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The Jensen Measure and
Errors in Variables: A Note

by

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Preliminary
Do Not Quote

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In the Journal of Business, Jensen (1969) developed a method of evaluating portfolio performance based on the Capital Asset Pricing Model. This method, now known as the Jensen measure, is used in the securities industry and is widely taught in business schools. The measure adjusts returns for beta risk by computing the distance a portfolio return lies above or below the securities market line. It is commonly thought that this is superior to measuring raw returns because the latter are positively related to the beta of the portfolio.

Although this approach has been criticised in a number of papers,¹ nobody has really examined whether the method, in its actual implementation, does what it is supposed to do - remove systematic differences in returns due to risk so as to leave only those due to differences in information. To answer this, we analyzed the mutual fund data to which Jensen first applied the measure. (This data, consisting of the annual returns of 115 mutual funds from 1945-1964 appeared in the same article, fifteen years ago.) If the Jensen measure does what it claims to, it should, at the very least, produce a performance measure that is less sensitive to beta than raw returns.

Table 1, which presents the correlation and covariance matrices for the Jensen measure (α), estimated beta ($\hat{\beta}$), and raw returns (R), along with two OLS regressions for Jensen's data, suggests that this is not the case. As can be seen, estimated beta is better at explaining the Jensen measure than at explaining raw returns. The reason for this is an errors in variable problem. $\hat{\beta}$ is not the true risk measure, but the true risk measure plus some random noise. This could be because the CAPM is not true or because the index is not the

¹Roll (1978) argues that the Jensen measure is sensitive to the choice of a market index and is inconsistent with the mathematical implications of a mean-variance efficient market portfolio. Verrecchia (1980) and Dybvig and Ross (1981) suggest that it may not measure performance properly when portfolio managers have timing ability.

actual market portfolio, but a good part of it is due to sampling error. The strong negative correlation between α and $\hat{\beta}$ is a classic regression towards the mean problem. Those funds with betas above one have, on average, overestimated betas, and those with beta below one tend to have underestimated betas.

This pattern can be eliminated by recalculating beta with a Bayesian adjustment, such as that described in Vasicek (1973), but this misses the point. What is surprising here is not the negative correlation (which Jensen noted on pp. 225-227), but its magnitude in comparison with the raw return-beta correlation. If our estimates of beta are so bad that we have a harder time explaining raw returns than "risk-adjusted" returns, Bayesian adjustments merely mask the bad betas, they don't eliminate the problem. Indeed, if we recalculate a new beta for each fund as a weighted average of the old beta of the fund and the mean of all of the funds' betas², we only eliminate this negative correlation for weights on the old fund beta that are less than .3. For the .3 weighting scheme, each new fund beta lies so close to the mean beta of all the funds as to make a risk discrimination meaningless.³

Whether Bayesian adjusted or not, applying a misestimated beta to obtain the same repeated measures of portfolio performance over a set of periods is likely to induce spurious "persistent performance". At first glance, this would be an argument for reestimating beta each period and then examining persistence. While this eliminates the bias in measuring persistent performance, it is at the

²The α 's are then recalculated, too.

³The standard deviation of the new betas is .0655. With a 10% market risk premium, this would imply that approximately 95% of the funds would have annual expected returns within 1.3% of the mean fund annual return.

cost of efficiency. The above evidence suggests that even when betas are estimated over long periods of time, they are quite bad. Using less data will create noisier betas, making it virtually impossible to detect any true performance.

The extent of the measurement error in Jensen's betas can be approximated by calculating the percentage of the cross-sectional variation in beta that is due to estimation error. Let the true betas of a group of mutual funds be denoted by the vector $\underline{\beta}$ and their estimated betas by $\hat{\underline{\beta}} = \underline{\beta} + \underline{z}$, where \underline{z} is the random estimation error vector. For simplicity, we will assume that there is no true mutual fund performance, that mutual fund returns follow a two fund separating distribution, and that we observe the ex-ante efficient portfolio, which has a realized excess return (above the risk-free rate) of \tilde{r}_m . Under these assumptions, the vector of realized excess returns of a group of mutual funds, \underline{R} , satisfies

$$\underline{\tilde{R}} = \underline{\beta} \tilde{r}_m + \underline{\varepsilon} \quad (1)$$

Because we observe only a finite sample of realized returns, we misestimate $\underline{\beta}$ and instead observe

$$\underline{\tilde{R}} = \underline{\alpha} + \hat{\underline{\beta}} \tilde{r}_m + \underline{\tilde{e}} \quad (2)$$

Taking sample averages of (2) yields

$$\bar{\underline{R}} = \underline{\alpha} + \hat{\bar{\underline{\beta}}} \bar{\tilde{r}}_m, \quad (3)$$

(where bars over random variables denote their sample means). Rearranging (3) and substituting (1) implies that the observed Jensen measure satisfies

$$\begin{aligned}\underline{\alpha} &= \bar{R} - \hat{\beta} \bar{r}_m \\ &= \bar{\varepsilon} - \underline{z} \bar{r}_m.\end{aligned}\tag{4}$$

Let $\underline{\alpha}'$ and $\hat{\beta}'$ denote the vectors of deviations of $\underline{\alpha}$ and $\hat{\beta}$ from their cross-sectional averages, so that the sums of the elements of $\underline{\alpha}'$ and $\hat{\beta}'$ are each zero. (This notational convention will apply for other variables as well.) Then, the cross-sectional covariance between the observed Jensen measure and observed beta is

$$\sigma(\underline{\alpha}', \hat{\beta}') = \sigma(\bar{\varepsilon}', \hat{\beta}') + \sigma(\bar{\varepsilon}', \underline{z}') - \bar{r}_m \sigma(\underline{z}', \hat{\beta}') - \bar{r}_m \sigma(\underline{z}', \underline{z}'),$$

where $\sigma(\underline{x}, \underline{y})$ denotes the inner product of \underline{x} and \underline{y} divided by n . For finite samples, the expectation of this is

$$-\bar{r}_m \sigma(\underline{z}', \underline{z}'),$$

which is minus the market risk premium times the (cross-sectional) variance of \underline{z} . Thus, an unbiased estimate of the sampling error is

$$\sigma(\underline{z}', \underline{z}') = -\sigma(\underline{\alpha}', \hat{\beta}') / \bar{r}_m.\tag{5}$$

From Table 1, the numerator is .00309. If we assume, as Jensen did, an 8.91% annualized market risk premium in this period, the quotient is .03468, which implies a standard deviation of .186 for the measurement error in beta. If we divide the former by .047682, the cross-sectional variation in beta, we find that 72.7% of this variation is due to variation in measurement error, \underline{z} .

It is also possible to explicitly estimate the standard deviation of the measurement error of the Bayesian adjusted betas. This is the theoretically

implied mean-squared prediction error from a regression of true betas on Jensen's betas. It equals

$$\begin{aligned} \left[\frac{\sigma(\underline{\beta}', \underline{\beta}') \sigma(\underline{z}', \underline{z}')}{\sigma(\underline{\beta}', \underline{\beta}') + \sigma(\underline{z}', \underline{z}')} \right]^{1/2} &= \left[\frac{(\sigma(\hat{\underline{\beta}}', \hat{\underline{\beta}}') - \sigma(\underline{z}', \underline{z}')) \sigma(\underline{z}', \underline{z}')}{\sigma(\hat{\underline{\beta}}', \hat{\underline{\beta}}')} \right]^{1/2} \\ &= \left[\frac{(.047682 - .03468)(.03468)}{.047682} \right]^{1/2} = .094 \end{aligned}$$

We should emphasize that these results are not restricted to Jensen's data alone. Mains (1977) replicates this study with a larger sample of returns for Jensen's set of mutual funds, computes different betas for two subperiods, and improves the treatment of cash distributions. But, as can be seen in the correlations of Table 2, he barely mitigates the problem. As with Jensen's betas, we can estimate the degree of inefficiency: 45.0% of his cross-sectional variance in beta is due to measurement error and the standard deviation of this measurement error is .130.

It should also be emphasized that this is not a critique of the CAPM, so much as a critique of risk-adjustment based performance measures. While it is possible that the estimated factor loadings for the Arbitrage Pricing Theory of Ross (1976) are closer to the true risk measures, we suspect that they are not sufficiently close to eliminate much of this errors in variables problem.

References

- Dybvig, Philip and Stephen Ross (1981), "The Simple Analytics of Performance Measurement," unpublished working paper, Yale University, New Haven, CT.
- Jensen, Michael (1969), "Risk, the Pricing of Capital Assets, and the Evaluation of Investment Portfolios," Journal of Business 42, 167-247.
- Mains, Norman (1977), "Risk, the Pricing of Capital Assets, and the Evaluation of Investment Portfolios: Comment on Jensen" [20], Journal of Business 50, 371-384.
- Roll, Richard (1978), "Ambiguity When Performance is Measured by the Securities Market Line," Journal of Finance 33, 1051-1069.
- Ross, Stephen (1978), "Mutual Fund Separation in Financial Theory -- The Separating Distributions," Journal of Economic Theory 17, 254-286.
- Verrecchia, Robert (1980), "The Mayers-Rice Conjecture: A Counterexample," Journal of Financial Economics 8, 87-100.

Table 1

Correlation and Covariance Matrices⁴ for the Betas ($\hat{\beta}$), Average Annual Raw Returns (R), and Average Annual Jensen Measures (α) of the 115 Mutual Funds in Jensen (1969). (Covariances are in Parentheses.)

	$\hat{\beta}$	R	α
$\hat{\beta}$	1(.04762)	.30(.001158)	-.64(-.003090)
R	-	1(.000321)	.55(.000217)
α	-	-	1(.000493)

Regression Coefficients in
Univariate Ordinary Least Squares Regressions
Using Jensen's Mutual Fund Data.

Dependent Variable	R	α
Constant	.75	.45
Regression Coefficient on $\hat{\beta}$.24	-.65
T-statistic	(3.3)	(-8.8)

⁴The computed correlation between $\hat{\beta}$ and α differs from the correlation reported in Jensen. Jensen claims it is -.68.

Table 2

Correlation and Covariance Matrices for the Betas ($\hat{\beta}$), Raw Net Returns (R), and Jensen Measures (α) of the 70 Mutual Funds in the Mains (1977) Study. (Covariances are in Parentheses.)

	$\hat{\beta}$	R	α
$\hat{\beta}$	1 (.037589)	.55 (.001774)	-.48 (-.001507)
R	-	1	.45 (.000119)
α	-	-	1 (.000256)