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Situated Mathematics Instruction:  
The Interaction of Teacher Knowledge, Beliefs, and Policy Interpretation

A dissertation submitted in partial satisfaction of the  
requirements for the degree Doctor of Education

in

Teaching and Learning

by

Andrea Lynn Barraugh

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2011

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The dissertation of Andrea Lynn Barraugh is approved, and it is acceptable in  
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Chair

University of California, San Diego

2011

## **DEDICATION**

To the students in the classrooms across America who deserve to understand mathematics.

To the teachers who opened their classrooms and themselves to me for this study. Your commitment to the profession and to your students is inspiring.

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#### **PROFESSIONAL ORGANIZATIONS**

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American Education Research Association (AERA)  
National Council for Teachers of Mathematics (NCTM)

## **ABSTRACT OF THE DISSERTATION**

Situated Mathematics Instruction:  
The Interaction of Teacher Knowledge, Beliefs and Policy Interpretation

by

Andrea Lynn Barraugh

Doctor of Education in Teaching and Learning

University of California, San Diego, 2011

Professor Amanda Datnow, Chair

Students in the U.S. demonstrate a pattern of knowing mathematics as procedures rather than as a discipline of principled reasoning. International studies trace this pattern to cultural regularities within classroom mathematics instruction. Current researchers seeking to understand the expression of mathematics instruction explore dyadic, and sometimes mediational, relationships between teachers' mathematical knowledge for teaching, their beliefs about teaching and learning mathematics, their social interactions, and their interpretations of curricular materials. However, most prior studies have not taken the school, district, or policy context into account as a mediating factor. Using a

situative perspective, this study incorporates these factors into a comprehensive exploration of what plays into teachers' mathematics instruction.

This study used a combination of quantitative and qualitative methods to gather information about the factors influencing mathematics instruction of seven upper elementary grade teachers in a single school district. It used validated surveys and assessments to measure teachers' knowledge for teaching mathematics and beliefs about teaching and learning mathematics. Observational data were also analyzed using pre-established rubrics to characterize the mathematical quality of teachers' instruction. Additionally, teacher interviews were conducted to uncover teachers' interpretations of policies and triangulate knowledge and beliefs data. The data were analyzed quantitatively and qualitatively using within and cross-case approaches.

Analysis of the data revealed that the mathematical quality of the teachers' instruction was determined by dynamic interaction between each teacher's beliefs, knowledge, policy interpretation, and context-specific factors. Teachers' mathematical knowledge for teaching was related to their mathematical explanations and responses to student math productions during instruction. However, the study also found that curricular materials, teachers' interactions with their colleagues, and local policies played a critical role in mediating instructional quality. Knowledge, beliefs, and interpretation of curriculum and accountability policies interacted differently depending on the teachers' social and institutional contexts. Implications for practice are discussed.

## **CHAPTER I: INTRODUCTION**

### **Research Foundations**

As researchers, policy makers, and educators in the United States search for underlying causes of low student performance on national and international mathematics assessments (NAEP, 2008; TIMSS, 1999, 2003, 2007) and rue the lack of principled mathematical knowledge<sup>1</sup> developed in mathematics classrooms (Ball, 1993; Cobb, 1988; Greeno, 1992; Harel, 2008a, b, c; Lampert, 1990, 1992; Schoenfeld, 1989; Simon, 1986), it is critical to investigate the quality of mathematics instruction and factors contributing to it. Most would agree that the quality of instruction teachers provide in the classroom influences the quality of student learning that takes place (Hill, Rowan, & Ball, 2005; Kersting, Givvin, Sotelo, & Stigler, 2009). An increasing number of studies suggest that student learning is most affected by teachers themselves, not by standards, programs, class sizes, schools, or curricula (Nye, Konstantopoulis, & Hedges, 2004). However, as will be demonstrated in this dissertation, contextual factors play more primary roles in the quality of teachers' mathematics instruction than previous research predicts.

Among current research, studies highlighting the critical role of the teacher focus on direct dyadic relationships such teacher's knowledge and instruction (Ball, Thames, & Phelps, 2008; Carpenter, Fennema, Peterson, & Carey, 1988; Fennema et al., 1996; Hill, Rowan, & Ball, 2005; Hill, Blunk, Charalambous, Lewis, Phelps, Sleep, Ball, 2008; Kersting et al., 2009; Ma, 1999; Shulman, 1986, 1987) or teachers' beliefs and instruction

---

<sup>1</sup> Principled math knowledge includes understanding key ideas and concepts with an emphasis on reasoning (Lampert, 1986; Greeno, 1992; Spillane & Zeuli, 1999).

(Hoyles, 1992; Raymond, 1997; Skott, 2001; Stipeck, Givven, Salmon, MacGyvers, 2001). These studies would benefit from a greater emphasis on the mediated nature of the relationships and on the role played by social and institutional contexts. From a situative perspective, knowing and doing are mediated. What a teacher knows and does is a product of his/her own cognition, interactions with other people, and interactions with the environment (Daniels, 2008; Greeno, 1998; Putnam & Borko, 2000; Spillane, 2001; Wenger, 1998).

Current researchers (Ball et al., 2008; Carpenter et al., 1988; Fennema et al., 1996; Hill et al., 2005; Hill et al., 2008; Kersting et al., 2009) take a cognitive perspective as they search for ways to measure teachers' *mathematical knowledge for teaching*<sup>2</sup> (MKT) and understand its relationship to the quality of mathematics instruction and student learning. These studies foreground teacher knowledge and background other factors that mediate knowledge, instruction, and student learning. While recent findings show a correlation between teacher knowledge and student learning (Hill et al., 2005; Kersting et al., 2009) and a predictive relationship between teacher knowledge and the quality of mathematics instruction (Hill et al., 2008; Kersting, personal communication, January 20, 2010), further studies are needed to examine these relationships. Moreover, more research is needed to understand the knowledge-instruction-learning dynamic

---

<sup>2</sup> *Mathematical knowledge for teaching* (Ball et al., 2008) is a construct derived from Shulman's (1986, 1987) pedagogical content knowledge (PCK). PCK is defined as the intersection of what a teacher must know about subject matter and about pedagogy in order to make specific content accessible for students (Shulman, 1986, 1987). MKT is a mathematics-specific version of PCK which includes math content knowledge common to everyday people and specialized content knowledge specific to making content understandable to others.

situated within teachers' historically and culturally derived beliefs, social interactions, and institutional contexts.

Through their training and experiences, teachers develop strongly held beliefs about how to teach and how students learn. I define beliefs as deeply held personal truths about the world. They develop from episodes in life (Eraut, 1985; Nespor, 1987; Nisbett & Ross, 1980) and have a strong evaluative and judgmental nature (Nisbett & Ross, 1980). While not static, beliefs are difficult to change (Pajares, 1992). Beliefs are critical mediators of how MKT comes to be expressed or repressed in classroom instruction. Any explanation of the relationship between teacher knowledge and classroom instruction or learning that does not consider the role of teachers' beliefs in shaping the use of knowledge is incomplete.

Likewise, as members of educational institutions, teachers are situated within institutional policy contexts steeped in state standards, required curricula, and high stakes accountability measures. For the purposes of this study, "institutional policy" means the top-down messages regarding content standards, curricular materials and their use, and assessment constructed at federal, state, or district levels. Within this context, teachers are active agents of policy implementation as they decide what policies mean and to what degree they will implement them (Coburn, 2001). Inherent in the teacher-policy relationship is the teacher-text relationship (McClain, Quin Zhao, Visnovska & Bowen, 2009; Remillard, 2009) within which the teacher is the agent of curriculum implementation, and the textbook serves as a tool to support teachers as active decision makers. Therefore, one must consider how teachers make sense and implement policy

messages and how their perceptions interact with their knowledge and their beliefs as they design and enact mathematics instruction. While some researchers examine teacher sensemaking at the individual level (Cohen & Ball, 1990; Spillane, Reiser, & Reimer, 2002; Spillane & Zeuli, 1999), others study it as a collective group activity (Coburn, 2001; Coburn, 2008; Hill, 1999; Lin, 2000; Spillane, 1998; Spillane, 1999; Yanow, 1996). I chose to focus at the individual level while also considering the influence of collective thinking and action.

In a study of teacher implementation of curriculum, Spillane (2002) touts the value of studying individual teachers' sensemaking processes including their schema, prior knowledge, beliefs, experiences, knowledge assimilation, and emotions. In alignment with Lakoff's (1980) theory that humans filter new information through culturally constructed cognitive frames or worldviews (Jennings, 1996; Spillane & Jennings, 1997), Spillane (1999) theorizes as teachers construct understandings and implement policy, they adjust the input to fit within their current cognitive models of content, pedagogy, and learning. Understanding the interplay between the sense teachers make of policies and their cognitive models as they meld into instructional decisions and practice is critical in understanding how mathematics instruction comes to be.

While studies of mathematical knowledge for teaching, teacher beliefs, or teachers' institutional policy sensemaking illuminate different factors contributing to the quality of classroom instruction, no study explicitly incorporates all three factors and examines their interaction. Studying the interplay of teachers' knowledge, beliefs, and

policy interpretation within institutional settings bears a more thorough explanation for how instruction comes to be expressed than any study done to date.

### **Personal Experience**

The importance of studying the interplay between knowledge, beliefs, and institutional policy interpretations is validated by my experience as an elementary and middle school mathematics teacher. In this capacity, I am acutely aware of variations in the characteristics of different teachers' mathematics instruction. Through collegial interactions I observe different levels of mathematics content and pedagogical knowledge, distinct variations in teachers' beliefs about how mathematics should be taught, and the increasingly strong role institutional policy plays in the content and structure of classroom mathematics lessons.

Currently, teachers' interpretations of curriculum and accountability policies seem to overshadow the other factors. When I plan with other teachers, they typically have a textbook, a list of content standards, or a test in front of them. Their goal seems focused on checking off pages or skills covered rather than on student learning. They demonstrate a need to cover content and produce higher test scores in response to what they believe is expected of them. Their interpretations represent filtration of policies through multiple agents (curriculum leaders, principals, committees) until certain information and expectations become privileged within schools (Coburn, 2001). Teachers are the final filters of policies and the agents who decide how to interact with the materials to produce classroom practice.

My initial interest, guided by recent research (Hill, Schilling, & Ball, 2004; Hill et al., 2008; Jacobs, Lamb, Philipp, Schappelle, 2009a, b; Kersting et al., 2009; Sherin & van Es, 2005) and my experience as a mathematics professional developer, was in furthering the field of teacher pedagogical content knowledge and its relationship to classroom instruction and student learning. I hoped to find ways to improve teachers' mathematics content knowledge and pedagogical knowledge so that they would be better prepared to make in-the-moment decisions that furthered the thinking of individual students. However, with the current state of high-stakes student accountability measures, institutional policies are playing a greater role in shaping classroom instruction than any other time in my 19 year career. I find myself wondering if teachers with high levels of knowledge for teaching mathematics and with strong reform-oriented beliefs can continue to enact research-based instructional strategies in the current climate of standards, standardized curriculum and accountability measures.

Like all public school districts across the United States, the district I teach in is affected by federal and state accountability policies. The federal No Child Left Behind Act of 2001 (NCLB) holds states accountable for students' yearly academic growth through annual standardized testing. While standardized testing is not new in American education, it is the accountability part of NCLB that shapes the policy context differently and filters into classroom instruction. Schools can be shut down if their students do not show adequate yearly growth. Some districts choose to publicly publish test scores by teacher.

The schools in my district are typically high-performing which lessens the threat of the policy as compared to lower-performing districts; however the pressure to increase test scores is ever present. While the intent of NCLB is to ensure all students are learning, the reality of the policy as it pertains to mathematics is a system where process-focused national standards (National Council of Teachers of Mathematics, 2000) have been translated into skill-focused, easily measured, state standards (California State Board of Education, 1999) in an effort to standardize curriculum and more readily track student progress. It hinders classroom instructional practice which might support students in developing principled mathematical knowledge.

At a district level, NCLB (2001) and its resulting skill-focused state standards is translated into the adoption of textbooks that align with the standards and are often similar in skill presentation to the state tests. In my district, further measures are in place to increase success on state tests. Pacing guides reflect district calculations of content that must be covered before the state test in order to ensure the highest scores possible. Quarterly district benchmark assessments hold teachers accountable to complete specific textbook chapters by certain dates. Additionally, three times per year, students in first through ninth grade take the Measures of Academic Progress (MAP) test, an online, multiple-choice assessment which tracks their mathematics progress across their academic career in the district (Northwest Evaluation Association, 2011). As evidence of the influence of the state testing accountability pressure, the district runs a correlational analysis of student test scores on the MAP test as predictive of their California Standards Test scores. The data is intended to inform teachers' instructional decisions. Most

significant about these assessments are their match in format and skill-emphasis to the state tests. Their creators do not attempt to balance the lack of alignment between skill-based state standards and concept, process-focused national standards (NCTM, 2006). They measure student progress on basic skills so they can perform well on state tests. This scenario is typical of public schools under the rule of NCLB. It is also the situation within which teachers must sift through policy to make decisions about what and how to teach math content to students.

Recently, I was among a community of reform-oriented professional developers who have committed their careers to make sure children experience mathematics in meaningful ways. It included principals, math coaches, math teachers, and district curriculum leaders. Many individuals I talked to expressed frustration that, because of the urgent accountability climate created by NCLB (2001), they were teaching and leading in ways that were incongruent with their beliefs and their knowledge of successful mathematics instruction. Some teachers reported they were required by their principals to work through test-preparation worksheets for 45 minutes per day. Others reported spending precious school funds on worksheet-based programs that would improve student test scores. Principals were discouraged by what they had to require their teachers to do in order to increase test scores. They did not believe the format-driven and procedure-based approach was helping students become strong mathematical reasoners and problem solvers. However, against their own beliefs and knowledge of how children learn mathematics, these educators found themselves doing to children exactly what they knew would inhibit the development of mathematical understanding.

### **Research Question**

My own daily experiences in California schools combined with my interactions with other professional educators across the nation bring to light the timeliness and the importance of examining how knowledge, beliefs, and interpretations of policy interact to create classroom instruction and, in turn, affect student learning. Combining my experiences with current research on classroom instruction, the following research question along with its component parts emerge:

How do teachers' knowledge for teaching mathematics, beliefs about teaching and learning mathematics, and the sense they make of institutional curriculum and accountability policies interact in the expression of mathematics instruction?

- What is the relationship between teachers' knowledge for teaching mathematics and the mathematical quality of their instruction?
- What role do teachers' beliefs about teaching and learning mathematics play in their expression of mathematics instruction?
- How does the sense teachers make about institutional curriculum and accountability policies interact with their beliefs and knowledge in the expression of mathematics instruction?
- What role does institutional context play in teachers' mathematics instruction?

In this dissertation, I describe a study I conducted which addresses these questions. I selected a group of seven teachers and measured their mathematical knowledge for teaching, the quality of their mathematics instruction, their beliefs about

teaching and learning mathematics, and their interpretations of institutional curriculum and assessment policies. Additionally, I examined the role of curricular materials as well as social<sup>3</sup> and institutional<sup>4</sup> contexts in teachers' expression of mathematics instruction. Through analysis of the data collected, I explain the interplay between personal and contextual factors as mathematics instruction is expressed.

Selecting a theory through which these questions can be answered requires consideration of the teacher's internal cognitive structures as well as how they develop, change, influence and are influenced by the knowledge of others, and contribute to action in institutional settings. It requires the acknowledgement that knowledge is not static but is constantly changing -- new knowledge from interactions with others and materials becomes assimilated and selectively accommodated into current cognitive structures (Piaget, 1985). The theory must allow for teachers to act and think differently according to different situations. To focus only on the teacher's cognition and action, separate from context, would be to ignore the role of situation in what teachers come to know and do.

### **A Situative Perspective**

Taking into account the complex interactions between individuals and the environment and how they interplay with teacher knowledge and action, I take a situative perspective to study the phenomena of instructional decisions and classroom instruction.

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<sup>3</sup> *Social context*, for the purposes of this dissertation, refers to teachers' collegial interactions.

<sup>4</sup> *Institutional context* refers to elements of the institutional structure such as curriculum, policies, and time constraints.

From this perspective, the physical and social contexts in which activity occurs are integral elements of the activity (Putnam & Borko, 2000).

Current studies of the relationship between teacher knowledge and instruction (Hill et al. 2008; Kersting et al., 2009), while claiming a situated cognition perspective, place greater emphasis on cognition. From the cognitive perspective, characteristics of the individual teacher (knowledge and beliefs) are directly related to the instruction produced – context is not a defining factor in the enactment of knowledge. Both Hill and Kersting use quantitative analysis techniques to search for correlations between knowledge and action. The assumption is that a teacher with high levels of knowledge can produce high quality instruction across any context. Hill acknowledges that teachers' experience-derived beliefs mediate the enactment of knowledge, making her study a starting place for considering a more complex relationship between knowledge and instruction.

Using a situative perspective, the development and enactment of a teacher's knowledge is mediated and situation specific. That is to say that knowledge does transfer across situations but is also distributed across individuals and materials within situations (Greeno, 1998). The teacher, other individuals, and the environment interact and instructional action results. An instructional decision a teacher makes while working in a low-performing school, supplied with specific materials, and interacting with specific individuals may be quite different than a decision the same teacher would make in a high performing school interacting with different individuals, even if the materials are the

same. The teacher's knowledge does play a role in what the teacher does, but it is mediated and changes as a result of social and environmental interactions.

Integral to studying the phenomena of the creation of classroom instruction is the sense teachers make of the institutional policy messages. For example, when content standards are given to a teacher, the standards themselves serve as an artifact distributing knowledge to the teacher and the teacher in turn makes sense of the standards and shares in the distribution of that knowledge. How the teacher interprets the standards and how they come to be a part of classroom instruction is determined by what a teacher knows, the sense the teacher makes of the policy expectations, socially constructed norms for use of the standards, and personal beliefs about both the role of policy and the content of the standards. The sense that is made is a product of the interaction between the individual and the socio-historically influenced environment -- the teacher's cognitive processes (schema, prior knowledge, beliefs, experiences, assimilation, emotions), socially constructed understandings, and the institutional context interact in a way that generates instruction through the teacher (Spillane, Reiser, & Reimer, 2002).

Like other researchers in the field, in this study, I measure individual teachers' mathematical knowledge for teaching and the quality of their mathematics instruction. However, this study is different in that I systematically analyze the relationship between knowledge and instruction in terms of situational influences including institutional policies, textbooks, curricular resources, colleagues, and past experiences -- all existing within socio-historical contexts. The unit of analysis is the teacher in the context of situated instructional decision making.

The results from this study provide evidence that teachers' knowledge, beliefs, and interpretation of policies interact and are enacted differently depending on the context. Social and institutional contexts play primary mediational roles. For the teachers in this study, knowledge about how to make mathematics comprehensible to students appears to play a secondary role to the interaction between the institutional context and the sense they make of policies. As such, the teacher's instructional characteristics may be more reflective of accountability policies and curricular materials than of what the teacher truly knows how to do in the context of classroom mathematics instruction. Likewise, the cases in this study reveal that teachers' beliefs are mediated by social and institutional contexts, resulting in instruction reflective of the beliefs of a colleague or of curricular materials rather than of the teacher's own beliefs. This study adds new knowledge to the field of mathematics instruction by emphasizing the principal role of context in teachers' expression of mathematics instruction.

Through understanding aspects of the teacher and the environment as they influence instruction, this study provides new information about how to prepare prospective teachers for induction into the world of teaching as well as how to support practicing teachers in professional development. For example, knowing that policy interpretation could overpower mathematical knowledge for teaching or beliefs may cause teacher education instructors to spend time preparing prospective teachers to learn how to work within the institutional policy aspects of the profession. Likewise, understanding the interaction of these elements could help professional developers attend to more than just increasing teacher knowledge but instead incorporating how to apply

that knowledge within the boundaries of their current institutional setting. Too often, efforts to improve instruction focus exclusively on the implementation of curricular materials or on increasing teacher knowledge. This study highlights the multidimensionality of factors that influence instruction.

## CHAPTER II: LITERATURE REVIEW

### Introduction

In this review of literature, I examine the fields of pedagogical content knowledge, teacher beliefs, and teacher sense making of institutional policies as they relate to teacher instructional decision making and the resulting classroom mathematics instruction. I will first consider each domain by itself as it exists in current literature. I argue that each domain shows promise in understanding teachers' instructional practice but considering their interactions will provide a more complete picture of how classroom instruction comes to be. In doing so, cognition, social interaction, and context are considered as mutually constitutive elements.

### Pedagogical Content Knowledge

A growing body of research focuses on measuring teachers' *mathematical knowledge for teaching* (MKT) as it relates to mathematics instruction and student learning (Ball et al., 2008; Hill et al., 2005; Kersting et al., 2009; Ma, 1999). Because MKT is a construct derived from Shulman's (1986, 1987) framework of pedagogical content knowledge (PCK), I will first define and review research on pedagogical content knowledge. I will then transition into a review of current studies focused on the more mathematically-specific iteration of PCK, mathematical knowledge for teaching.

PCK is defined as the intersection of what a teacher must know about subject matter and about pedagogy in order to make specific content accessible for students (Shulman, 1986, 1987). Currently, researchers seek to understand the relationship

between pedagogical content knowledge and student learning as mediated through instructional practice (Carpenter et al., 1988; Fennema et al., 1996; Hill et al, 2005; Hill et al, 2008; Kersting et al., 2009; Ma, 1999).

While it seems a reasonable presumption that what a teacher knows about subject matter and how to teach it affects classroom instruction, to infer causality is to overlook the tenets of situated cognition – the mutually constituted interaction between the individual and the environment. Most studies of teachers’ knowledge examine a direct relationship between PCK or MKT and instruction or a triangular, mediational relationship between teachers’ knowledge, instruction, and student learning (Hill et al., 2008; Kersting et al., 2009; Ma, 1999). Such studies, although informative, are limited through their omission of the role of the teachers’ interactions with culturally derived artifacts (policies, instructional material) and with other people. Nonetheless, much can be gained from understanding teachers’ knowledge about how to teach subject-specific content and what it affords or how it constrains instruction (Greeno, 1998; Hill, 2008).

It is the intersection of content knowledge and pedagogical knowledge that Shulman (1986, 1987) proposed as a missing paradigm, “refer(ring) to a blind spot with respect to content” (1986, p. 7), in the study of teacher knowledge. His assertion was supported by others who claimed the two knowledge types were inherently linked in effective teaching and learning (Grossman, 1990; Leinhardt & Greeno, 1986). Shulman proposed that PCK encompasses understandings such as what topics students find interesting or difficult, the common misunderstandings within the concepts, the most useful representations to teach certain concepts, and the analogies, metaphors, or

examples that make content comprehensible to students. As he continued to use PCK as a framework for thinking about teacher knowledge and instruction, he stated:

As you begin to feel the difference between what it means to know and understand something yourself and what it takes to help someone else come to know and understand it, and as you begin to recognize the complexity of that process, you have come a very short distance into studying the problem of learning and teaching (Shulman, 2000).

For Shulman, it is the complex inner workings of knowing, understanding, and teaching subject matter that compose the field of pedagogical content knowledge.

In order to provide clarity when discussing pedagogical content knowledge, and later in this chapter when differentiating it from mathematical knowledge for teaching, it is important to examine the evolution of its definition. Shulman's original framework (1986) for teachers' knowledge is divided into three categories: content knowledge (the amount and organization of subject specific knowledge in the mind of the teacher), pedagogical content knowledge (knowledge of how to make content comprehensible to students), and curricular content knowledge (knowledge of the full range of programs and materials available for specific topics). The categories in Shulman's proposal are not the only categories that scholars include in the teacher knowledge framework. Shulman (1986, 1987, 2000) himself modified his categories in multiple publications. However, of these categories, pedagogical content knowledge has the closest connection to instructional practice and student learning. Because of its centrality to understanding how the knowledge of a teacher is enacted in the classroom, this review of literature focuses on PCK and then on the construct of MKT.

Like the alterations to Shulman's teacher knowledge framework, the term "pedagogical content knowledge" has undergone a cumulative process of definition, redefinition, and interpretation. Shulman (1987) describes what makes pedagogical content knowledge unique to teaching as "the capacity of a teacher to transform the content knowledge he or she possesses into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the students" (p.15). He emphasizes that the ability of the teacher to reason his/her way from personal understanding of subject matter into the minds and motivations of the students is what transforms content knowledge into pedagogical content knowledge. While valuing pedagogy, Shulman claims that without a strong understanding of the topic to be taught, flexible and interactive instructional practices are not available to teachers.

Although Shulman's PCK helped to merge two separate knowledge bases, content and pedagogy, some researcher argued that it did not match the way in which knowledge is constructed (McEwan & Bull, 1991). From the constructivist perspective, knowledge is not passively received but is actively formed by the knower. The more the teacher knows about the student's understanding, the greater the chance for effective teaching to occur (Resnick, 1989; Reynolds, 1992). Based on this argument, Cochran, DeRuiter, and King (1993) proposed adding to the definition of PCK by increasing the emphasis on the role of understanding the student. They explained that knowledge of how to teach content must build on an understanding of what the student brings to the interaction including student abilities, learning strategies, developmental levels, attitudes, motivations, and prior knowledge. Cochran echoes the call of other researchers

(Grossman, 1990; Carpenter et al., 1988), that PCK must include knowledge of student understanding, conceptions, and misconceptions. Although Cochran et al. (1993) conceded that Shulman does address the characteristics of the student, they claimed Shulman's emphasis on subject matter and pedagogy "veils" (p. 266) the importance of the teacher's understanding of students.

The field's increased emphasis on student learning reflects a shift in researchers' thinking about which components of PCK might play more influential roles. As a result, understanding student thinking has become known as one of the most critical components of PCK. For example, Shulman (2000) claims that "illusory understanding," people appearing to know something that they actually don't know, is one of the greatest enemies of teaching. He suggests it be combatted with strategies that reflect a strengthening of PCK, specifically through helping teachers understand student thinking and helping them to build reflection and social interaction into their instruction.

### **Mathematical Knowledge for Teaching**

Ball et al. (2008) and her colleagues added to Shulman's framework by focusing specifically on *mathematical knowledge for teaching* (MKT), a discipline-specific iteration of PCK. They placed even greater emphasis on the importance of understanding student thinking as well as understanding the vertical and horizontal interconnectedness of mathematics concepts.

Drawing upon these definitions in the literature, for the purposes of this review, teachers' mathematical knowledge will be referred to as *mathematical knowledge for teaching* (MKT). It is defined as the intersection of content and pedagogy including:

*common content knowledge* (knowledge of subject matter developed through studying mathematics), *specialized content knowledge* (applied mathematical content knowledge developed through the work of teaching – identifying patterns in student errors, unpacking mathematical ideas, explaining procedures, selecting and presenting representations, or analyzing unfamiliar mathematical claims), *knowledge of students* (developmental levels, background knowledge, common misconceptions, how knowledge is constructed, student thinking about content) and *horizon content knowledge* (understanding the interconnectedness of math ideas within the discipline). This mathematics-specific form of the definition can bring clarity to the discussion of how teachers' knowledge for teaching mathematics influences instructional practice and, in turn, student learning.

Although MKT focuses on the intersection of content knowledge and pedagogical knowledge, current policies such as the No Child Left Behind Act of 2001 (NCLB) prioritize content knowledge over pedagogy (U.S. Department of Education, 2003). Specifically, NCLB defines highly qualified teachers through coursework, degrees, and credentials. However, research provides evidence that coursework, degrees, and credentials are not consistent indicators of student achievement (Begle, 1979; Borko et al., 1992; Stein et al., 1990). These same studies also confirm that a teacher's knowledge of subject matter is necessary but, in the absence of pedagogical content knowledge, is not sufficient in helping the teacher make content accessible to students.

### **Pedagogical Content Knowledge and Mathematics Instruction**

With increasing conviction, the consensus of the field of teacher knowledge (Hill et al., 2008, Kersting et al., 2009, Ma, 1999) “is that there is a powerful relationship between what a teacher knows, how she knows it, and what she can do in the context of instruction” (Hill et al., 2008). This assertion is validated in Ma’s (1999) comparison of Chinese and American teachers’ knowledge and practice. Ma views teacher knowledge, classroom instruction, and student performance through a socio-cultural lens, taking into account the culturally and historically constituted ways in which teachers and students come to know mathematics in different countries. Chinese students’ consistent higher performance on conceptual math items on the Third International Math and Science Study (TIMSS, 1997) were early indicators that teachers’ instructional practice might be country-specific and responsible for the differences in student competencies. The TIMSS international classroom instruction video study (Hiebert & Stigler, 2000) provided evidence of a procedural focus in American classrooms and a conceptual focus in Chinese classrooms. With trends in student test results and typical classroom practices already established for each country, Ma set out to understand how teachers’ pedagogical content knowledge might also be country-specific and related to student learning and instructional practice.

In an effort to characterize the pedagogical content knowledge of teachers in each country, Ma conducted cognitive interviews with 72 Chinese teachers and 23 American teachers. In these interviews, she used teaching scenarios (*Mathematical Knowledge for*

*Teaching Assessment*, MKT, developed by Ball, 1990b and Hill et al., 2004) and analyzed each teacher's response. One such scenario is displayed below.

People seem to have different approaches to solving problems involving division with fractions. How do you solve a problem like this one?

$$\frac{3}{4} \div \frac{1}{2} =$$

Imagine that you are teaching division with fractions. To make this meaningful for kids, something that many teachers try to do is relate mathematics to other things. Sometimes they try to come up with real-world situations or story-problems to show the application of some particular piece of content. What would you say would be a good story or model for  $1\frac{3}{4} \div \frac{1}{2}$ ?

**Figure 1:** Mathematical Knowledge for Teaching Interview Item

In this task, teachers must be able to compute an answer to  $1\frac{3}{4} \div \frac{1}{2}$  and to represent the mathematical relationships by putting them into a real life context. Both skills are part of Shulman's PCK – math content knowledge and representations that make content comprehensible. These tasks were difficult for American teachers who typically relied on memorized (or forgotten) computational procedures, which impeded their attempts to create a representation. Ma noted that without strong content knowledge, knowledge of pedagogy was not useful.

Overall, Ma found that American teachers tended to understand mathematics as a set of isolated procedures. Chinese teachers, on the other hand, tended to have a “profound understanding of fundamental mathematics” (PUFM). PUFM is defined by Ma as a deep, vast, and thorough understanding of mathematics evidenced by teaching and learning that shows connectedness, multiple perspectives, the power of basic ideas, and longitudinal coherence across topics that come before and after the topic that is at hand.

Additionally, when asked about how they would teach certain concepts, Chinese teachers typically explained, and reported sharing with their students, the interconnectedness of mathematics concepts. They thought about teaching math in “knowledge packages” – a way of viewing mathematical topics group by group rather than piece by piece. Conversely, American teachers tended to see each concept or skill as separate and teach students how to perform procedures rather than understand the mathematical concepts and their connections to other mathematical ideas. This difference was reflected in the TIMSS classroom video study and in the student results where Chinese students did well on items that required knowledge of mathematical connections while American students did poorly on the same items (TIMSS, 1997).

Ma found a strong connection between the way teachers’ in the different countries reported having learned mathematics and the depth and breadth of their mathematical knowledge, including their ideas about how to teach it. As a result, she claimed that the qualitatively different ways of knowing and teaching mathematics explains the differing characteristics of instruction between countries evident in the TIMSS video study

(Hiebert & Stigler, 2000) as well as the differing student performances on the TIMSS test (TIMSS, 1997).

While Ma's study provides strong evidence that teachers in China and America demonstrate qualitatively different PCK, the connections she draws between PCK, instructional practice, and student learning are based on logical arguments. She does not observe the teacher participants in their own classrooms nor does she assess their students' knowledge. Despite this limitation, Ma's findings provide insight into the trends of differing student performances and teaching practices between the two countries.

Similar to Ma's study, many other studies analyze teacher pedagogical content knowledge or instructional practice independently of one another or, through studying one element, make logical claims that it affected the other (Ball, 1990a; Heaton, 1992; Ma, 1999; Post, Harel, Behr, & Lesh, 1991; Putnam, Heaton, Prawat, & Remillard, 1992). Some studies, however, simultaneously incorporate measures of teacher PCK or MKT with observation-based analysis of instructional practice (Borko et al., 1992; Hill et al., 2008; Kersting et al., 2009; Leinhardt & Smith, 1985).

Hill et al. (2008) conducted one such simultaneous study designed to explore the relationship between teachers' mathematical knowledge for teaching (MKT) and the mathematical quality of their instruction (MQI). They used situated cognition's principle of affordances and deficits to explain what different facets of mathematical knowledge for teaching afford or how they constrain the activity of classroom instruction. While emphasizing a dyadic relationship between MKT and MQI, the researchers briefly

consider how teacher beliefs could mediate the relationship, but do not go into great detail in this area. Additionally, the researchers primarily take a cognitive perspective in their emphasis on teacher knowledge but do consider aspects of situation in their analysis.

In this study, Hill et al. (2008) used mixed methods to examine teachers enrolled in a year-long professional development program. The initial portion of the study operated on the premise that a direct relationship between MKT and MQI may exist. They had previously measured the mathematical knowledge for teaching of the 626 teachers participating in the program. They did this by doing a factor analysis of responses to a multiple-choice assessment (*Mathematical Knowledge for Teaching Assessment*, MKT, developed by Ball, 1990b and Hill et al., 2004) in which teachers read scenarios of student thinking and aspects of teaching and responded. For ten participants, the study was extended to examine classroom instruction as compared to their MKT assessment scores.

To measure the quality of each teacher's mathematics instruction, the researchers video-taped nine lessons over the course of a year and coded the transcripts by characteristics of the quality of math instruction (rate of math errors, appropriateness of responses to students, connections to larger math ideas, richness of mathematics, and lesson density of mathematical language). The resulting MQI scores were then compared to MKT scores through a correlational analysis. Specific characteristics of teachers' instruction did correlate with MKT scores (Spearman correlation range 0.30 – 0.83).

In the classroom, teachers who showed high mathematical knowledge for teaching tended to make few mathematical errors during instruction, helped students

connect classroom activities to math ideas and procedures, rarely misinterpreted a student's ideas, and rarely failed to respond productively to a student mistake. They also were able to show multiple models to demonstrate mathematical ideas and encouraged mathematical justification for student thinking. Instruction in the classrooms of these teachers was characterized as “rich mathematics” (p. 457).

On the other hand, teachers with low MKT scores tended to demonstrate instruction that was the opposite of their counterparts. They made many mathematical errors when explaining mathematical ideas (up to 89% of the mathematical statements were incorrect), repeated what was in the textbook in response to student ideas, were unable to use multiple models to demonstrate mathematical ideas, and did not include mathematical justification or proof as part of the instruction. Hill (2008) states,

[T]his demonstrates a substantial link between strong MKT and high mathematical quality of instruction. . . . high MKT teachers provide better instruction for their students. The symmetry of this relationship is striking; not only do high-knowledge teachers avoid mathematical errors and missteps, they appear able to deploy their mathematical knowledge to support more rigorous explanations and reasoning, better analysis and use of student mathematical ideas, and simply more mathematics overall. (p. 457)

Participants were sorted into convergent (cases where MKT score predicted the overall MQI), divergent (cases where MKT did not predict overall MQI), and mixed cases (cases where MKT/MQI variation was both convergent and divergent depending on which dimension was examined). Out of the participants, four teachers' scores diverged while six teachers' scores either converged or were mixed. Hill and colleagues claimed, “There is a powerful relationship between what a teacher knows, how she knows it, and what she can do in the context of instruction” (p. 496). Arguably, with 40 % of the cases

diverging, the study provides evidence that, while levels of MKT are related to instructional quality, other factors must be considered in combination with knowledge to account for mathematics instruction.

In order to understand their results, Hill et al. (2008) transitioned into the case study portion of their analysis. They selected two cases where MKT scores converged with MQI scores, two cases where MKT scores diverged from MQI scores, and one case where the MKT scores both converged and diverged from MQI scores. Using grounded theory (Glaser & Strauss, 1967), a practice in which theory is generated from analyzing data during research, they qualitatively analyzed previously collected background surveys and post-lesson debriefings, allowing factors to emerge which could help explain variations in the results.

They found that teacher beliefs (such as believing math had to be fun at the expense of dense, accurate instruction) could cause teachers with high MKT to have low instructional scores. They also found teachers with low MKT scores could earn high MQI scores if they did not deviate from the textbook lessons. Although grounded theory allowed the researchers to explore mediating factors not included in the original study design, it limited what they could find as the methods were not selected with the intent to uncover these factors. As such, teacher beliefs only came up in one of the case studies because an initial statement the teacher made provided insight into her beliefs. Additionally, the study was not designed to examine institutional factors and therefore no conversations on that topic took place. Interestingly, mediating factors were only explored when the cases diverged. Arguably, from a situative perspective, contextual

factors played an important role in mediating knowledge and instruction of all of the teachers and thus need to be investigated in every case.

### **Teacher Beliefs**

In order to understand other factors that contribute to mathematics instruction, a more intentional investigation of teachers' beliefs is warranted. The field of teacher beliefs is vast with varying interpretations of what *beliefs* mean. Beliefs, in their simplest definition, are deeply held personal truths about the world. Thompson (1992) argues for an interactive view of beliefs and instruction where beliefs influence instruction but experiences with students and the environment during instruction likewise influence beliefs. While beliefs are often viewed as static, they have been proven to be susceptible to change when an experience requires restructuring of current schemata to accommodate new, contradictory information (Thompson, 1992). However, Thompson claims it is more common for individuals to alter new information so that it may be assimilated into current schemata. Historically, beliefs have been viewed as strong indicators of decision making (Bandura, 1986; Dewey, 1933; Pajares, 1992). In this dissertation, I argue that beliefs can only partially account for instructional decisions.

The term "beliefs" is often used interchangeably in the literature with the terms orientations, perceptions, perspectives, conceptions, and identities. Beliefs are thought to comprise an entire system within individuals. Borrowing from Philipp's (2007) review of literature on teacher beliefs and affect, this study will define beliefs and belief systems as follows:

**Beliefs**—psychologically held understandings, premises, or propositions about the world that are thought to be true. Beliefs are more cognitive, are felt less intensely, and are harder to change than attitudes. Beliefs might be thought of as lenses that affect one’s view of some aspect of the world or as dispositions toward action. Beliefs, unlike knowledge, may be held with varying degrees of conviction and are not consensual. Beliefs are more cognitive than emotions and attitudes. (p. 259)

**Beliefs System**—a metaphor for describing the manner in which one’s beliefs are organized in a cluster, generally around a particular idea or object. Beliefs systems are associated with three aspects: (a) Beliefs within a beliefs system may be primary or derivative; (b) beliefs within a beliefs system may be central or peripheral; (c) beliefs are never held in isolation and might be thought of as existing in clusters. (p. 259)

Because beliefs are personal and do not require group consensus, they are idiosyncratic and often lead individuals with similar knowledge to make different decisions (Nespor, 1987). Rokeach (1968) warns that beliefs cannot be directly observed or measured but are inferred from a combination of the actions, talk, and intentions of individuals. Thompson (1992) concurs that teachers’ explanations of their beliefs cannot stand alone but must be compared to their instructional practice and mathematical behavior.

In order to coherently discuss the interactions between beliefs and knowledge in the production of mathematics instruction, it is essential to distinguish between the two constructs. Historically, researchers have claimed that beliefs appear to develop from episodes in life (Eraut, 1985; Nisbett & Ross, 1980; Nespor, 1987) and have a strong evaluative and judgmental nature (Nisbett & Ross, 1980). While cognitive knowledge does have an evaluative component and its development is influenced by beliefs, it is more open to evaluation and change. Nespor (1987) distinguishes beliefs from cognitive

knowledge in claiming that beliefs are part of episodic memory where cognitive knowledge is semantically stored.

Thompson (1992) adds to the discussion by claiming beliefs and knowledge differ by their degree of *conviction* and *consensuality*. For example, you can strongly hold a belief but knowledge is rarely discussed in this way. Likewise, beliefs can exist without social agreement while knowledge is typically considered accepted only once agreed upon as true by others.

Philipp (2007b) provides a more nuanced view of the beliefs and knowledge claiming that they differ in the way they are held, “one person’s belief may be another person’s knowledge” (p. 267). He states,

For researchers to know how one holds a notion may be as important as knowing what one holds as the notion. Two people who hold contradictory beliefs about something may have more in common with each other than with a person who holds one of those conceptions as knowledge. (p. 267)

Philipp means that an idea held as a belief is negotiable but an idea held as knowledge is often considered *truth*. Since studies about pedagogical content knowledge tapped into the relationship between knowledge and instruction, it makes sense to examine the relationship between beliefs and instruction before considering how they these inter-related constructs interact.

### **Relationship between Teacher Beliefs and Instruction**

In a belief/instruction comparison study, Stipeck, Givvin, Salmon, MacGyvers, (2001) found that measuring teacher beliefs can be predictive of instruction characteristics. The researchers studied 21 fourth through sixth grade teachers in

elementary schools within one city. They wanted to uncover how the instruction of teachers who held traditional beliefs about teaching would compare to the instruction of teachers who held inquiry-based beliefs. Borrowing from Thompson (1992), Stipeck defines teachers with traditional beliefs as those who believe “Knowing mathematics means being skillful and efficient in performing procedures and manipulating symbols without necessarily understanding what they represent” (p. 214). The research team hypothesized teachers holding this belief would “emphasize performance and proficiency (speed), convey to students that mistakes were something to be avoided, thus producing a high-risk environment, and be relatively controlling of students’ mathematics activity” (p. 217). They describe teachers who hold inquiry-oriented beliefs about mathematics as those who view mathematics as a dynamic discipline and see mathematics as a tool for solving problems. They predicted teachers with inquiry-oriented beliefs would actively engage students in activities supporting the construction of mathematical conceptualizations and would encourage reasoning, creativity, knowledge application, and social interaction around mathematical ideas.

In Stipeck et al.’s (2001) study, beliefs were measured using a survey of 57 statements to which participants marked the degree of their agreement using a Likert-type scale. The responses were scored on a scale of 1 to 6 (1 = strongly disagree; 6 = strongly agree). The survey statements were designed to elicit teachers’ beliefs along the dimensions of orientation toward math teaching and learning, extrinsic versus intrinsic motivation, and self-efficacy in teaching mathematics.

Classroom instruction was measured using videotapes of at least two instructional periods per teacher. The videos were analyzed using dimensions of practice codes and scales ranging from 1 to 5 (1 = not at all like this teacher; 5 = very much like this teacher). Some of the dimensions were: emphasizes performance outcomes; emphasizes speed; type of environment the teacher fostered; emphasis on autonomy; emphasis on student effort; focus on understanding; enthusiasm and interest in mathematics. The researchers also corroborated observation scores with teachers' self-perceptions on a questionnaire.

As predicted, results showed that certain clusters of beliefs were strongly associated in stable belief systems within individuals. These systems divided along the lines of traditional orientation and inquiry orientation to teaching. Similarly, associations between beliefs and practice fit the predictions of the researchers – traditional beliefs yielded traditional practices and inquiry-based beliefs yielded discovery-based practices. While these findings validate claims that beliefs are an important teacher characteristic influencing instruction, I wonder how the data and, perhaps the findings, would have differed if interviews of the teachers were used. The beliefs a teacher indicates in a Likert survey might be quite different from what that same teacher displays in a more open-ended interview.

### **Discrepancies between Beliefs and Instruction**

Through a situated cognition perspective, the closer the data collection tool can get to the teacher's actual teaching activities, the closer the information provided will be to the teacher's in-context thinking (Hoyles, 1992; Raymond, 1997; Skott, 2001). Using

mixed methods, Raymond (1997) provides a case study in which comparing data obtained via survey with classroom observation and interview data reveals a mismatch between a novice teacher's beliefs and practice. Joanna, a novice teacher, was studied over a 10 month period spanning spring of her first year of teaching to winter of her second year of teaching.

Raymond's data collection tools were: an introductory phone interview, six monthly 1-hour audio-taped interviews, five classroom observations in five separate months, analysis of lesson plans, concept mapping in which the teacher modeled the relationship between beliefs and practice, and a questionnaire regarding mathematics beliefs and factors that affect teaching practice. He found Joanna held traditional beliefs about the nature of mathematics. However, she held nontraditional beliefs about teaching and learning mathematics. His observations revealed Joanna's traditional classroom practice to be teacher-centered, skill-focused, and non-interactive.

Through analyzing his data, Raymond concluded that the discrepancy between Joanna's beliefs about teaching and learning and her practice stemmed from her concerns about institutional constraints: lack of time, limited resources, accountability for standardized test scores, classroom discipline. He claimed her espoused beliefs about teaching and learning represented how she wanted to teach mathematics but her teaching practice reflected her response to the constraints. The situative perspective taken in this study allows for the apparent inconsistency between beliefs and practice to become more understandable through contextual constraints. While analyzing classroom practice through the lens of beliefs provided valuable insight into individual and environmental

variables affecting classroom instruction, Raymond's study would provide a more complex understanding of Joanna's instructional decisions were it to consider Joanna's levels of pedagogical content knowledge.

From the studies reviewed, we can see that teacher knowledge can be connected to instructional practice (Hill et al., 2008; Kersting et al., 2009) and teacher beliefs are related to classroom instruction (Stipeck et al., 2001), yet Raymond (1997) provides evidence (albeit a single case study) which indicates the relationships might have more of a contextual element than indicated in previous studies. Considering how the teacher thinks and acts within institutional boundaries may bring the field a step closer to understanding how classroom mathematics instruction is created.

### **Teacher Sensemaking**

In order to consider the role of institutional policies in teachers' expression of classroom instruction, it is important to examine how teachers interpret institutional policy messages and take action based on these interpretations or "sensemaking."

### **Sensemaking Theory**

Sensemaking theory places emphasis on human agency within the individual and social activities of institutions. Among the research in this field, there are studies of individual teachers' sensemaking of policy (Cohen & Ball, 1990; Spillane & Zeuli, 1999), studies of individual and collective sensemaking of policy within social, professional, and institutional contexts (Coburn, 2001, 2008; Hill, 1999; Lin, 2000; Spillane, 1998, 1999; Yanow, 1996), and studies of how teachers make sense of

curricular materials (Cohen, 1990; Connelly & Clandinin, 1986; Heaton, 1992; McCutcheon, 1981; Putnam, 1992; Remillard, 2005; Sosniak & Stodolsky, 1993). As we seek to understand how teachers' interpretations of policy and curricular materials interact with their mathematical knowledge for teaching and beliefs, we must first understand how sensemaking occurs and how to identify sensemaking in action.

Through the lens of sensemaking theory, researchers examine how teachers take the chaos which constitutes a given situation and attempt to organize it in a way that makes sense through their pre-existing cognitive frameworks or "world views" (Coburn, 2001; Porac, Thomas, & Baden-Fuller, 1989; Vaughan, 1996; Weick, 1995). Teachers do not simply react to external stimuli such as policy messages, collegial interactions, or student interactions, but filter input through their own knowledge, beliefs, and experiences, shaped by social interactions, before taking action based on their interpretations (Spillane, 2002; Weick, 1995, 2005).

According to Weick (2005), "Sensemaking is about the interplay of action and interpretation rather than the influence of evaluation on choice. When action is the central focus, interpretation, not choice, is the core phenomenon" (p. 409). Using a finer grained examination, interpretation is contingent upon each teacher's schema, prior knowledge, beliefs, experiences, cognitive assimilation, and emotions (Spillane, 2002). Understanding how teachers' interpretations of institutional policies mediate their knowledge and beliefs and interplay with the instructional actions they take will contribute to a multi-faceted understanding of teachers' instructional decisions.

Sensemaking theorists warn, however, that it is hard to capture sensemaking and as it is often hidden in the micro-moments of experience (Porac et al, 1989; Weick, 1995, 2005). Weick (2005) expresses the value of attending to the significance of small moments in the following statement:

Students of sensemaking understand that the order in organizational life comes just as much from the subtle, the small, the relational, the oral, the particular, and the momentary as it does from the conspicuous, the large, the substantive, the written, the general, and the situated. To work with the idea of sensemaking is to appreciate that smallness does not equate with insignificance. Small structures and short moments can have large consequences. (p. 410)

When applied to teaching, this statement draws attention to all of the small moments that happen on a daily basis that might contribute the sense that the teacher makes of new situations. It is within this stream of new situations and small moments that Weick and colleagues (2005) identify characteristics of the act of sensemaking which can guide analysis of individual and collective action. First, they claim, sensemaking is about noticing signals of the state of the world being different than expected. Then it is about bracketing the signals into categories which can be labeled and pondered. Noticing and bracketing are guided by the mental models an individual has developed from previous experience. Identifying when an individual notices this tension between expectation and reality and the resulting cognitive action that occurs seems only identifiable through talking with the individual after it has occurred.

Weick (2005) claims sensemaking relies on action and talk as cycles rather than linear sequences – individuals take action which seems appropriate but as the situation and cues evolve, new actions are needed and different actions seem more appropriate. As

such, sensemaking is social in nature as it is distributed across other individuals, and the environment. It includes both retrospective and prospective thinking -- sense is made after an action has occurred, is forward oriented and is based on pre-existing beliefs and experiences. Additionally, individuals make presumptions about the input which guide action-- individual sensemaking begins with “immediate actions, local context, and concrete cues” and presumptions which guide future actions are made based on these elements. According to Weick (2005), “Sensemaking is central because it is the primary site where meanings materialize that inform and constrain identity and action” (p. 409). In sum, what we learn from this literature on sensemaking is that in order to understand the actions of individuals, we must come to understand how they filter external stimuli through internal cognitive constructs. To make the invisible visible, researchers of sensemaking must observe individuals as they make decisions across a range of situations within their institutional roles.

### **Sensemaking of Policies through Individual Knowledge and Beliefs**

Integral to sensemaking are the knowledge and beliefs of the individual as they interact with policy in specific situations. Spillane and Zeuli (1999) used case studies across nine school districts to examine patterns of practice which emerged in response to mathematics reforms. Central to the study was how the core ideas of the reform were interpreted and enacted by teachers.

The reform recommended principled mathematical knowledge be given more attention than it traditionally gets and procedural knowledge be a secondary goal. Principled math knowledge includes understanding key ideas and concepts with an

emphasis on reasoning (Greeno, 1992; Lampert, 1986; Spillane & Zeuli, 1999) while procedural knowledge emphasizes knowing computation procedures which are followed in a step by step application.

Spillane and Zeuli (1999) observed teachers during instruction, conducted interviews, and administered a survey about instructional practice. They found policy was profoundly altered during enactment as a result of different interpretations of reform principles at the level of the teacher. Teachers constructed different interpretations of the reform ideals and pursued different courses of action because they had different knowledge bases, beliefs, and experiences – cognitive frames through which the policies were interpreted differed.

Despite 21 of the 25 teachers self-reporting familiarity with reform principles, agreement with reform principles, and classroom instruction reflecting reform principles, upon observation only 4 teachers exemplified the development of principled mathematical knowledge in their classrooms. The researchers noticed a pattern in 10 of the 25 classrooms – the activities were designed to elicit thinking and reasoning but the classroom discourse patterns belied the integrity of the activities and made them procedural. Additionally, they noticed that 11 of the classrooms' discourse norms and mathematical tasks were oriented toward procedural knowledge, clearly contradictory to reform recommendations.

So how does one account for the discrepancy between teachers who believed they were teaching through reform ideals and the reality that most were not? While explaining why the patterns existed was not the emphasis of the study, Spillane and Zeuli imply a

combination of differing beliefs about teaching and different levels of understanding about what the reform policies really mean can partially account for the differential instruction. In other words, the interaction of knowledge, beliefs, and sensemaking appear to affect enactment of policy in instruction. Further studies are needed to explicitly analyze the interaction of these factors.

### **Sensemaking of Policies through Social Interaction**

While Spillane & Zeuli (1999) focused on the individual teacher's sensemaking, Coburn (2001) provides a qualitative case study in which she examines sensemaking as a social activity culminating in collective interpretations of policy. In this study, sensemaking is brought to the surface through sustained observations, in-depth interviews, and iterative coding. To define the study, Coburn sought to find the processes by which teachers make sense of policy messages within the context of enacting the 1995 *Every Child a Reader* initiative. For one year, she observed and interviewed teachers in one California school in their daily routines and professional communities. Her analytic unit was the group or collectivity as she attempted to create an account of the processes which produce collective sensemaking.

She discovered teachers' sensemaking was most affected by whom they were working with and the conditions of their conversations. Teachers tended to demonstrate a convergence of worldviews and practice within groups (grade level or self-selected) over time. The dominant personalities within a group tended to influence the worldviews and practice of the less dominant personalities. Additionally, "in-facing" conversations between teachers about matters close to practice tended to elicit deeper engagement and

opportunities to challenge worldviews. These informal conversations influenced instruction. “Out-facing” conversations included topics assigned by administration or the district to satisfy policy needs. These conversations had little to no effect on classroom instruction or on challenging worldviews. Most important to note here is that sensemaking extended beyond internal constructs and was influenced by social context.

Attending to Coburn’s methodology in making sensemaking visible is useful for further research. Her study included multiple methods for data collection. She observed meetings, professional development sessions, and informal hallway conversations, paying attention to conversation content, nature of interactions, and evidence of worldviews. She supplemented the observations with semi-structured interviews to capture teachers’ worldviews, reflections on practice, and perspectives on the process of reading instruction reform. She selected five teachers who represented a range of reading practices to observe more closely in a series of full-day observations in an attempt to uncover the relationship between collegial conversations and teachers’ reading practice. In order to understand the institutional reading environment, Coburn used record data and interviews with district personnel, state personnel, and local professional development providers. Through these methods, she was able to capture the processes of sensemaking within a single institutional context.

### **Sensemaking of Curricular Materials**

Another view of sensemaking focuses on teachers’ interpretations and enactment of curricular materials and the mutually influential relationship between teachers and materials (McLaughlin, 1976). For the purposes of this review, curricular materials will

be used synonymously with “curriculum” and “textbook,” referring to the mandated mathematics program adopted by the school district and/or other printed or published teacher resources intended for instruction with students (Remillard, 2005). The assumption in this paper is that teachers interact with the curricular materials and make meaning of them.

With the national pressure to raise student test scores and give all students equal access to curricula, more and more school districts expect their teachers to follow the textbook as closely as possible. Yet, as agents of both curricular implementation and enactment of their own beliefs, individual teachers make sense of textbooks differently. Remillard (2005), in a literature review, identifies four perspectives on the relationship between teachers and curricula: 1) Teachers as following or subverting curricula (Komoski, 1977; Manouchehri & Goodman, 1998), 2) Teachers drawing on curricula as they design instruction (Connelly & Clandinin, 1986; Freeman & Porter, 1989; Kuhs & Freeman, 1979), 3) Teachers as interpreters of curricula (Cohen, 1990; Heaton, 1992; Putnam, 1992; Weimers-Jennings, 1990; Wilson, 1990, 2003), and 4) Teachers as engaged in a dynamic interrelationship with the curricula (Davenport, 2000; Lloyd, 1999; McLaughlin, 1976; Remillard, 2000; Van Zoest & Bohl, 2002). I adopt the fourth perspective on the relationship between teachers and curricula for my study of classroom instruction.

The dynamic relationship perspective stands apart from the rest in “its focus on the activity of using or participating with the curriculum resource and on the dynamic relationship between the teacher and the curriculum” (Remillard, 2005, p. 221). It is a

situative perspective based on the Vygotskian idea that all human activity is constructed from mediated action, including the use of tools, to interact with others and with the environment. It fits with the sensemaking approach to understanding teacher decision making as it gives weight to human agency and highlights the distribution of knowledge between agent and tool.

Lloyd's (1999) study of two high school teachers' interactions with a reform-oriented curriculum illustrates the interplay that occurs between agent and tool. One teacher, Mr. Allen, viewed the problems in the curriculum as challenging and too open to student interpretation. The other teacher, Ms. Fay, expressed frustration about the same problems' over-structuring and the limitations this structure placed on exploration. Both teachers experienced difficulty with student group-work but attributed causality differently depending on how they viewed the problems. They each adapted the curriculum and their own instruction according to their differing interpretations of the content and of student engagement in it. Although the author does not make this claim, it is clear that the sense each teacher made of the curriculum was dependent on his/her own worldviews, experiences, and knowledge of teaching mathematics.

Another series of studies supporting the dynamic relationship between teacher and curricular materials was done by Remillard (1999, 2000). In these studies, teachers using the same textbook had differing purposes for using the textbook (searching for activities versus identifying big ideas for planning guidance). The varying purposes, stemming from differing beliefs about use of textbooks, created a different relationship between

teacher and textbook. It also created diverse learning opportunities for children despite the uniformity of resource.

Sherin and Drake (2009) studied patterns of curriculum implementation across 10 elementary school teachers. Their study focused on teachers' *curriculum strategies*. This construct is founded on the premise that teachers read, evaluate, and adapt curriculum differently than one another but with patterns of predictability:

The idea is that teachers stand at the hub of a complex system that includes materials, students, and the rest of the instructional milieu. As they stand at this hub, they act as interpreters and evaluators of curriculum materials, and of the classroom events that transpire. A teacher's curriculum strategy is the stance that the teacher adopts as he or she plays this role of interpreter. (Sherin & Drake, 2009, p. 472)

They found that each teacher had a consistent pattern for how they read, evaluated, and adapted curriculum. The patterns were represented along a continuum and all teachers in the study fit onto the continuum. In all patterns identified, teachers had a mutually interactive relationship with the curriculum wherein their understanding of math content and instruction was as affected by the materials as was the implementation of the materials affected by the teachers. While mediating factors were not explored in the study, Sherin and Drake predicted teachers' knowledge and beliefs played a role in teachers' *curriculum strategies*. This study supports the idea that teachers have a dynamic relationship with instructional materials (Davenport, 2000; Lloyd, 1999; McLaughlin, 1976; Remillard, 2000; Van Zoest & Bohl, 2002).

The studies reviewed in this section suggest an interactive relationship exists between teacher and curricular materials which is strongly influenced by individual

cognitive frames. In this dissertation, I use a situative perspective in order to include the teachers' interpretations of the materials as well as the materials as artifacts of knowledge themselves. To study classroom instruction without considering the dynamic relationship between agent and tool (i.e. textbook) would lead to an incomplete analysis.

### **Next Steps**

As the literature reviewed here shows, the development of classroom mathematics instruction must be studied in terms of the teacher's mathematical knowledge for teaching, the strongly held beliefs of the teacher, and the sense the teacher makes of institutional policies and curricular materials. Each of these elements interacts and affects one another. No studies to date examine all of them in a purposeful investigation in the context of classroom instruction. The next step for this research field is an integrative study in which teacher cognition and its relationship to instruction is considered in social and institutional contexts.

## **CHAPTER III: METHODOLOGY**

### **Study Focus**

The goal of this study is to extend the current body of research focused on factors influencing the quality of classroom mathematics instruction. Research in this field frequently focuses on individual characteristics of the teacher: teacher knowledge as it is related to instruction (Hill, 2005; Kersting, 2008) or teacher beliefs as they are related to instructional decisions (Stipeck et al., 2001). Occasionally beliefs are examined as mediators of knowledge and instruction (Hill, 2008) or institutional constraints are examined as mediators of beliefs and instruction (Raymond, 1997). The study I designed is more comprehensive in terms of the range of factors considered than studies done to date. It explicitly examines the interaction of teachers' knowledge, beliefs, and perceptions of curriculum and assessment policies as they generate mathematics instruction.

### **Mixed Methods Research Design**

I conducted a mixed method study using both qualitative and quantitative research practice to examine how teachers' personal and contextual factors interact and influence perception and instructional action. The explicit focus on institutional factors (teachers' interpretations of policies and school context) adds knowledge to the field. The study is guided by a situative perspective (Greeno, 1998; Putnam & Borko, 2000; Spillane, Reiser, & Reimer, 2002) and by sensemaking theory (Spillane & Zeuli, 1999; Coburn, 2001; Remillard, 2005).

The study is designed to answer the question: How do teachers' knowledge for teaching mathematics, beliefs about teaching and learning mathematics, and the sense they make of institutional curriculum and accountability policies interact in the expression of mathematics instruction? Furthermore, the question is broken down into sub-questions:

- What is the relationship between teachers' knowledge for teaching mathematics and the mathematical quality of their instruction?
- What role do teachers' beliefs about teaching and learning mathematics play in their expression of mathematics instruction?
- How does the sense teachers make about institutional curriculum and accountability policies interact with their beliefs and knowledge in the expression of mathematics instruction?
- What role does school context play in teachers' mathematics instruction?

The purpose of the study is to build upon current theory regarding classroom mathematics instruction. The data collection focused explicitly on the knowledge, thoughts, and actions of teachers situated within institutional school settings. I selected data collection techniques to capture both the observable (classroom practice) and less tacit constructs (knowledge, beliefs, and sensemaking). The prevalent use of qualitative data collection techniques (video observations, interviews, written responses) was an intentional choice as it allowed for a focus on "naturally occurring, ordinary events in natural settings" (Miles & Huberman, 1994). I was able to collect evidence of teachers' instruction and perceptions primarily in the participants' classrooms, the natural setting in

which the activity of teaching occurs. As Miles & Huberman (1994) claim, collecting and analyzing data qualitatively is particularly fitting for locating the meaning people place on elements of their lives, understanding their perceptions, and connecting them to the social world. I also collected and analyzed data quantitatively through assessments and surveys to capture the less tacit constructs of knowledge and beliefs. By analyzing evidence both quantitatively and qualitatively, I was able to identify relationships between factors as well as understand the theory underlying the relationships. Additionally, as recommended by Eisenhardt (1989), the use of multiple data sources allowed triangulation of evidence to confirm or disconfirm relationships.

The data collection techniques included a combination of assessments and surveys to measure teachers' knowledge, video recorded classroom observations to capture mathematics instruction, a survey to identify beliefs about teaching and learning mathematics, and semi-structured interviews as a source of information about teachers' beliefs, interpretation of policies, school context characteristics, and more detailed information about their knowledge. Details regarding the methods of data collection and measurement are provided in the corresponding sections in this chapter. The variety in data sources coupled with quantitative and qualitative reduction and analysis methods created an opportunity to identify and understand the complex interaction of factors underlying instructional action.

I used an integrated case-oriented and variable-oriented approach to data analysis (Yin, 2009). The synthesis of the two approaches helps to attend to both the particular and the universal (Silverstein, 1988). A situated perspective affords the examination of

individuals as they interact with their environment in socio-historical contexts.

Individual case analyses allow for particular actions of individuals to be explained. A case-oriented analysis provides “specific, concrete, historically-grounded patterns common to small sets of cases” (Miles & Huberman, 1994, p. 174). Supplementing a case-oriented analysis with a variable-oriented approach allows for identifying relationships among variables across cases.

In this chapter, I will describe the data collection instruments and the techniques I used to reduce and analyze the data I collected with them. In Chapter Four, as I report the findings, I will describe how I analyzed the variables at a group level to search for patterns between variables. And in Chapter Five, I will explain how I searched for relationships between variables across cases, and then looked within each case as I iteratively built an explanation for the ways variables interacted.

### **District Context**

The suburban Southern California school district in which I conducted this study is the Sunrise Unified School District, SUSD (pseudonym). I selected this district due to access and unique aspects of it which allowed for context to be highlighted in the study. I am an employee in the district and believed district leadership would grant me access to the schools and teachers. That did bear out as multiple superintendents met with me within days of my request and immediately approved the study. They also emailed principals asking them to allow me to conduct the study in their schools. The school context variations within the district suited the study’s questions about the role of school context in classroom mathematics instruction. The district is primarily an academically

high performing and socioeconomically upper middle class district. However, it has two schools located near large federally subsidized low income housing developments. As will be described later, school context differences did provide important information to address key research questions.

As revealed on its website, SUSD has 25 elementary schools, 6 middle schools, 5 comprehensive high schools and one continuation high school. It serves approximately 33,000 students and employs approximately 1,500 teachers. Demographically, the student body of the district is 56.7% White, 15.8% Asian, 10.7% Hispanic, 7.2% Filipino, and very small percentages of African American, American Indian, and Pacific Islander (SUSD State of the District Report, 2010). SUSD has a high school graduation rate of 97% with some of the highest standardized test scores in the state. As I will explain later, its climate of academic success breeds varying levels of classroom pressure depending on how each teacher interprets his/her school's vulnerability.

### **Positionality**

As an elementary and middle school mathematics teacher in the Sunrise Unified School District, I have firsthand experience with current district policies on curriculum, instruction, and assessment. When conducting the study, I was cognizant of the need to be aware of how my own beliefs, knowledge, and experiences could bias my interpretation of data. When recently teaching within SUSD, I found my own instruction to be negatively affected by curriculum and assessment policies. District benchmark assessments, aligned with the adopted textbook, made me feel pressured to use a skill-based textbook more than I believed was beneficial to students' development of

principled mathematical knowledge. I did not want my experiences with district policies to influence my participants or my interpretation of their data. I wanted to maintain interpretive validity so the representations I constructed of participants' "mental, conceptual, ideological, and emotional understandings" (Eisenhart, 2006, p. 574) were truly those of the teachers. My study design reflects this desire to minimize the role my own filters play in interpretation and give voice to the teachers. I designed semi-structured interviews allowing teachers to lead; I collected knowledge and instructional quality portions of data using pre-existing, validated instruments; I measured beliefs using a pre-established instrument; and I analyzed the interviews using a priori codes based on theoretical frames already established in the field as well as emergent codes arising from recurrent themes. While my observations and perspective as a teacher shaped the study topic, the data collection and analysis design supported the generation of findings arising from the lived experiences of the participants.

Another aspect of my positionality stems from ten years in the SUSD as a teacher and leader. Although I was on leave from the district during the data collection period, the way teachers perceived me is important to consider. Four of the participants met me for the first time when they joined the study. They viewed me as a colleague. Three of the participants knew me from my role as an Administrative Teacher on Special Assignment (assistant principal role) in their school. My role as a teacher colleague was advantageous as the teachers seemed comfortable sharing their thoughts and concerns on a collegial level. However, there were instances where I had to ask them to be explicit about their ideas and not to assume I knew what they were talking about. Additionally,

two of the teachers who knew me as an administrator initially were worried about sharing their criticisms of policies as they didn't want to "get in trouble." I spent time before the study began and during interviews helping them understand that their identities would not be revealed and it was safe to share their views. My positionality did not seem to affect the content of instruction as none of the teachers knew my philosophy about mathematics instruction and all followed district pacing guides and textbook lessons. In all, I think my insider positionality proved to be much more of an asset rather than a hindrance in eliciting detailed, honest information regarding teachers' thinking and reasoning about their mathematics instruction.

## **Participants**

### **Participant Selection**

The participants in this study included seven fourth and fifth grade math teachers in SUSD. Using purposeful criterion sampling techniques (Mertens, 2005), I sent a series of emails inviting all 85 fifth grade math teachers in SUSD to participate. All participant solicitation emails were sent in September and October of 2010 and all data were collected in September through December of the same year. Although grade level is not a critical aspect of this study, I originally chose fifth grade teachers for two reasons: 1) SUSD superintendents expressed a concern regarding math instruction at the fifth grade level, and 2) I have taught the grade level myself and am familiar with the content and how students come to understand or misunderstand the concepts. In order to maintain a consistent setting for the instructional activity, I did not include special education teachers in the study.

Although I hoped the sample would include ten fifth grade math teachers distributed across school contexts (i.e., low income<sup>5</sup>/low performing<sup>6</sup>, low income/high performing, high income/low performing, and high income/high performing), low participation rates prevented such a sample. Instead, the sample included seven teachers from fourth and fifth grades with representation from three of the four school contexts. The sample does not include teachers from the high income, low performing school context. I believe multiple factors contributed to my difficulty in getting teachers to participate in the study: 1) timing of data collection at the beginning of the year when teachers are busy establishing routines, 2) the amount of teachers' time required to participate, 3) the intimidating nature of being videotaped during instruction, and 4) the high stakes climate of accountability within the district.

The sample size was large enough to allow for quantitative and qualitative instructional quality comparisons across variations in teachers' knowledge, beliefs and school contexts. Additionally, the fact that all the teachers are in the same district allowed their interpretation of district policies to be examined in terms of school context variations. The size of the sample limits the generalizability of the findings, however reasons for the low participation highlighted context-specific variations in teachers' sense

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<sup>5</sup> *Low or high income*, for the purposes of this study, is a relative measure of the socioeconomic resources of the families in the school. According to the California Department of Education testing accountability criteria, "*Socioeconomically disadvantaged* is defined as a student both of whose parents have not received a high school diploma OR a student who participates in the free or reduced-price lunch program" (CDE Glossary, 2005). In this study, low income schools are those where more than one-third of the student population fits the socioeconomically disadvantaged criteria.

<sup>6</sup> *Low or high performances* refer to schoolwide scores on mathematics portion of the Standardized Testing and Reporting Program, California Standards Tests (2011). Low performance is defined for this study as not meeting the district's 80% proficiency goal; high performance is defined as exceeding the district's 80% proficiency goal (District Local Education Agency Plan, 2009).

of accountability pressures as well as the constraints created by state budget cuts. Additionally, it drew attention to the commitment of the teachers who did choose to participate.

When I presented this study to the district superintendents and mathematics leaders, they warned it was going to be hard to get volunteers. The climate at the time consisted of extremely sparse budgets and increased demands on teachers for student performances. Morale was not high and teacher participation in activities outside classroom duties was low. They predicted it would be especially difficult to get participants from the single low income, low performing school based on past patterns. No principals were willing to let me attend faculty meetings to explain the study as budget cuts had resulted in fewer meetings and less time to take care of their own business. After sending three separate rounds of emails over three weeks, the first five teachers who volunteered to participate were from schools set in mid to high income neighborhoods with high state test scores. They reported choosing to participate because they wanted to support research and to support me, a colleague in the district.

I sent additional emails to teachers in schools set in lower income neighborhoods. One participant from a low income and high performing school joined the study. She explained that she chose to participate because she loved her school and wanted its population represented. Because no fifth grade teachers from the single low income, low performing school in the district volunteered, I emailed fourth grade teachers at that school and had one positive response. She explained that she had never focused on her math instruction and was afraid of being videotaped yet she saw this as a nonthreatening

opportunity to grow as a teacher. She tried to convince her colleagues to participate but none wanted to have their math instruction studied. I repeatedly emailed and contacted the principal and teachers at the single high income, low (math) performing school in the district. No teachers responded and the principal did not want attention drawn to a math curriculum problem which she reported had been corrected. Her response seemed to reflect the high risk accountability environment in which she works. Despite the difficulty I experienced in finding teachers willing to participate, the teachers who did volunteer were highly committed to the study. The variability in their individual characteristics, experiences, and school contexts proved to be fertile ground for answering the questions proposed in the study.

### **Participant Sample and School Contexts**

Table 1 details the four schools and seven teachers by school socioeconomic and academic performance contexts. All data in the table was obtained from each school's School Accountability Report Card (SARC) (California Department of Education, 2010).

**Table 1:** Academic and Demographic School Data, 2009-2010

| School<br>(SES/API)                     | Teachers<br>(pseudonyms) | Grade | Socioeconomically<br>Disadvantaged Students<br>(schoolwide %) | English<br>Language<br>Learners (ELL)<br>(schoolwide %) | 2009-2010<br>Percentage of<br>Students<br>Proficient or<br>Advanced in<br>Math (CST) |
|-----------------------------------------|--------------------------|-------|---------------------------------------------------------------|---------------------------------------------------------|--------------------------------------------------------------------------------------|
| Pacific Shore<br>(High/High)            | Bill                     | 5     | 3                                                             | 15                                                      | 93                                                                                   |
|                                         | Fran                     | 5     |                                                               |                                                         |                                                                                      |
|                                         | Hannah                   | 5     |                                                               |                                                         |                                                                                      |
| High Bluff<br>Elementary<br>(High/High) | Barbara                  | 5     | 4                                                             | 19                                                      | 91                                                                                   |
|                                         | Kelly                    | 5     |                                                               |                                                         |                                                                                      |
| Little Rock<br>(Low/High)               | Liz                      | 5     | 41                                                            | 33                                                      | 84                                                                                   |
| Hillside<br>(Low/Low)                   | Hally                    | 4     | 49                                                            | 36                                                      | 76                                                                                   |

*Note.* SES = Socioeconomic Status of the school families (According to my own categorization which is relative within this district: Low SES = more than 30% of the students qualify for the Federal Free and Reduced Lunch Program; High SES = Less than 5% of the students qualify for the Federal Free and Reduced Lunch Program.)

API = Academic Performance Index (a California Standards Test score indicating the school's performance on standardized tests)

CST = California Standards Test

Patterns seen in Table 1 show as the schoolwide percentage of socioeconomically disadvantaged students increases, the percent of English Language Learners (students learning English as a second language) increases and the percent of students proficient in mathematics decreases. It also shows that the study included more teachers in the higher income, higher performing schools.

## **School Contexts**

To understand how the population and performance contexts of each school play into teachers' instructional decisions, this section describes similarities and differences between the sample schools. The schools with higher percentages of socioeconomically disadvantaged students, Little Rock and Hillside, are similar in size, approximately 600 students in each school. They are demographically similar in their percentage of economically disadvantaged students, 40 % and 49% respectively (SARC, 2010). They are also similar in their percentage of students who speak English as a second language, 33% and 36 % respectively. The predominant primary language of the ELL students at both schools is Spanish. Their ethnic compositions differ substantially from the typical schools in the Sunset Unified School District in that they have a higher percentage of ethnic minority students. Their ethnic compositions also differ from each other. Hillside's largest ethnic group is Hispanic, 40% of the total student population, followed by 32% White and 6% Asian. In contrast, Little Rock's largest ethnic student group is White, 36%, followed by 21% Filipino and 16% Hispanic. In this study, subgroup performances on state tests played into the pressure felt by teachers in the different school contexts.

Hillside and Little Rock are similar in that both have a history of struggling to meet NCLB (2001) academic growth targets on the CST. However, they are vastly different in that Little Rock has been engaged in intensive schoolwide reform efforts for

10 years and has received a *California Public Schools Ranking*<sup>7</sup> of 10, 10 for two consecutive years; Hillside, on the other hand, with a 2009-2010 ranking of 6, 5, is under district pressure to replicate Little Rock's reform. Little Rock's Academic Performance Index<sup>8</sup> (API) score is high while Hillside's API is the lowest among the elementary schools in the district. In the 2008-2009 school year, every subgroup at Little Rock met its state-required academic growth targets. At Hillside, the Hispanic/Latino and the ELL populations did not meet their academic growth targets. The demographic similarities make these schools comparable while the academic performance differences place Hillside under much greater district pressure. As will be discussed in Chapter Five, the different levels of pressures felt by teachers at these schools helped to highlight the role of policy context and teacher sensemaking in instructional action.

Adding to the variation in school contexts, Pacific Shores and High Bluff are the two high income, high performing schools in the study. They both have low numbers of students from socioeconomically disadvantaged families, 3% and 4% respectively. They also both have approximately half their student population classified as White and one-third classified as Asian. Their number of English Language Learners is approximately half of Hillside and Little Rock and there are a variety of languages spoken.

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<sup>7</sup> *California Public Schools Ranking*: A system wherein California Schools receive a numeric range of 1 – 10 for their performance on the STAR testing as compared to other schools in the state and to 100 other statistically matched schools. A 1 for the statewide rank means the school scored in the lowest 10% of the schools in the state. A 1 for the similar schools ranking means the school scored in the bottom 10 of the 100 statistically similar schools.

<sup>8</sup> Academic Performance Index (API) is a California Public Schools Ranking System based on school performances on the STAR California Standards Tests. The range of scores possible is from 200 - 1000. The California Department of Education has set the goal that all schools will score at least 800 (California Department of Education, 2011).

Pacific Shores and High Bluff are academically ranked highly within the district according to API scores. Out of all schools, elementary, middle, and high, Pacific Shores is ranked the second highest in the district while High Bluff is the fourth highest. Pacific Shores received a 10, 3 California Public School Ranking and High Bluff received a 10, 5 ranking. As described previously, the rank of 10 means both schools achieved API scores within the top 10% of the schools in California. The 3 and 5 (respectively) rankings indicate Pacific Shores scored in the bottom 30 of the 100 statistically similar comparison schools and High Bluff scored in the bottom 50 of schools statistically similar to them. As will be reflected in Chapters Five and Six, while the similar schools rankings are not high, they did not surface as a source of pressure for the teachers.

### **Participants' Educational Backgrounds and Teaching Experiences**

The teachers who volunteered for this study varied in their educational backgrounds and teaching experience. While it is not expected that variations in these factors will play a large role in the quality of each teacher's instruction (Begle, 1979; Hill, 2010), examining the factors establishes a general profile of the teachers in the study. Table 2 displays the range of backgrounds and teaching experience of the participants.

**Table 2:** Participant Background and Experience Summary

| Participant | Grade Taught | Years of Teaching | College Math Classes | Highest Degree |
|-------------|--------------|-------------------|----------------------|----------------|
| Liz         | 5            | 6-10              | 3-5                  | MA             |
| Barbara     | 5            | 6-10              | 3-5                  | MA             |
| Kelly       | 5            | 11-15             | 3-5                  | MA             |
| Bill        | 5            | 11-15             | 6 or more            | MA             |
| Hannah      | 5            | 16-20             | 3-5                  | MA             |
| Hally       | 4            | 16-20             | 3-5                  | MA             |
| Fran        | 5            | 20+               | 3-5                  | MA             |

The data in Table 2 shows a sample of teachers who are moderately to highly experienced. They all took at least 3 – 5 math classes at a college level and earned masters’ degrees. Additionally, they all hold California multiple subject teaching credentials. Their similarities in experience and education levels allow possible effects of background variables to be minimized. The high number of college math courses Bill took is a variable to consider when analyzing differences in knowledge levels and instructional quality. Additionally, while including a less experienced teacher may have provided an interesting contrast to the others, the low numbers of new teachers with jobs resulting from California budget shortfalls prevented finding such a participant. In

summary, this study contains a sample of experienced fourth and fifth grade math teachers across a variety of academic and demographic school contexts.

### **Data Collection, Reduction, and Analysis by Variable**

I used a combination of both quantitative and qualitative data collection and analysis techniques to measure teachers' mathematical knowledge for teaching (MKT), beliefs about teaching and learning mathematics, and interpretation of curriculum and assessment policies. Using pre-established assessments, surveys, observation protocols, analysis instruments validated by current researchers, and independently designed interviews provided a rich collection of data. Data collected with each instrument had potential to be both quantitatively and qualitatively analyzed providing an opportunity for identifying patterns and searching for meaning within each teacher's written and oral expressions. Table 3 displays the constructs measured and sources through which data were collected. The main data sources for each construct are marked but other sources often contributed important information to triangulate findings.

**Table 3:** Constructs Measured and Data Collection Sources

| Constructs            | Data Sources |          |                           |                     |                            |
|-----------------------|--------------|----------|---------------------------|---------------------|----------------------------|
|                       | MKT          | CVA      | Lesson Video Observations | IMAP Beliefs Survey | Semi-Structured Interviews |
| Teacher Knowledge     | <b>X</b>     | <b>X</b> |                           |                     |                            |
| Instruction           |              |          | <b>X</b>                  |                     |                            |
| Beliefs               |              |          |                           | <b>X</b>            | <b>X</b>                   |
| Policy Interpretation |              |          | <b>X</b>                  |                     | <b>X</b>                   |
| Institutional Context |              |          | <b>X</b>                  |                     | <b>X</b>                   |

*Note.* Mathematical Knowledge for Teaching Assessment (Hill et al., 2006)

CVA: Classroom Video Analysis Instrument (Kersting, 2010a)

IMAP Beliefs Survey: Integrated Mathematics and Pedagogy Beliefs Survey (Philipp & Sowder, 2003)

The data collection strategies displayed in Table 3 are detailed in the next sections.

### **Mathematical Knowledge for Teaching**

In order to measure teachers' mathematical knowledge for teaching I used two separate instruments which complemented each other. I used the Mathematical Knowledge for Teaching (MKT) Assessment (Hill et al., 2006) and the Classroom Video Analysis (CVA) instrument (Kersting et al., 2009). By using the instruments in combination I was able to quantitatively compare teachers' knowledge levels and qualitatively analyze written responses to gain a more detailed understanding of each teacher's knowledge about teaching mathematics. I used the numeric knowledge scores from the multiple-choice MKT Assessment to sort the participants by knowledge for teaching mathematics levels. I used the scores from the written responses on the CVA to

quantitatively triangulate the outcome of the MKT Assessment as well as the written responses themselves to qualitatively unveil reasons for varying scores. Both instruments provided ample data to gain insight into differing levels of teachers' knowledge for teaching mathematics.

### **The Mathematical Knowledge for Teaching Assessment.**

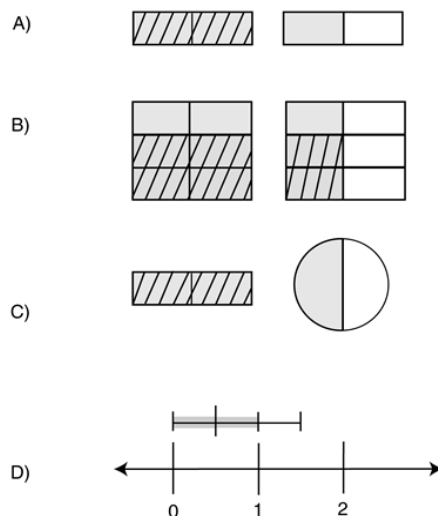
The Mathematical Knowledge for Teaching Assessment (MKT) (Hill et al., 2006) was developed in the Learning Mathematics for Teaching Project. It measures teachers' knowledge for teaching mathematics through a multiple choice assessment. Hill et al. (2004) provide empirical evidence that the MKT is both reliable and valid (reliability coefficients in .70 - .80 range). MKT scores have shown a small correlation to student learning gains on standardized tests; an MKT score increase of one Standard Deviation was associated with a student learning Standard Deviation gain of .06 (Hill et al., 2005). They have also shown a strong correspondence to the quality of classroom instruction (Hill et al., 2008).

The survey-based items comprising the instrument were designed using theories about teacher knowledge (Ball & Bass, 2003; Grossman, 1990; Shulman, 1987; Wilson, Shulman, & Richert, 1987) and the role of teacher knowledge in enacting quality mathematics instruction (Ball & Bass, 2000; Cohen & Ball, 1999). After attending an instrument dissemination workshop (Hill & Phelps, 2010), I was able to select from a bank of items (scales) specified by mathematics strand (geometry; number concepts and operations; proportional reasoning; probability, data, and statistics; and patterns, functions, and algebra) and by grade level (elementary; middle school; or 4-8). I selected

a 14-item scale designed to measure elementary school teachers' common content knowledge, specialized content knowledge, knowledge of content and students, and knowledge of content and teaching in the strand of number concepts and operations. Selecting this scale was appropriate for this study because the fifth grade curriculum pacing guide in SUSD indicated the focus during the data collection time frame would be in this strand. I did not select additional scales out of respect for the time commitment already required of teachers participating in the study.

The items on the MKT present classroom scenarios and teachers are asked to answer questions about the math content, student thinking, or about teaching decisions. Although I am unable to append the assessment itself to this dissertation as the items are considered secure, Figure 2 shows a released item.

6. At a professional development workshop, teachers were learning about different ways to represent multiplication of fractions problems. The leader also helped them to become aware of examples that do not represent multiplication of fractions appropriately. Which model below cannot be used to show that  $1\frac{1}{2} \times \frac{2}{3} = 1$ ? (Mark ONE answer.)



**Figure 2:** MKT Released Item (2008) – Specialized Content Knowledge

This item assesses a teacher's specialized content knowledge for selecting mathematically appropriate representations in order to make math content conceptually accessible to students (Ball & Bass, 2003; Shulman, 1986, 1987). Answers A, B, and D all show representations where the whole/unit is the same (an important concept for early development of fraction understanding) and show how  $1\frac{1}{2}$  of  $\frac{2}{3}$  can be represented or, vice versa, what  $\frac{2}{3}$  of  $1\frac{1}{2}$  looks like. Answer C is the incorrect model as it uses two different wholes/units which would be a misleading representation to show students. This specialized content knowledge is needed in the act of teaching.

Figure 3 shows another item from the MKT which assesses teachers' knowledge of content and student thinking.

2. Imagine that you are working with your class on multiplying large numbers. Among your students' papers, you notice that some have displayed their work in the following ways:

| Student A                                                                                    | Student B                                                                                     | Student C                                                                                                  |
|----------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------|
| $\begin{array}{r} 35 \\ \times 25 \\ \hline 125 \\ +75\phantom{0} \\ \hline 875 \end{array}$ | $\begin{array}{r} 35 \\ \times 25 \\ \hline 175 \\ +700\phantom{0} \\ \hline 875 \end{array}$ | $\begin{array}{r} 35 \\ \times 25 \\ \hline 25 \\ 150 \\ 100 \\ +600\phantom{0} \\ \hline 875 \end{array}$ |

Which of these students would you judge to be using a method that could be used to multiply any two whole numbers?

|             | Method would work for all whole numbers | Method would NOT work for all whole numbers | I'm not sure |
|-------------|-----------------------------------------|---------------------------------------------|--------------|
| a) Method A | 1                                       | 2                                           | 3            |
| b) Method B | 1                                       | 2                                           | 3            |
| c) Method C | 1                                       | 2                                           | 3            |

**Figure 3:** MKT Released Item (2008) – Knowledge of Content and Students

In this scenario, all three students have decomposed one or both of the multiplicands and distributed the parts. All methods will work for all whole numbers. Student A has decomposed 35 into  $30 + 5$  and distributed it:  $5 \times 25 = 125$  and  $30 \times 25 = 750$ . Student B uses the traditional algorithm of multiplying  $5 \times 35$  and then  $20 \times 35$ . Student C uses a method often called partial products where each number is decomposed into parts and each resulting part is multiplied by the other parts. It is another example of the distributive property of multiplication. Often teachers teach and accept only one strategy for number operations causing students to lose opportunities to make sense of numbers. Understanding and being able to respond appropriately to students' approaches to doing mathematics has been shown to be a critical aspect of teachers' knowledge for teaching mathematics (Carpenter et al., 1988; Fennema et al., 1996). Additionally, encouraging and understanding more than one way to solve a problem is seen as helping students become flexible problem solvers, develop flexibility with numbers, and learn to persevere (Carpenter & Lehrer, 1999; National Governor's Association, Common Core State Standards, 2010). The MKT multiplication item in Figure 3 assesses teacher knowledge in these areas.

The items displayed in Figures 2 and 3 demonstrate the common classroom scenario approach used in the MKT assessment. Using classroom instruction scenarios, the survey's designers consider the items to be situated in authentic contexts. Hill, Rowan, and Ball (2005) claim that, "A key feature of our measure is that it represents the knowledge teachers use in classrooms, rather than general mathematical knowledge" (p. 387). The MKT also contains an option for conducting cognitive interviews using the

assessment items. Although cognitive interviews would have provided more insight into teachers' knowledge for teaching mathematics, I did not choose to use them as my data collection plan was already extensive and time consuming for teachers.

While the MKT assessment is a valuable tool in measuring and comparing teachers' mathematical knowledge for teaching, it has some limitations. First, the multiple choice format restricts the range of responses to the teaching scenarios. Second, although generated from common student misconceptions and teacher challenges, the contrived scenarios are limited as they are missing real classroom interactions. Third, the scores generated from the MKT are meant to be used to make statements regarding variations in content knowledge among *groups* of teachers numbering 60 or more. The authors warn of the loss of statistical power as the group size decreases (Hill, Ball, & Phelps, 2005). With the small sample size in this study, the power of the MKT scores was limited as a statistical analysis was not possible. However, my use of the scores mirrored Hill's (2008) use in which MKT scores were compared across individuals and used to analyze their role in mathematics instruction. For this purpose, the scores provided important information.

An additional limitation of the MKT instrument is the inability to analyze the teacher responses qualitatively. It is not possible to make valid comparisons of how individuals answered certain problems as there were different versions of the assessment and similar items were not always comparable in levels of difficulty. Because the MKT assessment provided strictly quantitative data, I needed another instrument which would also provide qualitative data and allow me to understand teachers' MKT scores in more

detail. I used the Classroom Video Analysis (CVA) survey, detailed later in this chapter, to provide qualitative data.

### **MKT data collection, reduction, and analysis.**

Teachers participated in the MKT assessment using the Learning Mathematics for Teaching Project's online delivery, the Teacher Knowledge Assessment System (Hill et al., 2005). They logged in to the system with their own personal access code and took the assessment once, at the beginning of the study. With the Elementary Number Concepts and Operations (2004) scale I selected, my participants were given one of two forms, Form A or Form B. Teacher scores from the assessment were available to me but teachers did not see their own results.

Hill (2010) describes the design of the MKT scales as strongly reliable in that slight variations in knowledge levels can be detected between participants. Teacher responses to the assessment are recorded as raw data, as correct or incorrect, and IRT scores (Item Response Theory). The IRT scores are calculated using a conversion table in which items are given weighted value based on their difficulty level (Hill et al., 2004; Hill et al., 2005). MKT scores are reported as logits, "standard unit used to linearize otherwise nonlinear measures" (Hill et al., 2004). Ninety-five percent of participants on the MKT score between -2 and +2 standard deviations on a normal curve (Hill, personal communication, April 14, 2011). Their scores represent a combination of common content and specialized content knowledge.

In addition to being able to view the scores online, I also had access to the forms showing all items. I used the IRT scores as estimates of teachers' knowledge levels for

teaching mathematics. As a quantitative data source, the knowledge scores allowed me to search for within and cross-case patterns across other variables which had been quantitatively reduced in the study.

Additionally, I used the actual forms to examine items missed by individual teachers. If the content of an item coincided with content in the participant's lessons, I compared their item response to the knowledge they displayed in instruction. For example, three of the teachers who completed Form B demonstrated in the assessment a lack of understanding of how to draw a model representing a fraction as division (secure item so I cannot display it here). This missing knowledge surfaced in the classroom when all three teachers were able to explain equivalent fractions conceptually but could only demonstrate equivalent decimals by using a long-division procedure. The model they did not understand on the MKT was the exact model they needed to make the concept understandable and mathematically connected for students. Differing items between the forms prevented direct comparison on specific item responses across participants. Although analyzing the items missed on the MKT assessment provided an explanation for some aspects of instructional quality, it was unusual for an item on the assessment to perfectly align with the content of instruction. Therefore, while the MKT did have qualitative data, the quantitative data, IRT scores, proved most useful in analysis.

### **Classroom Video Analysis Survey.**

As a complement to the MKT, I also measured teachers' knowledge for teaching mathematics using the Classroom Video Analysis (CVA) instrument (Kersting et al.,

2009; Kersting, 2010a). The CVA uses video clips with open ended responses instead of multiple choice scenarios to better capture the decision making and complexities of a real classroom context. I used this instrument to triangulate data collected with the MKT. It generates data that can be analyzed both quantitatively and qualitatively. On the CVA, teachers watch an online series of 2 – 5 minute video clips of real classroom mathematics instruction. After watching a clip, they write a response to an open ended prompt:

Discuss how the teacher and the student(s) interact around the mathematical content.

Figure 4 shows a screenshot of what teachers encounter when taking the CVA survey.



**Figure 4:** CVA Screen for Division of Fractions Video Clip

As this image shows, the participant reads about the context of the video clip, watches the classroom video, and then responds to the prompt in writing. Kersting et al. (2009) reported, “We reasoned that the analysis teachers produced would reveal not only

their knowledge as it relates to the instructional segment depicted in the video clip, but also their ability to bring that knowledge to bear on a classroom situation” (p. 6). The open-ended aspect of the CVA seems to give more room for teachers to express their knowledge than does the MKT. However, its open-ended format could also cause researchers to make judgments based on what a participant does not write. In this sense, the MKT is more precise when determining what a participant does or does not know.

CVA video clip responses are scored on a scale of 0-2 (indicating degree of presence and sophistication) along four rubrics: 1) mathematical content, 2) alternative teaching strategies, 3) student thinking/understanding, and 4) level of interpretation. Kersting and colleagues (2009) tested the CVA on a sample of 64 teachers. The results showed good score reliability with high internal consistency ( $\alpha = .93$ ). The researchers also tested the validity of the measure by comparing it to the same teachers’ scores on the MKT. The scores given to CVA responses correlated significantly with MKT scores ( $r(56) = .53, p = .01$ ). Initial findings show aspects of the CVA correlate with student learning and CVA scores are predictive of instructional quality (Kersting et al., 2009).

My purpose in using the CVA was to triangulate the MKT scores as well as to collect qualitative data, the written responses, which could provide greater insight into how knowledge levels are related to teacher beliefs and instructional action. When selecting the video clips for this study, I was aware that the CVA video clips focused on fraction lessons. While ideally, participants would respond to video clips that were aligned to the content they were teaching, during the data collection window, the participants were not always teaching fraction concepts. This was not a problem when it

came to comparing the numeric scores from the MKT and CVA as the instruments correlated (Kersting et al., 2009). However, it did limit the applicability of the written responses as compared to the actual observed lessons. For example, responses which focused on general pedagogical strategies such as using manipulatives and creating visuals could be compared to the teacher's actual instruction. However, responses which focused on specific math content such as using a consistent whole when working with fractions could not be compared to a teachers' instruction in a lesson on similar triangles. Nonetheless, the CVA helped to triangulate information gathered from the MKT assessment. The qualitative data helped explain the relationships between teachers' knowledge, instruction, and sometimes beliefs.

#### **CVA data collection, reduction, and analysis.**

In this study, participants responded to nine video clips which I selected from the collection of fifteen fraction clips (developed from TIMSS, 1997) Kersting made available to me. I selected nine clips instead of all fifteen to be respectful of teachers' time. I selected fractions because the only other collection of clips was proportional reasoning from middle school classrooms. I wanted the clips to reflect the content and age group my teachers taught. The clips I selected covered a range of fraction concepts: meaning, addition, comparing, division, and equivalence.

Using the previously described scoring scheme (Kersting, 2010a), I scored each participant's responses along instrument domains. Table 4 displays a rubric for the domain of Mathematical Content. The examples are those provided with the instrument, not examples from the study I conducted.

**Table 4:** CVA Mathematical Content Rubric

| Scores | Rubric Description                                                                                                                                                                                                                                                                                                                                                                                                                        | Response Examples                                                                                                                                                                                                                                                                       |
|--------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 0      | <p>No mathematical content</p> <p><i>naming</i> objects/math tools (e.g., geoboard) is insufficient (a <i>mathematical evaluation</i> of tools can count toward the mathematical content score)</p> <p>Respondents may talk about mathematical thinking/reasoning, but unless it explicitly mentions the problem at hand, it is not a form of content. Likewise, using the words “conceptual” and “procedural” alone is insufficient.</p> | <p>“The student had to do no mathematical reasoning and I am still not sure that the student really understands why her model is wrong.”</p>                                                                                                                                            |
| 1      | <p>Present without elaboration</p> <p>There is inclusion of math, but it does not go beyond the immediately observable.</p>                                                                                                                                                                                                                                                                                                               | <p>“The teacher is using a great model to represent <i>breaking area into pieces</i>. This uses a lot of skills and the students are having to think critically. In her assistance of this student she did not give her the answers, but she did guide her in the right direction.”</p> |
| 2      | <p>Present with elaboration</p> <p>Math that is beyond what is immediately observable</p>                                                                                                                                                                                                                                                                                                                                                 | <p>“This student is clearly having some trouble with the definition of a “unit”, which is quite common.”</p>                                                                                                                                                                            |

*Note.* Rubric obtained from CVA scoring manual, p. 2.

As the rubric shows, participant answers are generally not elaborate on the CVA but specific elements of a teacher’s focus serve as a proxy for knowledge. For example, the response which received a 2 shows awareness of a common area of confusion specific to the study of fractions, indicative of specialized content knowledge (Ball et al., 2008). The response which received a 0 shows a lack of focus on specific math content and instead a focus on general pedagogy. After scoring all CVA responses, I averaged domain scores by teacher. Using raw scores rather than calculating IRT scores was the

recommended data reduction technique due to my small sample size (Boyles, personal email, April 20, 2011). These results are discussed in Chapter Four.

Similar to Kersting's correlation study (2009), I found CVA domain scores to distribute across participants similarly to their MKT scores. Participants with lower MKT scores tended to also have lower CVA scores, as I will explain later. However, I found the 0 – 2 scale range on the CVA rubrics were not sensitive enough to degrees of separation between qualitatively different responses. In contrast, 95% of MKT scores tend to fall between -2 and +2 standard deviations, a range with more precision than the CVA. As a result, I found the CVA scores to be most useful in indicating relative domain strengths within and across the cases. For example, knowing that a teacher had a high CVA score in attending to student thinking and understanding allowed me to go beyond an overall knowledge score and hypothesize that their instruction might reflect attention to student thinking. Such predictions guided a comparative search for relationships between variables. I compared CVA knowledge scores to instructional quality domains (which I will discuss in the next section).

I also used the participants' responses on the CVA to look for explanations of relationships that appeared to exist in the quantitative data. For example, during a cross-case analysis of factors, I noted the CVA Mathematical Content domain showed a possible relationship to the quality of mathematical connections and math language used during instruction. I was then able to analyze the written CVA responses for evidence of teachers' knowledge about the connectivity of mathematics as well as for their use of mathematical language. In this sense, I used the written responses differently than

originally intended by Kersting et al. (2009) but in a way that added depth and richness to the study.

### **Mathematical Quality of Instruction**

In order to capture the quality of lessons teachers taught every day and to gain insight into differences in how their ideal instruction would look, I asked teachers to allow me to observe two typical math lessons and one that represented their vision of an ideal lesson. These instructions did not matter as all seven teachers stuck to district pacing guides and textbooks for all three lessons. It was common for teachers to say they did not have time to design an ideal lesson within the pacing and accountability pressures they felt. The observed lessons were teacher-selected as the teachers signed up for the days and times they wanted to be observed. The lessons they chose to have me observe were typically influenced by their scheduling constraints rather than any intentional focus on lesson content.

Measuring the quality of teachers' mathematic instruction was a five step process: 1) Video-recording classroom lessons from start to finish, 2) Transcribing the lessons, 3) Practicing scoring using the instrument training materials (Hill, 2010), 4) Analyzing and quantifying specific aspects of the lesson quality, and 5) Reviewing the videos and transcripts and qualitatively developing a list of instructional characteristics specific to each teacher. In order to give structure to the measurement, I selected the Mathematical Quality of Instruction (MQI) instrument (Hill et al., 2008). Another instrument developed by Kersting (2010b), the Quality Math Instruction instrument (QMI), was available for my use but I selected the MQI due to its applicability across lessons with

varying math content. Analysis of the information gained from the data reduction took the form of a variable analysis within and then across cases.

### **Mathematical Quality of Instruction instrument.**

To measure the quality of teachers' mathematics instruction, I video-recorded three whole-class mathematics lessons per teacher (Details on the video recording procedures are described below). The *Mathematical Quality of Instruction* (MQI) instrument (Hill et al., 2008) provided a research-based framework through which characteristics of teachers' mathematics instruction could be identified and analyzed. This classroom-observation-based instrument allowed for quantitative analysis in that numeric scores were assigned and averaged across instructional dimensions. The scores allowed instructional comparisons between and across participants. They also made a relational analysis between MQI and MKT scores possible. The video footage and resulting transcripts permitted a qualitative analysis of each teacher's mathematics instruction.

Supported by previous research in using classroom observation to assess teacher knowledge for teaching and to analyze its relationship to in-class performance (Leinhardt & Smith, 1985; Borko et al., 1992), and certain that other instructional quality measures were biased toward instructional approaches (Sawada, D. et al. 2000; Horizon Research, 2005), researchers at the Learning Mathematics for Teaching Program developed the MQI so it would be "focused solely on teachers' knowledge as it is used in classroom instruction" (LMT, 2006, p.6) . Their goal was to be able to "quantify the quality of mathematics in instruction" (LMT, 2006, p.3).

Hill et al. (2008) emphasize that their instrument was intentionally designed to not give preference to a particular type of teaching (e.g., reform-oriented). They claim it quantifies the quality of the mathematics as it appears in instruction, regardless of the teaching method. They define *quality of mathematics in instruction* as “the extent of the key mathematical characteristics, including accurate use of mathematical language, the avoidance of mathematical errors or oversights, the provision of mathematical explanations when warranted, the connection of classroom work to important mathematical ideas, and the work of ensuring all students access to mathematics” (Learning Mathematics for Teaching, 2006, p. 3). While the researchers claim the framework underlying the instrument domains is not biased toward a particular pedagogical philosophy, it does place high value on evidence of research-based instructional elements such as encouraging multiple solution methods, students interacting with one another’s ideas, and the formulation and discussion of generalizations. I would argue that the presence of a framework through which the lesson is scored unavoidably places higher value on specific types of instruction.

Critical to this instrument is video-taped observations of each teacher during their mathematics instruction and detailed rubrics for lesson analysis. The LMT researchers recommend a rule of thumb for the number of observations necessary to use their instrument with reliability: “Four observations would be a safe bet for making inferences about the mathematical quality of teaching; three would be okay, two would likely be dicey” (LMT, 2006. No page numbers provided). In this study, I observed each teacher

three times as it was within instrument recommendations and was less inconvenient for teachers than four observations.

The MQI instrument protocol requires video recording a math lesson from the moment instruction begins until the moment it ends. Upon reviewing the video footage, each lesson is divided into seven-minute segments (a 56 minute lesson would have 8 segments). Each segment is scored along 35 different rubrics categorized under 9 domains: 1) Format of the segment, 2) Classroom work is connected to mathematics, 3) Mode of instruction, 4) Richness of the mathematics, 5) Working with students and mathematics, 6) Errors and imprecision, 7) Student participation in meaning-making and reasoning, 8) Lesson level binary codes (focused on lesson structure and organization), and 9) Overall MQI and MKT. For example, the sub-domains under the *Richness of Mathematics* dimension include: linking or connections, multiple procedures or solutions, explanations, developing math generalizations, and mathematical language. Each of these subcategories has its own rubric. Each seven minute segment receives a score from each rubric. The rubrics attend to duration of an instructional element in the segment (none, some, most/all), presence of an instructional element (yes or no), or the quality of an instructional element in a segment (low, mid, or high). I assigned numeric values to each of the rubric scores to allow for quantitative analysis. The domain and sub-domain scores are averaged across a lesson to form lesson-level scores and across all three lessons to form teacher-level scores. A holistic overall MQI score is also given to each lesson and averaged across lessons for each teacher.

### **MQI data collection, data reduction and analysis.**

Recording and analyzing teachers' mathematics instruction was the most complex and time consuming aspect of this study. Across the seven participants, I observed twenty-one lessons ranging from 25 to 90 minutes in length. I used a Panasonic HDC-TM700 32GB camcorder with a Sony UWP-V1 portable wireless lavalier microphone for the teacher's voice and a Rode NTG-2 shotgun microphone for the students' voices. In order to capture the teacher's actions, the students' actions, and anything the teacher projected on the screen at the front of the room, I positioned the camera on a tripod at the back, center of the room. I zoomed in on anything the teacher wrote that was projected for students to see so that it could later be analyzed. When the teacher provided direct instruction, I kept the camera focused on him/her in order to record gestures that could enhance explanations and to watch where his/her attention was focused. If a student raised a hand, I turned the camera to that student when he/she was called on by the teacher. Frequently, I panned the class (excluding students who did not consent to be video recorded) to gain a sense of what students were doing during direct instruction. During group work, I removed the camera from the tripod and followed the teacher. If a teacher was helping an individual student, I recorded that interaction as well as zooming in on the student work in order to contextualize the exchange as well as for later analysis of the teacher's instructional choices.

Once I completed recording a lesson, I uploaded the videos to my personal computer and for security purposes, in compliance with human subjects protection agreements, I deleted them from the camcorder. I used *Inqscribe* transcription software (<http://www.inqscribe.com/>) to transcribe the lessons. This was highly effective software

to use for the purposes of analyzing the data as the video attaches to the transcript and I was able to rewatch any segment or interaction by just clicking on that part of the transcript.

I transcribed one lesson per participant myself in order to develop a deeper connection with each teacher's instructional characteristics which I may have missed when recording the lesson. I also was aware of the interpretive role the transcriptionist plays in the text representation of the interaction (Lapadat & Lindsay, 1998; Tilley, 2003). The attachment of video to the transcript during my entire analysis did minimize the risk of the transcriptionist's interpretation altering the meaning of act. During the transcription, I kept anecdotal records regarding my impressions of the way the teacher facilitated classroom discourse and responded to students. I also noted areas of the transcript I should revisit later as they seemed to exemplify particular aspects of instructional quality or to capture the essence of a particular teacher's mathematics instruction. These identified segments were helpful later when I examined MQI domain scores as they validated what was highlighted by the MQI and provided rich exemplars of instructional quality. After completing collection of preliminary qualitative instructional quality data, due to the extensive time demanded of transcription, I hired a professional transcriptionist to complete the remaining 14 lessons.

After the lessons were transcribed, I conducted a mixed quantitative/qualitative analysis of each of them. Viewing the lessons and reading the transcripts gave me a holistic interpretation of each teacher's mathematics instruction. Creating lists of lesson characteristics that were less bounded than those within the MQI framework allowed me

to consider additional aspects of instruction that may not surface in the MQI. The MQI lesson-level and teacher-level numeric scores assigned along each domain and sub-domain allowed me to identify strengths and weaknesses in each teacher's instruction within a common framework across all participants. They also allowed me to search for relationships between specific aspects of instruction and other factors such as teacher knowledge, beliefs, curricular materials, and policy interpretation. Additionally, I calculated average scores across the entire participant sample to establish a general profile of the teachers in the study. In all, the multiple approaches to reducing and analyzing the lesson videos provided extensive usable information for the within and cross-case analyses.

## **Teacher Beliefs**

### **Integrated Mathematics and Pedagogy Beliefs Survey.**

Using the *Integrating Mathematics and Pedagogy* (IMAP) Web-Based Beliefs Survey (Philipp & Sowder, 2003) (Appendix A), I collected evidence of teachers' beliefs about knowing, learning, and teaching mathematics according to a conceptually-oriented beliefs framework. According to Philipp and Sowder (2003), the survey designers, the survey is designed to "go beyond the typical approach of using Likert scales because we believed that beliefs are best assessed in context" (Philipp & Sowder, 2003, p. 4). The survey targets beliefs about content specific instructional decisions with items like the MKT scenarios and real contexts with video clips like the CVA. Teachers participated in the survey by going online and responding to the 25 different items. Several of the

participants reported that participation in the survey made them think more deeply about instructional decisions they made in their own classrooms.

The belief framework measured by the survey is displayed below (Philipp and Sowder, 2003).

**Beliefs about Mathematics**

1. Mathematics is web of interrelated concepts and procedures (and school mathematics should be too).

**Beliefs about Learning and/or Knowing Mathematics**

2. One's knowledge of how to apply mathematical procedures does not necessarily go with understanding of the underlying concepts.
3. Understanding mathematical concepts is more powerful and more generative than remembering mathematical procedures.
4. If students learn mathematical concepts before they learn procedures, they are more likely to understand the procedures when they learn them. If they learn the procedures first, they are less likely ever to learn the concepts.

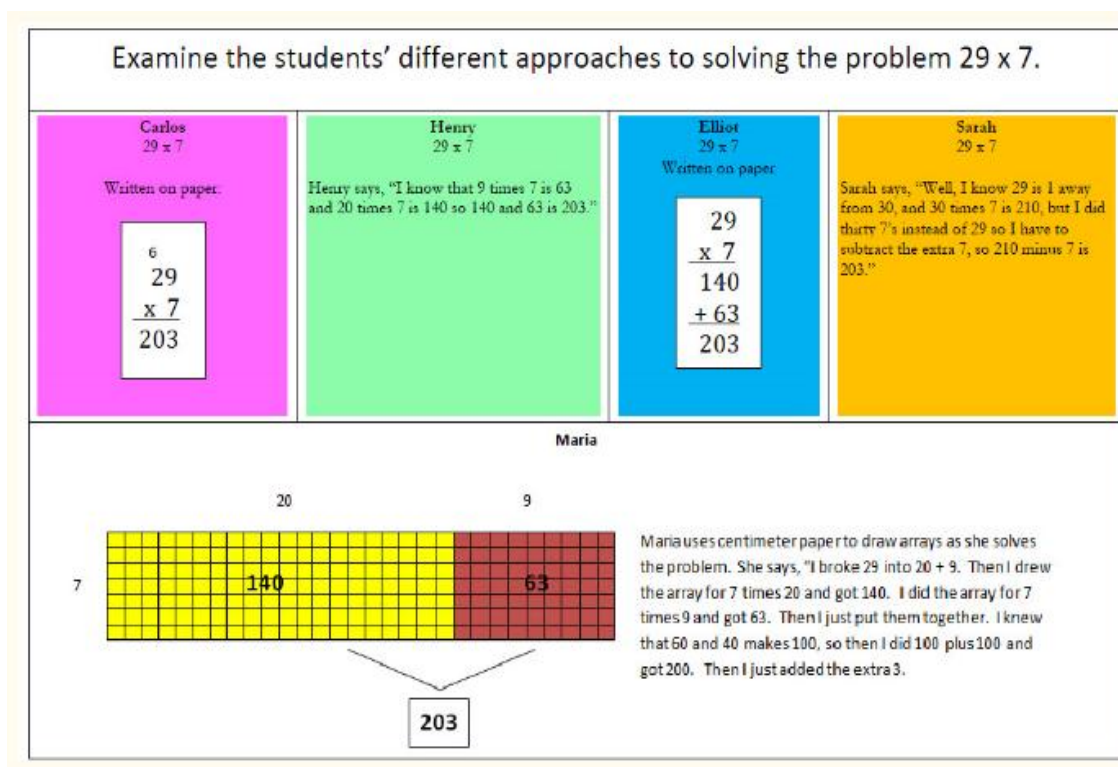
**Beliefs about Children's (Students') Learning and Doing Mathematics**

5. Children can solve problems in novel ways before being taught how to solve such problems. Children in primary grades generally understand more mathematics and have more flexible solution strategies that adults expect.
6. The ways children think about mathematics are generally different from the ways adults would expect them to think about mathematics. For example, real-world contexts support children's initial thinking whereas symbols do not.
7. During interactions related to the learning of mathematics, the teacher should allow the children to do as much of the thinking as possible.

The beliefs detailed in this framework are closely linked to current research in supporting student understanding in mathematics (Carpenter et al., 1988; Fennema et al.,

1996; Franke, 2011; Hiebert et al., 1997; Jacobs et al., 2009; Lampert, 1992). I measured the first six beliefs but cut items specific to Belief 7 out of the survey. I did this because the survey needed to be shortened in order to honor teachers' time. I felt Beliefs 5 and 6 provided strong insight into the domain of *Belief about Children's Learning and Doing Mathematics*. Additionally, I felt I could capture evidence of Belief 7 in the context of classroom observations.

Because the original survey was tailored to kindergarten through third grade mathematics content, I modified some of the items so they were relevant to fourth and fifth grade mathematics. For example, one item asked teachers to sequence strategies they would teach students for adding double digit numbers. I altered the item to show strategies for teaching double digit multiplication. Figure 5 shows the altered item which measures Belief 1. Belief 1 focuses on teachers believing that mathematics is a web of interconnected ideas.



**Figure 5:** Modified IMAP Beliefs Survey Item Measuring Belief 1

When viewing this item, teachers were asked to select and explain which strategies they would share with students and in which order they would focus on the strategies when teaching a unit on multi-digit multiplication. After selecting and sequencing the strategies, they wrote explanations for their choices. Following the collection of all participant responses, I analyzed the data according to instrument specifications.

### **IMAP Beliefs Survey data reduction and analysis.**

In the IMAP Beliefs Survey (Philipp & Sowder, 2003), each belief has two or more items that pertain to it resulting in 17 different rubrics for the 7 different beliefs. Teacher responses are scored according to item-specific rubrics which range in scale from 0 – 2, 0 – 3 or 0 - 4. Each rubric value is associated with the degree of evidence that

exists in a response for a specific belief. These relative values range from *no evidence* to *strong evidence* of the belief.

To exemplify how teachers' responses were reduced both quantitatively and qualitatively, the item in Figure 5 will continue to serve as the exemplar. Explanations on this item received scores of 0 – 3 depending on the degree of consideration they showed for the mathematical connections between strategies. Scores on this survey indicate the degree of evidence teachers provide that they hold specific beliefs, not whether or not they definitely do or do not hold the beliefs. For example, a response showing weak evidence of Belief 1 on the exemplar item earns a 0 score indicating “no appreciation for the interconnectedness of procedures and concepts. A decision to share only one or two strategies is interpreted as evidence of this lack of appreciation. Such respondents tend to express a definite preference for Carlos's strategy” (Philipp & Sowder, 2003, p. B1-S3.2). A score of 3 indicates strong evidence of Belief 1: appreciation for the interconnectedness of concepts and procedures; valuing of multiple (all) strategies and explained using mathematically appropriate reasons; seeing mathematics concepts as a web; not showing a preference for Carlos's (the first student's) strategy noted above.

The rubrics are ordinal in nature and relative to the item meaning that a 3 on one rubric may not indicate the same degree of evidence as a 3 on a different rubric.

Therefore, the instrument contains a more qualitative means for establishing the final level of evidence which exists for a certain belief: the *Rubric of Rubrics*. The Rubric of Rubrics establishes the final belief score when all scores for a specific belief are

considered. There is a separate Rubric of Rubric for beliefs represented by two or three survey items. Scores are not averaged. Figure 6 displays the permutations used to make the final determination of the degree of evidence that a teacher holds Belief 1.

**Belief 1 Rubric**

| S3.2          | S3.3          |
|---------------|---------------|
| Segment Score | Segment Score |
| 0             | 0             |
| 1             | 1             |
| 2             | 2             |
| 3             |               |

| Belief Score | Permutations of (S3.2, S3.3) |
|--------------|------------------------------|
| 0            | (0,0) (1,0)                  |
| 1            | (0,1) (1,1) (2,0)            |
| 2            | (0,2) (1,2) (2,1) (3,0)      |
| 3            | (2,2) (3,1) (3,2)            |

**Figure 6:** Permutations for Belief 1 Based on the Rubric of Rubrics

Figure 6 shows that for Belief 1, there were two items, S3.2 and S3.3. Different permutations of scores on these items result in different belief scores. A teacher with a (3,0) combination showed strong evidence of Belief 1 on item 3.2 but no evidence of Belief 1 on item 3.3. Using the instrument's two-item rubric of rubrics, these two scores indicate overall *evidence*, neither strong nor weak, that the teacher holds Belief 1 (final score of 2).

Philipp provided me with the training materials for using the IMAP Beliefs Survey which I used to calibrate my scoring until I became more proficient in the use of the instrument. I scored each teacher's responses and calculated final scores regarding the degree of evidence for each belief by teacher. I used the numeric scores to search for

patterns when comparing beliefs scores with knowledge (MKT) and instruction (MQI) scores. I returned to participants' written responses when I sought explanations for certain patterns and relationships. The combination of using both quantitative and qualitative methodology helped to create a thorough picture of the role teachers' beliefs played in their mathematics instruction.

### **Semi-structured interviews: beliefs and policy interpretation.**

Guided by sensemaking theory, I conducted two semi-structured qualitative interviews of each teacher. See Appendix B for the interview protocol. My goal was to obtain the voice and perception of the participants about their experiences working in their current school contexts. Questions included a focus primarily on teachers' experiences with curriculum and assessment policies vis-à-vis their instruction and their beliefs about teaching and learning mathematics. The interviews were the main instrument in the study that specifically focused the sense teachers made of policies. They also served as a complementary source of information to the IMAP Beliefs Survey. I conducted the interviews immediately after each teacher taught a math lesson so reflections on instruction and on the role of instructional resources and policies would be contextualized in a current activity. Because I planned to code and categorize the content of the interviews, I asked many clarifying questions that made meanings clear with regard to the categories to be used (Kvale, 1996). Inherent in the interview design was my goal to gather uninterpreted descriptions of participants' perceptions of their lived experiences (Kvale, 1996) as mathematics educators.

### **Interview data reduction and analysis.**

Prior to using any structured analysis techniques, I transcribed the interviews, listening to the voice and stories of the teachers in a holistic sense (Tobin, 1989). I kept anecdotal records of my impressions and of key topics that arose in the interviews. Next, using HyperResearch (Version 2.7) software for qualitative analysis, I coded the interviews using a priori coding (Kvale, 1996) based on the theories underlying the study's research questions. While the interviews were designed to bring out teachers' perspectives on policies and to tap into their beliefs, I included codes for different aspects of teacher mathematical knowledge for teaching, for their beliefs about different aspects of teaching and learning mathematics, and about the sense they made of curriculum and assessment policies. However, as I read and coded the interviews, the need for additional codes emerged and some of the a priori codes were unnecessary. For example, I expected different aspects of teachers' MKT to show up in their statements as they reflected on their lessons. However, evidence of teachers' knowledge was rare therefore I collapsed the knowledge codes into one category. Table 5 displays the final list of codes which were needed to categorize the teachers' statements within the study's parameters:

**Table 5:** Semi-Structured Interview Codes

| Code Categories | Codes                                                                                                                                                                                  |
|-----------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Knowledge       | Mathematical knowledge for teaching                                                                                                                                                    |
| Beliefs         | IMAP Belief 1, 2, 3, 4, 5, 6, 7<br>How to teach math<br>How students learn math<br>The subject of math                                                                                 |
| Policy          | Assessment<br>Curricular materials<br>Pacing<br>Standards<br>Time                                                                                                                      |
| Other           | Background info<br>School context<br>Perspective on math instruction<br>Professional development<br>Ideal lesson<br>Differentiation<br>Other influences<br>Role of parents<br>Students |

The first way I used the coded interview data was to pull out the evidence of each of the IMAP beliefs. I analyzed the statements qualitatively, determining whether the teacher's words supported or weakened the profile of beliefs established by the IMAP Beliefs Survey. I did not attempt to quantify the data by counting occurrences of words or of statements related to the beliefs. A single statement could hold as much power in validating or undermining an IMAP result as could five similar statements.

I also used teachers' statements to analyze their interpretation of district policies. Again, using HyperResearch, I extracted the statements and examined them by teacher. I characterized their interpretations based on a holistic reading of their statements. In this

way, I was able to establish what curricular materials each teacher believed they were expected to use, who they perceived themselves accountable to regarding curricular policies, how they interpreted policies on textbook implementation, who they felt accountable to for student performances and what that accountability looked like, and the role they perceived assessment and accountability played in their curricular and instructional decisions. Additionally, I compared these interpretations within and across school sites to better understand the role social interaction and school context played in the sense they made of institutional policies.

As will be described in Chapter Four, the textbook used by a teacher revealed a lot about the role of teacher beliefs, social interactions, and school context in curricular decisions. Interview data, in combination with the IMAP results, provided a means for me to examine the interaction of these factors. By identifying the core philosophical ideals forwarded in a textbook, I established a framework against which I could compare each teacher's beliefs. I took into account all belief statements a teacher made in the interviews and the belief profile established in the IMAP Beliefs Survey to determine to what degree a teacher's beliefs aligned with the textbook they were using and the one they were not using (two programs were in place in SUSD). Each teacher's alignment with the programs was rated as no alignment, weak alignment, partial alignment, alignment, or strong alignment. Details of this rating system are detailed in Chapter Four. I used qualitative data display techniques (Miles & Huberman, 1994) to organize the belief/text alignment data by school, teacher, and program used. As a result, I was able to identify patterns, such as teachers whose instruction mirrored their textbook but

their beliefs were not aligned with it. In these cases, given the muted role beliefs appeared to be playing in mathematics instruction, the need arose to inquire into what other factors might be mediating curricular materials and instructional practice. I was able to then return to the quantitative data for within and cross-case variable analyses to search for other relationships that might provide an explanation. Then I used lesson and interview transcripts as well as knowledge data to search for explanations.

### **Limitations**

While this study will add new data and ideas to theory about factors contributing to the generation of mathematics instruction, its generalizability is limited by its small sample size. That said, in the qualitative portion of the study, the goal is to generalize to theory, not to populations. I believe the study contributes important information which expands the theory on factors influencing classroom mathematics instruction. Moreover, the findings from this study could inform a larger, multi-year, funded study.

Another limitation of the study is its focus on instruction but not on student learning. Because the study already was large in scope for a dissertation project, I chose to focus on instruction and explore the connection to student learning in a follow-up study. I relied on instructional quality/student learning connections found in other studies (Hill et al., 2005; Kersting et al., 2009) to justify placing my focus on instruction only.

The location of the study in a high income, high performing school district also could draw criticism. It could be argued that institutional pressures may be felt differently in SUSD than in schools in different economic contexts and under state program

improvement mandates. However, the study did include variety in school contexts. Not all schools in SUSD fit the overall district profile.

Finally, findings from this study could be scrutinized because I was the only scorer on the CVA, MQI, and IMAP Beliefs measures. While I spent extensive amounts of time (approximately 20 hours per instrument), going through training materials and trying to score data the way the instrument designers intended, without calibration with other scorers there is a chance that others would score my data differently. I did my best to minimize this possibility and feel confident that my scores were well calibrated with the extensive examples and practice data provided in the instrument training manuals.

### **Summary**

In this study, I used mixed methods to examine how teachers' knowledge, beliefs, policy interpretation, and context-specific factors interact in teachers' expression of mathematics instruction. The purpose of the study was to build an explanation and expand current theory. I specifically collected data about teachers' knowledge for teaching mathematics, beliefs about teaching and learning mathematics, and the sense they make of curriculum and accountability policies. I also measured the quality of teachers' mathematics instruction. The study was guided by a situative perspective, bringing the role of institutional and social contexts into focus. In analyzing the data, I used an integrated variable-oriented and case-oriented approach (Yin, 2009).

The study was conducted in the Sunrise Unified School District, a high performing suburban district in Southern California. The sample consisted of seven

teachers from fourth and fifth grades in four different schools representing varying school contexts.

In order to measure teachers' knowledge specific to the act of teaching, I used two instruments: the Mathematical Knowledge for Teaching (MKT) Assessment (Hill et al., 2006) and the Classroom Video Analysis (CVA) instrument (Kersting et al., 2009). Both instruments provided quantitative and qualitative data, ensuring triangulation and rich information about teachers' knowledge. For measuring teachers' beliefs, I used the IMAP Beliefs Survey (Philipp & Sowder, 2003) coupled with semi-structured interviews. Through these measures I was able to develop a profile of each teacher's beliefs about teaching and learning mathematics. I collected information about how teachers made sense of curriculum and assessment policies through two semi-structured interviews. During the interviews, participants also provided information about their team, school, community, and district contexts. Through the combination of quantitative and qualitative data collection and analysis techniques, I developed a profile of the sample group, a profile of each teacher, and I identified relationships between variables across teachers.

In Chapter Four, I will describe the knowledge, beliefs, instructional quality, and interpretation of curriculum and assessment policies at a group level. In Chapter Five, I will explain findings from the cross-case comparison of variables followed by a within case analysis of each participant. The analysis in both chapters contributes to building an explanation for the complex interaction of the variables under study.

## **CHAPTER IV: EXAMINING GROUP CHARACTERISTICS**

### **Overview**

The content and quality of mathematics instruction in classrooms across the United States plays a critical role in the resulting mathematical literacy of students (Hill et al., 2005; Kersting et al., 2009). In this chapter, the overall quality of instruction demonstrated by the study's seven participants is broken down to describe characteristics common to this group of teachers. Additionally, the teachers' overall knowledge, beliefs, and the sense they make of curriculum and accountability policies are explored at a group level. In Chapter Five, meaningful differences between the teachers are explored to reveal the interplay between personal factors and environmental factors in the generation of mathematics instruction.

This chapter begins with an analysis of the overall quality of the participants' instructional practice and transitions into key domains identified within the Mathematical Quality of Instruction (MQI) measure (Hill, 2010). As described in Chapter Three, the domains highlighted within the MQI framework are: 1) Duration of different formats and modes of instruction, 2) Richness of the mathematics, 3) Working with students and mathematics, 4) Errors and imprecision, and 5) Student participation in meaning-making and reasoning. The rationale behind the MQI domains can be traced to current research-based goals and recommendations for classroom mathematics instruction. The National Council of Teachers of Mathematics (2000, 2006) recommends all mathematics content in K-12 classrooms should be experienced by students in the context of key mathematical processes: problem solving (Schoenfeld, 1989, 1992), reasoning and proof (Ball and

Bass, 2003; Harel, 2008a; Lampert, 1990), communication (Bransford, 2000; Cobb, Wood, & Yackel, 1993; Hatano, 1991; Lampert, Rittenhouse, & Crumbaugh, 1996), connections (Ma, 1999), and representations (Brenner, Godman, & Vye, 1997). Engagement in these processes contributes to the development of mathematical understanding and principled mathematical reasoning (NCTM, 2006; Schoenfeld, 1989). Mathematical knowledge developed exclusively through the memorization of facts or procedures without conceptual development and engagement in mathematical processes is likely to be fragile (NCTM, 2006).

### **Mathematical Quality of Instruction**

Across the 21 observed lessons, classroom instruction varied in quality with some lessons containing elements of instruction that promoted mathematical reasoning while a larger portion focused on step by step procedural recall. Using the Mathematical Quality of Instruction instrument (Hill, 2010), the instructional quality of a lesson is sorted into one of three different quality levels depending on its overall characteristics. Figure 7 shows the rubric for the overall MQI lesson score. The lessons in this study all fell within the low and mid-quality levels. As will be discussed in Chapter Five, the lack of high quality instruction tended to reflect the mediational role of the institutional context more than it did the knowledge and beliefs of individual teachers.

| Overall MQI                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                                                                                                                 |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Low                                                                                                                                                                                                                                                                                                                                                                                                   | Mid                                                                                                                                                                             | High                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| Instruction is characterized by combinations of the following: <ul style="list-style-type: none"> <li>• Systematic errors (mathematical errors, in notation, in language)</li> <li>• Student confusion or misunderstandings</li> <li>• Unproductive teacher-student interactions around the content (e.g., teacher cannot understand student utterances)</li> <li>• Lack of directionality</li> </ul> | Instruction is relatively error-free but lacks the mathematical richness, appropriate use and discussion of procedures, and the sharp mathematical focus detailed under “high.” | Instruction is characterized by combinations of the following: <ul style="list-style-type: none"> <li>• Mathematical richness in terms of explanations, generalizations, use and connections of representations or multiple solutions</li> <li>• Appropriate use and discussion of procedures as well as of their applicability</li> <li>• Instruction is mostly error-free and has a clear and sharp mathematical focus and directionality that allows students develop the important mathematical ideas under consideration</li> <li>• Instruction is also characterized by positive teacher-student interactions around the content</li> </ul> |

**Figure 7:** MQI Overall Lesson Quality Rubric Levels

Fourteen out of the twenty-one lessons, 67%, fit into the “low” overall MQI level.

However, the descriptors in the rubric need elaboration to truly capture what was observed in those classrooms. For example, within the rubric, two of the possible characteristics of low quality instruction are systematic mathematical errors and student confusion. These characteristics do not directly apply to the fourteen observed lessons categorized as low-quality. It was uncommon for the teachers to make mathematical errors, and students did not display confusion as they applied step-by-step procedures. While it is fortunate that the lessons were easy for students to understand, the absence of rich math content and the absence of exploration of the mathematical ideas underlying

the procedures is problematic. The fact that these critical qualities of rich mathematics instruction were missing must be considered when classifying the instruction as low, mid, or high quality.

Another characteristic of low mathematical quality of instruction is “unproductive teacher-student interaction around content” (Hill et al., 2006, pp. 22). The teacher-student interactions in the observed lessons were productive in the sense they were characterized by reinforcement of the mathematical procedures forwarded by the teachers. Teachers asked questions such as, “What is the next step?” or “What number should go here?” or “What is the correct answer?” Most students met the lesson’s procedural performance objectives. However, the interactions were unproductive in supporting student reasoning about the underlying mathematics of the procedures being practiced.

The final characteristic of lesson-level low quality mathematical instruction is lack of directionality in the lesson. The fourteen low-quality lessons typically included one to two skills such as rounding whole numbers and decimals and writing numbers in standard and expanded forms. Each lesson showed some evidence of periodic directionality, but none showed whole-lesson directionality in terms of research-recommended mathematical understanding goals.

Seven of the twenty-one observed lessons, 33%, fit into the mid-quality-level of mathematics instruction on the MQI rubric. Like the lower-quality lessons, these were relatively free of mathematical errors. They differed from the lower-quality lessons in that the teachers provided more complex tasks and encouraged more than one path to a

solution. These differences allowed for higher quality teacher interactions with students. Their interactions with students were similar to the lower quality lessons in that they were focused on getting a single correct answer but they were different in that students were encouraged to explain why they chose particular ways to solve problems. In this sense, the productivity of the interactions was not focused only on procedures but on why particular procedures were selected, and on rare occurrences, mathematically why a procedure worked. Because the cognitive demand of explaining their thinking was greater, the students in the mid-quality lessons actually had more confusion than those in the lower-quality lessons. However, once students worked through an explanation, the clarity of mathematical understanding they gained appeared greater than those who just replicated procedures. Like the lower-quality lessons, in the mid-quality lessons there were many ideas incorporated into a lesson and there was directionality within each segment. However, these lessons also lacked a cohesive, well-connected mathematical focus.

None of the lessons observed contained high-level rich mathematics instructional characteristics. The key missing characteristics were generalization of mathematical ideas and explicit connections between mathematics representations, concepts, and procedures. They also lacked a sharp math concept focus.

While the application of the overall MQI rubric provides an initial characterization of the observed instruction, analysis of instruction within the MQI domains provides more fine-grained information. Additionally, by examining the sub-

domains comprising each domain, a more specific accounting of the quality characteristics of the observed mathematics instruction emerges.

### **Modes of Instruction**

Using the MQI duration codes to measure instruction, interaction, and tasks occurring in the seven sample classrooms, common “modes of instruction” (Hill, 2010) come into view. Table 6 displays the MQI duration scores characterizing the sample group’s instructional modes.

**Table 6:** MQI Modes of Instruction Scores for Observed Lessons (n=21)

| Mode of Instruction         | M (SD)      | Range (1-3) |
|-----------------------------|-------------|-------------|
| Direct Instruction          | 2.83 (0.35) | 1.81-2.88   |
| Whole Class Discussion      | 1.05 (0.06) | 1.00-1.17   |
| Working on Applied Problems | 1.33 (0.52) | 1.00-2.20   |

Several statements can be made using the data in Table 6. First, direct instruction, where the teacher controls the delivery of the mathematical content, is the primary mode of instruction. Second, whole class discussions, where students interact with one another’s ideas, are rare. And third, the mathematical tasks given to students are most often lacking applied contexts.

Students in these classrooms typically stay in their seats the whole period, listen to the teacher guide them through rules governing how to work certain problem types, provide answers or solution methods when the teacher calls on them, listen to the teacher

evaluate their responses, and do context-free practice problems heavily guided by the teacher. While the seven mid-level lessons provided some instruction contrary to these generalizations, overall these characteristics describe central tendencies of the observed lessons.

### **Direct Instruction**

Overall, direct instruction is the primary mode of instruction of the participants in this study. According to the Mathematical Quality of Instruction (MQI, 2010) instrument, direct instruction is defined as the teacher controlling the delivery of mathematical content. It includes high amounts of teacher talk and low amounts of student talk or student practice. As Table 6 shows, using the MQI's scale of 1(low) - 3 (high) to identify the duration of instruction for each seven minute segment in a lesson, the mean duration of direct instruction for the sample group was 2.53 with a range of 1.81 - 2.88. A 1 means no direct instruction occurs, a 2 means direct instruction occurs for only part of the segment, and a 3 means direct instruction occurs for the majority of the segment. Combining the segments comprised of almost all direct instruction with portions of those where direct instruction makes-up part of the segment, it is reasonable to estimate that 75% of all instruction occurring in the sample classrooms is direct. This disproportionate amount of direct instruction focused on skill acquisition is contrary to research on how children develop mathematical understanding (Bransford, Goldman, & Vye, 2005; Fennema et al., 1996; Hiebert & Stigler, 2000; Jacobs et al., 2009; Lampert, 1986; Schoenfeld, 1989).

## **Whole Class Discussions**

Predictably, with the majority of lessons filled with teacher talk, the absence of whole-class discussions around mathematics was noteworthy in the 21 lessons observed. According to Hill (2010, p. 4), as cited in the MQI Manual, whole class discussion means “‘The teacher is in charge of the class, just as in direct instruction. However, the teacher is not primarily engaged in delivering information or quizzing. Rather, he or she has students share their thinking, explain the steps in their reasoning, and build on one another’s contributions. ... [This mode of instruction] gives students the chance to engage in sustained reasoning’” (Chapin et al., 2003, p. 17 as cited in Hill, 2010). A key feature of whole-class discussions is student interaction with one another’s ideas. Students listen and respond to the mathematical contributions of their peers; they do not just list different ways they solved problems. As noted previously, the low-level lesson scores in this domain represent lessons which contained no whole-class mathematical discussion. The mid-level lesson scores represent lessons which contained student explanations of why a procedure was chosen but few instances of students responding to one another’s’ ideas.

Using the MQI duration scale of 1 (none) – 3 (most or all) indicating the amount of a segment during which whole-class discussion occurs, Table 6 shows the group average of the 21 lessons in this study was 1.05 with a range of 1.00 – 1.17. The low mean and narrow range for whole-class discussions indicates it was rare for students to publically interact with one another’s ideas in the observed lessons. Discourse in all classrooms demonstrated the Initiate-Respond-Evaluate (Mehan, 1979) pattern where the teacher initiates a question, a student responds to the teacher, and the teacher evaluates

whether the student's contribution is right or wrong. This pattern of classroom discourse is not reflective of current recommendations that students should listen to and respond to one another's mathematical ideas (Bransford, 2000; Cobb et al., 1993; Hatano, 1991; Lampert, 1998; Lampert et al., 1996). In turn, it is not supportive of the development of mathematical understanding and reasoning.

### **Working on Applied Problems**

Critical to the initiation of whole-class discussions are tasks with embedded mathematics worth discussing (Hiebert et al., 1997; NCTM, 2006). The tasks students most often engaged in across the sample classrooms were context-free although six out of the twenty-one lessons, 29%, did contain some applied problems. Hiebert et al. (1997) define worthwhile tasks as those where the path to an answer is not immediately apparent and the mathematics is intellectually intriguing. The applied tasks observed in this study were often typical textbook word problems where there is a single answer and often a pre-designated way to solve them. If the teacher encouraged multiple solutions and led discussion about the mathematics in the problem, the problems were given credit as being "applied." Using the MQI duration scales of 1 (none) – 3 (most or all), as Table 6 shows, the average duration of work on applied problems across the 21 lessons was 1.33 with a range of 1.00 – 2.20 and a median of 1.06. The low mean coupled with the low median reflects the five classrooms in which tasks were consistently context free. The high end of the range reflects the two classrooms in which students sometimes worked applied problems.

### Richness of the Mathematics

The average richness of the mathematics in the instruction of the 21 observed lessons fell into the low quality range, averaging a 1.31 on the MQI's 1(low) -3 (high) scale. The richness of the mathematics is measured according to the subscales comprising the construct. The subscales and resulting group scores are displayed in Table 7.

**Table 7:** MQI Richness of Mathematics Scores for Observed Lessons (n=21)

|                                     | M (SD)      | Median | Range (1-3) |
|-------------------------------------|-------------|--------|-------------|
| Overall Richness of the Mathematics | 1.31 (0.22) | 1.26   | 1.10 – 1.76 |
| Linking or Connections              | 1.11 (0.17) | 1.26   | 1.00 – 1.49 |
| Explanations                        | 1.39 (0.37) | 1.25   | 1.00 – 2.12 |
| Multiple Procedures or Solutions    | 1.17 (0.23) | 1.08   | 1.00 – 1.63 |
| Developing Math Generalizations     | 1.03 (0.06) | 1.00   | 1.00 – 1.13 |
| Mathematical Language               | 1.85 (0.40) | 1.62   | 1.39 – 2.46 |

### Linking and Connections

The first subscale within the MQI's framework contributing to the overall richness of the mathematics in a lesson is *Linking or Connections*. This code focuses on the explicit connections made by teachers and/or students among different mathematical representations and/or ideas. Understanding the interconnectedness of mathematical ideas was noted in Ma's (1999) study as a weakness in the knowledge levels of American

teachers which could account for their students' low performances on international math assessments. A low score (1) on this subscale means either no connections are made or those made were pro-forma. A mid-level score (2) means some connections are made but they are not consistently high quality. A high score (3) means links and connections are consistently made and sustained through the lesson. These connections include detailed discussion of how two or more mathematical ideas are related, explicit links between a representation and a mathematical idea, and explicit links showing the correspondence between multiple representations. As shown in Table 7, the average linking or connections score across the 21 lessons was 1.11. This low average and the lesson range of 1.00 – 1.49 are indicative of the infrequent presence of mathematical connections in the lessons.

### **Explanations**

According to Franke (2011), student learning gains in mathematics are positively associated with the amount of mathematical explanations students express in classroom lessons. Explanations by the students and/or the teachers occurred more frequently than connections, but were typically characterized as low quality. Most frequently, the explanations were provided by the teachers rather than by the students. Lesson segments were scored on a 1- 3 scale: 1 meaning that no explanations occurred or they occurred but they were strictly procedural steps; 2 meaning explanations contained elements of generalizability and understanding *why*.; 3 meaning the explanations are detailed, move beyond particular problems in their generalizability, and they give meaning to steps and procedures. As Table 7 displays, the mean scale score for the explanation code across the

21 lessons was 1.39 with a range of 1.00 – 2.12. The mean indicates a high percentage of the lesson segments contained procedural explanations of *how* to solve problems while a much smaller percentage of the lesson segments contained explanations addressing *why* aspects of mathematics.

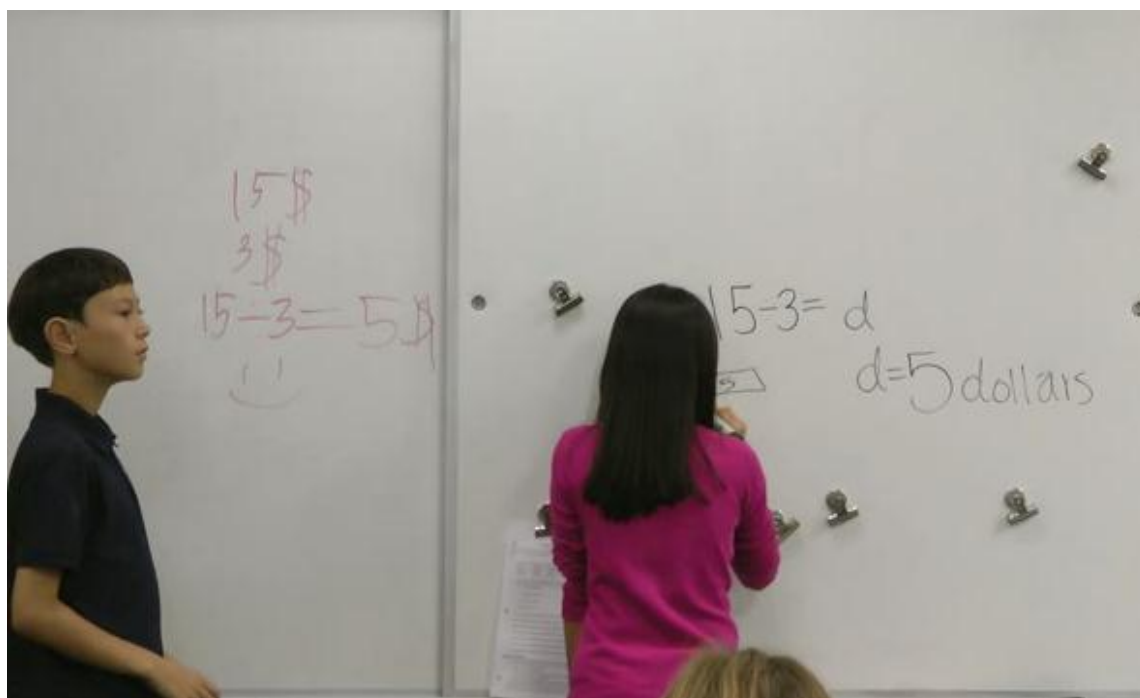
### **Multiple Procedures or Solution Methods**

Not surprisingly, in many of the observed classrooms where direct instruction was dominant and where teacher explanations focused on how to follow procedures to solve problems, instances of students grappling with problems and exploring multiple procedures or solutions were minimal. According to current research and recommendations regarding children's development of mathematical understanding, children need opportunities to explore different methods for solving mathematical problems and to develop perseverance in their attempts (Carpenter, 1999; National Governors' Association, 2010). Likewise, it is important for students to see the solutions of others and to be able to comparatively analyze the methods. The opportunity to generate different methods of solving problems allows students to develop knowledge structures likely to support the mathematical understandings they are constructing (Carpenter, 1999).

On the MQI's 1(low) – 3 (high) scale for multiple procedures or solutions, the observed lessons scored an average of 1.17. The range was from 1.00 – 1.63 demonstrating some occurrences of mid and high level performances but an overall low quality result. A 1 on the scale indicates there is no evidence of multiple solution methods being considered. A 2 indicates more than one method may have been

considered in the lesson but comparison between methods remained surface-level. A 3 indicates multiple procedures or solution methods were presented and accompanied by explicit discussions and comparisons.

For the purpose of clarifying this construct, Figure 8 shows an instance where the teacher asked two students to solve a problem on the whiteboard at the front of the class. The ensuing dialogue is categorized as mid-quality instruction as the teacher facilitates the sharing of multiple strategies, highlights what each strategy means, and presents another strategy but it is not scored as high-quality as she does not explicitly help students compare the strategies for efficiency, clarity, accuracy, and/or ease of use. To be fair, she may not have done a comparison because both students generated a similar strategy. The problem they are working is: *Anna has \$15 in the bank. Her sister Bonita has one-third as much money. How much money does Bonita have in the bank?*



**Figure 8:** Two Students Showing their Solutions to a Problem

In this image, the students have similar approaches to representing the mathematical relationships in the problem. They both divide Anna's fifteen dollars by three to get  $d$ , the amount of dollars Bonita has in the bank. The transcript below illustrates how the teacher guides an analysis of the two solutions by focusing on what the numbers mean in relation to the problem. The teacher also adds another solution strategy to consider how multiplication can be used instead of division. In the transcript, Student 1 is the student on the left and Student 2 is the student on the right.

**Student 1:** Well since it said one-third I guessed it would be dollars so then I wrote 15 divided by 3 equals 5 and a smiley face.

**Teacher:** What is your answer?

**Student 1:** Five dollars.

**Teacher:** Five dollars. And where does the dollar sign go when you're writing 5 dollars?

**Student 1:** On the other side.

**Teacher:** Yes. So tell me where the three dollars came from.

**Student 1:** Well since it's one-third I sort of knew already in my head.

**Teacher:** Ok. Add to what he's telling us while he's fixing his five dollars. Student 2, tell us what you did.

**Student 2:** I did 15 divided by 3 since Anita has 15 dollars and Bonita had one-third as much as her so I just knew that it would be a three here also? And then it equals  $d$  and then I drew this picture and I did 3 boxes and then that had to equal 15 so I did 5 in each one and then B had to be 5. So the answer is 5 dollars.

**Teacher:** Why did B only get one box?

**Student 2:** Because she had one-third as much as she did and she had 3.

**Teacher:** Ok. Alright. So where'd the 3 come from now that you're thinking about it some more, Student 1? The 3 dollars?

**Student 1:** It came from the one-third.

**Teacher:** It came from the one-third. Is that dollars or is that just 3? I think it might just be 3. What would I do if I wanted to write this as a multiplication equation? They both divided which I think is what naturally

a lot of us would do, but how would I write it in a multiplication equation?  
Student 3.

**Student 3:** 3 times d equals 15?

**Teacher:** Ok. 3 times d dollars equals 15. Because you knew that should be one-third. Is there another way to write it? Student 4?

**Student 4:** Fifteen times one-third equals d.

**Teacher:** Equals d dollars. Now multiplying by one-third is the same as dividing by 3, which is why you guys, Student 1 and Student 2, came up and intuitively knew that they just needed to divide by 3. (Classroom Observation, 9/24/10)

In this exchange between teacher and students, multiple solutions to the same problem are highlighted:  $15 \div 3 = D$  and  $15 \times 1/3 = D$ . The intentional elicitation of and attention to multiple solutions exemplifies a mid-level score as opposed to the majority of the lessons observed where only one way to solve each problem is considered.

### **Math Generalizations**

While the consideration of multiple solutions was infrequently present in the observed lessons, the development of math generalizations was present even less frequently. The MQI defines this code as “the teacher and/or the students develop mathematical generalizations by examining instances or examples, then making a general statement” (Hill, 2010, p. 9). Developing math generalizations is a key aspect of developing principled reasoning in mathematics (Harel 2008a; Schoenfeld, 1989). Yet, in the observed lessons, there were only 4 out of 163 seven-minute segments, 2%, in which students participated in developing mathematical generalizations. The low occurrence of this essential aspect of instruction is reflected by a 1.03 mean score on the MQI with a narrow range of 1.00 – 1.13. A 1 on this scale means no math generalizations existed in the segment. A 2 means generalizations are developed but are

not clear or complete. A 3 means math generalizations are developed, capture the essence of the mathematics under study, and are complete and clear. In the observed classrooms, the four instances of generalizations were not clear or complete.

### **Mathematical Language**

One positive aspect of the observed lessons' mathematical richness was teacher and student use of mathematical language. The mathematical language code refers to "fluent use of technical language, explicitness about mathematical terminology, and supporting students' use of mathematical terms" (Hill, 2010, p. 10). A segment scoring a 1 is characterized as the teacher using non-mathematical terms to describe mathematical ideas, being sloppy with terminology, or using little to no mathematical language. A segment scoring a 2 is characterized by the teacher accurately using mathematical language to convey content. A high-level score, 3, is differentiated from a mid-level score by the teacher's fluent and dense use of mathematical language as well as the intentionality with which he/she presses students to accurately use mathematical terminology. The lessons observed in this study had an average mathematical language score of 1.85 with a range of 1.39 – 2.46. Together, these scores represent a mid-quality central tendency of mathematical language used in the classrooms. Relative to the other richness of mathematics subscales, the use of mathematical language is a strength for the teachers observed.

### **Working with Students and Mathematics**

Similar to the low richness of mathematics scores, the observed lessons demonstrated low quality scores in the ways the teachers worked with students to develop

mathematical understanding. This domain is critical as the quality of interactions between the teacher and students around mathematical ideas is positively related to generative instruction and to student learning (Franke, Loef, Carpenter, Levi, & Fennema, 2001; Hill et al., 2005; Kersting, personal communication, June, 2010).

According to Hill et al. (2010),

This category captures whether teachers can understand and respond to students' mathematically substantive productions (utterances or written work) or mathematical errors. By mathematically substantive productions, we mean questions, claims, explanations, solution methods, ideas, etc. that contain substantial mathematical ideas. By students' mathematical errors, we mean those incorrect student productions that offer opportunities for discussing and addressing pertinent mathematical ideas. (p. 12)

The MQI breaks the domain of *Working with Students and Mathematics* into two subscales: 1) remediation of student errors and difficulties and 2) responding to student mathematical productions in instruction. Table 8 displays the overall domain and sub-domain scores for the teachers' work with students across all observed lessons.

**Table 8:** MQI Overall Working with Students and Mathematics Scale Scores for Observed Lessons (n=21)

|                                                               | M (SD)      | Median | Range (1-3) |
|---------------------------------------------------------------|-------------|--------|-------------|
| Working with Students and Mathematics                         | 1.24 (0.24) | 1.15   | 1.00 – 1.70 |
| Remediation of Student Errors and Difficulties                | 1.22 (0.19) | 1.19   | 1.00 – 1.60 |
| Responding to Student Mathematical Productions in Instruction | 1.27 (0.31) | 1.15   | 1.00 – 1.80 |

As Table 8 shows, the average score for the dimension of Working with Students and Mathematics is 1.24 on the MQI's 1-3 scale. This mean score is reflective of lessons where students make infrequent errors but when they do make errors, teacher remediation is light and procedural.

### **Remediation of Student Errors**

In the 21 lessons observed, the procedural remediation of student errors did not appear to help students understand the underlying mathematics in the problems they worked. For example, in the following transcript, a student raises his hand to ask for help with rounding a number and the teacher is providing support.

**Teacher:** Now, what number did it say to round?

**Student:** Three

**Teacher:** There we go. (He underlines it on the paper) Now, when we round, we're going to round that three, which number do we look to?

**Student:** The right.

**Teacher:** The right. And that is a . . . (Teacher draws arrow to number on right)

**Student:** Two.

**Teacher:** Is that bigger or less? . . . (Observation, 1 Oct. 2010)

In this transcript, the teacher guides the student in a step-by-step procedure to round a number. The questions he asks require procedural recall and do not support the student in developing a sense about the numbers. The teacher does not ask the student about her understanding of how to round a number nor does he draw a number line to help the student conceptualize what rounding means.

Reflective of the typicality of the example interaction around rounding rules, the lessons in this study had an average *Remediation of Student Errors and Difficulties* score of 1.22 on the MQI scale of 1- 3. A 1 on this subscale indicates students did not make any errors (indicating lack of rigor in the mathematics) or the teacher's remediation is brief, procedural, and/or confusing. A 2 on this subscale indicates the teacher occasionally remediates student errors and the remediation, although procedural, is extensive. A 3 on this subscale indicates the teacher remediates student errors conceptually and at length. The range of remediation scores by teacher were 1.00 – 1.60 indicating some instances of mid-level remediation but an overall lesson characterization of either no student errors due to low cognitive demand or brief procedural remediation.

### **Responding to Student Math Productions in Instruction**

Teacher responses to student thinking appeared to guide students through prescribed procedures rather than capitalize on and encourage student thinking. The observed lessons had an average *Responding to Student Mathematical Productions in Instruction* score of 1.27 with a maximum at 1.80. Current research emphasizes the student learning value of the teacher being able to understand the mathematics behind a student's thinking and being able to respond in the moment with the appropriate next question or idea for that student (Franke et al., 2001; Jacobs et al., 2009a, 2009b). This type of responsiveness to student thinking in instruction did not occur in the observed lessons. A typical example of the low quality response most teachers in this study had to student thinking is displayed in the transcript below. In the interaction, the teacher is in the middle of providing direct instruction to the whole class on how to rename fractions

as decimals. She explains to students they need to rename the fraction so that its denominator is a power of ten by multiplying the numerator and denominator by two. One student becomes inquisitive about the possibility of dividing to get a power of ten. Her question shows she is thinking about the inverse relationship between multiplication and division. It also shows her interest in inquiring beyond the stream of information the teacher is directing at the students. Instead of making the student's idea part of instruction, the teacher acknowledges the idea but does not allow it to divert class attention away from the procedure she is presenting.

**Student 1:** So when we're converting fractions into decimals, we don't divide? Do we ever divide?

**Teacher:** You can. Yes, but I want to teach this step first to make sure that you understand this method because this method won't always work but this is a good first method to try.

**Student 1:** So you try multiplication first and if that doesn't work, then you go to division?

**Teacher:** Then we'll cover that, yes. You're just too smart, you're jumping ahead of me. (laughs) Okay, I am going to give you three problems that I'd like you to work out on your own. Okay? ( $\frac{4}{20}$   $\frac{1}{2}$   $\frac{3}{25}$ ) Please do those three on your own.

**Student 2:** Can you divide?

**Teacher:** Not yet. I want you to use this method right now. Eventually we will get to the point where you can choose whichever method is best for you. But since we are focusing on this specific method today, I want you to try it with this method. (Observation, 9/27/11)

Several statements can be made about the teacher's decisions in this interaction.

First, she has a clear idea of the procedure she wants students to master – *multiply* to get a power of ten in the denominator. Second, she does not deviate from her pre-determined lesson path. Third, the student is being sent a negative message about

thinking beyond the lesson parameters. One of the fractions given to the students,  $\frac{4}{20}$ , can clearly be simplified to  $\frac{2}{10}$  using division, which is an easy fraction to write as a decimal, 0.2. While the teacher states they will learn different methods later, when the student asks about using division to simplify fractions again the next day, the teacher does not model the inverse operation for simplifying fractions. Instead, she models using long division, dividing the numerator by the denominator, to get a decimal. She does not help the student see the reciprocal relationship between multiplication and division, she does not have the student compare strategies, and she does not acknowledge the possibility of simplifying a fraction using division. It is unclear if the teacher understands that the student is referring to using division to simplify a fraction as a step in converting it to a decimal. However, understanding a student's thinking and connecting it to the math at hand is a critical aspect of the *Responding to Student Math Productions in Instruction* construct. This example demonstrates the typical low-quality, procedural response to student thinking that was a tendency of the observed lessons.

### **Errors and Imprecision**

Within the observed lessons, mathematical errors and imprecision in instruction were uncommon. Hill (2010) defines this domain as “teacher errors or imprecision in language and notation, uncorrected student errors, or the lack of clarity/precision in the teacher's presentation of the content” (p. 16). Table 9 displays high scores (reverse scoring adapted from the MQI manual for clarity) meaning errors were infrequent.

**Table 9:** MQI Overall Errors and Imprecision Scale Scores for Observed Lessons  
(n=21)

|                                                                 | M (SD)      | Median | Range (1-3) |
|-----------------------------------------------------------------|-------------|--------|-------------|
| Overall Errors and Imprecision                                  | 2.88 (0.13) | 2.91   | 2.60-3.00   |
| Major Mathematical Errors or<br>Serious Mathematical Oversights | 2.94 (0.08) | 2.96   | 2.80 – 3.00 |
| Imprecision in Language or Notation                             | 2.80 (0.21) | 2.85   | 2.35 – 3.00 |
| Lack of Clarity                                                 | 2.86 (0.29) | 2.97   | 2.22 – 3.00 |

**Major Mathematical Errors or Serious Mathematical Oversights; Imprecise Use of Mathematical Language; Lack of Clarity**

For the purpose of clarifying the error construct, an example of a *Major Mathematical Error* (as it occurred in several of the observed lessons) is a teacher referring to the simplification of fractions as “reducing fractions.” This is an error because the value of the number is not getting smaller or being “reduced” when it is renamed. For example, simplifying  $\frac{2}{4}$  to  $\frac{1}{2}$  is really just writing an equivalent fraction. Using a term that indicates a reduction in the value represented by  $\frac{2}{4}$  is misleading and creates conceptual confusion among students. An example of *Imprecise Use of Mathematical Language or Notation* is calling a right angle a “little square” or calling the denominator of a fraction the “bottom number.” The subscale *Lack of Clarity* refers to vague, incomplete, or muddled presentation of the mathematical content which severely distorts the essence of the mathematics (Hill, 2010). An example of muddling the content arose in one of the observed lessons when the teacher was having students use pencils to

represent geometric figures. She intended for the pencil to represent a line segment but she confused the students by selecting a sharpened pencil which they interpreted as the arrow indicating it goes on indefinitely. The more she tried to explain the physical finiteness of the material, the more muddled and abstract it became until the mathematics was lost in the confusion. Overall, however, major errors, imprecise language, and lack of clarity were uncommon occurrences in the observed lessons. Mathematical accuracy and precision were strengths of the participants in this study.

### **Student Participation in Meaning Making and Reasoning**

A key component of high quality instruction which allows students to develop principled mathematical reasoning is the opportunity to reason and make sense of mathematics (Lampert, 1986; Greeno, 1992; Harel, 2008a; Spillane & Zeuli, 1999). Table 10 displays the low scores, on a the MQI's 1 – 3 scale, representing the lack of opportunities students in the observed classrooms had to explain their thinking as well as question and reason through the mathematics.

**Table 10:** MQI Overall Student Participation in Meaning Making and Reasoning Scale Scores for Observed Lessons (n=21)

|                                                               | M (SD)      | Median | Range (1-3) |
|---------------------------------------------------------------|-------------|--------|-------------|
| Overall Student Participation in Reasoning and Meaning Making | 1.12 (0.15) | 1.10   | 1.00 – 1.41 |
| Students Provide Explanations                                 | 1.19 (0.25) | 1.07   | 1.00 – 1.77 |
| Student Mathematical Questioning and Reasoning                | 1.02 (0.03) | 1.00   | 1.00 – 1.05 |
| Enacted Task Cognitive Activation                             | 1.16 (0.18) | 1.19   | 1.00 – 1.48 |

In developing mathematical reasoning, as previously mentioned, the more opportunities students have to formulate their ideas and explain their thinking, the greater their learning gains (Franke, 2011). An important aspect of creating opportunities for this type of reasoning is enacting tasks, regardless of their initial formulation, in a way that promotes thinking and understanding (Hiebert et al., 1997; Sierpinska, 1999). The MQI accounts for this critical aspect of mathematics instruction through the Student Participation in Meaning Making and Reasoning domain and subscales. The MQI describes the domain as “students’ involvement in tasks that ask them to ‘do’ mathematics and the extent to which students participate in and contribute to meaning-making and reasoning” (Hill, 2010, p. 19). This type of participation usually occurs in the acts of reasoning, explanation, and question-asking.

While all mean and median scores in this domain accurately represent the overall minimal opportunities students in the study have to participate in reasoning and meaning

making, the larger range of 1.00 – 1.77 on the subscale of students providing explanations represents variation which will be explored in Chapter Five. In all, across the 21 lessons observed it was uncommon for students to engage in principled mathematical reasoning.

Instructional quality levels across all 21 lessons and the MQI domains tended toward low to mid-levels. However, characterization of the lesson quality is just the beginning of understanding how teachers' mathematics instruction comes to be expressed. Next, the potential mediating factors that contribute to the instruction will be described and compared.

## **Mathematical Knowledge for Teaching**

### **The MKT Instrument**

Because current research positively associates different aspects of teacher knowledge with the quality of their instruction and student learning gains (Hill et al., 2005; Kersting et al., 2009), the next step in this analysis is to characterize the participants' mathematical knowledge for teaching. Using the Mathematical Knowledge for Teaching (MKT) assessment (Hill et al., 2006), the participants in this study demonstrated a relatively narrow range of knowledge for teaching mathematics (compared to Hill et al., 2008). The mathematical knowledge for teaching construct is comprised of *common content knowledge*, knowledge that someone who knows mathematics would know, and *specialized content knowledge*, knowledge that someone who teaches mathematics should know (Hill et al., 2008).

MKT scores are calculated using Item Response Theory and are reported as logits, “standard units used to linearize otherwise nonlinear measures” (Hill et al., 2004, p. 14). Ninety-five percent of scores on the MKT assessment fall between -2 and +2 standard deviations (Hill, personal communication, April 14, 2011). Scores are based on participant responses to fourteen multiple choice questions embedded in classroom scenarios (described in-depth in Chapter Three). Participant MKT scores are displayed in Table 11.

**Table 11:** Participant MKT Scores

| Participant | MKT Score |
|-------------|-----------|
| Hally       | 0.28      |
| Hannah      | 0.51      |
| Bill        | 0.95      |
| Fran        | 0.95      |
| Kelly       | 1.14      |
| Barbara     | 1.39      |
| Liz         | 1.43      |

The knowledge levels of the participants in this study ranged from 0.28 – 1.43. To put these results in a larger context, Hill’s participants (2008) had knowledge scores ranging from -0.71 – 1.50. Comparatively, the teacher sample I studied had less lower-knowledge teachers. This could be attributed to the fact that there were no new teachers in my sample and all teachers had extensive experience teaching fourth or fifth grade mathematics. Despite the absence of teachers with low knowledge levels, Hill’s (2008) findings predict variation in instructional quality based on relative differences in

knowledge levels: higher knowledge teachers should demonstrate instruction with higher mathematical quality and vice-versa. In this study, that is only partially true which will be explored further in Chapter Five where meaningful differences in instructional quality by teacher knowledge levels and other factors are explored.

### **The CVA Survey**

To triangulate the information gathered in the MKT assessment and to provide insight into different aspects of their knowledge, I also asked participants to respond to nine video clips of classroom mathematics lessons as part of the Classroom Video Analysis Survey (Kersting et al., 2009). What they notice when watching complex classroom interactions provides different information about knowledge than can be gathered on a multiple choice assessment. As discussed in Chapter Three, the prompt teachers were given was the same as the prompt seen by Kersting's participants: "View the clip and discuss how the teacher and the student(s) interact around the mathematical content." Additionally, because my pilot study showed teacher hesitance in making suggestions, I verbally told participants to feel free to make suggestions for improvement. The added prompt did not seem to alter the outcome; some participants made suggestions on some clips and not on others and vice-versa with about the same frequency as my pilot participants.

In Kersting's (2009) tests, the scores on the CVA correlated significantly with teachers' scores on the MKT. With the exception of one outlier, as predicted in Kersting's (2009) correlation study, teachers' with higher scores on the MKT tended to score higher on the CVA dimensions and vice versa. An additional benefit to using the

CVA is the written responses provide data which can be qualitatively analyzed for meaningful differences in what teachers notice and discuss about mathematics instruction.

CVA responses are scored on a scale of 0 (low) – 2 (high) along four different dimensions of analysis: 1) analysis of mathematical content (MC), 2) analysis of student thinking (ST), 3) suggestions for improved teaching strategies (SI), and 4) depth of interpretation (DI). A 0 score on any of the first three dimensions indicates the lack of presence of the dimension in the participant's response. A score of 1 means the participant mentioned the dimension in the response but it did not go beyond the observable. For a score of 2, comments within a dimension have to be specifically tied to mathematical content. The scores in Table 12 reflect the average scores of the teachers across the CVA dimensions and the range of averages between the teachers.

**Table 12:** Teacher Knowledge Scores across the CVA Dimensions (n=7)

| Knowledge Domains           | M (SD)      | Range (0-2) |
|-----------------------------|-------------|-------------|
| Mathematical Content        | 0.91 (0.45) | 0.22 – 1.55 |
| Suggestions for Improvement | 0.83 (0.23) | 0.56 – 1.22 |
| Student Thinking            | 0.76 (0.41) | 0.00 – 1.11 |
| Depth of Interpretation     | 1.19 (0.48) | 0.56 – 1.78 |

The mean scores displayed in figure 12 indicate a central tendency of the teachers to have slightly below mid-level knowledge for teaching mathematics. The ranges draw attention to some extremely low scores of 0 (student thinking) and some higher scores nearing 2 (mathematical content and depth of interpretation) which warrant further analysis. The overall domain scores also indicate a relative weakness among the teachers

in noticing and discussing student thinking. In my analysis of this data, I focus on the first three domains because, in the absence of enough participants to do a statistical analysis, they are most directly comparable to observable instructional characteristics.

### **CVA: Analysis of mathematical content.**

Kersting et al. (2009) report the subscale of Mathematical Content had the highest correlation with the MKT scale ( $r(223)=.608, p<.01$ ). They suggest that similar mathematical knowledge is accessed when answering MKT items and when analyzing math content in the classroom videos. In keeping with Hill's claims that MKT scores are predictive of instructional quality, it is logical to think that the CVA's Mathematical Content scores would also be predictive of instructional quality. However, Kersting reports that no CVA subscales are predictive of teacher instructional performances on the MQI measure (personal communication, 7/11). With that in mind, I scored the responses using the instrument rubrics but also noted qualitative differences that might help explain instructional variation.

When discussing the classroom videos, teachers tended to either not discuss the mathematical content or to describe what was observable but not discuss the underlying mathematical concepts, resulting in many scores of 0 or 1. However, on occasion, some teachers did discuss the underpinnings of the core mathematical ideas leading to some scores of 2. The maximum score of 1.55 in this dimension stands out as it is the highest of the teacher averages across the first three dimensions. What did that teacher notice and say about math content and how is it qualitatively different than the person who scored a 0.22? How does it relate to classroom instruction? Is there information in the

response to help explain Kersting's (2010) claim that teachers' attention to mathematical content in the video clips is related to student performances? Table 13 displays a comparison of two different levels of teachers' responses to a classroom video. Both teachers watched a video clip in which the classroom teacher is guiding students in the use of fraction circles to understand division of fractions.

**Table 13:** Comparison of High Scoring vs. Low Scoring CVA Content Response

| Teacher with mean content score of 0.22                                                                                                                                                                     | Teacher with mean content score of 1.55                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| This response is a 0.                                                                                                                                                                                       | This response is a 2.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| Great use of hands-on manipulatives. I think I would have had them write down the problem on their papers, so they could see and make fractions at the same time (maybe that is what they ended up doing?). | <p data-bbox="802 745 1398 867">It is good the teacher chose to engage this particular partnership, since the fraction circle pieces were formed into creative "face" designs, so it looked to me that they were not engaged in the mathematical thinking.</p> <p data-bbox="802 884 1398 1005">Once engaged by the teacher, the two students worked together well, and seemed to piggyback on each other's thinking. The girl was excited to solve the "fraction circle puzzle." It was a good visual.</p> <p data-bbox="802 1022 1398 1144">I had to watch the video three times to see exactly how he was proving that dividing by 4 is the same as multiplying by <math>\frac{1}{4}</math>, although it was not being presented as a rule.</p> <p data-bbox="802 1161 1398 1274">It would probably take a number of similar problems and discoveries for the student to derive that rule on their own, but once they did, hopefully they would own why it is so.</p> |

The teacher who scored a 0 did not focus on math content, but instead focused on general pedagogy – the use of manipulatives and the need to write down a problem. The low response belongs to the teacher who had the lowest mean score, 0.22, across the video clips. She also had some of the lowest instructional quality scores. The teacher who scored a 2 looked beyond the use of the manipulatives to what the underlying mathematical concept was – developing a concrete model for division of fractions, which also made clear the relationship between multiplication and division. The high scoring

response belongs to the teacher who had the highest mean score, 1.55, across the video clips. This teacher also had some of the highest instructional quality scores. Although Kersting's findings (2008) do not predict a statistically significant correlation between the knowledge displayed along this dimension and instructional quality, a qualitative analysis of the content of responses may reveal connections to instruction undetectable with the quantitative analysis. In Chapter Five, I qualitatively examine what teachers notice about math content and how it specifically relates to their lessons.

### **CVA: Suggestions for improvement.**

The Suggestions for Improvement (SI) domain is a critical domain to examine closely as Kersting et al. (2009) found a statistically significant correlation between what teachers suggest and instructional quality. Participants in this study showed a tendency within the SI domain to generate mid to low level scores as indicated by the 0.83 mean, and less high level scores than in the content domain. Table 14 shows three different levels of teacher discussions on the SI domain. Participants are responding to a video clip where the teacher has asked students to share 132 cupcakes among six people. She is leading a class discussion about creating a conceptual model for the problem as well as grappling with the difference between  $\frac{1}{6}$  of 132 and, as a student suggests,  $132 \div 6$ .

**Table 14:** Comparison of Different Level Suggestions for Improvement Responses

| Score: 0                                                                                                                                                                                                                                                                                                    | Score: 1                                                                                                                                                                                                                                                                                                                                        | Score 2                                                                                                                                                                                                                                                                                                                                                                                                                               |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| The teacher was working hard to get the kids to engage their thinking more, both the students who knew the answer and the others who were struggling. I like the questioning strategies she used, and her enthusiasm made the girl volunteer feel more comfortable in showing her thinking on the overhead. | I like how the teacher had the student explain how she got the answer, because the kids could probably relate better to the student's explanation than the teacher's. The only suggestion I would have is to break it down to a smaller number, like 12 cupcakes, instead of 132, and let them apply the same principles to the larger problem. | I like how she is encouraging her students to ask questions and think critically about fractions in order to understand the meaning in a deeper way. I think it would be beneficial to review the meaning of numerators and denominators with them. That vocabulary would allow the students to then explain that you are dividing 132 into six equal parts. They could also see that the one is referring to one of those six parts. |

The teacher who scored a 0 focuses on general aspects of pedagogy and does not make any suggestions for improvement. The teacher who scored a 1 suggests starting with a smaller number and, in doing so, shows awareness of how children learn. The teacher who scored a 2 delves into issues of content by suggesting more emphasis on the meaning of each part of a fraction as related to the problem. It is this deeper emphasis on content in the suggestions that predicted higher quality instruction in Kersting et al.'s (2009) study. Yet, as explained in Chapter Three, in this study I did not use the same instructional quality measure as Kersting. Therefore, the Suggestions for Improvement construct did not align with the instructional quality measure I did use (MQI, Hill, 2010) and Kersting's correlational relationship was not quantitatively documented. However, as will be described in Chapter Five, qualitative analysis of the suggestions teachers made helped to explain other mediational relationships.

### CVA: Analysis of student thinking.

It was less common for the participants in this study to discuss student thinking in their analysis of the classroom video clips than it was for them to discuss teachers' instructional methods. In approximately 40% of the written responses students were not mentioned as indicated by the range's minimum of 0. As Table 12 displays, the mean score for student thinking was 0.76 reflecting the large numbers of mid to low level scores. Table 15 shows examples of the different levels of response within the ST domain. In these responses, the participants are responding to a clip in which the teacher is trying to explain to a student why  $\frac{1}{2} + \frac{1}{2} = 1$  and why you cannot just add the numerator and denominator, resulting in  $\frac{2}{4}$ . She uses a drawing of a pizza to assist with the explanation.

**Table 15:** Comparison of Different Levels of Student Thinking Responses

| Score: 0                                                                                                                                        | Score: 1                                                                                                                                                                                                                                                                                                                                                                                                    | Score: 2                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
|-------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| This was more teacher led than student led. I think that she should have had him do the work, and explain it to her, than the other way around. | The teacher does a wonderful job interacting directly with a student that has an issue adding fractions. She is able to get to eye level and work individually with the student at their level. The student still seemed to have a limited understanding of fractions. This student from the brief view from the video probably needs some more concrete lessons with fractions (i.e. fraction strips etc.) | <p>I think the kid learned a rule for adding fractions, but I didn't hear interaction between teacher and student that would make me think that the student understands why you don't add the denominators. I think with further problems (and ones with bigger numbers), this student may likely make the same mistake because he doesn't understand what's really happening when you add two fractions with like denominators.</p> <p>I would likely have helped the student see that when the denominators are the same, it is like the "unit". For example, if I add one watermelon and one more watermelon, I get two watermelons. The watermelon part did not change. The denominator is the unit. When the denominators (units) are different, you cannot add them without changing to a common denominator (unit). For example, I cannot add one watermelon and one strawberry, but if I changed the units to "fruits," then I could end up with two fruits.</p> |

As the examples show, the level 0 response shows no mention of student thinking, level 1 mentions lack of student understanding but is not specific, and level 2 shows specific attention to whether the student understands why you should not add denominators as well as what confusion this could cause in the future. In Chapter Five, I will discuss qualitative differences in teachers' knowledge indicated by these responses as related to other mediating factors and instructional quality.

### **Teacher Beliefs**

Because the premise of this study is that knowledge alone cannot explain qualitative differences in teachers' mathematics instruction, I explore teachers' beliefs about math teaching and learning as a likely mediating factor. Historically, beliefs have been viewed as strong indicators of decision making (Bandura, 1986; Dewey, 1933; Pajares, 1992). From a situative perspective, while a teacher's beliefs about teaching and learning mathematics may influence instruction, interactions with students and the classroom as well as institutional environment can mutually influence beliefs (Thompson, 1992). In this study, I measured teacher beliefs about the subject of mathematics, teaching mathematics, and how children learn mathematics in order to examine how teachers' beliefs might interact with other factors in the expression of mathematics instruction.

I used two measures to characterize the epistemological and pedagogical mathematics beliefs of each teacher: 1) The Integrating Mathematics and Pedagogy (IMAP) Web-Based Beliefs Survey (Philipp & Sowder, 2003) and 2) Semi-structured interviews (Appendix B). The IMAP Beliefs Survey provides a structured framework of

conceptually-oriented beliefs through which teachers' survey responses to classroom scenarios and videos were compared. The semi-structured interviews provide evidence of espoused beliefs as well as confirming or disconfirming information regarding the IMAP Belief Survey results. Interview data will be used in future chapters to help explain the role beliefs play in the act of classroom instruction and will not be discussed here. The beliefs yielded by the survey, as displayed in the IMAP Beliefs Survey Manual (Philipp & Sowder, 2003), are listed below.

### **Beliefs about Mathematics**

1. Mathematics is web of interrelated concepts and procedures (and school mathematics should be too).

### **Beliefs about Learning and/or Knowing Mathematics**

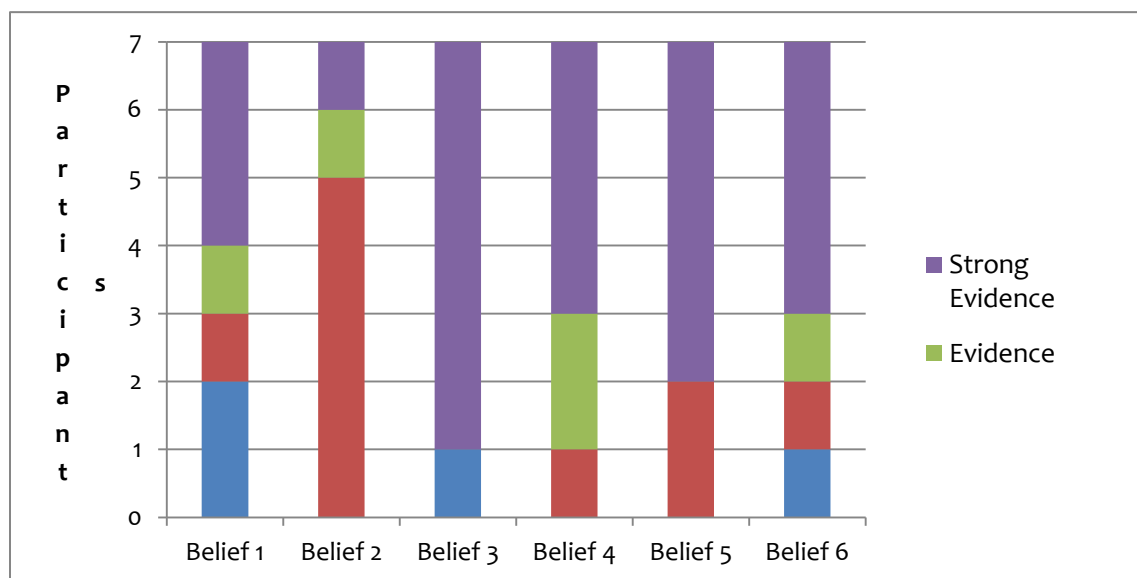
2. One's knowledge of how to apply mathematical procedures does not necessarily go with understanding of the underlying concepts.
3. Understanding mathematical concepts is more powerful and more generative than remembering mathematical procedures.
4. If students learn mathematical concepts before they learn procedures, they are more likely to understand the procedures when they learn them. If they learn the procedures first, they are less likely ever to learn the concepts.

### **Beliefs about Children's (Students') Learning and Doing Mathematics**

5. Children can solve problems in novel ways before being taught how to solve such problems. Children in primary grades generally understand more mathematics and have more flexible solution strategies that adults expect.
6. The ways children think about mathematics are generally different from the ways adults would expect them to think about mathematics. For example, real-world contexts support children's initial thinking whereas symbols do not.

7. During interactions related to the learning of mathematics, the teacher should allow the children to do as much of the thinking as possible.

The teachers in this study participated in all survey items measuring beliefs 1 – 6 (not belief 7, as I shortened the survey to honor teacher time constraints). Figure 9 shows the number of participants, by strength of evidence, holding each belief.



**Figure 9:** Distribution of Participants by Strength of Evidence that They Hold Each Belief

At a glance, the graph shows that, with the exception of Belief 2, each belief has at least 4 participants who show evidence to strong evidence of holding it. This is noteworthy as the beliefs represent a reform-oriented, conceptual instruction perspective on mathematics which is contrary to the characteristics of most of the observed instruction.

### **Beliefs about Mathematics**

As Ma (1999) noted, teachers in the United States tend to understand mathematics as a set of isolated procedures rather than seeing the interconnectedness of the concepts. The IMAP Beliefs Survey results show that four of the seven teachers demonstrate evidence to strong evidence in believing mathematics is a web of interrelated concepts and procedures (Belief 1) while three of the teachers showed weak to no evidence of this belief. Occurrences of linking or connections between concepts were rare in the observed lessons. The role Belief 2 played in this outcome will be explored in Chapter Five.

### **Beliefs about Learning and/or Knowing Mathematics**

Teachers' beliefs about how math is learned are similar among participants in this study and do not appear to directly reflect instructional practice. There is strong evidence that six of the seven teachers believe students are best served by developing conceptual understanding in mathematics (Belief 3). Yet, contrary to this belief, most teachers in the study provided procedural instruction. Belief 2 is important to pay attention to as it represents the belief that knowing a procedure does not mean one understands the underlying mathematics. Five of the teachers showed weak evidence of this belief which is more aligned with the strong procedural instruction component in the observed lessons. Finally, six of the seven participants in the sample group show evidence to strong evidence of believing concepts should be learned before procedures are introduced (Belief 4). As with the Belief 3 results, the observed instruction belies the evidence that teachers hold Belief 4. The prevalent mismatch between beliefs and instruction suggests other factors are mediating the relationship.

### **Beliefs about Students' Learning and Doing Mathematics**

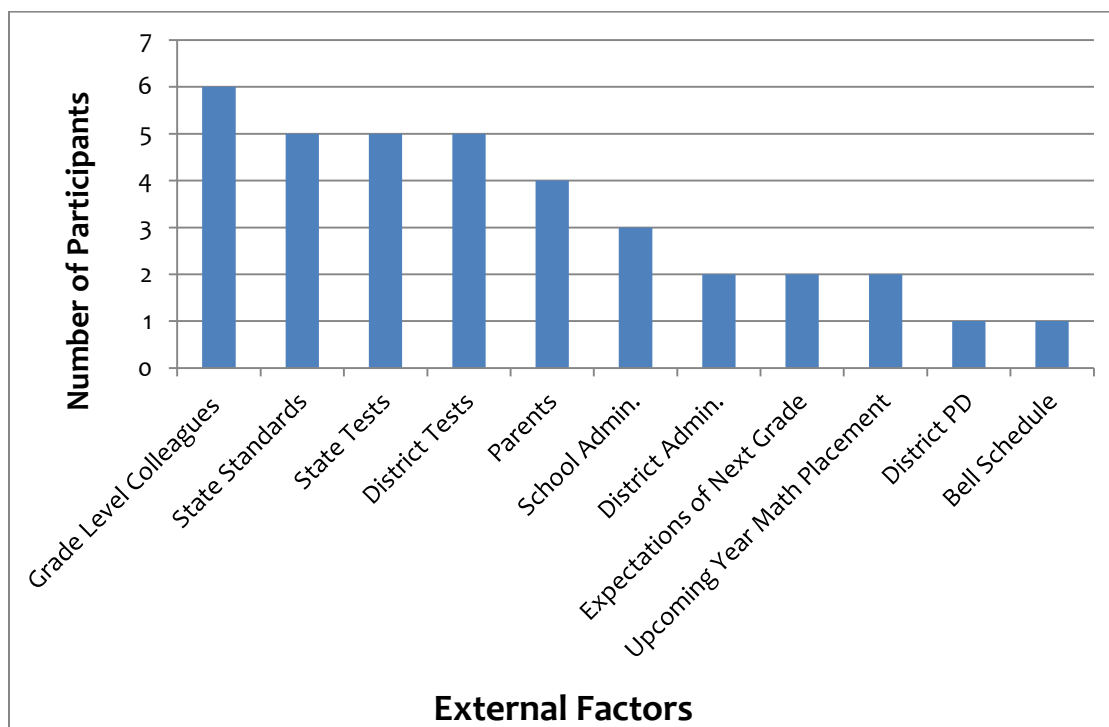
In keeping with belief findings that are contrary to classroom practice, five of the seven teachers in this study demonstrate strong evidence that students can solve problems in novel ways before being taught strategies for solving the problems (Belief 5). Yet, in the observed lessons teachers most often modeled how to solve different types of problems and then had students practice. Five of the seven teachers also showed evidence to strong evidence of believing children think differently about mathematics than adults and their understanding can be supported through the use of applied problems (Belief 6). More teachers provided evidence of this belief than used applied problems in their instruction. This information suggests beliefs are likely mediated by other factors rather than directly affecting instruction.

### **Teacher Sensemaking of Curricular and Instructional Policies**

While personal factors such as knowledge and beliefs may play crucial roles in the expression of math instruction, a situative perspective focuses attention on aspects of the environment that may interact with these personal factors. Likewise, sensemaking theory emphasizes human agency within individual and social activities of institutions. It focuses attention on interpretation as it interplays with action (Weick, 2005). Analyzing the sense teachers make of institutional policies combines thinking with context as policies are interpreted and action is taken. Specifically, I focus on the sense teachers made of policies around curriculum use and student performance accountability. I used semi-structured interviews combined with lesson observations to gain insight into how teachers interpret and enact policies and how this impacts their math instruction.

There is currently strong emphasis on teacher, school, district, and state accountability for student performances as evidenced in federal policies such as No Child Left Behind (2001 ) and The Race to the Top (2009). As described in previous chapters, these policies are high stakes with rewards and consequences attached. In addition to extensive systems of state, district, and school-level testing, state standards and state-approved/district-adopted textbooks are major aspects of policies that affect classroom teachers and their students. In this study, teachers provided evidence that the way they interpret district curriculum use policies play a large role in their classroom instruction. Additionally, the pressure they feel regarding test score accountability, who they feel it from, and how they make sense of it are important considerations in understanding how mathematics instruction comes to be expressed.

Through the fourteen semi-structured interviews conducted in this study, two per participant, the participants self-reported various contextual factors that influence their curricular and instructional decisions. Figure 10 displays the factors and the number of participants who discussed them throughout their interviews.



**Figure 10:** Teacher-Reported External Factors Influencing Curricular and Instructional Decisions

### Collegial Interaction

In the interviews, six of the seven participants discussed grade level colleagues at their own school sites as highly influential in their curricular and instructional decisions. They talked about making decisions together regarding what their students instructionally needed and about which curricular materials they would use. Pacing curriculum together was another activity teachers did together. One teacher, who was at a site with two other participants, discussed following the curricular recommendations of the more experienced teachers. They also mentioned hearing one another talk about teaching a topic a certain way and trying it in their own classrooms. Combining what the teachers said and the similarity in content and instruction at each site but differences across sites,

it is clear that there is a strong school-based social component to the sense teachers make of policies and what they do in their classrooms.

### **Role of collegial interaction in textbook selection.**

An example of the role school-site-specific social interaction plays in policy interpretation and, in turn curricular decision making, is the textbook used at each school site. Teachers account for their textbook selection by explaining interactions with their grade level teammates. Understanding the district curriculum policy context in which these decisions were made is an important first step in contextualizing the thinking and actions of the teachers.

In the Sunrise Unified School District, the *Houghton Mifflin California Math* (Hill, R. et al., 2009) program was selected as the core adoption from the state approved textbook list. Despite the adoption, district leadership was opposed to the traditional procedural instruction forwarded by the program (personal email with SUSD Math Leader, 2/8/11). While this textbook did not align with the district's espoused conceptual understanding math philosophy, adopting from the approved list entitled the district to receive state textbook funds. It also helped SUSD comply with the California Williams Act (2004) which states every child must have a state-approved textbook to use at home and at school. Because the program was misaligned with the math philosophy of the district curriculum leaders, teachers were instructed to keep the *California Math* books in their cupboards and instead use a different, non-state approved program which the district also acquired. The district-selected program was *Houghton Mifflin Math Expressions* (Fuson, 2009), a program emphasizing conceptual and social mathematics learning.

Despite the program-use mandate, the core program used in the classrooms varied by school. In four of the classrooms observed, *California Math* was used as the core program and in three of the classrooms *Math Expressions* was used. Exploring how certain programs came to be selected highlights the mediational relationship between contextual factors, teacher sensemaking, and instructional action.

From a sense-making perspective, teachers do not simply react to policies (external stimuli) but filter the messages through their pre-existing cognitive frameworks and social experiences before taking action. Therefore, despite the district's assertion that the use of *Math Expressions* was a non-negotiable mandate, teachers' interpretations of that message varied greatly. Table 16 displays the textbooks used at each school site and how teachers reported interpreting the district curriculum policy.

**Table 16:** Interpretation of District Curriculum Policy by School

| School Site   | Program Used     | Interpretation of Policy                                                                                                                                               |
|---------------|------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Pacific Grove | California Math  | Interpret the district policy as inflexible and know they are “renegades.” Feel they have principal support to make their own choices as long as test scores are high. |
| Little Rock   | California Math  | Interpret the district policy as flexible as long as they can justify the curricular materials they use through standards alignment and testing outcomes.              |
| High Bluff    | Math Expressions | Interpret the district policy as moderately flexible. They feel free to use both programs as needed but primarily stick with <i>Math Expressions</i> per the mandate.  |
| Hillside      | Math Expressions | Interpret the policy as inflexible. They feel no freedom to vary from the mandated program.                                                                            |

As Table 16 shows, teachers' interpretations of district policies vary by school site indicating a site-based social influence. As will be discussed later in this chapter, parental expectations and the curriculum-use stance of school administrators were site-specific and contributed to policy interpretations. Additionally, Chapter Five explores how different individual teacher factors (knowledge and beliefs) came to bear on these interpretations and, ultimately on curricular decisions. It also examines how the social status assigned to a teacher made his/her knowledge or beliefs more influential in the decision making process.

### **Standards and Assessments**

Predictably, given the current accountability climate, standards and assessments were mentioned in five of the seven participants' interviews as playing a major role in what they teach and how they teach it. Teachers discussed state standards as their map of the content they must teach. They use them as a guide to help decide what lessons to teach and which ones to cut from their textbooks. A statement by one of the teachers, and echoed by the other four teachers, reflects the strong role the state standards play:

We feel the liberty to make changes with our curriculum and we'll discuss it as a grade level, what's the best thing. I'm not going to throw a random lesson in just because I love it or think it's fun. I mean, I stick to what the grade level, what the expectations are. Now, will I bring some creativity and hopefully some love of learning into those lessons? Yes. I'll try to spice them up and make it so they're entertaining and the kids enjoy math and you know learn to love it and feel successful with it, but as far as the concepts that need to be taught, we stay pretty true to the standards.  
(Interview, 28 Sept. 2010)

In the interviews, only five of the participants espoused similar fidelity to the standards, but in all seven classrooms and across the 21 observations, 100% of the

content in the lessons was reflective of state standards. A conclusion can be drawn that all of the teachers in this study interpret the state and district policies around state content standards as mandated which they fully enact.

The accountability measures that accompany the state standards support the teachers' interpretation. The same five teachers who discussed the role of standards in their lesson content also discussed the role of state tests in their instruction. One teacher succinctly stated what the other four teachers cumulatively said in their interviews. The transcript below is his response to a follow-up interview question asking him to clarify a statement he made about testing controlling instructional pacing.

It [STAR testing] controls us beyond what we have control over. What I'm saying by that is your whole textbook is geared toward that. The whole district mantra is, and programs are geared to doing well with that. You know, the whole environment, the whole culture is geared to that. It's bigger than just what you're doing in your classroom; it's a whole environment that is addressed. (Interview, 7 Oct. 2010)

In following with the above statement, five of the teachers felt the April administration of the state test controlled their curriculum pacing; it caused them to have to accelerate the pace of their instruction to ensure all standards were covered prior to the test. One teacher referred to the instructional result of too many standards and not enough time before testing as “machine gun teaching – lining the kids up and just popping off all the subjects right at ‘em in a blur of time” (Interview, 8 Dec. 2010).

To be clear, not all participants interpreted state or district policies in this way. One teacher who did not emphasize state testing in her interview stated that the tests were based on year-end standards and that she was teaching the standards and her students would know them by the end of the year. She emphasized she did not target completion

of the curriculum by the April tests but trusted if she did her job teaching the standards all year, students would know what they needed to know before they went to the next grade level. As demonstrated by these comments, teachers made sense of policies and pressures differently but standards and testing were commonly discussed as influential factors. In Chapter Five, school contexts will be discussed to help account for differing interpretations.

### **Parent Expectations**

Four of the teachers reported parent expectations played a large role in the content and pedagogy typical of their lessons. The differences between their discussions of parent expectations seemed to be related to their school context which will be explored in Chapter Five. One participant discussed parents expecting their students to master the skill of double digit multiplication by the end of the year. Quite differently, two of the teachers from a different school reported having to teach at an accelerated pace from the textbook as the parents watched other classrooms and complained if their child's class fell behind. One of these teachers stated,

Fifth grade is a time when we should be exploring things and talking about things and using those manipulatives. But, you know, we don't have time so we're just covering the material and then the parents are pushing, pushing, pushing to see results. And to them, results are 100% on a math test. And then, "Did my kid get this score on MAPs<sup>9</sup> so they can go in pre-algebra?" (Interview, 14 Oct. 2010)

Teachers from this environment interpreted parent expectations as related to the district's policy for tracking students into math classes. These same teachers felt

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<sup>9</sup> MAP is an acronym for Measures of Academic Progress. It is a computer adapted test published by the Northwest Education Association. Students in SUSD take MAP tests in math, language usage, and reading three times per year. The most influential use of these scores for fifth graders is in their sixth grade math placement.

expectations of teachers in the next grade influenced their instruction. The fourth teacher who commented on the role of parents was from a different school. She reported feeling pressured from parents of “outlier” students (special education students and identified Gifted and Talented students) wanting curriculum and instruction differentiated to meet their children’s needs. Because the sample size in this study is small, generalizations cannot be made about how different environments may be related to the role of parental expectations in classroom instruction but it is clear that how teachers interpret and act on those expectations play a key role.

### **School and District Administration**

In the high performing schools, the further removed the external factors were from the classroom, the less directly the teachers in this study discussed their relationship to classroom instruction. Three of the four teachers from the high performing schools who mentioned school administrators as influencing their instruction used general descriptions such as “my principal is happy with our test scores” or “my principal wants us to raise our test scores.” Among these participants, district expectations were alluded to but they were infrequently discussed in relation to what teachers did in the classroom. One teacher’s explanation of how district goals are related to what teachers do in the classroom seems to capture the heart of these teachers’ comments – that expectations are centered on student performances on state tests.

Bluntly, it’s down to one week, how you do on the STAR test. I mean it’s all based upon an API [Annual Progress Index]. That’s our goal. Now the district’s big goal is college readiness for all. I think it’s the A-G<sup>10</sup>

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<sup>10</sup> A-G college components are specific high school courses required for students to be eligible for admission into University of California and California State Universities.

college components for all the students, making sure those are all [met] and it filters down to all of us. So it's college readiness for all, but really it stems to getting all of our kids doing well on that STAR test. I think that's what everybody follows for the good or the bad of it. I don't think anybody is trying to misguide children or something, that's just the program set, and if that's what everybody says you have to do, like I said earlier, that's what they're going to do. You're going to be judged on that. (Interview, 7 Oct. 2010).

In this comment, the teacher discusses a climate of testing and accountability which was an underlying theme in all seven teachers' interviews, regardless of whether they mentioned a specific test or attributed pressure to a person or group of people.

The fourth teacher who discussed the role of the school administrator in her instructional decisions reported a much more influential relationship. She taught in the lowest performing school in the district and described the hands-on role the principal played in making sure teachers were implementing the district-mandated program. In an interview on December 3, 2011, she details the principal's role:

Well, (smiling), it's been pretty much clear to us that the adopted, the California, the Math Expressions, should pretty much be where we are. And we have, as a grade level, my team teacher and I sort of branched away but then we were sort of guided back. They want us to be able to be with fidelity and be true. (...) his expectation is that we are going to get through the pace that's provided by the district. That's clear.

The pressure this teacher experienced from her principal appears to be context-specific as no other teacher in the study reported a similar administrative influence.

### **Additional External Factors**

Two additional factors were mentioned by teachers at one site: district professional development and the bell schedule (time allotted for math). As will be discussed further in Chapter Five, district professional development was discussed as

helping the teacher understand the philosophy of the textbook and see its value. The bell schedule was discussed by one teacher as preventing her from teaching math as creatively as she teaches other subjects. Both factors appear to mediate these teachers' classroom mathematics instruction.

### Summary

This chapter focused on characteristics of the study participants at a group level. Instructional quality of the group was examined according to the domains and sub-domains of the Mathematical Quality of Instruction instrument (Learning Mathematics for Teaching, 2004). Knowledge levels were described using results of the Mathematical Knowledge for Teaching assessment (MKT, Hill et al., 2003) and the Classroom Video Analysis measure (Kersting et al., 2009). Teachers' beliefs were explored using the Integrated Mathematics and Pedagogy (IMAP) Beliefs Survey (Philipp & Sowder, 2003). The sense teachers' made of institutional curriculum and assessment policies was reported as a result of analyzing semi-structured interviews.

Overall, the participants in this study demonstrated instruction characteristic of the mid to low quality levels according to MQI rubrics. Instruction tended to be primarily directed by the teacher with minimal opportunities for whole-class discussion or work on applied problems. It was uncommon for teachers to support students in making connections between mathematical ideas or representations. Teacher and student explanations were characterized by telling *how* rather than *why* procedures worked. While there were some exceptions, teachers primarily advocated a single procedure to reach an answer without considering multiple possibilities. Opportunities to develop

math generalizations were almost nonexistent. However, density and accuracy of mathematical language use proven to be a strength of the lessons. Teachers worked hard to support students in mastering the standards but the support tended to be procedural with little emphasis on conceptual understanding or student thinking. Teachers in the study demonstrated the ability to provide instruction with few errors and minimal occurrences of imprecise or confusing language or notation. However, with such precision came minimal opportunities for students to grapple with and make sense of the mathematics. Student questions generally were about how to perform a procedure or which algorithm to use rather than about the mathematics underlying the problems and procedures.

According to the MKT, participants in this study demonstrated a narrower distribution of knowledge than did Hill et al.'s (2008) participants. There were no instances of low-level knowledge; participants instead scored in the mid to higher ranges. This may be explained by the fact that all teachers in this study were experienced in teaching their grade level. The CVA knowledge measure documented an overall tendency of the group to comment on observable aspects of mathematics content rather than discussing the underpinnings of the mathematics in video clips. When making suggestions for lesson improvements, they often suggested general pedagogical alterations or unelaborated ideas for making content more understandable. They paid minimal attention to student thinking when viewing the video clips.

The IMAP Beliefs Survey provided evidence that many of the participants held reform-oriented beliefs -- they valued conceptual understanding and tended to

instructionally sequence conceptual models before procedures. About half the participants showed evidence that they believed in the interconnectedness of mathematical ideas. Many believed students could solve problems in novel ways without being shown procedures. There was less evidence that participants believed applying a procedure did not indicate conceptual understanding. The fact that the constellation of beliefs reportedly held by the participants was so divergent from their actual instruction indicated the presence of other mediating factors.

Finally, in this chapter the sense teachers made of curriculum and accountability policies was investigated through data gathered during semi-structured interviews. Teachers' interpretations of district policies regarding textbook use varied by school site. Factors seeming to play into these interpretations included the academic performance context of the school, grade-level specific collegial interactions, curriculum standards and accountability, parental expectations, and the role of the administrators. A unique math program adoption situation in the district allowed these factors to stand out.

For the purpose of describing the results of the measures, the factors discussed in this chapter are presented in isolation. However, they actually exist in a network of deeply intertwined relationships. No single factor can account for the quality of classroom mathematics instruction. For example, a teacher may have high levels of knowledge for teaching mathematics conceptually but that knowledge may be mediated by an accountability context which rewards procedural instruction. Additionally, a teacher may hold reform-oriented beliefs but enactment of those beliefs may be hindered by low levels of mathematical knowledge for teaching. Or, a teacher's beliefs and

knowledge may be aligned but different beliefs and knowledge of a higher status colleague may override what the teacher is poised to do in mathematics instruction. Underlying all instruction is how it is situated within social and institutional contexts. The situated interaction of these factors across the cases will be explored in Chapter Five.

## CHAPTER V: CROSS-CASE FINDINGS

In this chapter, I look across the participants to examine the roles teachers' knowledge, beliefs, and policy interpretation play in math instruction. I use a quantitative variable oriented approach to identify relationships between factors. I use a qualitative multiple case study iterative approach to build explanations (Yin, 2009) of the context-specific interaction between teacher and environmental factors as they affect mathematics instruction. A situative perspective (Greeno, 1998; Putnam & Borko, 2000; Spillane et al., 2002) guides the analysis, expanding the field's current emphasis on individual factors to include their interplay with social and institutional factors.

I set out to answer the research question: How do teachers' knowledge for teaching mathematics, beliefs about teaching and learning mathematics, and the sense they make of institutional policy messages interact in the expression of teachers' mathematics instruction? In response to current research (Hill et al., 2008; Kersting et al., 2009), I explored the relationship between teachers' knowledge for teaching mathematics and the quality of their mathematics instruction. Taking into account recent studies concerning the role of beliefs (Raymond, 1997; Stipeck et al., 2001), I also examined the relationship between teachers' beliefs about teaching and learning mathematics and the quality of their mathematics instruction. Finally, considering research on teacher sensemaking (Coburn, 2001; Remillard, 1999, 2000, 2005, 2009; Sherin & Drake, 2004; Spillane & Zeuli, 1999) and with an eye on policy and institutional contexts, I examined the sense teachers make of curricular and accountability policies as it interacts with beliefs and knowledge in the expression of mathematics

instruction. In this chapter, with the addition of each factor, complex mediational relationships iteratively build and explanations for instructional action come into focus.

Looking across the cases in this study, I found that the mathematical quality of teachers' instruction is determined by dynamic interaction between personal and contextual factors. Through their instruction, interviews, and answers to survey questions, the seven participants showed that their knowledge, beliefs, and interpretation of curricular and accountability policies interact differently depending on their social and institutional contexts. No single factor can account for the quality of a teacher's mathematics instruction but instead different factors play larger or smaller mediational roles depending on the context.

These findings add new dimensions to the claims of Hill et al. (2008) and Kersting et al. (2009) regarding the relationship between teachers' knowledge for teaching mathematics and the quality of their classroom instruction. Hill et al. (2008) claim there is a powerful relationship between what a teacher knows and can do in the context of classroom mathematics instruction. Likewise, Kersting et al. (2009) identified a predictive relationship between specific aspects of teacher knowledge and the quality of their instruction. Both studies are cornerstones in the field and provide critical insight into the potential role the teacher's knowledge can play in classroom mathematics instruction. However, neither study examines the role of knowledge as it relates to key external factors within the institutional context. And neither systematically examines the role of teacher beliefs in the expression of instruction. In this chapter, I will describe the

cross-case findings from the study I conducted in which I systematically explored these dimensions.

### **Relationship between Knowledge and Instruction**

While research indicates strong links between teachers' knowledge and the quality of their classroom instruction (Hill et al., 2008; Kersting et al., 2009), in this study what initially appears to be a link becomes much more complex as other mediating factors are examined. Because I used Hill's (2010) Mathematical Quality of Instruction (MQI) instrument for measuring instructional quality, the results from this study are most readily compared to her findings. Hill found a statistically significant correlation between teachers' MKT and the way they responded to student math productions as well as to their overall mathematical errors. The study I conducted produced a similar result (albeit not through a statistical correlation analysis due to small sample size) showing a relationship between MKT and teachers' response to students' math productions. However, the results did not show a link between MKT and teachers' mathematical errors in instruction. Instead, they showed a relationship between teachers' MKT and their mathematical explanations in instruction.

Table 17 displays MQI sub-domains appearing to relate to MKT levels. It should also be noted that, as the data in the table shows, there were only 2 out of 21 instructional quality sub-domains which showed a relationship to MKT. That leaves 19 other sub-domains with no noticeable relationship to the common content and specialized content knowledge measured by the MKT.

**Table 17:** Instructional Quality (MQI) Sub-Domains Related to Knowledge (MKT) Levels

| Participant | MKT<br>(0-2) | MQI Responding to<br>Students<br>(0-3) | MQI<br>Explanations<br>(0-3) |
|-------------|--------------|----------------------------------------|------------------------------|
| Liz         | 1.43         | 1.15                                   | 1.47                         |
| Barbara     | 1.39         | 1.47                                   | 1.47                         |
| Kelly       | 1.14         | 2.80                                   | 2.12                         |
| Bill        | 0.95         | 1.00                                   | 1.00                         |
| Fran        | 0.95         | 1.00                                   | 1.05                         |
| Hannah      | 0.51         | 1.00                                   | 1.25                         |
| Hally       | 0.28         | 1.45                                   | 1.35                         |

The data in Table 17 shows a difference between the instructional quality of teachers with MKT levels above and below 1.00. Teachers with MKT levels above 1.00 tended to demonstrate higher instructional quality in their responses to students' mathematical productions and in the mathematical explanations they provided to students. However, the teacher with the lowest MKT in this study, Hally, and the teacher with the highest MKT score, Liz, have MQI subscale scores that defy the pattern. While one might dismiss Hally's or Liz's scores as outliers, they represent important cases in identifying additional mediating factors which affect the expression of instruction. In Hill et al.'s (2008) study, cases such as Hally and Liz were classified as diverging from the MKT/MQI predictive hypothesis and, using grounded theory, explained through additional mediating factors such as beliefs or curricular materials. The study I conducted goes one step further in systematically investigating the role of teacher beliefs

and the role of institutional policies in the quality of math instruction. Over the course of the study, evidence emerged showing that although higher mathematical knowledge does afford teachers certain advantages in the quality of their math instruction, other mediating factors play a much larger role than hypothesized in previous research. Additionally, it will be shown in this chapter that the specific instructional quality scores which appear related to teachers' knowledge are inevitably intertwined with other mediating factors, particularly curricular materials.

### **Relationship between Curricular Materials and Instruction**

When examining variation patterns in instructional quality as they relate to different factors, one key pattern emerged: differences in the quality of instruction across classrooms appeared related to the curricular materials in use. In Table 18, the mean instructional quality scores which showed a relationship to the textbook used are displayed. The means are calculated across the teachers using each textbook. For example, the mean overall mathematical quality of instruction score for the teachers using *Math Expressions* was 1.67 while for the teachers using *California Math* it is 1.00.

**Table 18:** Relationship between Curricular Materials and MQI Domains

| Textbook         | Teachers | Overall MQI | Domain:<br>Working with<br>Students  | Domain:<br>SPMMR                      | Domain:<br>Richness of<br>Math  |
|------------------|----------|-------------|--------------------------------------|---------------------------------------|---------------------------------|
|                  |          |             | Responding to<br>Student<br>Thinking | Students<br>Explain their<br>Thinking | Multiple<br>Solution<br>Methods |
|                  | n        | M (SD)      | M (SD)                               | M (SD)                                | M (SD)                          |
| Math Expressions | 3        | 1.67 (0.34) | 1.57 (0.20)                          | 1.42 (0.25)                           | 1.36 (0.24)                     |
| CA Math          | 4        | 1.00 (0)    | 1.04 (0.07)                          | 1.03 (0.03)                           | 1.02 (0.04)                     |

*Note.* SPMMR means Student Participation in Meaning Making and Reasoning; All instructional quality scores are on a 1 (low) – 3 (high) scale.

The mean scores in Table 18 show that teachers using *Math Expressions* tended to generate higher MQI scores than teachers using *California Math* across several domains: Overall MQI, Richness of Mathematics, Working with Students and Mathematics, and Student Participation in Meaning-Making and Reasoning. It is noteworthy that the three sub-domains seemingly related to the textbook in use are also focal points of current research on students' development of principled mathematical reasoning. The skills of listening, understanding, and responding productively to student thinking have shown positive effects on student and teacher learning (Jacobs et al., 2009). The quality and quantity of opportunities students have to explain their mathematical thinking during classroom lessons have been linked to student learning gains (Franke, 2011). And finally, when students have opportunities to generate multiple solutions, or listen to and compare differing solutions from their peers, their mathematical understanding increases (Ball & Bass, 2003; Lampert, 1986).

So, it follows to question if the textbook materials in use specifically focus on those aspects of instruction and if they directly influence the quality of teachers' instruction. Could the solution to improving mathematics instruction be as simple as putting research-based curricular materials which enable high quality math instruction into the hands of teachers? Evidence in this study suggests the solution is not that simple. The relationship between curricular materials and the quality of instruction is not as direct as the data in Table 18 implies but is instead mediated by individual and environmental factors. A key consideration is the context in which the activity of instruction occurs.

### **District Curriculum Context**

In the Sunrise Unified School District, to understand the relationship between teachers' mathematical quality of instruction and the curricular resources used, it is essential to first understand the political and philosophical context surrounding the situation which resulted in two separate math programs being placed in each classroom. As discussed in Chapter Four, one of the programs is mandated for instructional use and one is present to meet legal requirements. Understanding the policy environment will help contextualize the decisions teachers made in selecting which program to use as the core program in each of their classrooms.

In a fiscal and philosophical move to both obtain state funds for textbooks and adopt a program that aligned with the philosophy of district-level math leadership, SUSD adopted two mathematics programs from Houghton Mifflin: *California Math* (2008) and *Math Expressions* (2009). *California Math* is a traditional textbook in the sense that it

has a single objective per lesson and each lesson consists of a model for how to procedurally solve a particular type of problem followed by independent practice problems. *Math Expressions* differs in that it tends to have mathematically inter-related objectives in a single lesson followed by an activity involving a conceptual representation, a search for patterns, and social interaction. The type of knowledge promoted by the lessons in *California Math* is primarily procedural in that students compute in a formulaic step-by-step manner. While *Math Expressions* does include a computational aspect, it emphasizes the development of conceptual understanding before formal procedural knowledge. The purchase of *California Math* secured state textbook funding for SUSD as it is on the list of state-approved core programs. The addition of *Math Expressions* met SUSD's philosophical requirement that math instruction in the classrooms focus on the development of conceptual before procedural math knowledge, mental math skills, and mathematical communication.

From the perspective of a SUSD district-level math curriculum coordinator, the programs are distinctly different:

We have adopted Math Expressions which doesn't have a hard textbook. It was offered as a companion program to California Math, the Houghton Mifflin program that we piloted. Math Expressions is much more conceptual, provides a strong student math talk component, and included multiple representations for students as they progressed from the concrete to the abstract. The students move quickly from using materials to making proof drawings to support their computation skills. The program is developed with a Cognitively Guided Instruction base which brings in the mental math skills to develop flexibility in number sense. It also provided a smooth transition for our students coming from Everyday Math, due to the similarities in instruction. For all of these reasons, we are asking teachers to use Math Expressions instead of California Math. California Math is a traditional math program taught in a very traditional way. (Personal email, 8 Feb. 2011)

In this email, the district curriculum coordinator articulates the district's intent to mandate the implementation of *Math Expressions*. She also echoes the *Math Expressions* author, Dr. Karen Fuson's, description of the program's philosophical underpinnings. According to Fuson, the program is rooted in recommendations from the *Children's Math Worlds Research Project* (CMW) (2003), a project in which she is the primary investigator. It is noteworthy that some of the previously cited researchers including Carpenter, Fennema, Hiebert, and Bransford, collaborated with Fuson on different aspects of the mathematical framework forwarded by CMW and underlying *Math Expressions*. It makes sense then that the introduction to *Math Expressions* claims to emphasize helping students build a conceptual foundation before being introduced to formal procedures. It cites student interactions and math talk as a staple of the learning environment advocated. It is notable that the principles underlying the development of *Math Expressions* are congruent with many of the principles underlying the different dimensions of the MQI (Hill, 2010).

According to district math leadership, SUSD did place the more traditional *California Math* program in classrooms to meet legal requirements, as noted below, but simultaneously mandated the implementation of *Math Expressions* in instruction.

Officially, California Math is on the list of state approved textbooks so it is our official "adopted text". But, the directive from LSS [Learning Support Services] is that core instruction will be from Math Expressions for the reasons listed above. The first year we asked all teachers and sites to work for fidelity with the implementation of Math Expressions. As always, it was quite difficult for some. Our current expectation is the same, although we are aware that more and more sites and teachers are teaching from California Math. California Math is in the classrooms because it is a requirement of the Williams Act [2004 Federal Act requiring a textbook for every student at home and at school]. We opted

to keep that state adopted text in the cupboard. (Or in the desks in some classrooms). (Personal email, 8 Feb. 2011)

In this context, with two programs present in each classroom, one mandated for use and one expected to not be used, the teachers in this study interpreted the curricular use policy in different ways. While the data presented in Table 18 indicates a strong connection between the core program in use and the quality of the math instruction, the beliefs, knowledge, and social interactions that mediated policy interpretation and enactment provide evidence that a more complex relationship exists.

### **Relationship between School, Curricular Materials, and Instruction**

In this study, the interpretation of policy and the resulting textbook serving as the core program in each classroom varied by school. The quality of instruction also varied by school. Table 19 shows instructional quality sub-domains which differ by school and appear related to the textbook used.

**Table 19:** Variation in MQI by School Site and Textbook Used

| School        |   | Textbook    | Overall MQI | Responding to Student Thinking | Students Explain their Thinking | Multiple Solution Methods |
|---------------|---|-------------|-------------|--------------------------------|---------------------------------|---------------------------|
|               | n |             | M           | M                              | M                               | M                         |
| High Bluff    | 2 | Expressions | 1.84        | 1.64                           | 1.47                            | 1.40                      |
| Hillside      | 1 | Expressions | 1.33        | 1.45                           | 1.31                            | 1.28                      |
| Little Rock   | 1 | CA Math     | 1.00        | 1.15                           | 1.00                            | 1.00                      |
| Pacific Shore | 3 | CA Math     | 1.00        | 1.00                           | 1.04                            | 1.03                      |

*Note.* All instructional quality scores are on a 1 (low) – 3 (high) scale.

School-site based decision making is historically part of the SUSD culture therefore the distribution of curricular programs by school-site displayed in Table 19 is not unexpected. However, it is the interactions of different factors that went into the decisions regarding which textbook to use that will explain the role each factor played. Additionally, the philosophical alignment between *Math Expressions* and the MQI (Hill et al., 2010) makes it predictable that teachers using the program would score higher on the MQI scales than those using *California Math*. It is noteworthy that Responding to Student Thinking and Explanations are the same two sub-domains that showed a quantitative relationship to teachers' MKT scores. The identification of a pattern between schools, textbooks, and instruction iteratively adds to the identified knowledge/instruction relationship as an explanation for how instructional action is built. An exploration of differing instructional quality scores between school sites and individual teachers who are using the same curriculum will highlight the roles of different factors in instruction.

### **Relationship between Curriculum Policy Interpretation and Curriculum Used**

One key factor in teachers' decision making regarding which program they should use was their interpretation of the district's curricular use policy. The teachers in the study varied in how they interpreted the district directive to use *Math Expressions* as the core program. The variations were distributed across school sites. Table 20 shows how the teachers at each school site interpreted the policy. It also shows the typical socioeconomic status of the families attending the school and the level of the school's

Academic Performance Index (API), contextual considerations which play a role in the external pressures that teachers experience.

**Table 20:** Variations in Curriculum Use Policy Interpretation by School Site

| School        | n | School SES/API | Program in Use   | Interpretation of District Policy                                 | Explanation                                                                                                                                                                                                          |
|---------------|---|----------------|------------------|-------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Hillside      | 1 | Low/Low        | Math Expressions | Expressions is the mandated program and the policy is enforced.   | They experience high pressure from principal to adhere to district mandate and use Math Expressions exclusively. If they use any part of CA Math, they know they are going against the direction of their principal. |
| High Bluff    | 2 | High/High      | Math Expressions | Expressions is mandated but teachers have choice with limits.     | The district wants them to use Math Expressions and they agree with the program's philosophy. They do fill some gaps in standards with CA Math.                                                                      |
| Pacific Shore | 3 | High/High      | California Math  | Expressions is mandated but no one from the district is checking. | Recognize the district mandate and know they are "renegades." Feel they have principal support to make their own choices if test scores are high.                                                                    |
| Little Rock   | 1 | Low/High       | California Math  | Choice between the two programs                                   | Believe staff can decide which program to use if they can justify it through standards alignment and testing outcomes.                                                                                               |

*Note.* SES = Socioeconomic Status of the school families; API = Academic Performance Index (a California Standards Test score indicating each school's academic performance)

Table 20 shows evidence that the sense teachers made of the district's mandate varied extensively by school. Interpretations ranged from the teacher at Little Rock believing that teachers at her school could use either math program to the teacher at Hillside believing the mandate to be inflexible. The low academic performance of Hillside and the resulting external pressure to increase student academic performances is

a contributing factor in this interpretation. Through analysis of survey and interview data, it becomes clear that the interpretations were shaped by teacher beliefs, school contexts, social interactions, and teachers' knowledge.

### **Relationship between Beliefs and Curricular Materials**

Teachers' beliefs about teaching and learning mathematics appear to play a primary role as mediators of policy interpretation and curriculum selection. However, from a situative perspective and from an analysis of the data collected in this study, the relationship also proves to be mediated by social and institutional factors. In order to understand the interplay between beliefs and other factors, first I will track the connection between beliefs and the selection of curricular materials.

Table 21 displays the degree of alignment that exists between teachers' beliefs and the underlying philosophy of each curricular program. Based on beliefs about teaching and learning mathematics coded in the interviews and scores on the IMAP Beliefs Survey (Philipp & Sowder, 2003), I rated each teachers' alignment with the programs as *no alignment*, *weak alignment*, *partial alignment*, *alignment*, or *strong alignment*. For example, teachers who espoused reform-oriented beliefs in the interview and showed evidence to strong evidence of similar beliefs in the IMAP Beliefs Survey received a *strong alignment* rating for *Math Expressions*. If they explicitly commented on disagreeing with the philosophy of *California Math* and their IMAP Beliefs profile concurred, they received a *no alignment* rating for *California Math*. If they talked about both the need for conceptual understanding and emphasized repeated practice of basic skills as well as demonstrated less evidence of the beliefs on the IMAP Beliefs Survey, I

scored them as *partial alignment* for both programs. If they espoused valuing repeated practice of basic skills and commented on disagreeing with aspects of *Math Expressions* as well as demonstrated weak to no evidence of most of the beliefs on the IMAP Beliefs Survey I scored them as *no alignment* with *Math Expressions* and *strong alignment* with *California Math*. *Weak alignment* and *alignment* represent mid-points between the three other markers.

**Table 21:** Alignment between Teacher Espoused Beliefs and the Curriculum

| School        | Program     | Participant            | Belief Alignment with Expressions | Belief Alignment with CA Math |
|---------------|-------------|------------------------|-----------------------------------|-------------------------------|
| High Bluff    | Expressions | Kelly<br>Barbara       | Strong<br>Partial                 | No<br>Partial                 |
| Hillside      | Expressions | Hally                  | Partial                           | Partial                       |
| Pacific Shore | CA Math     | Bill<br>Hannah<br>Fran | Weak<br>Partial<br>Partial        | Alignment<br>Partial<br>Weak  |
| Little Rock   | CA Math     | Liz                    | Alignment                         | Weak                          |

*Note.* The alignment scale is: alignment, weak alignment, partial alignment, alignment, or strong alignment.

The data in Table 21 indicates that two of the seven teachers have solid alignment between their beliefs and the curricular program in use. Kelly from High Bluff Elementary shows strong alignment with the philosophy of *Math Expressions* and her own beliefs about teaching and learning mathematics. Bill, from Pacific Shore Elementary, espoused beliefs in his interview which demonstrated clear alignment with the instructional approach supported by *California Math*. However, Liz demonstrates weak alignment between her beliefs and the curricular programs she is using. Additionally, Hannah and Fran at Pacific Shore went against the district mandate and

implemented *California Math* even though evidence of their beliefs would not predict such a decision. Because the unexpected curriculum choices strongly influenced instructional quality in these classrooms, the role of different factors as they affect curriculum decisions and interplay with instruction are investigated in the remainder of this chapter.

### **Context-Specific Variations in Factor Interaction**

It is the misalignment of individual factors one might expect to mediate action and the action actually taken that brings to the forefront the role of social and institutional context in privileging certain factors over others. Each school context in this study helps to highlight the situated nature of mathematics instruction. It is situated within the individual knowledge and beliefs of the teacher, within a classroom among students, within a grade level team, within a school, within a community, within a district, a state, and a nation and within the context of district, state, and national policies.

In the remainder of this chapter, relationships identified earlier between factors will be analyzed within individual cases. Additionally, the cases will be examined in interactive within-school clusters. Finally, an explanation will be constructed regarding the contextualized interaction of personal and environmental factors as math instruction comes to be expressed.

**Pacific Shore Elementary: Beliefs, Collegial Interaction, and Institutional Accountability Context**

The teachers at Pacific Shores Elementary provide an example of how collegial interaction, influenced by beliefs and professional status, affects instructional decision making. Additionally, the institutional accountability context in this high income/high performing school plays a prominent role in mediating the expression of classroom instruction. The three teachers at this school, Bill, Hannah, and Fran, demonstrate a common instructional profile. The variability in their beliefs and knowledge levels indicates other factors must be influencing their shared instructional characteristics. An examination of how personal and environmental factors interact among these teachers highlights the power of context in affecting instructional action.

After observing three different lessons for each teacher, using the Mathematical Quality of Instruction instrument (Hill, 2010), a common instructional profile for the fifth grades at Pacific Shores was identifiable. In these classrooms, instruction is characterized by high amounts of direct instruction specific to problems presented out of context. A typical lesson involves teachers presenting a procedure for how to solve a type of problem with the goal being students' replication of the procedure. Alternative solution methods are neither presented nor elicited from students. Students spend most of their time practicing application of procedures under the close supervision of their teachers. Math language is accurately used by the teachers to support the presentation of content in instruction. In alignment with the instruction presented, remediation of student errors tends to be procedural. The teachers are accurate in their presentation of content and provide clear procedural explanations. There are few occurrences of students or the

teacher explaining the mathematics underlying the procedures. Student questions in these classrooms tend to focus on *how* to apply procedures rather than *why* they work. There are stylistic differences between classrooms in how lessons are presented but the underlying pedagogy and content remain consistent.

As Table 19 shows, instructional quality characteristics across the Responding to Student Thinking, Students Explain their Thinking, and Multiple Solution Methods MQI domains appear related to the curriculum in use. All teachers at Pacific Shore Elementary used *California Math* and show similarly low scores across the three domains. However, the path from curriculum to classroom instruction is laden with mediational interactions precluding a direct relationship from being drawn between materials and instruction.

An examination of the knowledge and beliefs characteristics of the teachers reveals variability and therefore cannot account for the instructional similarity. Additionally, dissimilarity between their beliefs and knowledge causes one to question how the three teachers agreed upon the procedural, basic skills philosophy of the *California Math* textbook. An explanation of the teachers' curricular decisions and expression of instruction must be built from an examination of interaction of individual and environmental factors. To build this explanation, I will begin by establishing a profile of each teacher's beliefs and knowledge levels and will then examine the role they played in curriculum selection and how these factors interact within their social and institutional context.

**Bill: Beliefs and institutional accountability context.**

Bill is the mathematics leader on Pacific Shore's fifth grade team. He has established a professional reputation at his school as being a strong math teacher. This was made clear through my informal conversations with the principal as well as other teachers on Bill's team. When combining information from the IMAP Beliefs Survey (Philipp & Sowder, 2003) and from his interview, Bill demonstrated a strong philosophical belief that students learn mathematics through repeated practice on basic skills. Additionally, with a MKT score of 0.95, he showed mid-levels of mathematical knowledge for teaching relative to the participants in this study.

Contrary to the core beliefs described above, on the IMAP Beliefs Survey, Bill provided strong evidence of four of the six reform-oriented beliefs focused on the interconnected nature of mathematics, the generative nature of learning concepts before procedures, and the belief that students can solve problems in novel ways before being shown procedures. However, statements made in his interview were so powerful and the beliefs he expressed were so strongly embedded in his life experiences, they were given more weight when establishing a portrait of Bill's mathematics philosophy.

In an interview, Bill demonstrated how his traditional beliefs are aligned with the tenets underlying the *California Math* program:

There's two parts of the present adoption. There's the Math Expressions, which they would like us to use, and the California Math. So those are two different components. The Expressions I feel pushes algebra real quickly and to the expense of number sense, where you know there can be a point where you're trying to get kids so proficient in algebra that you start to invade other components of math that are very important and number sense is one of them. That in Math Expressions is the big flaw to me with it, and the second component that's a flaw is it's very basic, very

simple. Now the California Math is very simplistic. You saw some lessons I did in that today. It focuses on one or two items and it does a good job with that. It keeps it simple. The part I don't like about it is it doesn't provide enough challenge for all the kids. There are challenge aspects to it, and I will include those in my lessons, but I never assume that every kid understands everything because once you do they don't. (Interview, 6 Oct. 2010)

Noteworthy in his justification for his program choice is his emphasis on “number sense” which he later clarifies as computation with numbers such as identifying place value, adding, subtracting, multiplying, dividing, and math facts. Bill’s examples of number sense encompass only one aspect of NCTM’s (2000) understanding-based definition. NCTM recommends that teachers help students understand numbers, ways of representing numbers, relationships among numbers and number systems, understand the meaning of operations and how they relate to one another, and compute fluently and make reasonable estimates” (pp. 32). Bill’s strongly articulated emphasis on the computational fluency aspect of number sense aligns well with *California Math*.

Bill further explains the depth of his beliefs about teaching and learning mathematics when describing the components of an ideal lesson.

I think an ideal lesson is where you provide examples, you explain it to them, you give them practice time and when they’re practicing, you’re looking at what they’re doing and as you’re viewing what they’re doing, you try to intervene and make as many corrections as you can. (Interview, 6 Oct. 2010)

In accordance with these beliefs, during all three lessons observed, Bill demonstrated a lesson format aligned with the instructional format of *California Math*: present a skill or procedure and then provide individual support as students apply the skill to practice problems.

Bill used a sports metaphor to capture his core philosophy regarding the value of practicing basic math skills. By comparing high level football quarterbacks practicing short passes every day even though they can throw further to the need for students to practice basic math skills every day he emphasized his deeply held belief about how students learn mathematics: “I make sure every day we’re in the basics and every day I’m pushing kids. . . . You need to be manipulating numbers all the time. I really believe that” (Interview, 6 Oct. 2010). Bill’s explanation demonstrates his belief alignment with the philosophy of *California Math*. He believes that students become strong mathematicians by repeatedly practicing basic skills. Through his comments, Bill captures the “keep it simple” philosophy he has about developing students’ mathematical skills. His strongly held philosophy played a dominant role when he and his teammates agreed to override the district’s mandate to use a research-based math program. This philosophy also seemed to play a stronger role than Bill’s knowledge in the quality of his mathematics instruction as evidenced by the low mathematical quality of instruction scores characterizing Bill’s math lessons.

According to MKT/MQI correlational studies (Hill et al., 2008), Bill’s mid-level performance on the Mathematical Knowledge for Teaching (MKT) assessment should be predictive of mid quality instruction in the domains of Responding to Student Math Productions and Mathematical Errors. This proved true for the domain of Mathematical Errors. However, the fact that all the teachers in the study, regardless of knowledge levels, delivered mostly error-free instruction calls in to question the validity of the relationship. Bill earned a mid-level score, 0.95, on the MKT yet he did not show

evidence of mid-quality responses to student thinking in his instruction as Hill et al.'s (2008) results predict. A closer look at Bill's responses to MKT items reveals that he had difficulty on items requiring him to understand students' nontraditional computational strategies. This could partially explain the low level MQI scores along the domain. However, his instruction is more likely related to his belief that students need to be shown how to solve problems and then given time to practice the procedure. Instruction filtered through this belief does not elicit unconventional student math productions. As a result, his struggles on the MKT assessment are likely related to his lack of classroom experience eliciting and responding to student's mathematical strategies.

On the Classroom Video Analysis (CVA) measure of knowledge, Bill showed that he is aware of different models for making content conceptually accessible to students. He showed mid-level knowledge, 0.88 (on a 0-2 scale), in his focus on how the teacher makes the mathematics content understandable to students. For example, after watching a video clip of a teacher helping students use fraction circles to model  $\frac{2}{3} \div 4$ , Bill wrote, "The instructor does a good job of making the students create fractions with manipulatives. Then posing problems for the students to solve with the manipulatives. This is a good lesson to visually show the students how dividing fractions works." While Bill demonstrates appreciation for and understanding of conceptual instruction, his own mathematics instruction does not reflect this knowledge.

Bill's instruction is instead more aligned with his espoused traditional beliefs. This observation suggests that Bill's beliefs are highly influential in mediating his knowledge as Bill expresses mathematics instruction. One possible explanation for the

primary role Bill's beliefs played can be found in the policy context. Bill attributes his current instructional characteristics to his response to accountability demands:

Well, it comes top down. There's no secret about that. Whatever you're judged on as a professional in any industry, whether it's profit, wins/losses, you're going to focus on that. Because your principal, because that's their job, they're going to focus on it, and much of our funding in our state is based upon the API scores. I don't blame anybody in our district for focusing on that. That's the reality of the situation. So what happens is everybody kind of narrows their instruction down to that. Really, that's what's happened in math. You've had instruction that was really wide and it's kind of narrowed down to attend to a test not to attend to the needs of math always, like the application components. So I think that instruction over the years, in the fifteen years I've taught, it's narrowed down greatly. (Interview, 7 Oct. 2010)

In Bill's statement, he pragmatically describes his awareness and acceptance of the current policy context, including skill-focused state content standards and their accompanying skill-focused state tests. It helps explain the narrow skills focus of his instruction. It also helps explain why he might select a math program aligned with the California accountability policies. However, when he shifts his focus to what his current students are actually learning, Bill laments how his own teaching and philosophy has changed as a result of the institutional context. In the following statement, Bill is responding to a question about how he feels the "narrowing" of mathematical content he described affects students.

Oh, in the long run, I think it affects them greatly. I mean, you're building, it's like building a house upon sand. You know they don't have that, they might be able to do the math but can they apply the math? . . . I have not just forgotten how to teach like I used to, you know you slowly mold into something that follows a different path and that part of me, I think I need to get back to incorporating more of those lessons, more of those holistic lessons that approach the topic conceptually and applies it. .

. . but it doesn't fall in line with our goals as a school or as a district.  
(Interview, 7 Oct. 2010)

In this statement, Bill attributes a process of being molded by institutional structures and goals. The phenomenon he described exemplifies Weick's (2005) claim that individuals take action which seems appropriate within their cognitive frames but as the situation and cues evolve, new sense is made and new actions are needed and seem more appropriate. Unfortunately, I cannot go back in time to capture Bill's past instruction and beliefs and to track how the policy context may have influenced change in his beliefs and practice. However, I can say with certainty that the current assessment and accountability context supports the prominent traditional beliefs that are most strongly expressed in his interview. The beliefs that Bill has developed, bolstered by the accountability context, strongly mediated his selection of curricular materials. The curriculum, in turn, plays a prominent role in the format and content of his instruction. In a sense, the textbook is a tool through which Bill is able to enact his beliefs which are influenced by the policy context. Given Bill's status as the math leader on the team, his input in the curricular materials selection played an important role in the program that his colleagues use and the instruction they generate from those materials. He provides an example of Coburn's (2001) theory on collective interpretations of policy wherein teachers' sensemaking is most influenced by those with whom they work. Additionally, she claims dominant personalities within a group tend to influence the worldviews and practice of the less dominant personalities. That proves to be the case for Bill's colleagues. As such, social interaction was a critical vehicle upon which Bill's beliefs were carried across his team and mediated their instruction.

**Fran: Institutional accountability context and collegial interaction mediate knowledge and beliefs.**

Fran, the second teacher from Pacific Shore Elementary provides an example of how social interactions and the institutional accountability context can overpower individual beliefs and knowledge. Due to her strong reform-oriented beliefs and mid-level MKT, in a different team, school, district, state, time . . . Fran's instruction would likely look very different. However, like the other teachers on her team, Fran's instruction maintains the core content, structure, and pacing of the *California Math* program.

Fran's mathematics lessons exemplify the Pacific Shore instructional profile. While her overall instruction is attributable to the curricular materials in use, small variations in her instruction from that of her colleagues are attributable to her beliefs and knowledge. Of the three teachers, Fran tends to spend slightly less time on direct instruction and provides more time for students to work together. When she implements the *California Math* lessons, she adds additional questions and real-life applications which provide a higher level of cognitive activation for the students than is present in the textbook lessons.

On the MKT, Fran demonstrated knowledge similar in level to Bill, 0.95, but with no identifiable pattern in the types of items she was good at or struggled with. Like Bill, Fran's mid-level knowledge did not afford her the hypothesized mid-level instructional quality. Instead, with the exceptions noted above, she, along with the rest of the teachers at her site (and along with the other teacher in the study using *California Math*), demonstrated instructional quality in the lower ranges.

Contrary to her instructional characteristics, Fran's espoused beliefs tend toward misalignment with the repeated practice philosophy of *California Math*. On the IMAP Beliefs Survey, she showed strong evidence of holding four of the six reform-oriented beliefs. These beliefs include believing conceptual understanding is more powerful and generative than procedural knowledge and that students should learn concepts prior to procedures. In her interviews, she reported believing math should be about finding out "why" and pursuing interesting ideas. She believes math should be taught in interesting ways so that kids can think in ways they have never thought before. Creating an environment that is "slightly fun" where all students feel free to contribute and all students are learning is important to her. Additionally, she believes in differentiating instruction to meet the needs of different levels of learners. The only belief espoused by Fran which does align with the *California Math* philosophy is that students should review skills regularly to develop mastery. With a reform-focused constellation of beliefs, it would make sense for Fran to advocate for *Math Expressions* as the core curriculum and for her instruction to contain more conceptual characteristics. However, during the time of the study, the institutional accountability context, as described in Bill's case, carried more weight in the curriculum decisions.

With Fran's beliefs and knowledge not playing primary roles in affecting her expression of math instruction, the explanation-building focus shifts to other factors. One already noted factor is the role of curricular materials and what influenced their selection. Bill's system of beliefs and high status role on the grade level team can account for part of Fran's agreement to use *California Math*. However, the accountability context was high on Fran's list of reasons why she selected the program.

Pacific Shore's high academic performance status is accompanied by pressure.

For Bill the pressure was felt as an overall climate of accountability while for Fran, it was more specific. For Fran, accountability pressure came from the parents and the way she interpreted that pressure is related to the curricular materials she chose and the instruction she generated. For example, in an interview, Fran described the district's math placement policies, based on student test scores, as the root of parent pressure. She described feeling that she had to deviate from her beliefs about teaching and learning mathematics as a result.

And you know, especially in this community, the big thrust is math and advanced math and it's a symbol of status that your kid is in pre-algebra or they're taking AP Calculus. And now we're hearing from the high school about all the kids who are losing interest in math and are floundering because they missed those stepping stones that are so important down here -- the fundamentals about you know, this is, fifth grade is a time when we should be exploring things and talking about things and using those manipulatives. But, you know, we don't have the time so we're just covering the material and then the parents are pushing, pushing, pushing to see results. And to them results are 100% on a math test. And then, "Did my kid get this score in MAPs so they can go in pre-algebra?"  
(Interview, 10 Oct. 2010)

As Fran described, she modifies her instruction to respond to the fast-paced procedural focus she believes is necessary to support her students in getting high test scores which will place her students in high math tracks and satisfy the parents. This style of practice is validated through her students' high math testing performances and through the support of Bill who believes in practicing the basics. The primary mediators of the content and format of Fran's instruction are the accountability context and resulting curricular materials. However, Fran's beliefs and knowledge also play a role in mediating her in-the-moment instructional decisions. She adopts the basic skills focus of *California Math*

with questioning strategies focused on how to apply procedures rather than her espoused belief in *why*-oriented instruction. Yet, during instruction, she adds real-life examples to the lessons and incorporates games.

When analyzing Fran's Mathematical Quality of Instruction scores, it is clear that the decision to use *California Math*, which was mediated by the beliefs of a colleague and the institutional accountability context, is highly related to the overall low quality of her instruction. As with all participants in this study, Fran has strong scores in her use of mathematical language and in the accuracy and clarity of her instruction. Yet, she scores low in all reform-oriented aspects of her instruction.

**Hannah: Social interaction mediates beliefs and knowledge.**

Hannah, the third teacher from Pacific Shore Elementary in this study, provides an example of how a teacher's low knowledge levels may influence her beliefs and make her instruction more likely influenced by collegial interactions. Hannah moved from teaching third grade to teaching fifth grade one year prior to the study. She acknowledged not feeling completely comfortable with the fifth grade content. Unlike Bill who had a solid grasp on the mathematical knowledge necessary for teaching this grade level and was purposeful in his pedagogical decisions, Hannah seemed to be doing what she knew how to do without knowledge of alternative strategies for teaching the content.

Hannah's instruction reflects the Pacific Shore instructional profile with slightly nuanced differences from her colleagues in her explanations (she sometimes tries to figure out why a procedure works) and in her remediation of student errors (she goes

beyond procedural remediation and attempts to address the student's underlying confusion). Combining the results of the IMAP Beliefs Survey and Hannah's interviews produces a portrait of Hannah as a teacher who does not feel comfortable with her procedural instruction but may not know specifically how to change it.

On the IMAP Beliefs Survey, Hannah showed no evidence to weak evidence of five of the six reform-oriented beliefs. Yet, Hannah's relatively low MKT score, 0.51, may indicate the interference of knowledge and grade-level experience in her responses to the survey items. For example, after watching a video of a student being shown how to perform a division of fractions procedure by inverting and multiplying the second fraction, Hannah predicted the student would successfully be able to replicate the procedure three days later. When she watched the next video which took place three days later, Hannah was surprised to see that the student had forgotten the procedure. She wrote, "Oh no, she forgot the steps!" and predicted that it would be uncommon for students to forget the steps of a procedure such as this student had. All other teachers in this study, regardless of whether they initially thought the student could replicate the procedure, commented on their experience with the typicality of this student's error. Despite the fact that the item was designed to measure Hannah's belief about procedural instruction, Hannah's inexperience at this grade level, and related lower MKT, may have affected her response.

Hannah's interview provided additional information to support the hypothesis that Hannah's beliefs about teaching mathematics reflected on the IMAP Beliefs Survey may be more reflective of her MKT than to her true beliefs. In her interview, while her beliefs

statements stayed quite general, Hannah indicated discomfort about how she was teaching mathematics versus how she believed she should be teaching mathematics. When discussing the fact that her daily instruction involved fast-paced skills lessons trying to get the “finish line” (state testing), Hannah commented on how this environment affected her students. In the statement below, she also proposed that exploration activities would be a better instructional strategy.

I think maybe it affects them being able to think outside the box, so I think that they're only thinking the way that I'm teaching them and the book is teaching them, but I think that doing the explorations, giving them the way of going look I could try it this way, or just different techniques and different skills. (Interview, 28 Sept. 2010)

It is noteworthy that although Hannah was dissatisfied with the instruction she provided to her students, her alternative ideas remained quite general. Hannah appears unsure of how, specifically, she could go about making the math concepts she is teaching more understandable to students. This uncertainty seems reflective of lower specialized knowledge for teaching the math content. In additional responses, she said math learning should be fun and fast paced, including explorations, visual supports, and opportunities for students to feel successful.

The lack of specificity in her ideas about math instruction surfaced again in her CVA responses in which her comments about the lesson content averaged 0.22 (on a 0 – 2 scale). Her responses tended to focus on general pedagogy not specific to mathematics. For example, after viewing a video clip of a teacher supporting a student as he tries to represent two different fractions on a number line in a contextualized problem, Hannah wrote, “Wow – the teacher was very patient with the student. I like how she gave him a

lot of wait time, and helped him figure out the problem better – it looked like he was just about to figure it out.” Her response indicates awareness of general teaching practice (wait time) but not of mathematically specific pedagogy (specialized content knowledge). She does not comment on the effectiveness of the model or the specific understanding of the child, both of which would indicate higher levels of knowledge for teaching mathematics.

Due to Hannah’s discontent with her *California Math*-based procedural instruction, her unclear beliefs, and her relatively low levels of specialized content knowledge at this grade level, it is not discernable what role personal factors played in Hannah’s agreement to adopt the traditional program. However, when asked, Hannah attributed her curriculum selection to the wisdom of her more experienced colleagues and her basic skill-focused instruction to the accountability climate at her school. She said that she did not want to teach anything different than her colleagues as parents regularly compared what page in the math book each class was on and questioned her if she fell behind or did something different. In response to an interview question about the curriculum she was using in her classroom, Hannah explained how she had deferred the curriculum decision to her more experienced teammates, Bill and Fran.

The pacing is different and then the way it's...what skills you're doing during the year is different and just looking at it, I mean this was more me coming from the two teachers who have been teaching 5th grade longer. They said this is not...the other one they spent a lot of time on one skill and it was something that you didn't really need to spend a lot of time...and then it was just really weirdly paced. So we switched to the hard cover book and we prefer that. It's a lot quicker paced, but it's more like things they've already known we're just going a little bit further in depth so we just prefer that book over. (Interview, 28, Sept. 2010)

Hannah's espoused reason for using the program is less developed than Bill's and she attributes it to the reasoning of her more experienced colleagues. It is not clear which program Hannah would have chosen if the decision had been her own or if she had been among colleagues with different beliefs, but it is clear that her instruction is strongly influenced by the program in use. Its selection was influenced by her feeling of lesser status on her team which stemmed from inexperience at the grade level, lower knowledge levels, and underdeveloped beliefs about teaching and learning mathematics. Ultimately, her social interactions with her colleagues, perceived accountability pressure from parents, and the resulting textbook proved to be strong mediators between her knowledge, beliefs, and instruction.

### **Summary.**

The three cases considered here demonstrate how knowledge, beliefs, and interpretation of policy interact differently depending on the person and the situation. In studying the case of Bill, his beliefs and his interpretation of the policy context serve as strong mediators of knowledge and instruction. In studying Fran, collegial interactions and the accountability context come through as strong mediators of beliefs, knowledge, interpretation of policies, and instruction. And Hannah's case suggests the possibility that lower levels of knowledge create a situation where other factors become more dominant. Because all teachers at Pacific Shore demonstrated lower instructional quality relative to other teachers in the study, examining a teacher in a different school and different social context using the same curriculum helps clarify the role of curricular materials.

### **Little Rock Elementary: Accountability Context and Curricular Materials**

Liz, the only teacher from Little Rock Elementary, provides a comparison for the cases at Pacific Shore as she, too, used the *California Math* program but was situated in a very different school context. Liz is the one teacher in the study from a low income, high performing school. Little Rock Elementary is considered, within the district, a success story. School leaders attribute systemic changes focused on a college-going atmosphere and data driven instruction as turning the school around from the lowest performing school in the district to among the highest performing schools. The teachers in the school frequently administer the Measures of Academic Progress (MAP) (Northwest Evaluation Association, 2011) assessment, an online, adaptive test, which has been shown to be predictive of scores on the California Standards Test. They meet in grade-level teams regularly to discuss student progress and to plan additional academic support. This context of data and accountability is important in understanding Liz's decisions and instructional quality.

#### **Liz: Accountability context and curricular materials.**

Within this study, Liz demonstrates the highest MKT score and demonstrates strong evidence of holding reform-oriented beliefs. However, her instructional profile does not bear the characteristics that researchers would predict. The way Liz makes sense of the accountability context of her school and the selected curricular materials appear to mediate her beliefs and knowledge in her expression of instruction.

Liz's mathematics instruction, which I observed three times, is characterized by high amounts of direct instruction, clear procedural explanations, overt and precise

vocabulary use, and high student participation. However, her MQI scores in most domains paralleled those of the teachers at Pacific Shore: no emphasis on why procedures work, no visual representations, no contextualized problems, and questioning emphasizing the steps in a procedure. On the MQI, Liz's instruction is characterized by low scores across the domains with a few subscale scores which are higher. Her instructional strengths appear to be in the clarity of her explanations, 1.47, and in the density and fluency of her use of math language, 2.27. Data cited earlier indicated a possible pattern connecting teachers' mathematical explanations with their levels of MKT. This could indicate that Liz's high MKT score, 1.43, afforded her the ability to clearly explain mathematical ideas. Her high MQI score regarding use of mathematical language was common across the teachers in this district.

While Liz's instructional characteristics would lead one to believe she held traditional beliefs about teaching and learning mathematics, that is not the case. Liz's beliefs tend toward reform-oriented perspectives. On the IMAP Beliefs Survey, she showed evidence to strong evidence of five out of the six reform-oriented beliefs. In her interviews, Liz espoused the value of understanding mathematics conceptually before learning procedures, visualizing math concepts, and applying mathematics to real life. She said,

I do believe in using the shortcuts because I know that as the expectations and the skill levels increase they're going to need those fast methods for solving problems, but I always try to give them those deep understanding pieces first. (Interview, 12 Oct. 2010).

She also reported valuing critical thinking and problem solving as key elements of a strong mathematician. She solidified these claims when responding to a question regarding what a successful math student can do:

I think for a successful math student it's not just about getting a correct answer. It's about really being able to visualize and understand what that means, being able to picture if you're working with fractions what that would look like, being able to apply it to real life. It's not just about a memorization of a fact or a formula but understanding what that means and how it applies to life. (Interview, 28 Sept. 2010)

This statement demonstrates at least partial alignment between Liz's espoused beliefs with the philosophy of *Math Expressions*. It definitely does not demonstrate alignment with the repeated practice of decontextualized problems prevalent in *California Math* and characteristic of Liz's instruction. However, Liz, along with her team, selected *California Math* as their core program. Liz contextualizes this decision in the academic accountability climate of her school and the larger educational community. In a response to an interview question about the differences between the programs, Liz said,

I think there's a different philosophy involved and I think that Math Expressions tries to use more manipulatives, tries to add a lot more student leadership into the lessons and some of the concepts are taught in a really creative way that I really liked and valued, but the pacing is terrible. . . . The scope and sequence was also not conducive to covering all the standards before state testing and there are a lot of standards that were completely neglected. (Interview, 12 Oct. 2010)

As her statement reveals, Liz's regard for conceptual instruction is overshadowed by the skills upon which students will be tested. Her espoused beliefs are not enacted but are instead mediated by her interpretation of the data and accountability climate. This is

evident in her interview when she discussed what she appreciates about the *California Math Program*. She cites program strengths as the a solid skill foundation it builds, its vocabulary emphasis, its inclusion of easily accessible examples, repeated practice problems for each problem type, and daily skill review. This list of program strengths diverges from Liz's espoused conceptual beliefs.

Liz's mathematics instruction parallels the instruction of the teachers at Pacific Shore who use the same curricular program. At each site, different factors mediated the selection of curricular materials but the end result was the same – low quality, procedural mathematics instruction. Liz provides an example of a teacher with relatively high MKT and reform-oriented beliefs who adopts a procedurally-oriented textbook to accommodate the accountability environment. The context and materials mediate her knowledge and beliefs resulting in instructional characteristics different than one might expect.

#### **High Bluff Elementary: Social Interaction and Curricular Materials Mediate Beliefs.**

Two teachers from High Bluff Elementary, Kelly and Barbara, participated in this study. They provide validation of the social pattern observed at Pacific Shore Elementary: the teacher with the stronger beliefs and greater status as a math teacher will influence the decisions of colleagues (Coburn, 2001). They also demonstrate the interactivity between curricular materials, teacher beliefs, and teacher knowledge.

The instructional profile of the two fifth grade teachers at High Bluff Elementary contained some similarities but many differences from the profiles of the teachers at the previously discussed schools. Similar to the schools using *California Math*, both teachers

spent most of their lessons providing direct instruction. Unlike the other teachers, the problems students worked in these classrooms were contextualized (a characteristic of the problems in the *Math Expressions* textbook). Like the other teachers in the study, they both delivered clear, relatively error-free instruction. Yet, the questioning they used while implementing tasks generated higher levels of cognitive activation for students than did the teachers at Pacific Shore or Little Rock (users of *California Math*). Both teachers listened to and responded productively to student thinking during mathematics instruction at much higher levels of quality than the previously described teachers. Also similar to all teachers in this study, both Barbara and Kelly's instruction demonstrated a minimal presence of student-to-student interaction in class discussions, math generalizations, and higher level student questioning and reasoning.

Given the relationships identified between curricular programs and instructional quality, the first factor I will consider is the curricular program. Kelly and Barbara chose to implement the *Math Expressions* program. They both reported feeling they could have chosen a different program if they wanted but they believed in the philosophy of *Math Expressions* (Kelly, personal communication, 13 March, 2011; Barbara, personal communication, 16 March 2011). As previously described, the program emphasizes conceptual understanding before procedural knowledge, conceptual models, having students explain their thinking, and mental math strategies. Throughout separate interviews, Kelly and Barbara demonstrated different levels of philosophical alignment with this program. However, like Bill's role at Pacific Shore, Kelly's stronger philosophy and role as the math leader of her team plays a mediational role in Barbara's

curricular and instructional choices. Additionally, Kelly's professional development experience with *Math Expressions* seems to play a critical role in her own implementation of the program and the resulting quality of her lessons.

**Kelly: Beliefs, knowledge, curriculum, and professional development.**

Kelly is the first teacher I will discuss from High Bluff Elementary. Like Bill at Pacific Shore, Kelly is given the highest status as a math leader at her school site. She volunteers on math leadership committees and her teammates come to her when they need help. While Kelly and Barbara showed commonalities in their instructional characteristics, Kelly demonstrated higher quality instruction across all domains. She typically had the highest sub-domain instructional scores out of all participants in the study. Her greatest strengths were in her mathematical explanations, use of mathematical language, inclusion of multiple procedures, and responses to student math productions.

As has been demonstrated in this study, it does not appear that differences between Barbara and Kelly's MKT levels can account for the qualitative differences in instruction. Both teachers scored in the top half of the sample but Barbara, who had lower MQI scores relative to Kelly, produced a higher MKT score, 1.39, while Kelly, who showed higher MQI scores, scored slightly lower on the MKT, a 1.14. It should be once again noted that the MKT is designed to identify relationships among groups of 60 or more participants, therefore the lack of correspondence between the MKT/MQI scores should not be taken as a reflection of the instrument's validity. However, the CVA provides qualitative data which gives more specific insight into the knowledge/instruction relationship among my smaller sample size. Out of all the participants, Kelly

demonstrated the most detailed level of attention to mathematical content when viewing the classroom video clips. For example, she watched a video clip of a teacher directing a lesson in which he instructs students to create an equivalent fraction by multiplying the numerator and denominator by the same number. In the video, a student raises her hand and asks a question and the teacher provides an answer. Kelly responded,

I didn't feel like the girl understood at the end of the clip. She did say, "Oh, ok," but I think she was still unsure about the rule and why it worked. I didn't get to see the earlier problems that the class worked together and independently, so it's hard to tell how they derived that rule. If it were done to accentuate the meaning of what is happening when an equivalent fraction is made, I think the teacher could have referred back to some of that thinking to show how the rule works in every situation.

In this response, Kelly attends to student understanding, the meaning of the mathematics, and the generalizability of the mathematics. In Kelly's own instruction, she constantly checks in with students to assess understanding and reteaches in different ways to increase the chances for deeper understanding. Through this connection, Kelly's higher scores on the CVA provide an indication that her knowledge may play a role in her instructional quality. As Kersting et al. (2009) might say, perhaps the MKT measured inert knowledge while the CVA measured knowledge that is accessible and applicable in teaching situations (Bransford et al., 2000). However, once again, the relationship is not direct and other mediational factors must be considered.

The first additional mediating factors I will consider are Kelly's beliefs. In the interview statement below, Kelly explains her belief that students should understand the mathematics they are doing.

You know my hope is that they become risk takers and that they're willing to ask questions and they don't feel afraid of math or that "I don't like math." I hope it's presented in a way that they feel capable with it and that they understand...and I think the most important thing is that they understand what's really happening. I mean there was the girl who said, "Well I just multiply the denominator times the other side." You know, yes that totally worked on the four or five that we've done so far, but when the numerator isn't one that doesn't work so I didn't want her to think well that's all you just ever have to do. I mean that was kind of jumping ahead and she may not have even really got what I was trying to say but it is important to me that they know what is going on and why. I remember a 6th grade teacher of my own that actually taught us when dividing fractions that *ours is not to reason why, just invert and multiply*. It was a helpful little rule but you know it was like I was an adult taking teaching classes before I really understood what was going on with dividing fractions and I was a very capable mathematician. . . . It's like if you don't understand the point of the rule it's awfully hard to remember or if you happen to forget the rule can you recreate it from what you actually know? I just think mostly that they are willing to ask questions and that they are trying their best to understand what's really going on with whatever concept it is that we're learning so that it's not just a quick and fast computation and so that they could apply what they learn in later situations that aren't exactly the same. (Interview, 1 Oct. 2010)

As demonstrated in her interview, Kelly strongly believes students should understand the mathematics underlying their work. She validated these beliefs in her responses on the IMAP Beliefs Survey demonstrating strong evidence of all six of the reform-oriented beliefs.

One additional factor must be considered when accounting for the quality of Kelly's mathematics instruction. Kelly attended two district professional development courses (40 hours each) focused on understanding the philosophy and instructional strategies advocated by Fuson (2009), author of *Math Expressions*. This professional development likely mediated Kelly's instruction. In an interview, Kelly described how

the professional development she attended helped her to grasp the philosophy behind the curriculum:

Well I did the Expressions TLC one and two. I don't think I did three. You know I really liked that idea of understanding the philosophy behind the curriculum because...it's just like in my reading instruction, I don't use the [Houghton Mifflin] textbook but I have a strong understanding and philosophy of where I want...what I want the kids to know and what I want them to do, and so if I...it helps me make the decisions as to what I do each day. And in math its similar because even though I am using a textbook and I am basically following the lessons that they're offering with a good understanding of the philosophy of the text then I know well why are we spending all this time on these four word problems for example. There's a real focus on explaining their mathematical thinking and really trying to understand the things that are going on. It's not so much like here's the rule and you have to do a bunch of problems to show that you can compute. It's much more like you...we really want to understand what we're doing. Just...I think I said it today like if you can do it with  $2 \times 3$  and you really understand what's happening then when it's  $18 \times 27$  then it doesn't get confusing. (Interview, 24 Sept. 2010)

As Kelly explained, she feels that understanding a program's philosophy helps her make sense of the reasoning behind the lessons which, in turn, affects the quality of her implementation of it. Kelly's explanation of the philosophy of *Math Expressions* aligns with the instruction I observed in her classroom and with her espoused beliefs. It is not clear how much the professional development affected her knowledge and beliefs as the study took place after the she had completed the courses. However, it is clear that in Kelly's case, knowledge and beliefs play primary roles in her expression of instruction. The alignment between her beliefs and the philosophy of *Math Expressions* guided her selection of curricular materials. Her beliefs appear to support the continued development of mathematical knowledge for teaching along research-recommended trajectories. And, her higher instructional quality scores, relative to other participants in this study, reflect

that alignment. The strength of Kelly's beliefs also played a mediational role in her interactions with Barbara.

**Barbara: Beliefs, knowledge, curriculum, and collegial interaction.**

Throughout her interviews, Barbara, the second teacher at High Bluff Elementary, indicated respecting Kelly's faith in their core math program. It was clear that she placed great value on Kelly's mathematics instruction expertise. Barbara's instructional profile, as described earlier, is similar in many ways to Kelly's. Like other participants using *Math Expressions*, Barbara demonstrated instructional quality that was generally higher than the teachers using *California Math*. Her relative strengths, although not as strong as Kelly, were in the areas of providing explanations, explicitly using math language, remediating student errors, responding to student thinking, and enacting tasks in a way that provided cognitive stimulation for students.

Barbara's beliefs were not as strongly articulated and aligned with the principles underlying *Math Expressions* as were Kelly's, but her partnership with Kelly gave priority to certain aspects of her beliefs in selecting curricular materials. In her interviews, Barbara articulated believing students should have the opportunity to think about why they are doing certain steps when applying procedures and to connect math to real-life situations. She believes in making the math environment comfortable for all students. She is also opposed to wasting class time on mastery of basic math facts. In her IMAP Beliefs Survey, Barbara demonstrated evidence that her beliefs are partially aligned with the reform-oriented framework. She showed evidence that she believes conceptual understanding is more powerful than procedural knowledge and that

conceptual instruction should precede procedural instruction. She also validated her interview responses in the survey by demonstrating the belief that real life contexts support students' mathematical learning. These beliefs align with some of the tenets of *Math Expressions* and the beliefs of her colleague.

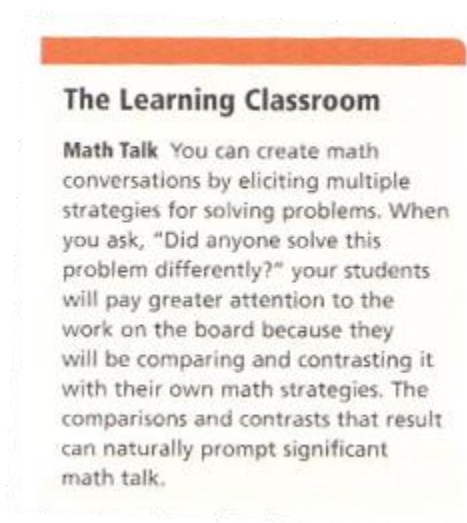
In contrast, in her interview, Barbara also indicated that she believes mathematics is a linear, logical, step-by-step, procedural discipline. She explained that her vision of a lifelong mathematician is, "Somebody who takes their time to read the question thoroughly and knows the steps of how to get it" (Interview, 9/27/10). When comparing writing instruction to math instruction, she said,

I just think that of all the things to do, writing is the toughest because you don't have a prescribed formula. . . . It just seems like in writing they've got all these other things to think about than they do when they're just doing a math problem. (Interview, 27 Sept. 2010)

Barbara's view of the linear nature of mathematics also surfaced in her responses to the IMAP Beliefs Survey. On the survey, she did not show evidence of believing that math is an interrelated web of concepts, that knowing how to apply a procedure is not a guarantee of understanding (although she does show evidence in the interview), or that children can solve problems in novel ways without being shown first. The absence of several reform-oriented beliefs coupled with her description of mathematics as a procedural, formulaic discipline indicate that her beliefs are also partially aligned with the philosophy of *California Math*.

Barbara's espoused belief in the philosophy of *Math Expressions* can partially be accounted for through the mediational role played by her social interactions with Kelly.

Additionally, Barbara's interactions with the curricular materials themselves appear to affect her espousal of beliefs and the quality of their enactment in her instruction. For example she comments on the program's emphasis on having students explain their thinking: "Well, I do like the piece in this particular program where they do this math talk thing" (Interview, 20 Sept. 2010). Figure 11 displays an example of the *Math Talk* she is referring to from the *Math Expressions Teacher's Manual* (2009).



**Figure 11:** Math Talk Excerpt from Math Expressions Teacher's Manual (pp. 82)

The *Math Talk* excerpt displayed in Figure 11 provides an example of the instructional guidance provided in the teacher's manual. Barbara reported carrying the manual around on her hip for the whole first year of the implementation to be sure she did and said what it recommended. Despite her desire to implement the program as the author intended, during the three lesson observations, Barbara's implementation of the strategies recommended in the manual showed gaps in her knowledge. These gaps prevented the higher levels of cognitive activation targeted by the author and may partially account for instructional quality scores that were lower than Kelly's. For

example, during one lesson, as recommended in Figure 11, she had four students draw representations on the whiteboard for how to solve a specific problem. However, when she tried to elicit math talk, it was primarily between her and the student who drew the representation (not between students as recommended in the manual and measured in the MQI). She became confused over the second student's explanation of his representation and ended up quickly brushing over the remaining representations herself. While the curricular materials helped Barbara add a student explanation and multiple strategy aspect to her instruction (both of which positively impacted her MQI domain scores), her implementation of the strategy was limited by her own understanding of student thinking. Her incomplete enactment of the Math Talk strategies in *Math Expressions* appears related to the findings of Spillane and Zeuli (1999) where 21 out of 25 teachers reported understanding reform curriculum but their enactment of it belied its integrity and made the activities procedural.

Barbara's average MQI score along the sub-domain of Students Explain their Thinking was 1.24, a low score but, arguably due to the mediational role of the materials, higher than the teachers using *California Math*. The low score reflects her attempts to ask students to explain their thinking which are characterized by asking students "how" they solved a problem rather than "why" they did what they did. This line of questioning leads to procedural knowledge rather than to her articulated goal of conceptual understanding.

While Barbara's choice to implement the *Math Talk* component of *Math Expressions* is attributable to the instructional materials and the influence of her partner,

the quality with which she implemented it seems to be reflective of her mathematical knowledge for teaching. As previously reported, Barbara scored high on the MKT assessment, 1.39. However, this high score was not predictive of equally high MQI scores. Instead, the CVA seems to qualitatively capture aspects of Barbara's knowledge which provide a better explanation for her instruction. In responding to the CVA video clips, Barbara frequently paid attention to student thinking (which may be reflective of the curricular materials mediating her knowledge). However, when she commented on student confusion, she did not articulate what it was about the underlying mathematics that may be causing the confusion. This information parallels how she attended to student thinking in instruction; she asked students to explain their thinking but attended more to correctness and to procedures rather than to concepts.

With further experience interacting with students and the curricular materials, and with the influence of Kelly, I predict Barbara's skill with incorporating student thinking into instruction will increase in quality. Likewise, with increased experience in listening to student thinking, it is likely that Barbara's specialized knowledge for teaching mathematics, as measured by the MKT and CVA, will increase. In this sense, Barbara's actions including interpreting district policy, selecting curricular materials, and enacting the curriculum in combination with her beliefs and knowledge represent a series of mediated actions. It cannot be said that her beliefs directly influence her instruction nor that her knowledge directly influences her instruction. But instead, the interaction between cognitive constructs and environmental factors generate a series of mediated actions which ultimately result in the instruction her students' experience.

### **Hillside: Accountability Context, Curricular Materials, Knowledge, and Beliefs**

Hillside Elementary and its one participating teacher, Hally, provide an example of the interaction between a powerful accountability context and a teacher's knowledge for teaching mathematics. The low performance and high pressure accountability climate at the school create a context characterized by an urgency to comply with policies. Within this context, policy interpretation and curricular materials play the strongest role in mediating instruction.

#### **Hally: Accountability context, curricular materials, knowledge, and beliefs.**

Hally, from Hillside Elementary, is a case of extremes. She is the only teacher in the study who teaches at a low income, low performing school and is the only fourth grade teacher. She is also the only teacher who reports receiving explicit direction from her principal regarding which curricular program she is required to use. Hally has the lowest mathematical knowledge for teaching levels in the study, yet, as reported earlier, she scored in the upper half of the sample along key MQI domains. Exploring the interaction of personal and contextual factors as they affect Hally's instruction highlights key dynamics that did not stand out among the other participants. The district's curriculum policy paired with the school's low performance context result in a high pressure environment focusing on implementation of mandated curricular materials with fidelity. These factors play a key role in mediating Hally's math instruction. Additionally, Hally's relatively low level of mathematical knowledge for teaching mediates the enactment of her beliefs.

Hally's math instruction shares many of the characteristics of the other teachers in the study who use *Math Expressions* as their core program. Her relative strengths are her focus on explicit use of mathematical language (affected by the high percentage of English Language Learners in her class), responses to student math productions, the level of student cognitive activation resulting from her implementation of tasks, and the amount and quality of explaining students are invited to do. However, possibly due to her lower MKT levels, Hally often provides unclear explanations, makes more errors in notation than other teachers in the study, and her lessons demonstrate overall lower clarity scores.

To contextualize Hally's institutional environment, at the time of the study Hillside was in "safe harbor." Safe Harbor means the school had previously been identified as a "Program Improvement" school (had not met state-mandated annual progress goals over two consecutive years) according to the federal policy of No Child Left Behind (2001), and the percentage of students testing "below proficient" on the California Standards Test in a target area during the second year had decreased by at least ten percent from the year prior. This performance improvement allowed the school to shed its "Program Improvement" status for one year. Under NCLB policy (2001), schools that do not show improvement while under "Program Improvement" are at risk of being shut down. The accountability context of Hillside was high pressure and the principal had been hired to lead the school out of "Program Improvement." As such, the institutional environment contributed to the principal's actions around district mandates and, in turn, the sense Hally made of the district's policies.

In an interview, Hally described her interpretation of the accountability environment in which she is situated and how it mediates her decisions regarding curricular materials. The statement is taken from an interview in which Hally discussed the low performance of her school and the role of the district in holding the school accountable for student performances.

So, from the district standpoint, obviously I think it would trickle down to my principal . . . "What are we going to do about this low API? How are we going to avoid getting into Program Improvement?" So, I'm sure the pressure is on for my principal, being that that is his job, making sure that we were all collaborating, making sure that we were using with fidelity all the curriculum, making sure that we weren't straying and using our own curriculum and resources that weren't research-based. So, it was made very clear to us that we should not stray off because Program Improvement would involve that we stay on. (Interview, 8, Dec. 2010)

The curriculum Hally refers to is *Math Expressions* which she reluctantly uses. She reports that any attempts to move away from it are met with the principal's guidance back. The constraint that she feels from the pressure to follow a curriculum and to ignore what she believes her students need is frustrating to Hally as she expresses in her interview:

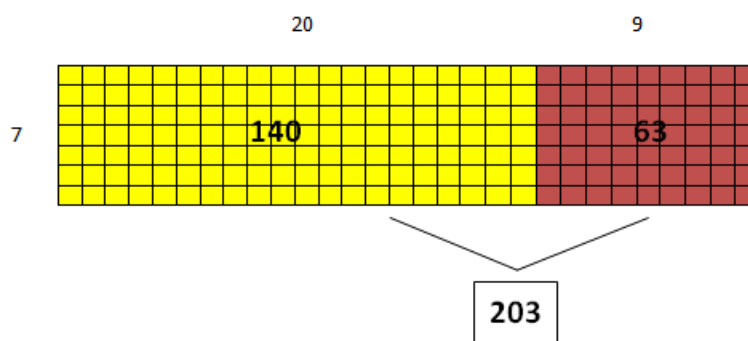
I think it's you know, not so much we're being bullied into it, I think it's they're wanting us to understand that the curriculum that was chosen from the district has research base and not that what we're using to supplement is not research based but a lot of times, as teachers, we'll make decisions and go outside and pull things that we have to show, you know, reteach the concept, but I think sometimes when we share that with administration, with principals, I want to be able to say, "Look, this is what I'm seeing, I'm going to fill in this gap, I'm going to go outside of it." And being able to have that trust, sometimes that trust isn't there. And so they're asking to just stay true, stay true, believe in this curriculum. Because I have shared that sometimes I'm, I lose faith, I lose faith in some of the practicality of the lessons and the pace. You know, kind of let me branch away and use what my professional decisions and the way I pace my things. But sometimes we get kind of pushed back to stay on the

track, you know, the curriculum that they want us to use. (Interview, 18 Nov. 2010)

As Hally later explained, in this excerpt she is referring to wanting to fill in “gaps” in the *Math Expressions* programs with traditional algorithms. Her principal wanted her to teach only what is presented in the program.

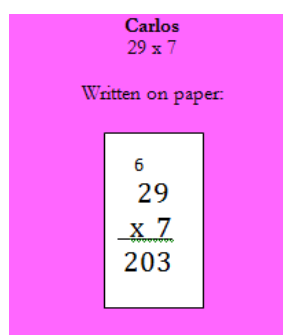
While the accountability context determines the curriculum Hally must use and influences her sense of autonomy, her beliefs and knowledge play critical roles in the implementation of the curriculum. In her interviews and survey responses, Hally demonstrates partial alignment between her beliefs and the math instruction philosophy of *Math Expressions*. Her MKT score, 0.28, indicates that her knowledge about the conceptual models required to teach from *Math Expressions* is low. The interaction between her knowledge and her beliefs play mediational roles between the curricular materials and her instruction.

In her interview, Hally described believing that vocabulary development should be central in math instruction. Additionally, contrary to research recommendations, she reported believing that teaching an algorithmic procedure prior to showing a conceptual model is a successful way to support student understanding. Yet, in the IMAP Beliefs Survey, she demonstrates a different perspective including showing evidence of holding five of the six reform-oriented beliefs. For example, on one item she is asked to instructionally sequence different strategies for solving the problem  $29 \times 7$  (Appendix A). The first method she says she would teach is the conceptual area model shown in Figure 12. She explains that this model builds on visual cues from place value which was previously taught.



**Figure 12:** Area Model of Multiplication which Hally would Teach First

The last method she states she says she would teach is the traditional algorithm shown in Figure 13.



**Figure 13:** Traditional Multiplication Algorithm which Hally would Teach Last

The discordance between the instructional sequencing beliefs she espoused in her interview (procedural to conceptual) and those which surfaced from the survey (conceptual to procedural) may be explainable by the influence of curricular materials and a high-pressure accountability context. Hally's familiarity with the area model from its presence in *Math Expressions* could account for why Hally answered the IMAP Beliefs Survey question the way she did -- a response to the pressure she feels from her principal to embrace the approach of the program. Despite demonstrating in the survey

that she would teach a conceptual model before the formal procedure, Hally reports implementing the opposite sequence in instruction:

Math Expressions is conceptual so we have to sort of go out of our comfort level to teach some of the things that are there. Especially with multi-digit multiplication they use the rectangular area boxes and it's a fantastic way to teach multi-digit multiplication but it's out of our comfort as teachers because that's not the way we were taught. So sometimes we'll go out of it to California Math which is a little bit more traditional and then jump back into Math Expressions. So we do a little bit of hopping around just to kind of fill in the gaps on our comfort levels on what we're teaching and it seems to work. (Interview, 3 Dec. 2010)

In this statement, Hally attributes her objection to the concept-first sequence to her lack of comfort with the area model. This discomfort appears to be related to her specialized knowledge for teaching mathematics as is revealed in her low overall MKT level of 0.28 and, specifically, in her response to an item on the assessment in which she was unable to identify different area models representing the same problem.

Hally's responses to the Classroom Video Analysis video clips provide further evidence that Hally may hold reform-oriented beliefs but that her low specialized mathematical knowledge may negatively mediate the enactment of those beliefs. On five of the nine video clips she watched, Hally suggested to improve the lessons through the use of conceptual models. With the convergence of philosophical indicators on Hally's CVA and IMAP Beliefs Survey responses, it is safe to say that Hally's beliefs are at least partially aligned with the philosophical tenets of *Math Expressions*. However her discomfort with the mathematical knowledge required to teach the program prevents her from enacting those beliefs. Additionally, the loss of autonomy and respect that she feels

from her interpretation of the principal's "guidance" negatively affects her motivation for wanting to learn something new. She expresses this sentiment in her interview:

I know that this particular curriculum, this is not a silver bullet. What a silver bullet is my kids, knowing what they need, having a teacher use the curriculum, and marrying it with something I know has been successful for me as a teacher which I know will be good for my students. That's the hybrid version of being able to successfully meet the students' needs. Just staying out of one book and not being able to have that leeway is hurtful for me and for my students. It's not what's best for kids as they always say, "What's best for children," you know. So it does affect how I teach, because since I'm not really crazy about this book, it sort of affects my feeling about teaching math. I'm not real crazy about getting into some of these lessons. (Interview, 8 Dec. 2010).

In this statement, Hally emphasizes both the role her knowledge and the curricular materials play as they interact to affect her sense of success as well as the role beliefs, curricular materials, and policies play as they interact to affect her instruction. The result is instruction that adheres to the domains in the MQI, producing higher results than teachers using *California Math* but relatively low results on the MQI scales (Overall MQI = 1.33 on 1, low – 3, high scale). Hally's discouragement and low levels of knowledge prevent her instruction from reaching high quality levels.

Hally's case demonstrates the complex interactions that occur between both personal and contextual factors as mathematics instruction is expressed. No single factor can be attributed to the quality of her mathematics instruction. Instead, in this situation, the curricular materials, the accountability context, and Hally's knowledge played larger roles than her beliefs. But all factors mediated each other in the enactment of Hally's mathematics instruction.

## Summary and Conclusion

In this chapter, I described the findings from a mixed methods analysis of both quantitative and qualitative data. I detailed the relationships indicated by a quantitative cross-case analysis of individual and environmental factors related to classroom mathematics instruction. Additionally, I built an explanation for the interaction of factors using a qualitative multiple case study iterative approach.

Using quantitative data, multiple variables showed relationships to teachers' mathematics instructional quality. First, teachers' *Mathematical Knowledge for Teaching* (MKT) showed a positive relationship to the *Mathematical Quality of Instruction* (MQI) domains of Responding to Student Productions and Explanations. Curricular programs used by a teacher also appeared to affect their instructional quality. Teachers using Houghton Mifflin's *Math Expressions* program (Fuson, 2009) demonstrated higher instructional quality scores than the teachers using the same publisher's *California Math* program (Hill, R. et al., 2008) in the domains of Responding to Student Productions, Student Explanations of their Own Thinking, and Multiple Solutions. When instructional quality scores were examined by school, it was noted that the curricular programs varied by school and so did instructional quality in the above mentioned MQI domains.

Noting that differing school contexts may affect teachers' interpretations of district curriculum use policies and the resulting programs selected, I examined the policy interpretations by teacher and by school. I found policy interpretations varied by school and were related to the school's academic performance context and other school-based contextual factors. The lower the academic performance context, the more rigid the

policy interpretation was. The similarities in policy interpretations by school indicated a social aspect to the sense teachers made of policies. To further explore reasons for school variance in policy interpretation, I examined alignment between individual teachers' beliefs about teaching and learning mathematics and the philosophy of the curricular program they used. I found that some of the teachers had selected core mathematics programs that were misaligned with their beliefs. This finding warranted analysis of what other factors may have played into their curriculum decisions which seemed to serve such a large role in the quality of their instruction.

I began the multiple-case study analysis with the teachers at Pacific Shore Elementary. Teachers there, Bill, Fran, and Hannah, demonstrated how interactions with their colleagues as well as the institutional accountability context often overpowered beliefs and knowledge in their mediational roles with curriculum selection and mathematics instruction. One teacher at Pacific Shores, Hannah, demonstrated how lower levels of specialized content knowledge prevented the enactment of her beliefs and elevated the role of collegial interactions in mediating instructional decisions. A case at Little Rock Elementary, Liz, demonstrated how the instruction of a high knowledge teacher holding strong reform-oriented beliefs was primarily mediated by the institutional accountability context and resulting curricular materials. At High Bluff Elementary, the case of Kelly demonstrated how alignment between beliefs, knowledge, and reform-oriented curricular material, bolstered by professional development, are a strong interactive recipe for instructional quality. And Barbara showed how collegial interactions and curricular materials played a primary role in instruction as they mediated

her knowledge and beliefs. At Hillside Elementary, Hally demonstrated that curricular materials can mediate low levels of knowledge and positively influence instructional quality (depending on alignment between the curriculum and the quality measure). She also demonstrated how policies are filtered differently depending on the climate of academic performance urgency felt at a school. Finally, she showed the human side of the current testing and accountability climate in her discouragement and loss of pleasure in teaching mathematics.

In all, this study has shown that there are identifiable relationships that exist between individual and environmental factors as mathematics instruction is expressed. However, no factor can be said to directly affect instruction. Every factor's influence on instruction exists in a mediated relationship. The nature of the mediational relationships is context dependent, giving priority in some circumstances to certain factors and in other circumstances to other factors. The participants in this study provided evidence that factors contributing to the quality of mathematics instruction are measurable and their interactions are traceable. However, a much larger sample of participants would be needed to identify generalizable patterns of interaction.

## **CHAPTER VI: CONCLUSION**

### **Summary**

The results of this study provide evidence that social and institutional context factors play principal roles in teachers' expression of mathematics instruction. Teachers' levels of knowledge for making mathematics concepts comprehensible to students and their beliefs about teaching and learning mathematics often were less influential than curricular materials, policies, or their colleagues in determining the characteristics of their mathematics instruction. In certain circumstances, alignment between curricular materials, knowledge, and beliefs allowed the enactment of what the teacher knew and believed about mathematics and instruction. However, such convergence was uncommon. Instead, reasons for teachers' instructional quality most often were traced to the mediational role of the institutional context rather than to personal factors.

I began this study because my own experiences as a classroom mathematics teacher compelled me to try to understand how mathematics instruction has become so procedurally standardized in America. How can it be, with the tremendous variability that exists among teachers' knowledge and beliefs, that with regularity we are mathematically misguiding children? In my last year as a K-12 classroom mathematics teacher, I almost succumbed to the procedural script of American mathematics instruction. It was that year which shaped this study and continues to help me make sense of the findings. It also provides direction for how to support teachers in changing their instruction to better help students develop principled mathematical reasoning.

The year before I conducted this study, I accepted a position teaching sixth grade math at a middle school. On the first day, I walked into the classroom holding strong reform-oriented beliefs and highly developed mathematical knowledge for teaching. Yet beneath my consciousness lay residue from years of experience in the American education system where a cultural lesson script of skill acquisition and repeated practice prevailed (Hiebert & Stigler, 2000; Ma, 1999). And all around me was a culture which validated those experiences: traditional textbooks, skill-based assessments, and a system of accountability. Sandwiched between these layers, my epistemological and pedagogical mathematical values were threatened. And, for the first time in my career, I perceived the institutional pressure to be so strong, the structures so rigid, and the within-system consequences my students could experience from my nonconformity so tangible, that I temporarily allowed my values to be overruled. While my falter was short-lived, it was that moment in my career as a mathematics educator that drove me to investigate the phenomenon of mathematics instruction and what influences it. At the foundation of my study design was the need to understand factors underlying this misguided instructional regularity. I was not satisfied with attributing instructional quality to teachers' individual characteristics alone. My experience told me that institutional context was intertwined in the enactment of these variables. I hoped studying which personal and contextual factors were most influential in teachers' mathematics instruction and how they interacted could help professional development and pre-service teacher programs adopt a more comprehensive approach to affecting instructional change.

The study I designed set out to investigate how teachers' knowledge for teaching mathematics, beliefs about teaching and learning mathematics, and the sense they make of institutional curriculum and accountability policies interact in the expression of mathematics instruction. Using a mixed methods approach, this study incorporated these factors into a comprehensive exploration of mediational relationships between them. Taking a situative perspective, I focused on the role teachers' interpretations of curriculum and accountability policies played within their institutional contexts. By analyzing the teacher in the context of situated instructional decision making, the interactions of individual and environmental factors were interpreted in institutional contexts.

I used validated surveys and assessments to measure teachers' knowledge for teaching mathematics and beliefs about teaching and learning mathematics. I analyzed classroom instruction video observations using pre-established rubrics to characterize the mathematical quality of teachers' instruction. Additionally, I conducted interviews to unveil teachers' interpretations of policies and triangulate knowledge and beliefs data. Case-oriented and variable-oriented data analysis approaches were used to identify relationships between variables and to build explanations. Findings from this study indicate social and institutional factors play critical roles in teachers' expression of mathematics instruction.

The teachers in this study provided answers for me as well as guidance for policy makers, researchers, and practitioners who aim to improve students' opportunities for truly learning mathematics. The teachers showed that in order to support them in

generating higher quality instruction, their knowledge, beliefs, and the sense they make of policies within their institutional contexts must receive equal attention. Additionally, their curricular materials play such a critical role in the content and format of their instruction that any attempt to affect instruction must include these resources. The most important contribution of this study to the field is the synthesis of studies which have focused on single variables as they are related to instruction into a more comprehensive study. It adds new knowledge to the field in shifting institutional and social contexts to the forefront of factors considered influential in mathematics instruction.

### **Connections to Prior Research and New Insights**

In Chapters Four and Five, this study's findings are continuously connected to research in the field. In this section I will highlight these connections more summatively. Findings of the study partially confirmed the knowledge/instruction link claimed in other studies (Hill et al., 2008; Kersting et al., 2009): teachers' mathematical knowledge for teaching (MKT) does play an important role in their mathematics instruction. However, it was far from the most influential factor in the mediational relationships which contributed to the participants' mathematics instruction. Within this study, higher levels of MKT appeared to afford teachers the ability to understand and respond to students' math productions during lessons. It also seemed to support teachers in providing explanations inclusive of the underlying mathematics. Kersting et al.'s (2009) study predicts such a relationship. Yet, Hally's case provided evidence that curricular materials could mediate the knowledge/instruction relationship indicating, when supported by curricular materials, teachers with low MKT could express higher quality instruction than

expected. This finding echoes Hill's et al. (2008) case study findings. However, Hill et al. (2008) did not predict a case such as Liz's where a teacher with high MKT found her instructional quality negatively mediated by curricular materials. While these relationships and cases focus on the MKT/instruction relationship, this study provided evidence that other variables play key roles in mediating the relationship.

Stipeck et al. (2001) found that teachers' belief orientations, traditional or inquiry oriented, were predictive of their mathematics instructional characteristics. In the study I conducted, I found this to be only partially true. In the cases of Bill (traditional) and Kelly (inquiry), their beliefs were aligned with their instructional characteristics for the most part. However, as is necessary with an explanation and theory building study, I searched for an explanation for this relationship. Bill explained the institutional accountability context had "molded" his beliefs and instructional focus. It can be claimed that the institutional context mediated his beliefs which affected his curriculum selection and his instructional quality. He showed that, in some cases, beliefs can predict instructional characteristics but that additional factors play into this relationship. Kelly provided an example also confirming Stipeck's claims while simultaneously adding a layer to his theory. Kelly's beliefs were aligned with the curricular materials that she used which inspired her to attend additional professional development and align her mathematics instruction. It was the alignment between the curricular materials and beliefs, aided by high levels of MKT, which seemed to most affect her instructional action. Hally provided a contradictory example in which her beliefs did not predict her instructional quality, but instead the mediation of the curricular resource (present because

of context-specific interpretation of institutional policy) increased the amount and quality of research-based elements in her lessons. And Liz's case contradicts Stipeck's claim altogether and instead provides support for Raymond's (1997) findings: discrepancies between beliefs and instructional characteristics are attributable to institutional constraints. Liz not only exhibited a constellation of reform-oriented beliefs but also had the highest MKT score, yet her instructional characteristics were traditional in nature. The institutional accountability climate proved to overshadow knowledge and beliefs as it mediated her instruction via policy interpretation and curriculum selection.

When the study's lens focuses on sensemaking within the network of factors influencing teachers' instruction, social and institutional contexts take an even more central role. Sensemaking focuses on human agency within individual and social activities of institutions. At an individual and group level, teachers' interpretation of district policy regarding which textbook to use appeared mediated by their perception of the accountability context at their school site as well as by the policy/curriculum/belief alignment of the more dominant members of their teams. Additionally, the school's academic performance context appeared to play a mediational role in how teachers interpreted curriculum use policies. Teachers in high performing schools tended to interpret the policies as more flexible whereas the teacher in the low performing school interpreted the curriculum use policies as more rigid. The study had only one teacher from a low performing school therefore indications from the data need further study with more participants

Among the teachers at two of the three high performing schools, policies appeared to be filtered through collegial interaction and ultimately contributed to instructional similarities. These findings support Coburn's (2001) claims that teachers' sensemaking is most affected by whom they work with and the nature of their conversations. In her case studies, she found that teachers' worldviews and practice tended to converge within their groups over time. Her data showed they converged to reflect the values and practice of the most dominant personality in the group. While I did not measure the nature of teachers' conversations, I did find Coburn's claims regarding the influence of dominant personalities to partially bear out. The cases of Kelly and Barbara provide evidence that Barbara's beliefs were changing toward Kelly's beliefs as mediated by her interactions with Kelly. However, her interactions with the curricular materials (selected largely due to Kelly's knowledge and beliefs) and students also affected her beliefs. It is hard to predict whether, without Kelly, Barbara's beliefs and instruction would be congruent with their current characteristics. The cases at Pacific Shore provide partial support for Coburn's claim also. Bill's highly regarded math teacher status on his team gave his beliefs preference in the team's curricular decision making. In the case of Hannah, her beliefs and practice did seem to converge with Bills, as Coburn's work predicts. However, data analysis indicated that her knowledge may have played a more primary and constraining role in enacting her beliefs in practice than it did in other cases. Additionally, contrary to Coburn's findings, Fran, who had worked with Bill for a decade, maintained her own reform-oriented beliefs. Her instructional practice, however, was converging with Bill's.

The final variable which emerged as instrumental in the quality of teachers' instruction was the curricular materials themselves. Their role was much larger than I anticipated at the beginning of the study. There was a pattern where teachers who used *Math Expressions* demonstrated higher instructional quality than those using *California Math* on the MQI in the key domains of Responding to Student Thinking, Students Provide Explanations, and Multiple Procedures and Solution Methods. Variability in beliefs or knowledge accounted for slight variations in instructional quality, but the material itself proved to have the greatest impact on instruction. Remillard (1999, 2000) found that instructional differences present among teachers using the same curricular materials was attributable to the teacher's instructional purpose and pattern for reading, evaluating, and adapting the curriculum. While I did not examine how teachers were reading and making sense of curricular materials, I can say that instructional differences between teachers using the same curricular material were negligible in comparison to the variations traceable to the differences in the curricular material itself. I would like to note that this study did produce evidence of the dynamic relationship perspective between teacher and text (Remillard, 2005). As an example, Barbara showed that she was learning from the instructional tips in the text as she implemented them with her students and was also affecting the curriculum itself in her implementation. However, based on the data from this study, it is safe to say that despite her relationship with the text, and despite her knowledge and beliefs, if Barbara were using the *California Math* textbook, she would have generated instruction with lower quality scores on the MQI.

In sum, this mixed methods study provides evidence that a complex network of mediated interactions affects the quality of teachers' mathematics instruction. Studies which focus on a single factor as it is related to instruction are highlighting only one portion of the network. In order to explain why certain aspects of a teacher's knowledge can predict certain quality characteristics of instruction, or why certain constellations of beliefs are sometimes predictive of instructional characteristics, one must first consider how the factors interact and then examine their relationship to the social and institutional contexts in which the action occurs. This study adds to the field of research on mathematics instruction by providing evidence of the context-dependent variation in roles that knowledge, beliefs, curricular materials, and policy interpretation play. It provides a possible explanation for why teachers who enter the American educational system intent on teaching one way end up teaching another. And it provides direction for a more comprehensive approach to professional development focused on improving the quality of teachers' mathematics instruction.

## **Implications**

### **Implications for Future Research**

This study yielded important new insights while also paving the way for future research. First, the relationships uncovered warrant a larger scale correlational study to confirm or disconfirm identified relationships as well as add to the possibility that patterns of interaction observed may be generalizable to a larger population. In addition to looking for relationships between variables, qualitative within and across case analytic approaches should be used to build explanations for the relationships.

Further research could also explore the role of school context in more depth. Finding groups of teachers representative of all quadrants of the low SES, low performing, high SES, high performing matrix may provide more information about the role that the current institutional accountability context plays. Additionally, district to district comparisons may further highlight institutional policy and teacher interpretation variables. Context-dependent variables could also be explored in a public versus private school comparison where the accountability policies differ and the hierarchy of stakeholder influence may take a different shape.

The unexpected primary role that curricular materials played in the quality of mathematics instruction draws attention to an aspect of the study which could be expanded. There is an entire field of research focused explicitly on the relationship between curriculum materials, teachers, and instruction (Remillard, 2009) which could help shape the investigation of curriculum's role in the network of interacting factors. For example, understanding how teachers interpret the content of curricular programs and the sense they make of textbook versus teacher agency in shaping instruction may help explain some of the mediational relationships identified in this study. To build such an explanation would require a larger sample size, variation in textbooks used across teachers and schools, and interviews with greater focus specifically on the teacher/textbook relationship.

Most importantly, this study relied on the supposition that instructional quality is related to student learning (Hill et al., 2005; Kersting et al., 2009) and yet did not explore these connections. While including student learning was beyond the scope of the study's

research questions, a larger scale version of this study should include measures of student performance. I highly recommend state testing results not be used as the measure. If the goal is for students to develop principled mathematical knowledge, and instructional quality is measured according to a framework meant to contribute to such knowledge development, then the measure of student learning must test principled mathematical knowledge.

### **Implications for Policy and Practice**

#### **Federal and state policy.**

Results from this study have profound implications for policy makers. Mathematical learning opportunities for students and the nature of their mathematical knowledge are highly affected by educational policy. Federal and state policy makers must pay attention to current research and to mathematics recommendations put forth by the National Council for Teachers of Mathematics (2000, 2006). Regardless of individual teacher characteristics, current policies appear to play a primary role in the prevalence of classroom instruction focused on the development of procedural competence rather than principled mathematical knowledge. The accountability context created by high stakes standardized testing is having the opposite effect of what was intended. While student test scores may be going up, the mathematical quality of their knowledge is not. Current efforts to nationally standardize content (National Governors' Association, 2010) and provide a more conceptual orientation to mathematics instruction must be accompanied by assessments which measure conceptual, principled mathematical knowledge. It is the responsibility of policy makers to be guided by

reliable research, not by their own beliefs or by politics, when developing policies which affect classroom mathematics instruction.

### **District leaders and administrators.**

One of the key findings in this study is that curricular materials and the philosophy behind them matter when it comes to the quality of classroom mathematics instruction. The teachers who had higher quality instruction in this study were using materials bearing a philosophy aligned with research and NCTM recommendations. When asking teachers to make decisions regarding curriculum adoptions, district mathematics leaders and administrators must facilitate a more research-based analytical process. That also means that they, themselves, must be apprised of current research in mathematics education which is endorsed by or aligned with NCTM. In the district I studied, when it is time for a new math adoption, one or two teachers per school pilot the programs in their classrooms and then all teachers vote. Like the decisions made among the teachers in the study I conducted, the votes typically represent the beliefs of whoever piloted the program. Additionally, in my experience, teachers select the program which will provide the best result within their accountability context. Missing from this equation is the inclusion of the teachers in a detailed analysis of the programs based on current research. In SUSD, although the district adopted both programs and mandated *Math Expressions*, the teachers voted for *California Math*. I believe these decisions should be made with a focus on research rather than on what feels comfortable or on what will produce high test scores. District leaders and administrators are in a position to

facilitate more thoughtful decision making around the clearly important curriculum decisions.

District leaders and administrators are also in a position to put assessments in place which preference the development of principled mathematics knowledge. While it is convenient to create alignment between district, state, and national accountability measures, when none of the measures address conceptual understanding, the absence of conceptual instruction is not surprising. Likewise, students who can reproduce procedures but don't understand mathematics is a predictable outcome. To be clear, I am not advocating for additional tests. However, in the current accountability climate the presence of district tests is a reality. Therefore I am suggesting the development of tests that measure more principled knowledge. Such tests may provide the support many teachers need to enact their reform-oriented beliefs.

Another implication for district leaders and administrators which arises from the findings is that they must eradicate policies which track students based on test scores. Tracking itself has many negative repercussions for students which are beyond the scope of this dissertation to discuss. As teachers in this study reported, the pressure they feel from parents and from the institution itself regarding what math track students test into negatively affects their curricular choices and their instructional decisions. In the current mathematics skill-based accountability climate, it contributes to discouraging them from teaching mathematics for conceptual understanding. Instead, students (with their parents) should be allowed to select the level of their math class. I have seen this practice

successfully enacted. The district has the power to remove that institutional pressure from teachers and perhaps positively affect mathematics teaching and learning.

Finally, as the cases of Bill and Kelly showed, collegial interaction can positively or negatively contribute to the quality of teachers' mathematics instruction.

Administrators need to be aware of the stronger personalities among their teachers and distribute them with intentionality. Coburn's (2001) research and the results of this study show that the more dominant member of a team can alter the practices (and sometimes beliefs) of colleagues. While there are many other academic subjects, personal factors, and logistical factors that complicate the task of assigning teacher placements, administrators can be strategic in the way they group teachers.

### **Professional development and pre-service programs.**

The interaction of multiple factors in affecting teachers' expression of mathematics instruction evidenced in this study draws attention to the need for multi-pronged approaches to programs which target teacher learning or instructional change. Critical to such an approach is situating teachers' learning within the institutional contexts in which they work. Targeting any of the factors discussed in this study by itself or out of context is unlikely to play a lasting role in teachers' instructional practice. For example, a professional development program which only targets the development of mathematical knowledge for teaching risks the development of knowledge that will not be activated within the institutional context (social and policy). Integral to a program that addresses institutional contexts is the consideration of state and district required curricular materials and assessments. Teachers must be given time and support in

determining how research-based practice can integrate into their current contexts. They also must be given opportunities and support to analyze the lessons within their textbooks according to their alignment with best practices in developing principled mathematical knowledge. Beliefs also have been shown to play a role in how instruction comes to be expressed. Key to addressing beliefs is reflection time. As Bill described in his interview, sometimes teachers just need the time and opportunity to reflect and make sense of their current practice and how it affects student learning.

Finally, results from the MQI provide direction for foci in programs supporting teachers. While the size of the sample makes it hard to say observed instructional characteristics are generalizable to a larger population, my experience as a teacher and professional developer as well as the characteristics of the American cultural lessons script (Hiebert & Stigler, 2000) support the findings. It was clear that instruction in the classrooms studied did not help students see the generalizability of the mathematics they were studying. They did not look for patterns or see the power of the mathematics they were learning. Professional development and pre-service teacher education programs should attend to this domain. Also, the absence of whole class discussions where students interact with one another's ideas indicates that teachers need support in the area of classroom mathematics discourse. It was unusual for students or teachers to consider multiple solution methods for solving problems or to inquire into the meaning underlying the procedures they did use. Teachers need support in how to build this instructional element into their practice, regardless of the curricular materials they use. Another key weakness in the observed instruction was explicit links between mathematical ideas or

representations. As Ma (1999) noted, this may be related to how the teachers have come to know mathematics; they see math content in discrete knowledge packages rather than as a series of interrelated ideas. However, with exposure and experience, classroom by classroom, that cultural pattern can change. A general shift needs to occur in classroom mathematics instruction so that students have opportunities to think and reason about mathematics rather than simply to acquire and practice mathematical skills.

### **Teachers.**

First and foremost, when I think about the implications of the findings from this research for teachers, I think about the intense institutional pressure teachers are under to help their students score well on state tests. It would be easy to say that it is impossible to provide conceptual instruction in the current accountability climate. It would be easy to blame one's instruction on institutional constraints. It is true that educational policies play a critical role in teachers' instructional decisions. However, in the end, it comes down to a teacher and a classroom of students. The teacher has agency and is not a passive recipient of policy.

We as teachers must empower one another to be informed when it comes to current research, reflective about how our practice is affecting student learning, and metacognitive about the role our own beliefs and knowledge are playing in our mathematics instruction. We must hold one another accountable for not succumbing to a system which subverts quality mathematics instruction but instead for letting our voices be heard in the policy context. We must be aware of the power of curricular materials in shaping our instruction and make sure that we are interrogating the materials and

thoughtfully applying what we know about teaching and learning to our use of the materials. We must seek out opportunities to grow professionally through informed interaction with one another and through ongoing professional development. It is up to us, the final mediators between policy, curricular materials, and students, to make sure our students have a chance to truly learn mathematics.

### **Final Remarks**

As findings from this study highlight, the act of generating mathematics instruction is complex but it is not a black box. The factors affecting instructional quality are identifiable and their interactions can be understood. If America's procedural mathematics lesson script is ever to be interrupted, and if America's students are to ever have a chance of understanding mathematics, improvement efforts must simultaneously emphasize the individual teacher and the educational policies as both interact with one another and external factors in their intuitional contexts. And improvement efforts must be based on research which seeks to support the development of principled mathematical knowledge.

# APPENDIX A

## IMAP Beliefs Survey

### Mathematics Background and Beliefs Survey

Examine the students' different approaches to solving the problem  $29 \times 7$ .

**Celia**  
29 x 7  
Written on paper

$$\begin{array}{r} 6 \\ 29 \\ \times 7 \\ \hline 203 \end{array}$$

**Henry**  
29 x 7  
Henry says, "I know that 9 times 7 is 63 and 20 times 7 is 140 so 140 and 63 is 203."

**Ellen**  
29 x 7  
Written on paper

$$\begin{array}{r} 29 \\ \times 7 \\ \hline 140 \\ + 63 \\ \hline 203 \end{array}$$

**Stash**  
29 x 7  
Stash says, "Well, I know 29 is 1 away from 30, so I know that 30 times 7 is 210. I subtract the extra 7, so 210 minus 7 is 203."

**Maria**

Maria uses centimeter paper to draw arrays as she solves the problem. She says, "I broke 29 into 20 + 9. Then I drew the array for 7 times 20 and got 140. I did this array for 7 times 9 and got 63. Then I just put them together, 140 plus 63 and it makes 203, so then I did the plus 63 and got 203. Then I just added the array 2."

**Directions:** If you were teaching your class multi-digit multiplication, which strategies would you like to see students share with the class? Select "yes" or "no" next to each student and then explain why or why not.

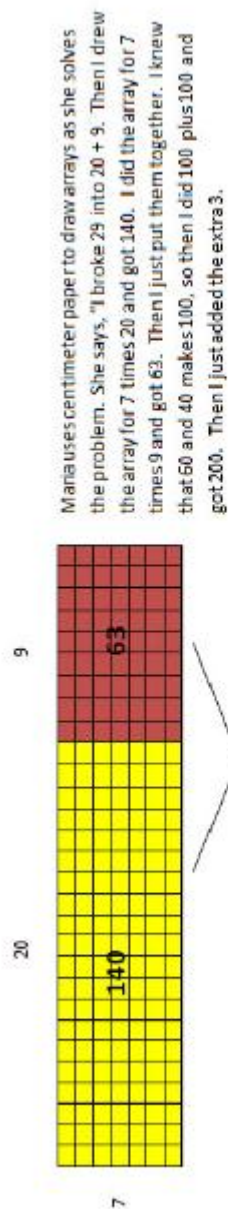
| Student/Approach | Yes                      | No                       | Responses | Explanation |
|------------------|--------------------------|--------------------------|-----------|-------------|
| Celia            | <input type="checkbox"/> | <input type="checkbox"/> |           |             |
| Henry            | <input type="checkbox"/> | <input type="checkbox"/> |           |             |
| Ellen            | <input type="checkbox"/> | <input type="checkbox"/> |           |             |
| Stash            | <input type="checkbox"/> | <input type="checkbox"/> |           |             |
| Maria            | <input type="checkbox"/> | <input type="checkbox"/> |           |             |

[Back](#)
[Reset](#)
[Next](#)

Examine the students' different approaches to solving the problem  $29 \times 7$ .

|                                                                                                                                      |                                                                                                                                        |                                                                                                                                                       |                                                                                                                                                                                                                       |
|--------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>Carlos<br/><math>29 \times 7</math></p> <p>Written on paper:</p> $\begin{array}{r} 6 \\ 29 \\ \times 7 \\ \hline 203 \end{array}$ | <p>Henry<br/><math>29 \times 7</math></p> <p>Henry says, "I know that 9 times 7 is 63 and 20 times 7 is 140 so 140 and 63 is 203."</p> | <p>Elliot<br/><math>29 \times 7</math></p> <p>Written on paper:</p> $\begin{array}{r} 29 \\ \times 7 \\ \hline 140 \\ + 63 \\ \hline 203 \end{array}$ | <p>Sarah<br/><math>29 \times 7</math></p> <p>Sarah says, "Well, I know 29 is 1 away from 30, and 30 times 7 is 210, but I did thirty 7's instead of 29 so I have to subtract the extra 7, so 210 minus 7 is 203."</p> |
|--------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

Maria



Mathematics Background and Beliefs Survey

Examine the students' different approaches to solving the problem  $29 \times 7$ .

Colby  
 $29 \times 7$

Writes on paper:  
$$\begin{array}{r} 6 \\ 29 \\ \times 7 \\ \hline 203 \end{array}$$

Henry  
 $29 \times 7$

Henry says, "I know that 9 times 7 is 63 and 20 times 7 is 140 so 140 and 63 is 203."

Elliot  
 $29 \times 7$

Writes on paper:  
$$\begin{array}{r} 29 \\ \times 7 \\ \hline 140 \\ + 63 \\ \hline 203 \end{array}$$

Sarah  
 $29 \times 7$

Sarah says, "I know 29 is 1 more than 30, and 30 times 7 is 210, but I did subtract the extra 7, so 210 minus 7 is 203."

Maria

20

9

63

140

7

203

Maria uses centimeter paper to draw arrays as she solves the problem. She says, "I broke 29 into 20 + 9. Then I drew the array for 7 times 20 and got 140. I did the array for 7 times 9 and got 63. Then I just put them together. I know that 20 and 40 makes 100, so then I did 100 plus 100 and got 200. Then I just added the extra 3."

Directions: Consider just the strategies on which you would focus in a unit on multi-digit multiplication. Over a several-week unit, in which order would you focus on these strategies? Explain each choice.

Sequence

First

Second

Third

Fourth

Fifth

Strategies Selection

Choose next

Choose next

Choose next

Choose next

Choose next

Explanation

Previous

Next

Reset

Back

Next

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## Mathematics Background and Beliefs Survey

Examine the students' different approaches to solving the problem  $29 \times 7$ .

**Sarah**  
29 x 7

Sarah says, "I know 29 is 1 away from 30, so 30 times 29 is 870 and then I subtracted 29 from that, so I have to subtract the area 7, or 210 minus 7 is 203."

**Elliot**  
29 x 7  
Writes on paper

29  
x 7  
-----  
140  
+ 63  
-----  
203

**Henny**  
29 x 7

Henny says, "I know that 7 times 2 is 14 and 20 times 2 is 40 and 6 is 120."

**Maria**

20

7

140

203

20 x 7 = 140

203

2.

Do you think Carlos could make sense of and explain Sarah's strategy? Why or why not?

Do you think that Carlos could make sense of and explain Elliot's strategy? Why or why not?

# Mathematics Background and Beliefs Survey

Place the following four problems in rank order of difficulty for children to understand.  
Explain your ordering (you may rank two or more items as being of equal difficulty).

NOTE: Easiest = 1

Please explain your ranking choices:

Select Rank

--Click Here--

--Click Here--

--Click Here--

--Click Here--

## 8. Problem

- a) Understand  $1/5 + 1/8$
- b) Understand  $1/5 \times 1/8$
- c) Which fraction is larger,  $1/5$  or  $1/8$ , or are they the same size?
- d) Your friend Jake attends a birthday party at which 5 guests equally share a very large chocolate bar for dessert. You attend a different birthday party at which eight guests equally share a chocolate bar exactly the same size as the chocolate bar shared at the party Jake attended. Did Jake get more candy bar, did you get more candy bar, or did you and Jake get the same amount of candy bar?

### 8. Problem

- a) Understand  $1/5 + 1/8$
- b) Understand  $1/5 \times 1/8$
- c) Which fraction is larger,  $1/5$  or  $1/8$ , or are they the same size?
- d) Your friend Jake attends a birthday party at which 5 guests equally share a very large chocolate bar for dessert. You attend a different birthday party at which eight guests equally share a chocolate bar exactly the same size as the chocolate bar shared at the party Jake attended. Did Jake get more candy bar, did you get more candy bar, or did you and Jake get the same amount of candy bar?

Place the following four problems in rank order of difficulty for children to understand.

Explain your ordering (you may rank two or more items as being of equal difficulty).

**NOTE: Easiest = 1**

Select Rank

--Click Here-- ▾

--Click Here-- ▾

--Click Here-- ▾

--Click Here-- ▾

Please explain your ranking choices:

Mathematics Background and Beliefs Survey

9.
- Consider the last two choices:
- c) Which fraction is larger,  $\frac{1}{5}$  or  $\frac{1}{8}$ , or are they the same size?

d) Your friend Jake attends a birthday party at which five guests equally share a very large chocolate bar for dessert. You attend a different birthday party at which eight guests equally share a chocolate bar exactly the same size as the chocolate bar shared at the party Jake attended. Did Jake get more candy bar, did you get more candy bar, or did you and Jake each get the same amount of candy bar?
- Which of these two items did you rank as easier for children to understand?
- ☐ Item c is easier than item d

☐ Item d is easier than item c

☐ Items c and d are equally difficult

Please explain your answer:



9.

Consider the last two choices:

c) Which fraction is larger,  $\frac{1}{5}$  or  $\frac{1}{8}$ , or are they the same size?

d) Your friend Jake attends a birthday party at which five guests equally share a very large chocolate bar for dessert. You attend a different birthday party at which eight guests equally share a chocolate bar exactly the same size as the chocolate bar shared at the party Jake attended. Did Jake get more candy bar, did you get more candy bar, or did you and Jake each get the same amount of candy bar?

Which of these two items did you rank as easier for children to understand?

☐ Item c is easier than item d

☐ Item d is easier than item c

☐ Items c and d are equally difficult

## Mathematics Background and Beliefs Survey

10.

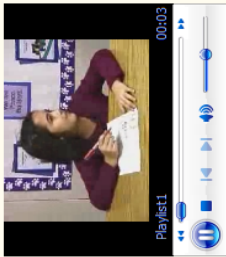
In a previous question, you were asked to rank the difficulty of understanding  $1/5 \times 1/8$ . By "understand", were you thinking of the ability to get the right answer?

- ☐ Yes  
☐ No

11.

Please describe your definition of *understanding* when it comes to mathematics.

## Mathematics Background and Beliefs Survey



Please watch the above video and write your reaction to this clip. Did anything stand out for you?

12.a

What do you think the child understands about division of fractions?

12.b

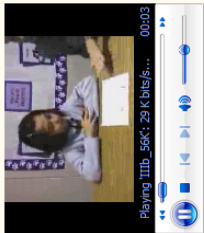
Would you expect this child to be able to solve a similar problem on her own 3 days after this session took place? Explain.

12.c

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## Mathematics Background and Beliefs Survey



Please watch another videoclip by clicking the play button above. Comment on what happened in this videoclip. (Note: This interview was conducted 3 days after the previous lesson on division of fractions.)

13.a

How typical is this child? If 100 children had this experience, how many of them would be able to solve a similar problem 3 days later? Explain.

13.b

Please provide suggestions about what the teacher might do to increase the likelihood that her students can solve a similar problem in the future.

13.c

Mathematics Background and Beliefs Survey

Lexi

$$\begin{array}{r} 56135 \\ -482 \\ \hline 153 \end{array}$$

Lexi says, "First I subtracted 2 from 5 and got 3. Then I couldn't subtract 8 from 3, so I borrowed. I crossed out the 6, wrote a 5, then put a 1 next to the 3. Now it's 13 minus 8 is 5. And then 5 minus 4 is 1, so my answer is 153."

Ariana

$$\begin{array}{r} 635 - 400 = 235 \\ 235 - 30 = 205 \\ 205 - 50 = 155 \\ 155 - 2 = 153 \end{array}$$

Ariana says, "First I subtracted 400 and got 235. Then I subtracted 30 and got 205, and I subtracted 50 more and got 155. I needed to subtract 2 more and ended up with 153."

14. Which child, Lexi or Ariana, shows the greater math understanding? Explain.

14.

|                                                                                                                                                                                                                                                                                                                          |                                                                                                                                                                                                                                                                                                                      |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>Lexi</p> $\begin{array}{r} 56135 \\ - 482 \\ \hline 153 \end{array}$ <p>Lexi says, "First I subtracted 2 from 5 and got 3. Then I couldn't subtract 8 from 3, so I borrowed. I crossed out the 6, wrote a 5, then put a 1 next to the 3. Now it's 13 minus 8 is 5. And then 5 minus 4 is 1, so my answer is 153."</p> | <p>Ariana</p> $\begin{array}{r} 635 - 400 = 235 \\ 235 - 30 = 205 \\ 205 - 50 = 155 \\ 155 - 2 = 153 \\ \hline 482 \end{array}$ <p>Ariana says, "First I subtracted 400 and got 235. Then I subtracted 30 and got 205, and I subtracted 50 more and got 155. I needed to subtract 2 more and ended up with 153."</p> |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

| Lexi                                                                                                                                                                                                                                      | Ariana                                                                                                                                                                        |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\begin{array}{r} 56135 \\ -482 \\ \hline 153 \end{array}$                                                                                                                                                                                | $\begin{array}{r} 635 - 400 = 235 \\ 235 - 30 = 205 \\ 205 - 50 = 155 \\ 155 - \frac{2}{482} = 153 \end{array}$                                                               |
| Lexi says, "First I subtracted 2 from 5 and got 3. Then I couldn't subtract 8 from 3, so I borrowed. I crossed out the 6, wrote a 5, then put a 1 next to the 3. Now it's 13 minus 8 is 5. And then 5 minus 4 is 1, so my answer is 153." | Ariana says, "First I subtracted 400 and got 235. Then I subtracted 30 and got 205, and I subtracted 50 more and got 155. I needed to subtract 2 more and ended up with 153." |

Here are the two approaches again so that you can refer to them to finish this section. For the remaining questions, assume that students have been exposed to both approaches.

15.a

Out of 10 students, how many do you think would choose Lexi's approach?

15.b

If 10 students used Lexi's approach, how many do you think would be successful in solving the problem 700-573? Explain.

16.a

Out of 10 students, how many do you think would choose an approach similar to Ariana's?

16.b

If 10 students used an approach similar to Ariana's, how many do you think would be successful in solving the problem 700-573? Explain.

17.

Which approach, Lexi's or Ariana's, would you prefer that your students use? Explain.

### Mathematics Background and Beliefs Survey

Read the following word problem:

*Leticia has 8 Pokemon cards. She gets some more for her birthday. Now she has 13 Pokemon cards. How many Pokemon cards did Leticia get for her birthday?*

18.

Do you think a typical first grader could solve this problem? Explain. (NOTE: The problem could be read to the child.)

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### Mathematics Background and Beliefs Survey

Here is another word problem. Again, read it and then determine whether a typical first grader could solve it.

*Miguel has 3 packs of gum. There are 5 sticks of gum in each pack. How many sticks of gum does Miguel have?*

19.

Do you think a typical first grader could solve this problem? Explain. (Note: The problem could be read to the child.)

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## Mathematics Background and Beliefs Survey



Please view the videoclip and then write your reaction to it. Did anything stand out to you?

20.a

Identify the strenghts of the teaching in this episode.

20.b

Identify the weaknesses of the teaching in this episode.

20.c

Do you think the child could have solved the problem with less help? Please explain.

20.d

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## **APPENDIX B**

### **Semi-Structured Interview Protocol**

#### **1. Typical vs. Ideal Lesson**

For this study, you have the opportunity to represent your instruction through 2 typical lessons and 1 lesson that represents your ideal of math instruction. Which category does each of yesterday's and today's lessons fit into? Explain.

- a. Was there anything that caught your interest in the lesson you taught today? Explain.

#### **2. Textbook**

The district adopted the Houghton Mifflin math textbook two years ago. What role does the textbook play in your planning and instruction?

- a. What are the district and school expectations regarding the role of the textbook in instruction?
- b. How does the textbook support or constrain you as a teacher?
- c. What do you feel are some of the strengths and weaknesses of this textbook?
- d. How would your ideal textbook be similar or different from the textbook you currently use?

#### **3. Assessment**

There are a number of assessments you administer to your students which are required by the state, the district, your school, and maybe even within your team. What assessments do you administer and what role do they play in your mathematics teaching?

- a. What do you do with the data from these assessments?
- b. What are the expectations from others regarding what you are supposed to do with this data?
- c. Describe the usefulness of the state and district assessments versus classroom level assessments (either textbook or ones you design yourself).
- d. If you were in charge, how would you change the assessment systems currently in place?

**4. Professional Development**

Describe any professional development the district has provided that is pertinent to your math instruction.

- a. What role has the training played in your math teaching?

**5. Other Factors**

Are there any other programs, policies, groups that you are a part of, people, or materials, or anything else that play into how you teach mathematics in your classroom? Please describe.

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