Title
Equation of State of Neutron Star Matter, Limiting Rotational Periods of Fast Pulsars, and the Properties of Strange Stars

Authors
Weber, F.
Glendenning, N.K.

Publication Date
1993-10-25

LBL-34783
UC-412

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

Presented at the NATO Advanced Study Institute,
Bodrum, Turkey, September 26–October 9, 1993,
and to be published in the Proceedings

Equation of State of Neutron Star Matter, Limiting Rotational Periods of Fast Pulsars, and the Properties of Strange Stars

F. Weber and N.K. Glendenning

October 1993

Prepared for the U.S. Department of Energy under Contract Number DE-AC03-76SF00098

LBL - 34783

EQUATION OF STATE OF NEUTRON STAR[†] MATTER, LIMITING ROTATIONAL PERIODS OF FAST PULSARS, AND THE PROPERTIES OF STRANGE STARS[†]

Fridolin Weber

Institute for Theoretical Physics, University of Munich, Theresienstr. 37/III, 80333 Munich, FRG, and Lawrence Berkeley Laboratory, Berkeley, CA 94720, USA

Norman K. Glendenning

Nuclear Science Division, Lawrence Berkeley Laboratory, University of California, Berkeley, CA 94720, USA

October 25, 1993

Presented at the
NATO Advanced Study Institute
HOT AND DENSE NUCLEAR MATTER
Bodrum, Turkey September 26 – October 9, 1993
To be published by Plenum Press in the Physics Series

[†]This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of Nuclear Physics, of the U.S. Department of Energy under Contract DE-AC03-76SF00098

EQUATION OF STATE OF NEUTRON STAR MATTER, LIMITING ROTATIONAL PERIODS OF FAST PULSARS, AND THE PROPERTIES OF STRANGE STARS

Fridolin Weber

Institute for Theoretical Physics, University of Munich, Theresienstr. 37/III, 80333 Munich, FRG, and Lawrence Berkeley Laboratory, Berkeley, CA 94720, USA

Norman K. Glendenning

Nuclear Science Division, Lawrence Berkeley Laboratory, University of California, Berkeley, CA 94720, USA

INTRODUCTION

One of the most challenging but also complicated problems of modern physics consists in exploring the behavior of matter under extreme conditions of temperature and/or density. Knowledge of its behavior is of key importance for our understanding of the physics of the early universe, its evolution in time to the present day, compact stars, various astrophysical phenomena, and laboratory physics. On earth, relativistic heavy-ion collisions provide the only means to learn about the behavior of dense nuclear matter. What is not widely appreciated is the fact that from the study of the properties of compact stellar objects, e.g. neutron stars, one too gains knowlege of the behavior of dense matter, as illustrated in Fig. 1.

Neutron stars are associated with two classes of astrophysical objects, – pulsars and compact X-ray sources. Matter in their cores possess densities ranging from a few times the density of normal nuclear matter (2.5×10^{14} g/cm^3) to about an order of magnitude higher, depending on mass. They thus contain matter in one of the densest forms found in the universe! The equation of state of the stellar matter decisively

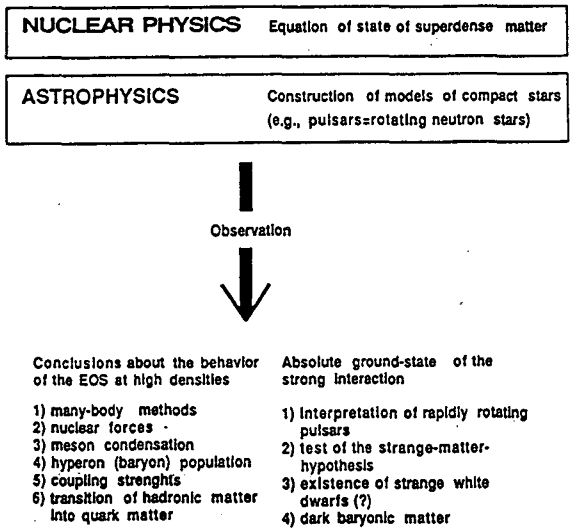

Figure 1: The marriage of nuclear physics with astrophysics enables one to learn about the behavior of the nuclear equation of state at high densities as well as the absolute ground-state of the strong interaction.

links neutron stars with nuclear and particle physics (plus various other branches of physics). It is the basic input quantity whose knowledge over a broad range of densities (ranging from the density of iron at the star's surface up to ~ 15 times the density of normal nuclear matter reached in the cores of massive stars) is necessary when solving the Einstein equations for the properties of neutron stars. The physical behavior of matter under such extreme densities as in the cores of massive stars is rather uncertain and the associated equation of state is only poorly known. The models derived for it differ considerably with respect to the dependence of pressure on density, which has its origin in various sources. To mention several are: (1) the many-body technique used to determine the equation of state, (2) the model for the nucleon-nucleon interaction, (3) description of electrically charge neutral neutron star matter in terms of either (a) only neutrons, (b) neutrons and protons in β-equilibrium with electrons and muons, or (c) nucleons, hyperons and more massive baryon states in β-equilibrium with leptons, (4) inclusion of meson (π, K) condensation, and (5) treatment of the transition of confined hadronic matter into quark matter. By means of marrying nuclear physics with astrophysics the compatibility of these phenomena with observed data on pulsars can be tested, which addresses an open fundamental problem of nuclear physics concerning the behavior of the high-density equation of state.

In these two lectures the following items will be treated:

- The present status of dense nuclear matter calculations and constraints on the behavior of the associated equation of state at high densities from data on rapidly rotating pulsars.
- Recent finding of the likely existence of a mixed phase of baryons and quarks forming a coulomb lattice in the dense cores of neutron stars.
- Review of important findings of recently performed calculations of rapidly rotating compact stars. These are constructed in the framework of general relativity

Table 1: Nuclear equations of state applied for the construction of models of general relativistic rotating neutron star models. Their tabulated representations, i.e. pressure versus energy and baryon density $P(\epsilon, \varrho)$, are given in Ref. [1].

Label	EOS	Description (see text)	Reference
	Relativistic field theoretical equations of state		
1	G_{300}	H, K=300	2
2	HV	H, K=285	3, 4
3	G^{DCM2}_{B180}	Q, K=265, $B^{1/4} = 180$	5, 6
4	G^{DCM2}_{265}	H, K=265	7
5	G^{π}_{300}	H, π, K=300	2
6	G^{π}_{200}	H, π, K=200	8
7	$\Lambda^{00}_{\rm Bonn}$ + HV	H, K=186	9
8	G^{DCM1}_{225}	H, K=225	7
9	G^{DCM1}_{B180}	Q, K=225, $B^{1/4} = 180$	5, 6
10	HFV	H, Δ, K=376	4
11	$\Lambda^{00}_{\rm HEA}$ + HFV	H, Δ, K=115	9
	Non-relativistic potential model equations of state		
12	BJ(I)	H, Δ	10
13	WFF(UV_{14}+TNI)	NP, K=261	11
14	FP(V_{14}+TNI)	N, K=240	12
15	WFF(UV_{14}+UVII)	NP, K=202	11
16	WFF(AV_{14}+UVII)	NP, K=209	11

theory for a representative collection of realistic nuclear equations of state which account for items (1)-(5) from above.

- Establish the minimum-possible rotational periods of gravitationally bound neutron stars and self-bound strange stars. Its knowledge is of fundamental importance for the decision between pulsars that can be understood as rotating neutron stars and those that cannot (signature of hypothetical self-bound matter of which strange stars are the likely stellar candidates).

- Investigate the properties of sequences of strange stars. Specifically, we answer the question whether such objects can give rise to the observed phenomena of pulsar glitches, which is at the present time the only astrophysical test of the strange-quark-matter hypothesis.

COLLECTION OF NUCLEAR EQUATIONS OF STATE

A representative collection of nuclear equations of state, which are determined in the framework of non-relativisitic Schroedinger theory and relativistic nuclear field theory, is listed in Table 1. It is this collection that has been applied for the construction of models of general relativistic rotating neutron star models. The specific properties of these equations of state are described in the third column of Table 1, where the following abbreviations are used: N = pure neutron; NP = n, p, leptons; π = pion condensation; H = composed of n, p, hyperons ($\Sigma^{\pm,0}$, Λ, $\Xi^{0,-}$), and leptons; Δ = Δ_{1232}-resonance; Q = quark hybrid composition, i.e. n, p, hyperons in equilibrium with u, d, s-quarks, leptons; K = incompressibility (in MeV); $B^{1/4}$ = bag constant (in MeV). Not all equations of state of our collection account for neutron matter in β-equilibrium (i.e. entries 13-16). These models treat neutron star matter as being composed of only neutrons (entry 14), or neutrons and protons in equilibrium with leptons (entries 13, 15, 16), which is however not the ground-state of neutron star matter predicted by theory[3]. The relativistic equations of state *account for all baryon states* that become populated in dense star models constructed from them. An inherent feature of the relativistic equations of state is that they do not violate *causality*, i.e. the velocity of sound is smaller than the velocity of light at all densities, which is not the case for the non-relativistic equations of state. Among the latter only the WFF(UV_{14}+TNI) equation of state does not violate causality up to densities relevant for the construction of models of neutron stars from it[13].

POSSIBILITY OF A MIXED PHASE OF BARYONS AND QUARKS IN THE CORES OF NEUTRON STARS

A special feature of the two equations of state denoted G^{DCM1}_{B180} and G^{DCM2}_{B180} is that these, additionally to hyperons, also account for the possible transition of baryon matter to quark matter at high densities. Model G^{DCM1}_{B180} is shown in Fig. 2, where pressure as a function of energy density (in units of the density of normal nuclear matter, ϵ_0 = 140 MeV/fm^3) is exhibited. Most interestingly, the transition of baryon matter to quark matter sets in already at a density $\epsilon = 2.3\,\epsilon_0$[5, 6] (arrow in Fig. 2), which lowers the pressure relative to confined hadronic matter. The mixed phase of baryons and quarks ends, i.e. the pure quark phase begins, at $\epsilon \approx 15\,\epsilon_0$, which is larger than the central density encountered in the maximum-mass star model constructed from this equation of state. Furthermore, one sees that the pressure in the mixed phase *varies with density* (i.e., is not constant!), which implies the existence of a mixed matter phase in neutron stars constructed from it. This finding, which goes back to recent work by Glendenning[5, 6], is in sharp contrast to all other investigations on this topic (for details, see[5, 6]). The reason for this lies in the fact that the transition between confined hadronic matter and quark matter takes place subject to the conservation of baryon charge and electric charge. Correspondingly, there are two chemical potentials, and the transition of baryon matter to quark matter is to be determined in three-space spanned by pressure and the chemical potentials of the electrons and neutrons (rather than two-space). This circumstance has not been realized in the numerous investigations published on this topic earlier. The only existing investigation which accounts for this properly has been performed by Glendenning[5, 6]. For an investigation of the structure of the mixed phase of baryons and quarks predicted by Glendenning, we refer to recent work by Heiselberg, Pethick, and Staubo[14]. This geometric structure of the mixed phase is likely to have dramatic effects on pulsar observables including transport properties and the theory of glitches.

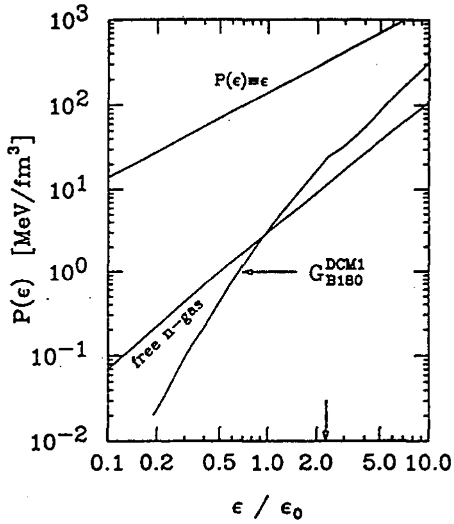

Figure 2: Pressure versus density of equation of state G_{B180}^{DCM1}. For comparison, the free neutron gas equation of state is shown too.

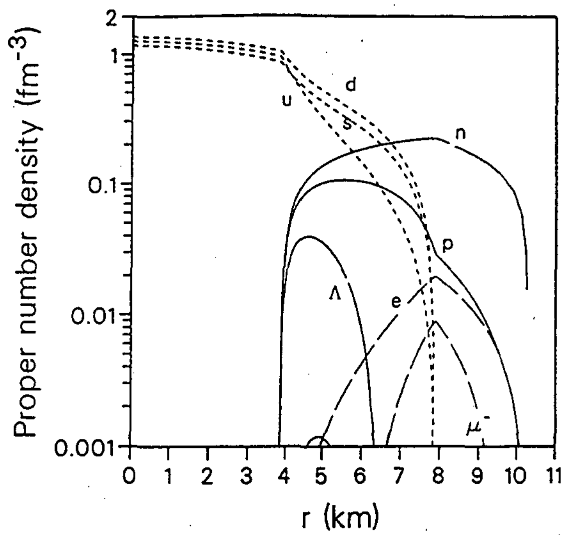

Figure 3: Baryon/Quark composition of a massive neutron star constructed for G_{B180}^{DCM1}

MODELS OF ROTATING COMPACT STARS IN GENERAL RELATIVITY

Neutron stars are objects of highly compressed matter so that the geometry of space-time is changed considerably from flat space-time. Thus for the construction of realistic models of rapidly rotating pulsars one has to resort to Einstein's theory of general relativity. In the case of a star rotating at its *absolute* limiting rotational periods, i.e. the Kepler (or mass-shedding) frequency, Einstein's equations,

$$\mathcal{R}^{\kappa\lambda} - \frac{1}{2}\, g^{\kappa\lambda}\, \mathcal{R} = 8\,\pi\, \mathcal{T}^{\kappa\lambda}(\epsilon, P(\epsilon)) \ , \tag{1}$$

are to be solved in combination with the general relativistic expression describing the onset of mass-shedding at its equator:[15, 16, 17]

$$\Omega_{\rm K} = \omega + \frac{\omega'}{2\psi'} + e^{\nu-\psi}\sqrt{\frac{\nu'}{\psi'} + \Big(\frac{\omega'}{2\psi'}e^{\psi-\nu}\Big)^2} \ . \tag{2}$$

The quantities $\mathcal{R}^{\kappa\lambda}$, $g^{\kappa\lambda}$, and $\mathcal{R}$ denote respectively the Ricci tensor, metric tensor, and Ricci scalar (scalar curvature). The dependence of the energy-momentum tensor $\mathcal{T}^{\kappa\lambda}$ on pressure and energy density, P and ϵ respectively, is indicated in Eq. (1). The quantities ω, ν, and ψ in Eq. (2) denote the frame dragging frequency of local inertial frames and time- and space-like metric functions, respectively. The primes denote derivatives with respect to Schwarzschild radial coordinate, and all functions on the right are evaluated at the star's equator. All the quantities on the right hand side of Eq. (2) depend also on $\Omega_{\rm K}$, so that it is not an equation for $\Omega_{\rm K}$, but a transcendental relationship which the solution of the equations of stellar structure, resulting from Eq. (1), must satisfy if the star is rotating at its Kepler frequency. (For more details,

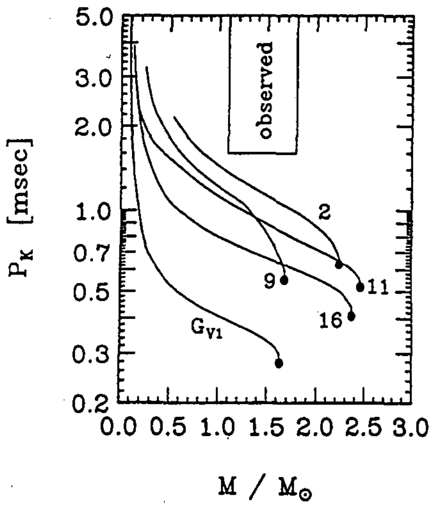

Figure 4: Kepler period versus rotational neutron star mass. The labeling of the curves is explained in Table 1, with the exception of G_{V1} which is explained in the text. Only pulsar periods $P > P_K$ are possible, which is consistent with the pulsar periods known to date.

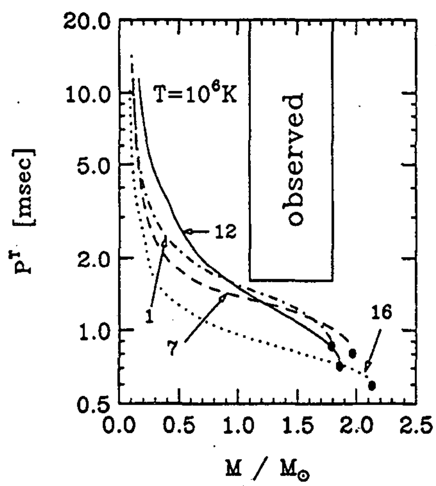

Figure 5: Rotational periods P^T at which instability against emission of gravity waves in cold neutron stars of temperature $T = 10^6$ K sets in versus mass. Stable pulsar periods are restricted to $P > P^T$, which is consistent with the pulsar periods known to date.

we refer to Ref. [16].)

NEUTRON STARS ROTATING AT THE KEPLER FREQUENCY

The computed general relativistic Kepler periods P_K ($\equiv 2\pi/\Omega_K$), defined in Eq. 2, are graphically depicted in Fig. 4 for a representative sample of equations of state of Table 1. The smallest rotational periods are obtained for the WFF(AV_{14} + UVII) equation of state (label 16). We find that the relativistic equations of state lead in general to larger rotational periods than the non-relativistic ones due to the somewhat larger radii of the associated star models[16]. The upper limit on the Kepler period is set by the relativistic HV (label 2) equation of state. The periods obtained from all other equations of state of Table 1 lie between curves "2" and "16". The rectangle in Fig. 4 covers both the approximate range of neutron star masses as determined from observations[18], as well as the measured rotational periods ($P \geq 1.6$ msec). One sees that all pulsar periods so far observed are larger than the absolute limiting Kepler values. The observation of pulsars possessing masses in the observed range but rotational periods that are smaller than say ~ 0.7 msec (depending on the star's mass) would be in contradiction to our collection of equations of state. Consequently the observation of such pulsars cannot be reconciled with the interpretation of such objects as rapidly rotating *neutron* stars. Half-millisecond periods, for example, are completely excluded for pulsars made of baryon matter. Therefore, the possible future discovery of a single sub-millisecond pulsar, rotating with a period of say ~ 0.5 msec, would give a strong hint that such an object is a rotating self-bound *strange star*, not a neutron star, and that 3-flavor strange quark matter is the true ground-state of the strong interaction, as pointed out by Glendenning[19, 20].

The fact that any successful model for the nuclear equation of state must accomodate pulsars with rotational periods of (at least) 1.6 msec and masses larger than typically 1.5 $M_\odot$ leads to an overall constraint on its density dependence (double constraint of fast rotation and a large enough neutron star mass): it must behave soft in the vicinity of the density of normal nuclear matter and intermediate nuclear densities in order to lead to small enough rotational pulsar periods, but rather stiff at high nuclear densities to account for large enough masses![9, 21, 22]

An investigation of the limiting rotational Kepler period of neutron stars that is performed *without* taking recourse to any particular models of dense matter but derives the limit only on the general principles that (a) Einstein's equations describe stellar structure, (b) matter is microscopically stable, and (c) causality is not violated has only recently been performed by Glendenning[20]. He establishes a lower bound for the minimum Kepler period for a $M = 1.442\, M_\odot$ neutron star of $P_{\rm K} = 0.33$ ms (equation of state denoted G_{V1} in Fig. 4). This curve sets an absolute limit on rotation on any star bound by gravity. The equation of state that nature has chosen need not be the one that allows stars to rotate most rapidly, so the above is a strict model independent limit.

GRAVITATIONAL RADIATION-REACTION DRIVEN INSTABILITY IN NEUTRON STARS

Besides the absolute upper limit on rotation set by the Kepler frequency, there is another instability that sets in at a lower rotational frequency, and which therefore sets a more stringent limit on stable rotation[23]. It originates from counter-rotating surface vibrational modes, which at sufficiently high rotational star frequencies are dragged forward. In this case, gravitational radiation which inevitably accompanies the aspherical transport of matter does not damp the modes, but rather drives them[24, 25]. Viscosity plays the important role of damping such gravitational-wave radiation-reaction instabilities at a sufficiently reduced rotational frequency such that the viscous damping rate and power in gravity waves are comparable[26]. The instability modes are taken to have the dependence $\exp[i\omega_m(\Omega)t + im\phi - t/\tau_m(\Omega)]$, where ω_m is the frequency of the surface mode which depends on the angular velocity Ω of the star, ϕ denotes the azimuthal angle, and τ_m is the time scale for the mode which determines its growth or damping according to the sign of τ_m. The rotation frequency Ω at which it changes sign is the critical frequency for the particular mode, m (=2,3,4,...). It is conveniently expressed as the frequency, Ω^ν_m (ν refers to the viscosity dependence, see below), that solves[23]

$$\Omega^\nu_m \; = \; \frac{\omega_m(0)}{m} \; \left[\tilde\alpha_m(\Omega^\nu_m) + \tilde\gamma_m(\Omega^\nu_m) \left(\frac{\tau_{g,m}}{\tau_{\nu,m}}\right)^{\frac{1}{2m+1}}\right] , \tag{3}$$

where

$$\omega_m(0) \; \equiv \; \sqrt{\frac{2\,m\,(m-1)}{2\,m\,+\,1}\;\frac{M}{R^3}} \tag{4}$$

is the frequency of the vibrational mode in a non-rotating star. The time scales for gravitational radiation reaction[27], $\tau_{g,m}$, and for viscous damping time[28], $\tau_{\nu,m}$, are given respectively by

$$\tau_{g,m} = \frac{2}{3}\,\frac{(m-1)\,[(2m+1)!!]^2}{(m+1)\,(m+2)} \left(\frac{2m+1}{2m(m-1)}\right)^m \left(\frac{R}{M}\right)^{m+1} R\,, \tag{5}$$

$$\tau_{\nu,m} = \frac{R^2}{(2m+1)\,(m-1)}\,\frac{1}{\nu}\,. \tag{6}$$

The shear viscosity is denoted by ν. It depends on the temperature, T, of the star ($\nu(T) \propto T^{-2}$). It is small in very hot ($T \approx 10^{10}$ K) and therefore young stars and larger in cold ones. A characteristic feature of the above equations (3) - (6) is that Ω^ν_m merely depends on radius and mass (R and M) of the spherical star model and its assumed temperature (respectively viscosity).

The functions $\tilde{\alpha}_m$ and $\tilde{\gamma}_m$ contain information about the pulsation of the rotating star models and are difficult to determine[23, 29]. A reasonable first step is to replace them by their corresponding Maclaurin spheroid functions α_m and γ_m[23, 29]. We therefore take $\alpha_m(\Omega_m)$ and $\gamma_m(\Omega_m)$ as calculated in Refs. [30, 31] for the oscillations of rapidly rotating inhomogeneous Newtonian stellar models (polytropic index n=1), and Ref. [23] for uniform-density Maclaurin spheroids (i.e. n=0), respectively. Managan has shown that Ω^ν_m depends much more strongly on the equation of state and the mass of the neutron star model (through $\omega_m(0)$ and $\tau_{g,m}$, see Eqs. (4) and (5)) than on the polytropic index assumed in calculating α_m[32].

Figure 5 shows the limiting rotational neutron star periods P^T ($\equiv 2\pi/\Omega^\nu_m$), i.e. the solution of Eq. (3), at which the gravitational radiation-reaction instability in stars set in. Considered are old (and therefore cold) neutron stars of temperature $T = 10^6$ K.[33] For such stars we find that the $m = 2$ mode is largest and thus is exited first. It therefore sets the limit on stable rotation for neutron stars of such temperatures. As in the case of rotation at the Kepler period, equation of state "16" here too leads to the smallest stable periods obtained for all equations of state of our collection. No neutron star can possess a stable rotational period lying below this curve. In other words, by taking the gravity wave instability into account, the minimum-possible stable rotational periods of cold pulsars are restricted to values larger than $\sim 0.8 - 1.4$ msec, depending on mass and equation of state. This is still in agreement with observation. The latter value, however, is already rather close to the rotational period of the fastest pulsars observed to date. For young and therefore hot pulsars, which are not considered here, the corresponding range of stable periods has been established somewhere else to be $\sim 1 - 1.6$ msec for $M \sim 1.5\, M_\odot$[13, 16, 33].

PROPERTIES OF STRANGE STARS

THE STRANGE-MATTER HYPOTHESIS

The hypothesis that strange quark matter may be the absolute ground state of the strong interaction (not ^{56}Fe) has been raised by Witten in 1984[34]. If the hypothesis is true, then a *separate* class of compact stars could exist, which are called strange stars. They form a distinct and disconnected branch of compact stars and are not part of the continuum of equilibrium configurations that include white dwarfs and neutron stars. In principle both strange and neutron stars could exist. However if strange stars exist, the galaxy is likely to be contaminated by strange quark nuggets which would convert all neutron stars that they come into contact with to strange stars[19, 35, 36]. This in turn means that the objects known to astronomers as pulsars are probably rotating strange matter stars, *not* neutron matter stars as is usually assumed. Unfortunately the bulk properties of models of neutron and strange quark stars of masses that are typical for neutron stars, $1.1 \lesssim M/M_\odot \lesssim 1.8$, are relatively similar (cf. Fig. 7) and therefore do not allow the distiction between the two possible pictures. The situation changes however as regards the possibility of *fast rotation* of strange stars. This has its origin in the different mass-radius relationships of neutron stars and strange quark stars (see Fig. 7)[37]. As a consequence of this the entire familiy of strange stars can rotate rapidly, not just those near the limit of gravitational collapse to a black hole as is the case for neutron stars. As an example, above the minimum possible rotational periods of maximum-mass neutron stars have been determined to be larger than ~ 0.8 msec; this is to be compared with $\approx (0.4 - 0.6)$ msec calculated for strange stars[38].

HADRONIC CRUST ON STRANGE STARS

At the present time there appears to be only one crucial *astrophysical test* of the strange-quark-matter hypothesis, and that is whether strange quark stars can give rise to the observed phenomena of pulsar glitches. In the crust quake model an oblate solid nuclear crust in its present shape slowly comes out of equilibrium with the forces acting on it as the rotational period changes, and fractures when the built up stress exceeds the sheer strength of the crust material. The period and rate of change of period slowly heal to the trend preceding the glitch as the coupling between crust and core re-establish their co-rotation. The existence of glitches may have a decisive impact on the question of whether strange matter is the ground state of the strong interaction.

The only existing investigation which deal with the calculation of the thickness, mass, and moment of inertia of the nuclear solid crust that can exist on the surface of a rotating strange quark star has only recently been performed by Glendenning and Weber[38]. Their calculations account for the fact that strange stars can possess a solid nuclear crust, which is suspended out of contact with the strange star via the strong electric field on its surface. The maximum density of the inner crust is strictly limited by the neutron drip density (about 4.3×10^{11} g/cm^3), since free neutrons, being electrically neutral particles, cannot exist in the star. These would be dissolved into quark matter as they gravitate into the core. The equation of state of such a

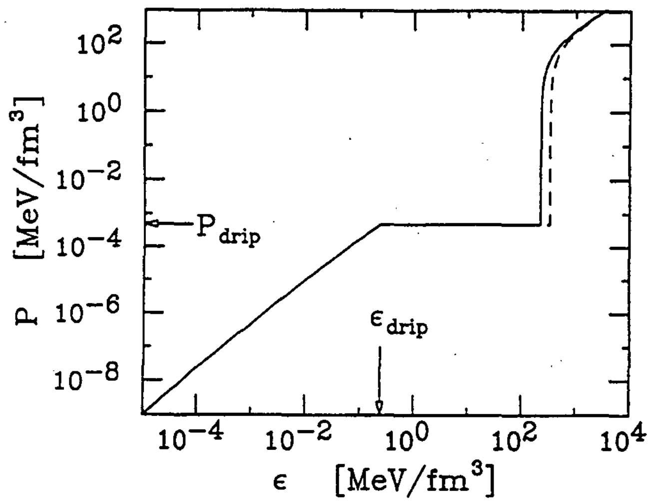

Figure 6: Equation of state of a strange star surrounded by a nuclear crust with inner density below neutron drip (see text). The sybmols $P_{\rm drip}$ and $\epsilon_{\rm drip}$ refer to drip pressure and drip energy density. The quark matter equation of state is calculated for $B^{1/4} = 145$ MeV (solid line) and 160 MeV (dashed).

strange star with crust is shown in Fig. 6.

The mass-radius relationship of strange stars with crust, which follows from the equation of state exhibited in Fig. 6, is shown in Fig. 7[38, 39]. Here the solid dots denote the maximum-mass star of the neutron star (NS) and strange quark star (SS) sequences. The arrows indicate the minimum-mass star of each sequence ('a': strange star, 'b': neutron star). White-dwarf-like strange star configurations ('sd': strange dwarfs) terminate at the crossed point labeled 'd'. The symbol 'wd' indicates the region of ordinary white dwarfs. A value for the bag constant of $B^{1/4} = 145$ MeV for which 3-flavor strange matter is stable has been chosen. This choice represents strongly bound strange matter with an energy per baryon $\sim$ 830 MeV, and thus corresponds to strange quark matter being absolutely bound with respect to ^{56}Fe.

Since the crust is bound by the gravitational interaction (and not by confinement, which is the case for the strange matter core), the mass-radius relationship of strange stars is qualitatively similar to the one for neutron stars. The radius being largest for the lightest and smallest for the heaviest stars in the sequence. Just as for neutron stars the relationship is not necessarily monotonic at intermediate masses. The radius of the strange quark core is proportional to $M^{1/3}$ which is typical for self-bound objects. This proportionality is only appreciably modified near the mass where gravity terminates the stable sequence. The sequence of strange stars has a minimum mass of $\sim 0.015\,M_\odot$ (radius of $\sim$ 400 km) or about 15 Jupiter masses (label 'a'), which is smaller than that of neutron star sequences, about 0.1 $M_\odot$ (label 'b')[40]. These low-mass strange stars may be of considerable importance since they may be difficult to detect and therefore may effectively *hide baryonic matter*. Furthermore, of interest to the subject of cooling of strange stars is the crust thickness of strange stars[41]. It ranges from $\sim$ 400 km for stars at the lower mass limit to $\sim$ 12 km for stars of mass $\sim 0.02\,M_\odot$, and is a fraction of a kilometer for the star at the maximum mass[38]. Those strange stars which result from solving the Oppenheimer-Volkoff equations for central star densities that are smaller than the corresponding central density of

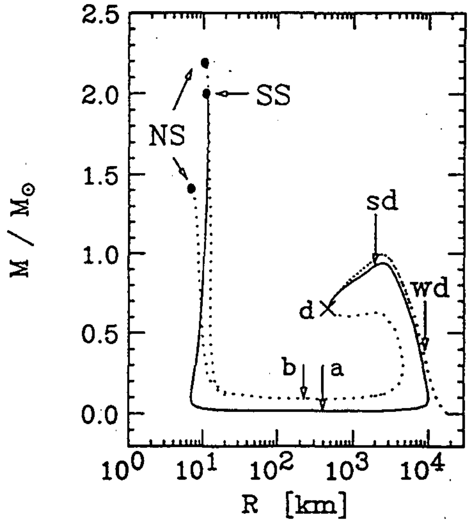

Figure 7: Mass-radius relationship of strange matter stars with nucler crust (strange stars and strange dwarfs, solid curve), and neutron stars and ordinary white dwarfs (dotted curve).

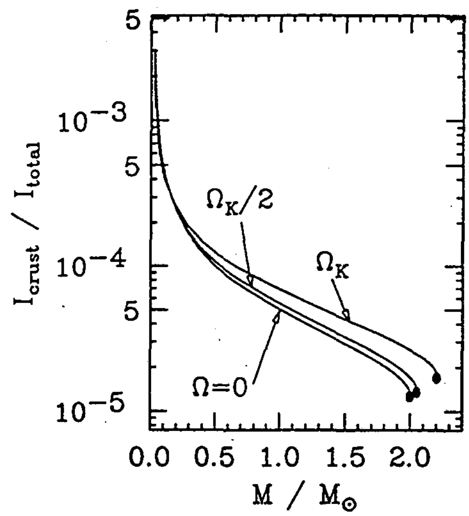

Figure 8: The ratio I_{crust}/I_{total} as a function of star mass. Rotational frequencies are shown as a fraction of the Kepler frequency, Ω_K[5].

the minimum-mass star, but larger than the smallest possible one (determined by $\epsilon = 3\,P_{drip} + 4\,B$, P_{drip} denotes the drip pressure) are shown too in Fig. 7. The cross refers to that particular star whose strange-matter-core radius has shrunken to zero, thus possessing mass and radius values of a white dwarf star[42].

The moment of inertia of the hadronic crust, I_{crust}, that can be carried by a strange star as a functin of star mass for a sample of rotational frequencies of $\Omega = \Omega_K, \Omega_K/2$ and 0 is shown in Fig. 8. Because of the relatively small crust mass of the maximum-mass models of each sequence, the ratio I_{crust}/I_{total} is smallest for them (solid dots in Fig. 8). The less massive the strange star the larger its radius (Fig. 7) and therefore the larger both I_{crust} as well as I_{total}. The dependence of I_{crust} and I_{total} on M is such that their ratio I_{crust}/I_{total} is a monotonically decreasing function of M. One sees that there is only a slight difference between I_{crust} for $\Omega = 0$ and $\Omega = \Omega_K/2$.

Of considerable relevance for the question of whether strange stars can exhibit glitches in rotation frequency, one sees that I_{crust}/I_{total} varies between 10^{-3} and $\sim 10^{-5}$ at the maximum mass. If the angular momentum of the pulsar is conserved in the quake then the relative frequency change and moment of inertia change are equal, and one arrives at[38]

$$\frac{\Delta\Omega}{\Omega} = \frac{|\Delta I|}{I_0} > \frac{|\Delta I|}{I} \equiv f\,\frac{I_{crust}}{I} \sim (10^{-5} - 10^{-3})\,f \ , \text{ with } \quad 0 < f < 1\,. \quad (7)$$

Here I_0 denotes the moment of inertia of that part of the star whose frequency is changed in the quake. It might be that of the crust only, or some fraction, or all of the star. The factor f in Eq. (7) represents the fraction of the crustal moment of inertia that is altered in the quake, i.e. $|\Delta I| = f\,I_{crust}$. Since the observed glitches have relative frequency changes $\Delta\Omega/\Omega = (10^{-9} - 10^{-6})$, a change in the crustal moment of inertia by less than 10% would cause a giant glitch even in the least favorable case (for more details, see[38]). Finally we find that the observed range of the fractional change in $\dot{\Omega}$ is consistent with the crust having the small moment of inertia calculated and the quake involving only a small fraction f of that, just as in Eq. (7). For this purpose we write[38]

$$\frac{\Delta\dot{\Omega}}{\dot{\Omega}} = \frac{\Delta\dot{\Omega}/\dot{\Omega}}{\Delta\Omega/\Omega}\frac{|\Delta I|}{I_0} = \frac{\Delta\dot{\Omega}/\dot{\Omega}}{\Delta\Omega/\Omega} f \frac{I_{\rm crust}}{I_0} > (10^{-1} \text{ to } 10)\, f \ , \tag{8}$$

which yields a small f value as before: $f < (10^{-4} \text{ to } 10^{-1})$. Here measured values of the ratio $(\Delta\Omega/\Omega)/(\Delta\dot{\Omega}/\dot{\Omega}) \sim 10^{-6}$ to 10^{-4} for the Crab and Vela pulsars, respectively, have been used.

SUMMARY

This work begins with an introduction of a representative collection of realistic models for the nuclear equation of state of neutron star matter. Specifically, the recent finding of the possible existence of a mixed phase of baryon matter and quark matter in dense neutron star matter is stressed. These equations of state are then applied for the construction of models of rapidly rotating neutron stars, whose masses and rotational periods are compared with observational data on pulsars. The indication of this work is that the gravitational radiation-reaction instability sets a lower limit on stable rotation for massive *neutron stars* of $P \sim 0.8$ msec. Lighter ones having typical pulsar masses of $\sim 1.45\ M_\odot$ are predicted to have mimimum rotational periods of ~ 1 msec. This finding may have very important implications for the nature of any pulsar that is found to have a shorter period, say below ~ 0.5 msec. At least for the studied broad collection of nuclear equations of state, rotation at such small periods is not allowed, and thus the interpretation of such objects as rapidly rotating neutron stars fails. Such objects, however, can be understood as rapidly rotating self-bound *strange stars*. The plausible ground-state state in that event is the deconfined phase of (3-flavor) strange-quark-matter. From the QCD energy scale this is as likely a ground-state as the confined phase. At the present time there appears to be only one crucial astrophysical test of the strange-quark-matter hypothesis, and that is whether strange quark stars can give rise to the observed phenomena of pulsar glitches. We demonstrate that the nuclear solid crust that can exist on the surface of a strange star can have a moment of inertia sufficiently large that a fractional change can account for the magnitude of pulsar glitches. Furthermore low-mass strange stars can have enormously large nuclear crusts (up to ~ 400 km) which might considerably alter the cooling rate of strange stars and enables such objects to be possible hiding places of baryonic matter.

If strange-quark-matter is the ground-state of baryonic matter at zero pressure then the conclusion that the confined hadronic phase of nucleons and nuclei is only metastable would be almost inescapable, which would have far-reaching consequences for laboratory nuclear physics, the early universe, and astrophysical compact objects.

Acknowledgement: This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of Nuclear Physics, of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

REFERENCES

1. F. Weber, Habilitation Thesis, January 1992, University of Munich, Munich, F.R.G.
2. N. K. Glendenning, Nucl. Phys. **A493** (1989) 521.
3. N. K. Glendenning, Astrophys. J. **293** (1985) 470.
4. F. Weber and M. K. Weigel, Nucl. Phys. **A505** (1989) 779, and references contained therein.
5. N. K. Glendenning, Nucl. Phys. B (Proc. Suppl.) **24B** (1991) 110.
6. N. K. Glendenning, Phys. Rev. D **46** (1992) 1274.
7. N. K. Glendenning, F. Weber, and S. A. Moszkowski, Phys. Rev. C **45** (1992) 844, (LBL-30296).
8. N. K. Glendenning, Phys. Rev. Lett. **57** (1986) 1120.
9. F. Weber, N. K. Glendenning, and M. K. Weigel, Astrophys. J. **373** (1991) 579.
10. H. A. Bethe and M. Johnson, Nucl. Phys. **A230** (1974) 1.
11. R. B. Wiringa, V. Fiks, and A. Fabrocini, Phys. Rev. C **38** (1988) 1010.
12. B. Friedman and V. R. Pandharipande, Nucl. Phys. **A361** (1981) 502.
13. F. Weber and N. K. Glendenning, *Neutron Stars, Strange Stars, and the Nuclear Equation of State*, in: Proceedings of the First Symposium on Nuclear Physics in the Universe, September 24-26, 1992, Oak Ridge, Tennessee, U.S.A., Ed. M. R. Strayer, IOP Publishing Ltd, Bristol, UK, 1993.
14. H. Heiselberg, C. J. Pethick, and E. F. Staubo, Phys. Rev. Lett. **70** (1993) 1355.
15. J. L. Friedman, J. R. Ipser, and L. Parker, Astrophys. J. **304** (1986) 115.
16. F. Weber and N. K. Glendenning, *Hadronic Matter and Rotating Relativistic Neutron Stars*, Proceedings of the Nankai Summer School "Astrophysics and Neutrino Physics", p. 64 - 183, Tianjin, China, 17-27 June 1991, Eds. D. H. Feng, G. Z. He, and X. Q. Li, World Scientific, 1993.
17. N. K. Glendenning and F. Weber, *Impact of Frame Dragging on the Kepler Frequency of Relativistic Stars*, (LBL-33401), 1993.
18. F. Nagase, Publ. Astron. Soc. Japan **41** (1989) 1.

19. N. K. Glendenning, Mod. Phys. Lett. **A5** (1990) 2197.

20. N. K. Glendenning, Phys. Rev. D **46** (1992) 4161.

21. N. K. Glendenning, *Strange-Quark-Matter Stars*, in: Proc. Int. Workshop on Relativ. Aspects of Nucl. Phys., ed. by T. Kodama, K. C. Chung, S. J. B. Duarte, and M. C. Nemes, World Scientific, Singapore, 1990.

22. J. M. Lattimer, M. Prakash, D. Masak and A. Yahil, Astrophys. J. **355** (1990) 241.

23. L. Lindblom, Astrophys. J. **303** (1986) 146.

24. S. Chandrasekhar, Phys. Rev. Lett. **24** (1970) 611.

25. J. L. Friedman, Phys. Rev. Lett. **51** (1983) 11.

26. L. Lindblom and S. L. Detweiler, Astrophys. J. **211** (1977) 565.

27. S. L. Detweiler, Astrophys. J. **197** (1975) 203.

28. H. Lamb, Proc. London Math. Soc. **13** (1881) 51.

29. C. Cutler and L. Lindblom, Astrophys. J. **314** (1987) 234.

30. J. R. Ipser and L Lindbolm, Phys. Rev. Lett. **62** (1989) 2777.

31. J. R. Ipser and L. Lindblom, Astrophys. J. **355** (1990) 226.

32. R. A. Managan, Astrophys. J. **309** (1986) 598.

33. F. Weber and N. K. Glendenning, Z. Phys. **A339** (1991) 211.

34. E. Witten, Phys. Rev. D **30** (1984) 272.

35. J. Madsen and M. L. Olesen, Phys. Rev. D **43** (1991) 1069, ibid., **44**, 4150 (erratum).

36. R. R. Caldwell and J. L. Friedman, Phys. Lett. **264B** (1991) 143.

37. N. K. Glendenning, *Supernovae, Compact Stars and Nuclear Physics*, invited paper in Proc. of 1989 Int. Nucl. Phys. Conf., Sao Paulo, Brasil, Vol. 2, ed. by M. S. Hussein et al., World Scientific, Singapore, 1990.

38. N. K. Glendenning and F. Weber, Astrophys. J. **400** (1992) 647.

39. F. Weber and N. K. Glendenning, *Fast Pulsars, Compact Stars, and the Strange Matter Hypothesis*, in: Proceedings of the 2nd International Conference on Physics and Astrophysics of Quark-Gluon Plasma, January 19-23, 1993, Calcutta, India, World Scientific.

40. G. Baym, C. Pethick, and P. Sutherland, Astrophys. J. **170** (1971) 299.

41. P. Pizzochero, Phys. Rev. Lett. **66** (1991) 2425.

42. N. K. Glendenning, Ch. Kettner, and F. Weber, *Unusual Configurations of Strange Stars*, in preparation (1993).

List of Figures

List of Tables

LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
TECHNICAL INFORMATION DEPARTMENT
BERKELEY, CALIFORNIA 94720