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A.A. Natale

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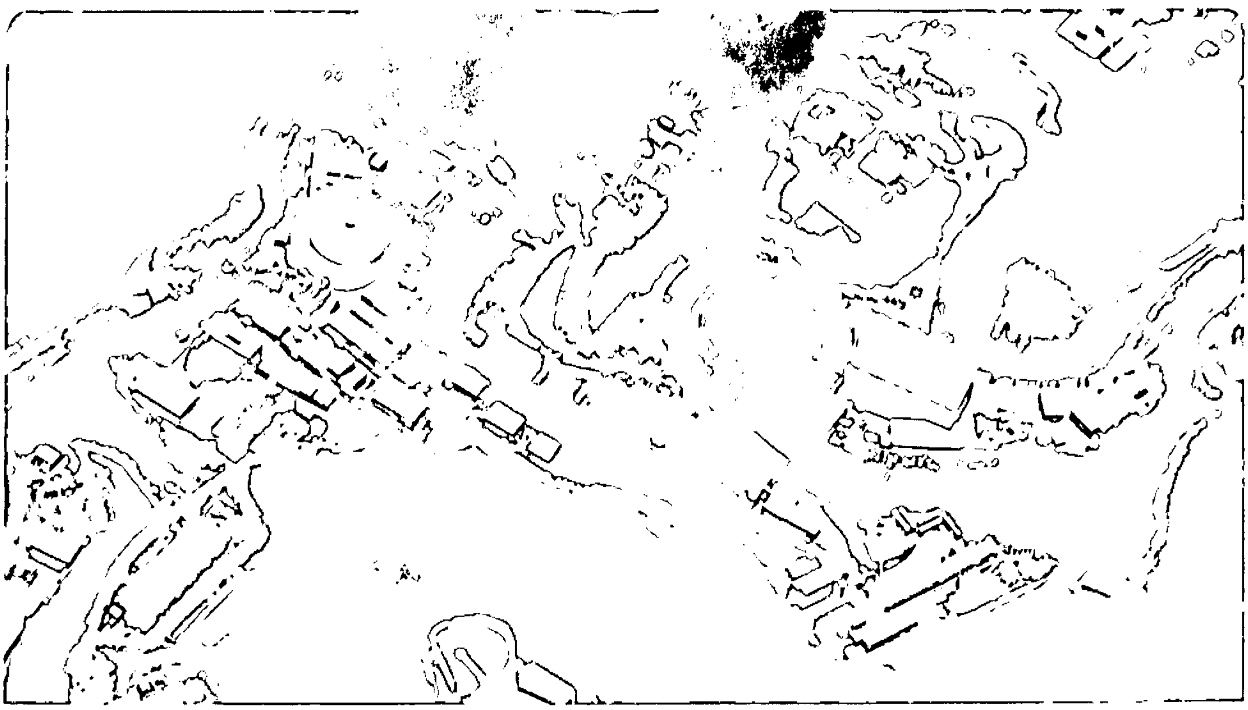

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FERMION MASSES AND THE EXTENDED TECHNICOLOR SCALE*

A. A. Natale†

Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

ABSTRACT

We discuss how the extended technicolor mass scale can be raised, avoiding the problems of flavor changing neutral currents in dynamically broken gauge theories.

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The calculation of fermion masses in gauge theories is a puzzle which has been remaining for several years. In theories with fundamental scalar fields the fermion mass spectrum is obtained by adjusting Yukawa couplings, in a range spanning already four orders of magnitude, from the mass of the electron to that of the b quark. This fact among others leads us to think that the elementary scalar fields are only a phenomenological approach to some new physics at higher mass scale.

There has recently been a great interest in models of dynamical symmetry breaking (dsb), where the Higgs boson is composite, and where "in principle" everything is calculable. The proposed models of dsb incorporate a new strong interaction:[1,2] Technicolor (TC), to provide the mass scale for the weak interactions. In order to explain the fermion masses one needs the introduction of yet another interaction, leading to the so called Extended Technicolor theories (ETC),[3] which couples the ordinary fermions to technifermions: A fermion mass term occurs via the diagram in Fig. 1.

The conventional fermion masses are believed to be related to the ETC boson masses by

$$m_f \simeq g^2_{ETC} \frac{\mu^3_{TC}}{M^2_{ETC}} , \qquad (1)$$

where the TC scale (μ_{TC}) is of $0(10^2)$GeV. To account for the known fermion masses the ETC gauge boson masses (M_{ETC}) have to be approximately 30 TeV.

In a realistic ETC theory we inevitably arrive at flavor changing neutral currents (FCNC), since the ETC gauge bosons connect fermions of same charge in different generations. These bosons give rise to rare transitions, in particular the $K_o - \bar{K}_o$ mass difference, which if mediated by a boson with a mass of 30 TeV, has a rate much too big to be reconciled with experiments.[4] Tentatives to build a suppression mechanism have failed.[5] There was also a proposal by Holdom to raise the ETC scale,[6] and this paper will discuss this second possibility.

The proposition to raise the ETC mass scale, is based on the fact that the standard renormalization group analysis gives the following behavior to the fermion self-energy[7]

$$\Sigma(p^2) \underset{p^2\to\infty}{\sim} \frac{\mu^3}{p^2}\left(\frac{p^2}{\mu^2}\right)^{\delta} , \qquad (2)$$

where

$$\delta = \frac{3c}{16\pi^2} g^2(p^2) .$$

For the general case of fermions in the representations R_1 and R_2, generating a mass in the representation R_3 ($R_3 \subset R_2 \times R_2$), c is given by

$$c = \frac{1}{2}\left[C_2(R_1) + C_2(R_2) - C_2(R_3)\right] .$$

($C_2(R_i)$ is the Casimir operator for the representation R_i).

If we use Eq. (2) to compute the fermion mass diagram of Fig. 1 we obtain[F1]

$$m_f \simeq \frac{g^2_{ETC}\,\mu^3_{TC}}{M^2_{ETC}}\left(\frac{M^2_{ETC}}{\mu^2_{TC}}\right)^{\delta} . \qquad (3)$$

However if we are in an asymptotically free theory δ is very small, and Holdom[6] was forced to assume that the technicolor interaction is not asymptotically free, in such a way that the β function has a fixed point g^*, controlling the high-momentum properties of the theory. Now, for reasonable values of g^*, the conventional fermion masses are compatible with high values of M_{ETC}, solving the FCNC problem. Actually, a quantitative analysis along the line of this proposal is not easy to carry out.

We want to show that this scenario can exist in the framework of an asymptotically free theory. To start with we recall that the renormalization group has another solution to the asymptotic self-energy,[7,14] given by

$$\Sigma(p^2) \underset{p^2\to\infty}{\sim} \mu\left(\frac{p^2}{\mu^2}\right)^{-\delta} , \qquad (4)$$

the so called irregular solution[F2].

Instead of computing the fermion masses by assuming g^2 constant, as done to obtain Eq. (3),[6] let us put Eq. (4) in the following form[14]

$$\Sigma(p^2) = \mu\left[1 + b\,g^2(\mu^2)\ln\frac{p^2}{\mu^2}\right]^{-\frac{3c}{16\pi^2 b}} , \qquad (5)$$

where b is the coefficient of g^3 in the β function. We then obtain[F3]:

$$m_f \simeq \mu_{TC} \frac{g^2_{ETC}(M^2_{ETC})}{g^2_{TC}(\mu^2_{TC})\, c} \left[1 + b g^2(\mu^2_{TC}) \ln \frac{M^2_{ETC}}{\mu^2_{TC}} \right]^{-\frac{3c}{16\pi^2 b}} \qquad (6)$$

Some comments about expression (6) are in order. The exponent $3c/16\pi^2 b$ is always bigger then 1/2 (to ensure the existence of Eq. (4)),[7,9] and normally takes values around 1 for many groups. The coefficient $bg^2(\mu^2_{TC})$ can be roughly estimated in the range of 0.1 to 1. The mass of the ETC gauge boson can vary from 10^7 GeV (enough to eliminate FCNC problems) to 10^{15} GeV (assuming a possible unification at this mass scale). This gives a suppression (of m_f related to μ_{TC}) of the order of 10^{-1} to 10^{-2}, due to the term in brackets. Now the fermion mass will depend strongly on the value of $g^2_{ETC}(M^2_{ETC})$, which can be of O(0.1 to 1), depending on the kind of breaking of the ETC group. We consequently obtained a suppression of $O(10^{-1}$ to $10^{-2})$ with respect to the TC scale, which means that we can explain fermion masses in the range 1 to 10^2 GeV.

We should ask about the light fermions. How they get a small mass? We can expect that the feed down mechanism of light fermion masses is provided by the electro-weak interactions, as proposed in early attempts to explain the e-μ mass ratio, where only the μ couples to a Higgs field.[10] Similar processes have also been discussed in TC models.[1,11]

We propose a very general scenario to technicolor theories, where, together with the standard model $SU(3)_c \times SU(2)_L \times U(1)$, we have a TC group, which could be a minimal $SU(2)_{TC}$, all of those embedded in a ETC group. The new feature is that we need some additional symmetry (discrete or continous), to prevent the coupling of technifermions to the light fermion generations, i.e., we want only the last generation to be connected to technifermions through the ETC bosons. To provide the isospin breaking within the last generation, we can also suppose that the d-fermion type pick up a mass only in the second order of ETC, as depicted in Fig. 2. Notice that variations of the ETC mass scale will not affect seriously the fermion spectrum (M_{ETC} can be much bigger than μ_{TC})[F4].

There is a second problem related to extended technicolor. Due to the presence of many massless fermions, after the breaking of the initial chiral symmetry, the theory is left with a large number of pseudo-Goldstone bosons (PGB), which gain masses when the electroweak interactions are turned on. In some cases these masses can be very small, which may give rise to significant flavor changing neutral interactions, even admitting a strong Cabibbo-like suppression.[4]

It is possible to see that the PGB masses (m_{PGB}) will be raised if computed with the help of Eq. (4). The PGB masses have been determined using chiral perturbation theory, spectral function sum rules and effective Lagrangians.[12] We would like to add another method to the above ones, the dynamical perturbation theory (DPT) proposed by Pagels and Stokar,[13] where m_{PGB} is obtained calculating diagrams as the one shown in Fig. 3 (the notation is the same as that in Ref. (13)).

The evaluation of m_{PGB} is strongly model dependent. We will compute the diagram of Fig. 3 at lowest order of DPT, for a PGB which was formed by an interaction characterized by a scale μ, with a "pion" decay constant "F_π" and coupling constant g. The radiative correction is due to a boson with a mass M_B, interacting weakly with a coupling g_B. Using Eq. (5), the PGB mass is given by

$$m^2_{PGB} \simeq g^2_B \left(\frac{\mu^2}{F^2_\pi}\right) \frac{M^2_B}{g^2\left(\frac{3c}{8\pi^2}-b\right)} \left[1 + b\,g^2(\mu^2)\ln\frac{M^2_B}{\mu^2}\right]^{-\frac{3c}{8\pi^2 b}} \qquad (7)$$

Using then Eq. (2), we get $m^2_{PGB} \propto g^2_B(\mu^2/F^2_\pi)(\mu^4/M^2_B)$. An analysis of Eq. (7) within the models discussed in the literature,[12] is enough to guarantee that the PGB masses are enhanced, to a level not dangerous to the present experiments[F5]. Notice that M_B can be (but is not necessarily) equal to M_{ETC}, bringing m_{PGB} very far away from the low energy phenomenology.

We conclude that there is not impediment to raise the ETC mass scale (in the case of asymptotically free theories having the asymptotic fermion self-energy given by Eq. (5)), to get rid of the FCNC problems in the extended-technicolor models. The only constraint that one must satisfy, is that the fermions in the last generation get masses through interactions with technifermions mediated by ETC gauge bosons, while the others (light fermions) pick up a mass by interacting weakly with those heavy fermions (as shown in Fig. 4).

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FOOTNOTES

F1. To compute m_f in Eq. (3), g^2 was assumed to be constant over the wide range of integration.

F2. It was recently argued that the irregular solution (Eq. (4)) is the only physical solution, by means of a renormalization group analysis of the dsb of QCD, based on the Nambu-Jona-Lasinio approach.[8] When used to compute the effective potential for composite operators, Eq. (4) leads to a deeper minimum.[9]

F3. To compute Eq. (6) it is helpful to use the following Mellin transform

$$\left[1+\alpha \ln \frac{p^2}{\mu^2}\right]^{-\beta} = \frac{1}{\Gamma(\beta)} \int_0^\infty d\sigma\, e^{-\sigma} \left(\frac{p^2}{\mu^2}\right)^{-\sigma\alpha} \sigma^{\beta-1}$$

F4. This only implies a large hierarchy between the TC and the ETC mass scales. A discussion on the hierarchy problem in dynamically broken gauge theories was made in Ref. (9). Even so, we can not rule out the possibility of breaking the ETC gauge group by fundamental scalar bosons invoking perhaps the presence of supersymmetry at high energies.

F5. For example, in the case discussed by Holdom, and Dimopoulos et al., (ref. (12)), where M_B is the mass of a leptoquark Pati-Salam boson ($M_B/g_B > 310$ TeV), which gives mass to a neutral color-singlet PGB, we can determine from Eq. (7) that $m_{PGB} \gtrsim O(1\ \text{TeV})$.

FIGURE CAPTIONS

Fig. 1. Diagram that generates fermion mass.

Fig. 2. Mass term in second order of ETC interactions.

Fig. 3. Lowest order contribution to the mass of the pseudo Goldstone bosons.

Fig. 4. In a model with m generations, the lighter fermion [n-1 (n-2,...)] acquires mass interacting weakly with the heavier [n(n-1,...)].

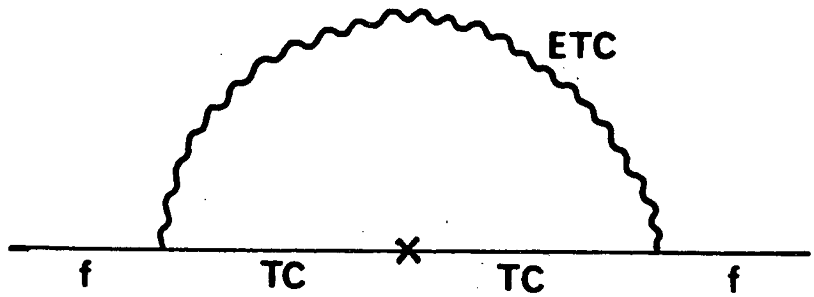

FIGURE 1

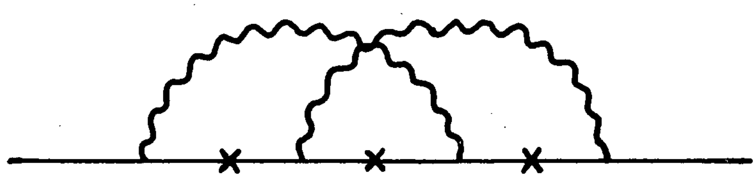

FIGURE 2

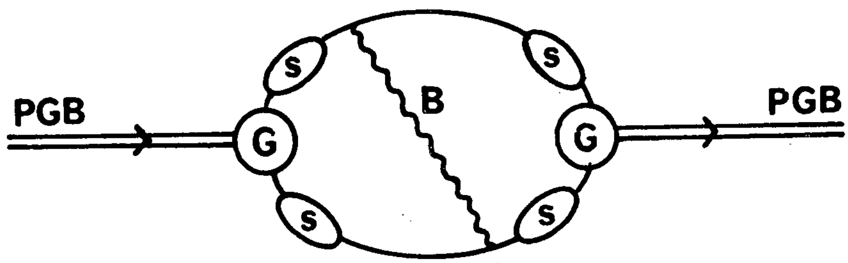

FIGURE 3

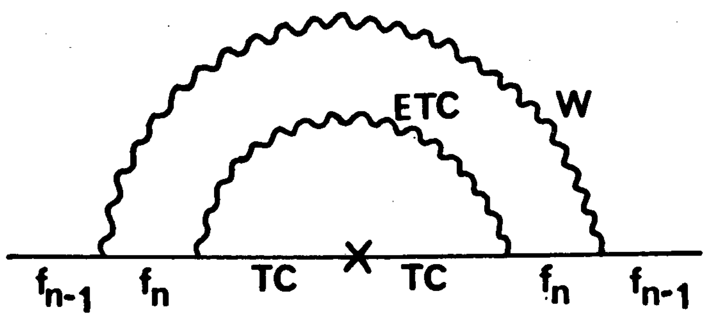

FIGURE 4

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LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720