additional terms introduced by the non-orthogonal dual Euler basis. Other joints such as the shoulder could see higher values of β , magnifying the error potential in the determination of $\|\mathbf{M}\|$, thus making the understanding and inclusion of the dual Euler basis even more critical.

4. Conclusions

To properly verify signals from commercially available software and confidently report findings from experimental data, it is crucial to understand the relationships between the different coordinate systems used to present biomechanical data. In this article, in an effort to empower the community with the ability to check software output data, a method has been presented to show how four distinct sets of moment components are related, how to convert between them, how to reconstruct the vector, and how to evaluate the magnitude of the vector. Of the four distinct sets, we note that the commonly used moment components in a joint coordinate system (i.e., M_{E_k}) require the dual Euler basis both for the reconstruction of the moment vector, $\mathbf{M} = M_{E_1}\mathbf{g}^1 + M_{E_2}\mathbf{g}^2 + M_{E_3}\mathbf{g}^3$, and the accurate determination of its magnitude $\|\mathbf{M}\|$. Other applications for the methods presented in this paper include affording the research community the ability to convert and compare moment signals reported in the literature into different bases when the appropriate angle data are also reported, as well as the ability for a research group to transition data between bases when this data was not originally included in the software output. The authors encourage the inclusion and use of the dual Euler basis in biomechanics education curriculum so that a more comprehensive understanding of the kinetic quantities described using the joint coordinate system can be achieved.

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Anderson, et al. 2007).

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Conflict of Interest Disclosure Statement

No potential conflict of interest was reported by the authors.

Supplemental Material

To make this work as widely accessible as possible, a detailed discussion of the transformations between the various set of basis vectors and components can be found in the supplementary material that accompany this article.

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