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Author
Teitz, Michael B

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LOCATIONAL STRATEGIES FOR COMPETITIVE SYSTEMS

Michael B. Teitz

Department of City and Regional Planning and
Center for Planning and Development Research
University of California, Berkeley

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1. Introduction

Since its presentation in a classic paper [2] in 1929, the Hotelling problem has repeatedly generated extensions in location theory. The problem has been expanded to encompass more than two participants, and extended into two-dimensional space [3]. It has been reformulated by Stevens [5] into game theoretic terms in a paper that comes close to treating the system problem. In its simplest form of two or more locators in a bounded linear market with totally inelastic demand of uniform density everywhere, it has become the basic model for introducing students to spatial competition [1].

But one generalization of the problem seems to have been missed. It is the case of location of competing firms with multiple component units, that is, system competition. In view of the relevance of Hotelling's formulation to retail location problems, this gap is curious, for systems of retail stores have dominated certain parts of consumer goods sales for many years. Neglect of such system problems, however, appears to be quite general in location theory [6]. We may therefore ascribe this specific instance to the more general phenomenon.

In this paper, we examine the Hotelling problem under some simple assumptions about the system character of the competing firms. The results are interesting enough to suggest further extensions.

2. System Competition on the Line: The Two-Firm Case with Fixed Prices and Variable Location

The simplest version of the Hotelling problem deals with two competing firms, A and B, distributing some product over a bounded linear market. We define the firms A and B to be representable as systems of points on a line, each point representing a branch or component of its parent system. For the present, the scale of each component of each firm will be left indeterminate. Similarly, we ignore the cost of operation of each firm.

For simplicity, let the market be represented by a straight line of unit length. Consumers are distributed uniformly along the line, and demand for the product has a constant and uniform density of one. Each consumer, therefore, has a fixed demand which he fulfills regardless of price. Let the price of the product be uniform at every branch of each firm and let the prices p_A, p_B charged by A and B be equal. The only variable left to influence a consumer's selection of a firm and branch, then, is distance from that consumer to the nearest branch of either firm. Assume that the cost of travel is linear with distance and borne by the consumer who behaves as a distance minimizer.

Each firm has some given number of branches, N_A, N_B, that it must locate in the market. Let this number be a fixed integer greater than or equal to one. We seek the optimal allocations and strategies for A and B under varying reaction procedures, optimizing behaviors, and number of branches. No two branches can locate at the same point. We assume some minimal distance between pairs of adjacent locators.

The simplest Hotelling problem, then, is a special case of the system problem for which $N_A = N_B = 1$. The standard example for this situation is the familiar case of ice-cream-vendor carts on a beach. We have expanded it to allow multiple cart ownership.

Consider a reaction procedure by which competitors change location alternately, each being allowed to shift not more than one branch. Each competitor seeks to maximize his aggregate share of the market in the short run, i.e., on his current move. Hotelling showed that a stable equilibrium results from this procedure, with both competitors located adjacent to the center of the market, each taking one-half. He further pointed out that this solution does not yield minimum aggregate travel. The minimum average travel solution, that is, the social optimum, would find each vendor located at a quartile point. Aggregate travel would be exactly one-half of that under competition. It is a classic illustration of the deviation of competitive and social optima.

The simplest case of system competition holds B constant with one branch and allows A more than one. We first examine the case where $N_A = 2$, $N_B = 1$. Consider the examples shown in Figure 1. Assume that A has the field to himself. He is indifferent to the location of his two branches under the conditions we have established and locates them at A_1, A_2 as shown in Figure 1.1. Now B arrives with his single branch. Where he locates is determined by the rules of behavior for the model. Let us assume short-run maximization of market share on the part of both A and B. The possible outcomes for B are easily established. If B locates just to the left of A_1, his share will be slightly less than $\overline{X_0A_1}$; if he locates anywhere between A_1 and A_2, his share will be $\frac{1}{2}\,\overline{A_1A_2}$; if he locates just to the right of A_2, his share will be slightly less than $\overline{A_2X_1}$. Since it is evident that in this case

$$\overline{X_0A_1} < \frac{1}{2}\,\overline{A_1A_2} < \overline{A_2X_1} \quad ,$$

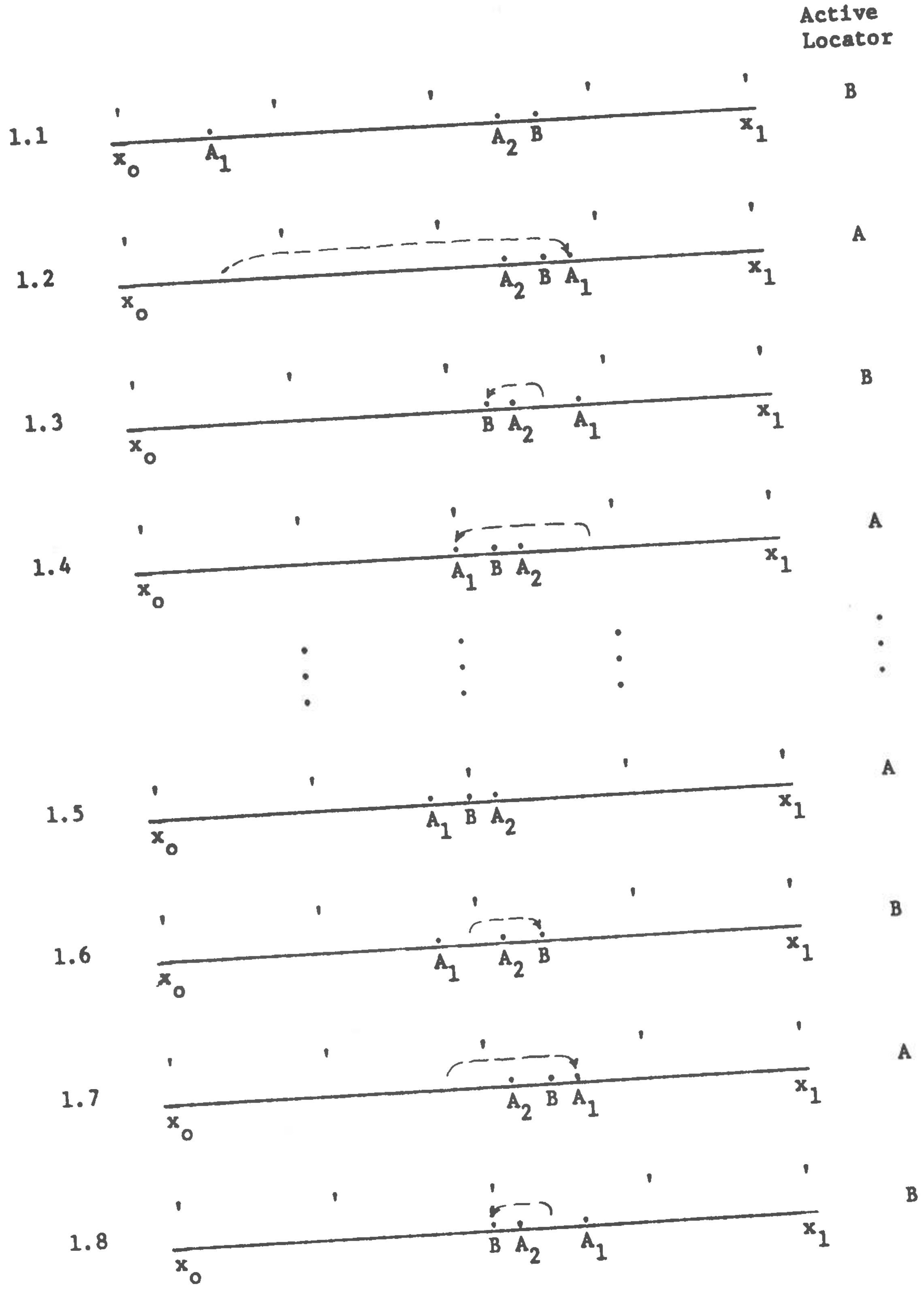

Figure 1.--System competition, short-run maximization; $N_A = 2$, $N_B = 1$.

B chooses the third location (Figure 1.1) and obtains approximately one-third of the market. What now is A's reaction? To achieve a maximal short-run gain he may move his branch A_1, and block B's access to the market by choosing a point immediately adjacent to B's right (Figure 1.2). Now A obtains virtually all the market. To respond, B has two choices, adjacent to the right of A_1 or the left of A_2. Clearly A_2 is superior, and he moves there, gaining about two-thirds of the market (Figure 1.3). A's response is to move A_1 to B's left, again recapturing the market (Figure 1.4). Thus, the situation resembles the original Hotelling formulation. The participants move first toward each other and then leapfrog towards the center. But once the center is reached, a peculiar equilibrium reigns. In Figure 1.5, the center has been reached by B. He is immediately bracketed by A_1 and A_2. Thus, B moves either to the right or left on his next move (Figure 1.6). The response of A is to move A_1 to B's right (Figure 1.7). B then must move back to the left (Figure 1.8), and so on *ad infinitum*. The result is a "dancing" equilibrium about the center.

It is difficult to evaluate the market shares in this instance. For the reaction procedure used here, it seems that half the time B takes about one-half the market. The rest of the time he gets virtually nothing. We could average these to calculate his "expected" share as one-quarter and A's as three-quarters.

The form of equilibrium arrived at by short-run maximization of market share is rather unsatisfactory. While not globally unstable, it does exhibit strong local instability. We are therefore encouraged to seek an alternative behavior rule to replace short-run maximization. One such rule suggests itself. Can a firm establish a location pattern so as to maximize that share of the market which is unassailable by any action his competitor might take? We may call such a strategy conservative long-run maximization. Evidently, it is a minimax strategy with B's share regarded as A's loss.

Consider the same situation as before ($N_A = 2_1$, $N_B = 1$). After locating as previously (Figure 2.1), A now meets B's response by moving to the quartile points (Figure 2.2-2.4). Once they are attained, A makes no further moves. B now has three choices. The two locations adjacent to the left of A_2 or to the right of A_1 each yield slightly under one-quarter of the market. Any location between A_1 and A_2 yields one-quarter exactly. Nothing B can do can improve on this share. Thus A with an advantage in branches of two-to-one, can gain a solid three-to-one advantage in market share. Furthermore, he can do no better by short-run maximization at equilibrium. It pays to branch out before your competition and stake out the territory.

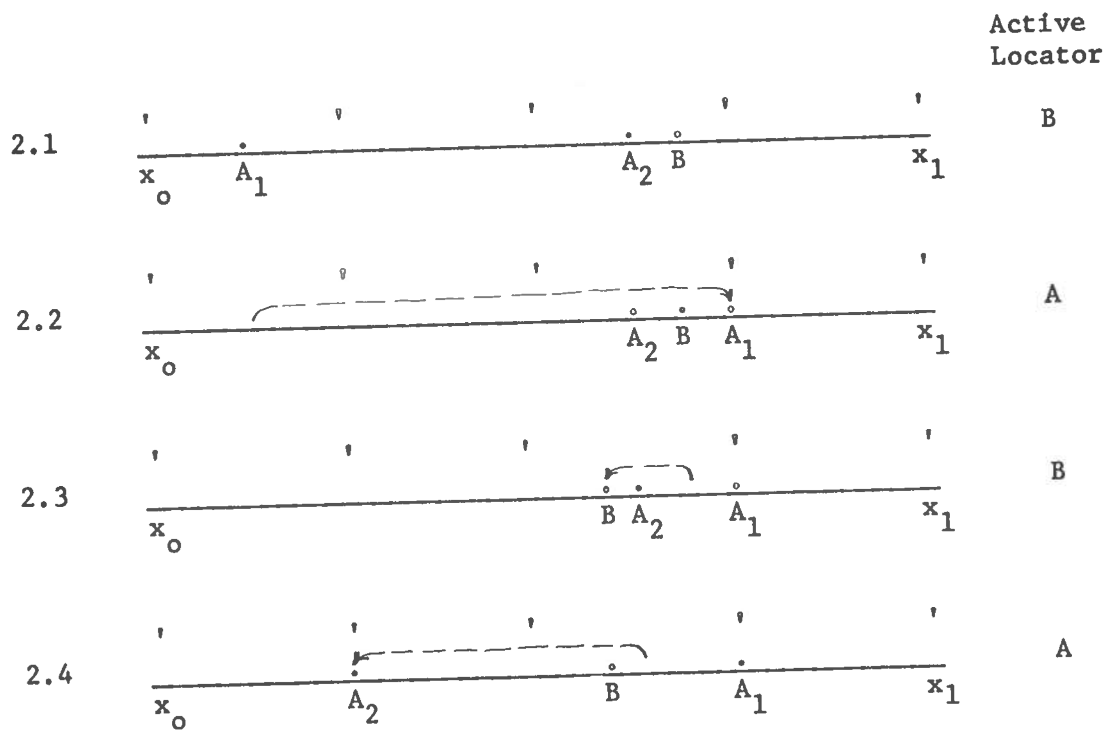

Figure 2.--System competition, conservative long-run maximization; $N_A = 2$, $N_B = 1$.

Conservative maximization appears to be an amenable behavior for analysis. We now need to examine its consequences for alternative systems. Consider first larger system sizes for A, holding B constant at one. Each possible arrangement of A's and B's branches may be represented by a permutation. Thus for $N_A = 3$, $N_B = 1$ we have

AAAB

ABAA .

All other arrangements are formally identical to one of these two. In fact, from A's viewpoint, a conservative maximation strategy should be independent of B's location, and therefore of the final pattern. We should be able to define it purely in terms of a desired location pattern for A alone. In the case of $N_A = 2$, we showed that the optimal pattern located branches at the quartiles. By analogous reasoning, we see that for $N_A = 3$, the locations for A should be at points 1/6, 3/6, and 5/6 of the way along the unit line. We will call these the odd sextile points. If A locates at these points (Figure 3), it is clear that no matter what B does he can obtain no more

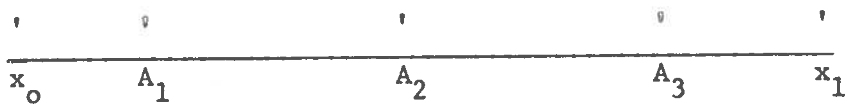

Figure 3.--Location at odd sextile points.

than one-sixth of the market. Thus a three-to-one advantage in branches gains A a five-to-one advantage in market share. Decentralization is still worthwhile. However, we should bear in mind that since the total market is fixed, a more appropriate measure would be the relative proportions of the market, M_A, M_B. These are given for various sizes of N_A, with $N_B = 1$, in Table I.

Table I

PROPORTIONATE MARKET SHARES WHEN $N_B = 1$

Number of Branches		Market Share	
N_A	N_B	M_A	M_B
1	1	0.50	0.50
2	1	0.75	0.25
3	1	0.83	0.17
4	1	0.87	0.13
5	1	0.90	0.10
.	.	.	.
.	.	.	.
.	.	.	.
n	1	$\frac{2n-1}{2n}$	$\frac{1}{2n}$

At the foot of Table 1 an expression for the market share where $N_A = n$ is given. More generally, we may state that for $N_B = 1$, the conservative maxima for A and B are:

$$M_A = 1 - \frac{1}{2N_A} \quad , \tag{1}$$

and

$$M_B = \frac{1}{2N_B} \quad . \tag{2}$$

If now we allow N_B to vary, a further set of similar results may be obtained. Consider the case $N_A = N_B = 2$. Suppose that A chooses the same conservative maximization locations as before for $N_A = 2$, namely, the odd quartiles. Wherever B locates his two branches, they will be unable to obtain more than one-half the market. The solution is quantitatively the same as for the original Hotelling problem, but the locations are scattered across the market. If N_A now grows to three while N_B remains at two, A's choice should again be the odd sextile points. Now B will receive two-sixths of the market, A four-sixths. Using similar reasoning, we construct Table II.

Table II

PROPORTIONATE MARKET SHARES WHEN $N_B = 2$

Number of Branches		Market Share	
N_A	N_B	M_A	M_B
2	2	0.50	0.50
3	2	0.66	0.33
4	2	0.75	0.25
.	.	.	.
.	.	.	.
.	.	.	.
n	2	$\frac{2n-2}{2n}$	$\frac{2}{2n}$

The general expression for $N_B = 2$ thus becomes

$$M_A = 1 - \frac{2}{2N_A} \quad , \tag{3}$$

$$M_B = \frac{2}{2N_A} \quad . \tag{4}$$

Suppose that $N_A = n$, $N_B = k$ ($k \leqq n$). If A locates his branches at the odd 2n-tile points, B must locate between them and can obtain at most k/2n as his share of the market. Thus, an entirely general expression for relative market shares will be:

$$M_A = 1 - \frac{N_B}{2N_A} \quad , \tag{5}$$

$$M_B = \frac{N_B}{2N_A} \quad . \tag{6}$$

With equation (5) we may plot M_A for various values of N_A holding N_B constant. Several such curves appear in Figure 4. A family of curves is defined by permissible values of N_B. Although N_A is strictly only an integer valued variable, the curves are drawn continuous for convenience. All are of course asymptotic to $M_A = 1.0$, but their slopes decrease rapidly with rising N_B. Why this is so may be simply explained by substituting $N_A = n$ and $N_B = n - 1$ into equation (5). The result is

$$M_A = 1 - \frac{n-1}{2n} = \frac{2n - (n-1)}{2n} = \frac{n+1}{2n} \quad ; \tag{7}$$

$$M_B = \frac{n-1}{2n} \quad . \tag{8}$$

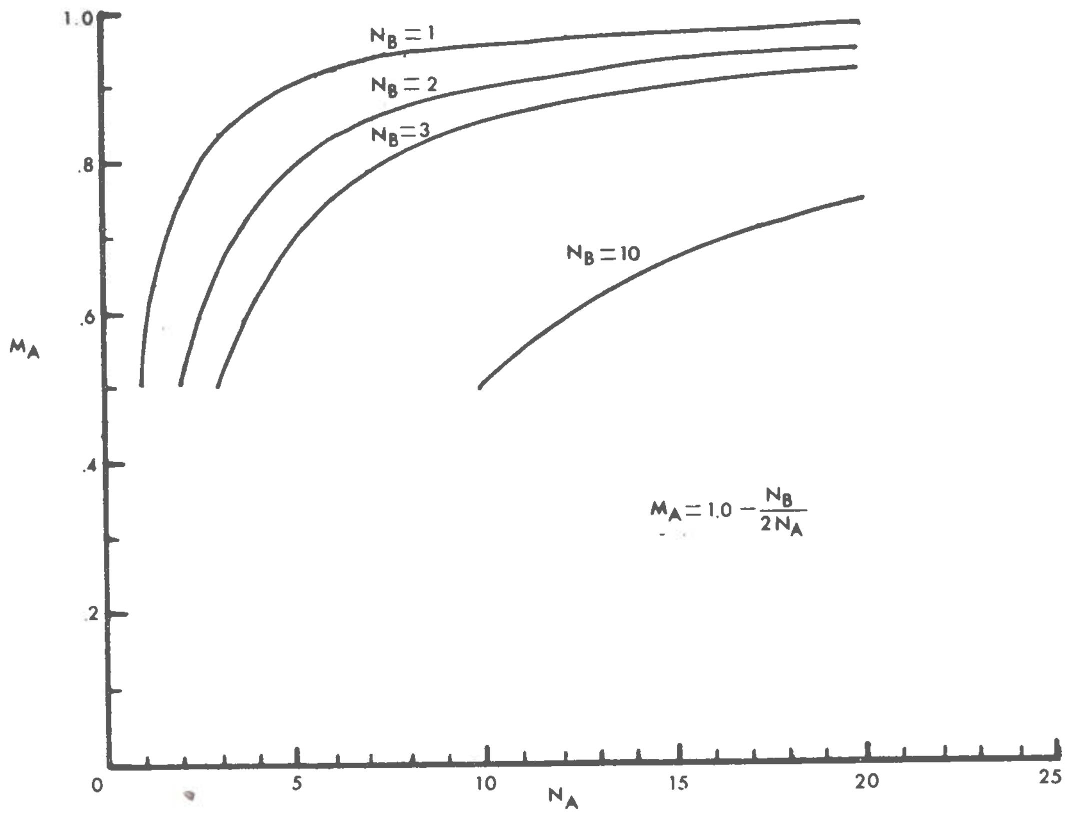

Figure 4.--Relative shares for A for N_{A1}, N_B varied.

Taking limits on equation (7), we have

$$\lim_{n \to \infty} M_A = .5 \quad , \tag{9}$$

and the curve is flat.

Most interestingly, we might note that the form of equation (5) allows us to substitute certain fixed values for market shares on the left-hand side and solve for the necessary relative orders of system size to achieve those shares. Values such that the resulting multiples are integral are given in Table III. Evidently, there are decreasing returns to relative system size in a fixed market.

Table III

RELATIVE SYSTEM SIZE FOR GIVEN MARKET SHARE

Proportionate Market Share Desired, M_A	Relative System Size
0.50	$N_A = N_B$
0.75	$N_A = 2\ N_B$
0.90	$N_A = 5\ N_B$
0.95	$N_A = 10\ N_B$
0.99	$N_A = 50\ N_B$

An interesting conclusion emerges from this exercise. Hotelling found that simple competition drove the participants to locate suboptimally from a social viewpoint. We now find that for systems of branches, competition drives the larger system to select the set of locations socially optimal for its own size. This feature persists independently of the relative size of the systems. Even a small gadfly can keep the big operator "honest."

The final result will not be the true optimum, but if the relative sizes of the systems are very different, it is close to the social optimum.

3. Equilibrium for Systems with Fixed Locations and Variable Prices

Even in Hotelling's original terms, the framework above is excessively simple. We have assumed a linear market, no more than two firms, zero cost, equal prices over firms and branches, and constant density and elasticity of demand. It would be desirable to relax all these assumptions simultaneously. However, that is beyond the scope of this paper. We shall therefore examine the consequences of relaxing some of them individually.

Consider first the question of price. Suppose that the competitors are located at fixed points on the unit line as before and A has two branches to B's one, located as shown in Figure 5. Assume a fixed transportation rate, s.

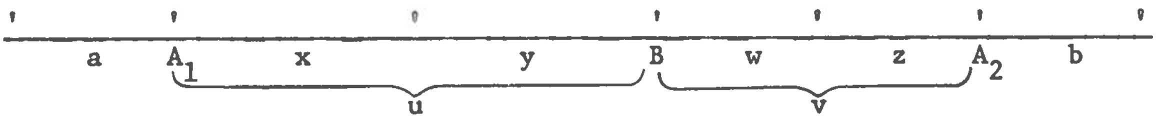

Figure 5.--Location of competitors' branches at fixed points.

Locations are fixed with distance u between A_1 and B, and v between B and A_2. Costs are zero. Each competitor now is free to vary his price in an attempt to maximize his market share. We seek an equilibrium set of prices and shares. Assume that A may vary prices between his two locations. The markets of A and B for any set of prices (p_{A1}, p_{A2}, p_B) will be given in terms of the unknowns x and z:

$$p_{A1} + sx = p_B + s(u-x) \quad , \tag{10a}$$

$$p_{A2} + sz = p_B + s(v-z) \quad , \tag{10b}$$

where

$$u - x = y \quad ,$$

$$v - z = w \quad ,$$

and

$$a + u + v + b = 1 \quad . \tag{11}$$

The total sales, then, of A and B are

$$q_A = a + x + z + b \tag{12a}$$

and

$$q_B = y + w \quad . \tag{12b}$$

Rearranging equations (10a) and (10b), we see that

$$x = \frac{u}{2} + \frac{1}{2s}(p_B - p_{A1}) \quad , \tag{13a}$$

$$y = \frac{u}{2} - \frac{1}{2s}(p_B - p_{A1}) \quad , \tag{13b}$$

and

$$z = \frac{v}{2} + \frac{1}{2s}(p_B - p_{A2}) \quad , \tag{13c}$$

$$w = \frac{v}{2} - \frac{1}{2s}(p_B - p_{A2}) \quad . \tag{13d}$$

Thus, the profit functions for the competitors may be written as

$$\Pi_A = p_{A1}q_{A1} + p_{A2}q_{A2} = p_{A1}(a + x) + p_{A2}(b + z)$$

$$= \left(a + \frac{u}{2}\right)p_{A1} + \frac{p_{A1}p_B}{2s} - \frac{p_{A1}^2}{2s} + \left(b + \frac{v}{2}\right)p_{A2} + \frac{p_{A2}p_B}{2s} - \frac{p_{A2}^2}{2s} \quad , \tag{14a}$$

and

$$\Pi_B = p_B(y + w) = \left(\frac{u + v}{2}\right)p_B + \frac{p_{A1}p_B}{2s} + \frac{p_{A2}p_B}{2s} - \frac{{p_B}^2}{s} . \tag{14b}$$

Each competitor endeavors to maximize his profit by varying his own price while treating his opponent's price as fixed. We seek an equilibrium consistent with such profit-maximization on the part of each.

For maximum profit, the conditions for the competitors are

$$\left.\begin{aligned} \frac{\partial \Pi_A}{\partial p_{A1}} &= 0 , \\ \frac{\partial \Pi_A}{\partial p_{A2}} &= 0 , \\ \text{and} \quad \frac{\partial \Pi_B}{\partial p_B} &= 0 . \end{aligned}\right\} \tag{15}$$

For the profit functions (equations 14a, 14b), the conditions (15) are given by

$$\frac{\partial \Pi_A}{\partial p_{A1}} = \left(a + \frac{u}{2}\right) + \frac{p_B}{2s} - \frac{p_{A1}}{s} = 0 , \tag{16a}$$

$$\frac{\partial \Pi_A}{\partial p_{A2}} = \left(b + \frac{v}{2}\right) + \frac{p_B}{2s} - \frac{p_{A2}}{s} = 0 , \tag{16b}$$

$$\frac{\partial \Pi_B}{\partial p_B} = \left(\frac{u + v}{2}\right) + \frac{p_{A1}}{2s} + \frac{p_{A2}}{2s} - \frac{2p_B}{s} = 0 . \tag{16c}$$

The equations (16a-c) may be solved for equilibrium values of p_{A1}, p_{A2}, and p_B yielding:

$$\hat{p}_{A1} = s\left(1 + \frac{1}{6}a - \frac{5}{6}b - \frac{1}{4}u - \frac{3}{4}v\right) , \qquad (17a)$$

$$\hat{p}_{A2} = s\left(1 - \frac{5}{6}a + \frac{1}{6}b - \frac{3}{4}u - \frac{1}{4}v\right) , \qquad (17b)$$

$$\hat{p}_{B} = s\left(1 - \frac{2}{3}a - \frac{2}{3}b - \frac{1}{2}u - \frac{1}{2}v\right) . \qquad (17c)$$

The second partials are all negative, so these values are associated with maxima on Π_1 and Π_2. If we substitute these values into equations (12) and (13), we find that the equilibrium sales quantities are

$$\hat{q}_A = 1 - \frac{1}{3}a - \frac{1}{3}b - \frac{1}{2}u - \frac{1}{2}v = \frac{1}{2} + \frac{1}{6}(a + b) \qquad (18a)$$

and

$$\hat{q}_B = 1 - \frac{2}{3}a - \frac{2}{3}b - \frac{1}{2}u - \frac{1}{2}v = \frac{1}{2} - \frac{1}{6}(a + b) . \qquad (18b)$$

Solutions for the individual components of $\hat{q}_A$ may be expressed as:

$$\hat{q}_{A1} = \frac{1}{2} + \frac{1}{12}a - \frac{5}{12}b - \frac{1}{8}u - \frac{3}{8}v \qquad (19a)$$

and

$$\hat{q}_{A2} = \frac{1}{2} - \frac{5}{12}a + \frac{1}{12}b - \frac{3}{8}u - \frac{1}{8}v . \qquad (19b)$$

Total profit at equilibrium for A and B may be derived from equations (17) and (19) as

$$\hat{\Pi}_A = \hat{p}_{A1}\hat{q}_{A1} + \hat{p}_{A2}\hat{q}_{A2} \qquad (20a)$$

$$\hat{\Pi}_B = \hat{p}_B\hat{q}_B . \qquad (20b)$$

The profit function is a quadratic in a, b, u, and v.

Two questions immediately suggest themselves: (1) Does this solution differ from that for three competitors on the line? (2) How does a single price for A's product affect the outcome?

To the first question, the answer seems to be negative. Although the profit function has only two equations (14a-b) instead of three for the three-competitor case, the portions of Π_A in (14a) corresponding to each of A's locations and prices are completely separable. The resulting conditions for maximization (16a-b) contain no interaction terms between p_{A1} and p_{A2}. Thus, they are exactly analogous to the three-competitor situation in which we would substitute the third competitor's price p_C for p_{A2}.

The answer to the second question is less self-evident. Assume that A is constrained to charge the same price p_A in each of his two outlets. If all locations and other characteristics are as before, we now have the equilibrium conditions:

$$p_A + sx = p_B + s(u - x) \tag{21a}$$

$$p_A + sz = p_B + s(v - z) \quad . \tag{21b}$$

Solving equation (21) and substituting for y and w, we have

$$\left.\begin{aligned} x &= \frac{u}{2} + \frac{1}{2s}(p_B - p_A) \quad , \\ y &= \frac{u}{2} - \frac{1}{2s}(p_B - p_A) \quad , \\ z &= \frac{v}{2} + \frac{1}{2s}(p_B - p_A) \quad , \\ w &= \frac{v}{2} - \frac{1}{2s}(p_B - p_A) \quad . \end{aligned}\right\} \tag{22}$$

From equations (21) and (22) we may formulate the profit functions

$$\Pi_A = p_A(a + x + z + b)$$

$$= \left(a + b + \frac{u}{2} + \frac{v}{2}\right)p_A + \frac{1}{s}\left(p_A p_B - p_A^{\,2}\right) , \tag{23a}$$

and

$$\Pi_B = p_B(y + w) = \left(\frac{u}{2} + \frac{v}{2}\right)p_B + \frac{1}{s}\left(p_B p_A - p_A^{\,2}\right) . \tag{23b}$$

Taking partial derivatives of Π_A and Π_B for p_A and p_B, respectively, and setting them equal to zero yields the following equilibrium prices:

$$\hat{\hat{p}}_A = s\left(1 - \frac{1}{3}a - \frac{1}{3}b - \frac{1}{2}u - \frac{1}{2}v\right) , \tag{24a}$$

and

$$\hat{\hat{p}}_B = s\left(1 - \frac{2}{3}a - \frac{2}{3}b - \frac{1}{2}u - \frac{1}{2}v\right) . \tag{24b}$$

Solving equation (22) for x, y, z, and w, we may then derive the equilibrium market quantities for A and B.

$$\hat{\hat{q}}_A = a + x + b + z = 1 - \frac{1}{3}a - \frac{1}{3}b - \frac{1}{2}u - \frac{1}{2}v . \tag{25a}$$

$$\hat{\hat{q}}_B = y + w = 1 - \frac{2}{3}a - \frac{1}{3}b - \frac{1}{2}u - \frac{1}{2}v . \tag{25b}$$

The profit function for each is again a quadratic in a, b, u, and v.

If we now compare equilibrium prices and quantities in each case, some interesting characteristics emerge. From equations (17) and (24), it is apparent that B's equilibrium price is unchanged by the imposition of uniform prices on A. Further, A's equilibrium price in the latter case is simply the mean of the equilibrium prices under differentiation. This is certainly not a self-evident result. For quantities, the outcome is also surprising. Comparing equations (18) and (25), we see that the total market shares of A and B remain unchanged at equilibrium regardless of the price policy chosen by A. It is also apparent that the quantities x, y, z, and w are no longer the same. Thus we would not expect profits to be unchanged. However, since the case of variable prices for A includes that of uniform prices as a subset of equilibrium solutions, it will not be possible for the latter to show a higher profit. The general result, then, will be

$$\hat{\hat{\Pi}}_A \leqq \hat{\Pi}_A \qquad \text{and} \qquad \hat{\hat{\Pi}}_B = \hat{\Pi}_B \quad . \tag{26}$$

Not much can be said about the stability of this equilibrium beyond Hotelling's original discussion. The competitors would gain by collusion to raise prices jointly above the equilibrium level, but there is always considerable incentive for one to cut his price in the hope that the other will not. We have introduced one further factor, namely, the difference in competitive power between A and B. Insofar as A controls greater resources, he may be able to outlast B in a price war and drive him completely from the market. On the other hand, in this model, the competitor who cuts his price most faces the greater cut in profit during the conflict. If, however, A has more outlets--say four to B's one--he can presumably cut the price at the ones adjacent to B and use the profit from the remainder to ride out the storm. Thus, at some cost in immediate income, he can utilize his system-power to drive out his weaker opponent. Such discriminatory behavior in gasoline and dry-cleaning price wars is often noted bitterly by independents, but it appears to be a logical concomitant of unequal system size.

4. Conclusion

Many variations might be played upon the general theme introduced here. We will take note of some of the more important for further investigation. While differences in system size imply parallel differences in power, they may also have negative effects. Certain kinds of costs arise in large systems--especially the cost of bureaucracy--that may not appear in small areas. This must be taken into account whenever there is any introduction of cost into the model, as must the more conventional cost-savings associated with scale economies.

Besides the question of cost, other difficult theoretical extensions involve demand behavior, space, and number of competitive systems. While we may examine these in the context of a computer simulation model, it is hard to derive analytical results for them. Nevertheless, such theoretical investigation is necessary for further understanding of the influence of the existence of systems on locational behavior.

One other area for further work is the performance of the model under alternative behavioral strategies. We have looked at system behavior under three strategies. Many others have been recognized and discussed [4]. How they would influence the outcome in competition among systems remains to be seen.

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