

UC Irvine

UC Irvine Electronic Theses and Dissertations

Title

Dispersion Engineering of Microwave and High-power Devices with Exceptional Points of Degeneracy

Permalink

<https://escholarship.org/uc/item/9mw2q35h>

Author

Abdelshafy, Ahmed Farghaly Abdelrahman

Publication Date

2021

Copyright Information

This work is made available under the terms of a Creative Commons Attribution License, available at <https://creativecommons.org/licenses/by/4.0/>

Peer reviewed|Thesis/dissertation

UNIVERSITY OF CALIFORNIA,
IRVINE

Dispersion Engineering of Microwave and High-power Devices with Exceptional Points of
Degeneracy

DISSERTATION

submitted in partial satisfaction of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

in Electrical and Computer Engineering

by

Ahmed Farghaly Abdelrahman Abdelshafy

Dissertation Committee:
Professor Filippo Capolino , Chair
Professor Alexander Figotin, Co-chair
Professor Michael Green
Professor Ozdal Boyraz

DEDICATION

To my grandmother...We will never forget you...
To my beloved parents and siblings

TABLE OF CONTENTS

	Page
LIST OF FIGURES	vi
LIST OF TABLES	xv
ACKNOWLEDGMENTS	xvi
VITA	xvii
ABSTRACT OF THE DISSERTATION	xxii
1 Introduction	1
1.1 Motivation	1
1.2 Exceptional Points of degeneracy (EPD)	1
1.3 Degenerate band edge (DBE)	4
1.4 Organization of the dissertation and contents	6
2 Exceptional Points of Degeneracy in Periodic Coupled Waveguides and the Interplay of Gain and Radiation Loss: Theoretical and Experimental Demonstration	9
2.1 Motivation	9
2.2 System Description of CTLs	13
2.2.1 State vector and wave propagation in CTLs	14
2.2.2 Evolution of waves in periodic coupled CTLs	16
2.2.3 Fourth-order exceptional points of degeneracy	19
2.3 Fourth Order EPD in Periodic CTLs	21
2.4 Experimental Verification of EPD Features in Periodic CTLs	29
2.5 Finite length CTLs with EPDs: giant resonance and radiation	31
2.6 EPDs With “Balanced” Gain And Loss	35
2.6.1 Case A: Series loss and shunt gain	37
2.6.2 Case B: shunt loss in both TLs, shunt gain in only one TL	38
2.7 Conclusion	41
3 Distributed Degenerate Band Edge Oscillator	43
3.1 Motivation	43
3.2 System Description	45

3.3	Distributed Degenerate Band Edge Oscillator	47
3.4	Frequency Stability of the Distributed DBE Oscillator	51
3.5	Conclusion	53
4	Exceptional Degeneracy in a Waveguide Periodically Loaded with Discrete Gain and Radiation Loss Elements	56
4.1	Motivation	56
4.2	System Description and Analytical Formulation	58
4.2.1	Trivial cases with vanishing gain or loss ($Y_r = 0$ or $g = 0$)	61
4.2.2	Symmetric gain and loss cases ($Y_r = g$)	62
4.2.3	Asymmetric gain and loss cases ($Y_r \neq g$)	65
4.3	EPD Oscillator	68
4.4	Conclusion	71
5	Degenerate Band Edge Slow Wave Structures for Electron Beam Driven Devices	72
5.1	Motivation	72
5.2	Four-eigenmode Super Synchronization Regime at the DBE	73
5.3	Four-Mode Degeneracy in a Double-Helix SWS	75
5.4	Four-Mode Degeneracy in a MTM coupled cavities SWS	77
5.4.1	Cold SWS dispersion	79
5.4.2	Interaction impedance	80
5.4.3	Computational Model	84
5.4.4	Hot Simulations Results	86
5.4.5	Experiments	90
5.5	EPDs for High Power Microwave and Terahertz Generation	96
5.5.1	Periodic double corrugated waveguide and the degenerate band edge oscillator (DBEO)	97
5.5.2	Cylindrical waveguide with periodically corrugated surface	102
5.6	Conclusion	102
6	An Accurate Traveling Wave Tube Analytic Model	105
6.1	Motivation	105
6.2	Mathematical Formulation	108
6.3	TWT hot eigenmodes based on PIC simulations	113
6.3.1	Eigenmode-like operation regime	113
6.3.2	Estimation of the amplified eigenmode in helical TWT	115
6.4	Connection between the analytical model and actual electron beam device .	124
6.5	Summary	128
	Bibliography	129
	Appendix A Numerical Parameters Used In Chapter 2	140
	Appendix B Wavenumber Perturbation in The Vicinity of The DBE	144

LIST OF FIGURES

	Page
1.1 Classification of the different structure-topologies that can exhibit exceptional points of degeneracy (EPDs).	2
1.2 (top) Periodic stack capable of supporting $k - \omega$ diagram with degenerate band edge. A unit cell L includes three layers: two birefringent layers A1 and A2 with different orientations ϕ_1 and ϕ_2 of the respective anisotropy axes in the X-Y plane, and one isotropic B layer. (below) $\omega - k$ dispersion diagrams in periodic structures showing the following features: (a) Standard regular band edge (RBE), between the pass and stop band of a single mode in a photonic crystal. (b) Degenerate band edge (DBE) and (c) split band edge (SBE) of a complex periodic structure. Pictures adopted from [1, 2]	5
2.1 Example geometries of waveguide microstrip lines on a grounded dielectric slab that support the fourth-order EPD, visible in the $(k - \omega)$ dispersion diagram at microwave frequencies. (a) and (b) Examples represent two cases of coupling, with proximity fields and with a physical connection, respectively. Results in Sections 2.3 and 2.6 are based on a CTL that models the geometry in (a), whereas the results in Sections 2.4 and 2.5 are based on the microstrip CTL geometry in (b). In the lossless case, the fourth-order EPD is called DBE; however, such structures exhibit both second- and fourth-order EPDs in the case of gain and loss balance.	11
2.2 Representation of the eigenvectors of the coupled waveguides near an EPD, schematically showing that the four independent vectors coalesce into a degenerate eigenvector at the EPD when one system parameter is varied. . . .	21

2.3	(First two columns) (a)–(c) Complex k plane showing the trajectory of the wavenumber k for increasing frequency near the fourth-order EPD condition for different values of the coupling capacitances, for a lossless CTL, and the (d)–(f) corresponding dispersion diagram showing the evolution from RBE to DBE and to SBE, respectively, when varying the coupling capacitance. Dots in the left panel plots indicate the starting point $\omega = 0.95\omega_d$ of the frequency sweep. (g)–(l) “Measures” of the closeness to the EPD calculated based on the four eigenvectors Ψ_n of the lossless and lossy periodic structure. (g)–(i) depicts $ \det(\underline{\mathbf{U}}) = 0$, while (j)–(l) depicts DH for different values of C_m . Any EPD necessarily has $ \det(\underline{\mathbf{U}}) = 0$ and the fourth-order degeneracy has $D_H = 0$. Mathematically speaking, only the lossless case with $C_m = C_{m,d}$ shows the fourth-order EPD (i.e., the DBE). Other values of C_m lead to the second-order EPDs (i.e., the RBEs). Losses perturb the RBE and the DBE. Solid lines represent the results for a lossless CTL, whereas the dashed lines represent the results for a CTL including radiation “loss,” in which EPD is no longer strictly manifested, but it can still occur in practice, depending on how small D_H is. Results are based on the microstrip geometry shown in Fig. 2.1(a) whose parameters are given in Appendix A. Here, $\omega_e \equiv \omega_d = 2\pi$ (4.03 GHz) and $k_e \equiv k_d = \pi/d$	23
2.4	(a) Microstrip unit cell of a periodic structure that exhibits a fourth-order EPD. The substrate is Rogers RO4360G2 with a dielectric constant of 6.15 and a height of 1.52 mm. (b) Fabricated unit cell under test with the four ports is attached to a VNA to extract the four-port S-parameters versus frequency used to calculate the eigenvalues. EPDs in the case of gain and loss balance.	31
2.5	(a) Dispersion relation and (b) hyperdistance measurement versus full-wave simulation of the unit cell of a periodic CTL exhibiting a fourth EPD. The results show that the fourth-order EPD occurs at two different frequencies in the range shown in this plot. Full-wave simulations are performed with Keysight ADS using the MoM.	32
2.6	Fabricated microstrip eight cell array with an EPD. The CTL is based on the unit cell described in Fig. 2.4.	33
2.7	(a) Dispersion relation for the unit cell of the periodic structure in Fig. 2.6 obtained using the full-wave FEM (Ansys HFSS) accounting also for radiation, ohmic, and dielectric losses. Simulations and measurements of the scattering parameters (b) S11 and (c) S21 for an eight-unit-cell array in Fig. 2.6, where a good agreement between full-wave simulations and measurements is shown at the resonance frequency associated with the EPD. The frequency of the resonance peak is estimated by (2.19).	34
2.8	Measurements versus simulations of the power loss parameter defined as $1 - S_{11} ^2 - S_{21} ^2$ varying as a function of frequency near the EPD resonance. . .	35

2.9	(a) Plot of the hyperdistance (18) to assess the occurrence of a fourth-order EPD in a CTL with losses, as a function of gain $-G$ to compensate for losses. The EPD is achieved in practical terms with $G = G_e = -9.54$ mS/m at which hyperdistance has the minimum value $D_H(\omega_d) = 0.078$ (keeping the angular frequency fixed at ω_d , at which the lossless CTLs develops a DBE). (b) Corresponding dispersion diagram for the lossy CTL (i.e., before introducing gain) where losses are modeled as series resistance R_n such that all TLs have $Q_{TL} = 100$. (c) Dispersion diagram after introducing the appropriate gain $G = G_e$ to achieve the “balanced gain and loss condition,” showing the typical flatness of a DBE is recovered.	39
2.10	(a) Dispersion diagram for a lossy CTL structure (i.e., before introducing gain) as in Fig. 2.9(a) but with one order of magnitude larger losses than in Fig. 2.9(a), that is, now all TLs have $Q_{TL} = 10$. (b) Corresponding dispersion diagram after introducing the gain $G_e = -93.4$ mS/m associated to minimum $D_H(\omega_d) = 0.18$	40
2.11	(a) Plot of the hyperdistance (18) to assess the presence of a fourth-order EPD in a CTL with losses, as a function of gain $-G$ to compensate for losses. The EPD is achieved in practical terms with a value of gain parameter $-G_e = 15.4$ mS/m (attached to TL1) at which hyperdistance has the minimum value $D_H(\omega_d) = 0.078$ (i.e., less than the predetermined threshold). Angular frequency is fixed at ω_d at which the lossless CTLs develop a DBE. (b) Corresponding dispersion diagram for lossy structure (i.e., before introducing gain) where losses are modeled as shunt $G_{L,n}$ such that all TLs have the same $Q_{TL} = 100$. (c) Corresponding dispersion diagram after introducing the gain associated to minimum D_H , showing nonideal EPD.	42
3.1	(a) Schematic of two coupled microstrip lines on a grounded dielectric substrate that support a fourth order EPD (the DBE) visible in the $(k-\omega)$ dispersion diagram. (b) Microstrip unit cell of the periodic waveguide that exhibits the DBE. (c) Real and imaginary parts of the wavenumbers of the four guided Floquet-Bloch modes obtained using the full-wave finite element method accounting for radiation, ohmic and dielectric losses.	48
3.2	(a) Loaded DBE oscillator consisting of 8 cascading unit cells (UCs) of microstrip-based CTLs shown in Fig.1(b). Active devices are placed between each two adjacent unit cells to the bias line. Two loads of 50Ω are attached at the two ends of the lower TL while the upper TL is terminated in short circuits. The oscillation starts for sufficiently large $g_m = 3$ mS. (b) Voltage waveform $v_L(t)$ monitored at a 50Ω load where steady state oscillation is observed in less than 30 ns. (c) The normalized spectrum $V_{L,N}(f)$ shows that oscillations occur at 3.03 GHz, that is very close to the DBE frequency of 3.02 GHz in Fig. 1(c). The spectrum is calculated by applying the Fourier transform in a time window from 35 to 100 nsec.	50

3.3	(a) Voltage and current distributions on top and bottom TLs of the distributed DBE oscillator shown in Fig. 2(a), when the bottom TL is terminated in two 50Ω loads, while the top TL is short circuited at its two ends. ($ V_n $ and $ I_n $ represent voltage and current time-harmonic waveform peak values, which are phasors' amplitudes, at steady state regime).	52
3.4	Average output power and oscillation frequency versus load resistance for the distributed DBE oscillator shown in Fig. 2(a), for $g_m = 3\text{ mS}$ and $g_m = 1.5\text{ mS}$. The stability of the oscillation frequency over a huge variation of the load resistance shows an important advantage of the proposed oscillator: the frequency of oscillation is almost the same, i.e. $\sim 3\text{ GHz}$, with a very slight shift that does not exceed $\pm 1\%$ ($\sim 30\text{ MHz}$).	54
3.5	Average output power and oscillation frequency versus (a) gain variation, and (b) capacitive loads variation. In (b) the capacitor C is parallel to each of the two load resistors $R_L = 50\Omega$, assuming $g_m = 1.5\text{ mS}$	55
4.1	(a) Unit cell schematic of a periodic waveguide, represented by its equivalent transmission line (TL), made of two segments with characteristic impedance Z_0 and loaded with shunt lossy element (Y_r) and shunt gain element ($-g$). (b) The relation between gain and loss to have an EPD. The shaded area represents the asymmetric cases where gain and loss relation to have an EPD is $gZ_0 = 4/(Y_r Z_0 \sin^2(\theta_A))$. The black-dashed curve represents one of the asymmetric cases when $\theta_A = \theta_B = (2m + 1)\pi/2$, also the PT-glide-symmetry case is depicted by the green dot in the intersection between the asymmetric case dashed curve and red line representing symmetric cases.	60
4.2	(a) Dispersion diagram of complex-valued wavenumber versus frequency for gain-radiation loss symmetric case, $Y_r = g = 2\sqrt{2}/Z_0 = \sqrt{2}/25\text{ S}$, with $3\theta_A = \theta_B = 3\pi/4$ at 3 GHz . Wavenumber degeneracies are observed at 3 GHz , 4.333 GHz , 7.715 GHz , 9 GHz , etc., where either $k = 0$ or $kd/\pi = \pm 1$. (b) A zoom-in version in the vicinity of the EPD at $(\omega_e, k_e) = (3\text{ GHz}, 0)$ shows that the complex-valued dispersion fits to the quadratic formula $(\omega - \omega_e) = h(k - k_e)^2$ denoted by red symbols with $h = 1.1162 \times 10^{14}\text{ m}^2/\text{sec}$	64
4.3	Dispersion diagram of complex-valued wavenumber versus frequency. Wavenumber degeneracies are observed at 3 GHz , 6 GHz , etc. where both wavenumbers vanish. The two wavenumbers are denoted by different colors, for the two different cases with $Z_0 = 50\Omega$ and $\theta_A = \theta_B = \pi/2$ at 3 GHz : (a) $Y_r = 20\text{ mS}$, $g = 80\text{ mS}$ corresponding to $Y_r Z_0 < 2$, and (b) $Y_r = 50\text{ mS}$, $g = 32\text{ mS}$ corresponding to $Y_r Z_0 > 2$	67
4.4	Complex-valued Bloch impedances Z_B showing the trajectory of Z_B evolution varying frequency where arrows represent the direction of frequency increasing. Degeneracies are observed at 3 GHz , 6 GHz , etc. where both wavenumbers vanish, i.e. $k = 0$. The two modes' Bloch impedance are denoted by different colors, matching different modes' colors in the dispersion diagram in Fig. 2 (a, b), for the two different cases with $\theta_A = \theta_B = \pi/2$ at 3 GHz : (a) $Y_r = 20\text{ mS}$ corresponding to $Y_r Z_0 < 2$, and (b) $Y_r = 50\text{ mS}$ corresponding to $Y_r Z_0 > 2$	68

4.5	EPD oscillator consisting of 8 cascaded unit cells (UCs) loaded with gain and loss (representing a radiating antenna) as shown in Fig. 4.1(a). Active gain devices are placed in each UC from the TL to the bias line (that acts as a ground for a.c. signals). (b) Voltage waveform $v_m(t)$ monitored at the Y_r load in the middle of the structure where steady-state oscillation is observed in less than 2 ns. (c) Normalized voltage spectrum $V_{m,N}(f)$ shows that oscillations occur at around 3 GHz, that corresponds to the EPD frequency of 3 GHz in Fig. 4.3(b).	70
5.1	Example of high-power electron beam driven device composed of a SWS that supports four degenerate eigenmodes. This degenerate condition is shown in the modal dispersion diagram of the “cold” periodic SWS, where four Floquet-Bloch eigenmodes coalesce at the single point (k_d, ω_d) that occurs at the band edge. Super synchronization is achieved when the electron beam velocity u_0 is matched to the phase velocity of the four modes at (k_d, ω_d) , leading to the super synchronization condition in Eqn. 5.1. Note that $k_d = \pi/d$ where d is the period of the SWS. Eigenmodes with purely real wavenumber are denoted by solid lines whereas complex wavenumbers are denoted by dashed lines. . .	74
5.2	(a) Double helix SWS exhibiting the fourth order eigenmode degeneracy at C-band. (b) Modal dispersion relation associated to the proposed “cold” double helix SWS in (a), showing the first two modes (only curves with purely real wavenumber are shown). The higher mode (red) exhibits a DBE at 6.14 GHz as shown from the typical characteristic fitting formula for this kind of degeneracy [3, 4]. (c) Interaction impedance of the DBE SWS calculated along the longitudinal axis, for the first two eigenmodes. The SWS unit cell dimensions are $r_{wg} = 6.6$ mm, $r_{h1} = 2.2$ mm, $r_{h2} = 2.4$ mm, the period is $d = p_{h1} = 2p_{h2} = 4$ mm, and the thickness and width of both helices is the same and equal to 0.1 mm and 1 mm, respectively.	76
5.3	A unit cell of the proposed MTM SWS: a disk-loaded circular waveguide with misaligned CeSRRs designed to facilitate the mode mixing required for multiple-mode interaction regime: (a) perspective view, (b) side view. (c) The two loaded disks inside the circular waveguide, each of which has a beam tunnel and two irises in the shape of half ring. All parameters and dimensions are reported in Table 5.1.	78
5.4	(Top) Dispersion diagram of the lowest order Floquet-Bloch eigenmodes of the cold periodic MTM SWS in Fig. 5.3, with dimensions reported in Table 5.1. (Below) A zoomed-in plot of the fourth mode (red curve) showing the higher order dispersion, associated to 4th order exceptional point of degeneracy, called degenerate band edge. (Inset) The longitudinal electric field map of the DBE mode at the transverse cross-section in the unit cell at $z = d/2$ shows that this mode has asymmetric pattern.	81

5.5	Dispersion diagram of the lowest order Floquet-Bloch eigenmodes of the cold SWS in Fig. 5.3. An electron-beam line of 490 keV (black dashed line) is plotted showing that the beam line intersects with the cold dispersion at different frequencies for each eigenmode. However, the dominating mode in the fully interacting system is the one that has the highest interaction impedance (i.e. third mode “purple”).	82
5.6	Interaction impedance described by Eq. (5.3) of the proposed SWS structure calculated at $(x, y) = (0, 0)$, i.e., along the beam axis for the lowest order fundamental Floquet-Bloch eigenmodes. The different colors match those of the modes of the cold SWS shown in Fig. 2. Note that the other Floquet harmonics have lower interaction impedance; thus, we do not plot them here since they are less likely to interact with the electron beam. Results are obtained from full wave simulations.	83
5.7	The electric and magnetic field maps of the third mode (normalized to their maximum) at the transverse cross-section in the unit cell at $z = d/2$, showing that this mode has almost a TM01-like mode profile.	84
5.8	Schematic showing the entire BWO structure, (top) 3-D perspective view of the structure (bottom) lateral view, i.e., the r - z cross-section at the $x=0$ plane. Starting from the left side, the BWO consists of: (i) annular cathode for annular electron beam generation; where the voltage is applied between the cathode and outer side of BWO using a coaxial circular waveguide port; (ii) MTM SWS with 11 unit cells, and total length =165 mm; (iii) metallic cylinder of radius $r_b = 12.85$ mm; (iv) canonical horn antenna to radiate the output microwave generated power, where (v) a circular waveguide port is placed at the end of the entire BWO (i.e. at the mouth of the horn antenna) to monitor the output power level.	85
5.9	(a) SINUS-6 applied voltage pulse in blue and the corresponding emitted current in red. (b) Simulated oscillating output microwave signal extracted from the TM01 mode of the horn output consisting of a circular waveguide port (Fig. 5.8). (c) Simulated output instantaneous power with a maximum around 88 MW, and (d) output spectrum obtained from a Fourier transform of the signal in the time window from 0 to 25 ns, showing a single frequency oscillation at 2.9 GHz which coincides with the zero-order approximation of the synchronization shown in the dispersion diagram in Fig. 5.5.	88
5.10	Phase space plot of electrons at different times in PIC simulations: (a) $t = 6$ ns; (b) $t = 10$ ns.	89
5.11	Simulated maximum instantaneous output power (blue trace) and RF conversion efficiency (red trace) versus the peak value of the applied voltage pulse for the MTM SWS shown in Fig. 5.8.	90
5.12	CAD drawing of the SINUS-6 electron beam accelerator showing the Tesla transformer (left), the pressurized spark gap switch (to the right of the Tesla transformer), and then connecting to the transmission line. The cathode and MTM SWS are affixed to the right end of the system within a solenoidal magnet.	91
5.13	(Top) CAD drawing the two CeSRRs comprising a unit cell of the MTM SWS. (Bottom) 3D printed cylindrical tube housing the 11-cell MTM SWS.	92

5.14	Photograph of the 3D printed, copper plated MTM SWS following 100 shots. Minimal damage was noted at the inlet (highlighted by the white ovals). . . .	94
5.15	Typical data recorded on a shot. Top left: electron beam current registered using a Rogowski coil; Top right: cathode voltage measured in the oil portion of the oil-vacuum interface registered using a capacitive divider; Bottom left: output of an S-band waveguide sensor registered using a crystal diode; and Bottom right: Fourier transform of the output of the waveguide sensor. . . .	95
5.16	Schematics of double corrugations inside a rectangular waveguide: (a) 3D perspective view; (b) Transverse cross section, in the x-y plane; and (c) y-z cross sections (i.e., a side view). This structure is adapted from [73] and will be used as a basis for a modified structure to realize a DBEO at THz frequencies (see Fig. 5.17).	98
5.17	Proposed double corrugated waveguide with DBE. (a) 3D perspective view; (b) Lateral view, i.e., the y-z cross section. Here the ridges along the z-direction have different heights h_1 and h_2 . (c) Top view, i.e., the x-z cross section showing the offset Δz in the z-direction between the two parallel corrugations. (d) Dispersion relation of the first few eigenmodes for the modified double corrugated waveguide showing normalized propagation wavenumber versus frequency. The second mode (green curve) exhibits a DBE at 661 GHz, proved by the fitting dispersion law $(\omega_d - \omega) \cong h_{\text{DBE}}(k - k_d)^4$ (blue dotted line). The beam line “black line” intersects with the DBE eigenwaves curve at around DBE frequency. This is used to conceive a DBEO.	99
5.18	Electric field E_z profile of the DBE eigenwave at 661.5 GHz; at different transverse cross sections (i.e., in the x-y plane, at different z coordinates) in the unit cell starting from $z = 0$: (a) $z = 0.1d$; (b) $z = 0.4d$; (c) $z = 0.6d$; and (d) $z = 0.9d$, where d is the period. Blue and red indicate negative and positive values of the E_z field, at a given time.	100
5.19	(a) Oscillating output signal versus time of the DBEO made of a modified double corrugated waveguide shown in Fig. 5.17. Steady oscillation regime is reached at approximately 7 ns. (b) The normalized spectrum of the output signal shows that the oscillations occur at 638 GHz, which indicates that the oscillations are approximatively at the DBE frequency. The spectrum is calculated in a time window from 14 to 15 ns.	101
5.20	Schematics of periodic surface lattice on a cylindrical waveguide’s inner wall, i.e., where inner radius is perturbed. (a) 3D perspective view; (b) longitudinal cut, and (c) transverse cross sections of such waveguide with corrugations, adapted from [5]. This structure can support a DBE. This specific geometry is compatible with annular electron beams. (d) Dispersion relation of modes in the waveguide showing an overmoded behavior and possible DBEs (showing in dotted lines) exhibiting flat dispersion in the mm-wave frequency range. Here, the parameters are mm, mm, and mm.	103

6.1	Complex-valued wavenumbers of the hot eigenmodes in the interactive electron beam-EM mode in the helical SWS considered in this chapter in Fig. 6.2(a). (a-b) Real-imaginary parts of the recovered dispersion by estimating the adjusted analytical model (γ) parameter using PIC simulations data (the black cross data point) and the frequency dependence of $w(\omega)$ and $\gamma(\omega)$ directly from the cold CST eigenmode solver in Fig. 6.10. A reasonable agreement is observed in the real part in (a) as well as the imaginary part in (b) between the recovered adjusted analytical model and PIC simulations data (the green dots).	108
6.2	(a) Schematic of a helix-based TWT amplifier. (b) (left) Perspective view and (right) front view of a unit cell of the TWT shown in (a) where the tape helix SWS in the circular waveguide supported by three equally spaced dielectric rods. (c) A solid electron beam flows along the axis of the helical conductor.	117
6.3	Dispersion diagram of the lowest order Floquet-Bloch eigenmodes of the cold SWS in Fig. 6.2. An electron beam (e-beam) line with particles' initial velocity about 0.2 times the speed of light to have a synchronization around the 12 GHz (blue dashed line) is plotted showing that the beam line intersects with the cold dispersion in the forward region.	117
6.4	(a) Simulated output RF port signal, for the no RF excitation case, shows no signal after some transient time ~ 10 nS, confirms the stability of the structure, and indicates the time required to reach steady state. (b) Simulated output RF port signal in green shows an amplification occurs to the input RF signal of 10 dBm input and 15 GHz in red, (c) frequency spectrum of the output RF signal shows that the output signal has the same frequency of the signal used to excite the input RF port but with a higher amplitude indicates the amplification operation.	119
6.5	(a) A snapshot in time at $t = 10$ nS of the transient $\mathbf{E}_z$ -field just outside the helix along the SWS after reaching steady state time. (b, c) Phasor $\mathbf{E}_z$ -field in terms of amplitude and phase along the SWS.	121
6.6	Spatial frequency components of longitudinal electric field associated to the hot structure obtained by performing FFT of phasor $\mathbf{E}_z(z)$ shown in Fig. 6.5 in the space window from $z = 0$ to $z = 0.08$ m.	121
6.7	(a) A snapshot in time at $t = 10$ nS of the transient electron beam particles energy along the SWS after reaching steady state time. (b, c) Complex-valued AC part of the phasor electron beam particles energy in terms of amplitude and phase. Red line in (b) represents the exponential curve $\chi e^{\alpha z}$ with $\alpha = 31.6 \text{ m}^{-1}$ and $\chi = 19.7 \text{ eV}$ which fits well to the magnitude growth of the AC part of electron beam particles energy.	123
6.8	Spatial frequency components of electron beam energy associated to the TWT obtained by performing FFT of phasor AC-Energy shown in Fig. 6.7 in the space window from $z = 0$ to $z = 0.08$ m.	124

6.9	Complex-valued wavenumbers of the eigenmodes in the interactive electron beam-EM mode in the helical SWS in Fig. 2 (a). (a) Real part of the recovered dispersion by estimating analytical model parameters using PIC simulations data, a reasonable agreement is observed in the real part. (b) Imaginary part of the dispersion shows a discrepancy between the recovered model and PIC simulations data.	126
6.10	(Blue) Normalized phase velocity and (Red) interaction impedance of the first forward mode of the SWS shown in Fig. 6.2(a) obtained from the cold full-wave simulation by analyzing wave propagation in the absence of the electron beam using the finite element-based eigenmode solver.	127
6.11	Comparison between imaginary parts of the wavenumbers of the hot eigenmodes for the helical SWS shown in Fig. 1(a) for two analytical models: (a) frequency-independent parameters (γ , v_0 , w) analytical model in (6.8), (b) frequency-dependent parameters ($\gamma(\omega)$, v_0 , $w(\omega)$) adjusted analytical model in (6.9).	128

LIST OF TABLES

	Page
5.1 Physical dimensions and parameters of the DBE-MTM SWS	79
5.2 Physical dimensions and parameters of the MTM SWS; the one used for the BWO relatization.	80
5.3 Dimensions and parameters of the double corrugated waveguide with DBE .	98

ACKNOWLEDGMENTS

I would like to express my sincere gratitude to my Ph.D. advisors, Professor Filippo Capolino and Professor Alexander Figotin, who have the qualities of a great mentor and prodigious scientist. With their inspiration and dedication, they have consistently conveyed a rigorous scientific approach which I absorbed over the five years I spent as a Ph.D. student at UC Irvine. Without their guidance and persistent help, this dissertation would not have been possible. In addition, I would like to thank my committee member, Professor Michael Green, who gave profound insights and feedback on RF oscillator concepts. I also thank Professor Ozdal Boyraz, who has provided helpful feedback and insights.

Additionally, I thank Professor Edl Schamiloglu at the University of New Mexico, and Professor Guido Valerio, at Sorbonne University, Ladislau Matekovits, at Politecnico di Torino for fruitful discussions and collaboration.

I thank my colleagues who were involved in this research, Mohamed Othman, Tarek Mealy for numerical analysis of EPD structures, Ahmed Al-Mutawa for experimental verification of DBE microstrip CTLs, Dmitry Oshmarin and Alireza Nikzamir for numerical analysis of oscillators, and also my lab mates Mohamed Nada, Hamidreza Kazemi, Farshad Yazdi, Robert Marosi, Kasra Rouhi, Ehsan Hafezi.

I enjoyed getting together my friends at UCI, Ahmed Khorshid, Mohamed Abdel Latif, Ahmed Ibrahim, Islam Farrag, Ahmed Mamdouh, Marween Zourgi, Mohamed Fouda, Yousef Hassan, Mohanad Odema, and the Egyptian community in Irvine, and around southern California and all of my friends which made my graduate school experience more enjoyable

I dedicate this work to my mom, my dad, my brother Hani, and my sister Manar whose love, motivation, and support have given me the strength to pursue my passions and thrive.

I am also grateful for the funding, the Air Force Office of Scientific Research (AFOSR) and the National Science Foundation.

The research work in this dissertation was mainly supported by the AFOSR FA9550-19-1- 0103, and NSF ECCS-1711975.

VITA

Ahmed Farghaly Abdelrahman Abdelshafy

EDUCATION

Doctor of Philosophy in Electrical and Computer Eng.	2021
University of California, Irvine	<i>Irvine, California, USA</i>
Masters of Science in Electronics and Communications Eng.	2016
Cairo University	<i>Cairo, Egypt</i>
Bachelor of Science in Electronics and Communications Eng.	2012
Cairo University	<i>Cairo, Egypt</i>

RESEARCH EXPERIENCE

Graduate Research Assistant	2017–2021
University of California, Irvine	<i>Irvine, California</i>

TEACHING EXPERIENCE

Teaching Assistant	2012–2016
Cairo University	<i>Cairo, Egypt</i>

JOURNAL PUBLICATIONS

- [J1] **A. F. Abdelshafy**, M. Othman, F. Yazdi, M. Veysi, A. Figotin and F. Capolino, “Electron-Beam-Driven Devices With Synchronous Multiple Degenerate Eigenmodes,” in *IEEE Transactions on Plasma Science*, vol. 64, no. 8, pp. 3126-3138, 2018.
- [J2] **A. F. Abdelshafy**, M. Othman, D. Oshmarin, A. Al-Mutawa and F. Capolino, “Exceptional Points of Degeneracy in Periodically-Coupled Waveguides and the Interplay of Gain and Radiation Loss: Theoretical and Experimental Demonstration,” in *IEEE Transactions on Antennas and Propagation*, vol. 67, no. 9, pp. 6909-6923, 2019.
- [J3] H. Kazemi, M. Y. Nada, T. Mealy, **A. F. Abdelshafy** and F. Capolino, “Exceptional points of degeneracy induced by linear time-periodic variation,” in *Physical Review Applied*, vol. 11, no. 1, p. 014007, 2019.
- [J4] T. Mealy, **A. F. Abdelshafy** and F. Capolino, “Exceptional Point of Degeneracy in Backward-Wave Oscillator with Distributed Power Extraction,” in *Physical Review Applied*, vol. 14, no. 1, p. 014078, 2020.
- [J5] **A. F. Abdelshafy**, D. Oshmarin, M. Othman, M. Green, and F. Capolino, “Distributed Degenerate Band Edge Oscillator,” in *IEEE Transactions on Antennas and Propagation*, vol. 69, no. 3, pp. 1821-1824, 2020.
- [J6] A. Alleluia, **A. F. Abdelshafy**, P. Ragulis, A. Kuskov, D. Andreev, M. Othman, B. Martinez-Hernandez, E. Schamiloglu, A. Figotin, and F. Capolino, “Experimental Testing of a 3D-Printed Metamaterial Slow Wave Structure for High Power Microwave Generation,” in *IEEE Transactions on Plasma Science*, vol. 48, no. 12, pp. 4356-4364, 2020.
- [J7] **A. F. Abdelshafy**, T. Mealy, E. Hafezi, A. Nikzamir, and F. Capolino, “Exceptional Degeneracy in a Waveguide Periodically Loaded with Discrete Gain and Radiation Loss Elements,” in *Applied Physics Letters*, vol. 118, no. 22, p. 224102, 2021.
- [J8] T. Mealy, **A. F. Abdelshafy**, and F. Capolino, “High Power Backward Wave Oscillator using Folded Waveguide with Distributed Power Extraction Operating at an Exceptional Point,” in *IEEE Transactions on Electron Devices*, vol. 68, no. 7, pp. 3588-3595, 2021.
- [J9] K Rouhi, R Marosi, T Mealy, **A. F. Abdelshafy**, A Figotin, and F. Capolino, “Exceptional Degeneracies in Traveling Wave Tubes with Dispersive Slow-Wave Structure Including Space-Charge Effect,” in *Applied Physics Letters*, vol. 118, no. 26, p. 263506, 2021.
- [J10] T. Mealy, **A. F. Abdelshafy**, and F. Capolino, “Exceptional Point of Degeneracy in Linear-Beam Tubes for High Power Backward-Wave Oscillators,” in *arXiv preprint arXiv:2005.08912*.
- [J11] **A. F. Abdelshafy**, F. Capolino, and A. Figotin, “An Accurate Traveling Wave Tube Analytic Model,” in *arXiv*, December, 2021.

[J12] D. Oshmarin, **A. F. Abdelshafy**, A. Nikzamir, M. Green, and F. Capolino, “Experimental Demonstration of a New Oscillator Concept Based on Degenerate Band Edge in Microstrip Circuit,” in *arXiv preprint arXiv:2109.07002*, 2021.

CONFERENCE PUBLICATIONS

[C1] **A. F. Abdel-Rahman**, T. M. Abufadl, “Particle Wave Interaction Using Cavity Modal Expansion With an Application on Simulation of Virtual Cathode Oscillator,” *2014 IEEE International Conference on Plasma Sciences (ICOPS)*, Washington DC, May 24-29, 2014.

[C2] **A. F. Abdel-Rahman**, T. M. Abufadl, “Cherenkov Radiation in Dielectric-Loaded Waveguides and Cavities,” *2015 IEEE International Conference on Plasma Sciences (ICOPS)*, Antalya, Turkey, May 24-28, 2015.

[C3] **A. F. Abdel-Rahman**, T. M. Abufadl, “Transient Analysis of Circular Waveguide Probe Excitation Using Cavity Modal Expansion,” *2015 IEEE Internat. Symp. on Antennas and Propagat. - USNC/URSI National Radio Science Meeting*, Vancouver, Canada, 19-24 July, 2015.

[C4] M. Othman, M. Veysi, F. Yazdi, M. Nada, **A. F. Abdelshafy**, A. Figotin and F. Capolino, “Exceptional Points of Degeneracy in Coupled Modes: Theory and Applications,” *Metamaterials 2017, The 11th International Congress on Artificial Materials for Novel Wave Phenomena*, Marseille, France, Aug. 28-31, 2017.

[C5] M. Othman, **A. F. Abdelshafy**, A. Figotin, and F. Capolino, “Exceptional points of degeneracy for enhanced interaction in multimode electron beam devices,” *IEEE International Vacuum Electronics Conference*, Monterey, California, April 24-26, 2018.

[C6] **A. F. Abdelshafy**, M. Othman, D. Oshmarin, M. Nada and F. Capolino, “Exceptional Points of Degeneracy in a Linear Array Oscillator with Gain and Loss Balance,” *2018 IEEE Internat. Symp. on Antennas and Propagat. - USNC/URSI National Radio Science Meeting*, Boston, MA, 8-13 July, 2018.

[C7] H. Mohamad, T. Zheng, M. Casaletti, Z. Ren, **A. F. Abdelshafy**, F. Capolino, Guido Valerio, “Degenerate Band Edge Condition in Substrate-Integrated Waveguides,” *2018 IEEE Internat. Symp. on Antennas and Propagat. - USNC/URSI National Radio Science Meeting*, Boston, MA, 8-13 July, 2018.

[C8] M. Y. Nada, M. Othman, F. Yazdi, D. Oshmarin, **A. F. Abdelshafy**, F. Capolino, “Unique Charactersitics and Applications of Systems With Exceptional Points of Degeneracy,” *2018 IEEE Internat. Symp. on Antennas and Propagat. - USNC/URSI National Radio Science Meeting*, Boston, MA, 8-13 July, 2018. [Invited Talk to a special session, Functional Material Platforms Enabling Exotic Scattering Phenomena]

[C9] M. Y. Nada, **A. F. Abdelshafy**, T. Mealy, F. Capolino, “Exceptional points of de-

generacy in coupled mode structures: theory and applications,” *SPIE Optics + Photonics (International Society for Optics and Photonics)*, San Diego, CA, Aug. 19-23, 2018. [Invited talk]

[C10] M. Y. Nada, A. F. Abdelshafy, T. Mealy, F. Capolino, “General conditions to realize exceptional points of degeneracy and applications,” *International Congress on Advanced Electromagnetic Materials in Microwaves and Optics (Metamaterials’2018)*, Espoo, Finland, August 27–September 1, 2018.[Invited talk]

[C11] M. Y. Nada, **A. F. Abdelshafy**, T. Mealy, F. Yazdi, H. Kazemi, A. Figotin, F. Capolino, “Various Topologies of Coupledmode Structures Exhibiting Exceptional Points of Degeneracy,” *IEEE International Conference on Electromagnetics in Advanced Applications (ICEAA)*, Cartagena de Indias, Colombia, Sept. 10–14, 2018.

[C12] M. Y. Nada, H. Kazemi, **A. F. Abdelshafy**, F. Yazdi, D. Oshmarin, T. Mealy, A. Figotin, and F. Capolino, “Microwave circuits with exceptional points and applications in oscillators and sensors,” *In 2018 18th Mediterranean Microwave Symposium (MMS)*, Istanbul, Turkey, October 31 - November 2, 2018.

[C13] T. Zheng, M. Casaletti, **A. F. Abdelshafy**, F. Capolino, Z. Ren and Guido Valerio, “Design of Substrate Integrated Waveguides Supporting Degenerate Band-Edge Resonances,” *IEEE European Conference on Antennas and Propagation*, Krakow, Poland, March 31-April 5, 2019.

[C14] **A. F. Abdelshafy**, T. Mealy, H. Kazemi, M. Othman and Capolino, “Novel Regime for Antenna Array Oscillators Based on Exceptional Point of Degeneracy,” *IEEE European Conference on Antennas and Propagation*, Krakow, Poland, March 31- April 5, 2019.

[C15] **A. F. Abdelshafy**, T. Mealy, A. Figotin and F. Capolino, “Low Starting Current Oscillator Based on the Degenerate Band Edge in a Double Helix Slow Wave Structure,” *IEEE International Vacuum Electronics Conference*, Busan, South Korea, April 28- May 1, 2019.

[C16] T. Mealy, **A. F. Abdelshafy** and F. Capolino, “Backward-Wave Oscillator with Distributed Power Extraction Based on Exceptional Point of Degeneracy and Gain and Radiation-Loss Balance,” *IEEE International Vacuum Electronics Conference*, Busan, South Korea, April 28- May 1, 2019.

[C17] D. Oshmarin, H. Kazemi, **A. F. Abdelshafy**, F. Capolino, “Pulse Generation using a Degenerate Band Edge Structure,” *2019 IEEE Internat. Symp. on Antennas and Propagat. - USNC/URSI National Radio Science Meeting*, Atlanta, GA, Jul. 7-12, 2019.

[C18] **A. F. Abdelshafy**, T. Mealy, D. Oshmarin, M. Y. Nada, H. Kazemi and F. Capolino, “Exceptional Points of Degeneracy induced in Uniform and Periodic Coupled Systems,” *2019 International Conference on Electromagnetics in Advanced Applications*, Granada, Spain, Sept. 9-13, 2019.

- [C19] H. Kazemi, M. Y. Nada, T. Mealy, **A. F. Abdelshafy**, F. Capolino, “Linear Time-Periodic Systems with Exceptional Points of Degeneracy,” *2019 International Conference on Electromagnetics in Advanced Applications*, Granada, Spain, Sept. 9-13, 2019.
- [C20] H. Kazemi, M. Y. Nada, R. Marosi, T. Mealy, **A. F. Abdelshafy**, and F. Capolino, “Theory of Periodically Time-Varying Induced Exceptional Points of Degeneracy,” *In 2019 Thirteenth International Congress on Artificial Materials for Novel Wave Phenomena (Metamaterials)*, Rome, Italy, Sept. 16-21, 2019.
- [C21] T. Zheng, M. Casaletti, **A. F. Abdelshafy**, F. Capolino, Z. Ren and G. Valerio, “Degenerate Band Edge Resonances in Air-filled Substrate Integrated Waveguide,” *2020 14th European Conference on Antennas and Propagation (EuCAP)*, Copenhagen, Denmark, March 15-20, 2020
- [C22] A. Alleluia, A. Kuskov, D. Andreev, E. Schamiloglu, **A. F. Abdelshafy**, M. Othman, A. Figotin, and F. Capolino, “A 3D-Printed Metamaterial Slow Wave Structure for High-Power Microwave Generation,” *IEEE International Vacuum Electronics Conference*, Monterey, California, April 20-23, 2020.
- [C23] T. Mealy, **A. F. Abdelshafy** and F. Capolino, “Backward-Wave Oscillator with Distributed Power Extraction Operating at an Exceptional Point of Degeneracy,” *IEEE International Vacuum Electronics Conference*, Monterey, California, April 20-23, 2020.
- [C24] E. Renzi, **A. F. Abdelshafy**, T. Mealy, L. Matekovits and F. Capolino, “A Concept for a Leaky Wave Antenna Oscillator With Second Order Degeneracy,” *2020 Internat. Symp. on Antennas and Propagat. - USNC/URSI National Radio Science Meeting*, Montreal, QC, Canada, July 5-10, 2020.
- [C25] E. Hafezi, **A. F. Abdelshafy**, T. Mealy, A. Nikzamir and F. Capolino, “Exceptional Points of Degeneracy in a Transmission Line Periodically Loaded with Gain and Radiation Loss,” *2021 United States National Committee of URSI National Radio Science Meeting (USNC-URSI NRSM)*, Boulder, CO, USA, Jan. 4-9 ,2021.

BOOK PUBLICATIONS

- [B1] **A. F. Abdelshafy**, M. Othman, A. Figotin and F. Capolino, “High Power Microwave Sources And Technologies Using Metamaterials,” Chapter[2]: Multitransmission Line Model for Slow-Wave Structure Interacting with Electron Beams and Multiple Synchronization, in *IEEE Press/Wiley MURI Book*.

ABSTRACT OF THE DISSERTATION

Dispersion Engineering of Microwave and High-power Devices with Exceptional Points of Degeneracy

By

Ahmed Farghaly Abdelrahman Abdelshafy

Doctor of Philosophy in Electrical and Computer Engineering

University of California, Irvine, 2021

Professor Filippo Capolino , Chair

In a world where high-speed communications and energy-aware electronic devices are becoming ubiquitous, efficient electromagnetic (EM) radiation sources remain vital. A precision oscillation frequency, high output power, and high efficiency are all coveted properties in EM oscillators, as well as low phase noise and a small form factor to fit into ultra-small but densely packed devices. This dissertation's main focus is on a new class of electromagnetic/RF devices, primarily oscillators, which rely on dispersion engineering utilizing the slow-light phenomenon associated with exceptional points of degeneracy (EPD).

An EPD is formed in the parameter space of the dynamical system when two, or more, physical eigenstates coalesce into a degenerate eigenstate and bifurcate when a parameter is varied. A coupled transmission line (CTL) theory is established which provides a paradigm for dispersion engineering concepts at radio frequencies based on EPDs. A special fourth-order EPD, so-called degenerate band edge (DBE) occurs in an infinitely long and lossless periodic system that, given proper operational condition, four of its eigenstates coalesce. In practice, since the DBE ideal condition of having an infinitely long and lossless periodic system is impossible, the operation of a finite-length periodic device near the DBE ideal condition can lead to unique and beneficial properties such as enhanced quality factor and

giant gain enchantment. A figure of merit is developed to assess the practical occurrence of fourth-order EPDs in CTLs with tolerances and losses.

First, experimental observation of the DBE is demonstrated in microstrip CTLs at microwaves even in the presence of fabrication tolerances and dissipative losses. We further introduce the “gain and loss balance” regime in CTLs as a mean of recovering an EPD in the presence of radiation and/or dissipative losses, without necessarily resorting to Parity-Time (PT)-symmetry regimes. Second, a new class of distributed oscillators is introduced by engineering the dispersion of two-coupled periodic waveguides to exhibit a DBE. The distributed DBE oscillator is realized in periodic CTLs with a unique mode selection scheme that leads to a stable single-frequency oscillation, even in the presence of load variation. We demonstrate its performance through full-wave transient simulations. Moreover, we introduce a general theoretical framework to exhibit EPDs for applications that incorporate discrete distributed coherent sources and radiation loss elements, such as a waveguide comprising of uniform lossless segments together with discrete gain and radiating elements.

Finally, we harness the dispersion engineering concept to conceive different DBE slow-wave structures (SWSs) for highly efficient electron-beam-driven oscillators. A DBE oscillator (DBEO) is demonstrated for sub-terahertz frequency where the DBE operation synchronization regime was manifested through particle in cell (PIC) simulations. Moreover, an accurate traveling wave tube analytic (TWT) model is constructed based on the Lagrangian field theory as a mean to recover the complex-valued dispersion diagram and the information of hot eigenmodes in a full-interactive TWT. A helical-based TWT analytical model was reconstructed and the agreement between the analytical model and CST-PIC simulations results was remarkable.

Chapter 1

Introduction

1.1 Motivation

Developing new principles for EM generation is becoming increasingly popular due to the escalating interest in improving electromagnetic radiation sources concerning efficiency, increasing output power, and reducing phase noise. In addition to frequency range, sources and oscillators must provide low power consumption, compact form factors, and low jitter/phase noise to ensure a reliable oscillation frequency and deter interference with other frequencies. Our dissertation examines several new aspects of highly efficient oscillators using novel dispersion engineering principles. We also propose an approach to estimating dispersion information in a traveling wave tube.

1.2 Exceptional Points of degeneracy (EPD)

ELECTROMAGNETIC guiding structures or resonators are characterized by their evolution equations in terms of the eigenmodes (eigenvalues and eigenvectors). Among many features

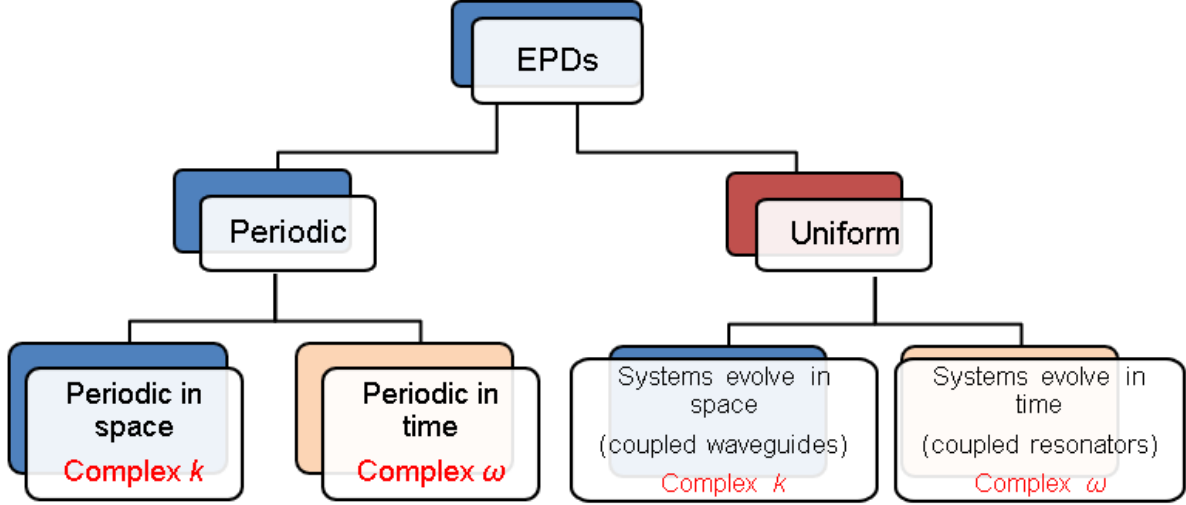


Figure 1.1: Classification of the different structure-topologies that can exhibit exceptional points of degeneracy (EPDs).

of the evolution of these eigenmodes, we explore the points in the parameter space of such system at which two or more eigenmodes coalesce into a single degenerate eigenmode [1, 6, 7, 8]. We denote these points as exceptional points of degeneracy (EPD), and the degeneracy order represents the number of coalescing eigenmodes. Such condition is simply referred to as “EP” in various works [9, 10, 11]; here, the “D” is used to stress the importance of the degeneracy. A general picture of possible EM structures that can exhibit EPDs and they have classified such structures into mainly four categories as shown in 1.1.

In this dissertation, we investigate EPD conditions of wave propagating eigenvectors in coupled-mode systems, i.e., guided wave structure supporting multiple eigenwaves. To avoid ambiguity; the EPD is a stronger degeneracy condition and its different condition than having simple degenerate modes in waveguides, in which two modes may have the same eigenvalue (wavenumber) but different field distributions (e.g. two TE₁₁ modes in a circular waveguide with orthogonal transverse field polarizations). In the simple degenerate case, i.e. TE₁₁ degenerate modes in a circular waveguide, the system matrix is diagonalizable which in contrast to the EPD condition where the system matrix is defective; we will quantify in chapter 2.

In Periodic guiding structures certain class of EPDs inherently exists so-called “regular” band edge (RBE) at which standing waves with zero group velocity are manifested at the band edge (i.e., the separation between pass and stop bands). RBE is a second-order EPD exists in periodic lossless/gainless structures which is manifested at the edge of the Brillouin zone. The RBE represents a point at which the two Floquet-Bloch eigenwaves with wavenumbers k and $-k + 2\pi/d$ coalesce (in their wavenumbers and eigenvectors) at a single frequency, where d is the period of the periodic structure [12, 1, 4]. Trivial degeneracies also occur at the cutoff frequency of waveguides or at zero frequency. Periodic structures in particular offer interesting degeneracies related to the electromagnetic band gap which would not be attainable in uniform waveguides. Higher order degeneracies can be found or engineered in periodic structures by the proper coupling between the supported modes; as will quantify in chapter 2.

EPDs also occur in Parity-Time (PT)-symmetric structures such as coupled waveguides and resonators when the system’s refractive index obeys $n(x) = n^*(-x)$ where x is a coordinate in the system [7, 13, 14, 15] and $*$ denotes complex conjugation. We point out that EPDs cannot occur in systems whose evolution is described by a Hermitian matrix. By tuning one of the system parameters e.g., frequency, gain and loss parameters, coupling, etc., the eigenvalues of a PT-symmetric system transitions from being entirely real valued to be complex, and the transition point, also called bifurcation point, is in fact an EPD [13, 16, 17]. In the radio frequency region, there has been some studies on EPDs and PT-symmetry in lumped circuits [18] and in a microwave cavity [19].

An EPD occurs in systems where the evolution of the system vector, in space or time, is described by a non-Hermitian matrix which can be imposed by periodicity [1, 20, 2, 21] or by having losses and gain in the system [6, 7, 8, 22, 23, 24, 25]. Moreover, in [26] EPDs have been found in nonreciprocal waveguides, between coupled topological modes, with a balanced distribution of loss and gain. EPDs of second order may occur with asymmetric

distributions of gain and loss in *uniform* CTLs [6]. Furthermore, in [27] second order EPDs in uniform CTLs with loss and gain have been investigated from the bifurcation theory point of view.

Despite the wide spectrum of systems that exhibit such strong degeneracy conditions, i.e., EPD, our focus is on the periodic structures for their compatibility with microwave, RF, and electron-beam devices; as will be quantified in Chapters 2, 3, and 5.

1.3 Degenerate band edge (DBE)

Degenerate band edge is a fourth-order EPD occurs in periodic-lossless-gainless structures when four eigenmodes of the supporting eigenwaves coalesces into one degenerate mode. Prof. Figotin and Dr. Vitebskiy have proposed interesting properties of photonic crystals made of multilayer dielectric structures [12, 1, 4, 2]. By cascading anisotropic dielectric layers and engineering the misalignment angle between the dielectric layers (A1 and A2 in 1.2) to form a one-dimensional periodic structure, higher order EPDs are achievable. Such structures support two modes simultaneously (two in each direction, for a total of four).

The EPDs conditions are manifested in the dispersion diagram, that relates frequency to the Floquet-Bloch wavenumber of eigenwaves supported by the periodic structure, as singular points called the regular band edge (RBE, second-order EPD), in 1.2(a) the degenerate band edge (DBE, fourth-order EPD), in 1.2(b), or the split band edge (SBE, double second-order EPD), in 1.2(c). The waves associated to those special singular points, in their vicinity, exhibit almost vanishing group velocity so-called slow wave phenomena [28]. The vanishing group velocity is promising for gigantic enhancement in the gain and quality factor of Fabry-Pérot cavities in microwaves and optical devices.

In mathematical terms, propagating and evanescent fields are described by a four dimensional

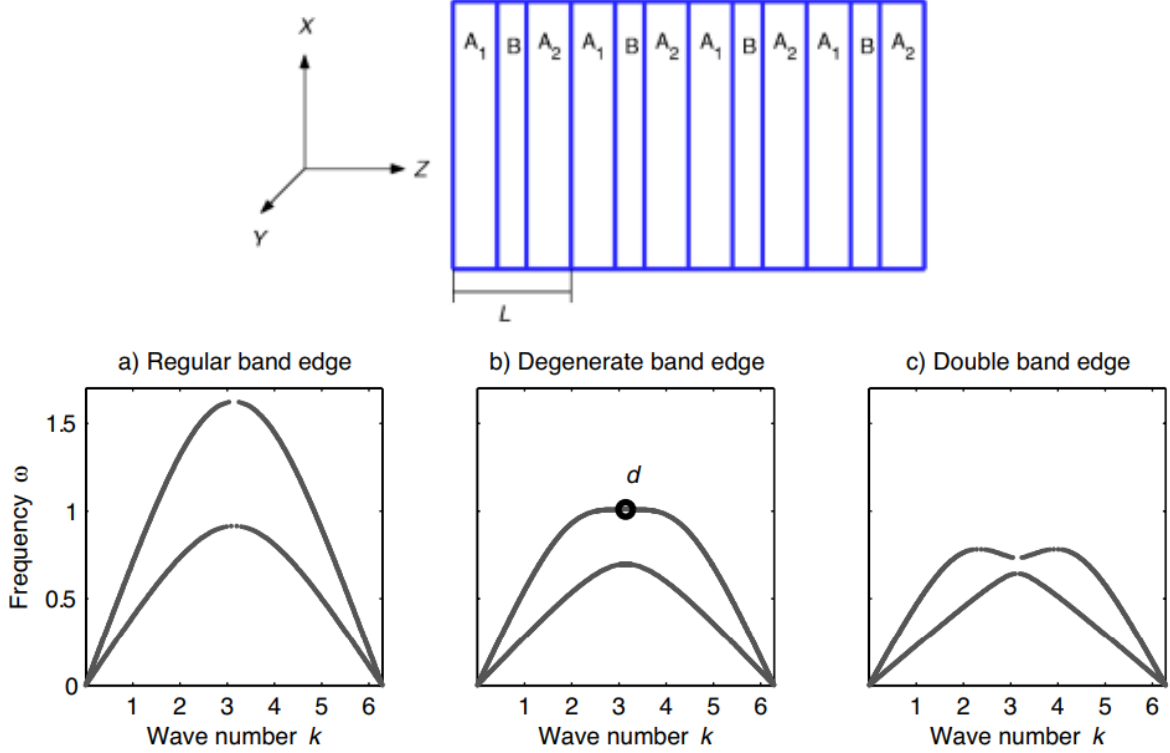


Figure 1.2: (top) Periodic stack capable of supporting $k - \omega$ diagram with degenerate band edge. A unit cell L includes three layers: two birefringent layers A_1 and A_2 with different orientations ϕ_1 and ϕ_2 of the respective anisotropy axes in the X - Y plane, and one isotropic B layer. (below) $\omega - k$ dispersion diagrams in periodic structures showing the following features: (a) Standard regular band edge (RBE), between the pass and stop band of a single mode in a photonic crystal. (b) Degenerate band edge (DBE) and (c) split band edge (SBE) of a complex periodic structure. Pictures adopted from [1, 2]

state vector Ψ containing transverse electric and magnetic fields. The EM system is described as the evolution of its multidimensional state vector as $\frac{\partial}{\partial z} \Psi(z) = -j \underline{\mathbf{M}}(z) \Psi(z)$. In periodic structures, the transformation across the unit cell is considered using the transfer matrix $\underline{\mathbf{T}}(z + d, z)$ such as $\Psi(z) = \underline{\mathbf{T}} \Psi(z + d)$. The corresponding eigenvalue problem for modal propagation is $\underline{\mathbf{T}} \Psi = \lambda \Psi$; more details are provided in chapter 2. The DBE condition occurs the transfer matrix is similar to a Jordan matrix, and that the four eigenvalues and eigenvectors are all coalesce at the DBE. This is corresponding to having only one eigenvector, with multiplicity equal to four in the case of DBE, hence a generalized eigenvalue (i.e., generalized eigenvectors) problem should be considered in this case; this will be elaborated in details in chapter 2.

Throughout this dissertation, we will advance the theory of EPDs with fourth-order and second-order, demonstrate various structures supporting the different orders of EPDs also using the gain and loss balance concept, as well as in lossless-gainless periodic structures. We will also devote a significant part of this dissertation on the applications of EPDs to develop a very stable oscillators in different technologies and for different frequency range.

1.4 Organization of the dissertation and contents

The dissertation is organized into Chapters that involve the theory and applications of dispersion engineering in the microwave, RF and electron beam devices.

Chapter 2. We present in this chapter a transmission line theory of exceptional points of degeneracy (EPD) in coupled-mode periodic guiding structures. We have also experimentally demonstrated for the occurrence of a fourth order EPD (the DBE) in microstrip CTLs at microwave frequencies, first through four-port measurements of a unit cell leading to the DBE dispersion relation and then through the transmission characteristics of a finite-length CTL. A figure of merit is also introduced, we call it hyperdistance, to estimate the effect of perturbations like imperfect coupling, presence of losses and any other perturbation that may arise from fabrications or numerical simulation on the EPD. Furthermore, in CTLs that experience significant radiation and dissipative losses, we show that the so-called “gain and loss balance” condition leads to recovering an EPD even in schemes that do not necessarily imply PT-symmetry.

Chapter 3. We present an application to the degenerate band edge DBE-based structures. The DBE structure in the cavity made of a periodic waveguide strongly synchronizes a discrete set of oscillators resulting in an overall single mode of oscillation. The single oscillation frequency, in close proximity to the DBE frequency, has been theoretically demonstrated

through full-wave transient simulations. Results demonstrate the stability of the oscillation frequency over a very wide range of load or gain variation. This scheme based on the DBE concept would be valuable in devices that require several coherent sources, in power combining, with a possible extension to distributed radiating active antenna arrays at microwaves and millimeter waves.

Chapter 4. We explore periodic structures with simultaneous existence of gain and loss. We demonstrate that a periodic waveguide loaded with gain and radiating elements exhibits EPDs. We summarize the different conditions for having EPDs, i.e. symmetric and asymmetric cases. The discrete radiation admittances considered in this chapter represent the input admittances of a periodic array of antennas. Such a phenomenon may pave the way to a new class of active traveling-wave antennas and also in array antennas with all elements oscillating and synchronized.

Chapter 5. We explore metallic waveguide structures and show various designs of the fourth order EPD at microwave and terahertz frequencies. We demonstrate the degenerate band edge (DBE) condition via full-wave simulations, namely the degeneracy of four Bloch modes, in three different realization i) double helix slow wave structure, ii) MTM coupled cavity metamaterial, iii) double corrugated rectangular waveguide. We show a very flat wavenumber-frequency dispersion diagram. We correlate our finding to theoretical and simulation results utilizing various techniques including dispersion synthesis, and oscillation frequency in degenerate band edge oscillators (DBEO) via particle in cell (PIC) simulations.

Chapter 6. In this chapter, we present a mean to recover the dispersion information in traveling wave tubes (TWTs). The approach utilizes an analytical model for TWTs based on the Lagrangian field theory upgrading its constants to be frequency-dependent. Such model gives accurate predictions of the dispersion relation the frequency dependent gain/amplification. The approach utilizes primary parameters of the SWS recovered by using the standard procedures without any “curve fitting”. We implement the construction of the model for a

helical-based TWT and found an excellent agreement between with the particle in cell (PIC) simulations.

Chapter 2

Exceptional Points of Degeneracy in Periodic Coupled Waveguides and the Interplay of Gain and Radiation Loss: Theoretical and Experimental Demonstration

2.1 Motivation

The focus of this chapter is on constructing a paradigm for dispersion engineering in coupled transmission lines (CTLs) based on exceptional points of degeneracy (EPDs). We develop a theory for fourth-order EPDs consisting of four Floquet-Bloch eigenmodes coalescing into one degenerate eigenmode. We present unique wave propagation properties associated to the EPD and develop a figure of merit to assess the practical occurrence of fourth-order

EPDs in CTLs with tolerances and losses. We experimentally verify for the first time the existence of a fourth EPD (the degenerate band edge), through dispersion and transmission measurements in microstrip-based CTLs at microwave frequencies.

In addition, we report that based on experimental observation and the developed figure of merit, the EPD features are still observable in structures that radiate (leak energy away), even in the presence of fabrication tolerances and dissipative losses. We investigate the “gain and loss balance” regime in CTLs as a mean of recovering an EPD in the presence of radiation and/or dissipative losses, without necessarily resorting to Parity-Time (PT)-symmetry regimes. The versatile EPD concept is promising in applications such as high intensity and power-efficiency oscillators, spatial power combiners, or low-threshold oscillators and opens new frontiers for boosting the performance of large coherent sources.

ELECTROMAGNETIC guiding structures or resonators are characterized by their evolution equations in terms of the eigenmodes (eigenvalues and eigenvectors). Among many features of the evolution of these eigenmodes, we explore the points in the parameter space of such system at which two or more eigenmodes coalesce into a single degenerate eigenmode [1, 6, 7, 8]. We denote these points as exceptional points of degeneracy (EPD), and the degeneracy order represents the number of coalescing eigenmodes. Periodic guiding structures enable the occurrence of a fundamental class of EPDs at the so-called “regular” band edge (RBE) at which standing waves with zero group velocity are manifested at the band edge (i.e., the separation between pass and stop bands). EPDs also occur in Parity-Time (PT)-symmetric structures such as coupled waveguides and resonators when the system’s refractive index obeys $n(x) = n^*(-x)$ where x is a coordinate in the system [7, 13, 14, 15] and $*$ denotes complex conjugation. We point out that EPDs cannot occur in systems whose evolution is described by a Hermitian matrix (see Section 2.2). Thus, an EPD occurs in systems where the evolution of the system vector, in space or time, is described by a non-Hermitian matrix which can be imposed by periodicity [1, 2, 21] or by having losses and gain in the system [6, 7, 8]. Moreover, in [26] EPDs have been found in nonreciprocal waveguides, between

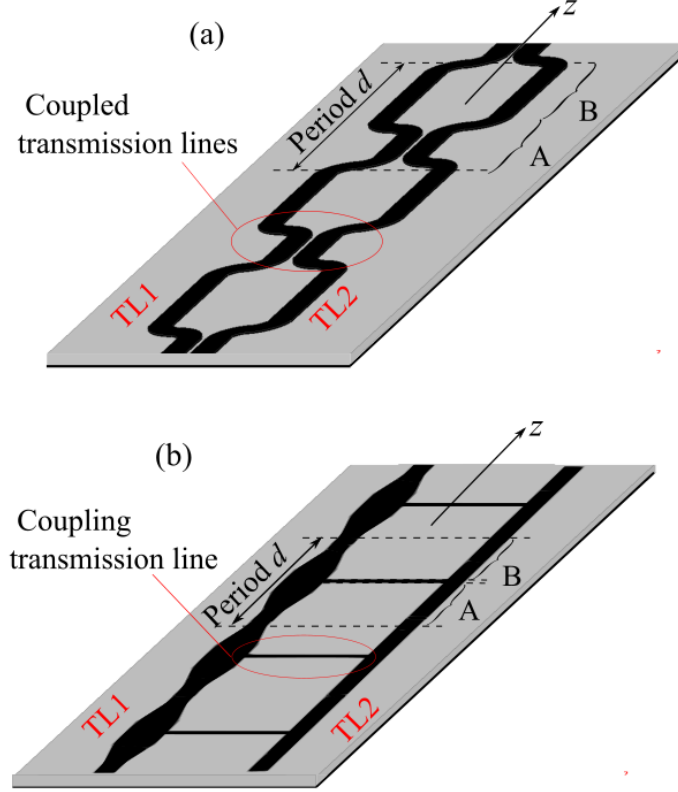


Figure 2.1: Example geometries of waveguide microstrip lines on a grounded dielectric slab that support the fourth-order EPD, visible in the $(k - \omega)$ dispersion diagram at microwave frequencies. (a) and (b) Examples represent two cases of coupling, with proximity fields and with a physical connection, respectively. Results in Sections 2.3 and 2.6 are based on a CTL that models the geometry in (a), whereas the results in Sections 2.4 and 2.5 are based on the microstrip CTL geometry in (b). In the lossless case, the fourth-order EPD is called DBE; however, such structures exhibit both second- and fourth-order EPDs in the case of gain and loss balance.

coupled topological modes, with a balanced distribution of loss and gain. EPDs of second order can also occur with asymmetric distributions of gain and loss in *uniform* CTLs [6]. Furthermore, in [27] second order EPDs in uniform CTLs with loss and gain have been investigated from the bifurcation theory point of view. It is worth noting that an EPD affects the flatness of a $\omega - k$ dispersion diagram and in general the higher the EPD order the flatter the dispersion. This concept shall not be confused with the flattening of the group velocity versus frequency dispersion diagram, for instance discussed in [29] and references therein.

In this chapter, we investigate second and fourth order EPDs occurring in *periodic* guiding structures whose wave dynamics are represented by two periodic coupled TLs. In particular we are interested in the *fourth order EPDs* that occur when all four independent Floquet-Bloch eigenmodes coalesce in their eigenvalues and eigenvectors and form one single degenerate eigenvector [1, 12, 4] at the band edge as it will be clear later on in the rest of this chapter, in *absence of losses and gain*.

A *fourth order EPD* occurring at the band edge of a lossless structure is called degenerate band edge (DBE). This DBE condition is the basis for a possible enhancement of gain in active devices comprising DBE structures [30, 3]. Although the DBE concept was first shown in periodic layers of an isotropic materials where two independent polarizations are coupled throughout the structure [1, 4] other investigations were also carried out using microstrip lines for filtering and antenna applications [31, 32, 33]. Moreover, coupled silicon waveguides were also designed [34, 35], with potential application in low threshold and high efficiency lasers [30, 36]. However, in connection with experimental studies, the existence of EPD features was shown experimentally in periodic an isotropic layered media consisting of dielectric layers fabricated of low-loss microwave ceramic disks [37]; such geometry exhibits the split band edge [1, 2]. Also, in optical waveguides the giant resonance and giant Q factor scaling associated with DBE were observed [35, 38]. Moreover, in printed CTLs frozen guided modes associated to stationary inflection point were observed [39]. More recently, the authors proposed that the DBE can also be manifested in all-metallic periodically-loaded waveguides [40] and experimentally detected even in the presence of losses and tolerances [41]. The application of such waveguide in high power microwave oscillators based on electron beams demonstrates low starting (threshold) current and better threshold scaling with length compared to conventional backward wave oscillators [3, 42]. The same concept has been adopted for investigating ladder circuit oscillators [43, 44] with low threshold and potential power efficiency. Note that the DBE (i.e. fourth order EPD) occurs rigorously only in lossless structures as shown in [1, 30, 31, 41], but fourth order EPDs can also be achieved

in gain and loss balanced CTLs as will be shown in this chapter. This is different from the CTL regime discussed in [45] since there second order EPDs have been considered in uniform CTLs, while here we deal with *periodic* CTLs and the investigated EPD regimes are of the fourth order. It is also worth noting that the gain compensation condition, the so-called “gain and loss balanced condition” in CTLs investigated in this chapter to obtain fourth order EPDs does not necessarily mean PT-symmetric systems. Moreover, in the realization of EPDs with the gain and loss balanced condition, it is important to observe that losses can actually represent distributed radiation in a periodically opened waveguide structure. This concept paves the way for a new class of radiating array oscillators based on EPDs.

Besides the presented general formulation that is applicable to any CTL, we discuss in detail the two microstrip coupled transmission lines shown in Fig. 2.1. However, the conclusions drawn can be extended to many other geometries or structures since our formalism is general; operating from RF to optical frequencies. The rest of the chapter is organized as follows. In Section 2.2 we develop the theory of periodic CTLs and discuss the characteristics of EPDs in periodic coupled waveguides. In Section 2.3 we introduce the concept of hyperdistance and investigate the effect of losses and coupling on the dispersion diagram as well as the effect of perturbations. In Section 2.4 and 2.5 we show the experimental demonstration of EPDs in periodic structures with both infinite- and finite-length structures at microwaves based on coupled microstrip lines. In Section 2.6 we describe EPDs in CTL, based on gain and loss balance.

2.2 System Description of CTLs

We consider a pair of CTLs such that two independent modal fields are able to propagate along each z -direction (so a total of four independent modes are present when we consider propagation in both positive and negative z directions). At an EPD some of these modes

(two or four, depending on the case, in a system satisfying reciprocity) will no longer be independent and thus will coalesce. We refer entirely to the formulation in [45] for homogeneous (i.e., uniform) TLs that describe the field evolution using a CTL approach, assuming a time harmonic evolution as $e^{j\omega t}$. In this section we consider the general case of a periodic CTL that may or may not possess dissipative as well as radiation losses and/or gain.

2.2.1 State vector and wave propagation in CTLs

We start by representing the field amplitudes in the two CTLs by equivalent voltage and current vectors $\mathbf{V}(z) = \begin{bmatrix} V_1(z) & V_2(z) \end{bmatrix}^T$ and $\mathbf{I}(z) = \begin{bmatrix} I_1(z) & I_2(z) \end{bmatrix}^T$. It is convenient to define the four-dimensional state vector

$$\mathbf{\Psi}(z) = \begin{pmatrix} \mathbf{V}(z) \\ \mathbf{I}(z) \end{pmatrix} \quad (2.1)$$

that comprises voltages and currents at any coordinate z in the CTL. This technique has been employed in [1, 30, 46] to investigate the modal properties of photonic crystals and periodic waveguides. The system evolution along z is described by the first order differential equations [45]

$$\frac{\partial}{\partial z} \mathbf{\Psi}(z) = -j \underline{\mathbf{M}}(z) \mathbf{\Psi}(z) \quad (2.2)$$

where $\underline{\mathbf{M}}$ is a 4×4 CTL system matrix, where $\underline{\mathbf{M}}$ is given by

$$\underline{\mathbf{M}}(z) = \begin{bmatrix} \underline{\mathbf{0}} & -j \underline{\mathbf{Z}}(z) \\ -j \underline{\mathbf{Y}}(z) & \underline{\mathbf{0}} \end{bmatrix} \quad (2.3)$$

where $\underline{\mathbf{0}}$ is the 2×2 zero matrix, and $\underline{\mathbf{Z}}$ and $\underline{\mathbf{Y}}$ are the series impedance and shunt admittance matrices describing the per unit parameters of the CTL [45, 46, 47]. Note that in [45] we have

investigated the uniform CTL case, i.e., the case when $\underline{\mathbf{M}}$ is invariant along z . The properties described in this chapter are related to a periodic piece-wise variation of the system matrix $\underline{\mathbf{M}}(z)$.

The series impedance and shunt admittance matrices describing the per unit parameters of the CTL are defined as $\underline{\mathbf{Z}} = j\omega\underline{\mathbf{L}} + \underline{\mathbf{R}}$, and $\underline{\mathbf{Y}} = j\omega\underline{\mathbf{C}} + \underline{\mathbf{G}}$. The CTL per unit length inductance $\underline{\mathbf{L}}$ and capacitance $\underline{\mathbf{C}}$ matrices are 2×2 symmetric and positive definite matrices [48, 49], whereas the per unit length series resistance $\underline{\mathbf{R}}$ and shunt conductance $\underline{\mathbf{G}}$ matrices are 2×2 symmetric matrices [48, 49] accounting for losses and for small-signal linear gain introduced by negative resistance or conductance. Note that $\underline{\mathbf{R}}$ and $\underline{\mathbf{G}}$ are positive-definite matrices if and only if they only represent the losses [48, 49]. Moreover, we recall that the capacitance and conductance matrices have negative off-diagonal entries (see Ch. 4 in [49]). Cutoff conditions could be modeled by resonant series and shunt reactive elements as was done in [30, 46]. For instance, cut-off series capacitances (inductances) per unit length can model cutoff conditions for TM (TE) waves in the waveguide and could be included in the impedance (admittance) matrix (see Ch. 8 in [50]). However, for the sake of simplicity here we ignore cutoff conditions since we analyze microstrip lines in terms of their fundamental quasi-TEM modes that ideally do not have a cutoff frequency.

The homogenous solution of (2.2), assuming a certain boundary condition at $z = z_0$, namely $\Psi(z_0) = \Psi_0$ inside a *uniform* (i.e., z -invariant) CTL segment, is found by representing the state vector solution at any arbitrary coordinate z_1 via

$$\Psi(z_1) = \underline{\mathbf{T}}(z_1, z_0)\Psi(z_0) \quad (2.4)$$

where we define $\underline{\mathbf{T}}(z_1, z_0)$ as the *transfer matrix* which translates the state vector $\Psi(z)$ between the two points z_0 and z_1 . Within a uniform segment the transfer matrix is easily

calculated as

$$\underline{\mathbf{T}}(z_1, z_0) = \exp(-j(z_1 - z_0)\underline{\mathbf{M}}) \quad (2.5)$$

and the transfer matrix satisfies the group property $\underline{\mathbf{T}}(z_2, z_0) = \underline{\mathbf{T}}(z_2, z_1)\underline{\mathbf{T}}(z_1, z_0)$ and the symmetry property

$$\underline{\mathbf{T}}(z_1, z_0)\underline{\mathbf{T}}(z_0, z_1) = \underline{\mathbf{1}} \quad (2.6)$$

where $\underline{\mathbf{1}}$ is the 4×4 identity matrix.

The previous discussion identifies the transfer matrix of a uniform segment of a CTL, and it is used to describe periodic waveguides made of a cascade of uniform CTL segments.

2.2.2 Evolution of waves in periodic coupled CTLs

Let us assume a periodic CTL composed of two uniform segments A and B cascaded as shown in Figs. 1(a) and 1(b). The transfer matrix of each individual CTL segment is given by

$$\begin{aligned} \underline{\mathbf{T}}_A &\equiv \underline{\mathbf{T}}(z_0 + d_A, z_0) = e^{-j\underline{\mathbf{M}}_A d_A}, \\ \underline{\mathbf{T}}_B &\equiv \underline{\mathbf{T}}(z_0 + d_B, z_0) = e^{-j\underline{\mathbf{M}}_B d_B} \end{aligned} \quad (2.7)$$

where, $\underline{\mathbf{M}}_A$ and $\underline{\mathbf{M}}_B$ are defined in terms of the per-unit-length impedance and inductances of the segments A and B, respectively using (2.3), while d_A and d_B are the lengths of segments A and B respectively. The transfer matrix $\underline{\mathbf{T}}_U$ of the unit cell of the CTL shown in Fig. 2.1(a), is expressed as the product of the two transfer matrices of the individual segments of the unit cell calculated in (2.7) as $\underline{\mathbf{T}}_U = \underline{\mathbf{T}}_B \underline{\mathbf{T}}_A$. On the other hand, the transfer matrix of a unit cell of the periodic CTL in Fig. 2.1(b) incorporates an additional coupling matrix

due to the coupling microstrip. Particularly in Fig. 2.1(b), segments A and B are uncoupled while the coupling between TL1 and TL2 is introduced using a physical connection which is mediated through another transfer matrix denoted by the coupling transfer matrix $\underline{\mathbf{T}}_C$ (whose expression is omitted here for the sake of brevity). Hence, the unit cell transfer matrix is $\underline{\mathbf{T}}_U = \underline{\mathbf{T}}_B \underline{\mathbf{T}}_C \underline{\mathbf{T}}_A$.

The transfer matrix $\underline{\mathbf{T}}_U$ translates the state vector across a unit cell as

$$\Psi(z + d) = \underline{\mathbf{T}}_U \Psi(z) \quad (2.8)$$

where d is the period in Figs. 1(a) and 1(b). For an infinitely long stack of CTL unit cells, a periodic homogenous solution for the state vector exists in the form $\Psi(z) \propto e^{-jkz}$. This form denotes a Floquet-Bloch type solution where k is the complex Floquet-Bloch wavenumber and e^{-jkz} is referred to as the Floquet-Bloch multiplier. To find such Floquet-Bloch wavenumbers and the eigenvectors, we write the following eigenvalue equation

$$(\underline{\mathbf{T}}_U - \zeta_n \underline{\mathbf{1}}) \Psi_n(z) = 0 \quad (2.9)$$

where Ψ_n are the regular eigenvectors, corresponding to four eigenvalues (or Floquet-Bloch multipliers) given by $\zeta_n = e^{-jk_n d}$, with $n = 1, 2, 3, 4$. The eigenvalues can be readily obtained as solutions of the characteristic equation $\det(\underline{\mathbf{T}}_U - \zeta \underline{\mathbf{1}}) = 0$. We introduce the matrix $\underline{\mathbf{k}}$ as a 2×2 diagonal matrix, whose diagonal elements are the Floquet-Bloch wavenumbers with positive real values, i.e. $\underline{\mathbf{k}} = \text{diag}(k_1, k_2)$. We define the Brillouin zone in our periodic structure from $k = 0$ to $k = 2\pi/d$. Therefore $k_1, k_2, -k_1 + 2\pi/d$, and $-k_2 + 2\pi/d$ are the four modal wavenumbers of the four independent Floquet-Bloch modes in the periodic structure, inside the Brillouin zone. In fact, all the Floquet-harmonics whose wavenumbers are $k_{n,p} = \pm k_n \pm 2p\pi/d$ with $p = 1, 2, 3, \dots$ obey symmetry because we consider reciprocal systems (k and $-k$ are both solutions). We will also define $\underline{\mathbf{A}}$ as a diagonal matrix whose

diagonal elements are the eigenvalues $e^{-jk_n d}$ via

$$\underline{\Lambda} = \begin{pmatrix} e^{-j\underline{k}d} & \underline{\mathbf{0}} \\ \underline{\mathbf{0}} & e^{j\underline{k}d} \end{pmatrix} \quad (2.10)$$

(Note that $e^{-j\underline{k}d}$ is a diagonal matrix with elements $e^{-jk_1 d}$ and $e^{-jk_2 d}$.) Therefore, it follows that the transfer matrix $\underline{\mathbf{T}}_U$ is written as

$$\underline{\mathbf{T}}_U = \underline{\mathbf{U}} \underline{\Lambda} \underline{\mathbf{U}}^{-1} \quad (2.11)$$

where $\underline{\mathbf{U}}$ is 4×4 matrix that serves as a non-singular similarity transformation that diagonalizes $\underline{\mathbf{T}}_U$, and is computed using the four regular, normalized eigenvectors of $\underline{\mathbf{T}}_U$ as $\underline{\mathbf{U}} = [\underline{\Psi}_1 \mid \underline{\Psi}_2 \mid \underline{\Psi}_3 \mid \underline{\Psi}_4]$ meaning that the column vectors of $\underline{\mathbf{U}}$ are the regular eigenvectors of $\underline{\mathbf{T}}_U$ that are linearly independent. If (2.11) is satisfied with a non-singular $\underline{\mathbf{U}}$ we say that $\underline{\mathbf{T}}_U$ is *similar* to a diagonal matrix.

In principle, the matrix $\underline{\mathbf{T}}_U$ is non-Hermitian, but it satisfies some other important properties in the absence of gain and loss, where each constitutive CTL segment has a z -evolution matrix $\underline{\mathbf{M}}$ that is J -Hermitian and a Hermitian characteristic matrix $\underline{\mathbf{ZY}}$, mainly the J -unitarity property,

$$\underline{\mathbf{T}}_U^\dagger(z_1, z_0) = \underline{\mathbf{J}} \underline{\mathbf{T}}_U^{-1}(z_1, z_0) \underline{\mathbf{J}} \quad (2.12)$$

where $\underline{\mathbf{J}}$ is defined in eq. (2.9) in [45], and the dagger symbol $\dagger$ denotes the complex conjugate transpose operation. Importantly, the constitutive matrices $\underline{\mathbf{T}}_A$ and $\underline{\mathbf{T}}_B$ for instance, are diagonalizable in the absence of gain and loss in each segment, and therefore both $\underline{\mathbf{T}}_A$ and $\underline{\mathbf{T}}_B$ have a complete set of eigenvectors. In addition, in the lossless case their eigenvalues lie on the unit circle of the complex eigenvalue plane. The product $\underline{\mathbf{T}}_B \underline{\mathbf{T}}_A$, on the other hand, is not necessarily diagonalizable and indeed EPDs can manifest in the parameter space of

the periodic structure described by $\underline{\mathbf{T}}_U$, and the complex Floquet multiplier, which are the eigenvalues of $\underline{\mathbf{T}}_U$, may not lie on a circle in the complex eigenvalue plane even in lossless structures as we will show in Section 2.3. We recall that $\underline{\mathbf{T}}_U$ is a non-Hermitian matrix which is a necessary condition to realize EPDs, though it is not sufficient by itself since there are non-Hermitian matrices that can be diagonalized. Whereas, the sufficient condition occurs when the matrix $\underline{\mathbf{T}}_U$ becomes defective, i.e., similar to a matrix that contains a non-trivial Jordan block, as it will be shown in the next subsection.

2.2.3 Fourth-order exceptional points of degeneracy

We define the fourth order EPD as a point in the parameter space of the periodic CTL at which the four eigenvectors coalesce into a single degenerate eigenvector. At an *EPD* the matrix $\underline{\mathbf{T}}_U$ becomes defective, i.e., *it cannot be diagonalized*. At a fourth order EPD, the eigenvalue problem (2.9) does not provide a complete basis of independent eigenvectors: they coalesce into a single one, hence $\underline{\mathbf{U}}$ in (2.11) becomes singular. Strictly speaking, it means that it is not possible to find a non-singular similarity transformation as in (2.11) that diagonalizes $\underline{\mathbf{T}}_U$. It is well-known from linear algebra that an $m \times m$ Jordan block has a single eigenvector. Accordingly, $\underline{\mathbf{T}}_U$ must be similar to a matrix that contains a non-trivial Jordan block of order four to have a fourth order EPD. Such a Jordan block will have a single degenerate eigenvector, and three other *generalized* eigenvectors. The four linearly independent generalized eigenvectors in such defective system are found by solving the generalized Floquet-Bloch form

$$[\underline{\mathbf{T}}_U - \zeta_d \mathbf{1}]^q \Psi_q^g(z) = 0, \quad q = 1, 2, 3, 4 \quad (2.13)$$

where $\Psi_1^g(z) = \Psi_1(z)$ is a regular or ordinary eigenvector while $\Psi_2^g(z)$, $\Psi_3^g(z)$ and $\Psi_4^g(z)$ are the generalized eigenvectors of ranks 2, 3, and 4, respectively (see details of generalized

eigenvectors in Ch. 7 in [51]). A fourth order EPD in our CTL system occurs *if and only if* the transfer matrix $\underline{\mathbf{T}}_U$ is similar to a Jordan canonical matrix [12, 4] given by

$$\underline{\mathbf{T}}_U = \underline{\mathbf{S}} \underline{\mathbf{\Lambda}} \underline{\mathbf{S}}^{-1}, \quad \underline{\mathbf{\Lambda}} = \begin{pmatrix} \zeta_d & 1 & 0 & 0 \\ 0 & \zeta_d & 1 & 0 \\ 0 & 0 & \zeta_d & 1 \\ 0 & 0 & 0 & \zeta_d \end{pmatrix} \quad (2.14)$$

with $\underline{\mathbf{S}} = [\Psi_1^g | \Psi_2^g | \Psi_3^g | \Psi_4^g]$ being composed of one regular eigenvector and three generalized eigenvectors corresponding to a coincident eigenvalue ζ_d with the multiplicity of four, with $\zeta_d = \exp(-jk_d d)$ and $\underline{\mathbf{\Lambda}}$ in (2.14) is a 4×4 Jordan matrix. This is the highest order degeneracy that may be obtained in reciprocal and linear structures with only two coupled TLs, since it combines all supported waves (forward and backward, and/or propagating and evanescent). As the system is reciprocal the $\pm k_d$ symmetry must hold. Therefore if we require to have only one degenerate eigenvalue $\zeta_d = \exp(\pm jk_d d)$ with multiplicity of four, this implies that k_d must be either $k_d = \pi/d$ or $k_d = 0$. Therefore, the fourth order EPD can occur either at the Brillouin zone edge ($k_d = 0$) or at the center ($k_d = \pi/d$). Here we have defined the fundamental Brillouin zone (BZ) within the range from $k_d = 0$ to 2π . The evolution of the four eigenvectors near the fourth order EPD, varying either frequency or any other parameter in the CTL, is schematically depicted in Fig. 2.2. We also stress that such an EPD (i.e. 4th order EPD) occurs in an entirely passive structure without gain or loss. Later in Section 2.6, we show that it may also occur in the presence of gain and loss when a particular *balance* condition is reached. In the following section, we explore examples where three kinds of exceptional points manifest themselves in CTL structures.

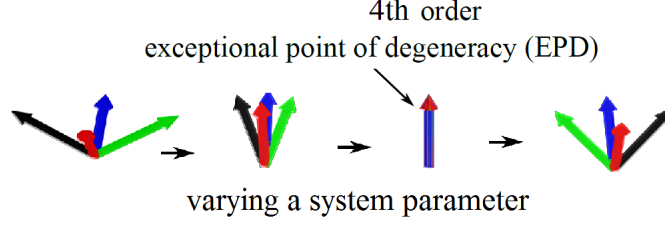


Figure 2.2: Representation of the eigenvectors of the coupled waveguides near an EPD, schematically showing that the four independent vectors coalesce into a degenerate eigenvector at the EPD when one system parameter is varied.

2.3 Fourth Order EPD in Periodic CTLs

The examples in Fig. 2.1(a) and (b) illustrate two coupled waveguide geometries that can potentially support a DBE. We recall that the DBE occurs in lossless and gainless structures. Therefore, under this assumption, when the CTL is arranged into a periodic structure like the ones shown in Fig. 2.1, both $\underline{\mathbf{M}}_A$ and $\underline{\mathbf{M}}_B$ are J -Hermitian, and the characteristic matrices $\underline{\mathbf{ZY}}$ and $\underline{\mathbf{YZ}}$ in (2.3) are Hermitian. The microstrip geometries in Fig. 2.1 constitute periodically cascaded segments of coupled/uncoupled microstrip lines that support four Bloch modes (two in each direction). In this section we elaborate on the periodic microstrip TL in Fig. 2.1(a). The other example in Fig. 2.1(b) will be utilized in Sections 2.4 and 2.5. In the following, we derive the dispersion relation for the fourth order EPD as well as introduce a figure of merit which we call hyperdistance to assess the quality of such EPD subject to any kind of perturbation, e.g., losses, frequency or structural perturbations like a coupling capacitance. We first consider lossless CTLs while at the end of this section losses in the CTL are also considered when discussing the hyperdistance concept.

A. Fourth order degeneracy

We first assume the periodic CTL to be lossless (no gain is introduced yet; we will investigate the case with gain and loss in Section 2.6), hence the matrix $\underline{\mathbf{M}}$ for each segment satisfies

the J -Hermiticity property [1] (see also [52], Ch. 6)

$$\underline{\mathbf{M}}^\dagger = \underline{\mathbf{J}} \underline{\mathbf{M}} \underline{\mathbf{J}}^{-1}, \quad \underline{\mathbf{M}} = -\underline{\mathbf{N}} \underline{\mathbf{M}} \underline{\mathbf{N}}^{-1} \quad (2.15)$$

where $\underline{\mathbf{J}}$ and $\underline{\mathbf{N}}$ are defined in eq. (2.9) in [45]; and as such each individual CTL segment has four modal solutions that have purely real propagation wavenumbers. Indeed, we recall that each of the constitutive CTL segments has the characteristic matrices $\underline{\underline{\mathbf{Z}}} \underline{\underline{\mathbf{Y}}}$ and $\underline{\underline{\mathbf{Y}}} \underline{\underline{\mathbf{Z}}}$ that are Hermitian and has real eigenvalues as proven in the Appendix of [45].

The periodic CTL has eigenvalues $\zeta_n = \exp(-jk_n d)$, with $n = 1, 2, 3, 4$, where k_n is the Bloch wavenumber, that is obtained from the solution of (2.9). In order to realize the fourth order EPD at a certain frequency, we impose the Jordan block similarity (2.14) on the transfer matrix of the periodic unit cell. There are several possible unit cell TL parameters (L , C) combinations that make $\underline{\mathbf{T}}_U$ satisfies (2.14). A set that models the geometry in Fig. 2.1(a) is provided as an example with parameters as in Appendix A, leading to a DBE at 4.03 GHz, which is the case shown in Fig. 2.3.

B. Dispersion perturbation analysis and Puiseux series

When a system parameter of the CTL (a geometry or electrical parameter like C_m , or frequency, or because of the introduction of losses) is perturbed by a small parameter δ , the perturbed eigenvalue is written as a perturbation of the degenerate eigenvalue in the neighborhood of an EPD eigenvalue $\zeta_e = \exp(-jk_e d)$ in terms of a fractional power expansion (also called Puiseux series [53, 28, 54]) in the perturbation parameter δ . Since in this section we focus on the fourth order EPD in the absence of losses and gain (i.e., on the DBE), the eigenvalue is $\zeta_d = \exp(-jk_d d)$ which can be related to CTL wavenumber k_d as

$$k_n(\omega) = k_d + a_1 e^{j(n-1)\frac{2\pi}{4}} \delta^{1/4} + a_2 e^{j(n-1)\frac{2\pi}{2}} \delta^{1/2} + \dots \quad (2.16)$$

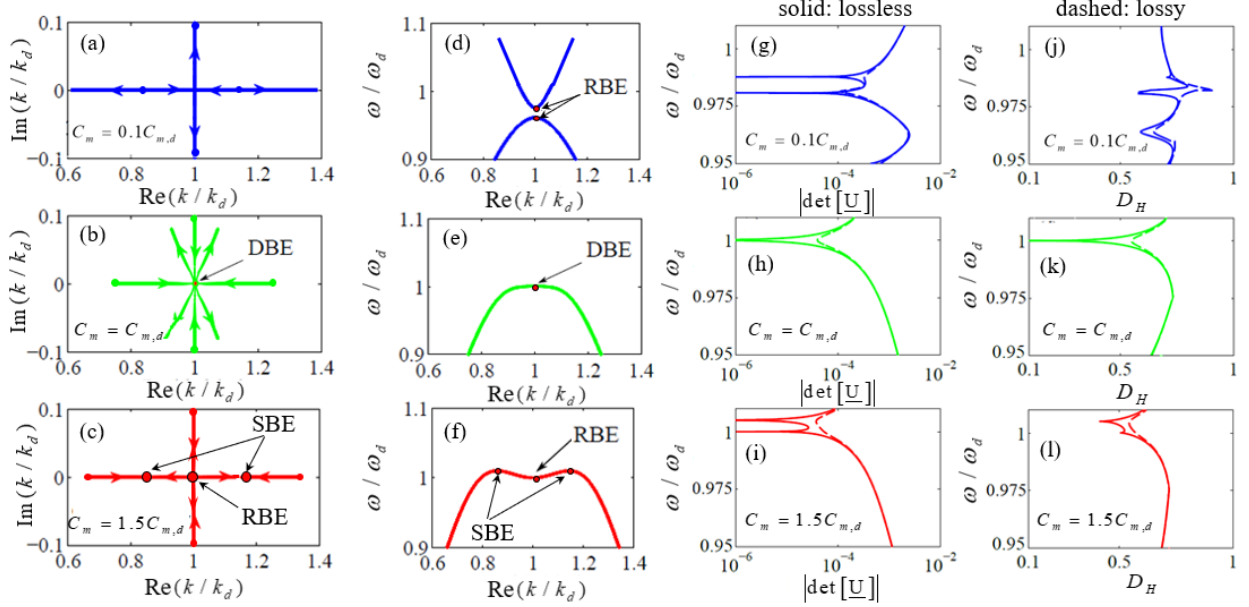


Figure 2.3: (First two columns) (a)–(c) Complex k plane showing the trajectory of the wavenumber k for increasing frequency near the fourth-order EPD condition for different values of the coupling capacitances, for a lossless CTL, and the (d)–(f) corresponding dispersion diagram showing the evolution from RBE to DBE and to SBE, respectively, when varying the coupling capacitance. Dots in the left panel plots indicate the starting point $\omega = 0.95\omega_d$ of the frequency sweep. (g)–(l) “Measures” of the closeness to the EPD calculated based on the four eigenvectors Ψ_n of the lossless and lossy periodic structure. (g)–(i) depicts $|\det(\underline{\mathbf{U}})| = 0$, while (j)–(l) depicts D_H for different values of C_m . Any EPD necessarily has $|\det(\underline{\mathbf{U}})| = 0$ and the fourth-order degeneracy has $D_H = 0$. Mathematically speaking, only the lossless case with $C_m = C_{m,d}$ shows the fourth-order EPD (i.e., the DBE). Other values of C_m lead to the second-order EPDs (i.e., the RBEs). Losses perturb the RBE and the DBE. Solid lines represent the results for a lossless CTL, whereas the dashed lines represent the results for a CTL including radiation “loss,” in which EPD is no longer strictly manifested, but it can still occur in practice, depending on how small D_H is. Results are based on the microstrip geometry shown in Fig. 2.1(a) whose parameters are given in Appendix A. Here, $\omega_e \equiv \omega_d = 2\pi$ (4.03 GHz) and $k_e \equiv k_d = \pi/d$.

(the proof is provided in Appendix B). Here a'_n s are the fractional series expansion coefficients for the n th eigenmodes, and $n = 1, 2, 3, 4$ provide the four possible quartic roots near the EPD. The perturbation factor δ could be, for example, defined as the normalized detuning of angular frequency from the exact DBE frequency, i.e. $\delta = (\omega_d - \omega)/\omega_d$, where ω_d is the DBE angular frequency. Note that in (2.16), we denote $\delta^{1/4}$ as the quartic root in the first quarter of the complex plane since we take the four different quartic roots into account using the exponential terms $\exp(j(n-1)\pi/2)$. Keeping only the lowest order terms in the fractional power expansion in (2.16) leads to the approximate form of the dispersion near the DBE

$$(\omega - \omega_d) \approx h(k - k_d)^4, \quad (2.17)$$

where h is a parameter that defines flatness of the dispersion near ω_d . Indeed, from (2.16) and (2.17), by substituting $\delta = (\omega_d - \omega)/\omega_d$, it is clear that the flatness parameter is $h = \omega_d/a_1^4$, and it depends on the system parameters. The smaller the value of h the flatter the dispersion curve, indicating higher Q factor [55].

C. Dispersion relation for fourth order degeneracy

In Fig. 2.3 we show the ω – k dispersion diagram for a lossless CTL with circuit parameters in Appendix A, that model the geometry in Fig. 2.1(a), with three different values of the coupling capacitance per-unit-length C_m (note that the co-diagonal elements of $\underline{\underline{\mathbf{C}}}$ are $-C_m$). In Figs. 3(d, e, f) we only plot the modal curves that are related to modes with purely real wavenumber for the lossless CTL. Figs. 3(a, b, c) show the complex wavenumber evolutions varying frequency. In the following we discuss the occurrence of three possible EPDs in lossless CTL: the RBE, the DBE and split band edge (SBE) by varying C_m . We define $C_{m,d}$ as the C_m value at which the DBE occurs. Losses are considered at the end of the section and in the last two columns of Fig. 2.3 (curves with dashed lines).

In Fig. 2.3(e) we plot the $\omega-k$ dispersion relation of the two modes with purely real wavenumber. Since there are four solutions, for $\omega < \omega_d$ there are two other modes that are evanescent (not shown in Fig. 2.3(e) but shown in Fig. 2.3(b)) and, all four modes coalesce at ω_d . For $\omega > \omega_d$ all four modes are evanescent. Note that the periodic structure's dispersion diagram is periodic in $Re(k)$ with period $2\pi/d$, hence we plot the dispersion in the first Brillouin zone defined as $0 \leq Re[k(\omega)] \leq 2\pi/d$, and because of reciprocity wavenumbers are symmetric about the Brillouin zone center $k = k_d = \pi/d$. The so-called frozen mode regime associated with the DBE is related to the vanishing group velocity and has been discussed in previous publications (see [1, 4, 30, 39, 41, 42, 46, 56, 57], and references therein). In this section we focus on the characteristic of the fourth order EPD, the DBE, and what happens near it, and in Section 2.6 the effect of gain and loss on the DBE is analyzed.

The determinant of the similarity transformation $\underline{\mathbf{U}}$ of the transfer matrix in (2.11) is depicted in Figs. 2.3(g, h, i) varying frequency, for three choices of C_m : we observe that $\underline{\mathbf{U}}$ is singular ($\det(\underline{\mathbf{U}}) = 0$) only at the exceptional points. Note that there exist two different kinds of EPDs: the DBE (fourth order degeneracy, associated to four coalescing eigenvectors) that is the most interesting case and the main subject of the study in this chapter, and various RBEs (second order degeneracies associated to two coalescing eigenvectors). In Figs. 2.3(a, b, c) we show the evolution of complex wavenumbers in the complex $Re(k)$ - $Im(k)$ plane for increasing frequency (the direction of the arrows), for three values of the capacitance per-unit-length C_m . By varying C_m , three different situations are observed in the dispersion diagrams and they are discussed in the following. The proper coupling capacitance that allows the DBE is denoted as $C_{m,d}$. For small coupling distributed capacitance ($C_m < C_{m,d}$), the CTL exhibits only one lower and one upper RBE at $k = k_d$, as typical in periodic single TL at the band edge. In Fig. 2.3(a) trajectories of eigenmode coalesce twice at $k = k_d$ at two different frequencies (the upper and lower band edges) designating two distinct second order EPDs. At this second order EPD, two eigenmodes coalesce and this is in principle analogous to the second order EPD that occurs in the uniform CTL investigated in [45]

where a balanced gain and loss condition was necessary to develop an EPD. Therefore, a second order EPD can be realized either with a uniform CTL with balanced gain and loss as in [45] or simply by using a periodic TL. In the latter case the EPD occurs at the center of the Brillouin zone, i.e. at $k_d = \pi/d$.

Increasing C_m , such that $C_m = C_{m,d}$, we find the proper conditions for the fourth order EPD (the DBE) to be manifested. At the DBE, four Floquet-Bloch eigenmodes coalesce to a single degenerate eigenmode; as explained in Fig. 2.3(b) where four modal trajectories of complex k varying as a function of frequency intersect at a single point $k = k_d$. At the DBE frequency ω_d the dispersion diagram in Fig. 2.3(e) is very flat and thereby approximated by the quartic law (2.17). Note that this condition cannot occur in *uniform* CTL made of only two TLs with L and C distributed parameters, as in this chapter, since it requires the presence of a bandgap that is endowed by periodicity. (Other important cases with two uniform CTLs with different distributed parameters for backward waves and evanescent waves will be discussed in the future.)

Increasing the coupling such that $C_m > C_{m,d}$ will lead to altering the DBE to a split band edge (SBE) [2, 58, 59] that has been sometimes referred to as double band edge (DbBE) [33, 56] where three distinct EPDs are found in the dispersion diagram in Fig. 2.3(f), in the shown frequency range. Each EPD is associated with a second order degeneracy in the eigenmodes as seen in Fig. 2.3(c). At each EPD the dispersion diagram is flat, and it exhibits a vanishing group velocity. It can be observed that when $C_m < C_{m,d}$ or $C_m > C_{m,d}$, two EPDs are manifested, all of the second order, and either all of them on the same side of the stop band (Fig. 2.3(f)) or on both sides of the stop band (Fig. 2.3(d)). These EPDs are separated, yet for $C_m = C_{m,d}$ these second order EPDs coalesce and form a fourth order EPD (DBE) (Fig. 2.3(e)).

D. Hyperdistance in four-dimensional complex vector space for identifying “vicinity” to a fourth order EPD

In order to distinguish a fourth order EPD among others, and in order to understand how “far” from the EPD a system is, one ought to define a figure of merit (or hyperdistance) to assess the quality of such EPD subject to any kind of perturbation, like losses, frequency detuning, or tolerances of CTL parameters. Here we focus on losses and coupling as the cause of EPD perturbation. Structures, such as that in Fig. 2.1 are naturally lossy due to dissipative losses (dielectric and ohmic losses) as well as radiation losses that limit the intrinsic quality factors of the constitutive components. Therefore, a perfect degeneracy condition like the DBE corresponding to a lossless structure does not exist in practice when losses are present but can be met in an approximate way and still retain the main features of the four coalescing eigenvectors. In case the periodic CTL is a radiating array, then radiation “losses” (i.e., distributed power extraction) are considered necessary. However, as we will clarify in Section 2.6, CTLs with (radiation) losses may rigorously exhibit EPDs when a gain is introduced in a *balanced fashion*. Just a few studies have shown the sensitivity of field enhancement in DBE structure to losses [47, 58, 60] and in general any imperfection can be thought as perturbations that affect the eigenvalues (wavenumber) in the way described by (2.16). Moreover, in designing realistic waveguides, numerical or experimental methods are used and numerical or systematic errors would require a quantitative measure for observing an “exact” EPD. Since our state vector in the CTL is four dimensional, we develop the concept of hyperdistance, denoted as $D_H(\omega)$, *between the four eigenvectors of the transfer matrix* of one-unit cell $\mathbf{T}_U$ to determine the closeness to an EPD. Various choices could be made for its definition, and here the hyperdistance that represents a figure of merit (FOM) is defined as

$$D_H = \frac{1}{6} \sum_{\substack{m=1, n=1 \\ m \neq n}}^4 |\sin(\theta_{mn})|, \quad \cos(\theta_{mn}) = \frac{\Re(\Psi_m, \Psi_n)}{\|\Psi_m\| \|\Psi_n\|} \quad (2.18)$$

with θ_{mn} representing the angle between two vectors Ψ_m and Ψ_n in a four-dimensional

complex vector space with norms $\|\Psi_m\|$ and $\|\Psi_n\|$. Angles are defined via the inner product $(\Psi_m, \Psi_n) = \Psi_m^\dagger \Psi_n$, where the dagger symbol $\dagger$ denotes the complex conjugate transpose operation, and D_H is defined to be always positive. This FOM yields a hyperdistance ideally equal to zero when all four eigenvectors in the CTL system coalesce, i.e., when the CTL system experiences a fourth order EPD. Mathematically this is described by the transfer matrix of the unit cell $\underline{\mathbf{T}}_U$ becoming similar to a 4×4 Jordan Block as in (2.14). Therefore, when losses (or any disorder of the structural parameters seen as perturbation) occur, the proposed FOM is not zero. When using numerical methods or measurements (see Sec. 2.4) we can assume that the EPD is met in practical terms when the FOM measure is less than a very small threshold value, i.e., $D_H < \varepsilon$, where ε is a small number. It is natural to question when such an EPD occurs in practical terms, i.e., how small ε shall be to claim an EPD is verified. Furthermore, it is also important to quantify how much losses or perturbations deteriorate the EPD.

To better illustrate this concept, we plot in Figs. 2.3(j), (k) and (l) the hyperdistance D_H varying as a function of frequency, for a lossless CTL (denoted by solid lines), i.e., for the three values of coupling capacitance C_m considered in Figs. 2.3(a, b, c). We can now compare the two FOMs introduced: the one associated to vanishing $|\det(\underline{\mathbf{U}})|$ in Fig. 2.3(g, h, i) and the one associated to vanishing D_H in Fig. 2.3(j, k, l). The first one vanishes when at least two eigenvectors coalesce, the second one is the proper one to describe a fourth order EPD because it vanishes only if all four eigenvectors coalesce. Indeed, we see that $|\det(\underline{\mathbf{U}})| \rightarrow 0$ at any EPD, whereas $D_H \rightarrow 0$ only at the DBE frequency ($\omega = \omega_d$) and only when $C_m = C_{m,d}$, i.e., only in Fig. 2.3(k).

The two FOMs are also observed when losses are introduced in the CLT as distributed series resistance (dashed line in Figs. 2.3(g)-(l)). Losses are assumed to be in both TLs with a quality factor Q_{TL} of 1000, defined as $Q_{TL} = \omega L/R$. We also see that the perturbation due to losses can deteriorate the DBE, i.e. D_H is now non-vanishing at $\omega = \omega_d$, when $C_m = C_{m,d}$,

in Fig. 2.3(k). This is in agreement with perturbation theory of eigenmodes leading to (2.16), implying that a small δ parameter leads to a significant change in the eigenvalues since $|\delta|^{1/4} \gg |\delta|$ when $|\delta| \ll 1$; which takes place in the close “vicinity” of the fourth order EPD. We again stress that the FOM D_H provides a more quantitative identification of the fourth order degeneracy compared to the simpler measure $|\det(\underline{\mathbf{U}})|$. The deviation in the ideal EPD conditions due to losses can be seen in Fig. 2.3(j, k, l) by comparing dashed lines to solid lines for the different kinds of EPDs in Figs. 2.3(d), (e) and (f) respectively. In a radiating array the limitation in the occurrence of such degeneracies is due *radiation “loss”* (it is natural to introduce gain in the structure to balance the losses and potentially recover the exceptional point as we will show in Section 2.6). However, the question remains whether we can observe the EPD in experiments even in the presence of losses or not. The concept of hyperdistance developed previously is very useful to decide how “near” a system is to an EPD and therefore if an EPD occurs in practice. Furthermore, the concept of hyperdistance is helpful in determining if introducing gain in some parts of the system is useful in compensating for losses, i.e., in defining the concept of “gain and loss balance” as it will be shown in Section 2.6.

2.4 Experimental Verification of EPD Features in Periodic CTLs

In this section we experimentally verify the existence of the EPD in the microstrip example in Fig. 2.1(b). First, instead of fabricating a long (multi-unit cell) coupled microstrip, we experimentally demonstrate the occurrence of EPDs by performing a measurement on a single unit cell fabricated as shown in Fig. 2.4(a). The unit cell is fabricated on a grounded substrate (Rogers substrate RO4360G2) with a dielectric constant of 6.15, height of 1.52 mm, and with a loss tangent of 0.0038. The TL appearing on the top of the figure was

designed to have a characteristic impedance of 50 Ohms and all the unit cell dimensions are reported in Fig. 2.4(a). To confirm the existence of EPDs in the unit cell, we perform scattering (S)-parameter measurements at the four ports of the unit cell using a four-port Rohde & Schwarz Vector Network Analyzer (VNA) ZVA 67 as shown in Fig. 2.4(b). From the S-parameters measurements, we then retrieve the transfer matrix $\underline{\mathbf{T}}_{\text{U}}$ as shown from well-known conversion tables [61, 62], for the range of frequencies shown in Fig. 2.5. The four wavenumbers are derived and plotted in Fig. 2.5 by solving (2.9) for complex k , at any frequency shown in the plot. The dispersion based on measurements shown in Fig. 2.5(a) is in a good agreement to the results from full-wave simulations of the S-parameters performed using Keysight Technologies ADS based on the Method of Moments (MoM). The full-wave simulation included the effects of the SMA connectors. The dispersion shows several frequencies at which the four wavenumbers have values very close to each other denoting the occurrence of the 4th order EPD, with some perturbation due to ohmic, dielectric and radiation losses. Indeed losses, fabrication tolerances, effect of connectors and other realistic factors affect ideal EPD conditions. Even though mathematically speaking the EPD is not verified exactly, in practical terms, the EPD's prominent features can be well-preserved as we discuss in the following.

In order to assess the existence of the EPD in practical terms, we utilize the hyperdistance concept developed in Section 2.3 for the first time. We use the transfer matrix $\underline{\mathbf{T}}_{\text{U}}$ of the unit cell in Fig. 2.4(a), based on measurements, to calculate the four eigenvectors $\underline{\Psi}_n$, with $n = 1, 2, 3, 4$, of the eigenvalue problem in (2.9). First, the eigenvectors are used to calculate the hyperdistance D_H in (2.18) that is supposed to vanish when approaching a 4th order EPD. Then, they are used to build the similarity transformation matrix $\underline{\mathbf{U}}$ in (2.10) and hence $|\det(\underline{\mathbf{U}})|$ that will tend to zero when at least two eigenvectors become dependent. We plot both D_H and $|\det(\underline{\mathbf{U}})|$ FOMs in Fig. 2.5(b) based on measurements and compare them to the same FOMs obtained from full-wave MoM simulations. We can see that at different frequencies, 2.81 GHz, and 3.1 GHz occurring at $kd = \pi$ the simulated and measured D_H

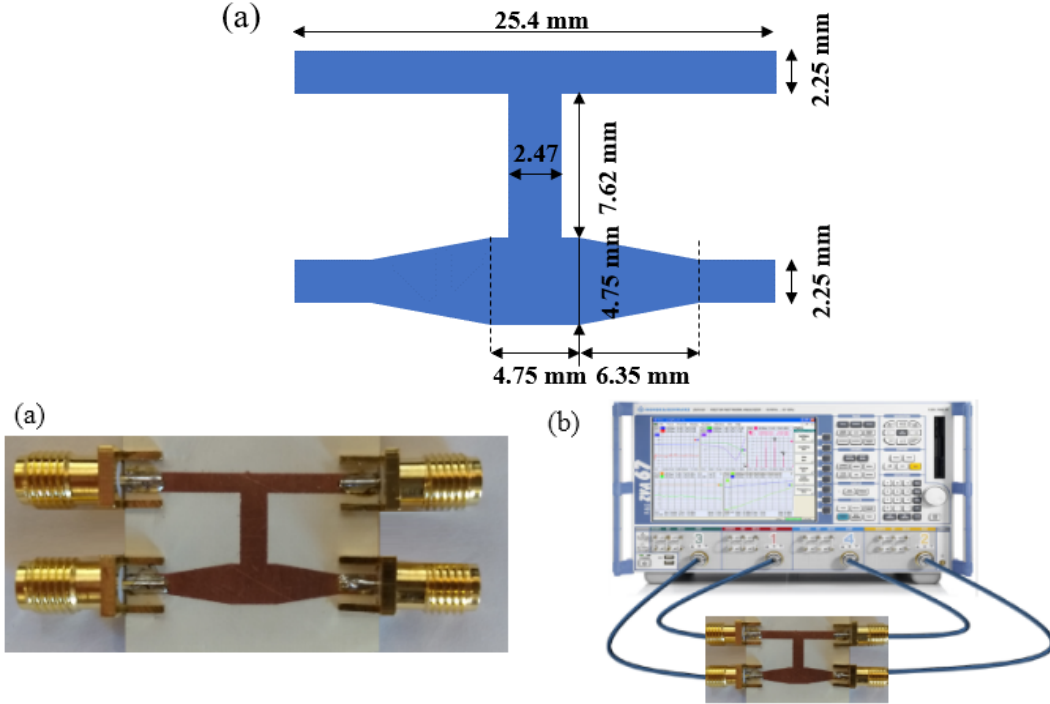


Figure 2.4: (a) Microstrip unit cell of a periodic structure that exhibits a fourth-order EPD. The substrate is Rogers RO4360G2 with a dielectric constant of 6.15 and a height of 1.52 mm. (b) Fabricated unit cell under test with the four ports is attached to a VNA to extract the four-port S-parameters versus frequency used to calculate the eigenvalues. EPDs in the case of gain and loss balance.

show a minimum denoting vicinity of a 4^{th} order EPD despite the presence of radiative and dissipative losses. This demonstrates the EPD properties of eigenvector coalescence is observed at microwaves.

2.5 Finite length CTLs with EPDs: giant resonance and radiation

We investigate the properties of a finite length periodic CTL as in Fig. 2.6, with a fourth order EPD. In principle, the coalescence of four eigenmodes into one degenerate eigenmode at the DBE causes a quartic dispersion relation (2.17) and leads to a vanishing group velocity

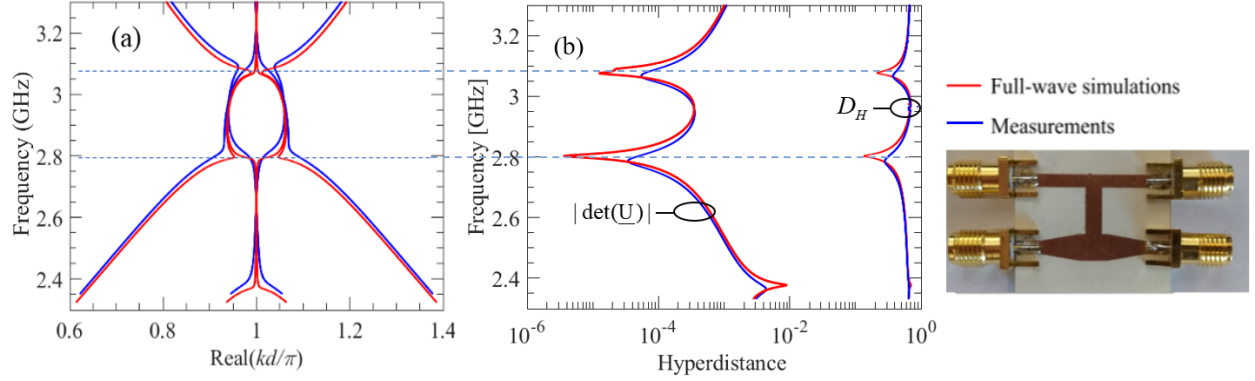


Figure 2.5: (a) Dispersion relation and (b) hyperdistance measurement versus full-wave simulation of the unit cell of a periodic CTL exhibiting a fourth EPD. The results show that the fourth-order EPD occurs at two different frequencies in the range shown in this plot. Full-wave simulations are performed with Keysight ADS using the MoM.

in the infinitely-long periodic structure in the absence of losses. The presence of losses and structural variations/perturbations leads to a non-zero group velocity and renders the hyperdistance (2.18) non-vanishing as seen from the experiment in Fig. 2.5. In addition, even in a lossless structure, waves in a *finite-length* CTL with EPD do not have zero-group velocity since the peak of transmission in Fig. 2.7(c) occurs at a frequency close to the DBE one. Nonetheless, the finite-length CTL develops very interesting resonance features related to the EPDs as we discuss in the following.

Consider the finite length microstrip CTL made of a finite number N of unit cells as shown in Fig. 2.6. The microstrip and substrate's parameters are the same as those used in relation to Figs. 2.4 and 2.5. We perform full-wave simulations using Ansys HFSS utilizing the finite element method (FEM) solver to retrieve the scattering parameters. The dispersion diagram of such structure is shown in Fig. 2.7(a), obtained by simulating a single unit cell. The 4th order EPD is evident at 2.81 GHz. The 4-port, N -cell, finite-length CTL is terminated with coaxial connectors at both extended sides of TL1, the length of the extended part on each side equals 50 mm. Whereas TL2 is terminated in short circuits at the beginning of the 1st unit cell and at the end of the 8th unit cell, as seen in Fig. 2.6. We are interested in the case where the excited fields inside the CTL create a standing wave because of the Fabry-Pérot

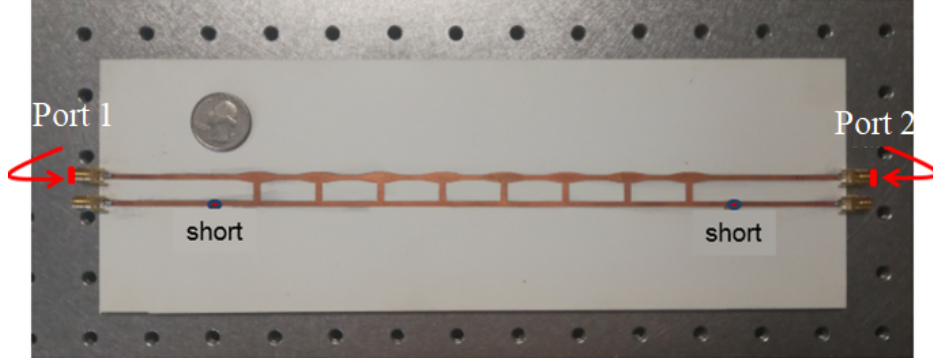


Figure 2.6: Fabricated microstrip eight cell array with an EPD. The CTL is based on the unit cell described in Fig. 2.4.

cavity (FPC) resonance near the EPD frequency (see details in [1, 30]). At such condition constructive interference of the four synchronous eigenmodes leads to a sharp transmission resonance [1, 30]. Such Fabry-Pérot resonance closest to the DBE frequency occurs at the angular frequency denoted by $\omega_{r,d}$, referred hereafter as DBE resonance, characterized by a peak in the transmission coefficient $|S_{21}|$ and a dip in $|S_{11}|$. The DBE resonance frequency is approximately given by [1]

$$\omega_{r,d} \approx \omega_d - h \left(\frac{\pi}{Nd} \right)^4. \quad (2.19)$$

Therefore $\omega_{r,d}$ approaches ω_d for large number N of unit cells. Thanks to the DBE degeneracy condition, a large enhancement in the energy stored is expected because we approach the zero- group velocity conditions when $\omega_{r,d} \approx \omega_d$ and causing a giant enhancement in the Q -factor [30, 63].

We show in Fig. 2.7(b) and (c) FEM full-wave simulations as well as measurement of the magnitude of the scattering parameters S_{21} and S_{11} . The full-wave simulation included the effects of the SMA connectors. Results show good agreement between simulation and measurement. Results also demonstrate the occurrence of the DBE resonance at 2.775 GHz, that is close to the DBE frequency of 2.81 GHz. The transmission resonance at the DBE

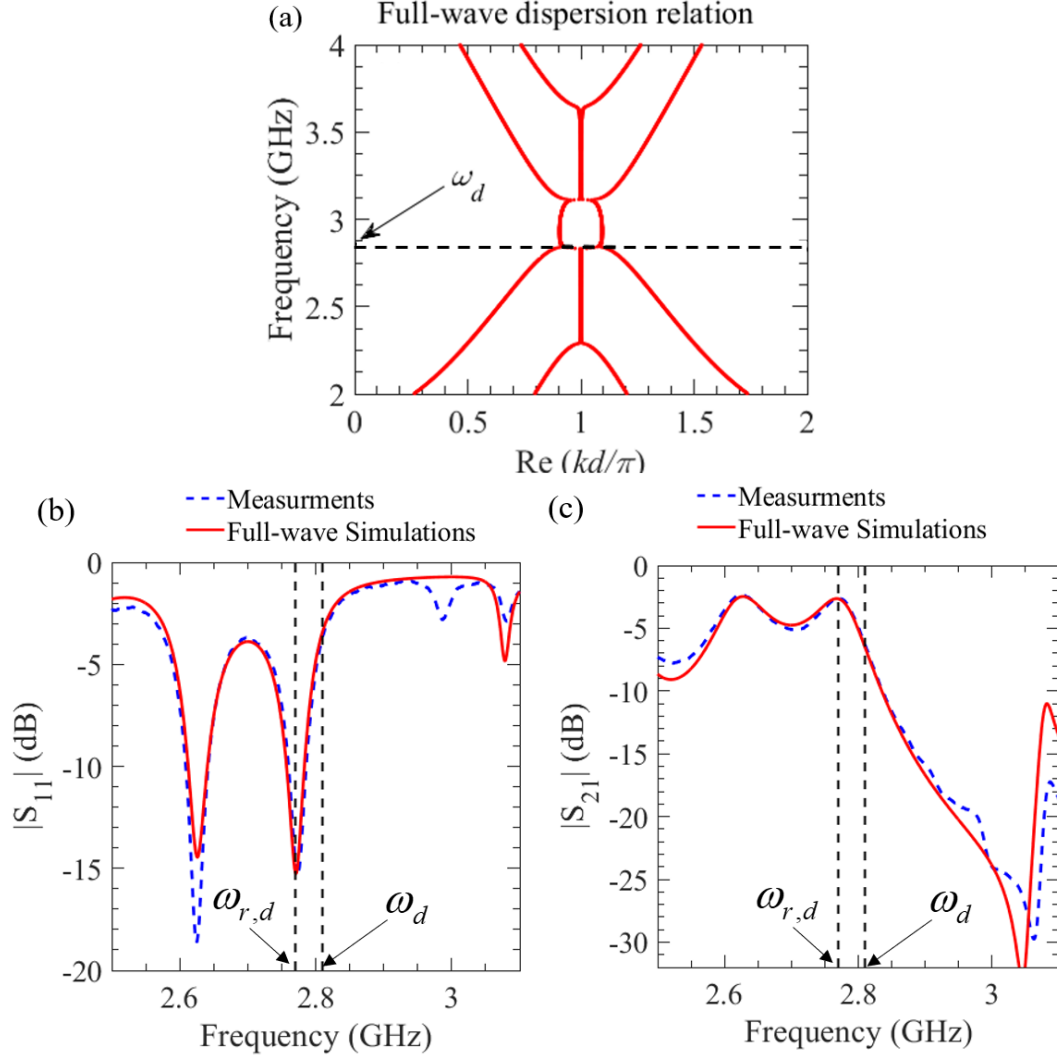


Figure 2.7: (a) Dispersion relation for the unit cell of the periodic structure in Fig. 2.6 obtained using the full-wave FEM (Ansys HFSS) accounting also for radiation, ohmic, and dielectric losses. Simulations and measurements of the scattering parameters (b) S_{11} and (c) S_{21} for an eight-unit-cell array in Fig. 2.6, where a good agreement between full-wave simulations and measurements is shown at the resonance frequency associated with the EPD. The frequency of the resonance peak is estimated by (2.19).

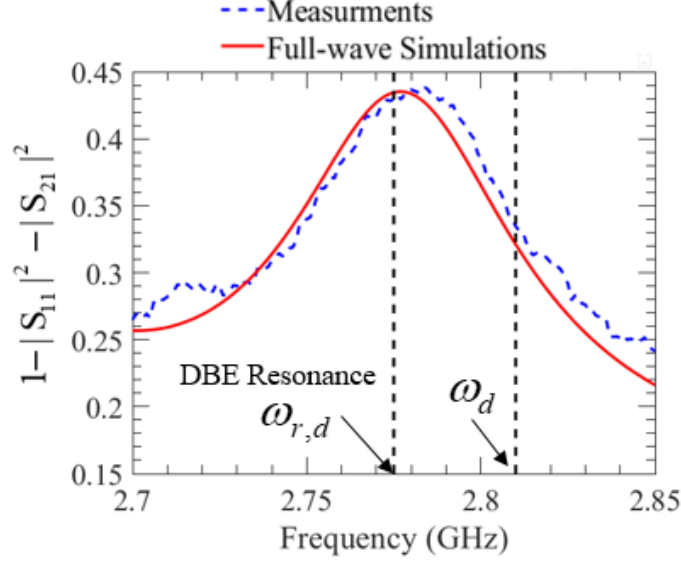


Figure 2.8: Measurements versus simulations of the power loss parameter defined as $1 - |S_{11}|^2 - |S_{21}|^2$ varying as a function of frequency near the EPD resonance.

has a transmission coefficient $|S_{21}|$ of -3.6 dB.

It is important to point out that such transmission resonance is affected by losses; both dissipative and radiative. Indeed, the exact mathematical DBE condition is not met as seen in Fig. 2.5 and as discussed in Section 2.4. Nonetheless, the DBE resonance is still observed in Fig. 2.7. We emphasize that most of the losses incurred by this CTL are in fact radiation losses. The power loss factor, shown in Fig. 2.8, is defined as $1 - |S_{11}|^2 - |S_{21}|^2$ combining both effects of dissipative Ohmic loss as well as radiation. Results show good agreement between FEM full-wave simulations and measurements.

2.6 EPDs With “Balanced” Gain And Loss

The RBE and DBE (EPDs of order 2 and 4, respectively) exist in structures that are lossless, indeed the presence of losses would inhibit the existence of the DBE, i.e., it would degrade

the degree of coalescence of the four eigenmodes. This phenomenon is monitored by the hyperdistance parameter D_H that increases with increasing losses. And this is the reason why we have observed a non-ideal DBE dispersion relation in Fig. 2.5. Increasing radiation losses (e.g., for antenna applications) will further degrade the DBE. Here our aim is to provide a mean to recover EPDs deteriorated by losses by strategically incorporating a proper distribution of gain in each unit cell. The perfect condition that allows the existence of an EPD is named perfect “*loss and gain balance*.” (For perfect EPD condition we mean the exact mathematical condition described in Section 2.2 that leads to $D_H \rightarrow 0$). The balancing between gain and loss implies that both the z -evolution matrices $\underline{\mathbf{M}}_A$ and $\underline{\mathbf{M}}_B$ are not J -Hermitian, and that the characteristic matrices $\underline{\mathbf{ZY}}$ and $\underline{\mathbf{YZ}}$ in (2.3) are non-Hermitian. This does not happen for the DBE case, therefore we will not refer to this condition as DBE. This “loss and gain balance” condition is different from the situation previously studied in PT-symmetric literature [45, 7, 8, 64, 6] in two aspects: (i) The EPD studied here is of fourth order; (ii) the gain compensation condition for a fourth order EPD in periodic CTLs does not necessarily mean that the system is PT-symmetric [45, 7, 6, 65], i.e., it does not mean that one TL has gain and the other has loss of exact symmetry and magnitude. Note that gain compensation of losses may, in various circumstances, not lead to a perfect EPD as will be shown in the study case B. However, in many practical cases the *exact* (i.e., perfect) condition may not even be necessary. Indeed, when D_H is sufficiently small we can observe qualitative features related to the EPD even though the structure does not possess the precise mathematical EPD condition. Furthermore, we point out that gain/loss compensation in a unit cell does not necessarily imply absolute instability or self-oscillation in a finite length structure; indeed loading effect must be taken into account in the stability criterion especially near these EPDs as done in [42].

The following discussion is based on the periodic coupled transmission line as in Fig. 2.1(a), where all parameters are given in Appendix A, investigated with the theoretical tools in Section 2.3. We will study different configuration for losses and incorporating gain.

2.6.1 Case A: Series loss and shunt gain

Losses in the CTL are assumed to be in the form of a per unit length series resistance in both TLs, representing radiation and/or dissipation losses. Here, gain is introduced in the periodic CTL using a negative per-unit-length conductance G in both TLs (therefore $-G$ is positive with units of Siemens/m). Note that here, we assume that such gain is introduced in both TLs, i.e., each TL has a self-negative conductance G in addition to a series resistance R_n associated to radiation and metal losses for the n^{th} TL, with $n = 1, 2$. Losses are described using the quality factor $Q_{TL,n} = \omega L_n / R_n$ associated to the per-unit-length TL parameters, and here we assume that all TLs have the same quality $Q_{TL,n} = Q_{TL}$ for simplicity. Accordingly, here each constitutive CTL segment has a z -evolution matrix $\underline{\mathbf{M}}$ that doesn't satisfy the J -Hermiticity and a characteristic matrix $\underline{\mathbf{ZY}}$ that is non-Hermitian. A simple procedure is developed to estimate the value of the needed G in the CTL to “recover” the fourth order EPD occurring at more or less the same angular frequency as that would occur in the lossless structure $\omega = \omega_d$. Note that here for “recovering” an EPD we mean that the required hyperdistance D_H shall be smaller than a specific low threshold ε , that is arbitrarily predetermined (at a lossless 4th order EPD, i.e. at a DBE, this D_H vanishes). Therefore, we find the value of the optimum distributed gain parameter, namely G_e such that the $D_H < \varepsilon$ at $\omega = \omega_d$, where here we assume $\varepsilon = 0.1$ as an example. In Fig. 2.9 we show the dispersion diagram of a CLT with losses and with compensating gain, as well as the hyperdistance D_H plotted in Fig. 2.9(a) versus gain parameter $-G$ at $\omega = \omega_d$. We observe that for all TLs with $Q_{TL} = 100$, a chosen values $G = G_e = -9.54$ mS/m, leads to a minimum $D_H < 0.1$ which indicates that the fourth order degeneracy has been achieved in practical terms, i.e., the four eigenvectors Ψ_n are almost parallel. Indeed, we plot the dispersion relation of the lossy structure (i.e. before introducing gain) in Fig. 2.9(b). Then we chose the best G , i.e. $G = G_e$, and in Fig. 2.9(c) we plot the dispersion with such gain and loss balance condition. It is clear that the dispersion in Fig. 2.9(c) is very similar to the

ideal case with no loss and no gain plotted in Fig. 2.3(e).

Additionally, in Fig. 2.10 we show that this procedure is also useful to recover EPDs when the CTL is strongly perturbed by losses. Indeed in Fig. 2.10 the CTL and discussion is the same as for the case in Fig. 2.9, except that now the TLs have losses that are an order of magnitude larger than in Fig. 2.9, i.e. all TLs considered for the result in Fig. 2.10 have $Q_{\text{TL}} = 10$. It is obvious from Fig. 2.10(a) that the dispersion of the modes in the lossy CTL (i.e. before introducing gain) is strongly perturbed from the ideal case, indeed the DBE is not visible at all. Then, using the same procedure previously discussed, one can recover the 4th order EPD in practical terms as shown in Fig. 2.10(b). We recall one more time that these guiding structures are not PT-symmetric since gain and loss are not displaced in a symmetric fashion.

2.6.2 Case B: shunt loss in both TLs, shunt gain in only one TL

We analyze another CTL configuration that has losses and gain not satisfying PT-symmetry. Losses in the CTL are assumed to be in the form of a per unit length shunt conductance $G_{L,n}$ in both TLs, representing radiation and/or dissipation losses. Shunt per-unit-length losses are analogously described by a quality factor $Q_{\text{TL},n} = \omega C_n / G_{L,n}$, and in the following we assume that all TLs have shunt conductances $G_{L,n}$ such that $Q_{\text{TL},n} = Q_{\text{TL}} = 100$. Gain is introduced in the periodic CTL using a negative per-unit-length conductance G in only TL1 (therefore $-G$ is positive with units of Siemens/m). Note that this CTL configuration differs from the one in Case A in: (i) gain is introduced to only one TL, not in both TLs as in the previous case, and (ii) losses are introduced using a shunt conductance per unit length rather than a series resistance per unit length. Therefore, TL1 has a self-negative conductance G in addition to a $G_{L,1}$ associated to losses, whereas TL2 has only $G_{L,2}$ associated to losses. Similar to Case A, each constitutive CTL segment has a characteristic matrix $\mathbf{ZY}$ in (2.3)

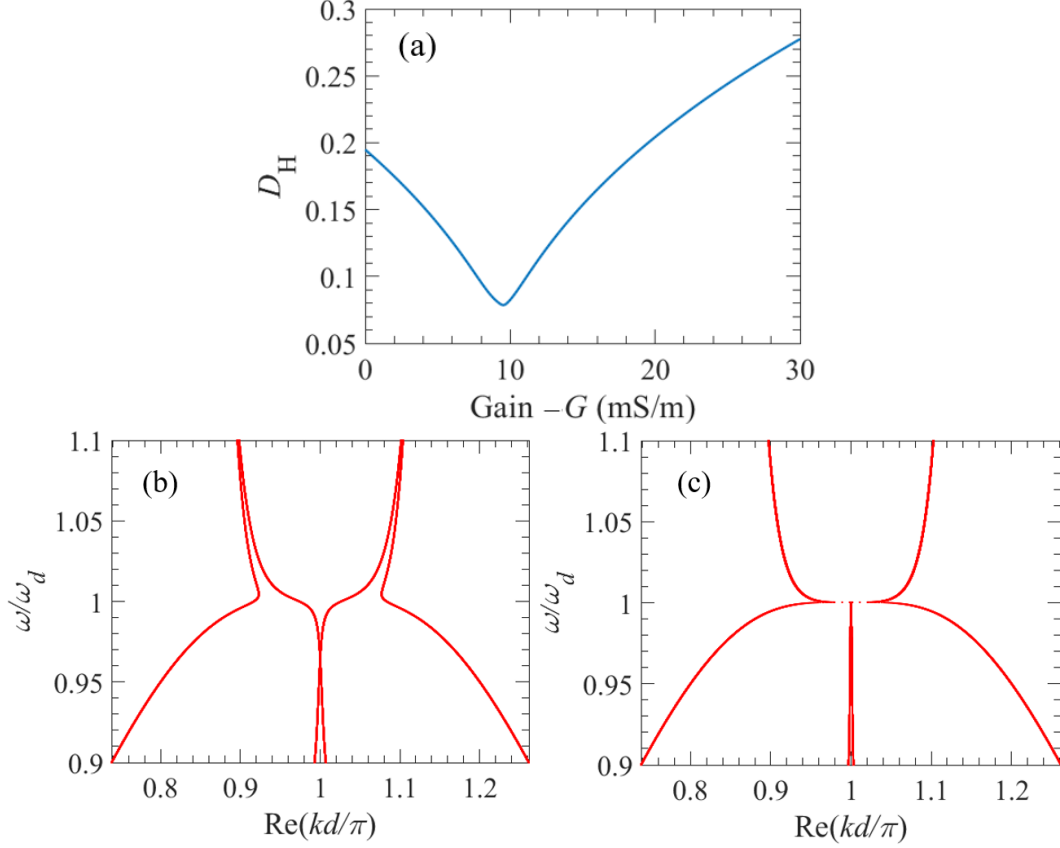


Figure 2.9: (a) Plot of the hyperdistance (18) to assess the occurrence of a fourth-order EPD in a CTL with losses, as a function of gain $-G$ to compensate for losses. The EPD is achieved in practical terms with $G = G_e = -9.54$ mS/m at which hyperdistance has the minimum value $D_H(\omega_d) = 0.078$ (keeping the angular frequency fixed at ω_d , at which the lossless CTLs develops a DBE). (b) Corresponding dispersion diagram for the lossy CTL (i.e., before introducing gain) where losses are modeled as series resistance R_n such that all TLs have $Q_{\text{TL}} = 100$. (c) Dispersion diagram after introducing the appropriate gain $G = G_e$ to achieve the “balanced gain and loss condition,” showing the typical flatness of a DBE is recovered.

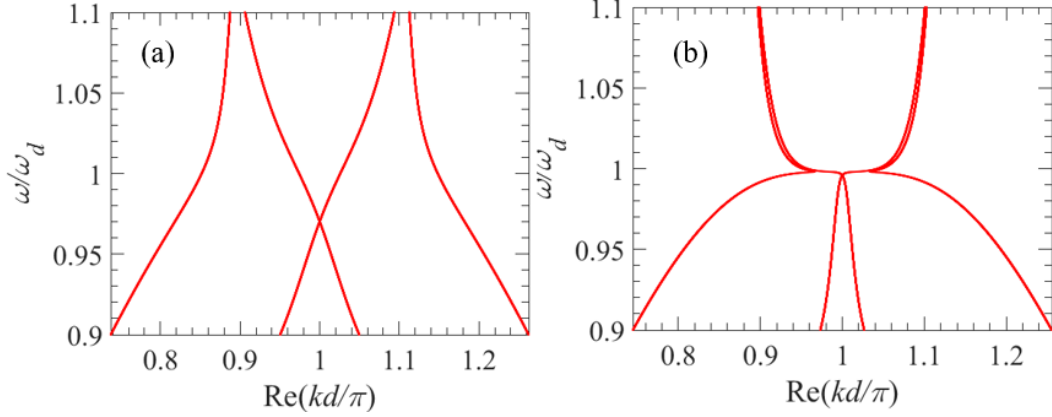


Figure 2.10: (a) Dispersion diagram for a lossy CTL structure (i.e., before introducing gain) as in Fig. 2.9(a) but with one order of magnitude larger losses than in Fig. 2.9(a), that is, now all TLs have $Q_{\text{TL}} = 10$. (b) Corresponding dispersion diagram after introducing the gain $G_e = -93.4$ mS/m associated to minimum $D_H(\omega_d) = 0.18$.

that is non-Hermitian, and the value of the needed gain G in TL1 to “recover” the fourth order EPD is determined with same procedure used in Case A. In Fig. 2.11(a) we plot D_H versus the gain parameter $-G$ at $\omega = \omega_d$ (that is the angular frequency at which the DBE for the ideal lossless structure is obtained) and we observe that for $G = G_e = -15.4$ mS/m we have minimum D_H and that such minimum has $D_H < 0.1$ which indicates that the fourth order has been achieved in practical terms. The dispersion relation of the lossy CTL, i.e., before introducing gain, is plotted in Fig. 2.11(b), and in Fig. 2.11(c) we show the dispersion after introducing the gain associated to minimum D_H . It is worth noting that, the gain compensation leads to a non-ideal EPD. However, in many practical cases the *exact* EPD may not even be necessary. Indeed in Fig. 2.11(c), one can see that the dispersion diagram is very similar to the one of the ideal DBE, therefore we can still observe qualitative features related to the DBE.

Note that, increasing the gain beyond the value of $G = G_e$ may lead to self-oscillations in the unit cell and would render the structure absolutely unstable. Such technique however can be very useful in realizing novel schemes for low threshold oscillators which we will investigate in the near future, and have been already explored in the areas of electron-beam devices

[3, 43] and lasers [36].

2.7 Conclusion

We have experimentally demonstrated for the first time the occurrence of a fourth order EPD (the DBE) in microstrip CTLs at microwave frequencies, first through four-port measurements of a unit cell leading to the DBE dispersion relation and then through the transmission characteristics of a finite-length CTL. We have also introduced the novel concept of a hyperdistance figure of merit to estimate the effect of perturbations like imperfect coupling, presence of losses and any other perturbation that may arise from fabrications or numerical simulation on the EPD. The hyperdistance concept is a way to measure the eigenvectors mutual “distance” in a multidimensional vector space, that should ideally vanish at the exact EPD condition. The smaller the hyperdistance the closer a system is to the EPD occurrence. Based on the defined figure of merit, our experimental verification has confirmed for the first time that the main EPD features of almost parallel eigenvectors can still exist in realistic periodic arrays that include radiation and dissipative losses. Furthermore, in CTLs that experience significant radiation and dissipative losses, we have shown that the so-called “gain and loss balance” condition leads to recovering an EPD, up to a level that can be quantified via the hyperdistance concept, even in schemes that do not necessarily imply PT-symmetry. The gain and loss balance condition scheme can be applied in principle for any amount of loss in a CTL, paving the way to use it in the design of active grid array antennas.

The capability of obtaining EPDs in CTLs with large radiation losses when adding gain in a proper manner in each unit cell also paves the way to use this multi-eigenmode degenerate scheme in the design of array oscillators and high-intensity spatial power combiners. Potential benefits may include low oscillation threshold, or even high-intensity radiation with high power efficiency, and spectral purity.

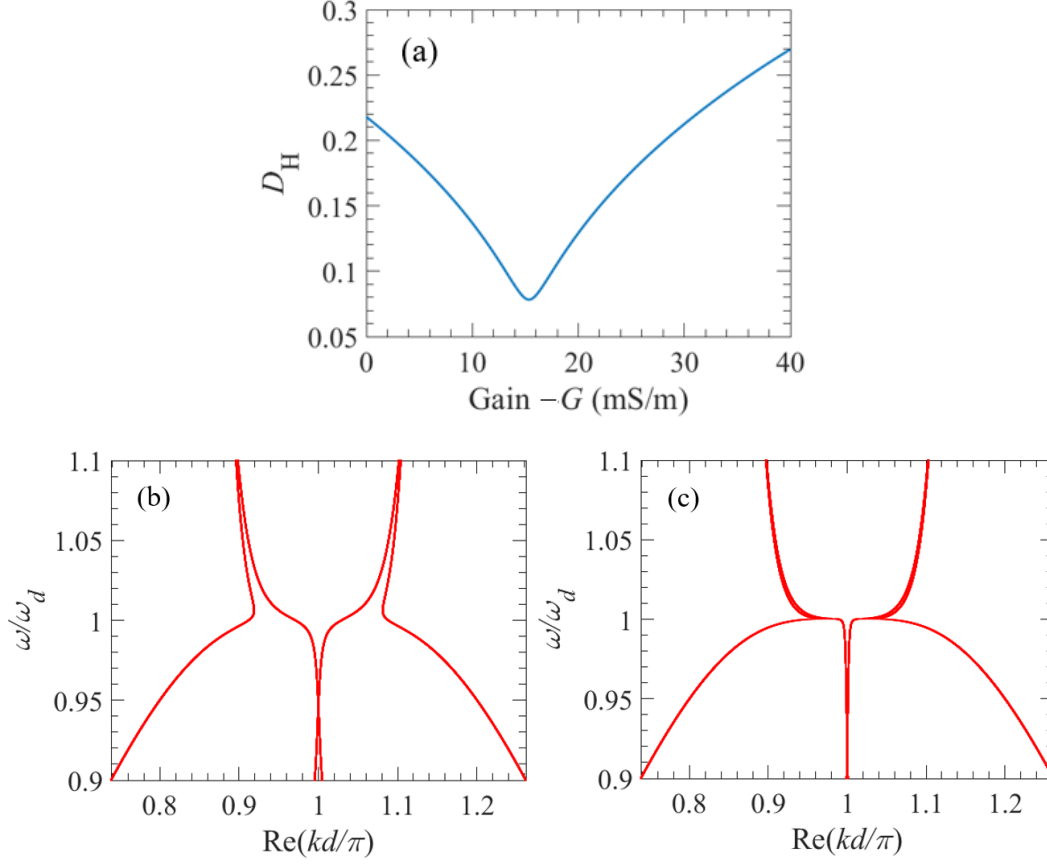


Figure 2.11: (a) Plot of the hyperdistance (18) to assess the presence of a fourth-order EPD in a CTL with losses, as a function of gain $-G$ to compensate for losses. The EPD is achieved in practical terms with a value of gain parameter $-G_e = 15.4$ mS/m (attached to TL1) at which hyperdistance has the minimum value $D_H(\omega_d) = 0.078$ (i.e., less than the pre-determined threshold). Angular frequency is fixed at ω_d at which the lossless CTLs develop a DBE. (b) Corresponding dispersion diagram for lossy structure (i.e., before introducing gain) where losses are modeled as shunt $G_{L,n}$ such that all TLs have the same $Q_{\text{TL}} = 100$. (c) Corresponding dispersion diagram after introducing the gain associated to minimum D_H , showing nonideal EPD.

Chapter 3

Distributed Degenerate Band Edge Oscillator

3.1 Motivation

We propose a new class of oscillators by engineering the dispersion of two-coupled periodic waveguides to exhibit a degenerate band edge (DBE). The DBE is an exceptional point of degeneracy (EPD) of order four, i.e., representing the coalescence of four eigenmodes of a waveguide system without loss and gain. We present a distributed DBE oscillator realized in periodic coupled transmission lines with a unique mode selection scheme that leads to a stable single-frequency oscillation, even in the presence of load variation. The DBE oscillator potentially leads to a boost of the efficiency and performance of RF sources, due to the unique features associated to the EPD concept. This class of oscillators is promising for improving discrete-distributed coherent sources and can be extended to radiating structures to achieve a new class of active integrated antenna arrays.

Oscillators are one of the fundamental components that exist in any radio frequency (RF) system. Typically, an RF oscillator is an amplifier with positive feedback mechanism utilizing a gain device with a selective resonance circuit that generates a single tone frequency. The negative conductance, i.e., the gain component, required for positive feedback can be obtained using transistors as in cross-coupled transistor pair [66], or by circuit topologies such as Pierce, Colpitts, and Gunn diode waveguide oscillators [67, 68]. In pursuance of improving the performance of RF and microwave sources, many research avenues are currently being investigated [69, 70, 71, 72, 73]. The focus of this chapter is on a new class of oscillators whose architecture features i) a cavity made of a periodic coupled-mode waveguide utilizing a special kind of degeneracy in its dispersion diagram, used as the passive circuit responsible for frequency selection, and ii) a set of distributed active devices incorporated in the cavity that provide the sufficient negative conductance to compensate the losses and thus to start the oscillation.

Generally, electromagnetic guiding structures or resonators are characterized by evolution equations that describe the spatial evolution of their eigenstates (eigenvalues and eigenvectors). We are interested in a very special degeneracy condition that occurs when two or more of these eigenstates coalesce into a single degenerate eigenmode at a certain point in the parameter space [1, 45, 7, 74]. Such points are called exceptional points of degeneracy (EPD), and the order of the EPD is determined by the number of eigenmodes that coalesce at this point. The dispersion relation of eigenmodes in such a guiding structure that exhibits an EPD of order n has the unique behavior in which $(\omega - \omega_e) \propto (\beta - \beta_e)^n$ in the vicinity of EPD, where ω and β are the angular frequency and the propagation constant, respectively, at the EPD they are denoted by the subscript e . In lossless waveguides, as in this chapter, this unique degenerate dispersion behavior is accompanied by supreme characteristics including the vanishing of the group velocity [4, 57] as well as the dramatic improvement in the local density of states [30] resulting in a robust increase in the loaded quality factor of the structure.

In this chapter we focus on the degenerate band edge (DBE), which is a fourth-order EPD manifested at the band edge of a lossless structure. At the DBE four eigenstates coalesce into a degenerate one and the dispersion relation near the DBE is characterized by $(1 - \omega/\omega_e) \approx \eta_e(1 - k/k_e)^4$ where η_e is a dimensionless flatness parameter. A finite-length waveguide made of a finite number N of unit cells forms a Fabry-Perot cavity (FPC) with a resonance very near the DBE frequency, see details in [1, 30, 20], and the field creates a standing wave. The resonance closest to the DBE frequency occurs at $\omega_{r,e}$ and for large N it is approximated by $\omega_{r,e}/\omega_e \approx 1 - \eta_e/N^4$. Associated to such DBE resonance, the cavity experiences a field enhancement at its geometrical center which leads to an enhancement in the Q -factor and less sensitivity to loads [45]. The motivation behind introducing a DBE-based oscillator is based on previous work related to high power microwave devices; it has shown an enhancement of gain in electron beam devices based on waveguide with a DBE [30, 3] and demonstrated a low starting (threshold) current and a unique threshold scaling with length compared to conventional backward wave oscillators [42, 3].

3.2 System Description

In this chapter, we present an example of a DBE oscillator based on two periodic coupled transmission lines (CTLs) as in one of the configurations proposed in [20] and shown in Fig.1(a). The DBE concept in CTLs was introduced in [75] for antenna minimization application. We first show the dispersion of the coupled waveguide where the DBE occurs at several points in the shown frequency range and beyond. Then, we consider a cavity made of a finite-length CTL where discrete distributed gain is introduced leading to a single-frequency of oscillation. We show the robustness of this new class of oscillators against load variation.

The passive waveguide (before the introduction of distributed gain) consists of two coupled microstrips over a grounded dielectric substrate engineered to exhibit a DBE, as shown

in Fig. 3.1(a). In such a waveguide there are two modal fields that can propagate along the $+z$ -direction, and two in the opposite one. Proper coupling among the four modal fields is required to exhibit a DBE. The realization of proper coupling is achieved either with proximity fields (inductive/capacitive coupling) or with a physical electric connection as in this chapter. For more detailed on the engineering of the proper coupling with an approximate LC model see [20] and references therein. The “corrugation” in TL1 (shown in Fig. 1(a)) is introduced to add another degree of freedom to our design space parameters.

The formulation that describes the field evolution using a coupled transmission line approach is found in [45, 20], assuming a time harmonic evolution $e^{j\omega t}$. It is convenient to define a four-dimensional state vector $\mathbf{\Psi}(z) = [\mathbf{V}(z), \mathbf{I}(z)]^T$, which comprises the voltages $\mathbf{V}(z) = [V_1(z), V_2(z)]^T$ and currents $\mathbf{I}(z) = [I_1(z), I_2(z)]^T$ in the two lines. The first order differential equations that describe the spatial evolution of the state vector in a uniform segment of the CTL are written as [45]

$$\partial_z \mathbf{\Psi}(z) = -j \underline{\mathbf{M}} \mathbf{\Psi}(z) \quad (3.1)$$

where

$$\underline{\mathbf{M}} = \begin{bmatrix} \mathbf{0} & -j\mathbf{Z} \\ -j\mathbf{Y} & \mathbf{0} \end{bmatrix} \quad (3.2)$$

is the 4×4 CTL system matrix, and $\mathbf{Z}$ and $\mathbf{Y}$ are the distributed series impedance and shunt admittance 2×2 matrices describing the per unit length parameters of the two CTLs [48, 45]. The solution of 3.1, assuming a certain boundary condition at $z = z_0$, is found as $\mathbf{\Psi}(z_1) = \underline{\mathbf{T}}(z_1, z_0) \mathbf{\Psi}(z_0)$ where $\underline{\mathbf{T}}(z_1, z_0)$ is the transfer matrix given by $\underline{\mathbf{T}}(z_1, z_0) = \exp(-j(z_1 - z_0) \underline{\mathbf{M}})$.

The periodic structure depicted in Fig. 3.1(a) has a unit cell composed of two uniform segments A, B and incorporates an additional coupling matrix due to the coupling microstrip.

The transfer matrix of a unit cell is expressed as the product of the individual segments' transfer matrices as $\underline{\mathbf{T}}_U = \underline{\mathbf{T}}_A \underline{\mathbf{T}}_C \underline{\mathbf{T}}_B$, where $\underline{\mathbf{T}}_A$ and $\underline{\mathbf{T}}_B$ are the transfer matrices of segments A and B, and $\underline{\mathbf{T}}_C$ is the coupling matrix representing the physical connection via a microstrip between the two lines.

3.3 Distributed Degenerate Band Edge Oscillator

The CTL shown in Fig. 3.1 is designed on a grounded substrate with relative dielectric constant $\epsilon_r = 2.2$, height of 0.508 mm, and with a loss tangent of 0.002. The complex wavenumber-frequency dispersion diagram shown in Fig. 3.1(c) is obtained using the finite element method, implemented in CST Studio Suite by DS SIMULIA. The dispersion diagram is constructed by extracting the S-parameters of a four-port unit cell and calculating the eigenmode using the associated transfer matrix based on the method discussed in [20]. The results show that various DBEs occur, at frequencies of 2.75 GHz and 3.02 GHz at $kd = \pi$, and 5.33 GHz and 5.59 GHz at $kd = 0$, where all four coalescing wavenumbers are *almost* real and equal to each other. Losses prevent the realization of a mathematically perfect DBE [20], which is evident from the non-vanishing imaginary part of the wavenumbers at the DBE frequencies in Fig. 3.1(c). However the main feature of the four coalescing eigenvectors is still retained as discussed in [20] using the concept of the eigenvector coalescing parameter (also called hyperdistance).

The distributed DBE oscillator is realized by incorporating discrete active components in a cavity made of a finite-length CTL exhibiting a DBE. Each active component is arranged between two adjacent unit cells to balance small distributed losses and compensate for the two load terminations, and hence start the oscillation as shown in Fig. 3.2(a). The high quality factor of such DBE cavities and the concept of DBE resonance has been already explained in [42, 55]. Gain is modeled using the non-linear cubic I-V characteristic $i(t) = -g_m v(t) + \zeta v^3(t)$

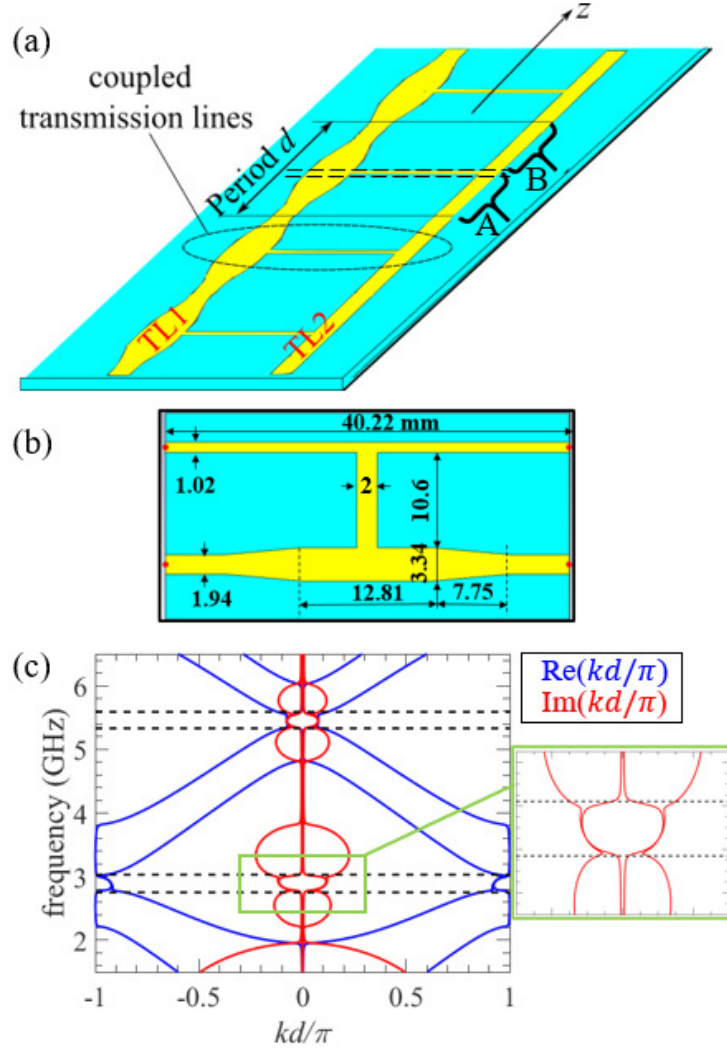


Figure 3.1: (a) Schematic of two coupled microstrip lines on a grounded dielectric substrate that support a fourth order EPD (the DBE) visible in the (k - ω) dispersion diagram. (b) Microstrip unit cell of the periodic waveguide that exhibits the DBE. (c) Real and imaginary parts of the wavenumbers of the four guided Floquet-Bloch modes obtained using the full-wave finite element method accounting for radiation, ohmic and dielectric losses.

of the active device [43] which can be practically implemented with circuits with amplifying devices, such as CMOS transistors or op-amps, with positive feedback. Here $-g_m$ is the small-signal slope of the I-V curve in the negative resistance region, and ζ is the third-order non-linearity constant that models the saturation characteristic of the device. To mimic the clipping mechanism in realistic active devices we choose the turning point (i.e., which is a point in the I-V curve of the active device above which it starts to behave as a passive one; more details are in [43]) $v_b = \sqrt{g_m/3\zeta}$ of the I-V characteristics to be 1 volt, and accordingly, we set $\zeta = g_m/3$. The smaller the ζ value the higher the output swing and the time required to reach steady state.

In general, active components are realized either as a single-ended or differential topology. The most common single-ended realizations are based on two-terminal diodes such as the Gunn diode [76, 77], IMPATT diode [78], and tunnel diode [79]. Such elements are mainly implemented in a compound semiconductor (III-V semiconductor) and used in high power oscillators [76]. On the other hand, a negative differential resistance is most commonly realized in CMOS technology as a cross-coupled transistor pair [66, 72]. As long as the cross-coupled pair is biased by a tail current source, a single-ended version can also be realized, with the other terminal connected to an appropriate dc voltage.

The following calculations and simulations are carried out using the time-domain transient solver implemented in Keysight Advanced Design System (ADS) software by means of scattering parameters obtained through full-wave simulation, where the excitation is modeled by thermal noise generation in the load resistors. The schematic of the proposed distributed oscillator is shown in Fig. 3.2(a) where the active devices are attached to the lower transmission line in between adjacent unit cells toward the ground (the bias line). For simplicity, we assumed that the gain $-g_m$ is equal in all the active devices. In fact, the overall performance of this kind of distributed oscillators can be improved by optimizing the distribution of the gain values of active devices along the finite structure.

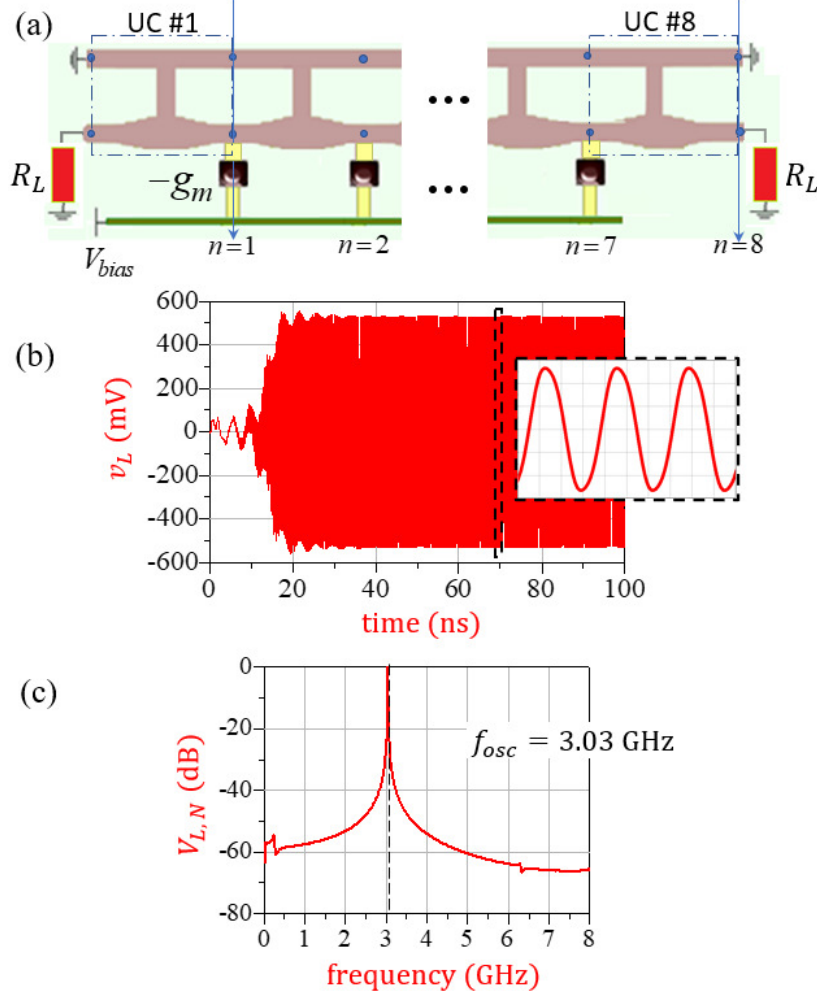


Figure 3.2: (a) Loaded DBE oscillator consisting of 8 cascading unit cells (UCs) of microstrip-based CTLs shown in Fig.1(b). Active devices are placed between each two adjacent unit cells to the bias line. Two loads of 50Ω are attached at the two ends of the lower TL while the upper TL is terminated in short circuits. The oscillation starts for sufficiently large $g_m = 3\text{ mS}$. (b) Voltage waveform $v_L(t)$ monitored at a 50Ω load where steady state oscillation is observed in less than 30 ns. (c) The normalized spectrum $V_{L,N}(f)$ shows that oscillations occur at 3.03 GHz, that is very close to the DBE frequency of 3.02 GHz in Fig. 1(c). The spectrum is calculated by applying the Fourier transform in a time window from 35 to 100 nsec.

To determine the oscillation threshold, which is the minimum value of the gain conductance g_m to start oscillations, we tested the finite-length loaded cavity shown in Fig. 3.2(a) with both ends of the lower TL terminated with $50\ \Omega$ and both ends of the upper TL terminated by short circuits to ground. The oscillation threshold is obtained by sweeping the gain $-g_m$ value until the oscillation starts. Accordingly, we report that the oscillation threshold is $g_{m,th} = 1\ \text{mS}$ for the 8 unit cells oscillator in Fig. 3.2(a). Note that, the oscillation threshold value depends on the length of the finite structure (i.e. number of unit cells) and the load values R_L . Therefore, since we analyze the effect of load variation, for the rest of the chapter we choose $g_m = 3\ \text{mS}$ to be sufficiently larger than the oscillation threshold $g_{m,th}$ for a large value of $R_L = 1\ \text{M}\Omega$. The waveform $v_L(t)$ at either load reaches a steady state in less than 30 ns as shown in Fig. 3.2(b). The oscillation frequency is determined by Fourier transforming $v_L(t)$ in the time window from 35 to 100 ns, shown in Fig. 3.2(c), and it clearly confirms the single-frequency oscillatory behavior despite the length of the cavity and the presence of seven active devices.

Fig. 3.3 shows the magnitude of voltage and current distributions in the loaded CTL cavity of Fig. 3.2(a). These voltages and currents are evaluated at nodes $n = 1, 2, \dots, 8$, in the presence of the active devices with $g_m = 3\ \text{mS}$. It can be observed that the voltage reaches its peak magnitude in the middle of the cavity; the voltage magnitudes in the lower TL are approximately four times larger than those in the upper TL.

3.4 Frequency Stability of the Distributed DBE Oscillator

An important advantage of the proposed DBE oscillator is the robustness of the oscillation frequency against a large variation in the load. This robustness is directly related to

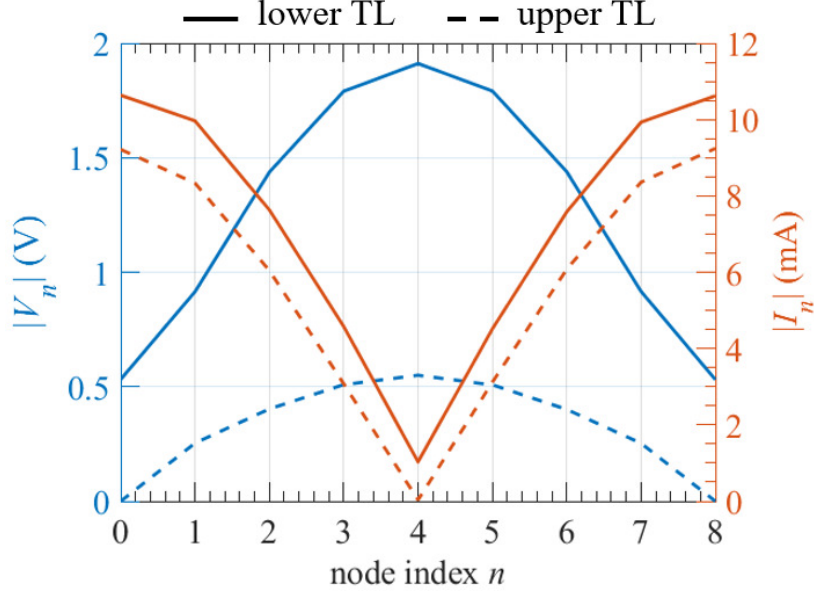


Figure 3.3: (a) Voltage and current distributions on top and bottom TLs of the distributed DBE oscillator shown in Fig. 2(a), when the bottom TL is terminated in two 50Ω loads, while the top TL is short circuited at its two ends. ($|V_n|$ and $|I_n|$ represent voltage and current time-harmonic waveform peak values, which are phasors' amplitudes, at steady state regime).

the strong DBE resonance associated to the cavity made of a finite-length CTL exhibiting a DBE [1, 30, 43]. This advantage has been shown only for a DBE-based double ladder lumped-element circuit oscillator with only one active device [43, 80]. Typically, the oscillation behavior is very sensitive to the output termination resistance variation resulting in a significant shift in oscillation frequency (e.g. mode jumping in ladder oscillators [69, 44]); in some cases the oscillation stops. Fig. 3.4 shows the effect of varying the load resistance on the oscillation frequency and on the average output power in the proposed distributed DBE oscillator, for $g_m = 3\text{ mS}$. The result shows a stable frequency of oscillation with a change of only 1% over a change of load resistance over 7 orders of magnitude. The same plot also shows the total output power on both loads as a function of the load resistance, where the maximum output power corresponds to $R_L = 150\Omega$. Note that, the distributed DBE oscillator also shows a stable frequency of oscillation when changing the gain as long as it exceeds the threshold to start the oscillation. This is shown by plotting again the output power and frequency of oscillation for the smaller conductance of $g_m = 1.5\text{ mS}$. For $g_m = 1.5\text{ mS}$,

the maximum output power occurs when $R_L = 50\ \Omega$ and the oscillation stops when the loads $R_L \geq 100\ \Omega$ (as $1.5\ \text{mS} < g_{m,th}|_{R_L=100\ \Omega}$). Moreover, the robustness of the oscillation frequency against a wide variation in the gain is shown in Fig. 5(a), by fixing the two load resistances to $R_L = 50\ \Omega$ and varying the gain conductance from $g_{m,th} < g_m < 9\ g_{m,th}$. It is clear from the red curve in Fig. 5(a) that increasing the gain in the proposed oscillator has a negligible effect on the oscillation frequency (i.e. less than 1% over the studied range while for $g_m > 9\ g_{m,th}$ the oscillation frequency jumps to a higher frequency at 7.6 GHz), whereas the output power (blue curve) increases by increasing the gain conductance. The stability analysis was repeated using a single load resistance, with the other end short circuited, which leads to very similar results and an even high stability of the oscillation frequency over load variation (not reported here for the sake of brevity).

To better understand the robustness of the oscillator to load variations, we add a capacitance C in parallel to each of the two load resistors $R_L = 50\ \Omega$ shown in Fig. 3.2(a) to imitate the effect of a parasitic capacitance. Fig. 3.5(b) shows the effect of varying such load capacitance on the oscillation frequency and on the average output power, for $g_m = 1.5\ g_{m,th} = 1.5\ \text{mS}$. The variation of the oscillations frequency is negligible for all shown C values.

3.5 Conclusion

It has been shown that the DBE in coupled periodic waveguides is useful to conceive new schemes for arrays of coupled oscillators. The DBE structure in the cavity made of a periodic waveguide strongly synchronizes a discrete set of oscillators resulting in an overall single mode of oscillation. The single oscillation frequency, in close proximity to the DBE frequency, has been theoretically demonstrated through full-wave transient simulations. Results demonstrate the stability of the oscillation frequency over a very wide range of load or gain variation, to confirm the stable single-frequency oscillation scheme dictated by the

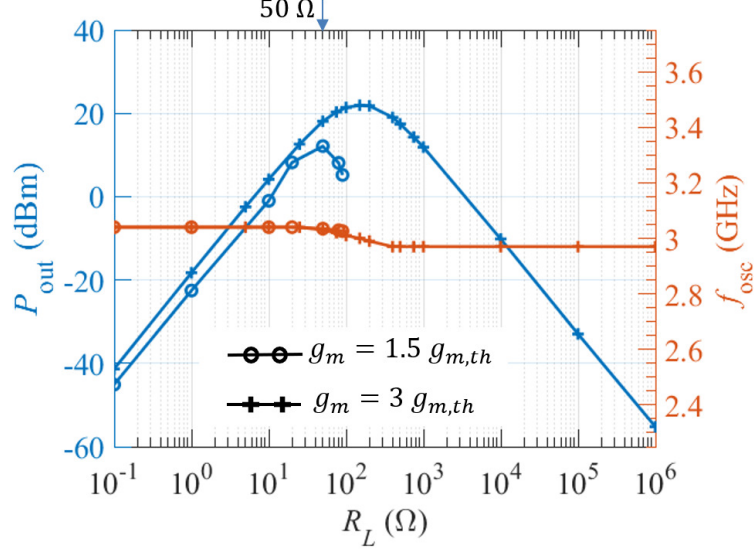


Figure 3.4: Average output power and oscillation frequency versus load resistance for the distributed DBE oscillator shown in Fig. 2(a), for $g_m = 3$ mS and $g_m = 1.5$ mS. The stability of the oscillation frequency over a huge variation of the load resistance shows an important advantage of the proposed oscillator: the frequency of oscillation is almost the same, i.e. ~ 3 GHz, with a very slight shift that does not exceed $\pm 1\%$ (~ 30 MHz).

modal degeneracy.

This new scheme of operation is promising for boosting the overall performance of RF sources, where potential benefits include spectral purity and high power efficiency. This scheme based on the DBE concept would be valuable in devices that require several coherent sources, in power combining, with a possible extension to distributed radiating active antenna arrays at microwaves and millimeter waves.

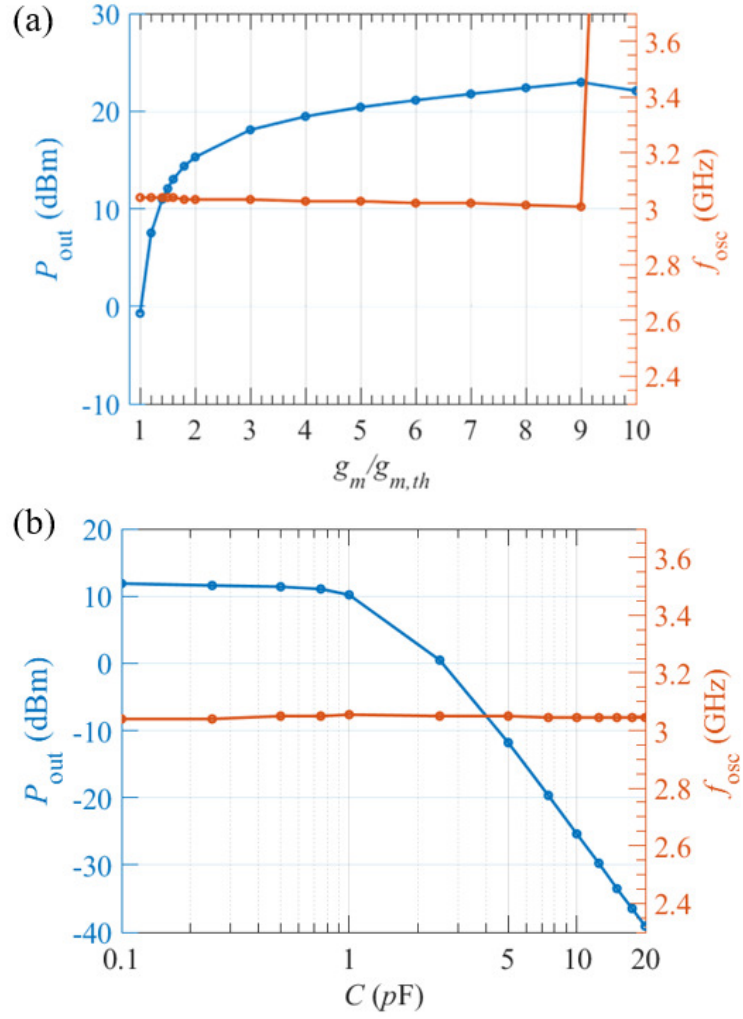


Figure 3.5: Average output power and oscillation frequency versus (a) gain variation, and (b) capacitive loads variation. In (b) the capacitor C is parallel to each of the two load resistors $R_L = 50 \Omega$, assuming $g_m = 1.5 \text{ mS}$.

Chapter 4

Exceptional Degeneracy in a Waveguide Periodically Loaded with Discrete Gain and Radiation Loss Elements

4.1 Motivation

We demonstrate that a periodic waveguide comprising of uniform lossless segments together with discrete gain and radiating elements supports exceptional points of degeneracy (EPDs). We provide analytical expressions for all possible conditions that guarantee the occurrence of an EPD, i.e., the coalescence of eigenvalues and eigenvectors. We show that EPDs are not only achieved using symmetric gain and radiation periodic loading, but they are also obtained using asymmetric gain and radiation loss conditions. We illustrate the characteristics of the degenerate electromagnetic modes, showing the dispersion diagram and discussing the

tunability of the EPD frequency. We show a special condition, we refer to it as parity-time (PT)-glide symmetry, which leads to a degeneracy that is occurring at all frequencies of operation. The class of EPDs proposed in this work is very promising for many applications that incorporate discrete-distributed coherent sources and radiation-loss elements; operating in the vicinity of such special degeneracy conditions leads to potential performance enhancement in a variety of microwave and optical resonators, antennas, and devices and can be extended to a new class of active integrated antenna arrays and radiating laser arrays.

Electromagnetic (EM) guiding structures or resonators are described by their eigenmodes' (eigenvalues and eigenvectors) evolution equations. Eigenmodes representing EM propagating waves in a multimodal waveguide may coalesce into a single degenerate eigenmode by varying at least one parameter of the parameter space (frequency, geometrical/physical parameters) of the waveguide system; this special point in the system parameter space is called an exceptional point of degeneracy (EPD) [53, 81]. At the EPD, two or more eigenstates of the system coalesce into a single degenerate eigenstate. Such condition is simply referred to as 'EP' in various works; here the 'D' is used to stress the importance of degeneracy [11]. The number of degenerated eigenstates is referred to as the order of the exceptional point. In the proximity of an EPD angular frequency ω , the eigenvalues λ associated with the coalescing eigenvectors change with respect to frequency as $(\omega - \omega_e) \propto (\lambda - \lambda_e)^n$, in which λ_e, ω_e , and n are the degenerate eigenvalue, EPD angular frequency, and order of EPD, respectively.

In general, an EPD occurs in a system where the space-time evolution of the system vector is characterized by a non-Hermitian matrix, which can be imposed also by periodicity in space [1, 41, 2, 20] or in time [21, 82] or by having losses and gain in the system [45, 20] including systems satisfying parity-time (PT) symmetry [81, 7, 8, 83]. The unique degenerate dispersion behavior is accompanied by supreme characteristics including the vanishing of the group velocity [4, 57] as well as the dramatic improvement in the local density of states [30] resulting in a robust increase in the loaded quality factor of the structure. The

EPD phenomenon has been proved to have various applications including high quality factor (Q) and low-threshold lasers [36], lasers in coupled ring resonators[84], and low-threshold oscillators [42, 3, 85]. Moreover, the deviation of the perturbed eigenvalues from the degenerate eigenvalue is large when a small perturbation to a system parameter is applied; so this sensitivity brings another class of applications in sensors [86, 87, 88, 89].

4.2 System Description and Analytical Formulation

In this chapter, we present an example of a waveguide that exhibits a second-order EPD by periodically loading a uniform waveguide with gain and radiating elements, as schematized in Fig. 4.1. We provide the analytical expressions for the second-order EPD conditions to occur in different loading configurations for the gain and radiating elements. The EPD condition is observed in the dispersion diagram and by the coalescence of the eigenvectors. We also describe the Floquet-Bloch impedance in the vicinity of the EPD, which can be important for matching and stability analysis. We conclude by showing a possible application as an array of radiation elements oscillating and radiating at the EPD frequency.

We consider a uniform waveguide that is periodically loaded with discrete gain and loss and that is schematically represented by its equivalent transmission line model (TL) [90, 91]; this model can be applied to waveguides operating from microwaves to optics, hence our formulation is general. We assume that the waveguide is periodically loaded with discrete shunt gain and resistive elements, as shown in Fig. 4.1(a). Indeed, it is customary to represent radiation from discrete points along a waveguide using resistive loads.

The periodic unit cell is divided into four parts: two uniform waveguide segments together with a discrete gain element and a discrete radiative element represented by its equivalent resistance. For simplicity the waveguide segments are assumed to have similar characteristic

impedance, but with possibly different electrical lengths $\theta_A = k_0 l_A$, and $\theta_B = k_0 l_B$ where l_A and l_B are the physical lengths of the waveguide segments A and B, respectively, and $k_0 = \omega/v_{ph}$ is the waveguide propagation constant, with v_{ph} being the phase velocity of a uniform-waveguide mode. It is convenient to define a system state vector as $\mathbf{\Psi}(z) = [V(z), I(z)]^T$, with T indicating the transpose action. Therefore, referring to Fig. 4.1(a), we use the transfer matrix of a shunt element $\mathbf{T}_{shunt}$ and lossless transmission line $\mathbf{T}_{TL}$ [62], and we form a relation between equivalent voltage and current between the two ends of a unit cell as $\mathbf{\Psi}_{n+1} = \mathbf{T}_U \mathbf{\Psi}_n$. The unit cell transfer matrix $\mathbf{T}_U$ is the result of the multiplication of four transfer matrices as

$$\mathbf{T}_U = \mathbf{T}_{shunt}(Y_r) \mathbf{T}_{TL}(\theta_B) \mathbf{T}_{shunt}(-g) \mathbf{T}_{TL}(\theta_A). \quad (4.1)$$

We look for solutions of the type $\mathbf{\Psi}_n \propto \mathbf{\Psi}_0 e^{-jkn d}$ satisfying the Floquet's condition $\mathbf{\Psi}_{n+1} = e^{-jkd} \mathbf{\Psi}_n$, where d is the waveguide period, k is the Floquet-Bloch wavenumber, and we implicitly assume the time convention $e^{j\omega t}$. Hence, the eigenmodes supported in such a system are described by the eigenvalue problem

$$[\mathbf{T}_U - \lambda \mathbf{I}] \mathbf{\Psi} = \mathbf{0}, \quad (4.2)$$

where $\mathbf{I}$ is the identity matrix of order two, $\lambda = e^{-jkd}$ is an eigenvalue and $\mathbf{\Psi}$ is the associated eigenvector. The eigenvalues are readily found by solving the characteristic equation $\det(\mathbf{T}_U - \lambda \mathbf{I}) = 0$, i.e., by finding the roots of the characteristic polynomial

$$\lambda^2 + \left[-2 \cos(\theta_A + \theta_B) - g Y_r Z_0^2 \sin(\theta_A) \sin(\theta_B) \right. \\ \left. - j Z_0 Y_r (1 - g/Y_r) \sin(\theta_A + \theta_B) \right] \lambda + 1 = 0, \quad (4.3)$$

where Z_0 is the characteristic impedance of the two uniform waveguide segments. To have

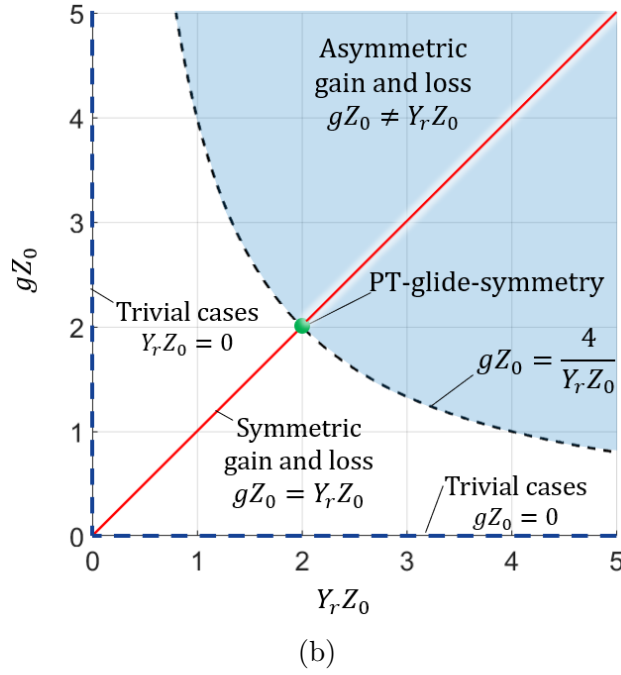
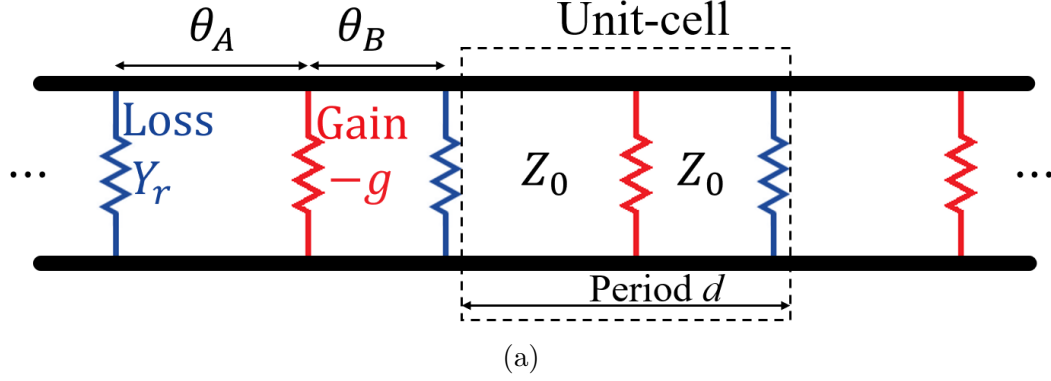


Figure 4.1: (a) Unit cell schematic of a periodic waveguide, represented by its equivalent transmission line (TL), made of two segments with characteristic impedance Z_0 and loaded with shunt lossy element (Y_r) and shunt gain element ($-g$). (b) The relation between gain and loss to have an EPD. The shaded area represents the asymmetric cases where gain and loss relation to have an EPD is $gZ_0 = 4/(Y_r Z_0 \sin^2(\theta_A))$. The black-dashed curve represents one of the asymmetric cases when $\theta_A = \theta_B = (2m+1)\pi/2$, also the PT-glide-symmetry case is depicted by the green dot in the intersection between the asymmetric case dashed curve and red line representing symmetric cases.

two identical roots in a second-order polynomial of the form of $\lambda^2 + a\lambda + b = 0$, $a^2 - 4b$ must vanish. Having $b = 1$ in the proposed system characteristic polynomial (4.3) indicates that the eigenvalues are $\lambda_1 = 1/\lambda_2 = e^{-jk_1d}$, which implies that $k_1 = -k_2$. Also, the necessary and sufficient conditions for having identical eigenvalues λ is $a = (-1)^p 2$; where p is an integer number indicating the positive and negative possible solutions of a . Therefore conditions that must be satisfied at the EPD frequency, related to the real and imaginary parts of a , read as

$$-2 \cos(\theta_A + \theta_B) - gY_r Z_0^2 \sin(\theta_A) \sin(\theta_B) = (-1)^p 2 \quad (4.4)$$

$$Z_0 Y_r (1 - g/Y_r) \sin(\theta_A + \theta_B) = 0. \quad (4.5)$$

The second condition, (4.5), is satisfied either by constraining the gain and radiation-element equivalent resistance (i.e., $1 - g/Y_r = 0$) or by constraining the TL segments' electrical lengths (i.e., $\sin(\theta_A + \theta_B) = 0$). Whereas, the first condition in (4.4) is used as the design equation for different possible cases that are leading to identical eigenvalues. The chart in Fig. 4.1(b) summarizes the required relation between gain and radiation loss to have an EPD, that are discussed next.

4.2.1 Trivial cases with vanishing gain or loss ($Y_r = 0$ or $g = 0$)

A trivial condition to satisfy EPD is by having $\theta_A + \theta_B = p\pi$, where p is an integer, besides having either $g = 0$ or $Y_r = 0$, represented by the blue-dashed vertical or horizontal line,

respectively, in the chart in Fig. 4.1(b). The EPD obtained for this case occurs at $k = \pi/d$, where d is the unit cell period. However, we do not focus on this trivial case as it is not suitable for applications that incorporate *both* discrete-distributed coherent sources and radiation loss elements.

4.2.2 Symmetric gain and loss cases ($Y_r = g$)

One possibility to satisfy the condition in (4.5), $\text{Im}(a) = 0$, is by enforcing balanced gain and radiation-loss, $g = Y_r$ represented by the solid-red line in the chart in Fig. 4.1(b). The normalized gain and radiation elements values that satisfy the other EPD condition (4.4) $\text{Re}(a) = 2(-1)^p$, yields

$$Y_r Z_0 = g Z_0 = \sqrt{\frac{2((-1)^p - \cos(\theta_A + \theta_B))}{\sin(\theta_A) \sin(\theta_B)}}. \quad (4.6)$$

It is important to mention that for any arbitrary choice of θ_A and θ_B , the term inside the root in (4.6) can always have a positive value by choosing the proper p value. The symmetrical gain and radiation-loss is a straightforward condition that leads to EPD where the introduced amount of radiation loss should be compensated with the same amount of gain in order to have neither a decaying nor a growing wave. Although, in this case, gain and loss loads are equal, the unit cell with $\theta_A \neq \theta_B$ does not classify as (PT)-symmetric condition, that could be defined based on the system's refractive index obeying $n(z) = n^*(-z)$, where z is a coordinate in the system and $*$ denotes complex conjugation [13, 7, 81]. Enforcing spatial symmetry in the unit cell by choosing equal electrical lengths $\theta_A = \theta_B$ leads to a unit cell that satisfies a possible definition of PT-symmetry as used in [92]. Indeed, the symmetric load case with $\theta_A = \theta_B$ satisfies the following

$$n(z + \frac{d}{2}) = n^*(z), \quad (4.7)$$

which holds the reflection between gain and loss by the complex conjugate operator $*$ and the translation along z by half a period. We define the PT-glide-symmetry condition as in (4.7) which can be used also to describe more complicated structures. In general, glide symmetry is a symmetry operation comprised of a reflection operation over a certain coordinate and translation along with that coordinate [93, 94, 95]. We refer to the condition in (4.7) as PT-glide-symmetry where the EPD condition is met at every frequency with $\lambda_e = e^{-jk_e d} = 1$; this condition is only valid for the ideal case discussed here where the gain and loss elements are assumed to be purely real valued and frequency-independent. In reality, active sources and radiation loss elements are dispersive (i.e., frequency-dependent), implying that in this case the EPD condition also has some frequency dependency. Note that for the reciprocal system we are studying, the EPD is only possible when k_e is purely real with the value of either $k_e d = 0$ or $k_e d = \pi$.

Figure 4.2 depicts the dispersion of a waveguide with symmetric loads and different electrical lengths. The waveguide exhibits EPD by satisfying the EPD condition in (4.6) at 3 GHz such that $3\theta_A = \theta_B = \pi/4$, $Z_0 = 50 \Omega$, and $Y_r = g = 2\sqrt{2}/Z_0 = \sqrt{2}/25 \text{ S}$.

In the rest of the chapter, we focus on the case with asymmetric gain and radiation-loss as it provides more flexibility in using different values of gain and radiation-loss. Indeed, the value of resistance of the radiating element cannot be set arbitrarily and the constraints depend on the specific design, while gain usually can be tuned by simply changing a biasing voltage.

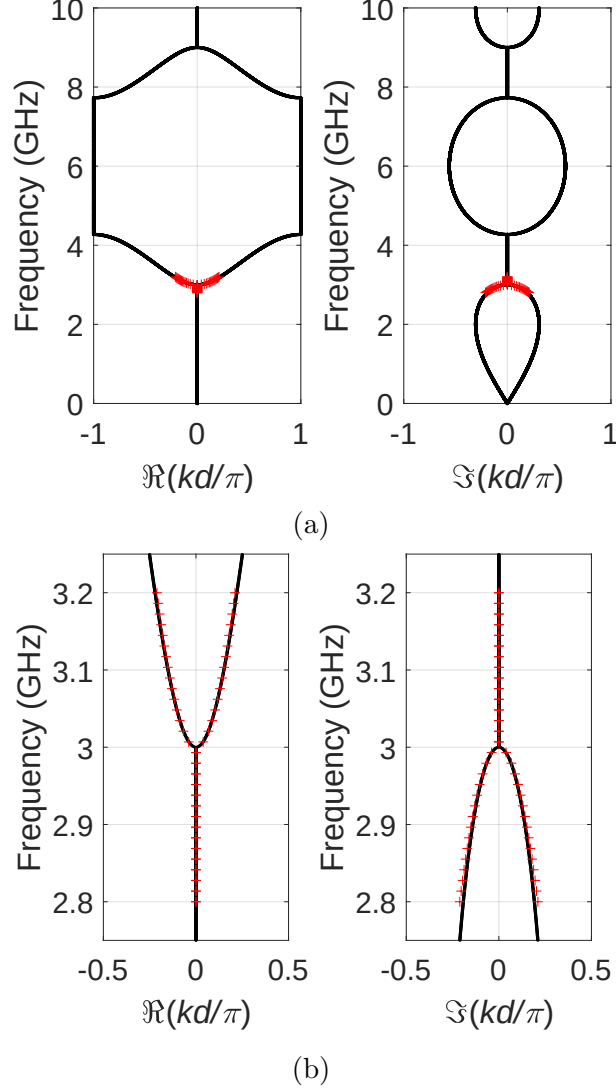


Figure 4.2: (a) Dispersion diagram of complex-valued wavenumber versus frequency for gain-radiation loss symmetric case, $Y_r = g = 2\sqrt{2}/Z_0 = \sqrt{2}/25$ S, with $3\theta_A = \theta_B = 3\pi/4$ at 3 GHz. Wavenumber degeneracies are observed at 3 GHz, 4.333 GHz, 7.715 GHz, 9 GHz, etc., where either $k = 0$ or $kd/\pi = \pm 1$. (b) A zoom-in version in the vicinity of the EPD at $(\omega_e, k_e) = (3 \text{ GHz}, 0)$ shows that the complex-valued dispersion fits to the quadratic formula $(\omega - \omega_e) = h(k - k_e)^2$ denoted by red symbols with $h = 1.1162 \times 10^{14} \text{ m}^2/\text{sec}$.

4.2.3 Asymmetric gain and loss cases ($Y_r \neq g$)

Condition (4.5) can be satisfied also for asymmetric gain and radiation-loss cases, represented by the shaded area in the chart in Fig. 4.1(b), by constraining the waveguide segments' electrical lengths as

$$\theta_A + \theta_B = p\pi, \quad (4.8)$$

where p is an integer number, in other words, the total length of the waveguide of a unit cell at f_e is an integer multiple of half wavelength. The other condition (4.4) forces the relation between the normalized gain and radiation-loss values to be

$$(gZ_0) = \frac{4}{(Y_r Z_0) \sin^2(\theta_A)}. \quad (4.9)$$

By forcing these two conditions (4.8,4.9), the degenerate eigenvalue of the eigenvalue problem in (4.2) is equal to

$$\lambda_e = e^{-jk_e d} = \begin{cases} (-1)^{p+1}, & \text{if } \theta_A \neq \theta_B \neq l\pi \\ (-1)^p, & \text{otherwise} \end{cases} \quad (4.10)$$

where l is an integer such that $0 \prec l \prec m$, and the degenerate eigenvector is $\Psi_e = I \begin{bmatrix} Z_{B,e}, & 1 \end{bmatrix}^T$, where

$$Z_{B,e} = -2Z_0 / (Y_r Z_0 + j2 \cot \theta_A), \quad (4.11)$$

is the Bloch impedance of the degenerate mode. Figure 4.3(a) depicts the dispersion of one waveguide that exhibits EPD by satisfying the conditions at 3 GHz such that $\theta_A =$

$\theta_B = \pi/2$, $Y_r = 20 \text{ mS}$, $Z_0 = 50 \Omega$, and $g = 80 \text{ mS}$ to satisfy the EPD condition in (4.9). The two complex wavenumbers are traced in two different colors such that one can observe the coalescence of the two complex wavenumbers at the EPD frequency and its harmonics (i.e. all meet the EPD conditions). Note that the EPD points are the transition points at which the complex wavenumbers alternate between the same sign for both parties of the complex wavenumber (i.e. real and imaginary parts) indicating growing waves to opposite signs indicating decaying waves.

Upon analyzing the modal dispersion equation, it can be proved that when the special case of $Y_r Z_0 = 2$, accordingly, $g = Y_r$ (i.e., symmetric case) and $\theta_A = \theta_B$ are met utilizing the aforementioned PT-glide-symmetry case, then the two eigenvalues (and also the eigenvectors) will be identical at every frequency. In Fig. 4.3(a) we show an example of the dispersion when $\theta_A = \theta_B$ when $Y_r Z_0 < 2$, that is true when using the aforementioned parameters, whereas in Fig. 4.3(b) we show an analogous example that exhibits EPD at the same frequency when the condition reads as $Y_r Z_0 > 2$ by selecting $Y_r = 50 \text{ mS}$ and $g = 32 \text{ mS}$ for the same $Z_0 = 50 \Omega$.

The periodic electromagnetic guiding structure is characterized by the modal dispersion equations (4.3). Each eigenmode is characterized by its eigenvalue $\lambda_i = e^{-jk_i d}$ related to the associated complex Floquet wavenumbers k_i and its eigenvector $\Psi_i = I_i \begin{bmatrix} Z_{B,i}, & 1 \end{bmatrix}^T$, with $i = 1, 2$ for the case under study here, where $Z_{B,i}$ is the i^{th} mode Floquet-Bloch impedance.

The evolution of the eigenmodes' complex Floquet-Bloch impedance varying frequency, which is directly related to the evolution of the eigenvectors Ψ_i , is shown next. The coalescence of the eigenvectors at the EPD is based on having a degenerate Floquet-Bloch impedance (i.e., $Z_{B,e} = Z_{B,1} = Z_{B,2}$). Figure 4.4 shows the trajectory of the complex Bloch impedance $Z_{B,i}$ for increasing frequency for two different cases: (a) $Y_r Z_0 < 2$ depicted in Fig. 4.4(a), and (b) $Y_r Z_0 > 2$ depicted in Fig. 4.4(b), associated with dispersion diagrams shown in Fig. 4.3(a,b), respectively. It is obvious from the traces shown in Fig. 4.4(a,b) that in general,

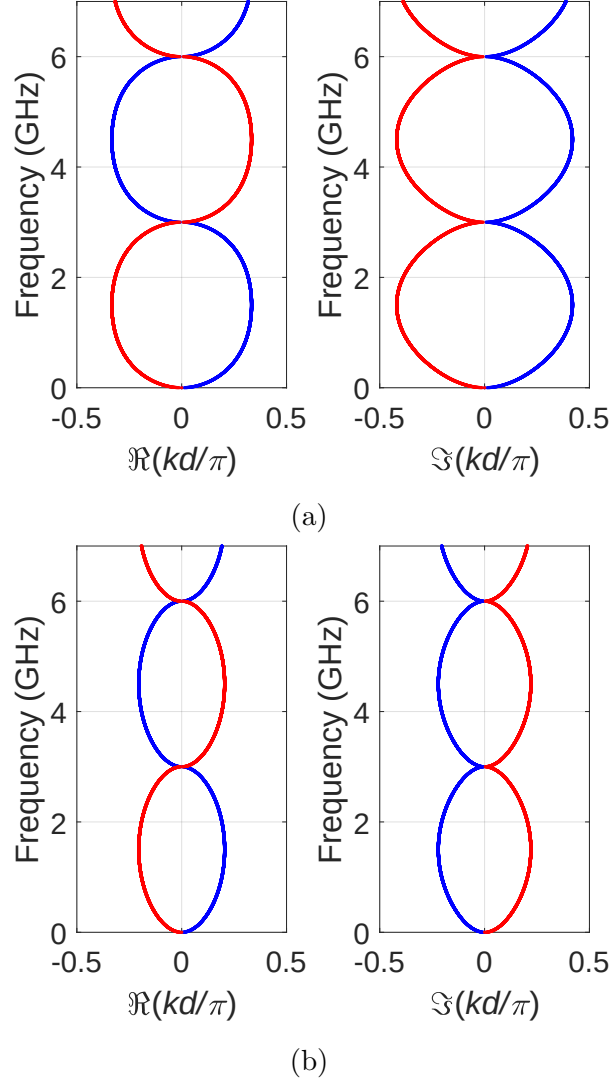


Figure 4.3: Dispersion diagram of complex-valued wavenumber versus frequency. Wavenumber degeneracies are observed at 3 GHz, 6 GHz, etc. where both wavenumbers vanish. The two wavenumbers are denoted by different colors, for the two different cases with $Z_0 = 50 \Omega$ and $\theta_A = \theta_B = \pi/2$ at 3 GHz: (a) $Y_r = 20 \text{ mS}$, $g = 80 \text{ mS}$ corresponding to $Y_r Z_0 < 2$, and (b) $Y_r = 50 \text{ mS}$, $g = 32 \text{ mS}$ corresponding to $Y_r Z_0 > 2$

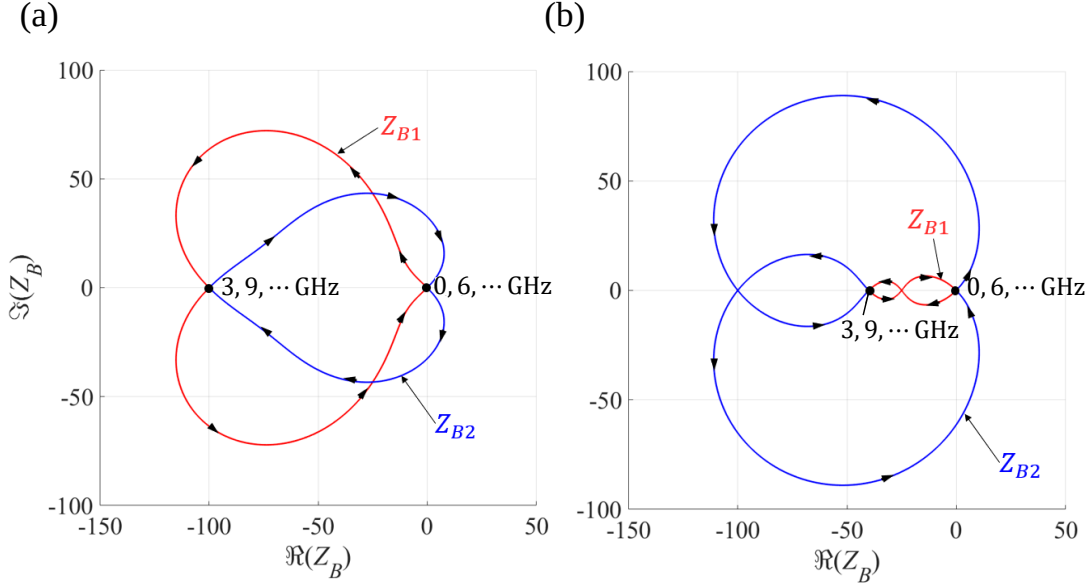


Figure 4.4: Complex-valued Bloch impedances Z_B showing the trajectory of Z_B evolution varying frequency where arrows represent the direction of frequency increasing. Degeneracies are observed at 3 GHz, 6 GHz, etc. where both wavenumbers vanish, i.e. $k = 0$. The two modes' Bloch impedance are denoted by different colors, matching different modes' colors in the dispersion diagram in Fig. 2 (a, b), for the two different cases with $\theta_A = \theta_B = \pi/2$ at 3 GHz: (a) $Y_r = 20$ mS corresponding to $Y_r Z_0 < 2$, and (b) $Y_r = 50$ mS corresponding to $Y_r Z_0 > 2$.

the Bloch impedances are complex conjugate to each other over the whole frequency range except at the EPD frequency 3 GHz and its harmonics 6, 9, ... GHz where they become purely real. At the EPD, the two impedances turn into one degenerate real impedance $Z_{B,e}$ either $-2/Y_r$ or zero.

4.3 EPD Oscillator

We describe succinctly some possible applications that incorporate discrete-distributed coherent sources and radiation elements (that are usually characterized by loss lumped elements, like the admittances Y_r). One application to the proposed EPD scheme is an active radiating oscillator that requires the incorporation of discrete-distributed coherent sources

and radiation loss elements. This active oscillator is realized in a cavity made of a finite-length waveguide exhibiting EPD with asymmetric gain and loss. As proof of the concept, and regardless of the specific implementation, the radiating elements are simply modeled as a distributed shunt radiation loss, whereas gain is modeled in each unit cell using non-linear cubic i - v characteristic $i(t) = -gv(t) + \zeta v^3(t)$ of the active device [43, 85] which can be practically implemented with circuits with amplifying devices, such as CMOS transistors or Op-Amps, with positive feedback. Here $-g$ is the small-signal slope of the i - v curve in the negative resistance region, and ζ is the third-order non-linearity constant that models the saturation characteristic of the device. We set the turning point $v_b = \sqrt{g/(3\zeta)}$ of the i - v characteristics to be 1 volt, and accordingly, we set $\zeta = g/3$.

We tested the finite-length loaded cavity comprised of 8 unit cells as shown in Fig. 4.5(a) in the time domain solver implemented in Cadence Virtuoso IC 616. The unit cell is chosen to have identical ideal TL segments with $Z_0 = 50\Omega$ and each has an electric length $\theta(3\text{GHz}) = \pi/2$. The gain and loss elements are chosen as $g = 32\text{ mS}$, $Y_r = 50\text{ mS}$ to satisfy the EPD condition in (4.9). Accordingly, we report that the oscillation occurs close to the EPD frequency and the waveform $v_m(t)$, at the load Y_r in the middle of the structure between the fourth and the fifth unit cell reaches a steady-state in less than 2 ns as shown in Fig. 4.5(b). The oscillation frequency is determined by taking the Fourier transform of $v_m(t)$ in the time window from 2 to 100 ns, shown in Fig. 4.5(c), and it confirms the oscillatory behavior around the EPD frequency 3 GHz and its odd harmonics (9, 15, ...) GHz since they all satisfy EPD conditions. Note that, operating in the vicinity of the EPD enhances the sensitivity of the system [86, 87, 89] which can be an effective way of controlling the directivity and beam angle of an antenna array or leaky-wave antenna, analogously to what was shown in [45] for EPD in a uniform (i.e., not periodic) coupled TL with balanced gain and radiation loss.

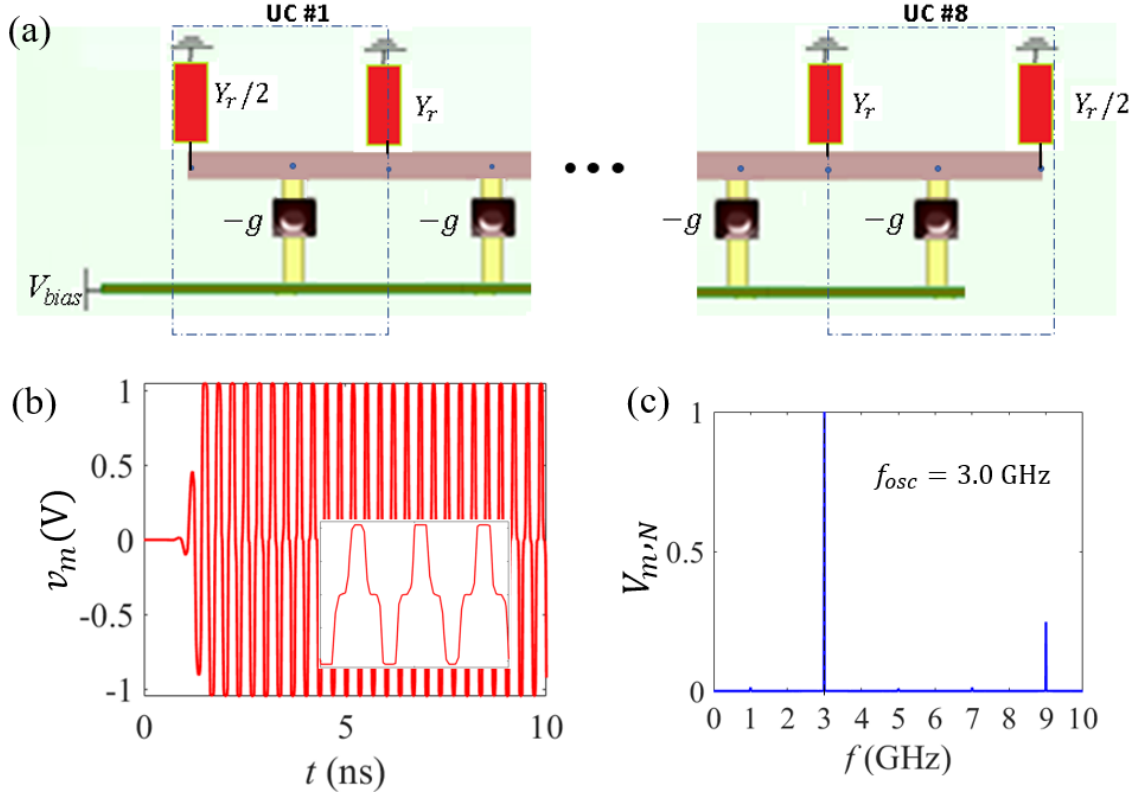


Figure 4.5: EPD oscillator consisting of 8 cascaded unit cells (UCs) loaded with gain and loss (representing a radiating antenna) as shown in Fig. 4.1(a). Active gain devices are placed in each UC from the TL to the bias line (that acts as a ground for a.c. signals). (b) Voltage waveform $v_m(t)$ monitored at the Y_r load in the middle of the structure where steady-state oscillation is observed in less than 2 ns. (c) Normalized voltage spectrum $V_{m,N}(f)$ shows that oscillations occur at around 3 GHz, that corresponds to the EPD frequency of 3 GHz in Fig. 4.3(b).

4.4 Conclusion

We have demonstrated that a periodic waveguide loaded with gain and radiating elements as shown in Fig. 4.1(a) exhibits EPDs. We have shown the different conditions for having EPDs summarized in Fig. 4.1(b) and also, importantly, we have demonstrated a case where the EPD condition is met at every frequency satisfying the PT-glide-symmetry condition. The theoretical framework developed applies to various structures operating from microwave to optical frequencies. The discrete radiation admittances considered in this chapter represent the input admittances of a periodic array of antennas. We have shown that EPDs occur at frequencies where the two TL wavenumbers vanish, leading to possible applications of broadside radiation in arrays of antennas periodically connected to the waveguide. Such a phenomenon may pave the way to a new class of active traveling-wave antennas and also in array antennas with all elements oscillating and synchronized.

Chapter 5

Degenerate Band Edge Slow Wave Structures for Electron Beam Driven Devices

5.1 Motivation

We present new slow wave structures (SWSs) for high-power electron-beam-driven oscillators based on the degenerate band edge (DBE). The proposed DBE operational regime is based on four degenerate eigenmodes all synchronized with the electron beam, and leads to an effective energy transfer from the electron beam to the four degenerate eigenmodes. This paves the way for a new class of high-power oscillators, which we have called degenerate band edge oscillators (DBEOs) that show an unconventional trend of the starting current and feature single frequency of oscillation with high spectral purity.

High power microwave generation using relativistic electron-beam driven vacuum devices is considered the sole technology for generating Mega- and Giga-watts of power used in radars,

satellite communications, and various other applications [96, 97, 98]. We are investigating a mean to improve the power efficiency generation utilizing EPD concept by proposing DBE slow wave structures (SWSs). In this chapter we propose different SWSs that offers a good interaction impedance besides its ability to exhibit higher order dispersion. The proposed BWOs is composed of SWSs exhibiting the fourth order degeneracy at which four guided radio-frequency modes interact with an electron beam leading to high power microwave generation. We show an example of the realization of such SWS which exhibits the DBE based on a double helix SWS. Another realization of such SWS is based on the coupled cavity metamaterial (MTM) SWSs. MTM SWS is composed of a circular waveguide with periodic loading of two complementary electric split-ring resonator (CeSRR) disks. A BWO using the proposed MTM SWS has been simulated using a particle-in-cell(PIC) solver and experimentally tested with University of New Mexico (UNM) group to evaluate its performance. Finally we show a realization of DBE-based SWSs for millimeter(mm) and Terahertz(THz) -waves power generation in periodic double corrugated waveguide. We show a possibility of high power oscillators at this range of frequencies, in particular, we demonstrate a degenerate band edge oscillator (DBEO) at 638 GHz based on PIC simulations.

5.2 Four-eigenmode Super Synchronization Regime at the DBE

The DBE occurs when four electromagnetic eigenmodes in a SWS coalesce into a single degenerate eigenmode, when varying frequency or any structural parameter of the waveguide [3, 4]. The DBE occurs in lossless periodic waveguides at a specific angular frequency and wavenumber denoted by ω_d and k_d , respectively (Fig. 5.1(b)). The DBE is also associated with a vanishing group velocity [3, 4]. The super synchronization operational principle for SWSs with DBE interacting with an electron beam is demonstrated in [3, 42]. In [42] a low

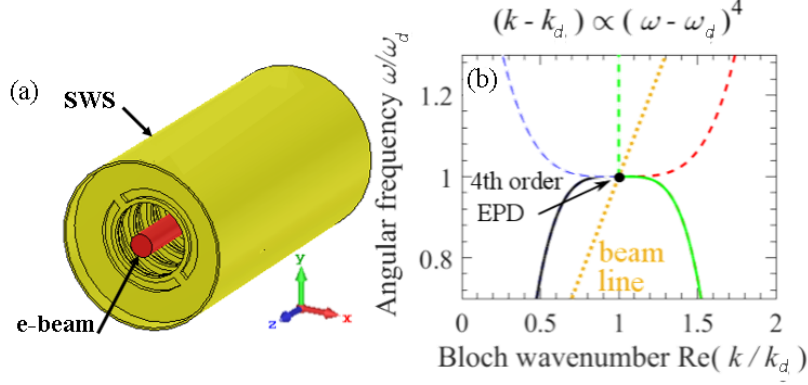


Figure 5.1: Example of high-power electron beam driven device composed of a SWS that supports four degenerate eigenmodes. This degenerate condition is shown in the modal dispersion diagram of the “cold” periodic SWS, where four Floquet-Bloch eigenmodes coalesce at the single point (k_d, ω_d) that occurs at the band edge. Super synchronization is achieved when the electron beam velocity u_0 is matched to the phase velocity of the four modes at (k_d, ω_d) , leading to the super synchronization condition in Eqn. 5.1. Note that $k_d = \pi/d$ where d is the period of the SWS. Eigenmodes with purely real wavenumber are denoted by solid lines whereas complex wavenumbers are denoted by dashed lines.

starting current oscillator was demonstrated using the generalized Pierce method [46, 99].

The DBE super synchronization regime mandates that the phase velocity of the four degenerate eigenmodes matches the average velocity of the electron beam u_0 , leading to the super synchronism condition

$$u_0 \approx \omega_d/k_d \quad (5.1)$$

shown graphically in Fig. 5.1(b), represented by the intersection at (k_d, ω_d) of the “cold” SWS dispersion in the fundamental Brillouin zone with the beam line (dotted yellow). Complex dispersions of “hot” SWSs accounting for the four modes-beam interaction are shown in [46, 3]. One of the consequences of having such a fourth order super synchronization condition in hot SWSs is the low oscillation threshold, i.e., the low starting beam current I_{st} which is defined as the minimum value of the beam current required to initiate oscillations. The estimation of this threshold current versus SWS length Nd , where N is the number of unit

cells of the periodic SWS, is carried out using the generalized Pierce method [46, 99, 3] and leads to the unconventional trend for large number of unit cells given by

$$I_{st} \propto N^{-5}. \quad (5.2)$$

5.3 Four-Mode Degeneracy in a Double-Helix SWS

A helix-based SWS typically consists of a metallic tape-helix inside a metallic waveguide; such SWSs have been used for decades for high power device sources. A simple modification consisting of a double-helix SWS supports a DBE when properly designed (Fig. 5.2). The double helix SWS is comprised of two concentric helices with two different radii and different pitches placed inside a cylindrical waveguide [100], whose unit cell is shown in Fig. 5.2(a). By properly engineering the coupling between the various modes supported, a four mode degeneracy (i.e., the DBE) is achieved. Proper coupling is realized by introducing a longitudinal misalignment between the two helices such that the outer helix pitch p_{h1} is two times the inner helix pitch p_{h2} . Fig. 5.2(b) shows the modal dispersion relation of the proposed “cold” SWS calculated using the Eigen mode solver implemented by CST Microwave Studio based on the finite element method. The dispersion relation shows that the second mode exhibits a DBE that locally fits the dispersion $\omega - \omega_d = h(k - k_d)^4$, where h is a design-related constant. The interaction with the electron beam in a SWS is quantified by the interaction impedance. Higher interaction impedance indicates stronger electromagnetic mode and electron beam interaction, and in general leads to higher efficiency of microwave generation. In Fig. 5.2(c) we plot the interaction impedance Z_p of the first two modes in the SWS versus frequency, using the definition of Z_p in Ch.2 of [101]. The DBE degenerate mode is shown to have a higher interaction impedance compared to that of the lower mode. This in turns may imply higher efficiency of microwave generation and oscillation purity.

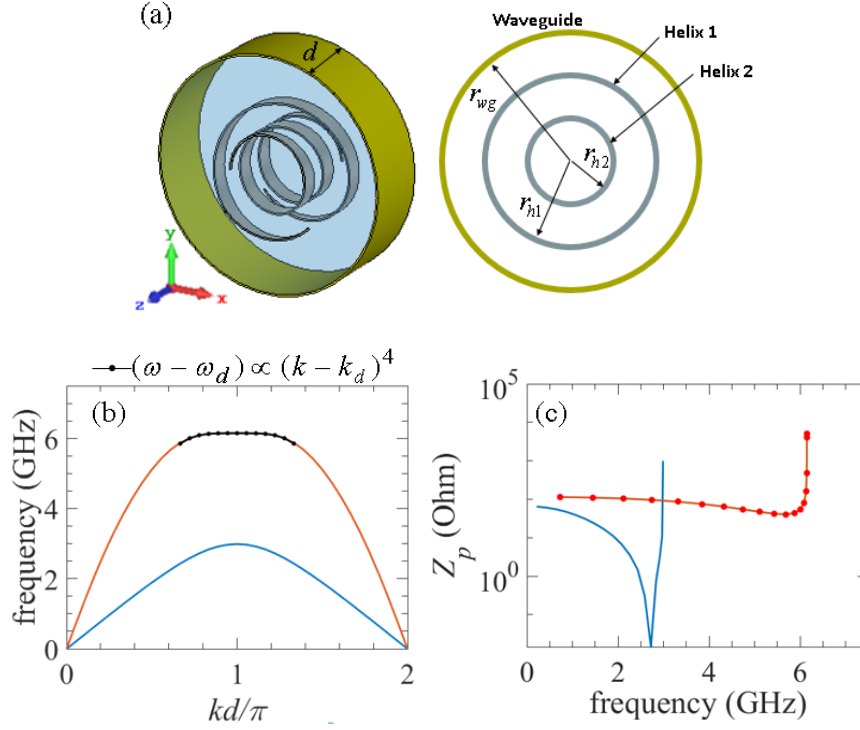


Figure 5.2: (a) Double helix SWS exhibiting the fourth order eigenmode degeneracy at C-band. (b) Modal dispersion relation associated to the proposed “cold” double helix SWS in (a), showing the first two modes (only curves with purely real wavenumber are shown). The higher mode (red) exhibits a DBE at 6.14 GHz as shown from the typical characteristic fitting formula for this kind of degeneracy [3, 4]. (c) Interaction impedance of the DBE SWS calculated along the longitudinal axis, for the first two eigenmodes. The SWS unit cell dimensions are $r_{wg} = 6.6$ mm, $r_{h1} = 2.2$ mm, $r_{h2} = 2.4$ mm, the period is $d = p_{h1} = 2p_{h2} = 4$ mm, and the thickness and width of both helices is the same and equal to 0.1 mm and 1 mm, respectively.

5.4 Four-Mode Degeneracy in a MTM coupled cavities SWS

A design of a backward wave oscillator (BWO) based on the MTM SWS where its unit cell is depicted in Fig. 5.3. The MTM structure proposed here is an extension of the all-metal complementary electric split-ring resonator (CeSRR) that has been studied in [102, 103]. This CeSRR based MTM SWS has several advantages starting from SWS miniaturization relative to conventional SWSs used in VEDs to the offered high interaction impedance. The main difference in our proposed SWS with respect to those in [102, 103], is that we have introduced two CeSRRs in the unit cell with two different resonant frequencies, and the two CeSRRs are tilted with respect each other. This asymmetric design by adding another CeSRR [104, 105], with different dimensions and tilt angle, was chosen for the following reasons. The first reason is related to long term research plan (beyond the scope of the present study) where the use of two CeSRRs, tilted with respect to each other, and with different resonant frequencies, is crucial in realizing higher order dispersion [3, 106] which is associated with exceptional points of degeneracy like the degenerate band edge (DBE), the split band edge (SBE) and the stationary inflection point (SIP) [1, 107, 20]. The use of multi-mode degeneracies in vacuum electronics attains some unique features related to HPM sources such as low starting (threshold) electron beam current and unique threshold scaling with the SWS length compared to conventional BWO, and large amplification in HPM amplifiers (for more details see [3, 106, 46, 42, 108] and references therein). Another reason was the importance to investigate the involved synchronization regime with several waveguide modes and the electron beam and how simulations would agree with experimental measurements in this very complex setup. The last reason is the investigation of the capabilities of the 3D printing technology in fabricating very complex and multimode SWS structures (e.g., ones that only 3D printing might be able to realistically fabricate) which are used for the design of HPM sources and oscillators.

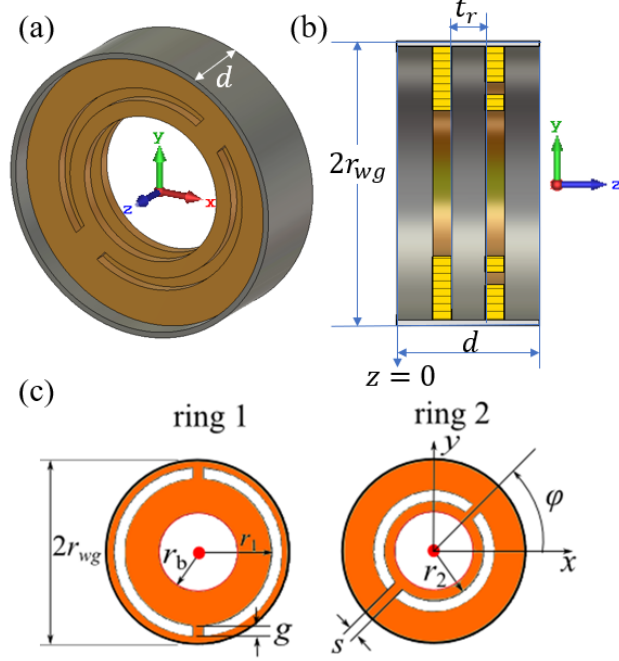


Figure 5.3: A unit cell of the proposed MTM SWS: a disk-loaded circular waveguide with misaligned CeSRRs designed to facilitate the mode mixing required for multiple-mode interaction regime: (a) perspective view, (b) side view. (c) The two loaded disks inside the circular waveguide, each of which has a beam tunnel and two irises in the shape of half ring. All parameters and dimensions are reported in Table 5.1.

A unit cell of the MTM SWS comprises *two* tilted rings and has period d . All the geometrical dimensions of the MTM SWS are reported in Table 5.1. The MTM SWS is biperiodic in that two types of periodic structural subassemblies, interchanging in one or more directions, comprise the SWS. The circular waveguide has radius r_{wg} and the two loading circular metallic rings have the same beam tunnel of radius r_b as well as two irises. The iris in each of the two rings in the unit cell has the shape of two half rings. Moreover, the two irises are different from ring 1 to ring 2, and they are misaligned by an angle φ as seen in Fig. 5.3(c). The metal rings' thickness considered here is the same for rings 1 and 2. The separation between rings 1 and 2 (i.e. the gap between rings) is t_r , as seen in Fig. 5.3. In the following analysis, we will assume a lossless structure, i.e., we ignore metal loss for cold test performance using full-wave simulations.

Symbol	Quantity	Value(μm)
r_{wg}	Waveguide radius	35 mm
s	Iris separation	4 mm
g	Iris width	4 mm
ϕ	Misalignment angle	45 degrees
h_s	Ring thickness	2 mm
t_r	Ring separation	0.5 mm
r_1	Iris 1 inner radius	18 mm
r_2	Iris 2 inner radius	28 mm
rb	Beam tunnel radius	16 mm
d	Period	15 mm

Table 5.1: Physical dimensions and parameters of the DBE-MTM SWS

5.4.1 Cold SWS dispersion

In Fig. 5.4 the $(\omega - k)$ dispersion diagram of the five lowest order eigenmodes is depicted for the designed DBE-MTM SWS in Fig. 5.3 and whose parameters are in Table 5.1. The dispersion is calculated using the eigenmode solver implemented in CST Suite Studio by DS SIMULIA based on the finite element method. The eigenmode solver enforces a phase shift across the structure period in the axial direction of propagation and solves for the eigenfrequencies. By repeating the simulation for different phase shifts, one can construct the dispersion diagram as the one shown in Fig. 5.4. In the dispersion diagram we show both forward and backward Floquet-Bloch propagating harmonics in the fundamental Brillouin zone that extends from $k = 0$ to $k = 2\pi/d$. It is worth noting that the plotted dispersion curves represent only the propagating part of the spectrum and thus they have a purely real-valued wavenumber k . In addition, it is worth emphasizing that the dispersion diagram obtained in Fig. 5.4 is for the cold SWS case (electron beam is absent). However, this cold dispersion is still helpful as a zeroth order approximation to establish synchronization in the fully interactive system (SWS and electron beam). This approximate picture of the interactive system is realized by plotting the “electron beam line” together with the cold dispersion diagram as shown in Fig. 5.4 in the black line. The DBE mode that is shown in this dispersion diagram has asymmetric (in the cross section) axial electric field distribution,

Symbol	Quantity	Value
r_{wg}	Waveguide radius	25 mm
s	Iris separation	5.7 mm
g	Iris width	2.2 mm
ϕ	Misalignment angle	45 degrees
h_s	Ring thickness	2 mm
t_r	Ring separation	3.5 mm
r_1	Iris 1 inner radius	15.7 mm
r_2	Iris 2 inner radius	20.7 mm
rb	Beam tunnel radius	12.85 mm
d	Period	15 mm

Table 5.2: Physical dimensions and parameters of the MTM SWS; the one used for the BWO realization.

as shown in Fig. 5.4 inset, which indicates a low interaction impedance. However, the DBE may also occur at higher frequencies than those shown here with symmetric axial electric field distribution. Furthermore, design variations may also lead to the DBE with the proper axial field distribution for interacting with the electron beam.

For the rest of this section, we will consider another design with the dimensions listed in Table. 5.2. The modifications made to the original structure with DBE are done for reasons first to make the structure fits the SINUS-6 accelerator hot setup at the University of New Mexico (UNM). The other reason was to have a strong axial field for sufficient interaction with the electron beam. The last reason is to investigate capabilities of fabricating the proposed DBE SWS using the 3D printing technology.

5.4.2 Interaction impedance

In Fig. 5.5 the $(\omega - k)$ dispersion diagram of the five lowest order eigenmodes is depicted for the designed MTM SWS in Fig. 5.3 and whose parameters are in Table 5.2; the one used for the BWO realization in the following.

As shown in Fig. 5.5, the electron beam line intersects with different eigenmodes at different

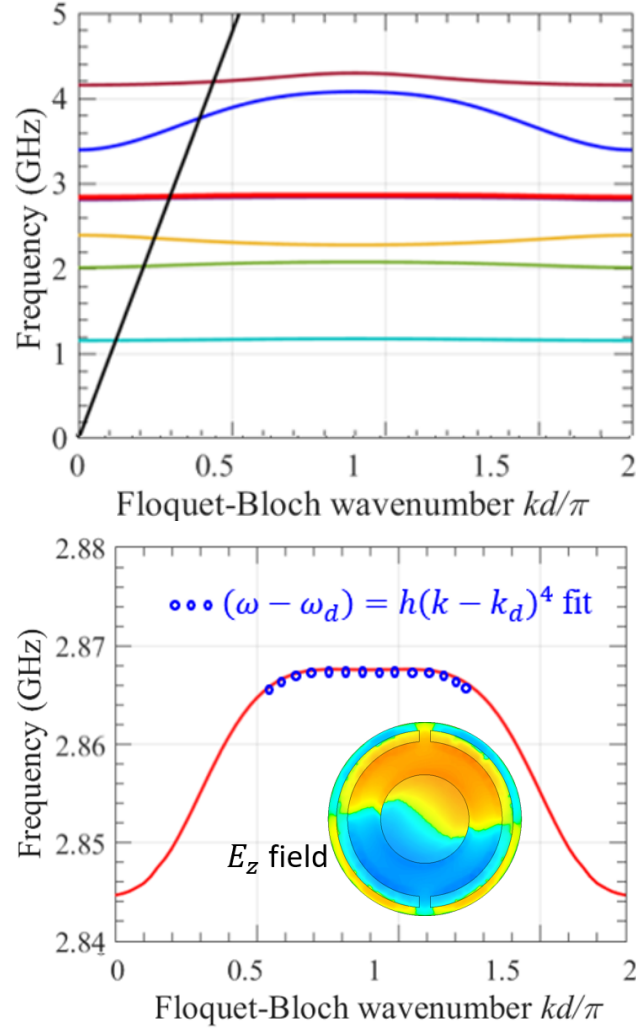


Figure 5.4: (Top) Dispersion diagram of the lowest order Floquet-Bloch eigenmodes of the cold periodic MTM SWS in Fig. 5.3, with dimensions reported in Table 5.1. (Below) A zoomed-in plot of the fourth mode (red curve) showing the higher order dispersion, associated to 4th order exceptional point of degeneracy, called degenerate band edge. (Inset) The longitudinal electric field map of the DBE mode at the transverse cross-section in the unit cell at $z = d/2$ shows that this mode has asymmetric pattern.

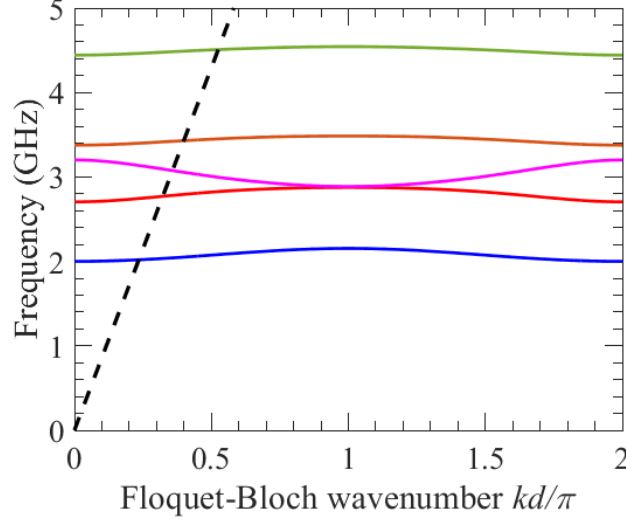


Figure 5.5: Dispersion diagram of the lowest order Floquet-Bloch eigenmodes of the cold SWS in Fig. 5.3. An electron-beam line of 490 keV (black dashed line) is plotted showing that the beam line intersects with the cold dispersion at different frequencies for each eigenmode. However, the dominating mode in the fully interacting system is the one that has the highest interaction impedance (i.e. third mode “purple”).

frequencies. To better quantify the interaction of different eigenmodes with the electron beam in a SWS, it is customary to calculate the interaction impedance because such impedance is responsible for high amplification and gain according to Pierce theory (see details in [96, 109, 97]). Higher interaction impedance indicates stronger enhancement of wave and electron beam interaction and leads to higher efficiency of microwave generation. The interaction impedance, with units of ohm, is defined as (see Ch.2 in [110] or Ch. 1 in [111])

$$Z_p(x, y) = \frac{|E_{zp}(x, y)|^2}{2\text{Re}(k_p)^2 P_r}, \quad (5.3)$$

where the subscript p designates the p^{th} Floquet-Bloch harmonic of the axial electric field.

The axial field is decomposed into Floquet-Bloch harmonics as

$E_z(x, y, z) = \sum_{p=-\infty}^{\infty} E_{zp}(x, y)e^{-jk_p z}$ in which $E_{zp}(x, y)$ is the p^{th} Floquet-Bloch harmonic axial electric field amplitude and k_p is the Floquet-Bloch harmonic wavenumber $k_p = k + 2\pi p/d$, with $p = 0, \pm 1, \pm 2, \dots$. Furthermore, P_r is the time-average power flow in the periodic SWS. All the aforementioned quantities are calculated using the eigenmode solver

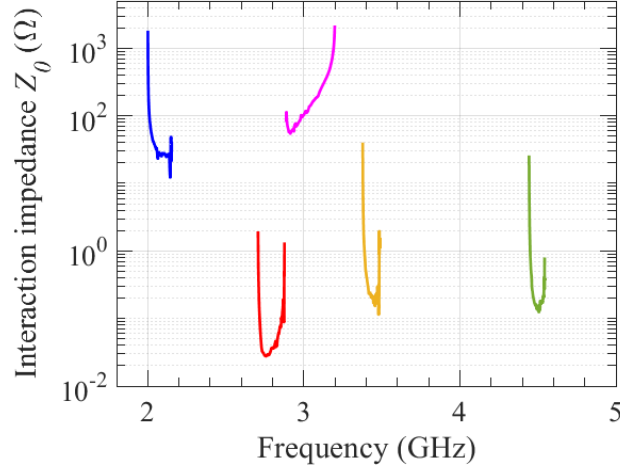


Figure 5.6: Interaction impedance described by Eq. (5.3) of the proposed SWS structure calculated at $(x, y) = (0, 0)$, i.e., along the beam axis for the lowest order fundamental Floquet-Bloch eigenmodes. The different colors match those of the modes of the cold SWS shown in Fig. 2. Note that the other Floquet harmonics have lower interaction impedance; thus, we do not plot them here since they are less likely to interact with the electron beam. Results are obtained from full wave simulations.

full-wave simulations implemented in CST Studio Suite by DS SIMULIA.

In Fig. 5.6 we show the interaction impedance for the first five fundamental eigenmodes (i.e. $p=0$). Note that higher order Floquet-Bloch harmonics can also be excited, however, they contribute less to the interaction with the beam since their interaction impedance is much smaller than the fundamental eigenmodes (not shown in Fig. 5.6 for brevity). It is clear from Fig. 5.6 that the third mode is the one that has the highest interaction impedance. Therefore, it is the mode that is most likely to couple with the beam and to efficiently harvest the energy from the electron beam. The electric field associated with the third mode has a very strong axial electric field component and resembles the TM_{01} mode of the waveguide with a slight azimuthal asymmetry caused by the misaligned irises, as depicted in Fig. 5.7, where we show the electric and magnetic field maps of the third mode.

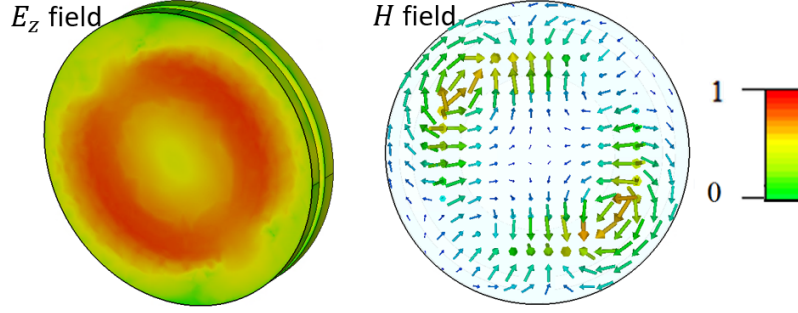


Figure 5.7: The electric and magnetic field maps of the third mode (normalized to their maximum) at the transverse cross-section in the unit cell at $z = d/2$, showing that this mode has almost a TM₀₁-like mode profile.

5.4.3 Computational Model

This Section is dedicated to describing the entire model used to investigate the performance of the proposed SWS as the interaction part of a pulsed power-driven BWO. Additionally, we briefly discuss the simulation setup carried out by CST Studio Suite (particle in cell solver) to model the experiment environment.

The PIC algorithm uses a 3D self-consistent solution to Maxwell's equation in the time domain in the presence of an electron beam current made by a finite number of emitted charged particles. It also accounts for saturation effects such as electron overtaking which is responsible for reaching a steady state in the oscillator.

Fig. 5.8 shows the proposed design for the MTM SWS-based BWO. The SWS consists of $N=11$ unit cells with total length 165 mm. The total length of the whole structure starting from the annular cathode ending in the output port at the mouth of the horn antenna (used to radiate the generated output power) is 460 mm. The PIC solver was used to quantify the performance of the proposed pulsed BWO. The cathode is chosen to be annular with inner and outer radii equal to 8.7 and 10.5 mm respectively. The motive behind using an annular beam instead of a solid beam is because it interacts with the axial electric field distribution of the interacting mode, shown in Fig. 5.7, which is concentrated near the inner radius of

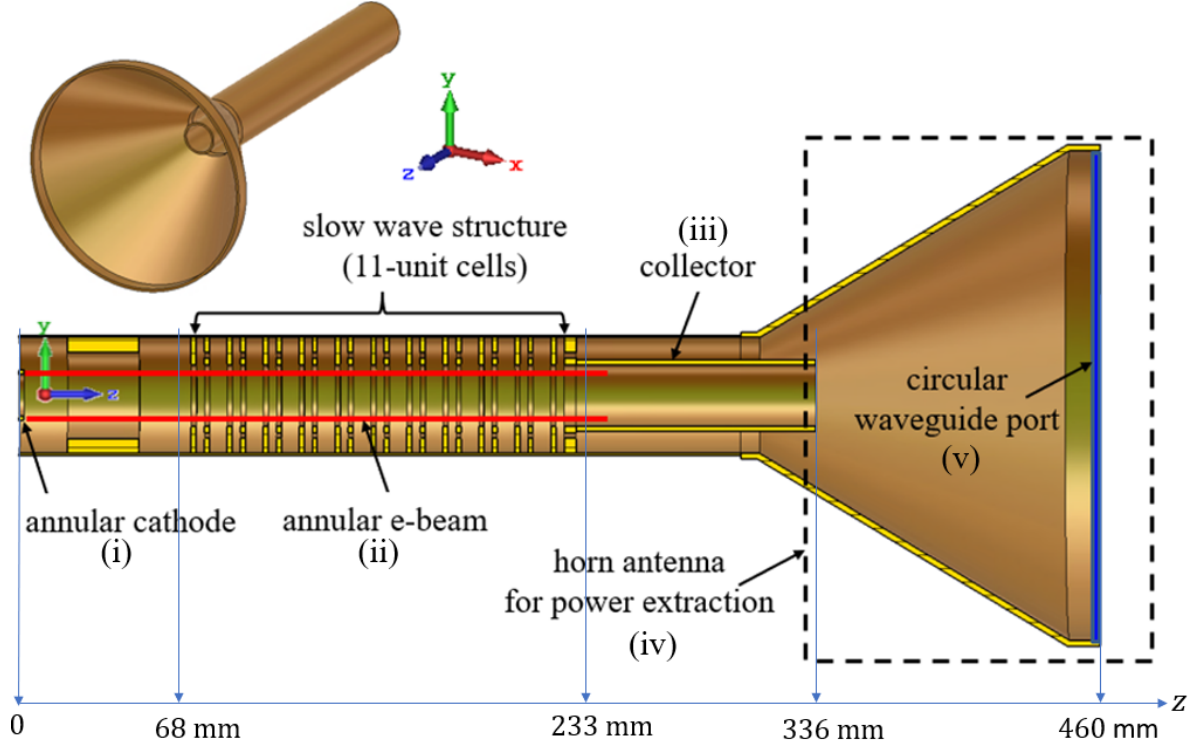


Figure 5.8: Schematic showing the entire BWO structure, (top) 3-D perspective view of the structure (bottom) lateral view, i.e., the r - z cross-section at the $x=0$ plane. Starting from the left side, the BWO consists of: (i) annular cathode for annular electron beam generation; where the voltage is applied between the cathode and outer side of BWO using a coaxial circular waveguide port; (ii) MTM SWS with 11 unit cells, and total length = 165 mm; (iii) metallic cylinder of radius $r_b = 12.85$ mm; (iv) canonical horn antenna to radiate the output microwave generated power, where (v) a circular waveguide port is placed at the end of the entire BWO (i.e. at the mouth of the horn antenna) to monitor the output power level.

the disks with a vanishing field at the center.

The annular electron beam emission is based on the explosive emission model in the PIC solver. The electron beam is excited by introducing a waveguide port between the outer surface of the cathode and the inner surface of the waveguide. This coaxial waveguide port is excited by a voltage waveform based on UNM's SINUS-6 accelerator, as shown in Fig. 5.9(a) (blue trace). The amplitude of the pulse is the maximum electron beam applied voltage (i.e. 490 kV), whereas the current is not forced but rather depends on the emission model.

A static axial magnetic field $B_z = 1.6$ T is used to ensure good confinement of the annular electron beam traveling in the axial direction. A metallic cylinder of radius equal to the beam tunnel r_b is introduced at the end of the SWS as the collector (i.e. beam collection tube and radio frequency coupler) as shown in Fig. 5.8 to prevent electron beam particles from hitting the output port.

The microwave output power is extracted out by introducing a horn antenna as the one shown in Fig. 5.8, where at the mouth of the horn antenna another circular waveguide port is placed to monitor the output power level.

5.4.4 Hot Simulations Results

This Section presents the hot simulation results associated with the detailed setup in previous Subsection. We investigate the interaction of the proposed SWS with a high-power relativistic electron beam and the consequent HPM generation. In the following we aim at generating a high power pulse; hence, we consider a 490-kV electron beam since we have found that this value provides high power and it is realizable in the experimental setup as will be shown in the following Section.

Figure 5.9(a) presents the electron beam applied voltage pulse (i.e., anode voltage with respect to the cathode's one) as well as the corresponding emitted current as a function of time. The voltage and current traces are obtained by placing a voltage monitor between the cathode surface and the BWO wall and a current monitor at a distance 3 mm from the emitting surface of the cathode, respectively.

In Fig. 5.9(b) we show the transient output electromagnetic signal obtained from the waveguide port monitor at the end of the structure, i.e., at the horn's aperture. Note that the delay in time between the instant when we apply the e-beam pulse and the instant when the

RF output pulse is observed at the output port is about 4 ns. This delay is related to the RF traveling time from the beginning of the SWS to the horn's aperture. We only report the output signal in the TM_{01} mode which carries the highest power among the various modes of the output circular waveguide port. The transient signal at the output waveguide port, with units of $(W^{1/2})$, shows that the output pulse reaches the waveguide port after ~ 5 ns. The instantaneous output microwave power is reported in Fig. 5.9(c), which is calculated as the output port signal (in Fig. 5.9(b)) squared. The maximum instantaneous output power reported here is ~ 88 MW. Moreover, we report the output signal normalized spectrum in Fig. 7(d) calculated in the time window between 0 and 25 ns showing a single oscillating frequency around 2.9 GHz. The output frequency is almost coinciding with the zeroth-order approximation of the synchronization frequency, shown in the dispersion diagram shown in Fig. 5.5 by the intersection point between the black dashed line that represents the 490 kV electron beam and the purple curve that represents the third Floquet-Bloch eigenmode.

An important metric of HPM generation is the radio frequency (RF) conversion efficiency. The RF conversion efficiency is calculated by dividing the maximum instantaneous output power by the peak input electron beam power (peak beam input power is obtained by multiplying the peaks of the applied voltage and current). We report that the RF conversion efficiency at this specific case studied in this section equals 5.5 %, using the peak input power calculated as $P_{in, \text{ peak}} = 490 \text{ kV} \times 3.3 \text{ kA} = 1617 \text{ MW}$. Yet, here we only aim at demonstrating the principle of operation and we used a high electron beam voltage to show the maximum output power in this case. Fig. 5.10 presents phase plots of electrons at different times in PIC simulations, showing strong bunching.

To study the effect of applied voltage on RF conversion efficiency we performed PIC simulations scanning the applied voltage, and evaluated the output extracted power for different values of electron beam applied voltage. Results are shown in Fig. 5.11, where the maximum instantaneous output power in the output circular waveguide port mode for various applied

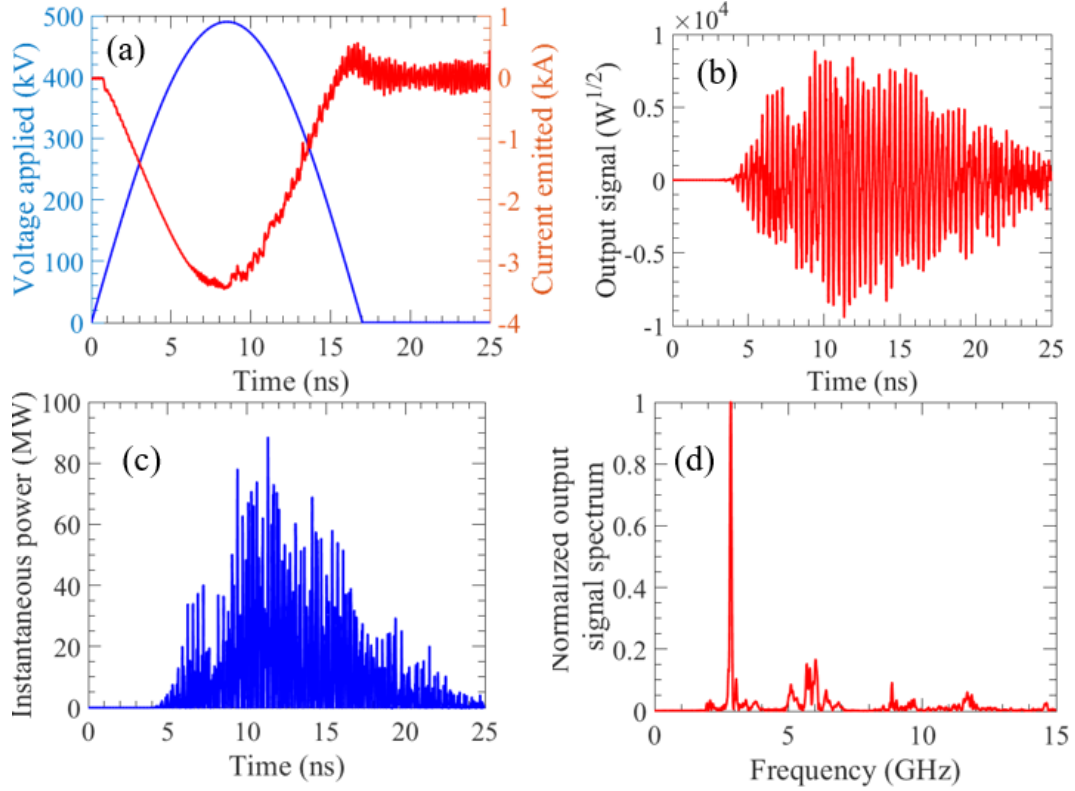


Figure 5.9: (a) SINUS-6 applied voltage pulse in blue and the corresponding emitted current in red. (b) Simulated oscillating output microwave signal extracted from the TM01 mode of the horn output consisting of a circular waveguide port (Fig. 5.8). (c) Simulated output instantaneous power with a maximum around 88 MW, and (d) output spectrum obtained from a Fourier transform of the signal in the time window from 0 to 25 ns, showing a single frequency oscillation at 2.9 GHz which coincides with the zero-order approximation of the synchronization shown in the dispersion diagram in Fig. 5.5.

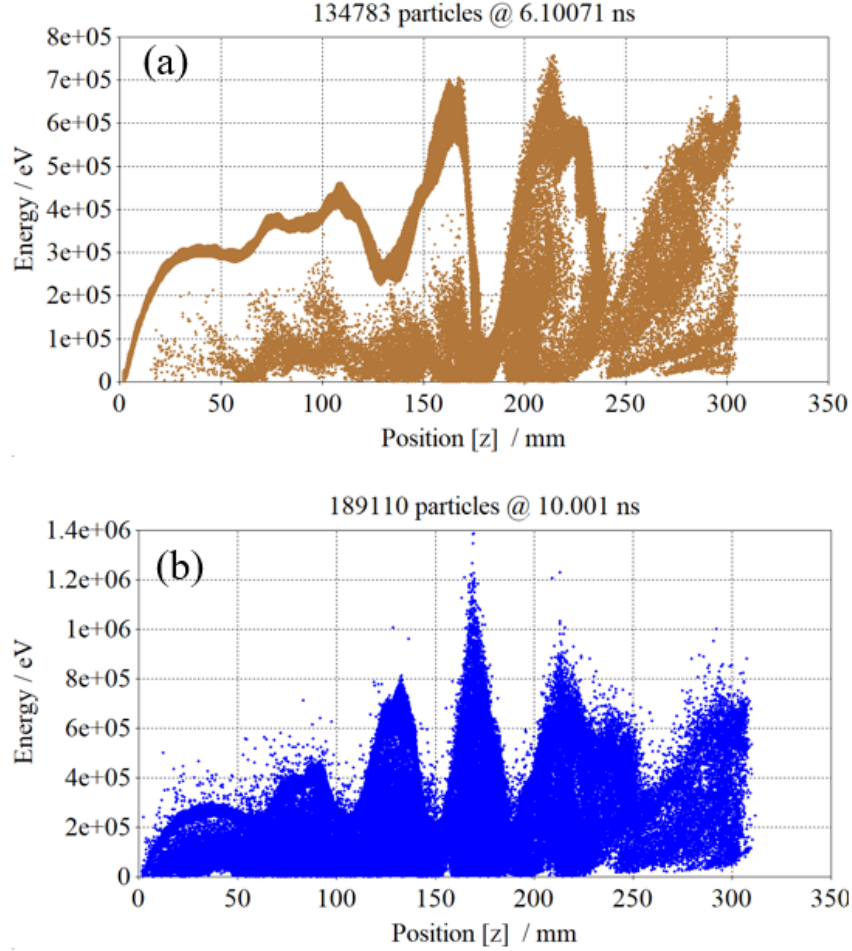


Figure 5.10: Phase space plot of electrons at different times in PIC simulations: (a) $t = 6$ ns; (b) $t = 10$ ns.

cathode voltages is reported. On the same graph we show the RF conversion efficiency evaluated for each applied voltage using the definition mentioned in the previous paragraphs. It is worth noting that, for each value of the applied voltage there is a specific value of the derived current which was taken into account in calculating the RF conversion efficiency. For the sake of brevity, these values of current corresponding to different values of maximum applied voltage are not reported here.

It is obvious from Fig. 5.11 that by decreasing the applied voltage, the efficiency improves to a certain value of such a voltage. The maximum RF conversion efficiency of almost 14% occurs when the electron beam applied voltage equals 200 kV where the maximum instant-

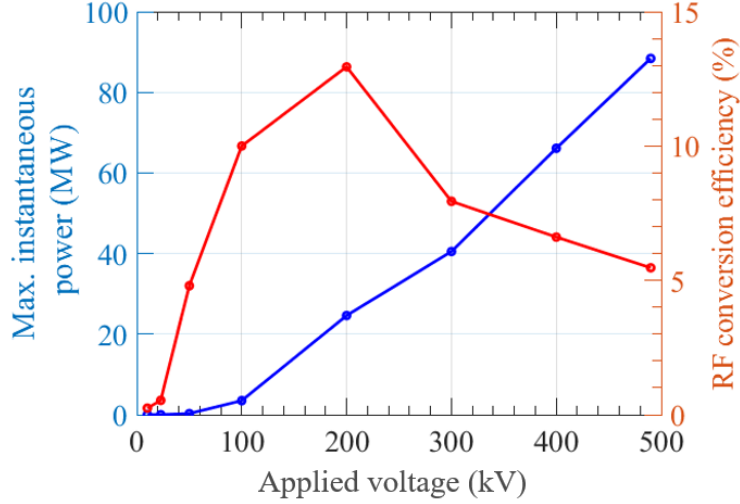


Figure 5.11: Simulated maximum instantaneous output power (blue trace) and RF conversion efficiency (red trace) versus the peak value of the applied voltage pulse for the MTM SWS shown in Fig. 5.8.

neous output power equals 25 MW. For applied voltage less than 200 kV the RF conversion efficiency starts to decrease. On the other hand, the maximum instantaneous output power monotonically increases with increasing applied voltage. A remarkable observation is that the output frequency obtained by performing a Fourier transform to the output signal like the one shown in Fig. 5.9(d) is stable for all applied voltages. Indeed, the output spectrum for different cathode voltages shows that the oscillation frequency is very stable around $2.9 \text{ GHz} \pm 1\%$ except for the lower beams below 25 kV where the oscillation frequency drops to 1.97 GHz.

5.4.5 Experiments

The SINUS-6 electron beam accelerator was selected for hot test studies of the UCI MTM SWS. The SINUS-6 can provide voltages up to 700 kV to a cathode with 130Ω load impedance in a 13 ns pulse. A solenoidal magnetic field is used to transport the electron beam. The solenoid can generate a magnetic field up to 3 T. A drawing of the SINUS-6 electron beam accelerator is shown in Fig. 9.

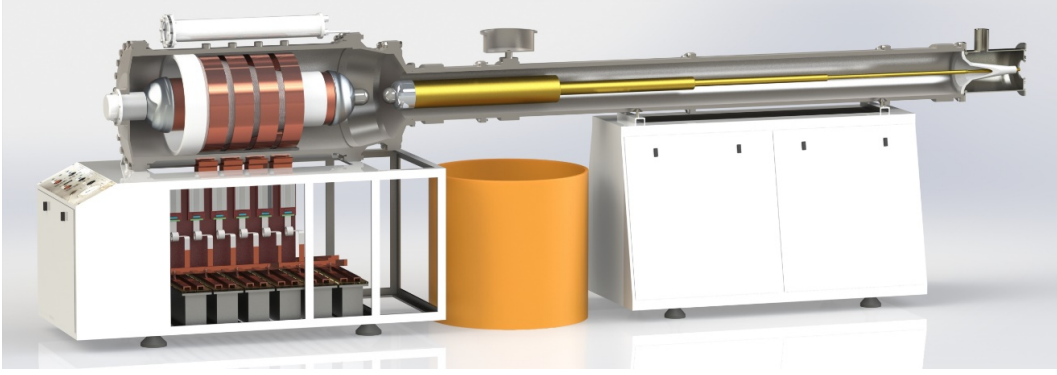


Figure 5.12: CAD drawing of the SINUS-6 electron beam accelerator showing the Tesla transformer (left), the pressurized spark gap switch (to the right of the Tesla transformer), and then connecting to the transmission line. The cathode and MTM SWS are affixed to the right end of the system within a solenoidal magnet.

Figure. 5.13(top) presents CAD drawings of the two CeSRRs that were 3D printed using Accura 60 material. The printed CeSRRs were then copper plated and assembled to form the 3D printed, copper plated MTM SWS (Fig. 5.13, bottom). The MTM SWS housing was then inserted into a 3D printed cylinder and assembled at the load end of the SINUS-6 accelerator. The solenoid was finally mounted around the MTM SWS assembly.

We should point out that, in this circumstance, 3D printing was the only manner in which we could have fabricated this MTM SWS. Utilizing traditional machining techniques would have been too challenging to produce a properly aligned final structure. And the choice of Accura 60 material and copper plating was determined by our previous experience in producing microwave components using this process. We find that the 3D printed, copper plated Accura 60 material allows us to achieve the usual base pressure in our system (described below).

To-date, available metal 3D printers use alloys whose conductivity is not suitable for microwaves. Furthermore, previous researchers developing 3D printed metallic structures found that a considerable amount of post-processing was required in order to improve the smoothness and uniformity of the structure. However, in a recent collaboration between the SLAC National Accelerator Laboratory and the Center for Additive Manufacturing and Logistic



Figure 5.13: (Top) CAD drawing the two CeSRRs comprising a unit cell of the MTM SWS. (Bottom) 3D printed cylindrical tube housing the 11-cell MTM SWS.

at North Carolina State University, a copper additively manufactured high efficiency X-band klystron output cavity with micro-cooling channels and weight optimized structure was successfully demonstrated [112]. This new capability will likely revolutionize the HPM community as well.

Initial experiments were performed to assess the vacuum compatibility of the 3D printed structures. A vacuum base pressure ranging from 7.6×10^{-6} Torr to 1.3×10^{-5} Torr was routinely achieved using a turbo pump. This base pressure is the same as achieved when using all metallic structures.

Once convinced that the MTM SWS and assembly is compatible with the vacuum base pressure necessary to operate the SINUS-6 accelerator, beam alignment was performed. In order to ensure proper generation of HPM from the MTM SWS it is important for the electron beam be properly aligned concentric within the SWS. This alignment involves adjusting the position of the solenoid.

Experimental hot tests were then performed using a cathode voltage of 490 kV with corresponding electron beam current of about 2.85 kA, which is the Fedosov current limit. Approximately 100 hot tests were performed and no damage was noted, except along the first ring at the inlet due to some cathode initiated damage (Fig. 5.14).

Fig. 5.15 presents a summary of the typical data acquired on a shot. For this shot the cathode voltage was 510 kV, the electron beam current was 2.85 kA, and the output frequency was 3.0 GHz. A magnetic field of 1.6 T was provided, as in the simulations.

In order to estimate the radiated power it is necessary to map out the output TM_{01} mode pattern and then integrate the power density over this pattern. A waveguide detector with a resistive sensor was used [113].

A summary of the experimental results is as follows. Following integration of the mode



Figure 5.14: Photograph of the 3D printed, copper plated MTM SWS following 100 shots. Minimal damage was noted at the inlet (highlighted by the white ovals).

pattern, we estimate that the MTM SWS was generating about 22 MW at 3.0 GHz when driven using a 490 kV, 2.85 kA electron beam. This efficiency is considerably less than the one predicted by PIC simulations in hot simulations subsection. Our explanations for the discrepancy are below.

The experiment employed features that were not present in the PIC simulations performed at UCI. One feature is that a metallic beam dump was used in the experiments (since the 3D printed, copper plated waveguide would not survive impact from spent electrons) and the collector was extended into the horn antenna, as shown in Fig. 16. This configuration might also be responsible for partial mode conversion of the TM_{01} mode into a TE_{11} -like mode, as suggested by the time-integrated photograph of the neon bulb array.

Nevertheless, with good agreement in frequency, and good order of magnitude agreement in radiated pulse levels, these results are encouraging. In addition, considering that the MTM

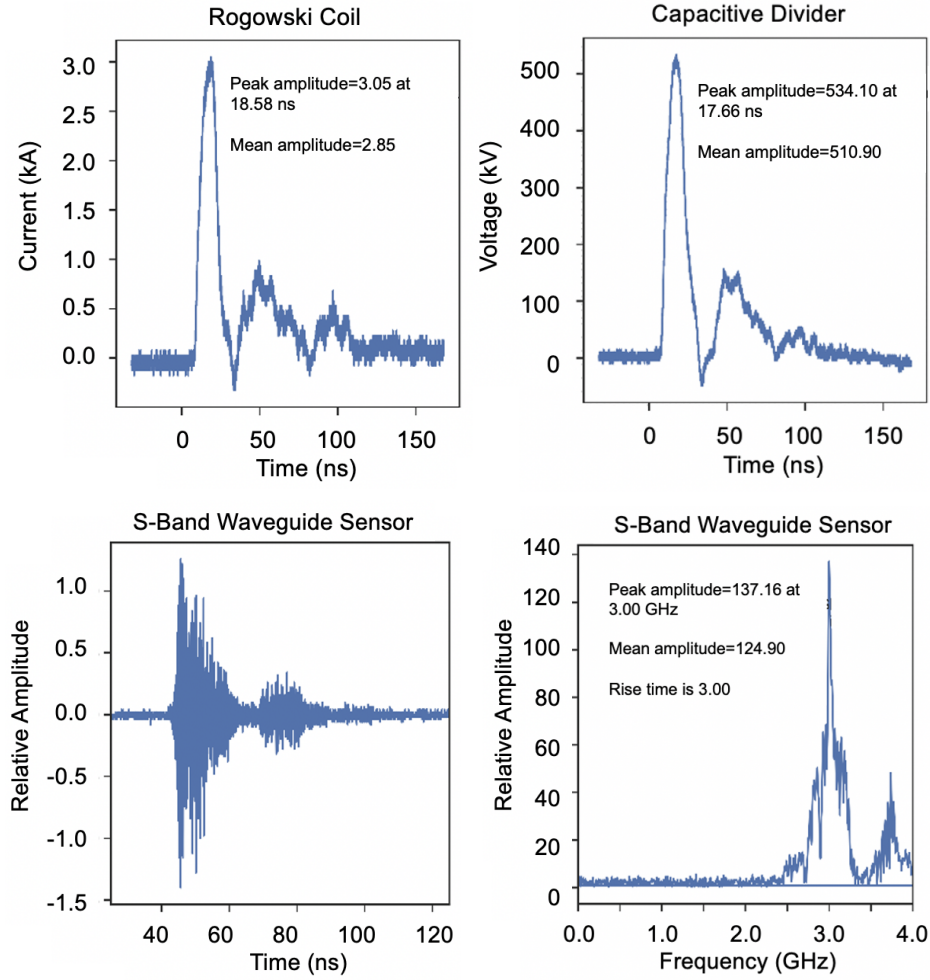


Figure 5.15: Typical data recorded on a shot. Top left: electron beam current registered using a Rogowski coil; Top right: cathode voltage measured in the oil portion of the oil-vacuum interface registered using a capacitive divider; Bottom left: output of an S-band waveguide sensor registered using a crystal diode; and Bottom right: Fourier transform of the output of the waveguide sensor.

SWS in experiment was 3D printed and copper plated suggests that this technology is a viable approach to rapid prototyping of pulsed power-driven HPM VEDs.

5.5 EPDs for High Power Microwave and Terahertz Generation

MM-waves and Terahertz generation has recently seen a growing interest and progress thanks to the advancement in microfabrication techniques [114, 115, 116] as well as in solid state devices [117, 118]. Amplifiers and oscillators in this frequency range are vital for many applications that require portable, compact and lightweight devices for imaging, sensing; and detection [119, 120]. They are also crucial for advancement of various fronts of science [115, 119, 121, 122, 123]. However, electron-beam-based vacuum devices typically produce high electromagnetic power that scales as the inverse of the frequency-squared [115, 116, 124]. This put a limitation on the attainable power when frequency increases, i.e., at mm-wave and THz frequencies, using vacuum sources. Moreover, realizing high interaction impedance and avoiding spurious resonances in TWTs and BWOs is a challenge at those frequencies due to losses, misalignment in fabrications, as well as the overmoded nature of some of those devices. Special dispersion engineering must be performed to attain higher levels of power and better efficiencies [96, 115, 116, 124]. The EPD framework may present an avenue for potential improvement in output power and efficiency compared to conventional devices at mm-wave and THz frequencies.

Here we present a preliminary investigation of SWSs at mm-waves and THz that exhibit an EPD. And in particular we demonstrate a DBEO at 638 GHz based on CST Particle Studio, 3D particle-in-cell (PIC) simulator.

5.5.1 Periodic double corrugated waveguide and the degenerate band edge oscillator (DBEO)

A potential SWS geometry at mm-wave or THz frequencies is the double corrugated rectangular waveguide [125, 126] shown in Fig. 5.16. Note that the SWS has two identical parallel sets of corrugations assembled in a rectangular waveguide such that they create a channel suitable for interaction with a cylindrical electron beam. In fact, the double corrugated waveguide may have several potential advantages as stated in [125, 126], in contrast to typical mm-wave and THz structures [127, 128]. The structure is here modified to exhibit the DBE as follows. We introduce misalignment between the two parallel corrugations. This misalignment is necessary to have the sufficient mode mixing between the eigenwaves supported by the corrugations so that a DBE occurs. Note that in the geometry proposed in [125] and shown in Fig. 5.16, the eigenwaves appear as symmetric and antisymmetric pairs [125]. The corrugation misalignment enables a mode-mixing scheme leading to new set of eigenwaves that exhibit a DBE. There are two degrees of misalignment introduced here: (i) the two parallel corrugations are misaligned by an offset Δz in the z -direction as shown in Fig. 9(c); (ii) the corrugation height alternates periodically, between two different values h_1 and h_2 as shown in Fig. 5.17(b).

We optimize the design parameters (Δz , h_1 , h_2) such that a DBE is manifested at 661 GHz. The resulting values are reported in Table 5.3.

Cold Simulations Results

Fig. 5.17(d) shows the dispersion relation of the first few order modes in the SWS calculated using the finite element method Eigenmode solver implemented by CST Microwave Studio. The dispersion relation shows that the second mode is asymptotically equivalent to the ideal dispersion $\omega - \omega_d = h(k - k_d)^4$ where here $h_{\text{DBE}} = 5.55 \text{ m}^4/\text{s}$. The lower frequency

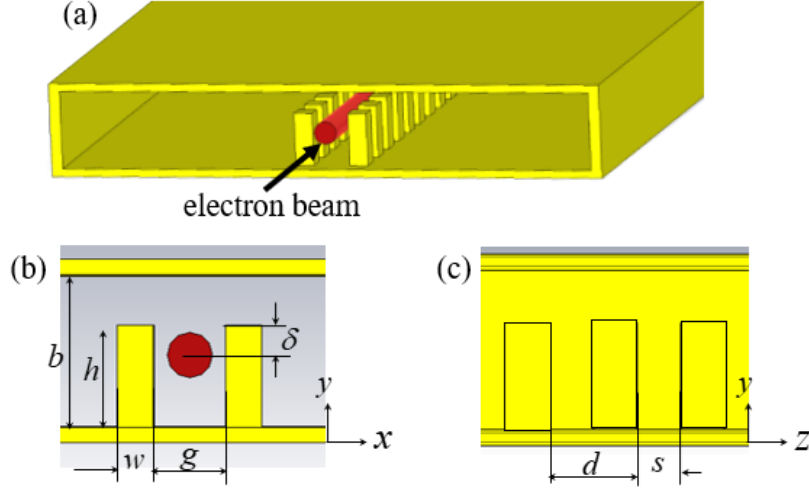


Figure 5.16: Schematics of double corrugations inside a rectangular waveguide: (a) 3D perspective view; (b) Transverse cross section, in the x-y plane; and (c) y-z cross sections (i.e., a side view). This structure is adapted from [73] and will be used as a basis for a modified structure to realize a DBEO at THz frequencies (see Fig. 5.17).

Symbol	Quantity	Value(μ m)
w_r	Ridge width	30
g	Ridge lateral separation	60
d	Period	70
s	Ridges longitudinal separation	35
w	Waveguide width	800
b	Waveguide height	115
Δz	Offset in z-direction	35
h_1, h_2	Heights of ridges	85, 64

Table 5.3: Dimensions and parameters of the double corrugated waveguide with DBE

mode exhibits a regular band edge (RBE), which is the standard band edge condition in periodic structures, whereas the higher order mode exhibits a DBE. In the following we consider the four-degenerate eigenwaves interaction regime by requiring that the velocity of the beam electrons must match the phase velocity of a Floquet harmonic of the four waveguide eigenwaves at the DBE frequency (i.e., condition (5.1)). In other words the designs starts by selecting a beam electron velocity u_0 such that the “beam line” intersects the “cold” structure dispersion diagram around the DBE point in Fig. 5.17.

In Fig. 5.18 we show the longitudinal electric field component E_z of the DBE mode at differ-

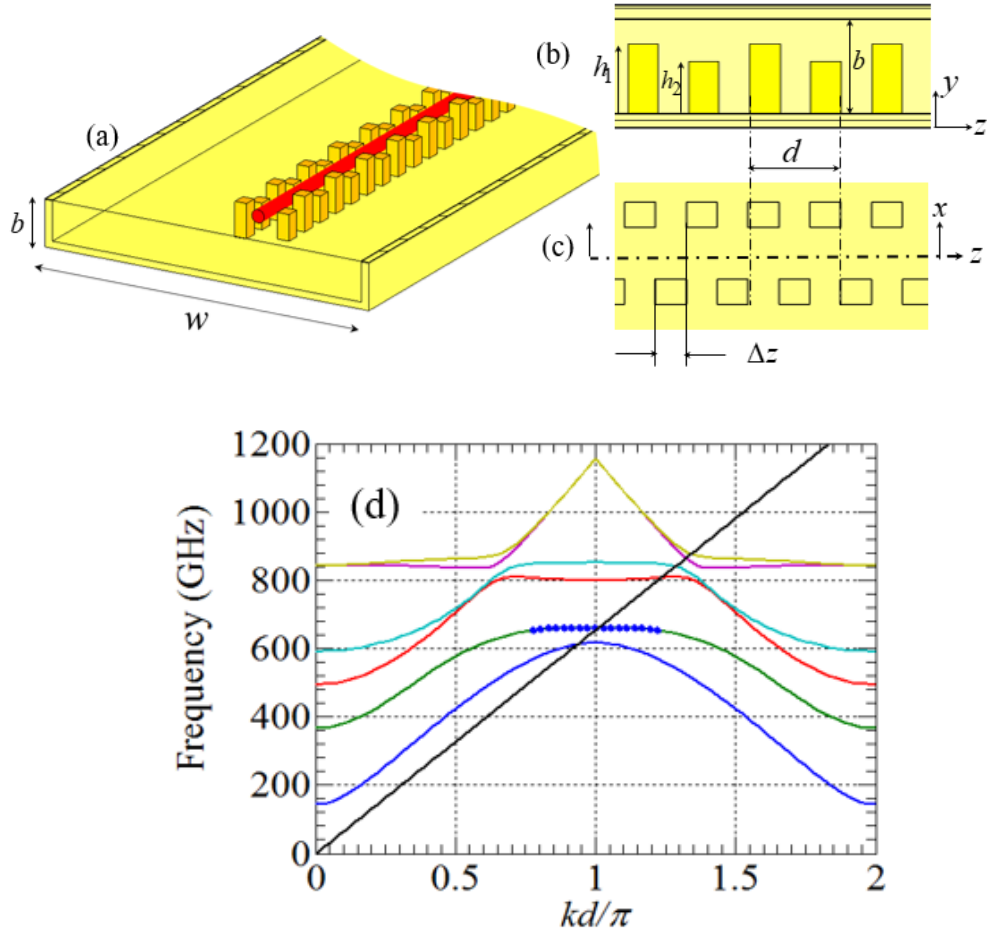


Figure 5.17: Proposed double corrugated waveguide with DBE. (a) 3D perspective view; (b) Lateral view, i.e., the y - z cross section. Here the ridges along the z -direction have different heights h_1 and h_2 . (c) Top view, i.e., the x - z cross section showing the offset Δz in the z -direction between the two parallel corrugations. (d) Dispersion relation of the first few eigenmodes for the modified double corrugated waveguide showing normalized propagation wavenumber versus frequency. The second mode (green curve) exhibits a DBE at 661 GHz, proved by the fitting dispersion law $(\omega_d - \omega) \cong h_{\text{DBE}}(k - k_d)^4$ (blue dotted line). The beam line “black line” intersects with the DBE eigenwaves curve at around DBE frequency. This is used to conceive a DBEO.

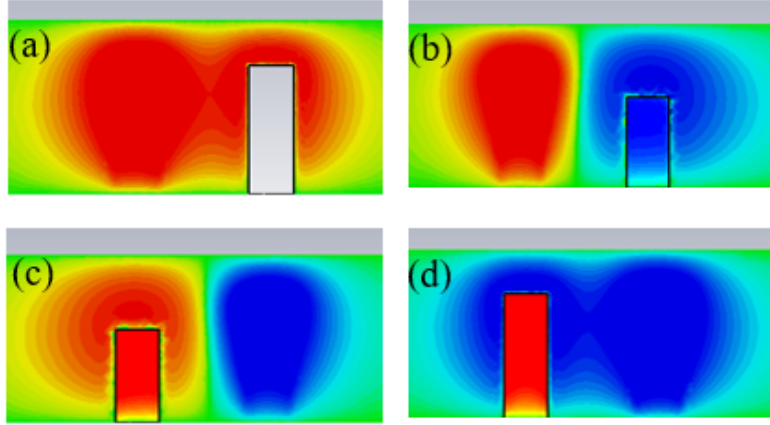


Figure 5.18: Electric field E_z profile of the DBE eigenwave at 661.5 GHz; at different transverse cross sections (i.e., in the x-y plane, at different z coordinates) in the unit cell starting from $z = 0$: (a) $z = 0.1d$; (b) $z = 0.4d$; (c) $z = 0.6d$; and (d) $z = 0.9d$, where d is the period. Blue and red indicate negative and positive values of the E_z field, at a given time.

ent transverse cross sections through the unit cell. The DBE mode field profile exhibits both symmetric and asymmetric features because it is a mix between symmetric and asymmetric modes due to the introduced misalignment. In summary the E_z field has alternating positive and negative values as necessary to interact with the electron beam.

Hot Simulations

The DBEO based on the double-corrugated SWS, to oscillate at a frequency in the neighbor of the DBE frequency is designed with a solid electron beam with voltage $V_0 = 77.13$ kV and current of $I_0 = 20$ mA. The beam radius is $r_{beam} = 25 \mu\text{m}$. We use a longitudinal magnetic field of 2 T and the number of unit cells is 90, corresponding to a SWS length of 12.6 mm. The metal of the SWS is considered to be copper with conductivity $\sigma = 5.9 \times 10^7 \Omega^{-1} \cdot \text{m}^{-1}$ as in [125]. The waveguide is terminated by a beam collector and the THz signal power is extracted from the cathode side through output coupling section identical to the setup in [127], for simplicity. In Fig. 5.19, we show results of DBEO using CST Particle Studio, PIC simulator and observe that the DBEO has a well-defined single frequency output signal at 638

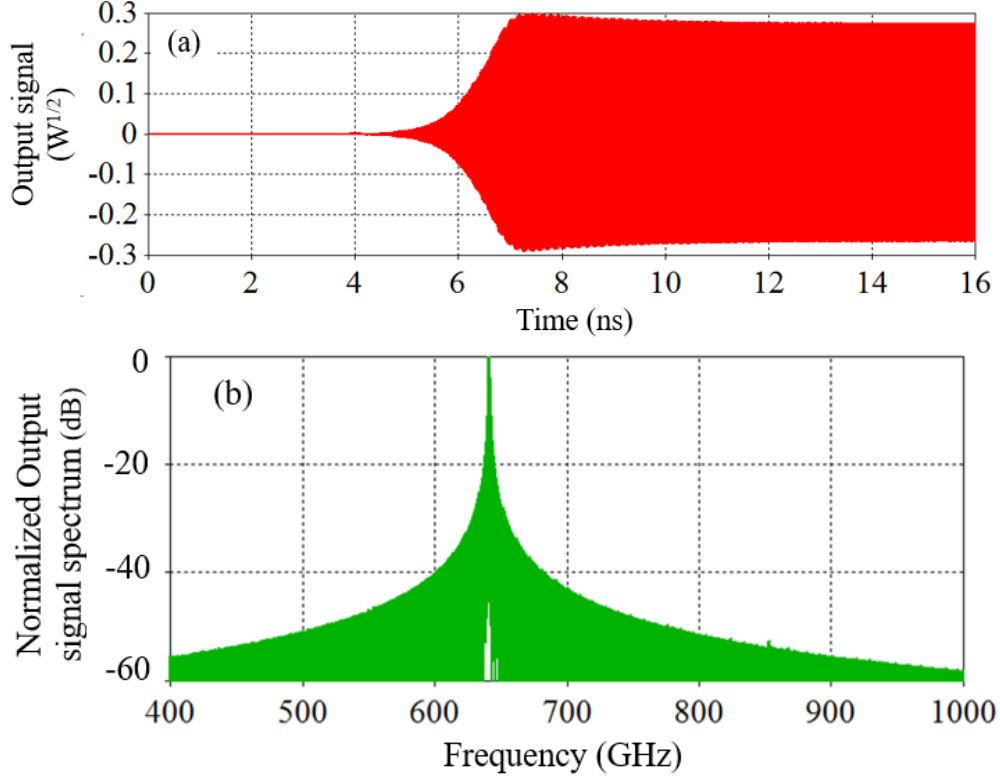


Figure 5.19: (a) Oscillating output signal versus time of the DBEO made of a modified double corrugated waveguide shown in Fig. 5.17. Steady oscillation regime is reached at approximately 7 ns. (b) The normalized spectrum of the output signal shows that the oscillations occur at 638 GHz, which indicates that the oscillations are approximately at the DBE frequency. The spectrum is calculated in a time window from 14 to 15 ns.

GHz. This frequency is in close proximity of the DBE frequency. And the small difference may be due to the fact that a finite length SWS exhibits a DBE resonance frequency at a frequency slightly lower than the DBE one, as observed in [42].

These preliminary results show a great promise in adopting such geometry to generate high power at THz frequencies; and more investigations are required to improve the interaction impedance between the beam and DBE eigenwaves, and also to better understand the role of metal losses at this frequency. Other geometries could also be investigated employing concepts in metamaterials [129] and multi-channel waveguides that can also designed to exhibit high orders of EPDs by increasing the number of channels.

5.5.2 Cylindrical waveguide with periodically corrugated surface

Another potential SWS in Fig. 5.20 is a cylindrical waveguide whose inner walls are periodically-patterned in both the longitudinal and azimuthal directions, and it is referred to as periodic surface lattice and has been proposed for BWO operation in [5, 130]. The cylindrical waveguide walls are periodically-patterned in both the z and ϕ -direction such that the inner radius of the waveguide is $r = r_0 + \Delta r \cos(\beta z) \cos(l\phi)$ where r_0 is the mean radius of the unperturbed cylindrical waveguide, Δr is the amplitude of the perturbation, l is an integer representing the number of azimuthal variations and $\beta = 2\pi/d$, with d being the lattice longitudinal period. Such corrugated waveguide allows coupling between the surface eigenwaves supported by the corrugations and the cylindrical waveguide eigenmodes as shown in [5]. Here we show the eigenmode dispersion relation between the propagation Bloch wavenumber and the angular frequency (i.e., the k - ω relation) of the periodic structure whose parameters are given in the caption of Fig. 5.20. The dispersion diagram in Fig. 5.20(d) has been calculated using the finite element method, implemented in the Eigenmode solver of CST Microwave Studio. The corrugated waveguide structure is overmoded as seen from Fig. 5.20(d) meaning that there are many propagating eigenwaves existing at different bands of frequencies especially in the range between 40 and 45 GHz. However, some eigenmodes can be seen to exhibit flat dispersion near the band edge at $kd=0$, as shown from the dotted fitting lines in the dispersion plot in Fig. 5.20(d). Such observation can be beneficial to engineer the output mode distribution and may lead to high efficient operation when synchronized with an electron beam at the DBE.

5.6 Conclusion

New SWSs that exhibit four degenerate electromagnetic guided modes are proposed. These four degenerate eigenmodes are all synchronized with the electron beam leading to a new

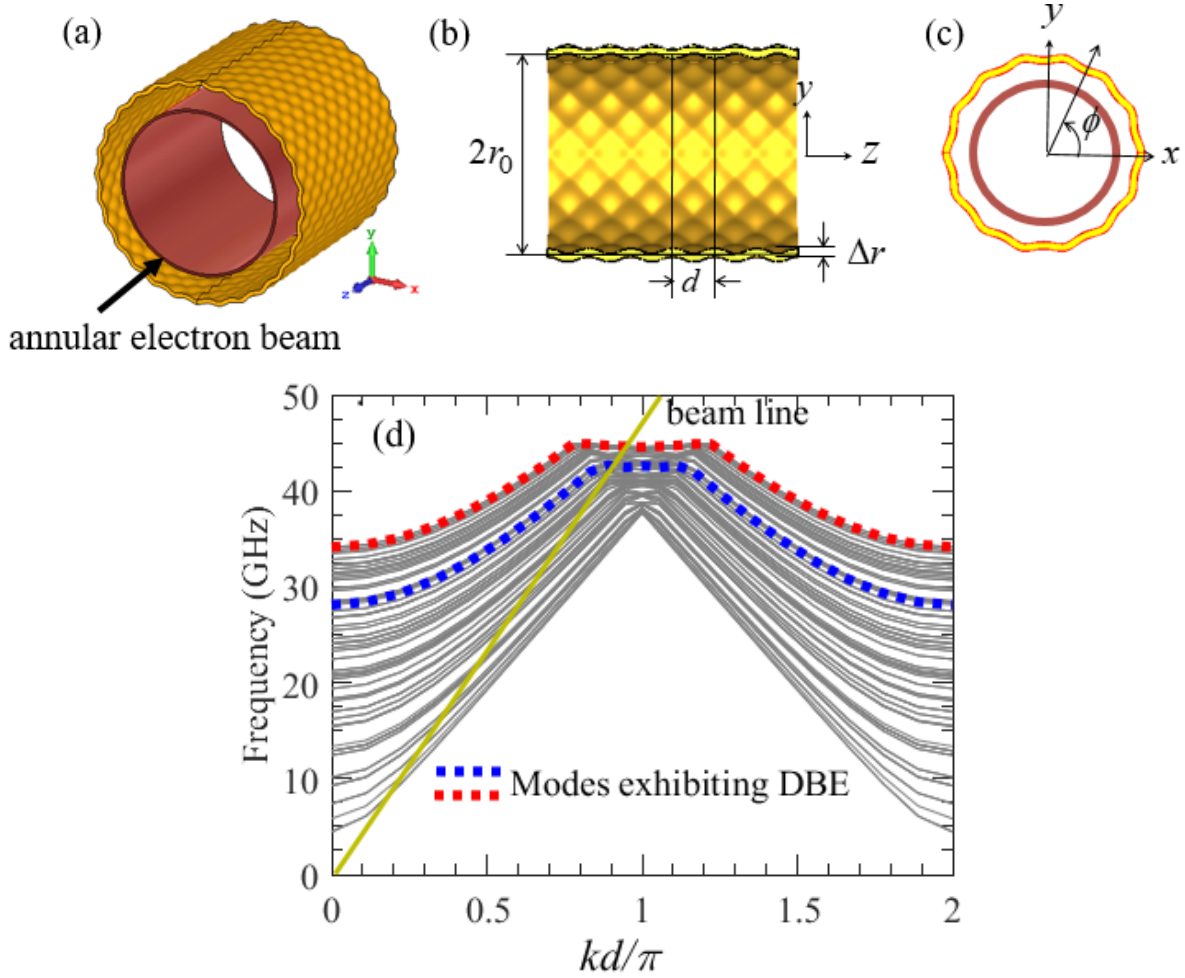


Figure 5.20: Schematics of periodic surface lattice on a cylindrical waveguide's inner wall, i.e., where inner radius is perturbed. (a) 3D perspective view; (b) longitudinal cut, and (c) transverse cross sections of such waveguide with corrugations, adapted from [5]. This structure can support a DBE. This specific geometry is compatible with annular electron beams. (d) Dispersion relation of modes in the waveguide showing an overmoded behavior and possible DBEs (showing in dotted lines) exhibiting flat dispersion in the mm-wave frequency range. Here, the parameters are mm, mm, and mm.

class of oscillators called degenerate band edge oscillators. This regime of operation causes enhanced interaction with the electron beam with potential advantages in terms of low starting current, high power efficiency, and single frequency of oscillation with excellent spectral purity. The preliminary example shown in this chapter, though not representing an optimized structure for high efficiency, shows the first examples where control of eigenmodes for both a fourth-order degeneracy and a proper longitudinal field distribution is possible.

Chapter 6

An Accurate Traveling Wave Tube Analytic Model

6.1 Motivation

We construct here an analytical model of traveling wave tube (TWT) based on the Lagrangian field theory upgrading its constants to be frequency-dependent. The frequency dependence of parameters of the TWT slow wave structure (SWS) is recovered from full-wave simulations by standard software (CST) without curve fitting. We implemented the construction for a helical-based TWT and found an excellent agreement between the model and particle in cell (PIC) simulations [131]. Our additional studies show that the proposed approach applies to different SWSs.

Vacuum electron devices (VEDs) are used widely for radar and satellite communications applications for several decades due to their high power operational capabilities and reliability[96, 109]. VEDs operation is based on the synchronous interaction of an electromagnetic (EM) wave in a slow wave structure (SWS) with the electron beam [132, 101].

We advance here an analytical model of traveling wave tubes (TWTs) based on the Lagrangian field as in [133, 134] upgrading its constants to be frequency-dependent. We refer to an eigenmode as “hot” if it is the one associated with the full-interactive TWT system and as “cold” if it is associated with the SWS (without the presence of the electron beam). The hot eigenmodes involve both the charge wave and the EM fields and may have a complex-valued wavenumber. The hot eigenmodes is one of the subjects of our studies. The commonly taken initial step in the studies of TWT eigenmodes is to consider the cold eigenmodes in the SWS in order to establish conditions providing synchronous interaction between the charge wave on the electron beam and the EM wave in the SWS. It is well known that the synchronization occurs when the speed of the EM wave in the SWS matches the speed of the beam electrons to facilitate effective energy transfer[101, 97, 109]. While the studies of the EM eigenmodes in the cold SWS are important for making good choices when designing the TWT, its actual efficiency is fully manifested only in hot eigenmodes. The hot eigenmodes, in particular, carry significant information on electron beam instabilities such as convective and absolute instability [135], which are crucial for electron beam energy harvesting (i.e., the energies transfer from the electron beam to the EM wave). The hot mode exponential growth in space is expressed through the relevant complex-valued wavenumber with non-zero imaginary part representing the TWT gain.

In most cases, a theoretical model or a computer simulation is used to model and design TWTs. Pierce’s classical small-signal theory has been widely used for modeling and designing TWTs for about seventy years [136, 101]. Pierce introduced the 3-wave theory and described the dispersion relation as a cubic polynomial (i.e., also know as 3-wave dispersion) [132] that is fully characterized by 4 dimensionless constants. Many earlier works are carried out to quantitatively evaluate those constants ([137, 138], and references therein). Other studies have extended the work by Pierce to theoretically model TWTs as in [135, 99]. Recently, a numerical eigenmode solver for hot eigenmodes in TWT systems was introduced based fully on PIC simulations in [139]. In spite of being excellent tools for initial design of

TWT, theoretical models are not reliable, and actual performance is determined by accurate simulations. In this research we present a mean to narrow the gap between the theoretical predictions and simulation results.

One of the focuses of our efforts here is to recover information about eigenmodes from a very reduced set of PIC simulations (as explained later on, we will need a PIC simulation at only one frequency). The biggest challenge in addressing this subject is the extraction of useful information about eigenmodes from PIC full-wave simulations is not by any means a simple straightforward problem. We address the problem by a thoughtful selection of special regimes of TWT operation in which the hot eigenmode features are manifested in the most pronounced and undisturbed form. This approach utilizes information of significant physical quantities obtained in PIC simulations, such as EM fields and electrons' energy, and it allows to extract the spectral information in the form of complex-valued wavenumbers as well as harmonics of the hot Floquet-Bloch eigenmodes..

The above mentioned spectral information allows in turn to reconstruct the analytical model parameters and its frequency dependence. Using then the analytical model, we obtain in particular the complex-valued $(\omega-k)$ dispersion relation of the hot eigenmodes in the TWT (i.e., in the beam-EM interactive system). This dispersion relation is compared side by side with data obtained from PIC simulations at various frequencies. As one can see from Fig. 6.1, the dispersion relations based on the recovered analytical model are in excellent agreement with three dimensional PIC multi-frequency simulations for the helical-based TWT shown in Fig. 6.2. The dispersion relations and the frequency dependence of the imaginary part of the complex-valued wave numbers provide valuable information on the TWT operation. For instance, the amplification region refers to the range of the frequencies where $\Im(k) > 0$ and the TWT gain is directly related to $\Im(k)$. The constructed analytical model with frequency dependence parameters accomplishes our primary goal to model TWT operations for a wide range of frequencies accurately as verified by the comparison with PIC simulations. Taking

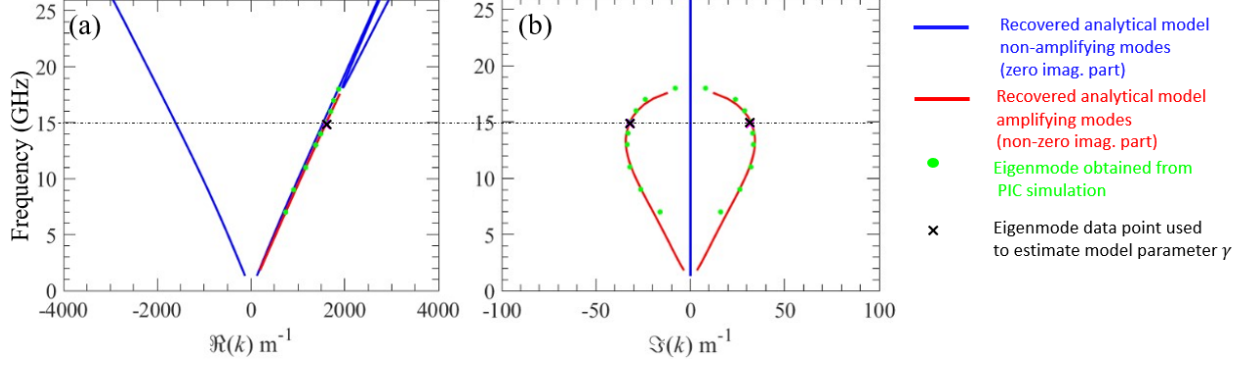


Figure 6.1: Complex-valued wavenumbers of the hot eigenmodes in the interactive electron beam-EM mode in the helical SWS considered in this chapter in Fig. 6.2(a). (a-b) Real-imaginary parts of the recovered dispersion by estimating the adjusted analytical model (γ) parameter using PIC simulations data (the black cross data point) and the frequency dependence of $w(\omega)$ and $\gamma(\omega)$ directly from the cold CST eigenmode solver in Fig. 6.10. A reasonable agreement is observed in the real part in (a) as well as the imaginary part in (b) between the recovered adjusted analytical model and PIC simulations data (the green dots).

into account the complexity of TWT operation and its simulation, the simplicity of the analytical model and its excellent agreement with PIC simulations is rather remarkable.

The remainder of this chapter is organized as follows. Section 6.2 presents a brief review of the analytical model used to describe the system and associated model parameters. Section 6.3 presents the proposed approach to obtain hot eigenmodes of interest in a TWT using a single PIC simulation (at one frequency). Section 6.4 demonstrates the connection between the analytical model and actual electron beam device based on eigenmode and dispersion relations analysis. Conclusions are presented in Section 6.5.

6.2 Mathematical Formulation

We briefly review the analytical model of TWTs used in this chapter. An effective mathematical model for a TWT was introduced by Pierce [136, 101]. This model can be considered as the simplest one that accounts for EM wave amplification and the electron beam energy conversion into microwave radiation in the TWT [98, 97, 109]. The Pierce model, also known

as the 4-wave theory of a TWT, is one-dimensional linear theory in which the SWS is represented by a lossless transmission line (TL), assumed to be homogeneous, that is, with uniformly distributed capacitance and inductance [132, 101, 97]. An approximation to the 4-wave theory is the 3-wave small-signal theory which laid the foundation for TWT design [101].

The analytical model used in this chapter is a generalization of the Pierce theory, based on the Lagrangian field framework presented in Refs. [133, 134], that, in particular, takes into account the space charge effects. The Lagrangian field theory in [133, 134], allows also to model more complex SWSs than the simple one represented by Pierce by involving more than one SWS mode and multi-stream beam. We provide below a concise summary of the simplest case of a single-stream electron beam coupled to a single TL representing the primary eigenmode of the SWS.

The state of the TWT system is described by variables $Q(t, z) = \int_{t_0}^t I(t', z) dt'$ and $q(t, z) = \int_{t_0}^t i(t', z) dt'$ where I and i are respectively the TL line and the electron beam currents, t_0 is the initial time. Variables Q and q represent the amount of charge that has traversed the cross-section of the transmission line and the electron beam, respectively, at point z , from time t_0 to time t . Then, following [134], we introduce the TWT-system Lagrangian $\mathcal{L}_{TB}$ as

$$\mathcal{L}_{TB} = \mathcal{L}_{Tb} + \mathcal{L}_B \quad (6.1)$$

where the Lagrangian components $\mathcal{L}_{Tb}$ and $\mathcal{L}_B$ are associated with the SWS and the electron beam respectively and are defined as follows

$$\mathcal{L}_{Tb} = \frac{L}{2} (\partial_t Q)^2 - \frac{1}{2C} (\partial_z Q + b \partial_z q)^2, \quad (6.2)$$

$$\mathcal{L}_B = \frac{v_0^2 \partial_z q \partial_t q}{2\beta} - \frac{2\pi}{\sigma_B} q^2. \quad (6.3)$$

Here, σ_B is the cross-section of electron beam and v_0 is the electron beam stream velocity. The symbols ∂_t, ∂_z represents the partial derivative with respect to time t and space z , respectively. The parameters L and C are the , respectively, distributed inductance and capacitance associated with the single TL. The term b in (6.2) describes how the electron beam couples to the TL. The representation of the coupling between an electron beam and a SWS goes back to Ramo [140]. The debunching effects are considered by the term $-2\pi q^2/\sigma_B$ in equation (6.3). The parameter β is the electron beam stream intensity and it equals $\sigma_B R_{sc}^2 \omega_p^2/(4\pi)$, where ω_p, R_{sc} are the corresponding plasma frequency and plasma frequency reduction factor, respectively. The Euler-Lagrange equations associated with the Lagrangian are the following system of second-order differential equations:

$$L \partial_t^2 Q - \frac{1}{C} (\partial_z^2 Q + b \partial_z^2 q) = 0, \quad (6.4)$$

$$\frac{1}{\beta} (\partial_t + v_0 \partial_z)^2 q + \frac{4\pi}{\sigma_B} q - \frac{b}{C} (\partial_z^2 Q + b \partial_z^2 q) = 0. \quad (6.5)$$

Since the beam parameters are assumed constant in space, we can make use of the dispersion relation to study the eigenmodes. With that in mind, we consider solutions of the form $q(z, t), Q(z, t) \propto e^{j(\omega t - kz)}$. In the case of spatially uniform (homogeneous) TL, the Fourier

transform, in time t and space variable z , of equations (6.4), (6.5) yields

$$\left(\frac{k^2}{C} - \omega^2 L\right) \hat{Q} + \frac{k^2}{C} b \hat{q} = 0 \quad (6.6)$$

$$\left[\frac{4\pi}{\sigma_B} - \frac{1}{\beta} (\omega - v_0 k)^2\right] \hat{q} + \frac{k^2}{C} (b^2 \hat{q} + b \hat{Q}) = 0 \quad (6.7)$$

where ω and k are respectively the frequency and the wavenumber, and $\hat{Q} = \hat{Q}(\omega, k)$ and $\hat{q} = \hat{q}(\omega, k)$ are the Fourier transforms of the system variables $Q(t, z)$ and $q(t, z)$, respectively. The above system of linear equations (6.6), (6.7) is of special interest to us as for it encodes important information on the TWT system including its dispersion relations and the structure of the eigenmodes. For every fixed ω , the linear system (6.6), (6.7) are viewed as a kind of eigenvalue problem, which is not the standard eigenvalue problem, where the k is an eigenvalue and the pair $\hat{Q}, \hat{q}$ forms an eigenvector, see details in [134]. Taking into account the significant role played by the velocities in electron flow interactions, we recast equations (6.6), (6.7) by substituting there $k = \omega/u$ where u is the phase velocity of a mode in the interactive system. We then solve the system of equations (6.6), (6.7) assuming non-trivial (non-zero) solutions and after elementary transformations we arrive at the following dimensionless form of the dispersion relation

$$D_s(u, \omega) = \frac{(v_0 - u)^2}{u^2} + \frac{\gamma}{w^2 - u^2} - \frac{R_{sc}^2 \omega_p^2}{\omega^2} = 0 \quad (6.8)$$

where $\gamma = \beta b^2/C$ is the system coupling parameter [134] with unit of [velocity]², and $w = 1/\sqrt{LC}$ is the cold SWS mode phase velocity (i.e., TL model). The Euler-Lagrange equations (6.4), (6.5) and the system of second-order differential equations (6.6), (6.7) are written in centimeter–gram–second (CGS) system, whereas the dispersion relation is dimensionless, hence in the following we will use SI units for convenience. In SI units, the parameters R_{sc}

and ω_p are given in [141]. The translation between the Lagrangian model parameters used in this framework and the parameters used in Pierce model is listed in the Supplementary Material of [141].

The dispersion equation (6.8) has three main parameters: (i) the electron stream velocity v_0 ; (ii) the SWS TL modal phase velocity w , and (iii) the system coupling parameter γ . Recovering the correct values of these parameters as functions of the angular frequency ω is not straight forward, and the purpose of this chapter is to develop an approach that uses values of these parameters without any recourse to “curve fitting” approximations but rather based on simple cold SWS full-wave simulations and a single three dimensional PIC simulation as explained next.

Analytical model adjustment

In this subsection, we present a way of improving the agreement between the analytical model previously discussed and real device data by introducing a phenomenological adjustment to the analytical model. The main physics-based modification is the introduction of frequency dependence of parameters w and γ (6.8) that were assumed to be constant in the analytic model. In other words, we adjust the analytical model by assuming that $w = w(\omega)$ and $\gamma = \gamma(\omega)$ in (6.8), and that yields

$$D_s(u, \omega) = \frac{(v_0 - u)^2}{u^2} + \frac{\gamma(\omega)}{w^2(\omega) - u^2} - \frac{R_{sc}^2 \omega_p^2}{\omega^2} = 0. \quad (6.9)$$

The frequency dependence of analytical model parameters (w and γ) is invoked as an ad-hock to the final characteristics equation in (6.8). In the following sections, we present an approach to recover the frequency dependent parameters by exploring solution sets of the dispersion equation (6.8) at one or more frequencies and comparing that to the hot complex-valued

eigenmode observations obtained based on PIC simulations.

6.3 TWT hot eigenmodes based on PIC simulations

Our goal here is to develop an approach of recovering TWT eigenmode information by a thoughtful selection of regimes of TWT operation. The selected regime carry TWT eigenmode information in the most pronounced and undisturbed form. The approach features estimates of the hot eigenmode information, such as (ω, k) , where ω and k are the angular frequency and the associated complex-valued wave number, respectively, based on the EM fields and particles information obtained in PIC simulation. In general, there is no straightforward way to extract an information about the eigenmodes from commonly performed PIC simulation.

6.3.1 Eigenmode-like operation regime

The eigenmode-like regime of operation is determined by a set of conditions that facilitate manifesting the eigenmode frequency and wavenumber through analyzing observable parameters in PIC simulations. . Those conditions include, first, limiting our setup to the spatial region where the cold SWS has a single dominant mode. Consequently, the 4-wave theory based on Lagrangian formalism in Sec. 6.2 is expected to provide an accurate account for the interaction between the electron beam and the wave in the SWS. Second, we would like to suppress backward waves by all means available including introducing a sever. Third, we choose TWT regimes of operation for which nonlinear effects are minimal (i.e., negligible). Suppose now that the above conditions are implemented. Then we carry out PIC simulation assuming that the complex time representation of a chosen observable s (for instance, electric or magnetic field) after reaching the steady state can be represented as follows:

$$s(t, z) = \sum_{i=1}^4 a_i \exp \{j (\omega t - k_i z)\}, \quad (6.10)$$

where $\exp \{j (\omega t - k_i z)\}$ represent the TWT hot eigenmodes, ω and $k_i = k_i(\omega)$ are the associated angular frequency and complex-valued wavenumber, respectively. The four modes are the resultant of the interaction between two EM waves in the cold SWS (along $\pm z$) along with two electron beam space-charge waves.

In a simple fully interactive system, four hot eigenmodes form the basis for two regions of operation

1. Amplification region: in this region the four modes are divided into two sets of modes first, amplifying/attenuating modes such that two modes wavenumbers are complex conjugate to each other (i.e. $k_1 = k_2^*$) second, oscillatory (unamplifying/unattenuating) modes such that their wavenumbers are real (i.e., $\Im(k_3) = \Im(k_4) = 0$, oscillatory mode varies harmonically in time and bounded in the entire space by a constant).
2. Non-amplified region: in this region the four modes are unamplifying/unattenuating modes as their wavenumbers are real (i.e. $\Im(k_{1:4}) = 0$).

Assumption 1: In the amplification region the amplifying eigenmode is the dominant one. Such that, for a large value of z and after reaching the steady state any observable physical quantity in (6.10) can readily be approximated as follows:

$$s(t, z) \approx a \exp \{j (\omega t - k_i z)\} \quad (6.11)$$

where $\Im \{k_i\} > 0$, so that the mode $\exp \{j (\omega t - k_i z)\}$ is amplifying and exponentially grow-

ing in space. In view of Assumption 1, the structure hot amplified eigenmode $\exp \{j (\omega t - k_i z)\}$ can be detected by considering the Fourier transformation in spatial variable of one or more of the observable physical quantities in the PIC simulation. The PIC algorithm uses a three dimensional self-consistent solution of Maxwell's equations in the time domain in the presence of an electron beam current made of by a finite number of emitted electrons. It also accounts for saturation effects such as electron overtaking. In the following subsection, we show an example of helix-based SWS and the estimation of the amplified eigenmode using observable physical quantities.

6.3.2 Estimation of the amplified eigenmode in helical TWT

We illustrate the efficiency of the approach described in the previous section by considering an example of a C-Band TWT amplifier shown in Fig. 6.2(a). A helix-based SWS typically consists of a metallic tape-helix inside a metallic waveguide; such SWSs have been used for decades for high power device sources and amplifiers [98, 97, 109]. Figure 6.2(a) shows an example of helix-based TWT optimized to operate at around 15 GHz, with a total length of about 18 cm comprised of 160 unit cells with period $d = 1.04$ mm. TWT unit cells are made of a helix metallic tape with an inner radius of $795 \mu\text{m}$, 0.2 mm thickness and 0.51 mm width. The metallic circular waveguide has a radius of 1.06 mm and the three equally spaced dielectric rods support that physically hold the helix are made of BeO dielectric with a relative dielectric constant of 6.5 . The input and output radio frequency (RF) signals of the structure are defined as input RF port and output RF port as shown in Fig. 6.2(a).

We show in Fig. 6.2(b) the dispersion relations of the modes in the periodic cold SWS. The cold SWS dispersion diagram indicates that the frequency range of the first quasi-TEM forward mode, which is the reasonable for amplification regime of operation, ends around 25 GHz.

The $(\omega-k)$ cold dispersion is calculated using the eigenmode solver implemented in CST Suite Studio by DS SIMULIA based on the finite-element method. The eigenmode solver enforces a phase shift across the structure period in the longitudinal direction of propagation and solves for the real-valued eigenfrequencies. The dispersion diagram is constructed by repeating the simulation for different phase shifts.

The cold dispersion diagram shows both forward and backward Floquet-Bloch propagating harmonics in the fundamental Brillouin zone that is here defined from $kd/\pi = 0$ to $kd/\pi = 2$. The plotted dispersion curves represent only the propagating part of the spectrum and thus, they have a purely real-valued wavenumber k . In addition, it is worth emphasizing that the dispersion diagram obtained in Fig. 6.2(b) is for the cold SWS case (electron beam is absent). However, this cold dispersion is still helpful as a 0th-order approximation to establish synchronization in the fully interactive system (SWS and electron beam). This approximate picture of the interactive system is realized by plotting the “electron beam line” together with the cold dispersion diagram as shown in Fig. 6.3 in the blue-dashed line. As shown in Fig. 6.3, the electron beam line intersects with the TEM eigenmode in blue around 12 GHz.

The hot simulation setup is carried out by CST Studio Suite (PIC solver). The PIC algorithm uses a 3-D self-consistent solution to Maxwell’s equation in the time domain in the presence of an electron beam current made by a finite number of emitted charged particles. It also accounts for saturation effects such as electron overtaking which is responsible for reaching a steady state in an oscillator.

The hot simulation is performed taking into account the eigenmode-like regime considerations by defining particles that are emitted based on the direct current (DC) emission model of the PIC Solver. Such TWT amplifier uses a solid linear electron beam with a radius of $560\text{ }\mu\text{m}$. The particles’ initial velocity is 0.2 times the speed of light ($v = 0.2c$) to have a synchronization around 12 GHz in the region where the cold SWS unit cell can be modeled

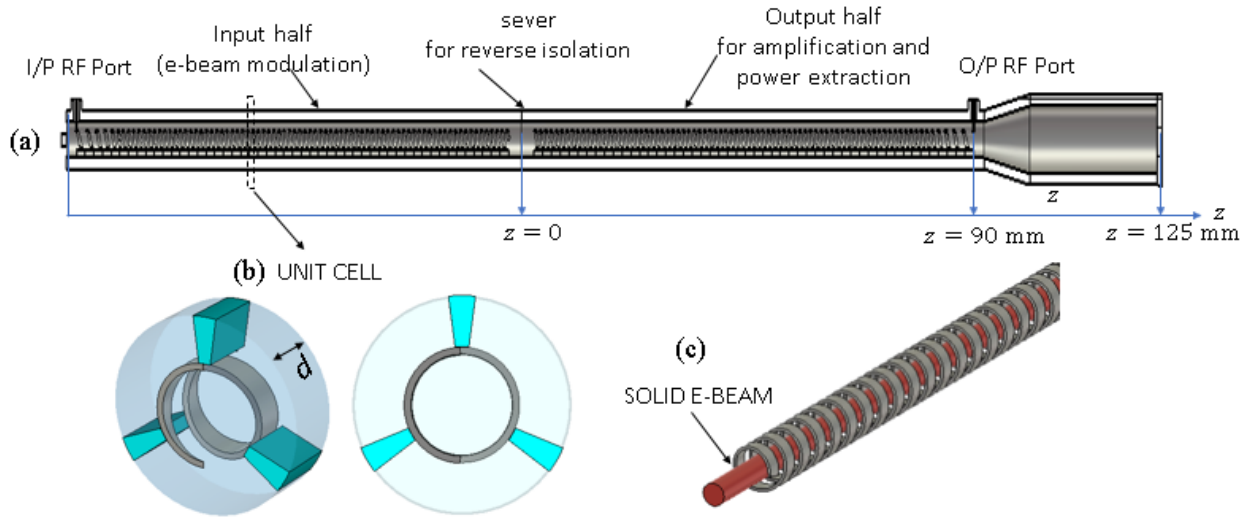


Figure 6.2: (a) Schematic of a helix-based TWT amplifier. (b) (left) Perspective view and (right) front view of a unit cell of the TWT shown in (a) where the tape helix SWS in the circular waveguide supported by three equally spaced dielectric rods. (c) A solid electron beam flows along the axis of the helical conductor.

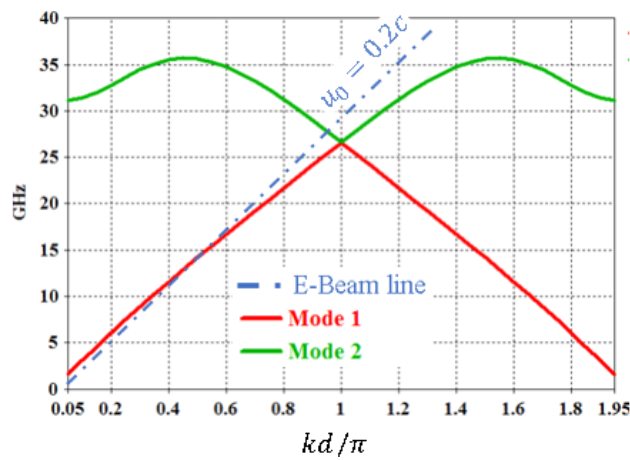


Figure 6.3: Dispersion diagram of the lowest order Floquet-Bloch eigenmodes of the cold SWS in Fig. 6.2. An electron beam (e-beam) line with particles' initial velocity about 0.2 times the speed of light to have a synchronization around the 12 GHz (blue dashed line) is plotted showing that the beam line intersects with the cold dispersion in the forward region.

by one cold eigenmode (in each direction) as shown in the cold dispersion and to have a good matching to minimize any rise of the backward (i.e., reflected) mode due to mismatch at the output RF port. The value of the emitted current was set to 10 mA that is less than the threshold current for oscillation, to minimize nonlinear effects. A static axial magnetic field $B_z = 0.64$ T is used to ensure a good beam confinement of the solid electron beam traveling in the axial direction. The total number of charged particles used to model the electron beam in the PIC simulation is about 8,901,500 while the whole space in the SWS structure is modeled using 6,640,704 mesh cells.

A hot simulation is performed to verify the stability of the structure and its immunity to oscillation by running the CST PIC for no RF excitation at the input RF port (red trace), and the signal observed at the output port (blue trace) is shown in Fig. 6.4(a). It is clear that in the case where there is no RF excitation and since the hot setup configuration, that ensures the stability of the hot structure, the output vanishes after passing the transient time.

Figure 6.4(b) shows the output (in blue) and input (in red) RF signals, indicating the amplification level for a single-tune sinusoidal excitation signal of frequency 15 GHz. The frequency spectrum of the output RF signal shown in Fig. 6.4(c) is obtained by applying Fourier transform to the output RF signal in the time window from 10 nsec to 25 nsec. By utilizing the eigenmode-like regime of operation in the hot setup, one can assume that any observable physical quantity after reaching the steady state resembles the hot amplified eigenmode as in (6.11).

To estimate the amplified hot eigenmode, we analyze the observable physical quantities obtained in the CST PIC simulations. The PIC solver simulates the complex interaction between the EM wave and electron beam using a large number of charged particles and it follows their trajectories in self-consistent electromagnetic fields computed on a fixed mesh. The physical quantities that represent the EM mode in the interacting SWS are electric and

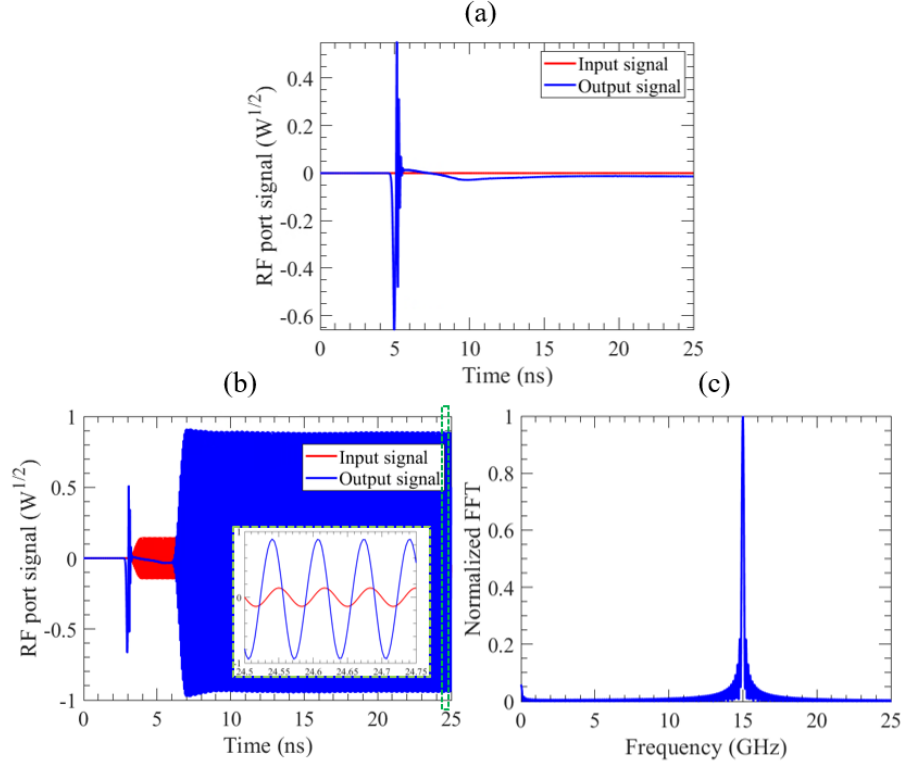


Figure 6.4: (a) Simulated output RF port signal, for the no RF excitation case, shows no signal after some transient time ~ 10 nS, confirms the stability of the structure, and indicates the time required to reach steady state. (b) Simulated output RF port signal in green shows an amplification occurs to the input RF signal of 10 dBm input and 15 GHz in red, (c) frequency spectrum of the output RF signal shows that the output signal has the same frequency of the signal used to excite the input RF port but with a higher amplitude indicates the amplification operation.

magnetic fields. Also, one can observe physical quantities related to the beam particles such as electron beam energy, momentum, and charge density.

Figure 6.5(a) shows a snapshot of the z -component of the transient electric field along the SWS after the steady state is reached (on the gap between the SWS and circular waveguide wall). The phasor-domain representation of the $E_z(z, t)$ versus z along the SWS is calculated as

$$\mathbf{E}_z(z) = \frac{1}{T} \int_{t=t_0}^{t=t_0+T} E_z(z, t) e^{-j\omega t} dt \quad (6.12)$$

where t_0 is any time instant after steady state is observed, and $T = 2\pi/\omega$. The phasor $\mathbf{E}_z(z)$ is depicted in Fig. 6.5(b, c) in terms of magnitude and phase, respectively.

It is apparent that there is more than one spatial frequency component in the field data shown in Fig. 6.5. Those spatial frequency components are obtained by taking the Fourier transform of the $\mathbf{E}_z(z)$. We report the normalized spatial spectrum in Fig. 6.6 showing different spatial frequency components where the fundamental one that carries most of the energy is $k = 1580 \text{ m}^{-1}$. Also, it is worth noting that, the plotted normalized spatial spectrum in Fig. 6.6 invokes other modes from PIC simulations that are not part of the analytical model eigenmodes solution. Those other modes are related to the Floquet-Bloch modes associated with the periodic SWS such as $k_n = k_0 + 2\pi n/d$ where n is an integer number representing the order of harmonics. For the particular illustrative example shown here, we report that $k_0 = 1580 \text{ m}^{-1}$ and $2\pi/d = 6041.5 \text{ m}^{-1}$ accordingly the corresponding -1 Floquet-Bloch harmonic $k_{-1} = -4461.5 \text{ m}^{-1}$, similarly for the negative fundamental $k_0 = -1580 \text{ m}^{-1}$ the corresponding $+1$ Floquet-Bloch harmonic is $k_1 = 4461.5 \text{ m}^{-1}$.

Similarly, we apply the same procedure to a physical quantity related to the electron beam such as particles energy. In reality, the energy of the electron beam particles do not have a single value in each z point usually depends on each particle transverse location, hence it may

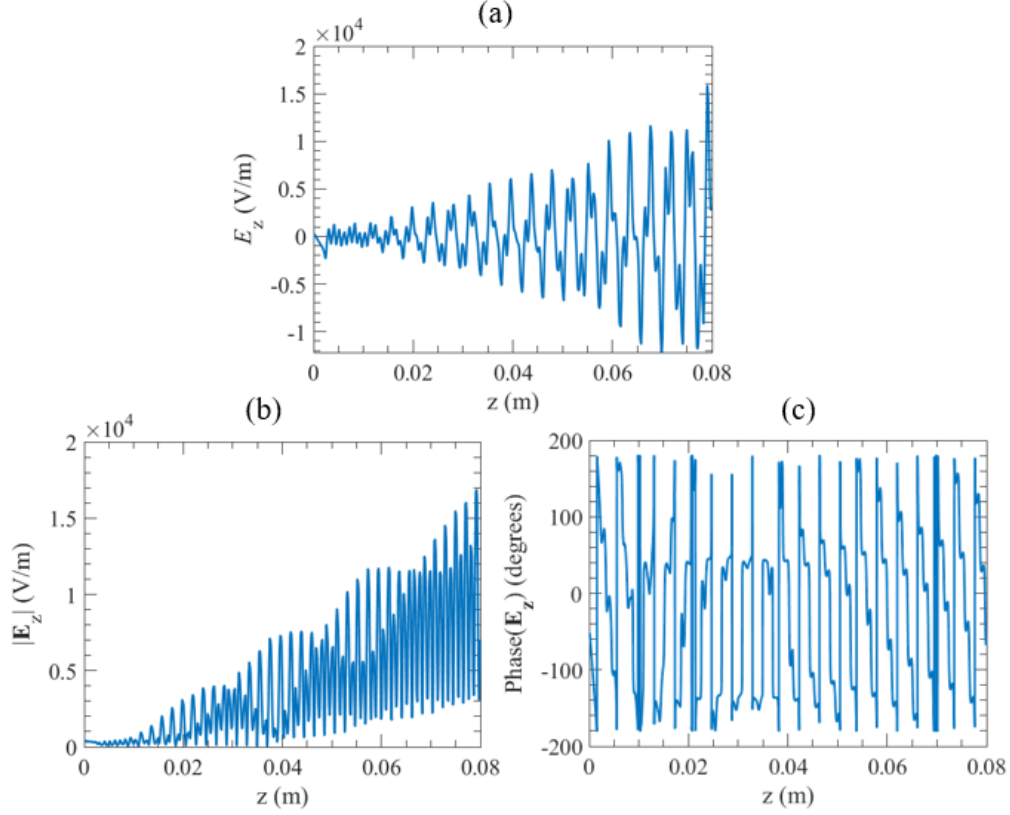


Figure 6.5: (a) A snapshot in time at $t = 10$ ns of the transient $\mathbf{E}_z$ -field just outside the helix along the SWS after reaching steady state time. (b, c) Phasor $\mathbf{E}_z$ -field in terms of amplitude and phase along the SWS.

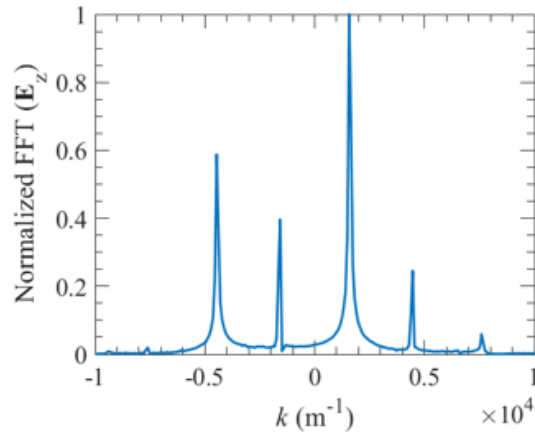


Figure 6.6: Spatial frequency components of longitudinal electric field associated to the hot structure obtained by performing FFT of phasor $\mathbf{E}_z(z)$ shown in Fig. 6.5 in the space window from $z = 0$ to $z = 0.08$ m.

be convenient to deal here with the average of all particles in each transverse cross section such that at each point z the particles energy is represented by a single value representing their average.

Figure 6.7(a) shows a snapshot of the average kinetic energy of electron beam particles along the SWS after the steady state is reached. The phasor-domain representation of small-signal kinetic energy (i.e., by subtracting the time-averaged kinetic energy) of electron beam “AC-Energy” is calculated similarly to the E_z in (6.12). The phasor small-signal kinetic energy is depicted in Fig. 6.7(b, c) in terms of magnitude and phase, respectively.

It is apparent that there is only one spatial frequency component in the electron beam data shown in Fig. 6.7, which is contrary to the electric field spectrum that involves Floquet-Bloch modes due to the periodicity nature of the SWS. We report the normalized spatial spectrum of phasor small-signal kinetic energy in Fig. 6.8 where its spatial frequency is $k = 1595 \text{ m}^{-1}$. This spatial frequency component is directly related to the eigenmode that is harvesting energy from the electron beam.

By analyzing the spatial frequency spectrum of such physical observables at a certain frequency one can estimate the dominant amplified eigenmode complex-valued wavenumber and the corresponding complex-conjugate attenuated eigenmode. For the helical TWT understudy and from the spatial frequency spectrum in Fig. 6.8 and the amplitude growth rate reported in Fig. 6.7(b) the estimated eigenmodes are $(\omega, k) \sim (15\text{GHz}, 1595 \pm j31.6\text{m}^{-1})$.

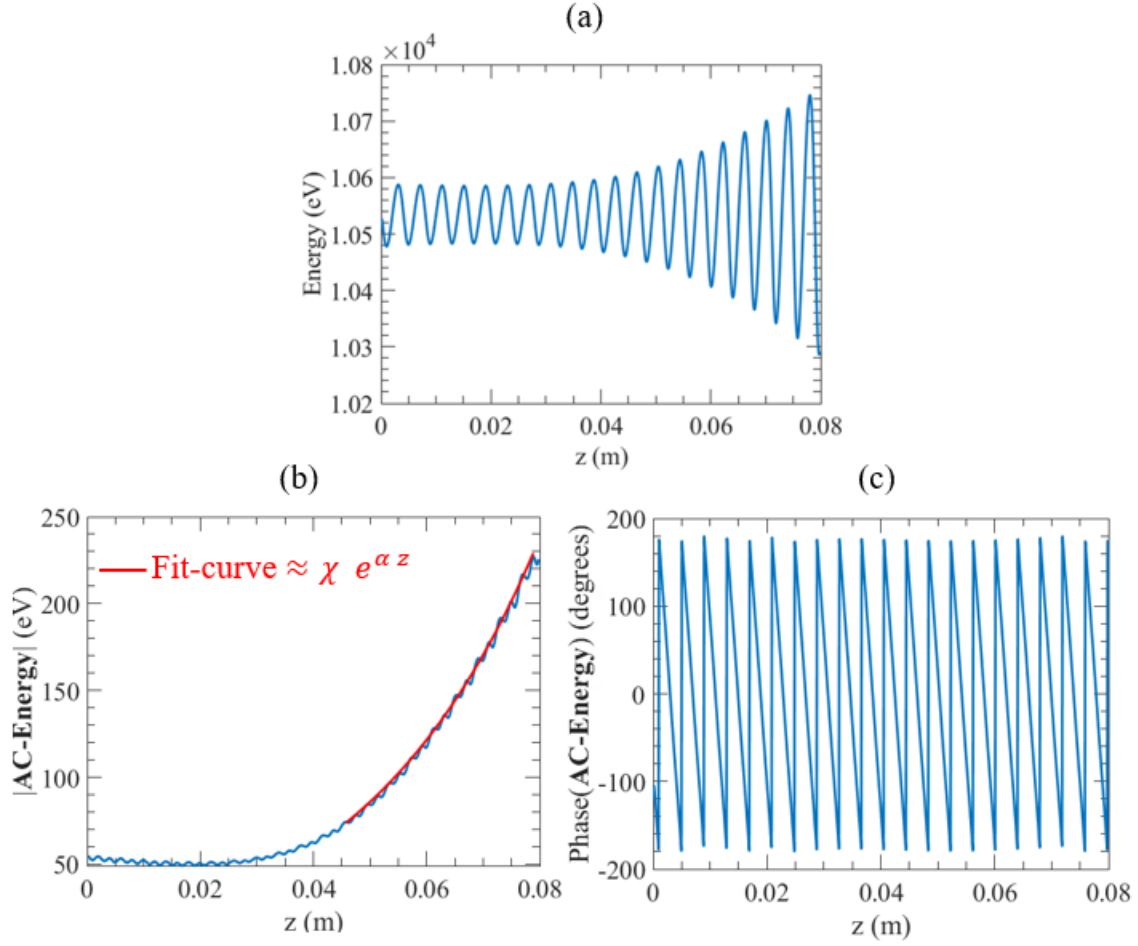


Figure 6.7: (a) A snapshot in time at $t = 10$ nS of the transient electron beam particles energy along the SWS after reaching steady state time. (b, c) Complex-valued AC part of the phasor electron beam particles energy in terms of amplitude and phase. Red line in (b) represents the exponential curve $\chi e^{\alpha z}$ with $\alpha = 31.6 \text{ m}^{-1}$ and $\chi = 19.7 \text{ eV}$ which fits well to the magnitude growth of the AC part of electron beam particles energy.

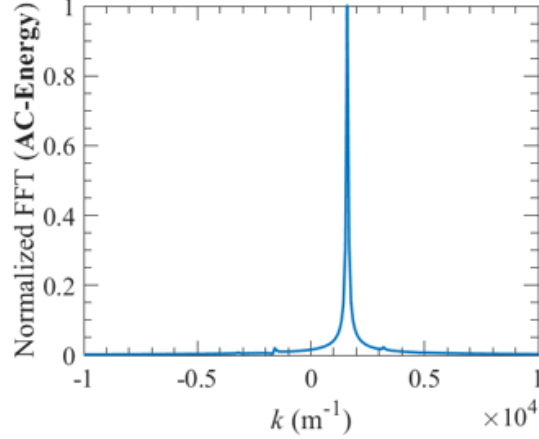


Figure 6.8: Spatial frequency components of electron beam energy associated to the TWT obtained by performing FFT of phasor **AC-Energy** shown in Fig. 6.7 in the space window from $z = 0$ to $z = 0.08$ m.

6.4 Connection between the analytical model and actual electron beam device

TWT observables are used to determine the analytical model parameters (γ, u_0, w) through PIC simulations. As to their qualification to be TWT observables we notice that by their very definition these are eigenmodes associated to the amplified eigenmode in the fully interactive system utilizing an eigenmode-like regime of operation. The wavenumber-frequency dispersion describing the eigenmodes in the TWT is determined by running multiple PIC simulations at different frequencies and then determining hot amplified eigenmode (ω, k) at each. Those eigenmodes can be observed through a series of PIC simulations each where the structure is excited by a single tone RF signal. Notice also that TWT observables give knowledge about the complex-valued wavenumbers k for certain frequency ω at which the TWT dispersion relations as well the TWT characteristic function $D_s(\omega, k)$ solutions attain instabilities (i.e. convective instability referring to growing and decaying waves with space) as discussed in Sec. 6.2.

To facilitate the computation of analytical model parameter γ we would like to optimize

γ to best fit growing solutions of the dispersion (ω, k_{D_s}) to the estimated amplified eigenmode through PIC simulations (ω, k_{PIC}) . This optimization process is applied only at the estimated frequency. The estimated frequency can be any frequency, in the vicinity of the synchronization frequency, within the amplification region. A common way to deal with this kind of problem is to look for analytical parameter γ that minimizes error between the analytical model and PIC data for the estimated frequency ω , by defining the following error function:

$$error = c_1 |\Re(k_{D_s} - k_{PIC})|^2 + |\Im(k_{D_s} - k_{PIC})|^2 \quad (6.13)$$

where c_1 is a weighting coefficient to equate the importance of the real and imaginary parts of the wavenumbers since there are orders of magnitude difference between the real and imaginary parts of k . In particular, given the aforementioned estimated eigenmode $(\omega, k) \sim (15\text{GHz}, 1595 + j31.6\text{m}^{-1})$, c_1 was set to be equal to the ratio between the imaginary and real parts of k , i.e., $c_1 = 31.6/1595$. Then the expression of the *error* function is minimized numerically by optimizing the analytical parameter γ .

For the particular illustrative example shown here, we perform the PIC simulations for different frequencies and follow to the approach described in Sec. 6.3 when estimating the hot amplified eigenmodes for each frequency. Figure 6.9 shows the recovered dispersion for the optimized analytical parameters as following $\gamma = 8.651 \times 10^{10} \text{ m}^2/\text{s}^2, v_0 = 0.2c, w = 0.2c$. The hot eigenmodes obtained from the PIC simulations are plotted by those green circle dots showing a reasonable agreement between the recovered dispersion and real data from PIC simulations for the real part of the dispersion in Fig. 6.9(a). Whereas, the imaginary part of the dispersion shown in Fig. 6.9(b) shows a discrepancy except at 15 GHz “black cross symbol” which is the CST data point used to optimize γ . The discrepancy at the other frequency points can be explained as a result of using a simple analytical model that has

only non-dispersive parameters (i.e., γ , v_0 , and w defined as frequency-independent which is not practically true).

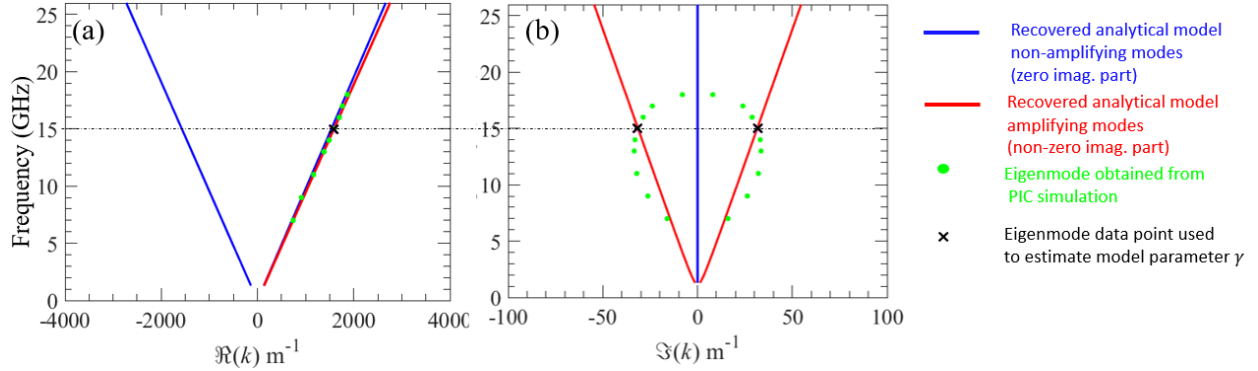


Figure 6.9: Complex-valued wavenumbers of the eigenmodes in the interactive electron beam-EM mode in the helical SWS in Fig. 2 (a). (a) Real part of the recovered dispersion by estimating analytical model parameters using PIC simulations data, a reasonable agreement is observed in the real part. (b) Imaginary part of the dispersion shows a discrepancy between the recovered model and PIC simulations data.

Adjusted analytical model

In this subsection, we show an improved matching between the analytical model and PIC results attributed to the adjusted analytical model. The adjustment is the replacement of constants w and γ with the corresponding frequency dependent functions $w(\omega)$ and $\gamma(\omega)$. The phase velocity frequency dependence $w(\omega)$ is obtained from the cold full-wave simulation by analyzing wave propagation in the SWS in the absence of the electron beam to get the $w(\omega)$ to be used in our analytical model recovery method. The frequency dependence of γ is also obtained from the cold simulation and relation $\gamma(\omega) = B/C(\omega)$ where $C(\omega)$ is the frequency-dependent equivalent capacitance of the TL model of the SWS as discussed in [141], and B is an adjustment constant which is evaluated by minimize the error function in (6.13). We simulated the helix slow-wave circuit by using the finite element-based eigenmode solver implemented in CST Microwave Studio and extracted the (i) cold-circuit EM phase velocity $w(\omega)$ normalized to the speed of light c as shown in Fig. 6.10 and (ii)

the interaction impedance $Z_c(\omega)$ of the coupling mode under interest (i.e., mode 1 “red trace” in the cold dispersion shown in Fig. 6.3). By using $w(\omega)$ and $Z_c(\omega)$, one obtains an equivalent frequency-dependent distributed inductance $L(\omega) = Z_c(\omega)/w(\omega)$ and capacitance $C(\omega) = 1/[Z_c(\omega)w(\omega)]$.

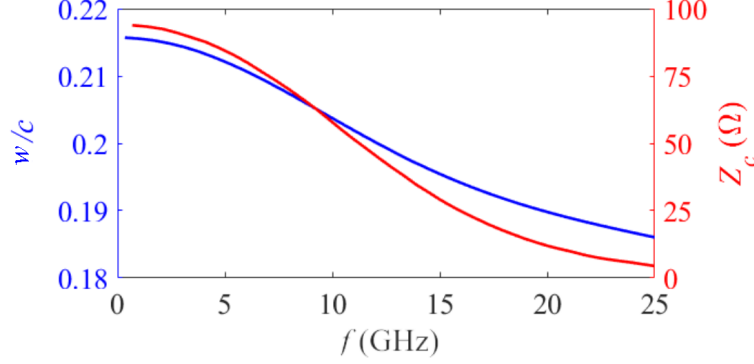


Figure 6.10: (Blue) Normalized phase velocity and (Red) interaction impedance of the first forward mode of the SWS shown in Fig. 6.2(a) obtained from the cold full-wave simulation by analyzing wave propagation in the absence of the electron beam using the finite element-based eigenmode solver.

Figure 6.1 depicts the recovered complex-valued wavenumbers dispersion for the optimized adjusted analytical parameters as following $B = 71.28 \text{ F} \cdot \text{m/s}^2, v_0 = 0.2c$ using $w(\omega)$ obtained from full-wave simulations. Note that, B is evaluated at only one frequency and then the frequency dependence $\gamma(\omega)$ is found from $\gamma(\omega) = B/C(\omega)$. The hot eigenmodes obtained from the PIC simulations are plotted by green circular dots showing a significant improvement in the agreement between the recovered dispersion and real data from PIC simulations.

In Figure 6.11, we plotted side by side the imaginary part of the recovered dispersion for the analytical model in Fig. 6.11(a), and the phenomenological adjusted one in Fig. 6.11(b) to emphasis the significant improvement in matching the model to real data obtained from CST PIC simulations.

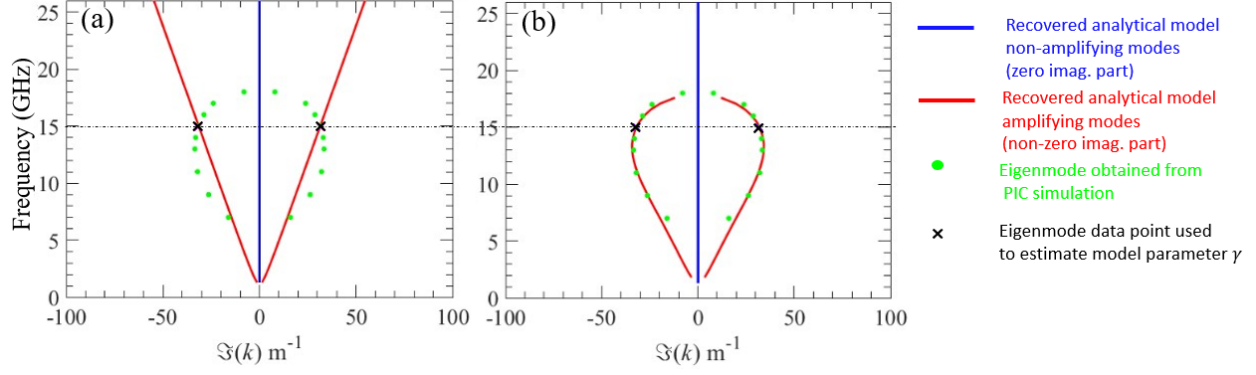


Figure 6.11: Comparison between imaginary parts of the wavenumbers of the hot eigenmodes for the helical SWS shown in Fig. 1(a) for two analytical models: (a) frequency-independent parameters (γ , v_0 , w) analytical model in (6.8), (b) frequency-dependent parameters ($\gamma(\omega)$, v_0 , $w(\omega)$) adjusted analytical model in (6.9).

6.5 Summary

An adjusted analytical model for TWTs is proposed that gives accurate predictions of the dispersion relation and the frequency dependent gain/amplification. The approach utilizes primary frequency-dependent parameters of the cold SWS (modal wave velocity and equivalent TL capacitance) recovered by using a standard procedure, and one PIC simulation of the TWT, at one frequency, to find the adjustment parameter B without curve fitting. . The proposed adjusted analytic model was tested against PIC simulation for a helical-based TWT operating in the GHz range and showed an excellent agreement. Our extensive preliminary studies suggest that the proposed approach can be applied to different kinds of TWTs including those based on the serpentine SWS.

Bibliography

- [1] Alex Figotin and Ilya Vitebskiy. Gigantic transmission band-edge resonance in periodic stacks of anisotropic layers. *Physical Review E*, 72(3):036619, September 2005.
- [2] Alex Figotin and Ilya Vitebskiy. Slow-wave resonance in periodic stacks of anisotropic layers. *Physical Review A*, 76(5):053839, 2007.
- [3] Ahmed F Abdelshafy, Mohamed AK Othman, Farshad Yazdi, Mehdi Veysi, Alexander Figotin, and Filippo Capolino. Electron-Beam-Driven Devices With Synchronous Multiple Degenerate Eigenmodes. *IEEE Transactions on Plasma Science*, 46(8):3126–3138, 2018.
- [4] Alex Figotin and Ilya Vitebskiy. Frozen light in photonic crystals with degenerate band edge. *Physical Review E*, 74(6):066613, December 2006.
- [5] I. V. Konoplev, A. J. MacLachlan, C. W. Robertson, A. W. Cross, and A. D. R. Phelps. Cylindrical periodic surface lattice as a metadielectric: Concept of a surface-field Cherenkov source of coherent radiation. *Physical Review A*, 84(1):013826, July 2011.
- [6] Mohamed A. K. Othman, Vincenzo Galdi, and Filippo Capolino. Exceptional points of degeneracy and $\mathcal{PT}$ symmetry in photonic coupled chains of scatterers. *Phys. Rev. B*, 95(10):104305, March 2017.
- [7] Christian E. Rüter, Konstantinos G. Makris, Ramy El-Ganainy, Demetrios N. Christodoulides, Mordechai Segev, and Detlef Kip. Observation of parity-time symmetry in optics. *Nature Physics*, 6(3):192–195, 2010.
- [8] A. Guo, G. J. Salamo, D. Duchesne, R. Morandotti, M. Volatier-Ravat, V. Aimez, G. A. Siviloglou, and D. N. Christodoulides. Observation of $\mathcal{PT}$ -symmetry breaking in complex optical potentials. *Phys. Rev. Lett.*, 103:093902, Aug 2009.
- [9] T. Stehmann, W. D. Heiss, and F. G. Scholtz. Observation of exceptional points in electronic circuits. *Journal of Physics A: Mathematical and General*, 37(31):7813, 2004.
- [10] Bo Zhen, Chia Wei Hsu, Yuichi Igarashi, Ling Lu, Ido Kaminer, Adi Pick, Song-Liang Chua, John D. Joannopoulos, and Marin Soljačić. Spawning rings of exceptional points out of Dirac cones. *Nature*, 525(7569):354–358, 2015.

- [11] M. V. Berry. Physics of nonhermitian degeneracies. *Czechoslovak journal of physics*, 54(10):1039–1047, 2004.
- [12] A. Figotin and I. Vitebskiy. Oblique frozen modes in periodic layered media. *Physical Review E*, 68(3):036609, September 2003.
- [13] R. El-Ganainy, K. G. Makris, D. N. Christodoulides, and Ziad H. Musslimani. Theory of coupled optical PT-symmetric structures. *Optics letters*, 32(17):2632–2634, 2007.
- [14] I. V. Barashenkov, L. Baker, and N. V. Alexeeva. PT-symmetry breaking in a necklace of coupled optical waveguides. *Physical Review A*, 87(3):033819, 2013.
- [15] James Gear, Fu Liu, S. T. Chu, Stefan Rotter, and Jensen Li. Parity-time symmetry from stacking purely dielectric and magnetic slabs. *Physical Review A*, 91(3):033825, 2015.
- [16] Z. H. Musslimani, Konstantinos G. Makris, R. El-Ganainy, and D. N. Christodoulides. Optical Solitons in P T Periodic Potentials. *Physical Review Letters*, 100(3):030402, 2008.
- [17] Ming Kang, Fu Liu, and Jensen Li. Effective spontaneous PT-symmetry breaking in hybridized metamaterials. *Physical Review A*, 87(5):053824, 2013.
- [18] Joseph Schindler, Ang Li, Mei C. Zheng, Fred M. Ellis, and Tsampikos Kottos. Experimental study of active LRC circuits with PT symmetries. *Physical Review A*, 84(4):040101, 2011.
- [19] S. Bittner, B. Dietz, U. Günther, H. L. Harney, M. Miski-Oglu, A. Richter, and F. Schäfer. P T Symmetry and Spontaneous Symmetry Breaking in a Microwave Billiard. *Physical review letters*, 108(2):024101, 2012.
- [20] Ahmed F Abdelshafy, Mohamed AK Othman, Dmitry Oshmarin, Ahmad T Almutawa, and Filippo Capolino. Exceptional points of degeneracy in periodic coupled waveguides and the interplay of gain and radiation loss: Theoretical and experimental demonstration. *IEEE Transactions on Antennas and Propagation*, 67(11):6909–6923, 2019.
- [21] Hamidreza Kazemi, Mohamed Y. Nada, Tarek Mealy, Ahmed F. Abdelshafy, and Filippo Capolino. Exceptional Points of Degeneracy Induced by Linear Time-Periodic Variation. *Physical Review Applied*, 11(1):014007, January 2019.
- [22] Ahmed F Abdelshafy, Tarek Mealy, Ehsan Hafezi, Alireza Nikzamir, and Filippo Capolino. Exceptional degeneracy in a waveguide periodically loaded with discrete gain and radiation loss elements. *Applied Physics Letters*, 118(22):224102, 2021.
- [23] Tarek Mealy, Ahmed F Abdelshafy, and Filippo Capolino. Exceptional point of degeneracy in a backward-wave oscillator with distributed power extraction. *Physical Review Applied*, 14(1):014078, 2020.

- [24] Tarek Mealy, Ahmed F Abdelshafy, and Filippo Capolino. High-power backward-wave oscillator using folded waveguide with distributed power extraction operating at an exceptional point. *IEEE Transactions on Electron Devices*, 2021.
- [25] Tarek Mealy, Ahmed F Abdelshafy, and Filippo Capolino. Exceptional point of degeneracy in linear-beam tubes for high power backward-wave oscillators. *arXiv preprint arXiv:2005.08912*, 2020.
- [26] S. Ali Hassani Gangaraj and Francesco Monticone. Topological Waveguiding near an Exceptional Point: Defect-Immune, Slow-Light, and Loss-Immune Propagation. *Physical Review Letters*, 121(9):093901, August 2018.
- [27] George W. Hanson, Alexander B. Yakovlev, Mohamed A. K. Othman, and Filippo Capolino. Exceptional points of degeneracy and branch points for coupled transmission lines—linear-algebra and bifurcation theory perspectives. *IEEE Transactions on Antennas and Propagation*, 67(2):1025–1034, 2019.
- [28] Alex Figotin and Ilya Vitebskiy. Slow light in photonic crystals. *Waves in Random and Complex Media*, 16(3):293–382, August 2006.
- [29] Albert Ferrando, Enrique Silvestre, Pedro Andrés, Juan J. Miret, and Miguel V. Andrés. Designing the properties of dispersion-flattened photonic crystal fibers. *Optics Express*, 9(13):687–697, December 2001.
- [30] Mohamed A. K. Othman, Farshad Yazdi, Alex Figotin, and Filippo Capolino. Giant gain enhancement in photonic crystals with a degenerate band edge. *Physical Review B*, 93(2):024301, January 2016.
- [31] Salih Yarga, Kubilay Sertel, and John L Volakis. Degenerate band edge crystals for directive antennas. *IEEE transactions on antennas and propagation*, 56(1):119–126, 2008.
- [32] Gokhan Mumcu, Kubilay Sertel, and John L Volakis. Miniature antenna using printed coupled lines emulating degenerate band edge crystals. *IEEE transactions on antennas and propagation*, 57(6):1618–1624, 2009.
- [33] Salih Yarga, Kubilay Sertel, and John L Volakis. A directive resonator antenna using degenerate band edge crystals. *IEEE transactions on antennas and propagation*, 57(3):799–803, 2009.
- [34] Justin R. Burr, Nadav Gutman, C. Martijn de Sterke, Ilya Vitebskiy, and Ronald M. Reano. Degenerate band edge resonances in coupled periodic silicon optical waveguides. *Optics Express*, 21(7):8736–8745, April 2013.
- [35] Michael G. Wood, Justin R. Burr, and Ronald M. Reano. Degenerate band edge resonances in periodic silicon ridge waveguides. *Optics Letters*, 40(11):2493–2496, 2015.
- [36] Mehdi Veysi, Mohamed A. K. Othman, Alexander Figotin, and Filippo Capolino. Degenerate band edge laser. *Physical Review B*, 97(19):195107, May 2018.

- [37] A. A. Chabanov. Strongly resonant transmission of electromagnetic radiation in periodic anisotropic layered media. *Physical Review A*, 77(3):033811, 2008.
- [38] Michael G. Wood, Justin R. Burr, and Ronald M. Reano. 7 nm/V DC tunability and millivolt scale switching in silicon carrier injection degenerate band edge resonators. *Optics Express*, 24(20):23481–23493, 2016.
- [39] Nil Apaydin, Lanlin Zhang, Kubilay Sertel, and John L Volakis. Experimental validation of frozen modes guided on printed coupled transmission lines. *IEEE transactions on microwave theory and techniques*, 60(6):1513–1519, 2012.
- [40] Mohamed Othman and Filippo Capolino. Demonstration of a Degenerate Band Edge in Periodically-Loaded Circular Waveguides. *IEEE Microwave and Wireless Components Letters*, 25(11):700–702, 2015.
- [41] Mohamed A. K. Othman, Xuyuan Pan, Yiorgos Atmatzakis, and Christos G. Christodoulou. Experimental Demonstration of Degenerate Band Edge in Metallic Periodically-Loaded Circular Waveguide. *IEEE Microwave Theory and Techniques*, 1611(8):9, 2017.
- [42] Mohamed AK Othman, Mehdi Veysi, Alexander Figotin, and Filippo Capolino. Low Starting Electron Beam Current in Degenerate Band Edge Oscillators. *IEEE Transactions on Plasma Science*, 44(6):918–929, 2016.
- [43] Dmitry Oshmarin, Farshad Yazdi, Mohamed AK Othman, Jeff Sloan, Mohammad Radfar, Michael M Green, and Filippo Capolino. New oscillator concept based on band edge degeneracy in lumped double-ladder circuits. *IET Circuits, Devices & Systems*, 13(7):950–957, 2019.
- [44] Jeff T Sloan, Mohamed AK Othman, and Filippo Capolino. Theory of double ladder lumped circuits with degenerate band edge. *IEEE Transactions on Circuits and Systems I: Regular Papers*, 65(1):3–13, 2017.
- [45] Mohamed A. K. Othman and Filippo Capolino. Theory of Exceptional Points of Degeneracy in Uniform Coupled-Waveguides and Balance of Loss and Gain. *IEEE Transactions on Antennas and Propagation*, 65(10):1–15, 2017.
- [46] Mohamed Othman, Venkata Ananth Tamma, and Filippo Capolino. Theory and New Amplification Regime in Periodic Multi Modal Slow Wave Structures with Degeneracy Interacting with an Electron Beam. *IEEE Transactions on Plasma Science*, 44(4):594–611, 2016.
- [47] Venkata Ananth Tamma, Alexander Figotin, and Filippo Capolino. Concept for Pulse Compression Device Using Structured Spatial Energy Distribution. *IEEE Transactions on Microwave Theory and Techniques*, 64(3):742–755, 2016.
- [48] Giovanni Miano and Antonio Maffucci. *Transmission lines and lumped circuits: fundamentals and applications*. Academic Press, 2001.

- [49] Clayton R. Paul. *Analysis of multiconductor transmission lines*. John Wiley & Sons, 2008.
- [50] Roger F. Harrington. *Time-harmonic electromagnetic fields*. McGraw-Hill, 1961.
- [51] Carl D. Meyer. *Matrix analysis and applied linear algebra*, volume 2. Siam, 2000.
- [52] Krešimir Veselić. *Damped oscillations of linear systems: a mathematical introduction*, volume 2023. Springer Science & Business Media, 2011.
- [53] Tosio Kato. *Perturbation theory for linear operators*, volume 132. springer, 1995.
- [54] Aaron Welters. On explicit recursive formulas in the spectral perturbation analysis of a Jordan block. *SIAM Journal on Matrix Analysis and Applications*, 32(1):1–22, 2011.
- [55] Mohamed Y. Nada, Mohamed A. K. Othman, Ozdal Boyraz, and Filippo Capolino. Giant Resonance and Anomalous Quality Factor Scaling in Degenerate Band Edge Coupled Resonator Optical Waveguides. *Journal of Lightwave Technology*, 36(14):3030–3039, July 2018.
- [56] Claudius Löcker, Kubilay Sertel, and John L. Volakis. Emulation of propagation in layered anisotropic media with equivalent coupled microstrip lines. *IEEE Microwave and Wireless Components Letters*, 16(12):642–644, 2006.
- [57] Nadav Gutman, C Martijn de Sterke, Andrey A Sukhorukov, and Lindsay C Botten. Slow and frozen light in optical waveguides with multiple gratings: Degenerate band edges and stationary inflection points. *Physical Review A*, 85(3):033804, 2012.
- [58] K-Y Jung and FL Teixeira. Numerical study of photonic crystals with a split band edge: Polarization dependence and sensitivity analysis. *Physical Review A*, 78(4):043826, 2008.
- [59] Heeso Noh, Jin-Kyu Yang, Ilya Vitebskiy, Alex Figotin, and Hui Cao. Giant resonances near the split band edges of two-dimensional photonic crystals. *Physical Review A*, 82(1):013801, 2010.
- [60] K-Y Jung and FL Teixeira. Photonic crystals with a degenerate band edge: Field enhancement effects and sensitivity analysis. *Physical Review B*, 77(12):125108, 2008.
- [61] L. B. Felsen and W. K. Kahn. Transfer characteristics of 2n-port networks. In *Proceedings of the symposium on millimeter waves*, pages 477–512, 1959.
- [62] David M. Pozar. *Microwave engineering*. John Wiley & Sons, 2009.
- [63] A. Figotin and I. Vitebskiy. Slow wave phenomena in photonic crystals. *Laser & Photonics Reviews*, 5(2):201–213, March 2011.
- [64] I. V. Barashenkov, G. S. Jackson, and S. Flach. Blow-up regimes in the PT-symmetric coupler and the actively coupled dimer. *Physical Review A*, 88(5):053817, 2013.

- [65] Ramesh Garg, Inder Bahl, and Maurizio Bozzi. *Microstrip lines and slotlines*. Artech house, 2013.
- [66] Behzad Razavi. *Design of integrated circuits for optical communications*. John Wiley & Sons, 2012.
- [67] George W. Pierce. Piezoelectric crystal resonators and crystal oscillators applied to the precision calibration of wavemeters. In *Proceedings of the American Academy of Arts and Sciences*, volume 59, pages 81–106. JSTOR, 1923.
- [68] Robert E. Collin. *Foundations for microwave engineering*. John Wiley & Sons, 2007.
- [69] Tetsuro Endo and Shinsaku Mori. Mode analysis of a multimode ladder oscillator. *IEEE Transactions on Circuits and Systems*, 23(2):100–113, 1976.
- [70] Ali Hajimiri and Thomas H. Lee. Design issues in CMOS differential LC oscillators. *IEEE Journal of Solid-State Circuits*, 34(5):717–724, 1999.
- [71] Hui Wu and Ali Hajimiri. Silicon-based distributed voltage-controlled oscillators. *Solid-State Circuits, IEEE Journal of*, 36(3):493–502, 2001.
- [72] Behzad Razavi and Razavi Behzad. *RF microelectronics*, volume 2. Prentice Hall New Jersey, 1998.
- [73] Ulrich L. Rohde, Ajay K. Poddar, and Georg Böck. *The design of modern microwave oscillators for wireless applications: theory and optimization*. John Wiley & Sons, 2005.
- [74] A. Guo, G. J. Salamo, D. Duchesne, R. Morandotti, M. Volatier-Ravat, V. Aimez, G. A. Siviloglou, and D. N. Christodoulides. Observation of P T-symmetry breaking in complex optical potentials. *Physical Review Letters*, 103(9):093902, 2009.
- [75] G. Mumcu, K. Sertel, and J. L. Volakis. Miniature antenna using printed coupled lines emulating degenerate band edge crystals. *IEEE Transactions on Antennas and Propagation*, 57(6):1618–1624, June 2009.
- [76] Jo Be Gunn. Microwave oscillations of current in iii–v semiconductors. *Solid State Communications*, 1(4):88–91, 1963.
- [77] BK Ridley and TB Watkins. The possibility of negative resistance effects in semiconductors. *Proceedings of the Physical Society*, 78(2):293, 1961.
- [78] W Shockley. Negative resistance arising from transit time in semiconductor diodes. *Bell System Technical Journal*, 33(4):799–826, 1954.
- [79] John Millar Carroll. *Tunnel-diode and semiconductor circuits*. McGraw-Hill Book Company, 1963.

- [80] Dmitry Oshmarin, Ahmed F Abdelshafy, Alireza Nikzamir, Michael M Green, and Filippo Capolino. Experimental demonstration of a new oscillator concept based on degenerate band edge in microstrip circuit. *arXiv preprint arXiv:2109.07002*, 2021.
- [81] W. D. Heiss. The physics of exceptional points. *Journal of Physics A: Mathematical and Theoretical*, 45(44):444016, 2012.
- [82] Kasra Rouhi, Hamidreza Kazemi, Alexander Figotin, and Filippo Capolino. Exceptional points of degeneracy directly induced by space-time modulation of a single transmission line. *IEEE Antennas and Wireless Propagation Letters*, 19(11):1906–1910, 2020.
- [83] Mohammad-Ali Miri and Andrea Alu. Exceptional points in optics and photonics. *Science*, 363(6422), 2019.
- [84] Hossein Hodaei, Mohammad-Ali Miri, Matthias Heinrich, Demetrios N. Christodoulides, and Mercedeh Khajavikhan. Parity-time-symmetric microring lasers. *Science*, 346(6212):975–978, 2014.
- [85] A. F. Abdelshafy, D. Oshmarin, M. A. K. Othman, M. M. Green, and F. Capolino. Distributed Degenerate Band Edge Oscillator. *IEEE Transactions on Antennas and Propagation*, pages 1–1, 2020. Conference Name: IEEE Transactions on Antennas and Propagation.
- [86] Jan Wiersig. Sensors operating at exceptional points: General theory. *Physical Review A*, 93(3):033809, 2016.
- [87] Hossein Hodaei, Absar U. Hassan, Steffen Wittek, Hipolito Garcia-Gracia, Ramy El-Ganainy, Demetrios N. Christodoulides, and Mercedeh Khajavikhan. Enhanced sensitivity at higher-order exceptional points. *Nature*, 548(7666):187–191, 2017.
- [88] J. Ren, H. Hodaei, G. Harari, A. U. Hassan, W. Chow, M. Soltani, D. Christodoulides, and M. Khajavikhan. Ultrasensitive micro-scale parity-time-symmetric ring laser gyroscope. *Optics Letters*, 42(8):1556–1559, April 2017.
- [89] Hamidreza Kazemi, Amirhossein Hajiaghajani, Mohamed Y Nada, Manik Dautta, Muhannad Alshetaiwi, Peter Tseng, and Filippo Capolino. Ultra-sensitive radio frequency biosensor at an exceptional point of degeneracy induced by time modulation. *IEEE Sensors Journal*, 21(6):7250–7259, 2020.
- [90] N Marcuvitz and J Schwinger. On the representation of the electric and magnetic fields produced by currents and discontinuities in wave guides. I. *Journal of Applied Physics*, 22(6):806–819, 1951.
- [91] Leopold B Felsen and Nathan Marcuvitz. *Radiation and scattering of waves*, volume 31. John Wiley & Sons, 1994.

- [92] Huizhou Wu, Xiangbo Yang, Yan Tang, Xiaopeng Tang, Dongmei Deng, Hongzhan Liu, and Zhongchao Wei. The scattering problem in pt-symmetric periodic structures of 1d two-material waveguide networks. *Annalen der Physik*, 531(9):1900120, 2019.
- [93] A. Hessel, Ming Hui Chen, R.C.M. Li, and A.A. Oliner. Propagation in periodically loaded waveguides with higher symmetries. *Proceedings of the IEEE*, 61(2):183–195, 1973.
- [94] Mohammad Bagheriasl, Oscar Quevedo-Teruel, and Guido Valerio. Bloch analysis of artificial lines and surfaces exhibiting glide symmetry. *IEEE Transactions on Microwave Theory and Techniques*, 67(7):2618–2628, 2019.
- [95] Adam Mock. Symmetry-engineered waveguide dispersion in pt symmetric photonic crystal waveguides. *JOSA B*, 37(1):168–180, 2020.
- [96] James Benford, John A. Swegle, and Edl Schamiloglu. *High power microwaves*. CRC Press, 2015.
- [97] Shulim E. Tsimring. *Electron beams and microwave vacuum electronics*, volume 191. John Wiley & Sons, 2006.
- [98] A. S. Gilmour. *Principles of traveling wave tubes*. Artech House, 1994.
- [99] V. A. Tamma and F. Capolino. Extension of the Pierce Model to Multiple Transmission Lines Interacting With an Electron Beam. *IEEE Transactions on Plasma Science*, 42(4):899–910, April 2014.
- [100] Ahmed F. Abdelshafy, Tarek Mealy, Alexander Figotin, and Filippo Capolino. Low starting current oscillator based on the degenerate band edge in a double helix slow wave structure. In *2019 International Vacuum Electronics Conference (IVEC)*, pages 1–2, 2019.
- [101] John Robinson Pierce. *Travelling-wave tubes*. Van Nostrand; Macmillan, 1950.
- [102] Yanshuai Wang, Zhaoyun Duan, Xianfeng Tang, Zhanliang Wang, Yabin Zhang, Jinjun Feng, and Yubin Gong. All-metal metamaterial slow-wave structure for high-power sources with high efficiency. *Applied Physics Letters*, 107(15):153502, 2015.
- [103] Xin Wang, Zhaoyun Duan, Xirui Zhan, Fei Wang, Shifeng Li, Shengkun Jiang, Zhanliang Wang, Yubin Gong, and BN Basu. Characterization of metamaterial slow-wave structure loaded with complementary electric split-ring resonators. *IEEE Transactions on Microwave Theory and Techniques*, 67(6):2238–2246, 2019.
- [104] Antonio Breno De Alleluia, Artem Kuskov, Dmitrii Andreev, Edl Schamiloglu, Ahmed F. Abdelshafy, Mohamed A.K. Othman, Alexander Figotin, and Filippo Capolino. A 3d-printed metamaterial slow wave structure for high-power microwave generation. In *2020 IEEE 21st International Conference on Vacuum Electronics (IVEC)*, pages 99–100, 2020.

- [105] Antonio Breno de Alleluia, Ahmed F. Abdelshafy, Paulius Ragulis, Artem Kuskov, Dmitrii Andreev, Mohamed A. K. Othman, Braulio Martinez-Hernandez, Edl Schamiloglu, Alexander Figotin, and Filippo Capolino. Experimental testing of a 3-d-printed metamaterial slow wave structure for high-power microwave generation. *IEEE Transactions on Plasma Science*, 48(12):4356–4364, 2020.
- [106] A. Muhammed Zuboraj, B. Kubilay Sertel, and C. John L. Volakis. Propagation of Degenerate Band-Edge Modes Using Dual Nonidentical Coupled Transmission Lines. *Physical Review Applied*, 7(6):064030, 2017.
- [107] Michael R. Lamont, Boris T. Kuhlmeier, and C. Martijn de Sterke. Multi-order dispersion engineering for optimal four-wave mixing. *Optics express*, 16(10):7551–7563, 2008.
- [108] Farshad Yazdi, Mohamed AK Othman, Mehdi Veysi, Alexander Figotin, and Filippo Capolino. A new amplification regime for traveling wave tubes with third-order modal degeneracy. *IEEE Transactions on Plasma Science*, 46(1):43–56, 2017.
- [109] A. S. Gilmour. *Klystrons, traveling wave tubes, magnetrons, crossed-field amplifiers, and gyrotrons*. Artech House, 2011.
- [110] John Robinson Pierce. Traveling-Wave Tubes. *Bell System Technical Journal*, 29(3):390–460, 1950.
- [111] Levi Schächter. *Beam-wave interaction in periodic and quasi-periodic structures*. Springer Science & Business Media, 2011.
- [112] Diana Gamzina, Michael Kozina, Apurva Mehta, Emilio A Nanni, Sami Tantawi, Paul B Welander, Timothy Horn, and Christopher Ledford. Copper reconsidered: Material innovations to transform vacuum electronics. In *2019 International Vacuum Electronics Conference (IVEC)*, pages 1–2. IEEE, 2019.
- [113] P Ragulis, Ž Kancleris, R Simniškis, and M Dagys. Resistive sensor for short-pulse hpm measurements. In *Proc. AMEREM*, page 1, 2018.
- [114] R. L. Ives. Microfabrication of high-frequency vacuum electron devices. *IEEE Transactions on Plasma Science*, 32(3):1277–1291, June 2004.
- [115] S. S. Dhillon, M. S. Vitiello, E. H. Linfield, A. G. Davies, Matthias C. Hoffmann, John Booske, Claudio Paoloni, M Gensch, P. Weightman, G. P. Williams, E. Castro-Camus, D. R. S. Cumming, F. Simoens, I. Escorcía-Carranza, J Grant, Stepan Lucyszyn, Makoto Kuwata-Gonokami, Kuniaki Konishi, Martin Koch, Charles A. Schmuttenmaer, Tyler L. Cocker, Rupert Huber, A. G. Markelz, Z. D. Taylor, Vincent P. Wallace, J. Axel Zeitler, Juraj Sibik, Timothy M. Korter, B. Ellison, S. Rea, P. Goldsmith, Ken B. Cooper, Roger Appleby, D Pardo, P. G. Huggard, V. Krozer, Haymen Shams, Martyn Fice, Cyril Renaud, Alwyn Seeds, Andreas Stöhr, Mira Naftaly, Nick Ridler, Roland Clarke, John E. Cunningham, and Michael B. Johnston. The 2017 terahertz science and technology roadmap. *Journal of Physics D: Applied Physics*, 50(4):043001, 2017.

- [116] John H. Booske. Plasma physics and related challenges of millimeter-wave-to-terahertz and high power microwave generation. *Physics of Plasmas*, 15(5):055502, February 2008.
- [117] L. A. Samoska. An Overview of Solid-State Integrated Circuit Amplifiers in the Submillimeter-Wave and THz Regime. *IEEE Transactions on Terahertz Science and Technology*, 1(1):9–24, September 2011.
- [118] H. J. Song and T. Nagatsuma. Present and Future of Terahertz Communications. *IEEE Transactions on Terahertz Science and Technology*, 1(1):256–263, September 2011.
- [119] John F. Federici, Brian Schulkin, Feng Huang, Dale Gary, Robert Barat, Filipe Oliveira, and David Zimdars. THz imaging and sensing for security applications—explosives, weapons and drugs. *Semiconductor Science and Technology*, 20(7):S266, 2005.
- [120] Roger Appleby and H. Bruce Wallace. Standoff detection of weapons and contraband in the 100 GHz to 1 THz region. *IEEE transactions on antennas and propagation*, 55(11):2944–2956, 2007.
- [121] B. B. Hu and M. C. Nuss. Imaging with terahertz waves. *Optics Letters*, 20(16):1716–1718, August 1995.
- [122] J. Martyn Chamberlain and Robert Miles. *New directions in terahertz technology*, volume 334. Springer Science & Business Media, 2012.
- [123] Ken B. Cooper, Robert J. Dengler, Nuria Llombart, Bertrand Thomas, Goutam Chattopadhyay, and Peter H. Siegel. THz imaging radar for standoff personnel screening. *IEEE Transactions on Terahertz Science and Technology*, 1(1):169–182, 2011.
- [124] Robert J. Barker, Neville C. Luhmann, John H. Booske, and Gregory S. Nusinovich. Modern microwave and millimeter-wave power electronics. *Modern Microwave and Millimeter-Wave Power Electronics*, by Robert J. Barker (Editor), Neville C. Luhmann (Editor), John H. Booske (Editor), Gregory S. Nusinovich, pp. 872. ISBN 0-471-68372-8. Wiley-VCH, April 2005., 1, 2005.
- [125] M. Mineo and C. Paoloni. Double-Corrugated Rectangular Waveguide Slow-Wave Structure for Terahertz Vacuum Devices. *IEEE Transactions on Electron Devices*, 57(11):3169–3175, November 2010.
- [126] C. Paoloni, D. Gamzina, L. Himes, B. Popovic, R. Barchfeld, L. Yue, Y. Zheng, X. Tang, Y. Tang, P. Pan, H. Li, R. Letizia, M. Mineo, J. Feng, and N. C. Luhmann. THz Backward-Wave Oscillators for Plasma Diagnostic in Nuclear Fusion. *IEEE Transactions on Plasma Science*, 44(4):369–376, April 2016.
- [127] Sudeep Bhattacharjee, John H. Booske, Carol L. Kory, Dan W. van der Weide, Steve Limbach, Sean Gallagher, John D. Welter, Mike R. Lopez, Ronald M. Gilgenbach, and R. Lawrence Ives. Folded waveguide traveling-wave tube sources for terahertz radiation. *Plasma Science, IEEE Transactions on*, 32(3):1002–1014, 2004.

- [128] J. Cai, J. Feng, and X. Wu. Folded Waveguide Slow Wave Structure With Modified Circular Bends. *IEEE Transactions on Electron Devices*, 61(10):3534–3538, October 2014.
- [129] A. Polemi, S. Maci, and P. S. Kildal. Dispersion Characteristics of a Metamaterial-Based Parallel-Plate Ridge Gap Waveguide Realized by Bed of Nails. *IEEE Transactions on Antennas and Propagation*, 59(3):904–913, March 2011.
- [130] I. V. Konoplev, A. J. MacLachlan, C. W. Robertson, A. W. Cross, and A. D. R. Phelps. Cylindrical, periodic surface lattice—Theory, dispersion analysis, and experiment. *Applied Physics Letters*, 101(12):121111, September 2012.
- [131] Ahmed F. Abdelshafy, Filippo Capolino, and Alexander Figotin. An accurate traveling wave tube analytic model. *arXiv*, December, 2021.
- [132] J. R. Pierce. Theory of the beam-type traveling-wave tube. *Proceedings of the IRE*, 35(2):111–123, 1947.
- [133] Alexander Figotin and Guillermo Reyes. Multi-transmission-line-beam interactive system. *Journal of Mathematical Physics*, 54(11):111901, 2013.
- [134] Alexander Figotin. *An Analytic Theory of Multi-stream Electron Beams in Traveling Wave Tubes*. World Scientific, 2020.
- [135] Peter Andrew Sturrock. Kinematics of growing waves. *Physical Review*, 112(5):1488, 1958.
- [136] J. R. Pierce. Circuits for traveling-wave tubes. *Proceedings of the IRE*, 37(5):510–515, 1949.
- [137] YY Lau and D Chernin. A review of the ac space-charge effect in electron–circuit interactions. *Physics of Fluids B: Plasma Physics*, 4(11):3473–3497, 1992.
- [138] David H Simon, Patrick Wong, David Chernin, YY Lau, B Hoff, Peng Zhang, CF Dong, and Ronald M Gilgenbach. On the evaluation of pierce parameters c and q in a traveling wave tube. *Physics of Plasmas*, 24(3):033114, 2017.
- [139] Tarek Mealy and Filippo Capolino. Traveling wave tube eigenmode solver for interacting hot slow wave structure based on particle-in-cell simulations. *arXiv preprint arXiv:2010.07530*, 2020.
- [140] S. Ramo. Currents Induced by Electron Motion. *Proceedings of the IRE*, 27(9):584–585, September 1939.
- [141] Kasra Rouhi, Robert Marosi, Tarek Mealy, Ahmed F. Abdelshafy, Alexander Figotin, and Filippo Capolino. Exceptional degeneracies in traveling wave tubes with dispersive slow-wave structure including space-charge effect. *Applied Physics Letters*, 118(26):263506, 2021.

Appendix A

Numerical Parameters Used In Chapter 2

Periodic CTL parameters used in Section 2.3 and 2.6. The periodic CTL used has the following parameters pertaining to the microstrip lines shown in Fig. A1(a) over a grounded dielectric slab with a dielectric constant of 2.2 and height of 1.5 mm, which provides a DBE at 4.03 GHz. All microstrips have a width of 1 mm. The coupled line segment has length $d_A = 10$ mm and separation gap 0.2 mm, while the uncoupled segment is made of an uncoupled TL1 with length $d_{B1} = 14$ mm and uncoupled TL2 length $d_{B2} = 10$ mm as shown in Fig. A1(b). Note that the use of the bended junction impacts the assumption of having a uniform segment of a TL1 by slightly increasing the self-inductance and capacitance due to bending [65]. Therefore, the corresponding equivalent CTL parameters of the microstrip line unit cells are as follows. For the coupled section: $C_{11} = C_{22} = 55.15$ pF/m, $L_{11} = L_{22} = 0.467$ μ H/m, $C_{12} = C_{21} = -C_m = -27.2$ pF/m, $L_{12} = L_{21} = L_m = 0.25$ μ H/m, are the matrix entries of the $\underline{\underline{\mathbf{C}}}$ and $\underline{\underline{\mathbf{L}}}$ matrices. For the uncoupled section: TL1 has $L = 0.61$ μ H/m, and $C = 48.54$ pF/m whereas TL2 has $L = 0.57$ μ H/m, and $C = 39.64$ pF/m. Most of the results in Section 2.3 are for lossless CTLs, losses are considered at the end of Sec. 2.3 and in the

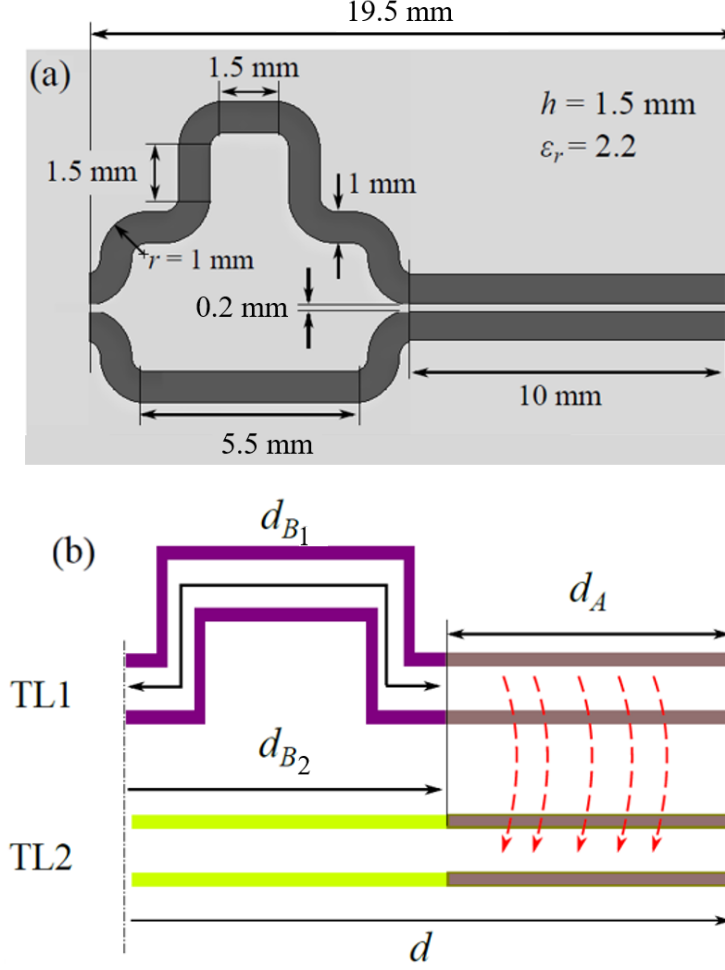


Figure A.1: Geometries of the microstrip configurations adopted in Section 2.3, that develops an EPD. (a), (b) the equivalent passive CTL system of the geometries in Sec. 2.3.

examples in Section 2.6 and in the full-wave simulation results shown in Fig. A3 for the structure in Fig. A1(a).

In Fig. A2, we show the dispersion diagram (for both real and imaginary parts of $k\omega$) using circuit simulator, which is based on predefined models for each piece of the circuit and using those models the system response is described by a system of equations solved using implicit integration methods, adopted in Keysight Technologies ADS (denoted by blue curves). Also, we plot the dispersion obtained using eq. (2.9) for the passive equivalent circuit whose circuit parameters mentioned before (denoted by red curves). Both show a good agreement around the DBE point which is clearly observed at $f_d=4.03$ GHz.

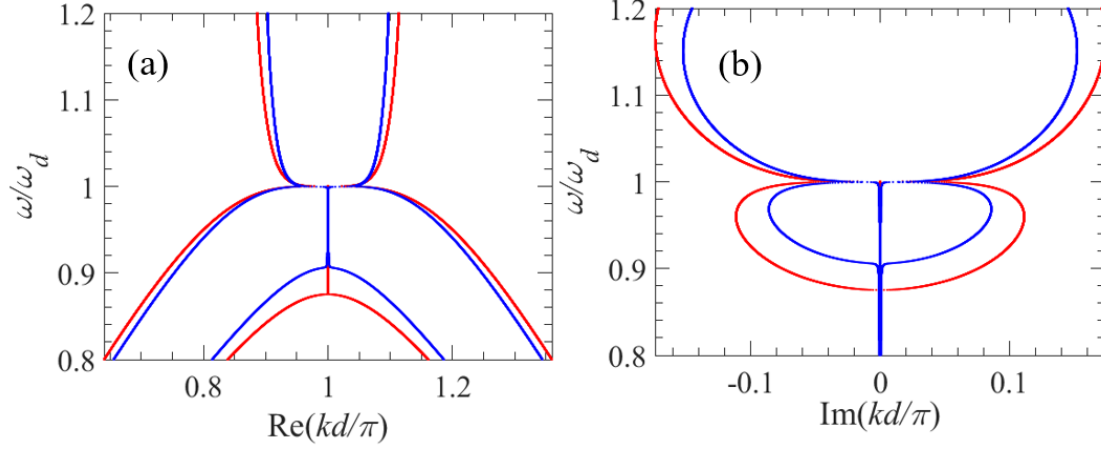


Figure A.2: (a) Real and (b) imaginary parts of the k - ω dispersion diagram obtained for microstrip CTL in Fig. A1(a) using two different methods. First, a circuit simulator (blue curves) adopted in Keysight Technologies ADS. Secondly, by solving eigenvalue equation (2.9) for the equivalent passive CTL system shown in Fig. A1(b) (red curves), whose circuit parameters (L , C) are stated in this Appendix. In both simulations, losses have been neglected.

Additionally, in Fig. A3 we plot the real and imaginary parts of the dispersion using MoM full-wave solver adopted in Keysight Technologies ADS. Losses considered in this full-wave simulation includes radiation, ohmic (i.e. copper has conductivity $\sigma = 5.8 \times 10^7$ Siemens/m), and dielectric (i.e. substrate used with dielectric constant of 2.2, height of 1.5 mm, and with a loss tangent of 0.002) losses. The dispersion shows a deformed flat dispersion at frequency 4.3 GHz which is slightly upshifted from the DBE frequency f_d obtained for the lossless case using circuit simulator and equivalent passive model in Fig. A2.

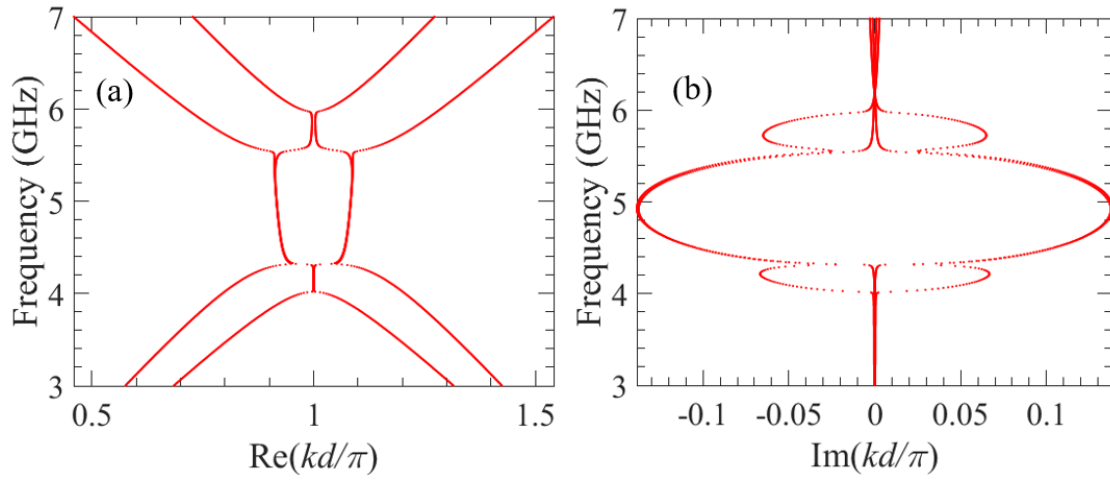


Figure A.3: (a) Real and (b) imaginary parts of dispersion diagram obtained for microstrip CTL in Fig. A1(a) using MoM full-wave simulation (Keysight Technologies ADS) accounting for radiation, ohmic and dielectric losses.

Appendix B

Wavenumber Perturbation in The Vicinity of The DBE

We show here the relation between the expansion of perturbed eigenvalues of periodic CTL $\zeta_n = \exp(-jk_nd)$ and the perturbed wavenumbers k_n , when a system parameter of the CTL is perturbed by a small perturbation parameter δ . In general, the perturbed eigenvalues ζ_n are written as a perturbation of the degenerate eigenvalue in the neighborhood of the DBE eigenvalue $\zeta_d = \exp(-jk_d d)$ in terms of a fractional power expansion (also called Puiseux series [53, 28, 54]) in the perturbation parameter δ as

$$\zeta_n(\delta) = \zeta_d + \alpha_1 e^{j(n-1)\frac{2\pi}{4}} \delta^{1/4} + \alpha_2 e^{j(n-1)\frac{2\pi}{2}} \delta^{2/4} + \dots \quad (\text{B1})$$

where α'_n s are the coefficients of the fractional series expansion of the n th eigenmodes, and $n = 1, 2, 3, 4$ provide the four possible quartic roots near the DBE. The perturbed wavenumber $k_n = jd^{-1} \ln(\zeta_n)$ can be obtained as well, by expanding $\ln(\zeta_n)$ around $\zeta_n = \zeta_d$ using the Taylor series expansion which is given by

$$\ln(\zeta_n) = \ln(\zeta_d) + \frac{\zeta_n - \zeta_d}{\zeta_d} - \frac{(\zeta_n - \zeta_d)^2}{2\zeta_d^2} + \dots \quad (\text{B2})$$

Accordingly, the perturbed wavenumber k_n is written as a perturbation of the degenerate wavenumber in the vicinity of the DBE eigenvalue $k = k_d$ as follows

$$k_n(\delta) = k_d + a_1 e^{j(n-1)\frac{2\pi}{4}} \delta^{1/4} + a_2 e^{j(n-1)\frac{2\pi}{2}} \delta^{1/2} + \dots \quad (\text{B3}) \text{ where}$$

$$\begin{aligned} a_1 &= j d^{-1} e^{j k_d d} \alpha_1 \\ a_2 &= j d^{-1} e^{j k_d d} \left(\alpha_2 - \frac{\alpha_1^2 e^{j k_d d}}{2} \right). \end{aligned} \quad (\text{B4})$$

Appendix C

Properties of System and Transfer Matrices of Periodic Coupled Transmission Lines

We summarize here some important properties associated with matrices related to the system description for the coupled transmission lines.

First, a system description is done by defining a state vector $\Psi(z)$ as in (2.1) that varies in a uniform segment as shown in (2.2) where $\underline{\mathbf{M}}$ is the constant system matrix. This system matrix entries in (2.3) are the impedance $\underline{\mathbf{Z}} = j\omega\underline{\mathbf{L}} + \underline{\mathbf{R}}$ and admittance matrices $\underline{\mathbf{Y}} = j\omega\underline{\mathbf{C}} + \underline{\mathbf{G}}$. It is clear from this representation that $\underline{\mathbf{Z}}$ and $\underline{\mathbf{Y}}$ are skew-Hermitian matrices, i.e., $\underline{\mathbf{A}}^\dagger = -\underline{\mathbf{A}}$, if and only if they are lossless/gainless (i.e. $\underline{\mathbf{Z}} = j\omega\underline{\mathbf{L}}$ and $\underline{\mathbf{Y}} = j\omega\underline{\mathbf{C}}$, where $\underline{\mathbf{L}}$ and $\underline{\mathbf{C}}$ are 2×2 symmetric and positive definite matrices [48, 49]). However, introducing gain and/or loss to the system makes $\underline{\mathbf{Z}}$ and $\underline{\mathbf{Y}}$ non-Hermitian. *Therefore*, if we consider only lossless/gainless systems, both $\underline{\mathbf{Z}} \underline{\mathbf{Y}}$ and $\underline{\mathbf{Y}} \underline{\mathbf{Z}}$ matrices are Hermitian (i.e. $\underline{\mathbf{A}}^\dagger = \underline{\mathbf{A}}$) as they are real symmetric matrices, which implies that they are diagonalizable and they have real

eigenvalues and real determinant [51].

Second, the system matrix $\underline{\mathbf{M}}$ in a lossless/gainless system described by (2.2), which has the skew-Hermitian matrices $\underline{\mathbf{Z}}$ and $\underline{\mathbf{Y}}$ as its entries (2.3), is a non-Hermitian matrix as

$$\underline{\mathbf{M}}^\dagger = \underline{\mathbf{M}}^T \neq \underline{\mathbf{M}} \quad (\text{C1})$$

but it satisfies the J-Hermiticity property (i.e., $\underline{\mathbf{A}}^\dagger = \underline{\mathbf{J}} \underline{\mathbf{A}} \underline{\mathbf{J}}$, where $\underline{\mathbf{J}}$ is any matrix satisfying $\underline{\mathbf{J}} = \underline{\mathbf{J}}^{-1} = \underline{\mathbf{J}}^\dagger$ [48]). This property is easily proved by applying the Hermitian operator to the $\underline{\mathbf{J}} \underline{\mathbf{M}}$ matrix, i.e., by evaluating the complex conjugate transpose $(\underline{\mathbf{J}} \underline{\mathbf{M}}(z))^\dagger = \underline{\mathbf{M}}^\dagger(z) \underline{\mathbf{J}}^\dagger$; then using (C1) it follows that

$$(\underline{\mathbf{J}} \underline{\mathbf{M}}(z))^\dagger = \underline{\mathbf{M}}^\dagger(z) \underline{\mathbf{J}}^\dagger = \underline{\mathbf{J}} \underline{\mathbf{J}} \underline{\mathbf{M}}^T(z) \underline{\mathbf{J}} = (\underline{\mathbf{J}} \underline{\mathbf{M}}). \quad (\text{C2})$$

Accordingly, $\underline{\mathbf{J}} \underline{\mathbf{M}}$ is a Hermitian matrix, hence $\underline{\mathbf{M}}$ satisfies the J-Hermiticity property $\underline{\mathbf{M}}^\dagger = \underline{\mathbf{J}} \underline{\mathbf{M}} \underline{\mathbf{J}}$.

Third, the periodic structure consists of at least two uniform segments A and B, and the state vector z -variation is described using the transfer matrix concept for each unit cell $\underline{\mathbf{T}}_U$ as in (2.8), that is the product of the transfer matrices of the two segments $\underline{\mathbf{T}}_U = \underline{\mathbf{T}}_B \underline{\mathbf{T}}_A$, each one given in (2.7). Within a uniform segment (either A or B), the system vector varies as in (2.4) and the transfer matrix is given by $\underline{\mathbf{T}}(z) = \exp(-j\underline{\mathbf{M}}z)$ as shown in (2.5), where the constant $\underline{\mathbf{M}}$ matrix at the exponent satisfies the J-Hermiticity properties as shown in (C2). Using the following expansion that defines the transfer matrix

$$\underline{\mathbf{T}}(z) = \exp(-j\underline{\mathbf{M}}z) = \mathbf{1} + (-j\underline{\mathbf{M}}z) + \frac{(-j\underline{\mathbf{M}}z)^2}{2!} + \dots, \quad (\text{C3})$$

the spatial variation is written as

$$\begin{aligned}
\partial_z \underline{\mathbf{T}}(z) &= \partial_z (\mathbf{1} + (-j\underline{\mathbf{M}}z) + \frac{(-j\underline{\mathbf{M}}z)^2}{2!} + \dots) \\
&= -j\underline{\mathbf{M}} + \frac{(-j\underline{\mathbf{M}})^2 2z}{2!} + \frac{(-j\underline{\mathbf{M}})^3 3z^2}{3!} + \dots \\
&= -j\underline{\mathbf{M}}(\mathbf{1} + (-j\underline{\mathbf{M}}z) + \frac{(-j\underline{\mathbf{M}}z)^2}{2!} + \dots) \\
&= -j\underline{\mathbf{M}}\underline{\mathbf{T}}(z).
\end{aligned} \tag{C4}$$

Accordingly, the transfer matrix is proven to satisfy the J-unitarity (i.e., $\underline{\mathbf{A}}^\dagger = \underline{\mathbf{J}}\underline{\mathbf{A}}^{-1}\underline{\mathbf{J}}$) following the proof in Appendix A of [12], based on the fact that $\partial_z[\underline{\mathbf{T}}(z)\underline{\mathbf{T}}^{-1}(z)] = (\partial_z \underline{\mathbf{T}})\underline{\mathbf{T}}^{-1} + \underline{\mathbf{T}}(\partial_z \underline{\mathbf{T}}^{-1}) = 0$ and the properties shown in (C2) and (C4).

Finally, we prove that the unit-cell transfer matrix $\underline{\mathbf{T}}_U = \underline{\mathbf{T}}_A \underline{\mathbf{T}}_B$, which translates the state vector across a periodic unit-cell is J-unitarity. Indeed, as just proved, both $\underline{\mathbf{T}}_A$ and $\underline{\mathbf{T}}_B$ are J-unitarity. Hence, $\underline{\mathbf{T}}_U$ also satisfies the J-unitarity property as it can be easily proven as follows:

$$\begin{aligned}
\underline{\mathbf{T}}_U^\dagger &= (\underline{\mathbf{T}}_A \underline{\mathbf{T}}_B)^\dagger = \underline{\mathbf{T}}_B^\dagger \underline{\mathbf{T}}_A^\dagger \\
&= \underline{\mathbf{J}} \underline{\mathbf{T}}_B^{-1} \underline{\mathbf{T}}_A^{-1} \underline{\mathbf{J}} = \underline{\mathbf{J}} \underline{\mathbf{T}}_U^{-1} \underline{\mathbf{J}}.
\end{aligned} \tag{C5}$$