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ABSTRACT

A proof of the factorizability of the residues of Regge poles, valid for a many-channel potential scattering problem, is given. Unitarity and certain other plausible assumptions about the S matrix allow the proof to be extended to the relativistic S-matrix theory.

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Recently Gell-Mann (private communication) has postulated that the residues of Regge poles of the S matrix for a many-channel problem are factorizable, viz., for

$$\beta_{ij}(E) = \lim_{J \rightarrow \alpha(E)} (J - \alpha(E)) S_{ij}(E), \quad (1)$$

where i, j label the channels and $\alpha(E)$ is the position of a pole, then

$$\beta_{ij}(E) = \gamma_i(E) \gamma_j(E). \quad (2)$$

Gell-Mann has given a proof of this equation based on the nonrelativistic Schrödinger equation.¹

In the course of a general study of analyticity in J for the nonrelativistic potential scattering problem we had also obtained a simple proof of this result, which is worth reporting, since the method, being based directly on the S matrix, can immediately be generalized to enable us to say something about the relativistic problem.

For the potential case (and for a wide class of potentials) it can be shown that the S matrix can be written in the form

$$S(J, E) = F_1(J, E) [F_2(J, E)]^{-1}, \quad (3)$$

where F_1 and F_2 are n -by- n matrices,² n being the number of channels, with F_1 and F_2 analytic functions of J . Thus, poles of $S(J, E)$ occur for

$$\det [F_2(J, E)] = 0. \quad (4)$$

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Except for accidental degeneracies the zeros in $\det[F_2(J, E)]$ are simple zeros. Since the elements of F_2 are analytic in J , it follows that the rank of F_2 is $n - 1$.

If we write

$$[F_2]^{-1} = \frac{G}{\det F_2}, \quad (5)$$

where G is the matrix of cofactors, then Sylvester's law of nullity tells us

$$r(G) + r(F_2) - n \leq 0, \quad (6)$$

where $r(A)$ means the rank of the matrix A . Thus

$$r(G) \leq 1, \quad (7)$$

i.e., all 2-by-2 cofactors of G are zero. Simple calculation then shows that the residues satisfy

$$\beta_{ii} \beta_{jj} = \beta_{ij} \beta_{ji}. \quad (8)$$

Apart from an irrelevant sign, Eq. (2) follows from Eq. (8) and the fact that S is symmetrical.

If, as seems plausible, the relativistic S matrix can also be written in the form of Eq. (3), with F_1, F_2 , analytic functions of J , then the generalization of the above to the relativistic case is immediate. In fact, however, we may not even need this, since we can proceed directly from unitarity. This tells us³ that for fixed real E , with $E_N < E < E_{N+1}$,

$$S_N^\dagger(J^*, E + i\epsilon) S_N(J, E + i\epsilon) = 1, \quad (9)$$

where S_N is the submatrix containing only those channels which are open in this region of E between the consecutive thresholds E_N and E_{N+1} . Thus,

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$$S_N(J, E + i \epsilon) = [S_N^\dagger(J^*, E + i \epsilon)]^{-1}. \quad (10)$$

We now assume that the S-matrix elements are meromorphic functions of J . Then Eq. (10) is of the form of Eq. (3) and we can prove the factorizability relation, provided that at a pole $J = \alpha(E)$ of $S_N(J, E + i \epsilon)$ none of the matrix elements of $S_N(J^*, E + i \epsilon)$ has a pole. The latter condition ensures that poles of $S_N(J, E + i \epsilon)$ correspond to zeros of $\det[S_N(J^*, E + i \epsilon)]$. Note that a further consequence is that all elements of S_N have in general the same poles.

There are two ways in which the above condition might be violated; namely, the function $\alpha(E)$ being real, or there being a pole at $\alpha^*(E)$ in addition to the one at $\alpha(E)$. To rule out the first, consider the matrix of residues, β , defined by

$$S(J, E) = \frac{\beta(E)}{J - \alpha(E)} + \text{regular part}, \quad (11)$$

where α is now real. Then Eq. (9) certainly requires

$$\beta^\dagger \beta = 0, \quad (12)$$

which is clearly impossible unless $\beta \equiv 0$. We cannot at present rule out the other possibility, but since there is no reason for any symmetry between $S(J, E)$ and $S(J^*, E)$, we believe that it can happen only accidentally, and for isolated values of E .

We have proven the factorizability relation, Eq. (2), for $E_N < E < E_{N+1}$, and for the corresponding submatrix S_N . The relation can obviously be continued to other regions of E , provided we do not cross any cuts. Further, by taking N sufficiently large we can include channels with arbitrarily high thresholds.

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It is a pleasure to acknowledge very many useful discussions on the subject of this note with the members of the S-matrix theory group at this laboratory, and at the University of California, Berkeley, Physics Department.

FOOTNOTES AND REFERENCES

- * This work was performed under the auspices of the U. S. Atomic Energy Commission.
1. Murray Gell-Mann, Factorization of Coupling to Regge Poles (California Institute of Technology preprint).
 2. The matrices F_1 and F_2 are generalizations of Jost functions; see, for example, R. G. Newton, J. Math. Phys. 1, 319 (1960).
 3. That the appropriate continuation of $S(J,E)$ satisfies unitarity for nonphysical J was shown by M. Froissart (unpublished); see also E. J. Squires, On the Continuation of Partial-Wave Amplitudes to Complex ℓ (UCRL-10033, Jan. 1962), submitted to Nuovo cimento.
 4. So far this has been proven only for one-channel problems and in a restricted region of J .

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