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A SEMI-ANALYTICAL SOLUTION FOR PARTIAL PENETRATION IN TWO-LAYER AQUIFERS

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ABSTRACT

A semi-analytical solution is presented to the problem of drawdown distribution in a two-layer aquifer when it is pumped from a well that is partially penetrating in one of the layers. The solution is used to illustrate the effects on aquifer behavior of partial penetration as well as the effect of a contrast in flow properties between the two layers. The validity of the solution has been verified against four available limiting cases. A method for analysing field data is proposed, and an example is given to illustrate the procedure.

INTRODUCTION

Most aquifers in nature are more or less heterogeneous. A very common type of heterogeneity is found in stratified formations where the hydraulic properties of the porous media change from one layer to another. It is of great interest to predict the behavior of such aquifers, when subjected to either withdrawal or injection operations.

Because of mathematical difficulties, the analysis of transient fluid flow in multi-layered aquifers has not received a great deal of attention. A rather simple case is that of a two-layer aquifer with no crossflow, i.e., the hydraulic connection between layers occurs only at the pumping well. Lefkovits, et al. [1961] solved this problem for a bounded reservoir composed of two or more horizontal layers, when the pumping well is fully penetrating and the rate of discharge is held constant. Papadopulos [1966] has studied the above case for two aquifers of infinite radial extent.

A more complex case of layered aquifer occurs when the layers are hydraulically connected throughout their interface. Katz [1960] and Russel and Prats [1962], using different methods, have handled this problem for a bounded reservoir composed of two or more horizontal layers with a pumping well that is fully penetrating and a fluid level that is kept constant in the pumping well (constant terminal pressure). Due to a convergence problem, Katz's solution does not lend itself to numerical evaluation when the radius of the well is less than ten times the thickness of the aquifer; and, consequently, it cannot be applied to groundwater problems.

A more practical case occurs when the rate of discharge, rather than the water level, is held constant. Jacquard [1960] has solved this problem when the pumping well penetrates the total thickness of the aquifer. So far, no numerical results have been obtained directly from his equations. Pelissier and Sequier [1961] have been able to invert the expression which Jacquard derived in the transform domain, to obtain the pressure history at the well only.

In addition to the above analytical studies, there have also been several numerical approaches to the layered aquifer problem. Vacher and Cazbat [1961] have used a finite difference method to obtain pressure distributions in a two layer system with cross flow when a fully penetrating well is pumped at constant rate. Javandel and Witherspoon [1968a, 1969] applied the finite element method to solve problems of flow in multilayered aquifers.

It often happens that the pumping well does not penetrate or is not open over the whole thickness of the aquifer. The problem of partial penetration in a multilayered aquifer is one of the most complex to handle analytically. Clegg and Mills [1969] have considered a two-layer aquifer where both layers have finite thickness, and the pumping well completely penetrates the top layer. They found that even for this special case, the final solution could only be obtained when both layers had the same formation parameters. In effect this converts the problem into a single layer, partial penetration problem that was solved much earlier by Hantush [1957].

Pizzi et al. [1965] used an electric analog model to study the effect of stratification on the performance of a well when it is only partially penetrating. This study revealed that the effect of stratification within the aquifer on the behavior of a partially penetrating well appeared to be like that of an extremely high, so called, "apparent skin factor." Kazemi and Seth [1969], have applied a finite difference technique to study the effect of anisotropy and stratification in a reservoir on pressure transient behavior of wells with restricted flow entry.

The above workers have been primarily interested in effects at the producing well because this is important in the field of petroleum engineering. In groundwater studies, however, one is often interested in the behavior of the aquifer away from the pumping well.

In this paper, we shall present a semi-analytical solution for drawdown distribution in a two layer aquifer drained by a well which partially penetrates only the top layer. The lower layer is considered to be very thick relative to the upper layer. Since evaluation of the final solution is quite difficult, numerical inversion of the Laplace transformation has been applied to evaluate some practical cases and to study the effect of the parameters involved. A typcial example where such a problem is commonly encountered involves relatively thin sands which overlay thick chalk in the London aquifer.

THEORY

Let us consider a mathematical model consisting of a system of a two-layered aquifer. As illustrated in Figure 1, each layer has its own flow properties and extends radially to infinity. The top layer has a finite thickness h and the lower one is relatively very thick so that mathematically it behaves as a semi-infinite medium. It is assumed that both layers remain saturated throughout the period of investigation. It is also assumed that the initial drawdown is zero throughout the system. We require that the interface between the two layers have a perfect hydraulic contact and the upper boundary of the top layer to be impermeable.

A well with infinitesimal radius has been placed in the top layer and is open along the length ℓ from the top of the aquifer. This well will be pumped at a constant rate Q over a period of time, t. The problem is to determine the drawdowns at any point of the system as a function of time.

The differential equation and initial and boundary conditions for this problem can be written as:

$$\nabla^2 s_i = \frac{1}{\alpha_i} \frac{\partial s_1}{\partial t} \qquad i = 1,2 \qquad (1)$$

$$s_i(r,z,0) = 0 \qquad (2)$$

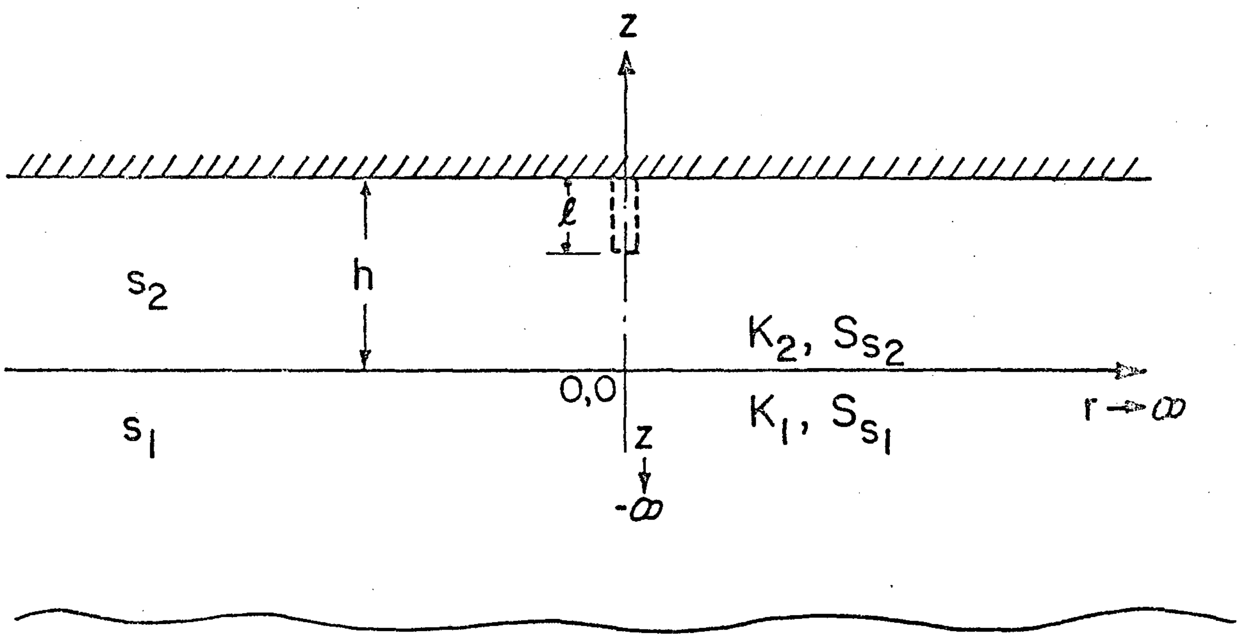

Fig. 1. Diagram of a two layer aquifer system with partially penetrating well.

$$s_1(r,0,t) = s_2(r,0,t) \quad (3)$$

$$K_1 \frac{\partial s_1}{\partial z} = K_2 \frac{\partial s_2}{\partial z} \qquad \text{at } z = 0 \quad (4)$$

$$\frac{\partial s_2}{\partial z}(r,h,t) = 0 \quad (5)$$

$$\lim_{r\to\infty} s_i(r,z,t) = 0 \quad (6)$$

$$\lim_{z\to-\infty} s_1(r,z,t) = 0 \quad (7)$$

$$\lim_{r\to 0} 2\pi K_2 r \int_{h-\ell}^{h} \frac{\partial s_2}{\partial r}\, dz = -Q \quad (8)$$

$\overline{s}_1$ and $\overline{s}_2$ as given below represent the drawdowns in the Laplace transform domain due to a continuous point sink with unit strength at the point ($z = z_0$, and $r = 0$), in layer 1 and 2 respectively.

$$\overline{s}_1(r,z,\eta) = \int_0^\infty \frac{\xi J_0(\xi r) M}{2\pi\eta\alpha_2} \frac{e^{\beta z - \gamma z_0}}{M\gamma+\beta}\, d\xi \quad (9)$$

$$\overline{s}_2(r,z,\eta) = \int_0^\infty \frac{\xi J_0(\xi r)}{4\pi\eta\alpha_2} \left\{ e^{-|z-z_0|\gamma} + \frac{M\gamma-\beta}{M\gamma+\beta}\, e^{-(z+z_0)\gamma} \right\} d\xi \quad (10)$$

$\overline{s}_1$ and $\overline{s}_2$, as given above, satisfy equations (1) through (4) as well as (6) and (7) in the Laplace transformed domain, (Javandel and Witherspoon, 1968b). An examination of the above two equations reveals that if we consider the whole system to have the properties of layer 2, drawdown in the top layer of this system is due to a sink of unit strength at the point $z = z_0$ as well as a sink at the point $z = - z_0$ but with a strength of $\frac{M\gamma-\beta}{M\gamma+\beta}$. Drawdown in the lower layer is due to a sink at $z = z_0/\sqrt{D}$ of strength $\frac{2M\beta}{M\gamma+\beta}$. Since $\overline{s}_1$ is expressed in the transformed domain in equation (9), the apparent location of this sink is at $z = \frac{\gamma}{\beta} z_0$. In this latter equation the whole system has the properties of the lower layer. One can now introduce the well known method of images to satisfy the existence of the no flow boundary at z = h. As a result, if we now set $A = \frac{M\gamma-\beta}{M\gamma+\beta}$ the following two equations are obtained which will also satisfy condition (5).

$$\bar{s}_1(r,z,\eta) = \int_0^\infty \frac{\xi J_o(\xi r)}{4\pi\eta\alpha_2\gamma} (1 + A)\left[e^{-\beta(\frac{\gamma}{\beta} z_0 - z)} + e^{-\beta[(2h-z_0)\frac{\gamma}{\beta} - z]} + \sum_{n=1}^{\infty} A^n\left\{e^{-\beta[(2nh+z_0)\frac{\gamma}{\beta} - z]} + e^{-\beta[(2n+2)h - z_0]\frac{\gamma}{\beta} + z\beta}\right\}\right] d\xi \tag{11}$$

$$\bar{s}_2 \int_0^\infty \frac{\xi J_o(\xi r)}{4\pi\eta\alpha_2\gamma}\left[e^{-\gamma|z-z_0|} + \sum_{n=1}^{\infty} A^n\left(e^{\gamma[z-(\ell nh+z_o)]} + e^{-\gamma[z+(2nh-z_o)]}\right) + \sum_{n=0}^{\infty} A^n\left(Ae^{-\gamma[z+(2nh+z_0)]} + e^{\gamma[z+z_0-h(2n+2)]}\right)\right] d\xi \tag{12}$$

Integrating equations (11) and (12) with respect to z_0 from h-ℓ to h, and adjusting for the strength of the sink, leads to the follwoing equations which represent drawdown distribution due to a well of infinitesimal radius operating at constant rate.

$$\bar{s}_1(r,z,\eta) = \int_0^\infty \frac{Q\xi J_o(\xi r)\ (1+A)}{4\pi\ell K_2\eta\ \gamma^2} e^{\beta z}\left[e^{-\gamma(h-\ell)} - e^{-\gamma(h+\ell)} + \sum_{n=1}^{\infty} A^n\left(e^{-\gamma[(2n+1)h-\ell]} - e^{-\gamma[(2n+1)h+\ell]}\right)\right] d\xi \tag{13}$$

$$\bar{s}_2(r,z,\eta) = \int_0^\infty \frac{Q\xi J_o(\xi r)}{4\pi\ell K_2\eta\gamma^2}\left\{e^{-\gamma(h-\ell-z)} - e^{-\gamma(h+\ell-z)} + f\right\} d\xi \tag{14}$$

for $z < h - \ell$

$$\bar{s}_2(r,z,\eta) = \int_0^\infty \frac{Q\xi J_o(\xi r)}{4\pi \ell K_2 \eta\gamma^2} \left\{2 - e^{\gamma(h-\ell-z)} - e^{-\gamma(h+\ell-z)} + f\right\} d\xi \tag{15}$$

for $z > h - \ell$

where

$$f = \sum_{n=1}^{\infty} A^n \Big(e^{\gamma[z+\ell-h(2n+1)]} - e^{\gamma[z-\ell-h(2n+1)]} + e^{-\gamma[z-\ell+h(2n-1)]} - e^{-\gamma[z+\ell+h(2n-1)]}\Big) \tag{16}$$

If we introduce the following dimensionless terms: $s_D = \frac{4\pi K_2 hs}{Q}$, $t_D = \frac{\alpha_2 t}{r^2}$, $r_D = r/h$, $\ell_D = \ell/h_1$ and $z_D = z/h$, equations 13 through 16 can be written

$$\bar{s}_{D_1} = \frac{1}{\ell_D} \int_0^\infty \frac{\xi J_o(\xi r_D)}{\eta\gamma^2} (1+A)\, e^{\beta z_D} \Big[e^{-\gamma(1-\ell_D)} - e^{-\gamma(1+\ell_D)} + \sum_{n=1}^{\infty} A^n \Big(e^{-\gamma[(2n+1)-\ell_D]} - e^{-\gamma[(2n+1)+\ell_D]}\Big)\Big] d\xi \tag{17}$$

$$\bar{s}_{D_2} = \frac{1}{\ell_D} = \int_0^\infty \frac{\xi J_o(\xi r_D)}{\eta\gamma^2} \left\{e^{-\gamma(1-\ell_D-z_D)} - e^{-\gamma(1+\ell_D-z_D)} + f_D\right\} d\xi \tag{18}$$

for $z_D < 1 - \ell_D$

$$\bar{s}_{2_D} = \frac{1}{\ell_D} \int_0^\infty \frac{\xi J_o(\xi r_D)}{\eta\gamma^2} \left\{2 - e^{\gamma(1-\ell_D-z_D)} - e^{-\gamma(1+\ell_D-z_D)} + f_D\right\} d\xi \tag{19}$$

for $z_D > 1 - \ell_D$

where

$$f_D = \sum_{n=1}^{\infty} A^n \Big(e^{\gamma[z_D+\ell_D-(2n+1)]} - e^{\gamma[z_D-\ell_D-(2n+1)]} + e^{-\gamma[z_D-\ell_D+(2n-1)]} - e^{-\gamma[z_D+\ell_D+(2n-1)]}\Big) \tag{20}$$

Analytical inversion of equations (18) and (19) is quite tedious and once obtained the results do not lend themselves easily to numerical evaluation.

Therefore, in order to draw meaningful results from these equations one can apply numerical methods of inversion.

NUMERICAL INVERSION OF RESULTS

Numerical inversion of the Laplace transform has long been used for solving all kinds of engineering problems [Bellman, et al. 1966]. Several different methods are available which can be employed for numerical inversion, depending on the characteristics of the function to be inverted and the degree of accuracy that is required. A brief review of some common methods together with their application to groundwater problems is given elsewhere (Javandel, 1976).

Here, a method after Bellman has been utilized for the inversion of $\bar{s}_D$. In this method, the inverse of $\bar{s}_D$ at a specified dimensionless time t_{D_i} may be obtained from the following formula:

$$s_D(t_{D_i}) = g(X_i) = \sum_{k=0}^{N-1} a_{ik}\, \bar{s}_D(k+1) \qquad i = 1,2,3...N \tag{21}$$

In the above equation, X_i are zeros of the shifted Legendre polynomial and

$$t_{D_i} = -\ln X_i \tag{22}$$

Extensive tables of the matrix a_{ik} are given by Bellman et al. [1966]. The zeros of the shifted Legendre polynomial are bounded between zero and unity, and thus, one would expect to cover a time range of $(0,\infty]$. In practice, however, only a small range of time is obtained. In order to expand the range of t_D, one may note that:

$$L\,\{s_D(at_D)\} = \int_0^\infty e^{-nt_D}\, s_D(at_D)\, dt_D = \frac{\bar{s}_D(n/a)}{a} \tag{23}$$

Hence, if in equation (21) one uses $\frac{\bar{s}_D(1/a)}{a}$, $\frac{\bar{s}_D(2/a)}{a}$, . . . in place of $\bar{s}_D(1)$, $\bar{s}_D(2)$, . . ., the values of t_D at which each numerical inversion is calculated would become:

$$t_{D_i} = - a \ln X_i \tag{24}$$

Throughout this study N=15 has been used in equation (21). Since the reliability of this method rests on the accuracy of the calculation of $\bar{s}_D(k+1)$, integration of equations (17) through (19) has been performed by a forty point Gauss-Laguerre quadrature formula. Elements of the series in equations (17) and (20) each represent the contribution of imaginary sinks above and below the top layer of the aquifer and therefore will vanish very rapidly when the sinks are at greater distances from the zone of interest.

With regard to the stability of the procedure, due to the unboundedness of the Laplace inverse operator, an arbitrary small error in calculating $\bar{s}_D$ can produce an arbitrarily large error in the value of s_D. However, the values of matrix a_{ik} have been accurately calculated and if one uses ordinary precautions, results can be obtained with any level of accuracy desired. No instabilities were observed in carrying out these numerical calculations.

VERIFICATION OF THE SOLUTION

Results obtained from equations 17 through 20 were verified against four limiting cases.

(1) The present solution should converge to the Theis solution if we set the depth of penetration of the pumping well equal to the total thickness of the top layer and the permeability of the lower layer vanishes. This has been checked analytically by letting $\ell_D = 1$ and $A = 1$ in equations (18) and (19). It has also been checked directly by letting $\ell_D = A = 1$ in the program and the results are shown on Figure 2. Agreement between the limiting case of the present solution and the Theis is excellent.

(2) When the permeability of the lower layer is set to zero, the solution should match the case of Hantush's [1957] solution for partial penetration in

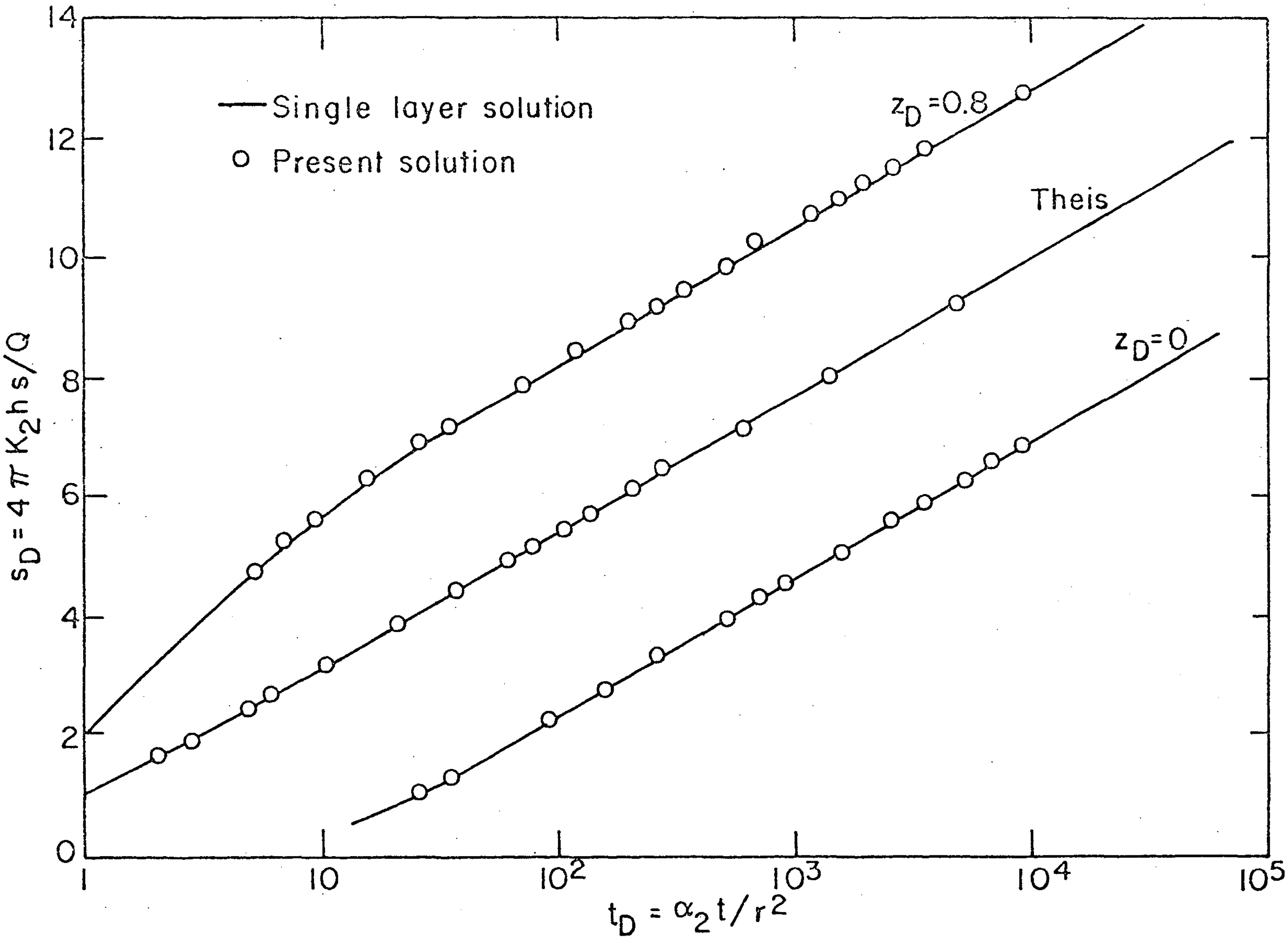

Fig. 2. Comparison of the present solution with the single layer partial penetration.

a single layer. This will lead to A = 1, and Figure 2 includes a comparison of our solution with the single layer solution for ℓ_D = 0.5, r_D = 0.1 and Z_D = 0.8 and 0.0.

(3) If flow properties of both layers are identical, then the solution should merge to the one given by Saad [1960], for a thick artesian aquifer. This can easily be verified by letting M = D = 1, which will lead to A = 0.

(4) When the pumping well penetrates all the way through the top aquifer and $K_1 << K_2$, then one would expect that, at least at early time, our solution should agree with the leaky aquifer theory of Hantush [1960] for an infinitely thick caprock. We examined this by letting r_D = 0.1, K_2/K_1 = 625, and S_{s_2}/S_{s_1} = 1 from which

$$\beta = \frac{r}{4h}\sqrt{\frac{K_1\ S_{s_1}}{K_2\ S_{s_2}}} = \frac{0.1}{4}\sqrt{\frac{1}{625}} = 0.001$$

Figure 3 shows the good agreement between Hantush's solution and ours for β = 0.001. As β increases, however, this agreement will only occur at early time. To demonstrate this, we set r_D = 1.5, K_2/K_1 = 10, and S_{s_2}/S_{s_1} = 1 for which β = 0.118. Figure 3 shows how results from the two solutions deviate as $t_D > 1$. These differences are to be expected because Hantush assumed vertical flow in the confining layer, and this will not hold when K_2/K_1 is as small as 10. Our new solution can thus be used to determine limiting conditions for the applicability of Hantush's [1960] leaky aquifer theory and the subsequent work on this problem by Neuman and Witherspoon [1969a, 1969b].

DISCUSSION OF RESULTS

From equations 17-19, we note that in order to investigate the variation of drawdown with time, we must consider the effects of five parameters: ℓ_D, r_D, z_D, M and D. It is not practical to attempt to tabulate solutions to these equations here, but an extensive table covering a wide range in the parameters is being prepared as a separate report. A limited number of results will be

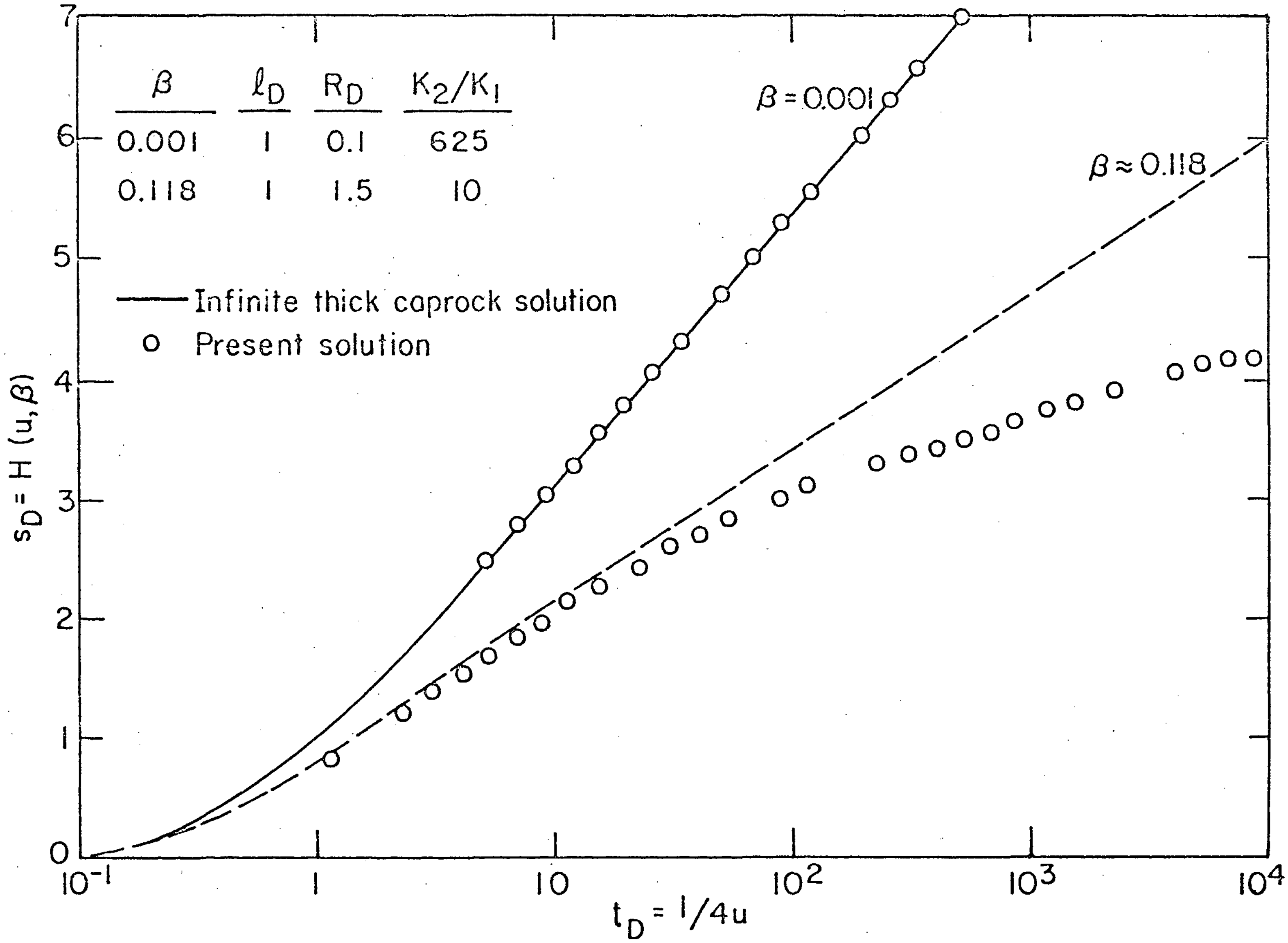

Fig. 3. Comparison of the present solution with the Hantush's infinitely thick caprock.

presented in the form of graphs to illustrate some interesting points.

At small values of time, drawdown in the aquifer (layer 2) is similar to that of the single layer partial penetration problem. This can be seen on Figure 4 where curves for z_D from 0.2 to 1.0 all coincide with single layer results for $t_D \leq 50$. Later on when the contribution of the lower layer arrives at the point, the amount of drawdown drops from its corresponding value for a single layer partial penetration problem. At larger values of z_D the effect of the lower layer is sensed at a later time which in effect causes a larger value for departure time. These results were obtained for $r_D = 0.1$ and Figure 5 shows the effect of increasing to $r_D = 0.5$. To avoid crowding the figure, the solution of the single layer partial penetration problem has been shown only for $z_D = 1$. We see that at greater distance from the pumping well, the family of nondimensional curves of the drawdown is more compact (note that the vertical scale on Figure 5 has been enlarged by a factor of two). This suggests that the effect of partial penetration diminishes when we go farther away from the pumping well. We may also note that the time of departure of the single layer solution from ours has been moved to a t_D about 25 times smaller than that for the case of $r_D = 0.1$. In fact an approximate formula for departure time may be given as

$$(t_D)_d = \frac{(1 - \ell_D + z_D)^2}{5 r_D^2} \quad .$$

When $\ell_D = 1$, which means the pumping well is open all the way through the top layer, the solution presented stands for the case of an aquifer which is overlain by a relatively thick leaky layer. This solution does not have the restriction that is considered in almost all available solutions for leaky aquifer, where the flow in the aquitard is considered to be one dimensional and vertical and in the aquifer itself horizontal. Figure 6 shows the variation of dimensionless drawdown versus dimensionless time at

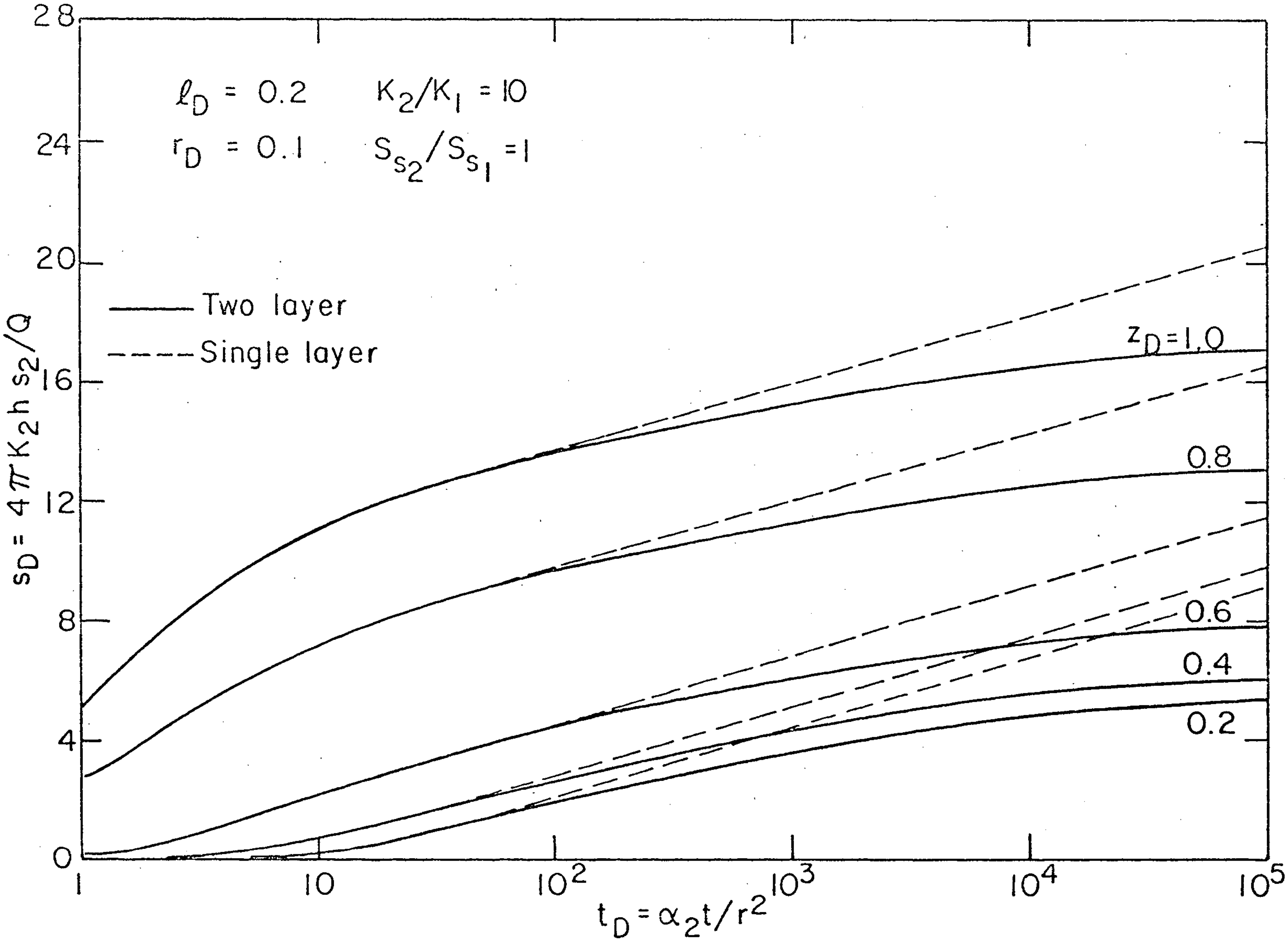

Fig. 4. Variation of dimensionless drawdown versus dimensionless time, for two-layered as well as single layer aquifer.

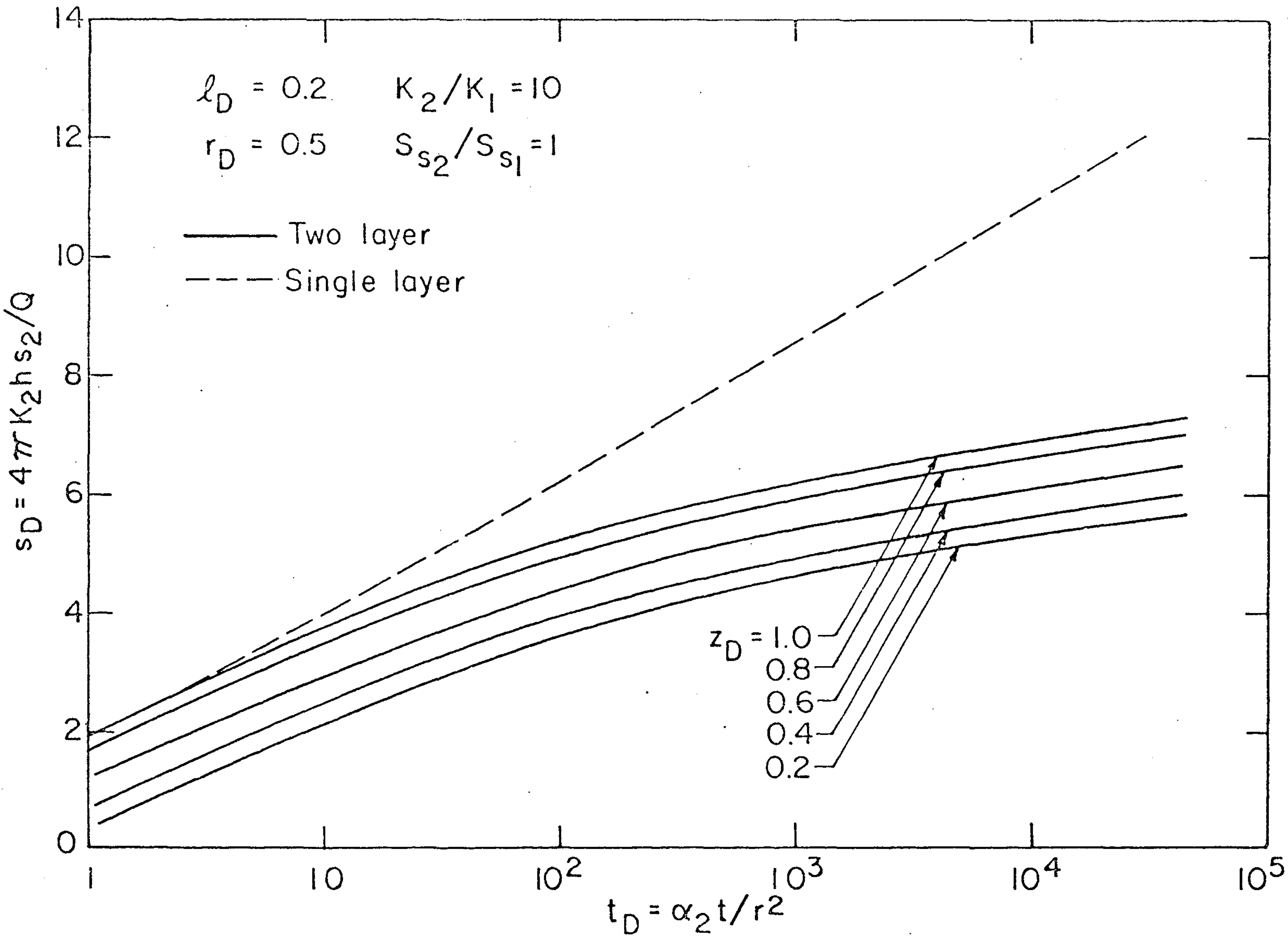

Fig. 5. Variation of dimensionless drawdown versus dimensionless time.

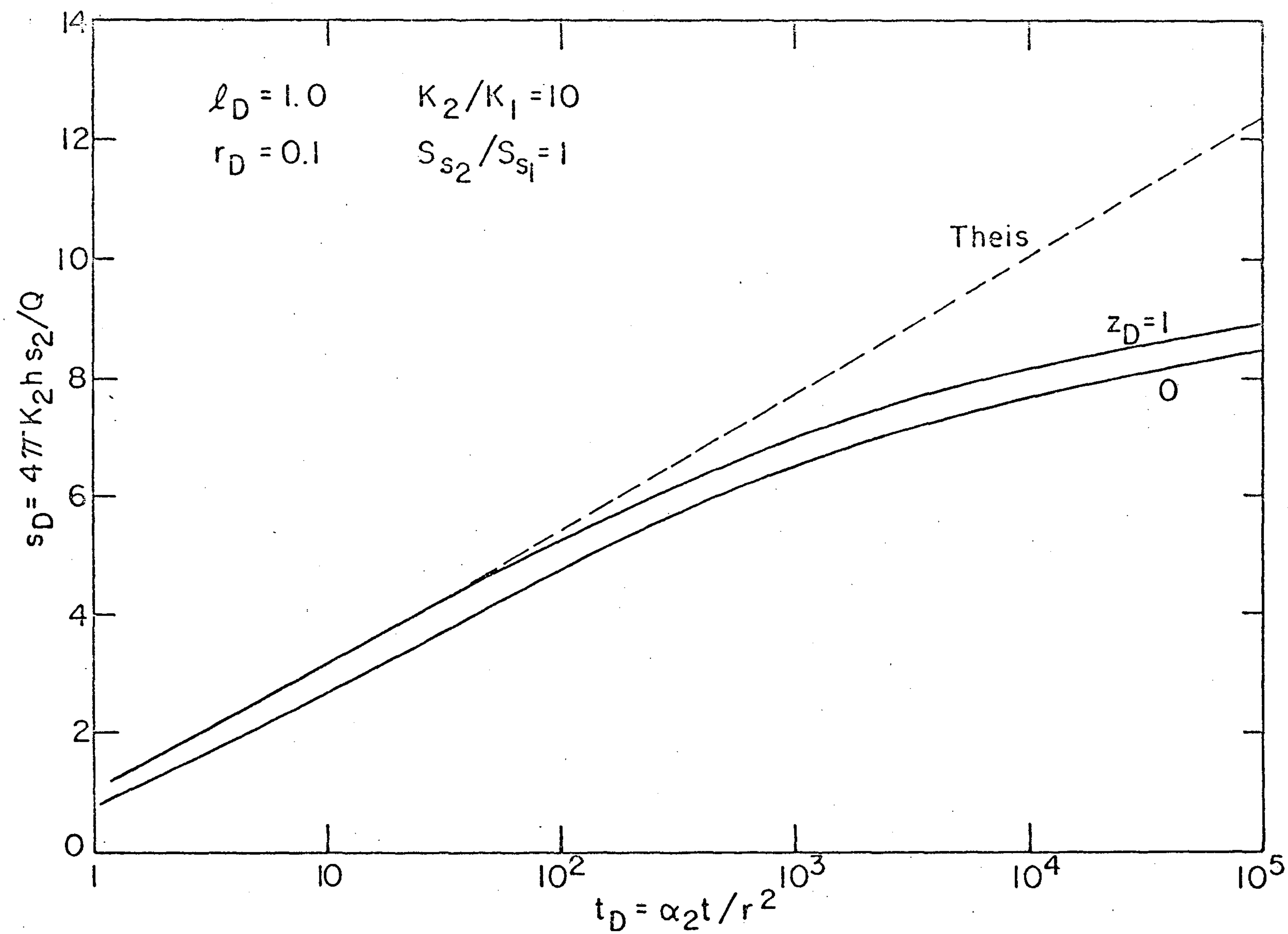

Fig. 6. Variation of dimensionless drawdown versus dimensionless time.

the top and bottom of the top layer, where $K_2/K_1 = 10$, at $r_D = 0.1$. Theis curve has also been shown for the sake of reference. This figure shows that in fact, equipotentials are not vertical in the aquifer.

In the case of single layer partial penetration, it was observed that for relatively large distances from the pumping well, (r greater than one and a half times the thickness of aquifer), the effect of partial penetration would vanish and the aquifer behaves as if the pumping well was fully penetrated, Hantush [1957] and Javandel and Witherspoon [1967]. Here, the same phenomena is observed, except for the fact that, the effect of leakage from lower layer will still be manifested. Figure 7 shows that for penetration depth of ℓ_D = 0.2 when $r_D = 1.5$ the corresponding curves essentially coincide with the case of full penetration i.e. ($\ell_D = 1.0$, $r_D = 1.5$).

As mentioned above, for earlier times after the starting of pumping the response of the aquifer is as if the lower layer were absent. However, later on the behavior is completely different and the amount of deviation from a single layer case depends on the contrast of permeability of the two layers. Figure 8 shows the effect of permeability contrast for the case of $\ell_D = 0.5$, $z_D = 0.4$, and $r_D = 0.1$. In order to illustrate the effectiveness of the lower layer in terms of leakage, Figure 9 has been prepared for the same parameters as of Figure 8. This figure shows the difference between drawdown in the single layer and that of the two layer relative to the single layer solution versus dimensionless time. The area under each curve, at a certain time, indicates the percent of the total volume of fluid drawn from the lower layer up to that time for that permeability ratio.

One may note that just for the sake of convenience, in the above cases, the ratio of specific storage in two layers has been assumed to be unity. Any other value can be easily applied, without introducing any complexity. For example Figure 10 shows the effect of contrast of the specific storage

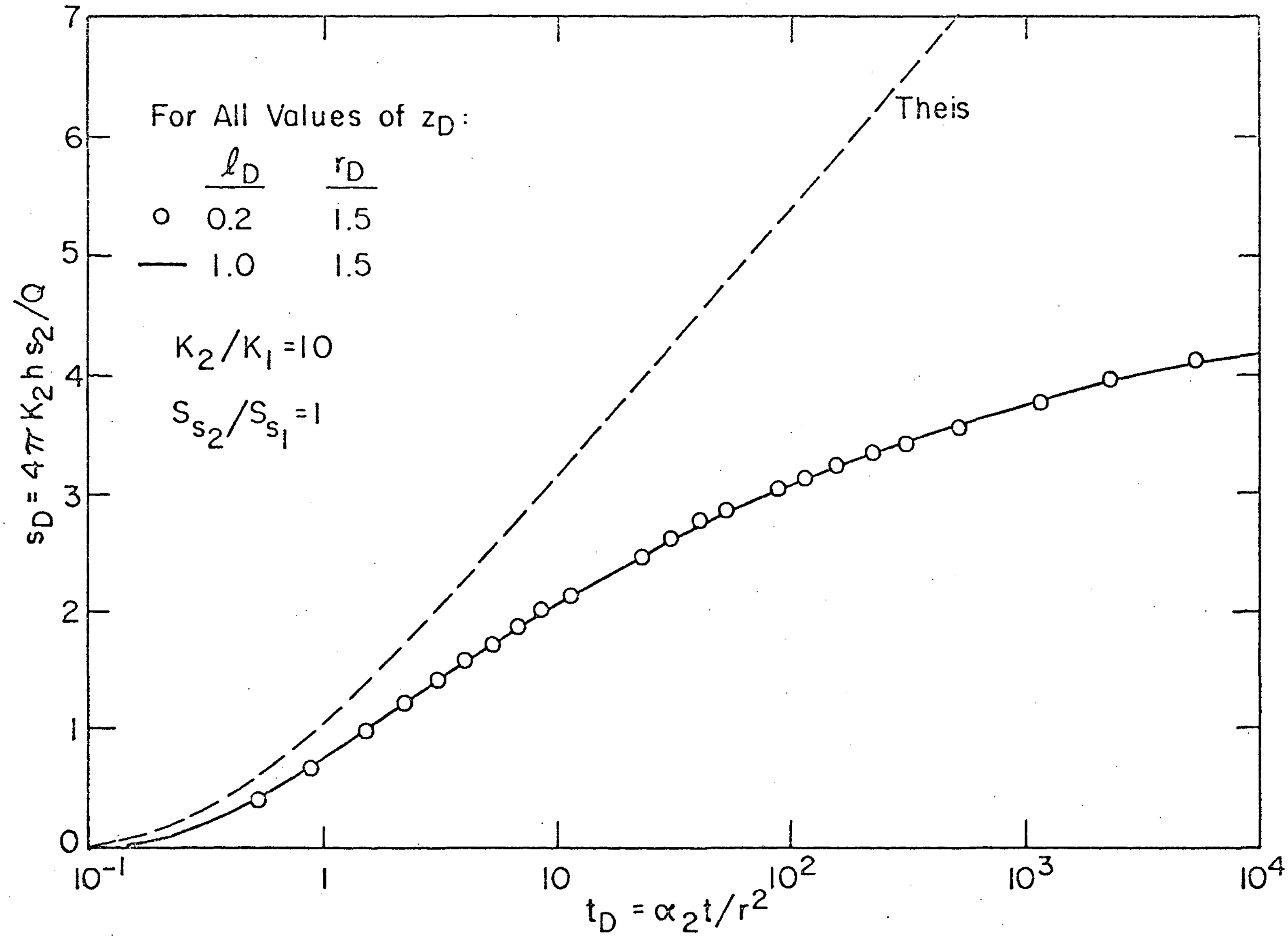

Fig. 7. Variation of dimensionless drawdown versus dimensionless time.

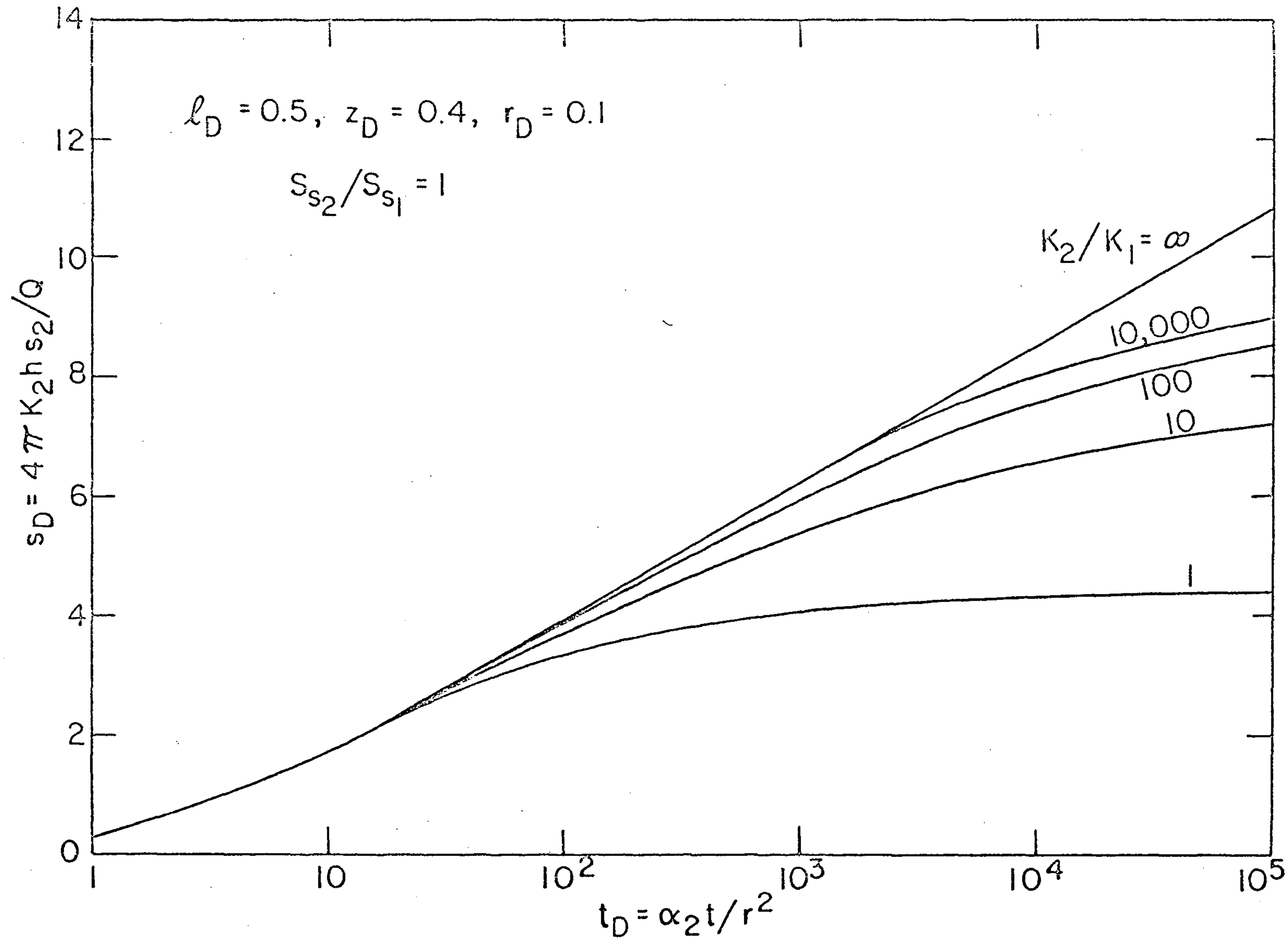

Fig. 8. Effect of permeability contrast.

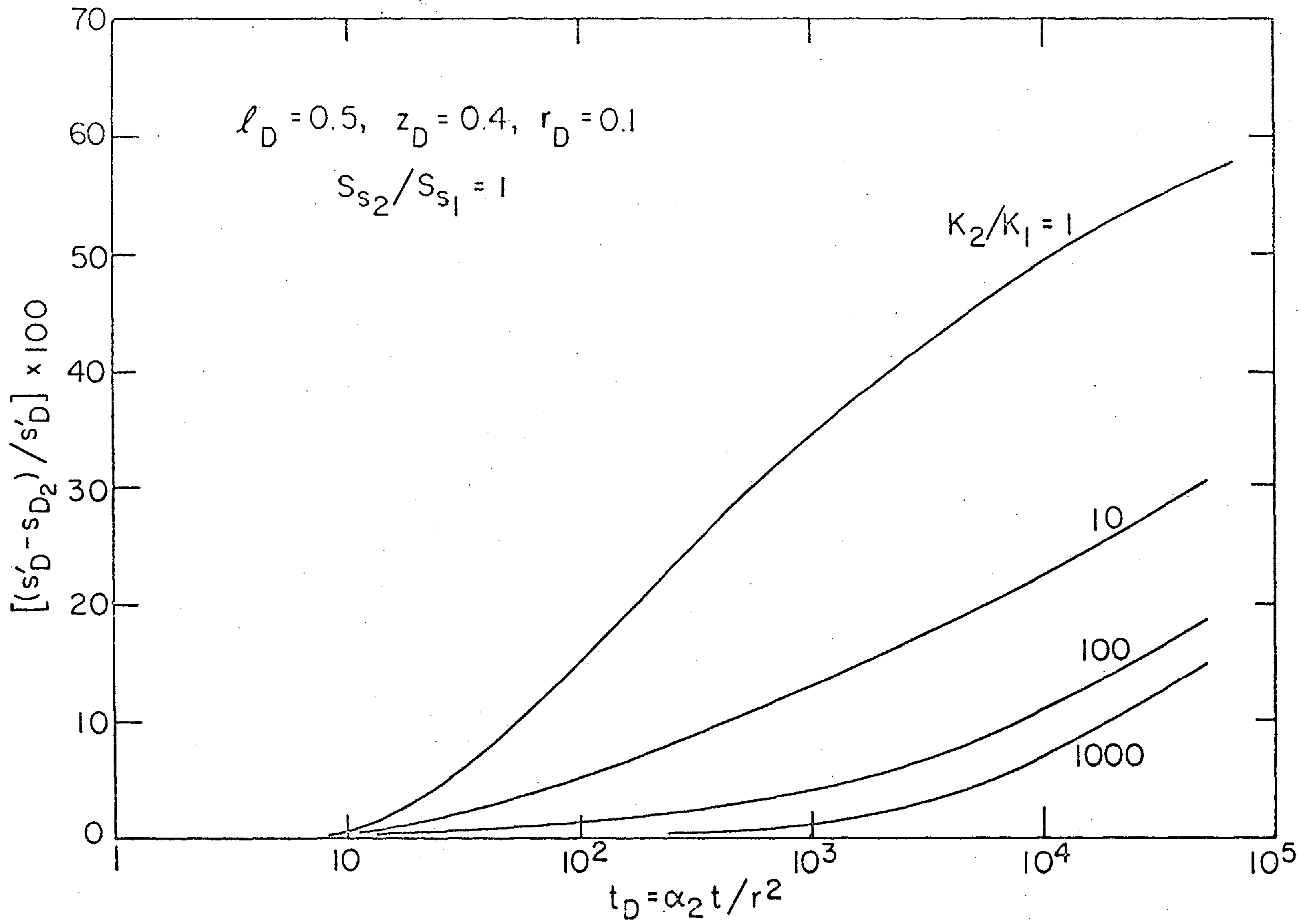

Fig. 9. Percentage of less drawdown due to leakage of lower layer.

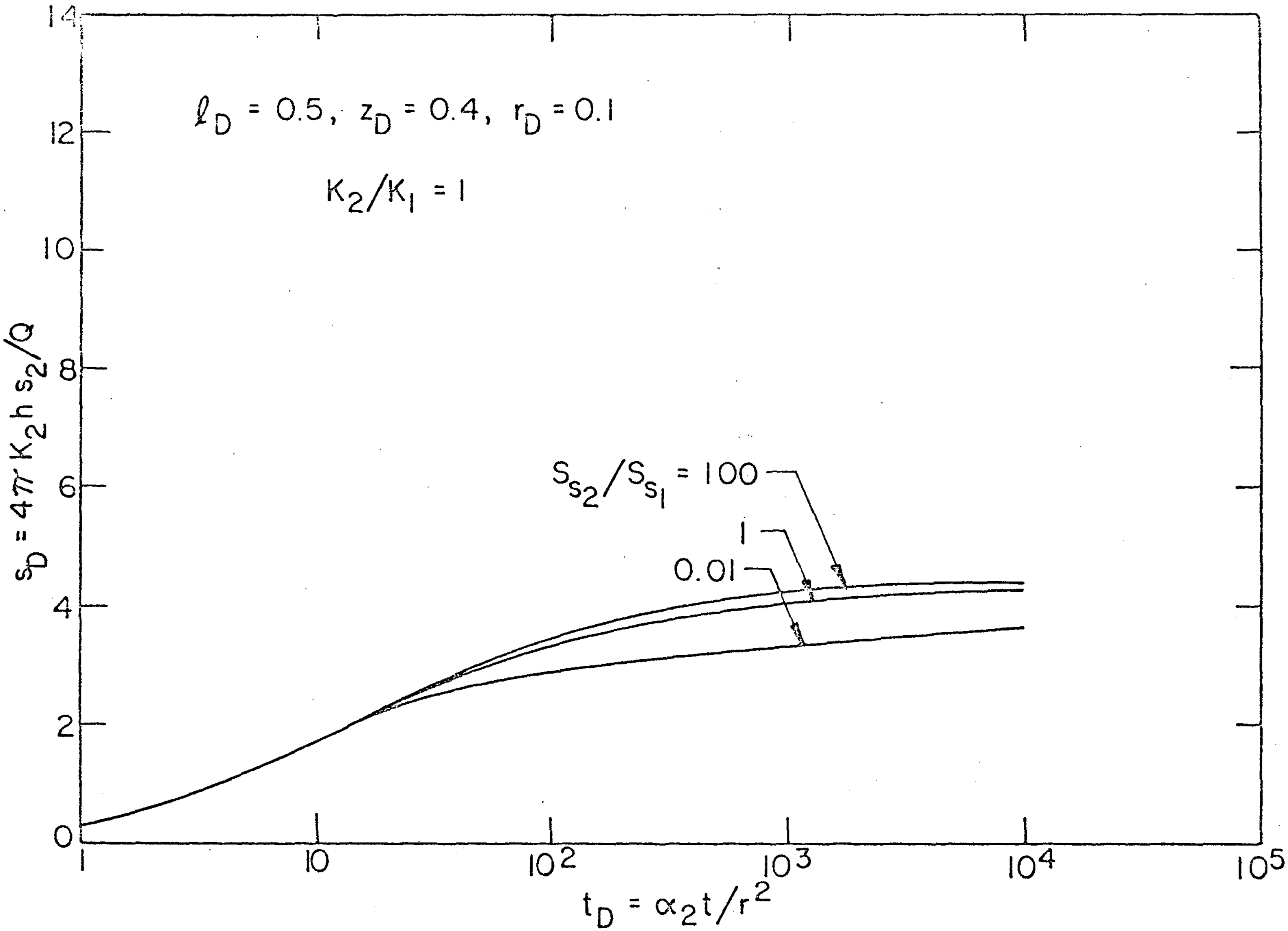

Fig. 10. Effect of specific storage contrast.

of two layers for $\ell_D = 0.5$, $z_D = 0.4$, $r_D = 0.1$, keeping permeability ratio equal to unity. Comparing Figures 8 and 10, and noting that the range of contrast of specific storage considered in Figure 10 is well beyond the limits usually observed in the field, indicates that permeability contrast plays much more important role. This can also be seen from equations (18) through (20). The term corresponding to the contrast of the specific storage appears only in f_D where its total effect compared to the first two terms of the integrand is of much smaller magnitude.

INTERPRETATION OF FIELD DATA

As discussed above, drawdown vs. time is a function of ℓ_D, r_D, z_D, M and D. To simplify the interpretation of field data, it was found that the problem is greatly simplified if observation wells are provided with the same depth and amount of penetration as the pumping well. There are two advantages in such an installation. First, it is usually simpler to construct an observation well over some part of the aquifer than to install a piezometer. Secondly, the solution for drawdown in such an observation well is much simpler than that for a piezometer as given by equations 17-20. This can be demonstrated by integrating equation 19 with respect to z_D and dividing the result by the length of the observation well which in this case is ℓ_D. In the Laplace transform domain, the solution takes the following form:

$$\bar{s}_{D_2} = \frac{1}{\ell_D^2} \int_0^\infty \frac{\xi J_o(\xi r_D)}{n\gamma^2} \left\{ 2\ell_D - \frac{1}{\gamma} + \frac{e^{-2\gamma\ell_D}}{\gamma} + \frac{\sinh^2(\gamma\ell_D)}{\gamma} \sum_{n=1}^{\infty} 4A^n e^{-2n\gamma} \right\} d\xi \quad (26)$$

Drawdown vs. time in equation 26 is now only a function of ℓ_D, r_D and M.

As was shown earlier on Figures 4 and 5, the drawdown behavior in the aquifer for small values of time is as though there were only a single layer. Therefore, one can choose the parameters ℓ_D and r_D and calculate the variation of dimensionless drawdown as a function of dimensionless time for various ratios of K_2/K_1. The difference between these values of drawdown and the corresponding drawdowns for a single layer solution which we shall call Δs_D, can then be plotted on semi-log paper for different values of K_2/K_1. Figure 11 shows such a family of curves for $\ell_D = r_D = 0.2$. When other values of r_D are needed, the same families of curves will result for the same value of ℓ_D except that there must be a shift in the time axis. As a result, the curves shown on Figure 11 are independent of r_D and when used together with type curves for a single layer partial penetration case can be employed to interpret field data for $\ell_D = 0.2$.

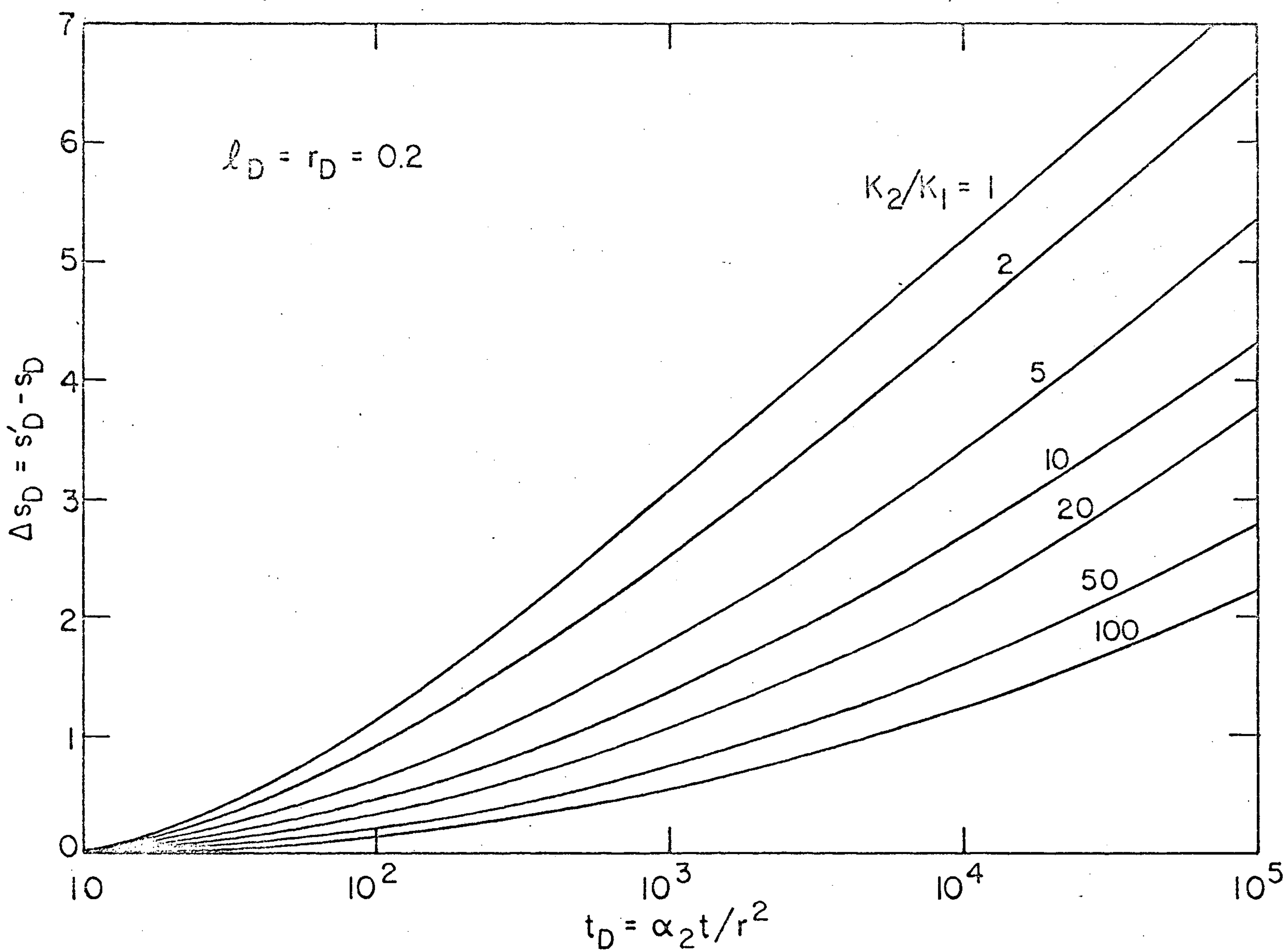

Fig. 11. Deviation of dimension drawdown of a two-layer aquifer from a single-layer one.

Naturally other families of curves must be generated for values of ℓ_D other than 0.2.

The following example will illustrate the procedure to be used in interpreting field data. Table 1 shows data for drawdown vs. time from a hypothetical field test where the aquifer consists of two layers. The top layer is 40 m thick and the bottom layer is very thick by comparison. Both the pumping and observation wells are completed in the top 8 m of the upper layer and the distance between the wells is 4 m. The rate of discharge is 0.02 m^3/sec. The problem is to determine the flow properties in both layers of the aquifer.

Table 1. Drawdown vs. time from two-layer pumping test.

time, minutes	drawdown, meters	time, minutes	drawdown, meters
2	0.70	180	2.24
3	0.94	360	2.50
5	1.22	720	2.60
10	1.58	1440	2.70
15	1.75	2160	2.79
20	1.85	2880	2.81
40	2.07	7200	2.91
60	2.18	14400	2.98
90	2.26	28800	3.03
120	2.30		

The following procedure should be used in the interpretation of these data.

1. Prepare a log-log plot as shown in Figure 12 for the average dimensionless drawdown for an observation well with $\ell_D = 0.2$ versus dimensionless time for $\ell_D = 0.2$ and $r_D = 0.1$ from a single layer solution of Hantush (1961).
2. Plot the data of Table 1 for drawdown versus time on another piece of log-log paper with the same scale per log cycle.
3. Find a match point by superposing the two plots being careful to use only early time data. The coordinates of the arbitrary match point chosen here are $t = 1600$ sec for $t_D = 10$ and $s = 0.2$ m for $s_D = 1$.

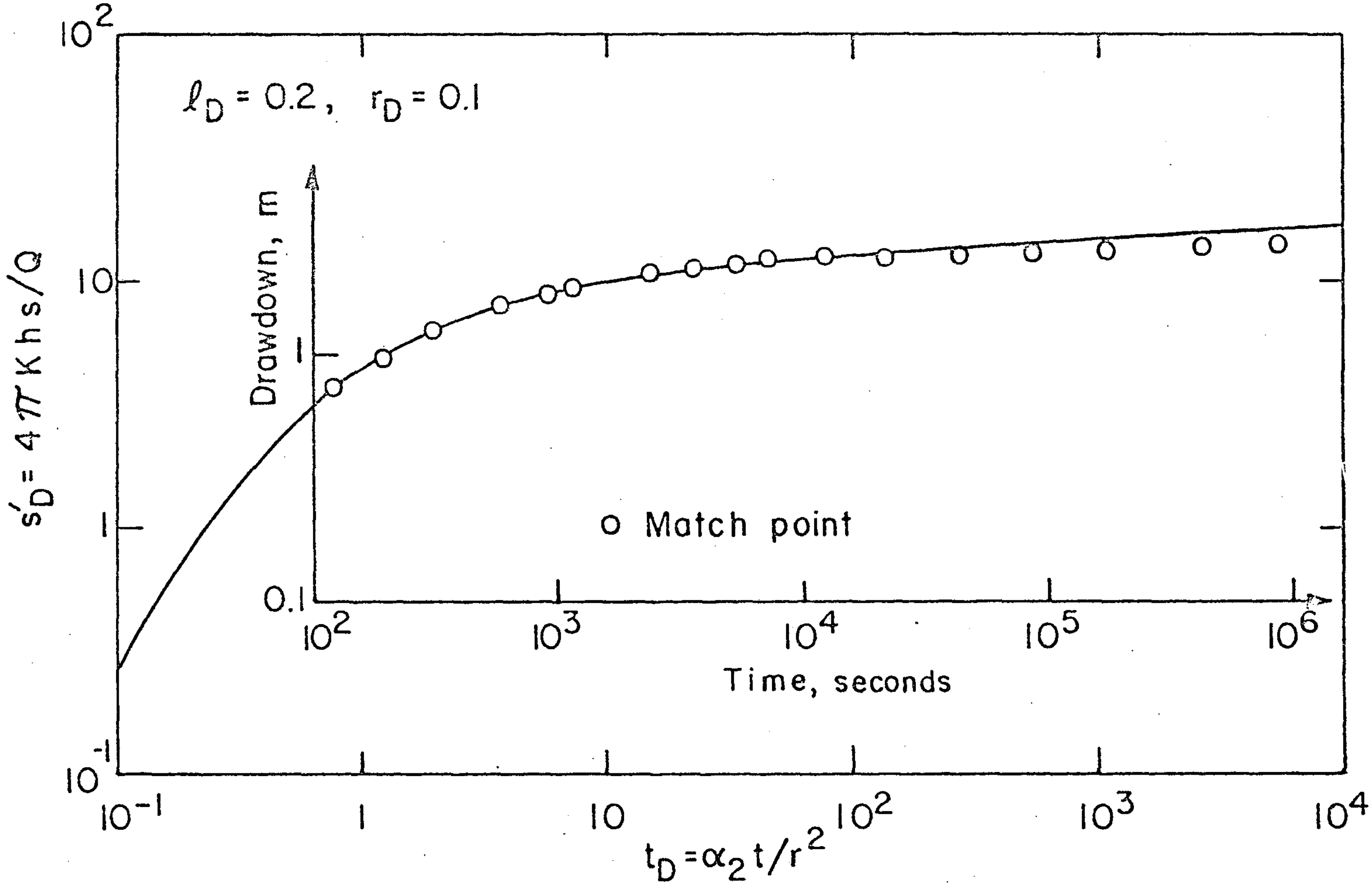

Fig. 12. Drawdown data compared with corresponding single layer type curve.

4. From the definitions of t_D and s_D, find K_2 = 0.0002 m/sec and α_2 = 0.1 m^2/sec.
5. With the two curves superposed, read the values of $s_D = s_D' - s_D$ for several different values of t_D.
6. Plot the values of Δs_D versus t_D on semi-log paper with the appropriate scale and superpose as on Figure 11.

It is necessary to shift the two curves in Stage 6 parallel to the t_D axis in order to obtain the best match with the curves of K_2/K_1. In this way one can then estimate the value for the permeability ratio. From the data tabulated in Table 1, we obtain a result of K_2/K_1 = 10. At this point two comments may be helpful. First, there must be an appropriately long period of pumping in order for the drawdowns to deviate from single layer type curves. Secondly, after the properties of the top layer have been determined, it may prove to be more accurate to convert the pump test data into dimensionless results and subtract them from corresponding values of s_D' for the single layer case. This should lead to a better result than will be obtained in reading Δs_D directly from the log-log results shown on Figure 12.

CONCLUSION

A semi-analytical solution has been presented to the problem of drawdown distribution in a two-layer aquifer when it is pumped from a well that is partially penetrating in one of the layers. The validity of the solution has been verified against four available limiting cases. Investigation of results revealed the following: 1) at small values of time, drawdown in the aquifer is similar to that of the single layer partial penetration problem; 2) as in the case of single layer partial penetration, here too, the effect of partial penetration diminishes when one goes away from the well beyond the distance of one and a half times of the thickness of the top layer;

3) available type curves for the single layer partial penetration together with families of curves such as those given on Figure 11 can be used for interpretation of pump test data in such a complex problem, resulting in hydraulic properties of both layers; 4) application of curves such as those given by Figure 9 would indicate the percentage of the total volume of fluid drawn from the lower layer at any given time through the pumping.

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NOTATION

Symbol	Definition	Dimensions
A	$\frac{M\gamma - \beta}{M\gamma + \beta}$	-
D	$\frac{\alpha_2}{\alpha_1}$	-
h	thickness of the top layer	L
$J_o(x)$	Bessel's function of the first kind and zero order	-
K_1, K_2	permeability of layers 1 and 2, respectively	L/T
ℓ	depth of penetration	L
ℓ_D	ℓ/h	-
M	K_2/K_1	-
Q	rate of discharge	L^3/T
r	radial distance	L
r_D	r/h	-
s_1, s_2	drawdown of layer 1 and 2, respectively	L
s_{D_i}	$4\pi K_2 h s_i/Q$	-
S_{s_1}, S_{s_2}	specific storage of layer 1 and 2, respectively	L^{-1}
s'_D	dimensionless drawdown for a single layer aquifer	-
t	time	T
t_D	$\alpha_2 t/r^2$	-
z	vertical coordinate	L
z_D	z/h	-
z_o	vertical coordinate of a point sink	L
α_1, α_2	diffusivity of layer 1 and 2, respectively	L^2/T
β	$(\xi^2 + \eta D)^{1/2}$	-
γ	$(\xi^2 + \eta)^{1/2}$	-
∇^2	Laplacian operator	-
η	Laplace transform parameter	-
ξ	Hankel transform parameter	-

REFERENCES

Bellman, R. E., Kalaba, R. E. and Lockett, Jr., Numerical Inversion of the Laplace Transform, Elsevier, New York, 1966.

Clegg, M. W. and M. Mills, "A study of the behavior of partially penetrating wells", Soc. Petrol. Engrs. J., 9 (2), 189-203, 1969.

Hantush, M. S., "Non-steady flow to a well partially penetrating an infinite leaky aquifer", Proc. Iraqi Scientific Soc., 1, 10-19, 1957.

Hantush, M. S., "Modification of the theory of leaky aquifers", J. Geophys. Res., 65 (11), 3713-3725, 1960.

Hantush, M. S., "Drawdown around a partially penetrating well", J. Hydraulic Div., Proc. ASCE, 87 (HY4), 83-98, 1961.

Jacquard, P., "Etude mathematique du drainage d'un reservoir heterogene", Rev. Inst. Francais du Petrole, XV (10), 1384, 1960.

Javandel, I., and P. A. Witherspoon, "Use of thermal model to investigate the theory of transient flow to a partially penetrating well", Water Resources Res., 3 (2), 591-597, 1967.

Javandel, I. and P. A. Witherspoon, "Application of the finite element method to transient flow in porous media", Soc. Petrol. Engrs. J., 8 (3), 241-252, 1968a.

Javandel, I. and P. A. Witherspoon, "Analysis of transient fluid flow in multilayered systems", Contribution 124, Water Research Center, University of California, Berkeley 1968b.

Javandel, I. and P. A. Witherspoon, "A method of analyzing transient fluid flow in multilayered aquifers", Water Resources Research, 5 (4), 856-869, 1969.

Javandel, I., "Application of numerical inversion of Laplace transformation for solving groundwater problems", Proc. of International Seminar on Regional Groundwater Hydrology and Modelling, Venice, Italy, 373-401, 1976.

Katz, M. L., Fluid Flow and Heat Transfer in Stratified Systems, Ph.D. Thesis, University of Michigan, 1960.

Kazemi, H. and M. S. Seth, "Effect of anisotropy and stratification on pressure transient analysis of wells with restricted flow entry", J. of Petrol. Tech. XXI(5), 639-647, 1969.

Lefkovits, H. C., P. Hazebrook, E. E. Allen, and C. S. Mathews, "A study of the behavior of bounded reservoirs composed of stratified layers," Soc. Petrol. Engrs. J., 2 (2), 43-58, 1961.

Neuman, S. P. and P. A. Witherspoon, "Theory of flow in a confined two-aquifer system," Water Resources Research, 5 (4), 803-816, 1969a.

Neuman, S. P. and P. A. Witherspoon, "Applicability of current theories of flow in leaky aquifers", Water Resources Research, 5 (4), 817-829, 1969b.

Papadopulos, I. S., "Nonsteady flow to multiaquifer wells", J. Geophys. Res. 11 (20), 4791-4798, 1966.

Pelissier, F. and P. Sequier, "Analyse numerique des equations des bicouches", Rev. Inst. Francois du Petrole, 16 (10), 1182-1187, 1961.

Pizzi, G., G. M. Ciucci and G. L. Chierici, "Quelques cas de remontées de pression dans des couches hétérogènes avec pénétration partielle étude par analyseur électrique", Rev. Inst. Francais de Pétrole, XX (12), 1811-1846, 1965.

Russel, D. G. and M. Prats, "Performance of layered reservoirs with cross flow, single-compressible-fluid case," Trans., AIME, (215), 53-67, 1962.

Saad, K. F., Nonsteady Flow Toward Wells which Partially Penetrate Thick Artesian Aquifers, M. S. Thesis, New Mexico Institute of Mining and Technology, 1960.

Vacher, J. P. and V. Cazbat, "Ecoulement des fluids dans les milieux poreux stratifies resultat obtenus sur le modele du bichouche avec communication", Rev. Inst. Francais de Petrole, XVI (10), 1147-1163, 1961.

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