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Publication Date

1981-07-01

**Report no.
UCB-SESM-81/05**

**STRUCTURES AND MATERIALS RESEARCH
Department of Civil Engineering**

**EXPERIMENTAL AND THEORETICAL STUDIES
ON STRESS RELAXATION OF CONCRETE
UNDER UNIAXIAL COMPRESSIVE STRAIN**

by

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July, 1981

**STRUCTURAL ENGINEERING LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA**

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ON STRESS RELAXATION OF CONCRETE
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Report No. UCB/SESM - 81/05
July 1981

ACKNOWLEDGEMENTS

The financial support for this investigation was provided by a donation from Dr. Roy Carlson for promoting research into the properties of concrete materials. The authors are grateful for this support and that of the Division of Structural Engineering and Structural Mechanics of the University which provided the main laboratory facilities used for carrying out the investigation.

The visit of Professor Koso Thomas was jointly sponsored by the University of California, Berkeley and the Senior Fulbright Program of the Council for International Exchange of Scholars, Washington, D. C.

Messrs. Steven Ernst and Tony Costa assisted with setting up of the electorical circuitry for the units in the test set-up, and Professor Lubliner made valuable suggestions during the writing up of the analytical sections of the work. Acknowledgement is also made of the contribution of Mr. Paulo Monteiro to the development of suitable creep functions used in the analysis of test data.

The manuscript was typed by Ms. JoAnn Kim and the drawings were prepared by Ms. Amy Pertschuk and Ms. Gert Weil.

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1. INTRODUCTION

1.1 General

The measurement of stress relaxation in concrete has not been previously accomplished under controlled conditions which eliminate the effects of shrinkage, autogenous volume change and temperature variation. Although there has been the need for such measurements to provide results against which creep-based relaxation theories could be compared, and from which formulations of time dependent stress variation for mass concrete structures could be determined, it has been difficult to develop techniques which will reliably maintain constant strain on a specimen while monitoring the stress relief as time elapses.

In the recent past, however, a number of innovative procedures have been developed for the measurement of strain under constant stress both in the laboratory [1, 2] and in the field. These measurements have formed the basis for generating creep functions which have been used directly for predicting stress patterns in concrete structures during ageing, or analysing stress distribution from known strain histories. The alternative procedure of computing stress relaxation from creep tests and employing the result to develop theories for time dependent analysis of structures are less cumbersome. Confirmation of accuracy is, however, difficult because of the absence of a satisfactory method for obtaining and checking such derived stress changes.

This report suggests appropriate laboratory equipment and instrumentation to achieve constant strain application on specimens, and gives measurements to quite high accuracies of stress changes occurring during relaxation of the specimen. Existing procedures for converting creep-time plots to relaxation plots are also examined and are compared with the curves obtained from direct measurement of stress using the relaxation equipment devised.

1.2 Objectives and Scope

1. To develop a reliable equipment set and instrumentation for imposing constant monotonic axial strain on sealed specimens and continuously monitoring and recording strain and stress levels.

2. To construct relaxation curves from creep data and compare these with the results from direct measurements, using the improved test methods and equipment.

The specimens were cast from two different mix proportions and cured under identical conditions to eliminate concrete strength variability as far as was possible. The scope of the tests covered considerations of creep and relaxation trends for the conventional loading ages of 28 and 90 days. In addition, a 7-day creep test was carried out, as well as 14 and 60-day relaxation tests. Ambient conditions were kept the same for all specimens during testing. Specimen dimensions, mix used for their preparation, compressive strength and elastic properties are given in Tables 1a and 1b.

2. REVIEW OF PREVIOUS STUDIES

2.1 General

Tests performed to study stress loss in concrete under constant strain are very few. The earliest study of note was the work by J. R. Shark [3], on the mechanics of plastic flow of concrete. Most of this work, like many others after it, employed reinforced or plain concrete beams under static lateral load and studied the relaxation under constant deflection. While direct measurement of relaxation under axial load is rare, the prediction of relaxation from creep data is common. Several researchers have made predictions which fall under two main groups: (1) those predictions based in a modified elastic theory and (2) those based on visco-elastic functions. Many of these researchers have found ways of verifying their predictions experimentally, but only a few research techniques have had continuous control of the deformation applied to the specimens tested.

The discussion which follows deals briefly with a number of known contributions to the subject.

2.2 Theories Based on Basic Elastic Concepts

Ross and Seed [4] recognized the variation in the Elastic modulus of concrete with time, by representing the residual stress σ_1 at any time t by the equation:

$$\sigma_1 = E_t \cdot \frac{\sigma_0}{E_0} \dots \dots \dots (2.1)$$

where, σ_0 = initial stress in the specimen,

E_0 = initial modulus of Elasticity, and

E_t = effective modulus of Elasticity at time t .

Defining a hypothetical creep c_1 under constant stress over an elapsed time t , the authors obtained an expression for E_t as follows:

$$E_t = \frac{\sigma_0}{e_t} = \frac{\sigma_0}{e_0 + c_1} ,$$

and,

$$\begin{aligned} \sigma_1 &= \frac{\sigma_0}{e_0 + c_1} \cdot \frac{\sigma_0}{E_0} \\ &= e_0 \cdot \frac{\sigma_0}{e_0 + c_1} \dots \dots \dots (2.2) \end{aligned}$$

Allowing for shrinkage, the authors introduced a shrinkage strain, s , and modified Eq. (2.1) to give a residual stress from relaxation alone as

$$\sigma_1 = E_t \cdot \left(\frac{\sigma_0}{E_0} - s \right) \dots \dots \dots$$

The authors were able to confirm their theory by experimental results.

The method neglects the effect of creep recovery when e_0 is kept constant.

Hansen [5] on the other hand, considered the effect of creep under decreasing stress, and proposed an expression for the elastic strain, e_1 , remaining after a period under load, as follows:

$$e_1 = e_0 - c_v \dots \dots \dots (2.3)$$

c_v = creep over period under decreasing stress.

Hansen defined this creep under interval $T_1 - T_0$ as the stress decreases from σ_0 to σ_1 as:

$$c_v \approx \frac{1}{2} (\sigma_0 + \sigma_1) \cdot \frac{c_1}{\sigma_0} \dots \dots \dots (2.4)$$

where,

c_1 = creep at T_1 under stress σ_0 applied at T_0 .

Hansen assumed E to be constant during the period of relaxation and wrote, on the basis of this assumption:

$$e_1 = \sigma_1 \cdot \frac{e_0}{\sigma_0} \dots \dots \dots (2.5)$$

Equating Eq. 2.3 with Eq. 2.5 and substituting for c_v , he obtained:

$$\sigma_1 = \left(\frac{2e_0 - c_1}{2e_0 + c_1} \right) \cdot \sigma_0 \dots \dots \dots (2.6)$$

which on inspection is incompatible with Eq. 2.2 for values other than $c_1 = 0$. As Neville [6] points out, this is absurd.

This expression has limited application. For example, c_1 cannot be greater than $2e_0$. The method is useful only when creep is small in comparison with the initial elastic strain. Another objection to the method is that the assumption of constant modulus of elasticity is erroneous and leads to an overestimation of the magnitude of stress loss.

McCoy [7] has suggested the use of the finite difference equation to improve results obtained by Hansen's method. He proposed that Eq. 2.6 should be re-written to reduce the stress range over which the average stress is calculated. His modification led to the following relationship:

$$\sigma_{i+1} \approx \frac{2e_0 - \Delta c}{2e_0 + \Delta c} \sigma_i \dots \dots \dots (2.7)$$

where,

$$\Delta c = c_{i+1} - c_i$$

The accuracy of this method will depend on the choice of Δc for the step type procedure. McCoy obtained higher accuracies of prediction with this modification than with Hansen's original method. In a review of these procedures, however, Neville pointed out that McCoy's final equation for residual stress gave a better indication of the decay of stress, as creep

increases indefinitely, than Hansen's equation (Eq. 2.6) which would ultimately give zero stress as creep increases.

2.3 Methods Employing Visco-elastic Functions

These methods isolate the elastic and viscous deformations which take place in concrete under long term loading. The viscous deformations are usually represented either by a suitable continuous function or by a series of functions developed from rheological models. Once defined for constant stress conditions, the deformations are used to derive relaxation functions for members under constant strain.

Arutyunyan [8] defined a deformation function:

$$\delta(t, \tau) = \frac{1}{E(\tau)} + C(t, \tau) \dots \dots \dots (2.8)$$

where,

$$\frac{1}{E(\tau)} = \text{instantaneous-elastic deformation}$$

and $C(t, \tau) = \text{creep deformation up to instant, } t$

$\tau = \text{age of the concrete at loading}$

$t = \text{time since casting}$

using Eq. 2.8 and the principle of superposition, a number of authors obtained expressions for total deformation under varying stress. In general, the expressions take the form:

$$\epsilon_x(t) = \sigma_x(\tau_1) \delta(t, \tau_1) + \int_{\tau_1}^t \frac{d\sigma_x(\tau)}{d\tau} \delta(t, \tau) d\tau \dots \dots (2.9)$$

$\sigma_x(\tau_1)$ being the stress applied at age τ_1 and $d\sigma_x(t)$ the stress change occurring at any age τ .

Integrating 2.9, Arutyunyan [8] arrived at the equation:

$$\epsilon_x(t) = \frac{\sigma_x(t)}{E(t)} - \int_{-1}^+ \sigma_x(\tau) \frac{\partial}{\partial \tau} \delta(t, \tau) d\tau \dots \dots \dots (2.10)$$

Arutyunyan treated Eq.2.10 as a Volterra integral equation for function $\sigma_x(t)$ with kernel

$$k(t, \tau) = \frac{\partial \delta(t, \tau)}{\partial \tau} = \frac{\partial}{\partial \tau} \left[\frac{1}{E(\tau)} + c(t, \tau) \right] \dots \dots \dots (2.11)$$

The solution of Eq. 2.10 in respect of $\sigma_x(t)$ reduces to:

$$\sigma_x(t) = \epsilon_x(t) E(t) + \int_{\tau_1}^t \epsilon_x(\tau) E(\tau) R(t, \tau) d\tau \dots \dots \dots (2.12)$$

where $R(t, \tau)$ is the resolvent of the kernel $E(t) k(t, \tau)$ and therefore the relaxation function of the concrete. The author recommended for the creep function $C(t, \tau)$ the relationship:

$$C(t, \tau) = \phi(\tau) f(t - \tau) \dots \dots \dots (2.13)$$

and used an approximation to $f(t - \tau)$ of the form:

$$f(t - \tau) = \sum_{k=0}^m B_k e^{-\gamma_k (t - \tau)} \dots \dots \dots (2.14)$$

B_k and γ_k being constants to be determined for particular characteristics of the material, but such that $B_0 = 1$, $\gamma_0 = 0$ and $\gamma_k > 0$ ($k=1,2,3,\dots,m$).

$\phi(\tau)$, is any uniformly decreasing function satisfying the condition:

$$\lim_{\tau \rightarrow \infty} \phi(\tau) = C_0$$

After evaluating a number of experimental data, Arutyunyan concluded that the function $\phi(\tau)$ can be represented in the general case by the expression

$$\phi(\tau) = C_0 + \sum_{k=1}^m \frac{A_k}{\tau^k} \quad (\tau > 0) \dots \dots \dots (2.15)$$

where C_0 is the limiting value of the creep function for the material and A_k are values given to reflect the nature and condition of ageing of the material, and obtained from fitting the creep function to experimental creep curves.

The method is tedious and would involve approximations to make the expressions manageable. In the analysis of experimental results discussed by Arutyunyan, he limited himself to only two terms of the series in Eqs. 2.14 and 2.15, employing

$$\phi(\tau) = C_0 + \frac{A_1}{\tau} \dots \dots \dots (2.16)$$

and $f(t-\tau) = 1 + B_1 e^{-\gamma_1(t-\tau)}$ to fit his creep curves, and used a variation function for the modulus of instantaneous deformation:

$$E(\tau) = E_0 (1 + \beta e^{-\alpha\tau}) \dots \dots \dots (2.17)$$

With these relationships established, the integral (2.12) becomes equivalent to the solution of linear differential equations with variable coefficients. The author treated this solution laboriously, introducing a supplementary influence function ϕ in the final solution for $\sigma_x(t)$ as follows:

$$\sigma_x(t) = \sigma_x(\tau_1) \left\{ 1 - \gamma E_0 r^{-(1-\rho)} \left(\frac{A_1}{\tau_1} + C_0 \right) \right. \\ \left. \times \tau_1^\rho e^{r\tau_1} [\phi(r\tau, \rho) - \phi(r\tau_1, \rho)] \right\} \dots \dots \dots (2.18)$$

Arutyunyan gave equations for all constants and provided a table of values for ϕ corresponding to given values of the variables of the function.

Other authors have approached the problem from the same type of basic deformation equations, but some have introduced simplifications to the creep and instantaneous deformation functions to reduce the mathematical operations involved in the solution of the residual stress equation. Some of the simplifications consider the modulus of Elasticity to be constant throughout the ageing process, and creep to be a function only of the time under load. These simplifications result in a deformation function of the type:

$$\delta(t-\tau) = \frac{1}{E_0} + C(t-\tau) \quad \dots \dots \dots (2.19)$$

Simplifications of this kind are rather drastic and are far from being representative of the true deformation phenomenon.

Another simplification employed has been to relate relaxation directly to creep by a formulation involving specific creep and a constant modulus of Elasticity thus,

$$\text{Log}_e \psi = -\phi \quad \dots \dots \dots (2.20)$$

where,

$$\psi = \text{relaxation ratio} = \frac{\sigma_t}{\sigma_0}$$

$$\phi = \text{creep coefficient} = \frac{c_t}{e_0} = E_0 c_s$$

c_s = specific creep, and given by:

$$c_s = \frac{\text{creep at time } t \text{ under constant stress}}{\text{Initial stress}}$$

This is the so-called "rate of creep equation" which is dealt with later under iterative procedures.

Klug and Wittman [9] however, have, like Arutyunyan, maintained the Volterra integral equation for the stress decay function and obtained the relationship for ultimate residual stress σ_{∞} , as:

$$\sigma_{\infty} = \frac{\sigma_0}{1 + \phi_{\infty}} \quad \dots \dots \dots (2.21)$$

where

ϕ_{∞} = ultimate creep coefficient.

The authors found from their experiments that the initial rate of stress loss was proportional to the creep velocity and that the constant or proportionality was the Elastic modulus of the specimen. They also found that their equations showed good agreement with their experimental results.

As an alternative to employing the basic algebraic concepts directly, a number of researchers have used rheological models to describe the relaxation process. Trost [10] used such a method to obtain results similar to those of Klug and Wittman, and Bulatov [11] devised models to provide equations describing the relaxation and deformation not only of concrete, but also of other solids, such as cement stone and rock.

A number of iterative procedures have been developed, which take account of the visco-elastic components of the deformation of concrete under sustained loading. The procedures are based on methods which have been used for estimating strain under varying stress. These are: 1) the rate of creep method, 2) the effective modulus method, and 3) the superposition method. These methods have been described in detail by Ross [12] who also applied the method to stress relaxation under constant strain.

In the rate of creep method, any specific creep-time curve relating to the material under study is used to determine the rate of

$$= \frac{\sigma_i - \sigma_{i-1}}{E} + C_{sp}(t_1 + \Delta t_1) - C_{sp}(t_2) \quad \sigma_i$$

Under constant strain conditions $\Delta \epsilon = 0$ and

$$\sigma_i = \frac{\sigma_{i-1}}{1+E [C_{sp}(t_1 + \Delta t_1) - C_{sp}(t_1)]} \quad \dots \quad (2.25)$$

This method neglects the effect of the real creep recovery and the variation in the modulus of elasticity with time. Various investigators have found the method unsatisfactory as it underestimates the residual stress when relaxation is calculated from creep. Brooks and Neville [13] attempted a modification of the method by defining the total strain recovered after a stress removal of magnitude $d\sigma$ as:

$$-\Delta \epsilon = \frac{d\sigma}{E} + dr \quad \dots \quad (2.26)$$

where,

dr is the creep recovered during the interval after the removal of the stress $d\sigma$, and

E is the modulus of elasticity at time t .

To obtain constant strain conditions during relaxation, the increment of creep over an interval of time dt , due an applied stress σ at time t , was equated with the total strain recovered, thus,

$$\sigma dc_s = - \left(\frac{d\sigma}{E} + dr \right) \quad \dots \quad (2.27)$$

dc_s = change in specific creep during dt

The creep recovery component was given by the expression:

$$dr = kr_{s_{\infty}} \cdot d\sigma$$

with

$k = \text{constant}$

$r_{s_{\infty}} = \text{specific limiting recovery which was assumed constant}$

This modification assumes k constant for all intervals and $r_{s_{\infty}}$ constant for ages at unloading. These assumptions are not correct, but they facilitated the solution of Eq. 2.29 by analytical procedures to give:

$$\text{Log}_e \psi = - \left[\frac{1}{E_0 \left(\frac{1}{E} + kr_{s_{\infty}} \right)} \right] \phi$$

which on making $E = E_0$ gave,

$$\text{Log}_e \psi = - \left(\frac{1}{1 + E_0 kr_{s_{\infty}}} \right) \phi \dots \dots \dots (2.30)$$

where,

ψ and ϕ have the same meaning as in Eq. 2.20.

The authors then changed the form of Eq. 2.30, to the following:

$$\text{Log} (1/\psi) = a + b \phi \dots \dots \dots (2.32)$$

for which they determined coefficients a and b to fit results from 14 sets of creep and relaxation tests, 8 of which were carried out by the authors themselves. Their final equation was given as:

$$\text{Log} (1/\psi) = 0.090 + 0.686 \phi \dots \dots \dots (2.32)$$

and predicted results to within 15% of observed values.

A version of the rate of creep method makes use of the concept of reversible and irreversible creep. According to the work of researchers,

England and Illston [14] there are three components of deformation under sustained load: 1) elastic strain which remains constant under the load, 2) irreversible creep which is independent of stress history and 3) delayed elastic strain or reversible creep which, under variable stress conditions, can be represented by the relationship:

$$C_r = P (1 - e^{-\frac{t}{Q}}) \dots \dots \dots (2.33)$$

where C_r = delayed elastic strain under a constant stress

P = limiting delayed elastic strain

Q = a constant representing the rate of occurrence

and t = time since change of load

Using two successive intervals of time in which the boundary values are T_1 , T_2 and T_3 , and the constant stresses over the intervals are σ_1 and σ_2 , the stress σ_2 at intervals T_3 - T_2 under constant strain will be:

$$\sigma_2 = \frac{\sigma_1/E + C_{r_2} (1 - \exp(\Delta C_{sp} T_2/Q))}{1/E + \Delta C_{sp} T_2 + C_{sp} T_\infty (1 - \exp(\Delta C_{sp} T_2/Q))}$$

where C_{r_2} = delayed elastic strain at time T_2

$C_{sp} T_\infty$ = limiting specific delayed elastic strain

$\Delta C_{sp} T_2$ = change in irreversible creep during interval T_3 - T_2

$P_2 = \sigma_2 C_{sp} T_\infty - C_{r_2}$ (limiting delayed elastic strain yet to come at start of interval T_3 - T_2)

The effective modulus method assumes a sustained modulus of elasticity, defined by:

$$E_t = \frac{1}{\epsilon + C_{sp}} \dots \dots \dots (2.34)$$

where ϵ is the specific elastic deformation, i.e. the elastic strain due to stress of 1 psi. The residual stress is then calculated from the relation:

$$\sigma_t = \epsilon_0 E_t$$

where ϵ_0 = the sustained strain

E_t = the declining effective modulus

The method underestimates stress relaxation as it neglects stress history.

Carlson [16] used a modification of this method in conjunction with the rate of creep method to provide a rapid determination of stress relaxation from creep data. In this method, intervals of time are chosen as in the rate of creep method, but the increase in creep over the interval considered is equated with the elastic strain arising from unloading by a stress $\Delta\sigma$ using E_t as the linear elastic constant. Thus, the stress relief over an interval t_1 to $t_1 + \Delta t$ will be given by:

$$-\Delta\sigma_i = E_t \sigma_{i-1} (C_{sp}(t_1 + \Delta t) - C_{sp}(t_1)) \dots \dots \dots (2.34)$$

An example of the procedure is given in Table 16. The method gives complete instantaneous recovery of strain on unloading and no creep recovery effects are carried over to subsequent time intervals. Stress relaxation is thus overestimated.

Although the superposition theory has been adopted by investigators using analytical procedures to derive relaxation equations, the theory has its most common application in interactive methods. In the method, Eq. 2.9, giving the total strain under a variable stress is written in the form:

$$\epsilon(t) = \sigma(\tau_1) \delta(t, \tau_1) + \sum_{\tau_1}^t \Delta\sigma(\tau) \delta(t, \tau) \dots \dots \dots (2.35)$$

The method requires the determination of strain-time relationships from creep experiments for different ages of loading. From these, specific creep curves are drawn. A number of authors represented these by algebraic expressions and used these instead of the direct specific creep curves in applying the method. The procedure is to choose a number of time intervals for which specific creep strains are determined from an experimental curve for the age of initial loading, or from relevant equations. Mid-interval creep strains are then deduced and the strain change $\Delta\epsilon_1$ over the first interval is neutralized by removing a stress of magnitude $\Delta\sigma_1$ computed from the equation:

$$-\Delta\sigma_1 = E_{t_1} \Delta\epsilon_1 \dots \dots \dots (2.36)$$

where E_{t_1} = sustained modulus of elasticity at time t_1 .

This negative stress removes not only the strain at the first interval, but also strains at succeeding mid-intervals equal to the creep which would normally have taken place at the interval under a constant load $\Delta\sigma_1$ applied at the start of interval 1. Other intervals are taken in succession and the net strain resulting for the algebraic sum of all strains occurring up to a particular interval from the initial stress application, is converted to stress change using Eq. 2.36. Residual stress at any time is then given by the stress at the interval preceeding that time minus the stress change at the interval being considered.

Some authors have applied variations of this method to relaxation problems, and a number of others [16 & 17] have employed equations defining creep over time at various ages of loading as well as furnishing equations for the sustained modulus of elasticity. The method, however, underestimates relaxation as it overlooks the fact that complete reversal of creep does not take place on removal of the load. An example of the method using manual computation is given in Table 16.

A review of various methods for the calculation of stress relaxation has been given by Bastgen [18]. The methods assessed included those of the German Standard DIN 1045, Hansen, McCoy, Dischinger [19], a modification of

Dischinger's equation, the methods of Klug [20] and Troot, the method of superposition, the method of Wittmann [21], the visco-elastic model of Pfefferle [22] and other visco-elastic theories. Bastgen found that the equations which best predicted actual behavior were those based on the principle of superposition [23, 25 & 35]. Further work by Bastgen [26] resulted in the development of three equations for easy calculation for the conversion of creep strains into residual stresses. The work was based on modifications of the theories of Wittman and Dischinger and the effective elastic modulus method. The three equations suggested are:

$$(1) \quad \sigma(t) = \sigma_0 \cdot \frac{E a_2 t}{e^{b_2 + t}} \dots \dots \dots (2.37)$$

$$(2) \quad \sigma(t) = \sigma_0 \cdot e^{\frac{-\phi(t)}{1 + 0.4 k_v(t)}} \dots \dots \dots (2.38)$$

$$(3) \quad \sigma_t = \frac{\sigma_0}{1 + \phi(t)} \dots \dots \dots (2.39)$$

2.4 Strain Controlling Techniques

Strain controlling techniques have been varied. Some investigators have used simple methods requiring stepwise adjustment of the load while others have employed standard testing machines which incorporate strain controlling devices.

Ross [12] used the stepwise adjustment technique to maintain constant strain, but found that in order to isolate the effects of creep under variable declining stress, the strain in the loaded specimen had to be allowed to vary by certain amounts to take account of shrinkage. The allowance for shrinkage was determined from measurements on a companion control specimen. This technique seems typical of a large proportion of the few investigations

to date. A slight refinement of this procedure was introduced in the work of Brooks and Neville [13]. These researchers used a dead load system consisting of a lever arm and a water tank. The quantity of water in the tank was gradually adjusted to provide a constant strain on the specimen. The load on the specimen was determined from measuring the depth of water in the tank periodically. During the test the specimen was submerged in water at a temperature of $23 \pm 1^\circ\text{C}$. The stress on the specimen was adjusted after a strain of 15×10^{-6} to 20×10^{-6} and a correction was applied for shrinkage/swelling which was measured independently on an unloaded companion specimen.

An example of techniques involving the use of large testing machines is that used by Tanigawa, Yamada and Tomika [27]. They used a highly stiff testing machine with a 100 ton testing capacity. By installing steel blocks parallel to the specimen, the machine was able to control either deformation or load on the specimen by the mechanical wedge action of the blocks. The tests carried out were aimed at studying short term stress relaxation effects. Such techniques would appear to be impracticable for long term testing.

The known techniques seem to lack that rare combination of simplicity and accuracy which should characterize research endeavors in most fields. This requirement has been paramount among the considerations which guided the development of the testing equipment for the investigation presented in this report.

3. TEST SPECIMENS

3.1 Preparation

3.1.1 Creep and Relaxation Test Specimens

Cylindrical specimens were used for all creep and relaxation tests, as well as for measurement of autogenous volume changes. These specimens were cast at the same time as a number of standard compression strength specimens which were used to determine compressive strength, modulus of elasticity and Poisson's ratio of the concrete in the specimens.

The creep and relaxation specimen moulds were made of cast iron and machine finished in two 18-inch longitudinal sections which when assembled around a 2-inch base plate gave an inside diameter of 6.000 inches, with a tolerance of ± 0.002 inch, and a length of 18 inches. An 8-inch Carlson strain meter was assembled in a central position along the axis of each mold by means of a wire which was attached to the top of the meter and stretched and anchored on a 5/16-inch by 8-inch mold bar brace across the open top end of the mould. Thereafter, the meter was checked for resistance to ground and its calibration compared against a standard mechanical gage. The lead wires from the strain meter were then passed through a centrally drilled hole in the mould's base plate and the hole sealed using an "O" ring. Immediately after casting, the meter positioning wire was cut and the anchoring bar removed. Thirty minutes after casting, the concrete in each mould was vibrated to dislodge and disperse any bleeding water which may have settled beneath and around the metal ends of the Carlson meter.

The top of each mould was then covered with Seran wrap and the specimen was taken from the casting room to a temperature and humidity controlled room (73°F and 50% relative humidity). When all bleeding water had been reabsorbed from the surface of the specimens, a conical shaped layer of mortar was placed on top of each specimen. The mortar for capping of the specimens was prepared from a mix of the fine aggregate and cement proportions used in making the original concrete. A 1 1/2-inch thick steel loading plate with pins on its underside to provide anchorage within the mortar, was laid on the mortar and worked into position with the help of a levelling

plate, until the loading plate was normal to the axis of the specimen. Excess mortar was removed and then the top of the specimen was sealed with Saran wrap. Each specimen was left to stand in the temperature and humidity controlled room until they were stripped the following day.

Within three minutes of removal from its mold, each specimen and its top and bottom plates was wrapped with a 1/16-inch thick butyl rubber sheet, allowing a 3-inch wide splice lap. The rubber sheet was coated with rubber cement to provide a tight seal of the concrete. To ensure as perfect a seal as possible, additional tightness was provided by two large hose clamps which sealed the rubber against the top and bottom end plates. The specimens remained in the 73°F, 50% R.H. room until they reached the desired age for testing. Two of the specimens used for the creep studies, and one of those used for the relaxation studies had, in addition to the Carlson gages, Whittemore gage points attached at 10-inch centers to opposite faces of the specimens. By this means, facility was provided for carrying out mechanical checks on the Carlson gages whenever necessary during the tests. Figures 1 and 2 show a stress relaxation and a creep specimen in their respective frames.

3.1.2 Standard Compression Strength Specimens

The specimens were cast in 6-inch diameter by 12-inch long sheet metal cans. After casting, the lids and joints of each can were sealed with silicon rubber to ensure that each specimen was internally sealed. The sealed specimens were then stored in a 100% R.H. room until just prior to testing when they were stripped, capped and completely wrapped with Saran wrap to ensure water retention during testing.

4. INSTRUMENTATION

4.1 General

The paramount concern in devising the instrumentation system and procedures was to obtain efficient performance of all elements used, as well as provide accurate and speedy recording of input and output data.

4.2 Creep Test Set-up

The creep specimens were tested in a loading frame capable of accepting two specimens at a time in tandem. The loading frame consisted of two 1-inch thick triangular header plates and three 1 1/2-inch steel rods which passed through equally spaced holes in the plates. The specimens were fitted centrally between the header plates with a pressure gage inserted between the top plate of the topmost specimen and upper header plate. This was done to ensure that the specimen remained under stress during the test. The assembly was held in position by bolts at the ends of the rods, as shown in Figure 2, care being taken to prevent eccentric loading on the specimens.

Leads from the two meters in the specimens in the frame were connected to designated channel input points in a 10-channel bridge switch. Output from the switch was fed into an SESM test set with a minicomputer link. This facility provided reading of gage outputs and the immediate computation of strain and temperature on manual selection of gages as well as strain and temperature reading modes, at prescribed time intervals. The two test specimens fitted with Whittemore points were used as top specimens in the loading frame. These enabled additional strain readings to be taken and checked against the automated readings during the tests.

4.3 Autogenous Test Set-up

The internal instrumentation and capping of these specimens were identical to those for the creep specimens. The objective of the test was to measure volume changes in terms of changes in uni-axial strain which takes place in the absence of stress. Specimens were tested from each batch of concrete mixed. One autogenous test specimen was positioned in a safe area near the creep specimens to which they referred, and the lead wires from the Carlson gages embedded in them were connected to selected channel points in the bridge switch. Readings and computations

were carried out at set intervals, commencing from the age at which the creep specimens were loaded.

4.4 Stress Relaxation Test Set-up

The relaxation tests were carried out on concrete cylindrical specimens of the type described in 3.1. One relaxation specimen at a time was positioned centrally in a frame similar to that for the creep tests, complete with pressure cell and header plates. The system arrangement is shown schematically in Figure 3. The Carlson meter lead wires from the specimen were brought into a bridge panel to constitute the active arm of an unbalanced Wheatstone bridge (see Figure 4). The other arm was formed by another Carlson strain meter, embedded in a companion specimen which remained unloaded throughout the duration of the test. This arm provided compensation for temperature and autogenous volume changes in the loaded specimen.

Pressure on the specimen was continuously measured by means of a 5000 psi strain gage pressure meter D (see Figure 3), which was connected to the system through E, using the bridge circuit as shown in Figure 4.

Constant strain in the specimen was maintained by providing for controlled leakage of oil from the system. An accumulator connected to the system facilitated the control of the leakage rate and relieved the pulses from the electric and hand pumps during their operation. It also insured that sudden loss of pressure through accidental leaks in the system did not occur. Details of the loading system for the stress relaxation tests are given in Section 4.6.

Output data from the tests were obtained through output amplifiers, connected to a digital voltmeter (DVM), F, and a speedomax recorder G. The DVM displayed input and output voltages from the strain meter and pressure meter circuits, on the operation of a selector switch. The speedomax recorder, on the other hand, gave a continuous trace of stress and strain on a vertically ruled chart.

A connection was made from the strain meter bridge to an independent Carlson test set through a multiple switch arrangement to permit the measurement of meter ratio and total meter resistances during the test. Connection

was also provided to the hydraulic pressure system to enable ready calibration of and checks on the recording instruments.

4.5 Loading System for Creep Tests

This consisted of a hand operated fluid pump with a pressure gage and hose attachment linking the pump when required to the hydraulic control unit. A typical unit consists of an accumulator, an electric pump, and an oil reservoir. The system was developed and perfected by Best, Pirtz and Polivka [1]. The desired pressure was preset on an electric pressure regulator, which activates the pump motor when there is any drop in pressure in the system.

4.6 Loading System for Relaxation Tests

4.6.1 General

The hydraulic control and loading unit for the stress relaxation tests was similar to the unit used for the creep tests. Figure 5 shows the unit with a hand operated fluid pump attached and Figure 6 shows the interior from a top-end view of the unit. The unit consisted of an accumulator LR3, and an electric pump LR5, housed in a frame fitted with pressure gages and an oil reservoir, all of which were connected to a load cell LR4, located below the specimen on the loading frame. Gage LR1 measured the accumulator pressure. Gage LR2, an Ashcroft direct reading bourdon tube gage, measuring to 3000 psi was connected to the system to keep a check on the system pressure during the test. The 5000 psi strain gage pressure meter (Sec. 4.4) was connected to the system near the load cell. This gave more representative measurements of pressure on the specimens. Oil was fed from the reservoir to the load cell LR4 through the accumulator. The reservoir itself was connected to the electric pump which was adjusted to control oil delivery to the system. All units were inter-connected by 1/4-inch diameter copper tubes, oil flow through which was controlled by valves. The hydraulic system was linked to a terminal tube and valve to facilitate connection to a manual fluid pump for application of initial pressure to the specimen at the time of loading.

4.6.2 Method of Load Application

A blackhawk hand operated fluid pump with pressure gage attached was used to deliver oil at the required pressure to the hydraulic system. Before pumping commenced, the oil reservoir in the hydraulic control unit was filled with oil to a reasonable level, and all valves in the system were set at their correct positions. Particularly, the accumulator valve was set at the closed position. Pumping proceeded until the pressure in the system was reasonably close to the pressure required for initial loading, when the accumulator valve was opened. Bringing the accumulator into operation in the system had the effect of absorbing shocks from the pumping and preventing accidental overstress during loading. Pumping was continued until the required stress on the specimen was reached. The applied stress was then monitored continuously and recorded as described in Section 6.2.

4.6.3 Strain Control

On application of full load, the rods compressing the relaxation test frame became strained to a level compatible with the strain in the specimen under load. As the specimen relaxed and stress dropped, the strain in the rods and in the specimen was kept constant by removing oil from the load cell and the accumulator through a special needle valve which had an angle of only one degree. Because of the nature of the connections in the system, some additional oil was able to escape from various joints in the system, but mostly from the edge of the packing gland of the load cell. When too much oil had escaped than was necessary to make up the difference, additional oil from the reservoir was pumped back into the system by the electric pump which was activated through the unbalance of the strain bridge circuit.

5. CALIBRATION OF RECORDING INSTRUMENTS

5.1 Digital Voltmeter (DVM)

The digital voltmeter was calibrated against known strains and pressures. For the strain calibration, a compact Carlson strain gage calibrator, fitted with a dial gage, which had a least reading of 0.0001-inch was used. Pressure calibration was accomplished with the aid of an Amthor dead weight pressure tester.

a. Strain Calibration

A Carlson meter with known characteristics was clamped in position in the Carlson strain gage calibrator frame and its lead wires were connected to the DVM. The input voltage to the strain meter bridge was then varied. For each voltage, known displacements were applied at intervals by turning the loading screw on the frame. Readings of displacements as indicated by the dial gage in micro-inch, and voltage, as displayed by the DVM were taken for each input voltage. A graph of micro-volts against strains was drawn from which the micro-volt reading per micro-strain was deduced for the input voltages studied.

b. Pressure Calibration

The Amthor dead weight pressure tester was connected to the hydraulic pressure line of the test system through valve H as shown in Figure 3. Selector switch J was then operated to link the output of the pressure bridge with the DVM. After checking the oil level in the tester and setting the input voltage to the circuit at the desired level and all valves to their correct positions, dead weights were added to the loading plate of the tester. Load in increments equivalent to 200 psi were then applied, and the corresponding microvolt readings on the DVM taken. Loading applied was chosen to cover the range of expected pressures to be recorded during the test. Input voltages used varied from 3.5 to 3.7 volts.

5.2 Speedomax Recorder Chart

Calibration of the speedomax recorder was carried out before each test. This two - input recorder had two pens. One was filled with green ink and the other with red ink.

Strain measurements were recorded by movement of the green pen on the specially ruled graph paper graduated in inches.

a. Strain Calibration

The Carlson strain gage calibrator described in 5.1a. was used. The lead wires from the Carlson gage were connected to the active arm of the strain meter bridge and a setting chosen on the strain output potentiometer balance. The instrument selector switch J (Figure 3) was then set for connection to the recorder before calibration commenced. Known displacements were applied to the Carlson meter for an input voltage to the bridge circuit of 1 volt. For each displacement applied as in Section 5.1a., the pen movement in inches was recorded. The calibration was repeated for a bridge input voltage of 2 volts. The procedure was again repeated for different output balance settings. The pen movement per microstrain was recorded.

b. Pressure Calibration

Pressure measurements were recorded by movement of the red pen. For pressure calibration the Amthor dead weight pressure tester was loaded in increments of 200 psi with the selector switch in position for output to the recorder. Movement of the red pen across the recorder graph paper was read at each increment of pressure for an applied voltage of 3.5 volts and a chosen pressure output balance setting. The calibration was repeated for other input voltages until a satisfactory range of pen movement was achieved, which gave about 1 inch of movement for every 50 psi change in stress over the stress range anticipated for each test.

6. TESTING PROCEDURE

6.1 Creep Tests

The creep tests required to provide the data for constructing relaxation curves were carried out at loading ages of 7, 28 and 90 days.

After setting up the specimens in tandem in the creep frame as described in Section 4.2, initial gages readings were taken at zero load. Control valves were then checked and an initial stress of 2,100 psi was applied to the specimens by means of a manual hand pump. Care was taken to ensure that the loading rate was uniform and kept at a level of 35 ± 5 psi, per second. At the end of the loading, controlling valves were set at desired open or closed positions and gage readings taken as close as possible to the instant of full load application. The loading technique applied has been perfected to achieve these initial readings within 10 seconds of attaining full applied load. The applied stress was checked against the reading of the pressure gage on the specimens. This pressure gage had been previously calibrated over the range of the expected applied load. Readings were taken subsequently at intervals of 1m, 3m, 5m, 10m, 30m, 3h, 5h, and 24h after loading, and later, daily readings were taken for a week, weekly readings for a month, followed by twice monthly readings until enough data had been recorded. Readings of the gages embeded in companion autogenous specimens were also taken. Each set of readings relating to a particular time interval was recorded on tape and later analysed with the aid of a computer.

6.2 Relaxation Tests

After the specimen was carefully centered in the stressing frame shown in Figure 1, a load was applied to it using the same manual pump and procedures adopted for the creep tests. As load was applied, close watch was kept on the speedomax recorder pens and the accumulator valve. As soon as the applied pressure was 100 psi short of the desired initial pressure, the accumulator valve was opened and the pressure brought up to the required level. The pressure and strain potentiometer balances were then quickly given final adjustments to bring the recorder pens to their correct positions

in the previously calibrated recorder scale. Automatic recordings of pressure and strain then commenced as the pens trace out stress and strain changes with time. Additional readings of voltage input and output from the pressure and strain bridge circuits were taken using the digital voltmeter. These readings together with those of the pressure gage attached to the hydraulic control unit were taken hourly for the first four hours and daily thereafter for 14 days, after which readings were taken every two days. Periodic measurements were also taken of resistance ratios and three-wire resistances of the active and compensating gages to keep a continuous check on the performance of the equipment. For this purpose, a Dekabox test set and a "Magnetic" null indicator wired independently of the relaxation instrumentation system, was used (Sec. 4.4). The set-up for these independent measurements can be seen in Figure 7.

7. EXPERIMENTAL RESULTS

7.1 General

There are two aspects of the investigation being reported. The first relates to the performance of the equipment devised for the measurement of relaxation. The second concerns the trend of the measurements and how these measurements compare with values derived from creep data. The details of testing carried out are given in Table 2.

7.2 Strain Control Records

The records taken during the tests show a highly successful strain control. Table 3 gives strain changes calculated from resistance ratios taken during the test of specimens R2, R3, R4, and R5. These changes are small. The stress and strain plots, shown in Figures 8 through 12, were obtained from the speedomax recorder and are even more striking. These show continuous strain records in which the strain is represented by a vertical trace which only marginally departs from a constant initial value. It was observed that during the earlier periods of loading when the pump was more frequently activated, there were short impulses in the system. These show up in the trace in the form of short horizontal lines. As time elapsed, this tendency seemed to have been less evident. Figures 8 through 12 show only the early stages of relaxation. The estimated maximum variation from initial strain based on the scale of the speedomax chart was 0.5×10^{-6} in./in. The variation indicated by the resistance ratio measurements was larger for R2 and R4. Even so, the order of change in these cases indicated that strain control was successful.

7.3 Stress Records

The residual stress in the concrete during each test was read from the Ashcroft direct reading bourdon - tube gage and the DVM, as well as being traced out on the chart of the Speedomax recorder. Very good correlation was obtained between the Ashcroft gage and the Speedomax chart values. Because of voltage fluctuation there were slight differences between the DVM readings and the two other readings. These differences were, however, never significant and were of the order of +2 to +5 psi, as can be seen from

Table 4. The Speedomax recording gave a reasonably gradual decay of stress for the first 30 minutes of relaxation. Thereafter, the instrument gave a step-like trace for a period of about three days before settling down again to a steady recording. Even during the period of step-like responses to stress loss, the results show steps not exceeding -7 psi. Periodic checks were made of pressure levels using a lead weight tester. These checks helped to ensure that the Ashcroft and the Speedomax instruments were operating within satisfactory level of accuracy. In order to obtain a fairly clear indication of stress loss, the calibration of the chart of the Speedomax had to be done to a reasonably large scale. This meant that only a stress drop of 500 psi could be accommodated on the chart's horizontal scale. Thus, the pressure indicator pen had to be reset at convenient points during the test to avoid any errors arising from the movement of the pen out of the range of the scale. The residual stress curves produced by the Speedomax recorder, therefore, show a break at every scale change. Full records of DVM readings are given in Tables 4a through 4e. These tables include equivalent stress values deduced from output voltages of the pressure bridge circuit.

On the whole, pressure recordings were trouble free as they were with the strain recordings. The time scale on the Speedomax chart was set at a vertical movement of 1-inch for every 20 minutes of expired time for the first 7 days of the test and then reduced to an inch of movement for every 200 minutes for the rest of the test period.

7.4 Creep Test Results

Results from creep tests were analysed by a CDC 6400 computer with the aid of a program which provided for outputs of temperature of specimens, creep strain, creep plus autogenous strains, elastic plus creep plus autogenous strains and average specific creep of the two specimens loaded in tandem in each test run. The results of computer calculations are shown in Tables 5 through 8 for tests performed under the program for this investigation. Tables 9 through 11 give data from another test program, for 28, 180 and 360-day old concrete made with Mix 1. The autogenous strain values given are based on zero strain at time of application of full load. Table 12 gives 183-day creep data for Mix 2.

7.5 Relaxation Test Results

Manual plots of residual stress against time since loading (elapsed time), are given in Figures 13 through 17. These have the characteristic shape of curves of decaying functions. For easy comparison of results, the curves are shown together in Figures 18 a&b, drawn to a scale which has been normalized with respect to the applied stress. As expected, the proportion of the initial stress remaining at a given elapsed time reduces with increasing age of the concrete at time of loading. The curves also confirm the behavior well established from creep results, that the rate of relaxation is greater during the periods immediately following load application than during later periods.

The stress values used in plotting these curves, were those reduced from DVM reading.

8. COMPARISON OF RELAXATION RESULTS WITH VALUES DERIVED FROM EXISTING STANDARD METHODS USING CREEP DATA

8.1 General Remarks

Figures 13-17 show the results of tests for specimens R1-R5 (see Tables 1b & 2). The plots of the theoretical curves were carried out using the equation:

$$\sigma_t = \sigma_0 - \sum_{i=1}^n \Delta\sigma_i, \dots \dots \dots (8.1)$$

where $\Delta\sigma_i$ is the stress relief necessary at time t_i to maintain constant strain over interval i to $i+1$.

$\Delta\sigma_n$ was calculated from the equation:

$$\Delta\sigma_n = E_n [\sigma_0 C(0,n) - \sum_{i=1}^{n-1} \Delta\sigma_i \psi(i,n)] \dots \dots \dots (8.2)$$

ψ is the deformation function defined in Section 9 and includes C , the creep function. Derivation of the relaxation function from these two functions is given in Section 9. The deformation functions adopted for the test series are given in Table 12. A logarithmic function was chosen, initially, as it gave a reasonably high correlation coefficient for these tests, although in giving a fit at early stages of creep.

8.2 Comparison of Theoretical and Experimental Curves

The experimental curves show lower residual stress values than those estimated by integration of creep functions derived from creep data. This underestimation is due to the use of derived creep functions as creep recovery functions. The effect of adopting this function is to apply over-estimated inelastic strain corrections at intervals following all ages at which stress was removed. Studies on creep recovery in concrete has shown that this assumption is incorrect. Creep recovery is a different phenomenon. For example, creep recovery in mass concrete is independent of load history while creep is not [28].

Bletzacker [29] has postulated that time dependent deformation is really only potentially recoverable because on removal of the applied stress, the reversed deformation and the time-dependent potential energy imposes a reverse deviatoric stress on the viscous elements. Thus, during the time required to recover the potential energy, further dissipation, relaxation and interelemental stress redistribution takes place resulting in a smaller net-recovery than the preceeding creep. Davies [30] also found that the principle of superposition was not directly applicable to the recovery process during unloading. Therefore, theoretical recovery based on this principle was always larger than actual recovery. Most importantly, Davis observed that elastic strain on unloading in normal weight concrete was always smaller than the elastic strain at the age of load application.

The results of the present investigations show clearly the effect of the combined elastic and creep strain lag during unloading of a previously stressed normal weight concrete specimen (see Figures 13-17). It was discovered that the error of prediction was smaller the greater the time which had elapsed since loading. The error also reduced as the age at which initial load was applied, increased. Table 13 gives details of measured and predicted residual stress values. The error is measured by the relative value, $\frac{\sigma_m - \sigma_c}{\sigma_o}$, σ_m being the measured residual stress, σ_c , the predicted stress and σ_o the initially applied stress.

It is seen, therefore, that the principle of superposition of creep as propounded by McHenry [31] in 1943 does lead to some inaccuracies in predicting relaxation from creep data, even when, as in the studies reported herein, the modulus of elasticity is varied with the age of the concrete. Larger inaccuracies have, of course, been reported [14] when variation of the modulus of elasticity with age had been neglected. It can be seen from Table 13 that the errors ranged from 8-14%, and will be considerably higher when σ_o is replaced by σ_m . However, an improvement of the prediction has been achieved through a closer study of the recovery phenomenon which provided the basis for the derivation of a creep recovery function from a modification of the creep function (see Section 9.3).

9. DERIVATION OF STRESS RELAXATION BY INTEGRATION OF THE BASIC CREEP EQUATION

9.1 General Formulation

Suppose that the stress causing deformation varies with time starting from a particular age at load application t_0 . The stress level in the concrete at that instant will be $\sigma(t_0)$, and the strain at any other time t will be $\epsilon(t)$. Using the principle of superposition we can determine the strain level at a particular time t_n , by solution of the equation:

$$\epsilon(t_n) = \sigma(t_0) \psi(t_0, t_n - t_0) + \int_{t_0}^{t_n} \frac{\partial \sigma(t)}{\partial t} \psi(t, t_n - t) dt \quad \dots (9.1)$$

where $\psi(t, t_n - t)$ is a function defining the total deformation in the concrete under an applied unit stress applied at age t , and maintained constant over an interval of time, $t_n - t$.

Adopting the property of a Kelvin material, the total deformation of concrete under a unit axial stress can be represented by the equation:

$$\psi(t, t_n - t) = \frac{1}{E}(t) + C(t, t_n - t) \quad \dots \dots \dots (9.2)$$

$\frac{1}{E}(t)$ gives the instantaneous - elastic deformation and $C(t, t_n - t)$ gives

the creep deformation over interval $t_n - t$ of the sustained load.

Functions for $E(t)$ and $C(t, t_n - t)$ can be established for various types and quality of concrete. No general function for each property is applicable to concrete, although attempts have been made [8, 17] to provide expressions for both, which would be applicable under a wide range of conditions. While a power expression has been found more suitable for expressions giving the variation of the elastic modulus with time, a logarithmic expression for creep incorporating the variable $(t+1)$ has been found more popular than other expressions. It has however, been found more appropriate to develop expressions for each set of concrete properties involved under conditions

of a project for which time-dependent relations are required, rather than adopting standard expressions based on a different set of conditions.

The expression adopted for $E(t)$ in the studies presented in this report was of the form:

$$E(t) = \alpha t^{-\beta}$$

and that for creep was of the form:

$$C(t, t_n - t) = \eta t \text{ Log } (1 + t_n - t)$$

Eq.9.1 can be expanded by integration of the last term to give:

$$\epsilon(t_n) = \sigma(t_n) \psi(t_n, 0) - \int_{t_0}^{t_n} \sigma(t) \cdot \frac{\partial \psi}{\partial t}(t, t_n - t) dt \dots \dots \dots (9.3)$$

and Eq.9.2 differentiated to give:

$$\frac{\partial \psi}{\partial t}(t, t_n - t) = \frac{\partial}{\partial t} \left[\frac{1}{E(t)} + C(t, t_n - t) \right] \dots \dots \dots (9.4)$$

This approach to the evolution of a relaxation function is impracticable in view of the nature of the variables in the proposed creep function, the character of the function itself and the objective of the study. An alternative approach was pursued instead, which involved maintaining Eq.9.1 in its original form and solving for stress changes with time by numerical methods. This avoided introducing modifications to the E and C functions to give close form solutions. The alternative approach used is described in Section 9.2.

However, an interpretation of the relaxation process can be given utilizing Eq.9.3 and derivations for application to deformation of structural elements can proceed from such interpretation.

If we define a deformation rate ϕ , in the following manner:

$$\phi(t_0, t_n - t) = \frac{\partial}{\partial t} \left[\frac{1}{E(t)} + C(t, t_n - t) \right]$$

we can write Eq.9.3 as follows:

$$\begin{aligned} \epsilon(t_n) &= \sigma(t_n) \psi(t_n, 0) - \int_{t_0}^{t_n} \sigma(t) \cdot \phi(t_0, t_n - t) dt \\ &= \frac{\sigma(t_n)}{E(t_n)} - \int_{t_0}^{t_n} \sigma(t) \cdot \phi(t_0, t_n - t) dt \dots \dots \dots (9.5) \end{aligned}$$

The stress at any point t_n can then be deduced, giving:

$$\sigma(t_n) = \epsilon(t_n) \cdot E(t_n) + \int_{t_0}^{t_n} \sigma(t) \cdot E(t_n) \phi(t, t_n - t) dt \dots (9.6)$$

when strain is held constant:

$$\sigma(t_n) = \epsilon_0 \cdot E(t_n) - E(t_n) \int_{t_0}^{t_n} \sigma(t) \cdot \phi(t, t_n - t) dt \dots \dots \dots (9.7)$$

$\sigma(t)$ is the function which defines the stress relief with time. The function $\phi(t, t_n - t)$ is a rate function which can be expressed in simple terms, to make the use of Eq.9.7 readily appealing.

9.2 Relaxation Function by Numerical Integration of Creep Function by Steps

Examination of Eq.9.1 shows that when $\epsilon(t_n)$ is kept constant throughout the period $t - t_n$ as will be required in the relaxation process, $\frac{\partial \sigma}{\partial t}$ becomes the change necessary in the initial stress, $\sigma(t_0)$, per unit time over the period, to maintain this constant strain.

Substituting in Eq. 9.1,

$$\epsilon(t_n) = \epsilon_0$$

$$\sigma(t_0) = \sigma_0, \text{ the initial stress and,}$$

$$\frac{\partial \sigma}{\partial t} = - \frac{\partial \sigma}{\partial t}, \text{ having regard for sense of stress change,}$$

we have:

$$\epsilon_0 = \sigma_0 \psi(t_0, t_n - t_0) - \int_{t_0}^{t_n} \frac{\partial \sigma(t)}{\partial t} \phi(t, t_n - t_0) dt \dots \dots \dots (9.8)$$

Using numerical procedures we can divide the time interval $t_n - t_0$, i.e., the time since application of initial stress into n intervals and represent the loss of stress over any interval i , as $\Delta\sigma_i$, thus:

$$\epsilon_0 = \sigma_0 \psi(0, n) - \sum_{i=1}^n \Delta\sigma_i \psi(i, n) \dots \dots \dots (9.9)$$

Rewriting Eq. 3 we have:

$$\epsilon_0 = \sigma_0 \psi(0, n) - \sum_{i=1}^{n-1} \Delta\sigma_i \psi(i, n) - \Delta\sigma_n \psi(n, n) \dots \dots \dots (9.10)$$

where,

$$\psi(n, n) = \frac{1}{E_n} + C(n, n) \dots \dots \dots (9.11)$$

$$\text{and } \psi(0, n) = \frac{1}{E_0} + C(0, n) \dots \dots \dots (9.12)$$

$$C(n, n) = 0 \dots \dots \dots (9.13)$$

Substituting Eqs. 5, 6, & 7 into Eq. 4 and transposing gives:

$$\Delta\sigma_n = E_n [\sigma_0 C(o,n) - \sum_{i=1}^{n-1} \Delta\sigma_i \psi(i,n)] \dots\dots\dots (9.14)$$

The residual stress at any time t_n can then be determined by summing up the stress changes from time o, when the load was applied to interval n.

$$\sigma_{t_n} = \sigma_o - \sum_{n=1}^n \Delta\sigma_n \dots\dots\dots (9.15)$$

This is the basis of a number of well known procedures [2,17] utilizing the step by step method of determining stress loss from known strain histories, using data provided from creep tests.

9.3 Modified Relaxation Function

The stress loss at the n^{th} interval given by Eq.(9.14) can be rewritten to incorporate a creep recovery function $\bar{C}(i,n)$ which is a modified creep function,(C). The relevant equation is given by:

$$\Delta\sigma_n = E_n [\sigma_0 C(o,n) + \sum_{i=1}^{n-1} \Delta\sigma_i \{ \frac{1}{E_{IR}(t)} + \bar{C}(i,n) \}] \dots\dots\dots (9.16)$$

E_{IR} is the modulus of instantaneous or elastic recovery. For simplicity we can combine both instantaneous and creep recovery effects into a deformation recovery function $\bar{\psi}$. Thus,

$$\Delta\sigma_n = E_n [\sigma_0 C(o,n) + \sum_{i=1}^{n-1} \Delta\sigma_i \bar{\psi}(i,n)] \dots\dots\dots (9.17)$$

It is now reasonably understood that the phenomenon of creep involves a complicated combination of factors related to the reactions of the constituents of the concrete to long term sustained loading. Certain constituents such as the cement paste, respond in a way which contributes a deformation that is irrecoverable. Intergranular reaction while under load is believed to cause internal structural readjustments which also contribute to irrecoverable creep of the material. Other constituents, such as aggregates and moisture produce effects which under certain conditions result in delayed elastic response, most or all of which are recoverable.

Equation 9.16 assumes that the time dependent component of the deformation, ψ , of Equation 9.2, can be separated into its recoverable and irrecoverable components, and that $\bar{\psi}$ represents the recoverable components. The quantity E_{IR} in the equation represents the modulus of instantaneous recovery which has been shown to be less than the modulus of instantaneous deformation $E(t)$. The recovery function can therefore be obtained by modifying the function ψ so that it includes a correction, not only for irrecoverable creep, but also for the difference between $E(t)$ and E_{IR} .

Introducing an irrecoverable term, R_t , which can simply be related to the basic deformation function, one can write, for the irrecoverable deformation at time t ,

$$R_t = \sigma \psi K e^{-\alpha(t-t_1)} \dots \dots \dots (9.18)$$

where α and K are constants depending on concrete material properties, and t_1 , the time at which the stress is removed.

The recovery function will therefore be given by:

$$\bar{\psi} = \psi [1 - K e^{-\alpha(t-t_1)}] \dots \dots \dots (9.19)$$

From earlier discussion, it is clear that recovery properties will depend on the mix of concrete. There have also been indications that they depend on aggregate properties. The present studies seem to indicate further that K

changes very little at the older ages when the type of concrete constituents are the same, but α varies with concrete strength and therefore with age, being smaller the older the age at which unloading takes place.

K and α were obtained by writing an error factor,

$$\eta = \frac{\bar{\psi} - \psi}{\psi} = -Ke^{-\alpha(t-t_1)}$$

and carrying out a regression analysis using data from measured and calculated stresses. The value $(\bar{\psi} - \psi)/\psi$ was obtained by deduction from values of $\frac{\sigma_m - \sigma_c}{\sigma_0}$,

using σ_0 to relate the error to the standard reference stress. This in turn was converted as shown in Table 14 to specific strain differences using the initial strain as the reference strain.

Table 15 shows values of α and K for the specimens used in this investigation. These coefficients were applied to Eq. 9.19, to obtain the corrected relaxation function. The results achieved by using this modification are discussed in Section 10.

10. CONCLUSION AND RECOMMENDATIONS

It has been demonstrated that the hydraulic strain controlling system devised for carrying out the stress relaxation tests, performed as well as could be expected. The degree of control achieved was remarkably high at $\pm 0.5 \times 10^{-6}$ in/in for most of the tests. The authors were able to perfect a system which can be reproduced quite cheaply, thereby eliminating the need for resorting to much larger investments to obtain high accuracy. The construction of stress relaxation curves from creep data has been improved by the use of a creep recovery function. This function is related to the creep function by the equation:

$$\phi = \{1 - Ke^{\alpha(t-t^1)}\} \psi$$

It is shown in Figures 13-17 that by using this function in the integral equation for residual stress, a prediction of stress relaxation in concrete was possible to within 9% of measured values.

The use of stress relaxation curves for determining stress history from a record of strain history should increase with the availability of an accurate method for constructing relaxation curves from creep data. There is no doubt that the creep tests will continue to be regarded, and be performed as an easier option in the process of determining time dependent effects of induced strains. There is no doubt that the procedure for utilizing creep directly in the determination of stress histories from strain histories is involved and laborious where the electronic computer is not readily available. An example of such a procedure is given in Table 18. The principle of super position is adopted and the creep values used have been obtained by interpolation of creep data covering a number of loading ages. The implications of the procedure are quite obvious. The use of relaxation curves will mean that a more direct process will be substituted for the intricate procedures of Table 18 in determining stress levels in mass concrete structures such as dams. An outline of a suitable procedure using stress relaxation curves is shown in Figure 20. The stress relaxation functions corresponding to strain changes over predetermined time

intervals are applied at times coinciding with the upper boundary of the interval. The stress at any time t_n is then given by:

$$\alpha(t_n) = \sigma_0 + \sum_{i=1}^n \Delta\sigma_{in}$$

The method is readily programable for computer operations.

It is clear from the investigations that the phenomenon of stress relaxation is somewhat different from that of creep. It is recommended, therefore, that more studies be encouraged which relate directly to stress relaxation behavior. The literature is conspicuously thin on a number of topics concerning factors which are presumed to influence stress relaxation behavior. Even these influences have not all been supported by experimental evidence.

There is a need to investigate further, the validity of using a logarithmic function to express the basic creep behavior. Although this particular form of expression is convenient because it gives a linear variation of deformation with the logarithmic value of expired time, it has been shown by a number of studies of creep data at the University of California at Berkeley, that such a function underestimates total strain for approximately the first five hours of creep. It has been discovered that where time is measured in days, both elastic and creep strains can be evaluated wrongly between zero and five hours. In the step by step integration process therefore, a smaller initial creep estimate can significantly reduce the residual strain close to the time of loading to an extent that will result in the application of a lower stress relief value and a consequently higher residual stress than is actually the case. An error in the initial operations during the early intervals can give rise to errors over the entire period under prediction. The correction for irrecoverable creep as expressed by Eq. 9.19 will therefore be smaller if a correction is made for the inaccuracy of the creep function at early ages. Figures 21-23 show the corrections applicable for some of the ages at which loading took place. Where required between ages, interpolation was done. Direct application of this method of correction was carried out for the results

of tests R2-R5. Quite good predictions were achieved (Figures 24-27).

It is assumed that the correction is necessary only at the beginning of the iteration, as most of the effects of high early creep may have arisen principally from the breakdown of the structure of the material at application of load. This tendency will be negligible at later ages.

It is recommended that a number of alternative forms for the creep curves be explored to provide a closer fitting for the first five hours or so of creep. Preliminary investigations of power functions indicate encouraging prospects. Table 13 gives the trial functions, and Figures 28-31 show the predictions of residual stresses using these functions. The error obtained is between 1-5%. This can be further corrected using the procedures outlined in Section 9.3.

It is further recommended for future investigations, that the computer program developed by the authors for predicting relaxation curves from creep data be modified to accept as direct inputs, values of creep strain and time under load.

TABLES

TABLE 1a
DETAILS OF CONCRETE MIXES USED IN SPECIMENS

Material or Relevant Data	Mix 1		Mix 2	
	Source or Type	Weight or Volume	Source or Type	Weight or Volume
Maximum Size Aggregate	3/4-inch	---	1 1/2-inch	---
Cement (lb.)	Type II	812	Type II	659
Pozzolan (lb.)	Airox	143	---	---
Water (lb.)	EBMUD	416	EBMUD	297
Sand (lb.)	Yuma	705	Callaway	1210
1 1/2-in. Aggregate	---	---	Crushed Limestone	829
3/4-in. Aggregate	Yuma	1805	Crushed Limestone	993
Water Reducing Agent	Daratard #7 (ml)	827	ZeeconR Type D (lb)	1.635
Slump (in.)	---	4 1/4	---	4 1/2
Air (%)	---	1.2	---	1.9
Temperature (°F)	---	68	---	73
Weights are based on S.S.D. and given for one cubic yard of concrete				

TABLE 1b
CONCRETE MECHANICAL PROPERTIES AT 73°F

Specimen	Dimension (in.) diameter x length	Mix No.	Unit Wt. #/cu.ft.	7-day Compressive Strength psi	7-day Modulus of Elasticity (10 ⁶ psi)	28-day Compressive Strength psi	28-day Modulus of Elasticity (10 ⁶ psi)
C1	6 x 18	1	145.3	4080	2.93	6380	3.30
C2	6 x 18	1	145.3	4080	2.93	6380	3.30
C3	6 x 18	2	149.8	3870	3.12	5040	3.50
C4	6 x 18	2	149.8	3870	3.12	5040	3.50
R1	6 x 18	1	145.3	4350	2.93	6380	3.30
R2	6 x 18	1	145.3	4350	2.93	6380	3.30
R3	6 x 18	2	149.8	3870	3.12	5040	3.50
R4	6 x 18	2	149.8	3870	3.12	5040	3.50
R5	6 x 18	2	149.8	3870	3.12	5040	3.50

NOTES

1. All specimens are sealed.
2. Strength and modulus of elasticity values for ages at which tests were carried out are given in appropriate tables of test results.
3. Compressive strength and modulus of elasticity tests were conducted on 6-in. by 12-in. control specimens.

TABLE 2
DETAILS OF TESTS CARRIED OUT

Specimen No.	Mix	Carlson Meter No.	Test Conducted	Age Stressed Days	Applied Stress psi	Initial Stress Strength Ratio	Time Under Stress Days	Test Temp. T°F
C1	1	1606 1609	Creep	7	2100	0.50	237	73
C2	1	1613 1614	Creep	28	2100	0.31	139	73
C3	2	3521 3989	Creep	28	2100	0.33	160	73
C4	2	3522 3531	Creep	90	2100	0.34	77	73
R1	1		Relax	14	1600	0.31	44	73
R2	1	1604	Relax	28	2075	0.31	60*	73
R3	2	3523	Relax	28	2050	0.36	20	73
R4	2	3526	Relax	60	2100	0.36	33	73
R5	2	3533	Relax	90	2080	0.34	40*	73

* Specimens were stressed for longer periods, but results reported cover only periods indicated.

CREEP DATA FROM OTHER TESTS

Specimen No.	Mix	Carlson Meter No.	Test Conducted	Age Stressed Days	Applied Stress psi	Initial Stress Strength Ratio	Time Under Stress Days	Test Temp. T°F
C5	1	1556 1599	Creep	28	2100	0.31	343	73
C6	1	1550 1598	Creep	180	2100	0.26	192	73
C7	1	1601 1605	Creep	372	2100	0.24	109	73
C8	2	3486 3487	Creep	183	2100	0.30	26	73

TABLE 3

STRAIN CHANGES (10^{-6} in/in) DURING RELAXATION TEST

AS MEASURED BY CARLSON STRAIN GAGE

Test No.		R2	R3	R4	R5
Days Under Load	Carlson Meter No.	1604	3528	3526	3533
	Applied Stress psi	2075	2050	2100	2030
(Initial)		(698.5)	(778.5)	(581.8)	(580.7)
1		---	+0.3	---	0.0
3		---	+0.3	+0.5	+0.5
10		---	0.0	+0.4	---
20		---	+0.3	---	---
30		---	---	---	-0.3
33		---	---	-0.3	---
37		---	---	---	---
40		-3.0	---	-0.4	-0.3
50		-3.0	---	---	---
60		-3.5	---	---	---

TABLE 4A
PART RECORD OF TEST RESULTS FOR SPECIMEN R1

Date	Time Hrs.	Elapsed Time			Pressure Recorder Chart psi	Pressure Ashcroft Gage psi	Digital Voltmeter Readings				Pressure Reduced From DVM V ₃ Reading *
							Strain Output Micro- volts V ₁	Strain Input Volts V ₂	Pressure- Output Micro- volts V ₃	Pressure Input Volts V ₄	
11/15/80	10.25	0	00	0	1600	1600	516	2.0285	+348	3.5003	1550
		0	00	4	1550	1550	516	2.0276	+248	3.5012	
		0	00	10	1505	1500	517	2.0272	+202	3.5031	
		0	00	45	1440	1440	518	2.0270	+ 28	3.5034	
11/16/80	11.10	0	00	00	1330	1333	512	2.0279	-130	3.5017	1439
		0	04	00	1185	1190	505	2.0302	+440	3.5010	
		1	00	00	1065	1065	509	2.0362	+176	3.4980	
		3	00	20	1000	1000	508	2.0263	+ 48	3.4972	
11/18/80	10.45	4	21	18	950	955	508	2.0517	- 44	3.4974	1190**
		6	21	45	925	930	527	2.0412	- 94	3.4973	
		8	21	20							
11/20/80	07.35	4	21	18	900	900	510	2.0311	-159	3.4972	1059
		14	04	27	850	860	512	2.0333	-240	3.4969	
		16	21	42	848	850	509	2.0259	-257	3.4979	
		19	03	55	830	830	508	2.0423	-306	3.4968	
11/22/80	08.15	24	21	46	810	815	509	2.0597	-328	3.4970	894
		31	21	37	800	800	510	2.0491	-338	3.4970	
		43	22	58	770	773	513	2.0409	-407	3.4911	
11/24/80	07.45										854
11/26/80	07.53										846
11/29/80	14.52										823
12/2/80	08.07										807
12/4/80	14.20										802
12/10/80	08.11										769
12/17/80	08.02										
12/29/80	09.23										

*Corrected for voltage fluctuations.
**DVM re-read: potentiometer setting changed.

TABLE 4B
PART RECORD OF TEST RESULTS FOR SPECIMEN R2

Date	Time Hrs.	Elapsed Time			Pressure Recorder Chart psi	Pressure Ashcroft Gage psi	Digital Voltmeter Readings				Pressure Reduced From DVM V3 Reading *
		d	h	m			Strain Output Micro- volts V ₁	Strain Input Volts V ₂	Pressure Output Micro- volts V ₃	Pressure Input Volts V ₄	
1/9/81	08.42	1	02	31	1665	2075	- 797	1.00011	4150	3.4946	2075
1/10/81	11.13	2	00	00	1595	1660	-7301	1.00002	3282	3.4575	1658
1/11/81	08.43	3	02	27	1535	1587	-7309	1.00005	3205	3.5185	1602
1/12/81	11.09	4	01	27	1565	1535	-7303	1.00004	3085	3.4890	1542
1/13/81	10.09	4	23	25	1460	1568	-7313	1.00010	3136	3.4917	1569
1/14/81	08.07	5	23	05	1430	1470	-7302	1.00006	2927	3.4970	1464
1/15/81	07.47	7	00	14	1415	1440	-7306	0.99972	2873	3.4948	1437
1/16/81	08.56	11	00	31	1332	1420	-7313	1.00019	2840	3.4974	1419
1/20/81	09.13	14	06	00	1295	1330	-7305	1.00022	2607	3.4311	1328
1/23/81	14.42	17	01	39	1275	1295	-7292	1.00029	2569	3.4619	1297
1/26/81	10.21	19	00	38	1260	1270	-7313	1.00069	2514	3.4655	1267
1/28/81	09.20	22	02	30	1237	1250	-7307	1.00044	2492	3.4507	1261
1/31/81	11.12	23	08	03	1225	1238	-7301	1.00021	2468	3.5017	1232
2/1/81	16.45	26	00	38	1235	1230	-7301	1.00038	2521	3.5334	1226
2/4/81	09.20	32	00	08	1200	1235	-7306	1.00042	2380	3.5697	1233
2/10/81	08.50	36	05	01	1190	1200	-7309	1.00035	2362	3.4940	1190
2/14/81	13.43	42	05	13	1150	1185	-7304	1.00013	2297	3.4947	1181
2/20/81	13.55	46	08	44	1140	1150	-7306	1.00026	2277	3.4945	1149
2/24/81	17.26	53	00	01	1117	1142	-7305	1.00018	2213	3.4835	1139
3/3/81	08.43	61	00	23	1092	1115	-7297	1.00009	2152	3.4964	1110
3/11/81	09.06	64	00	14	1090	1085	-7312	0.99962	2177	3.4945	1082
3/14/81	08.56	67	59	35	1065	1090	-7250	0.99986	2116	3.4806	1089
3/18/81	08.17					1065					1062

*Corrected for voltage fluctuations.

TABLE 4C
PART RECORD OF TEST RESULTS FOR SPECIMEN R3

Date	Time Hrs.	Elapsed Time			Pressure Recorder Chart psi	Pressure Ashcroft Gage psi	Digital Voltmeter Readings				Pressure Reduced From DVM V ₃ Reading *
		d	h	m			Strain Output Micro- volts V ₁	Strain Input Volts V ₂	Pressure Output Micro- volts V ₃	Pressure Input Volts V ₄	
3/26/81	11.41	0	00	00	0 2050	0 2050	+ 74 -6898	1.00045	365	3.7082	
3/26/81	18.15	0	06	34	1672	1670	-6899	1.00021	4557	3.7098	1670
3/26/81	09.12	0	21	31	1580	1585	-6909	1.00019	3712	3.7098	1581
3/27/81	09.10	1	21	29	1532	1530	-6919	1.00006	3514	3.7098	1534
3/28/81	12.07	3	00	26	1465	1460	-6913	1.00099	3093	3.3658	1459
3/29/81	09.50	3	22	09	1425	1420	-6913	1.00047	2943	3.3665	1416
3/30/81	09.16	4	21	35	1385	1386	-6961	1.00029	2856	3.3666	1385
3/31/81	13.47	6	04	06	1360	1360	-6921	1.00019	3070	3.6986	1364
4/1/81	11.41	7	00	00	1348	1348	-6918	1.00089	3024	3.6989	1342
4/2/81	11.41	8	00	00	1322	1324	-6920	1.00080	2974	3.6987	1324
4/3/81	11.15	8	59	34	1298	1300	-6914	1.00144	2935	3.6989	1303
4/4/81	13.47	10	02	06	1282	1280	-6917	1.00035	2897	3.7089	1284
4/5/81	11.41	11	00	00	1265	1268	-6920	1.00075	2854	3.7086	1270
4/6/81	11.51	12	00	10	1250	1250	-6918	1.00112	2824	3.7084	1255
4/7/81	12.10	13	00	29	1235	1240	-6916	1.00076	2791	3.7084	1240
4/8/81	11.41	14	00	00	1220	1225	-6923	1.00062	2756	3.7083	1227
4/9/81	11.41	16	00	00	1204	1208	-6921	1.00148	2728	3.7086	1208
4/11/81	11.41	18	00	00	1195	1193	-6927	1.00053	2685	3.7085	1190
4/13/81	13.19	20	01	38	1170	1172	-6929	1.00069	2623	3.6786	1169
4/15/81								1.00017	2578	3.6814	

* Corrected for voltage fluctuations.

TABLE 4D

PART RECORD OF TEST RESULTS FOR SPECIMEN R4

Date	Time Hrs.	Elapsed Time			Pressure Recorder Chart psi	Pressure Ashcroft Gage psi	Digital Voltmeter Readings				Pressure Reduced From DVM V ₃ Reading *
		d	h	m			Strain Output Micro- volts V ₁	Strain Input Volts V ₂	Pressure Output Micro- volts V ₃	Pressure Input Volts V ₄	
4/23/81	09.20	0	00	30	2100	2100	+4361	1.00024	2	3.6640	1880
	09.50				1880	1880	-1052	1.00010	3917	3.6640	1809
	14.20	0	04	30	1807	1806	-1015	1.00021	3769	3.6646	1673
4/24/81	08.15	0	22	55	1650	1650	-1017	1.00025	3486	3.6647	1580
4/25/81	09.37	2	00	17	1583	1585	-1017	1.00048	3290	3.6649	1547
4/26/81	09.45	3	00	25	1548	1540	-1017	1.00017	3225	3.6648	1518
4/27/81	09.38	4	00	18	1512	1517	-1016	1.00036	3162	3.6643	1494
4/28/81	09.53	5	00	33	1485	1490	-1016	1.00056	3112	3.6642	1467
4/29/81	09.20	6	00	00	1465	1465	-1015	1.00144	3057	3.6643	1439
4/30/81	10.34	7	01	14	1435	1440	-1017	1.00097	2998	3.6643	1435
5/1/81	10.30	8	01	18	1430	1435	-1018	1.00290	2989	3.6645	1420
5/2/81	14.50	9	04	38	1412	1420	-1019	1.00183	2958	3.6644	1407
5/3/81	14.23	10	05	03	1400	1400	-1018	1.00270	2932	3.6654	1392
5/4/81	17.55	11	09	25	1386	1390	-1017	1.00067	2902	3.6641	1389
5/5/81	17.00	12	08	30	1385	1385	-1018	1.00016	2895	3.6646	1370
5/6/81	09.20	13	00	00	1373	1363	-1019	1.00078	2855	3.6641	1364
5/7/81	09.32	14	00	12	1367	1370	-1019	1.00051	2842	3.6639	1350
5/9/81	09.20	16	00	00	1350	1350	-1018	1.00041	2814	3.6636	1335
5/11/81	09.30	18	00	10	1335	1335	-1019	1.00032	2782	3.6642	1317
5/13/81	18.00	20	08	40	1315	1315	-1019	1.00090	2744	3.6646	1307
5/15/81	18.05	22	08	45	1303	1300	-1019	1.00022	2724	3.6644	1296
5/17/81	09.20	24	00	00	1292	1290	-1019	1.00025	2707	3.6645	1279
5/19/81	14.15	26	04	55	1275	1275	-1018	1.00024	2664	3.6643	1240
5/26/81	09.35	33	00	15	1245	1245	-1015	1.00172	2583	3.6632	

* Corrected for voltage fluctuations.

TABLE 4E
PART RECORD OF TEST RESULTS FOR SPECIMEN R5

Date	Time Hrs.	Elapsed Time			Pressure Recorder Chart psi	Pressure Ashcroft Gage psi	Digital Voltmeter Readings				Pressure Reduced From DVM V ₃ Reading *
		d	h	m			Strain Output Micro- volts V ₁	Strain Input Volts V ₂	Pressure Output Micro- volts V ₃	Pressure Input Volts V ₄	
5/27/81	09.15	0	01	35			+ 4	1.00051	11	3.6605	
	10.50	0	02	35	2080	2080					
5/28/81	08.20	0	23	30	1675	1675	-203	1.00032	3560	3.6601	1675
5/29/81	15.30	2	04	40	1600	1600	-201	1.00019	3391	3.6597	1696
5/30/81	20.45	3	08	55	1560	1560	-203	1.00101	3307	3.6594	1556
5/31/81	15.10	4	04	20	1536	1540	-201	1.00172	3253	3.6593	1531
6/1/81	10.30	4	22	40	1520	1525	-202	1.00016	3225	3.6592	1518
6/2/81	12.07	6	00	17	1497	1500	-202	1.00125	3175	3.6588	1494
6/3/81	12.42	7	00	52	1478	1480	-204	1.00163	3132	3.6584	1474
6/5/81	12.19	9	00	29	1445	1450	-201	1.00301	3061	3.6578	1441
6/8/81	11.50	12	00	00	1415	1420	-199	1.00033	3013	3.6570	1419
6/10/81	11.50	14	00	00	1390	1395	-203	1.00145	2954	3.6570	1391
6/16/81	11.50	20	00	00	1340	1345	-202	1.00169	2840	3.6562	1338
6/21/81	19.00	25	07	10	1295	1300	-201	1.00033	2755	3.6548	1298
6/26/81	12.50	30	01	00	1268	1270	-200	1.00164	2688	3.6549	1267
6/30/81	11.56	34	00	06	1250	1255	-203	1.00170	2656	3.6565	1250
7/6/81	12.06	40	02	51	1227	1230	-202	1.00148	2604	3.6572	1226
7/16/81	10.47	49	58	57	1187	1197	-200	1.00129	2516	3.6552	1185
7/24/81	17.41	58	05	51	1165	1170	-207	1.00108	2473	3.6574	1164
8/3/81	18.25	68	06	45	1144	1145	-209	1.00128	2427	3.6479	1145

* Corrected for voltage fluctuations

TABLE 5

SPECIMEN METER NO. CHANNEL EL. MOD., MPsi										AGE AT LOADING		7 DAYS		SPECIMEN C1	
										TEST TEMPERATURE		73 DEG. F.			
										COMP. STRENGTH (7 DAY)		6000. PSI			
										APPLIED TEST STRESS		1400. PSI			
										LEVEL OF STRESS APPLIED		23.3 PERCENT OF COMP. STR.			
SPECIMEN METER NO. CHANNEL EL. MOD., MPsi										AGE AT LOADING		7 DAYS		SPECIMEN C1	
NO.1 1606 22-01 3.0										TEST TEMPERATURE		73 DEG. F.			
NO.2 1609 22-02 3.0										COMP. STRENGTH (7 DAY)		6000. PSI			
AUTOG.1 1601 70-00										APPLIED TEST STRESS		1400. PSI			
AUTOG.2 1601 70-01										LEVEL OF STRESS APPLIED		23.3 PERCENT OF COMP. STR.			
SPECIMEN METER NO. CHANNEL EL. MOD., MPsi										AGE AT LOADING		7 DAYS		SPECIMEN C1	
NO.1 1606 22-01 3.0										TEST TEMPERATURE		73 DEG. F.			
NO.2 1609 22-02 3.0										COMP. STRENGTH (7 DAY)		6000. PSI			
AUTOG.1 1601 70-00										APPLIED TEST STRESS		1400. PSI			
AUTOG.2 1601 70-01										LEVEL OF STRESS APPLIED		23.3 PERCENT OF COMP. STR.			

TABLE 7

AVERAGE TOTAL, ELASTIC, CREEP, AND AUTOGENOUS STRAINS. -- CORRECTED FOR TEMPERATURE CHANGE
 STRESS RELAXATION -- ROY CARLSON GRANT
 SPECIMEN: SEALED 6 IN. CONCRETE CYLINDER

[illegible]

TABLE 8

[illegible]

TABLE 8

TABLE 9

AVERAGE TOTAL, ELASTIC, CREEP AND AUTOGENOUS STRAINS, -- CORRECTED FOR TEMPERATURE CHANGE
BALD VERDE OPTION 1 CLASS F-1 F57530
SPECIMEN: SEALED 6 BY 16 IN. CONCRETE CYLINDER

Specimen C5

SPECIMEN METER NO. CHANNEL FL. MOD. MOD1 AGE AT LOADING : 24 DAYS
NO.1 1555 70-06 1.7 TEST TEMPERATURE : 73 DEG. F.
NO.2 1599 70-07 3.7 COMP. STRENGTH (24 DAY) : 6880 PSI
AUTOG.1 1601 70-00 APPLIED TEST STRESS : 2100 PSI
AUTOG.2 1605 70-01 LVRFL OF STRESS APPLIED : 10.6 PERCENT OF COMP. ST.

***** MICROSTRAIN (FROM TIME OF LOADING)*****																	***** AVERAGE *****	
DATE	TIME	CONCRETE	DAYS	AVG.	AVG.	TOTAL	ELASTIC	PLUS	CORR.	ELASTIC	PLUS	CORR.	ELASTIC	PLUS	CORR.	ELASTIC	PLUS	CORR.
***** SPECIMEN *****																	***** SPECIMEN *****	
***** NO.1 *****																	***** NO.2 *****	
***** DAYS *****																	***** DAYS *****	
***** SPECIMENS FULLY LOADED, AMPLIFIED TEST STRESS 2100 PSI *****																	***** SPECIMENS FULLY UNLOADED, ZERO AMPLIFIED TEST STRESS *****	
8-24-78	1000	0																
9-21-78	859	27.06																
9-21-78	859	27.06	-0.0007	71.7	0.00	0	0	0	0	0	0	0	0	0	0	0	0	0
9-21-78	859	27.06																
9-21-78	900	27.06	0	71.9	0.00	-571	-571	-571	-571	-571	-571	-571	0	0	0	0	0	0
9-21-78	904	27.06	-0.0029	72.0	1.00	-604	-604	-604	-604	-604	-604	-604	-74	-74	-74	-74	-74	-74
9-21-78	910	27.07	-0.0049	71.9	1.00	-610	-610	-610	-610	-610	-610	-610	-80	-80	-80	-80	-80	-80
9-21-78	1001	28.00	-0.0474	72.0	1.00	-611	-611	-611	-611	-611	-611	-611	-90	-90	-90	-90	-90	-90
9-21-78	1104	28.05	-0.0849	72.1	1.00	-641	-641	-641	-641	-641	-641	-641	-71	-71	-71	-71	-71	-71
9-21-78	1400	28.17	-0.2033	72.1	0.00	-641	-641	-641	-641	-641	-641	-641	-90	-90	-90	-90	-90	-90
9-22-78	925	28.09	1.0174	71.9	-1.00	-702	-702	-702	-702	-702	-702	-702	-130	-127	-124	-124	-124	-124
9-21-78	917	29.01	1.9701	72.1	-1.00	-727	-727	-727	-727	-727	-727	-727	-145	-141	-138	-138	-138	-138
9-24-78	1004	11.0	7.0	72.2	-1.00	-746	-746	-746	-746	-746	-746	-746	-174	-169	-171	-171	-171	-171
9-24-78	821	11.9	4.0	72.1	-2.00	-745	-745	-745	-745	-745	-745	-745	-192	-186	-189	-189	-189	-189
9-24-78	812	12.0	5.0	72.2	-2.00	-774	-774	-774	-774	-774	-774	-774	-205	-199	-202	-202	-202	-202
9-27-78	1701	14.1	5.1	72.1	-3.00	-772	-772	-772	-772	-772	-772	-772	-214	-211	-214	-214	-214	-214
9-24-78	804	14.9	7.0	72.0	-2.00	-798	-798	-798	-798	-798	-798	-798	-224	-218	-221	-221	-221	-221
10-5-78	1545	42.2	14.3	72.2	-3.00	-842	-842	-842	-842	-842	-842	-842	-278	-267	-272	-272	-272	-272
10-12-78	1537	49.2	21.3	72.3	-4.00	-845	-845	-845	-845	-845	-845	-845	-310	-296	-301	-301	-301	-301
10-17-78	1470	44.3	26.3	72.0	-6.00	-905	-905	-905	-905	-905	-905	-905	-324	-311	-320	-320	-320	-320
10-19-78	1920	56.4	24.4	71.4	-6.00	-912	-912	-912	-912	-912	-912	-912	-374	-359	-370	-370	-370	-370
10-26-78	2050	67.5	25.5	72.1	-7.00	-913	-913	-913	-913	-913	-913	-913	-385	-374	-384	-384	-384	-384
11-1-78	1516	69.2	41.1	72.2	-9.00	-947	-947	-947	-947	-947	-947	-947	-412	-400	-407	-407	-407	-407
11-14-78	811	81.9	54.0	71.8	-10.00	-972	-972	-972	-972	-972	-972	-972	-440	-428	-437	-437	-437	-437
11-28-78	1021	94.0	49.1	71.7	-12.00	-995	-995	-995	-995	-995	-995	-995	-472	-459	-468	-468	-468	-468
12-12-78	1070	110.0	82.1	71.4	-13.00	-1011	-1011	-1011	-1011	-1011	-1011	-1011	-497	-484	-493	-493	-493	-493
12-19-78	1514	117.2	89.1	71.5	-13.00	-1020	-1020	-1020	-1020	-1020	-1020	-1020	-516	-502	-511	-511	-511	-511
1-3-79	1047	173.0	104.1	70.7	-13.00	-1015	-1015	-1015	-1015	-1015	-1015	-1015	-531	-517	-526	-526	-526	-526
1-9-79	1159	178.2	110.2	71.5	-13.00	-1019	-1019	-1019	-1019	-1019	-1019	-1019	-544	-529	-538	-538	-538	-538
1-23-79	1444	152.2	124.2	71.4	-16.00	-1051	-1051	-1051	-1051	-1051	-1051	-1051	-584	-569	-578	-578	-578	-578
1-30-79	946	159.0	131.0	71.1	-16.00	-1055	-1055	-1055	-1055	-1055	-1055	-1055	-612	-597	-606	-606	-606	-606
2-6-79	1527	164.2	174.1	71.3	-15.00	-1057	-1057	-1057	-1057	-1057	-1057	-1057	-621	-604	-613	-613	-613	-613
2-13-79	1230	171.1	145.1	71.4	-15.00	-1041	-1041	-1041	-1041	-1041	-1041	-1041	-634	-617	-626	-626	-626	-626
2-20-79	911	180.0	152.0	71.1	-15.00	-1062	-1062	-1062	-1062	-1062	-1062	-1062	-654	-637	-646	-646	-646	-646
2-27-79	1501	187.2	159.1	71.7	-16.00	-1066	-1066	-1066	-1066	-1066	-1066	-1066	-670	-652	-661	-661	-661	-661
3-6-79	1404	194.2	166.2	71.8	-16.00	-1070	-1070	-1070	-1070	-1070	-1070	-1070	-683	-665	-674	-674	-674	-674
3-13-79	1104	201.1	171.2	71.8	-16.00	-1074	-1074	-1074	-1074	-1074	-1074	-1074	-687	-669	-678	-678	-678	-678
3-20-79	1540	208.2	180.3	72.0	-17.00	-1074	-1074	-1074	-1074	-1074	-1074	-1074	-688	-669	-678	-678	-678	-678
4-6-79	1532	221.2	195.1	71.4	-18.00	-1074	-1074	-1074	-1074	-1074	-1074	-1074	-691	-672	-681	-681	-681	-681
4-14-79	1012	237.0	209.1	71.4	-16.00	-1043	-1043	-1043	-1043	-1043	-1043	-1043	-701	-682	-691	-691	-691	-691
5-1-79	1023	250.0	222.1	71.4	-17.00	-1049	-1049	-1049	-1049	-1049	-1049	-1049	-704	-685	-694	-694	-694	-694
5-23-79	1450	272.2	244.2	71.9	-16.00	-1045	-1045	-1045	-1045	-1045	-1045	-1045	-711	-692	-701	-701	-701	-701
6-6-79	1433	284.2	254.2	72.2	-16.00	-1100	-1100	-1100	-1100	-1100	-1100	-1100	-714	-695	-704	-704	-704	-704
6-14-79	845	293.0	264.0	72.2	-16.00	-1103	-1103	-1103	-1103	-1103	-1103	-1103	-717	-698	-707	-707	-707	-707
7-1-79	1751	311.1	285.1	72.2	-17.00	-1104	-1104	-1104	-1104	-1104	-1104	-1104	-720	-701	-710	-710	-710	-710
7-17-79	1925	327.4	299.4	72.4	-17.00	-1113	-1113	-1113	-1113	-1113	-1113	-1113	-724	-705	-714	-714	-714	-714
8-14-79	1417	354.3	327.4	72.5	-17.00	-1120	-1120	-1120	-1120	-1120	-1120	-1120	-727	-708	-717	-717	-717	-717
9-11-79	707	371.0	343.0	72.6	-17.00	-1125	-1125	-1125	-1125	-1125	-1125	-1125	-731	-712	-721	-721	-721	-721
9-11-79	726	371.9	343.9	72.6	-17.00	-1125	-1125	-1125	-1125	-1125	-1125	-1125	-737	-718	-727	-727	-727	-727
9-11-79	727	371.9	343.9	72.6	-17.00	-1125	-1125	-1125	-1125	-1125	-1125	-1125	-737	-718	-727	-727	-727	-727
9-11-79	727	371.9	343.9	72.6	-17.00	-1125	-1125	-1125	-1125	-1125	-1125	-1125	-737	-718	-727	-727	-727	-727
9-11-79	729	371.9	343.9	72.6	-17.00	-1125	-1125	-1125	-1125	-1125	-1125	-1125	-737	-718	-727	-727	-727	-727
9-11-79	730	371.9	343.9	72.6	-17.00	-1125	-1125	-1125	-1125	-1125	-1125	-1125	-737	-718	-727	-727	-727	-727
9-11-79	732	371.9	343.9	72.6	-17.00	-1125	-1125	-1125	-1125	-1125	-1125	-1125	-737	-718	-727	-727	-727	-727
9-11-79	737	371.9	343.9	72.6	-17.00	-1125	-1125	-1125	-1125	-1125	-1125	-1125	-737	-718	-727	-727	-727	-727
9-11-79	826	371.9	343.9	72.6	-17.00	-1125	-1125	-1125	-1125	-1125	-1125	-1125	-737	-718	-727	-727	-727	-727
9-11-79	921	372.0	344.0	72.6	-17.00	-1125	-1125	-1125	-1125	-1125	-1125	-1125	-737	-718	-727	-727	-727	-727
9-11-79	1224	372.1	344.1	72.6	-17.00	-1125	-1125	-1125	-1125	-1125	-1125	-1125	-737	-718	-727	-727	-727	-727
9-31-79	1739	372.1	344.1	72.6	-17.00	-1125	-1125	-1125	-1125	-1125	-1125	-1125	-737	-718	-727	-727	-727	-727
9-1-79	1034	373.0	345.0	72.5	-17.00	-1125	-1125	-1125	-1125	-1125	-1125	-1125	-737	-718	-727	-727	-727	-727
9-2-79	1001	374.0	346.0	72.5	-17.00	-1125	-1125	-1125	-1125	-1125	-1125	-1125	-737	-718	-727	-727	-727	-727
9-3-79	1040	375.0	347.0	72.5	-17.00	-1125	-1125	-1125	-1125	-1125	-1125	-1125	-737	-718	-727	-727	-727	-727
9-4-79	1074	376.0	348.0	72.5	-17.00	-1125	-1125	-1125	-1125	-1125	-1125	-1125	-737	-718	-727	-727	-727	-727
9-5-79	1701	377.0	349.0	72.5	-17.00	-1125	-1125	-1125	-1125	-1125	-1125	-1125	-737	-718	-727	-727	-727	-727
9-6-79	1654	378.0	350.0	72.5	-17.00	-1125	-1125	-1125	-1125	-1125	-1125	-1125	-737	-718	-727	-727	-727	-727
9-7-79	1719	379.0	351.0	72.5	-17.00	-1125	-1125	-1125	-1125	-1125	-1125	-1125	-737	-718	-727	-727	-727	-727
9-14-79	1745	380.1	352.1	72.5	-17.00	-1125	-1125	-1125	-1125	-1125	-1125	-1125	-737	-718	-727	-727	-727	-727
9-21-79	090	390.0	362.0	72.4	-17.00	-1125	-1125	-1125	-1125	-1125	-1125	-1125	-737	-718	-727	-727	-727	-727
9-28-79	1510	400.2	372.2	72.5	-17.00	-1125	-1125											

TABLE 12

[illegible]

TABLE 13
DEFORMATION FUNCTION COEFFICIENTS

<u>Log Function</u> General deformation function is given by: $\psi = [\alpha t^{-\beta} + \eta t^{-\gamma} \text{Log} (1+t_n-t)] \times 10^{-6} \text{ strain for psi}$ $t = \text{age at loading, days}$ $t_n = \text{total age, days}$						
Concrete Type (Mix)	Unit Weight lb/cu. ft.	Coefficients				
		α	β	η	γ	
1	145.3	0.3292	0.0588	0.2572	0.3348	
2	149.8	0.3204	0.0441	0.3238	0.3380	
<u>Power Function</u> General deformation function is given by: $\psi = [\alpha t^{-\beta} + \eta t^{-\gamma} (t_n-t)^{\theta}] \times 10^{-6} \text{ strain per psi}$ $t = \text{age at loading, days}$ $t_n = \text{total age, days}$						
Concrete Type (Mix)	Unit Weight lb/cu. ft.	Coefficients				
		α	β	η	γ	θ
1	145.3	0.3292	0.0588	0.2441	0.3492	0.2496
2	149.8	0.3204	0.0441	0.24363	0.33986	0.30132

MEASURED AND PREDICTED RESIDUAL STRESSES

Specimen No.	Age, Days	$\frac{E_o}{E_t}$	Time Since Loading (days) $t-t_1$	Predicted Stress σ_c psi	Measured Stress σ_m psi	$\frac{m}{\sigma_o} c$ (q)	Correction q $\frac{E_o}{E_t}$
R1	14.0	1.000	0	1600	1600	---	---
	14.5	0.998	0.5	1480	1260	-0.1375	-0.1374
	15.0	0.998	1.0	1420	1200	-0.1375	-0.1372
	15.5	0.997	1.5	1380	1160	-0.1375	-0.1371
	16.0	0.996	2.0	1345	1120	-0.1406	-0.1400
	16.5	0.995	2.5	1305	1090	-0.1344	-0.1337
	17.0	0.994	3.0	1282	1065	-0.1356	-0.1348
	18.0	0.993	4.0	1242	1030	-0.1325	-0.1316
	22.0	0.985	8.0	1135	935	-0.1250	-0.1231
	26.0	0.978	12.0	1080	880	-0.1250	-0.1223
	30.0	0.972	16.0	1040	840	-0.1250	-0.1215
	34.0	0.966	20.0	1015	820	-0.1219	-0.1178
	38.0	0.964	24.0	995	810	-0.1156	-0.1114
	42.0	0.961	28.0	980	805	-0.1094	-0.1051
	46.0	0.956	32.0	975	800	-0.1094	-0.1046
R2	28.0	1.000	0	2075	---	---	---
	28.5	0.999	0.5	1940	1715	-0.1084	-0.1083
	29.0	0.998	1.0	1900	1670	-0.1180	-0.1107
	29.5	0.997	1.5	1860	1640	-0.1060	-0.1057
	30.0	0.996	2.0	1720	1600	-0.0578	
	30.5	0.995	2.5	1790	1570	-0.1060	-0.1055
	31.0	0.994	3.0	1765	1545	-0.1060	-0.1054
	32.0	0.993	4.0	1720	1510	-0.1012	-0.1005
	36.0	0.985	8.0	1610	1390	-0.1060	-0.1044
	40.0	0.978	12.0	1540	1320	-0.1060	-0.1037
	44.0	0.972	16.0	1490	1275	-0.1036	-0.1007
	48.0	0.966	20.0	1460	1245	-0.1036	-0.1001
	52.0	0.964	24.0	1430	1230	-0.0964	-0.0929
	56.0	0.961	28.0	1415	1210	-0.0989	-0.0950
	60.0	0.956	32.0	1400	1205	-0.0940	-0.0899
	64.0	0.954	36.0	1380	1200	-0.0867	-0.0827

TABLE 14 (cont'd)

R3	28.0	1.000	0	2050	2050	---	---
	28.5	0.999	0.5	1960	1750	-0.1029	-0.1023
	29.0	0.999	1.0	1810	1590	-0.1073	-0.1072
	30.0	0.998	2.0	1704	1500	-0.0976	-0.0974
	32.0	0.994	4.0	1585	1410	-0.0854	-0.0849
	34.0	0.991	6.0	1575	1360	-0.0756	-0.0749
	38.0	0.987	10.0	1432	1280	-0.0741	-0.0731
	42.0	0.982	14.0	1378	1240	-0.0673	-0.0668
	48.0	0.977	20.0	1325	1180	-0.0707	-0.0691
	54.0	0.972	26.0	1291	1168	-0.0600	-0.0583
	58.0	0.968	30.0	1271	1148	-0.0600	-0.0581
R4	60.0	1.000	0	2100	2100	---	---
	60.5	0.9996	0.5	1990	1760	-0.1095	-0.1093
	61.0	0.9993	1.0	1908	1650	-0.1229	-0.1228
	62.0	0.9986	2.0	1813	1580	-0.1110	-0.1108
	64.0	0.9972	4.0	1708	1510	-0.0943	-0.0940
	66.0	0.9958	6.0	1646	1465	-0.0862	-0.0858
	70.0	0.9932	10.0	1560	1400	-0.0810	-0.0804
	74.0	0.9908	14.0	1522	1360	-0.0794	-0.0764
	80.0	0.9874	20.0	1472	1320	-0.0724	-0.0715
	86.0	0.9842	26.0	1437	1280	-0.0748	-0.0735
	90.0	0.9823	30.0	1408	1260	-0.0705	-0.0693
R5	90.0	1.000	0	2080	2080	---	---
	90.5	0.9998	0.5	1940	1720	-0.1100	-0.1060
	91.0	0.9995	1.0	1911	1680	-0.1060	-0.1050
	92.0	0.9990	2.0	1826	1620	-0.0990	-0.0989
	16.0	0.9972	6.0	1674	1495	-0.0860	-0.0850
	102.0	0.9945	12.0	1580	1410	-0.0813	-0.0808
	108.0	0.9919	18.0	1526	1350	-0.0846	-0.0839
	114.0	0.9896	24.0	1490	1310	-0.0865	-0.0856
	118.0	0.9880	28.0	1471	1290	-0.0870	-0.0859
	122.0	0.9867	32.0	1454	1265	-0.0909	-0.0897
	126.0	0.9853	36.0	1440	1250	-0.0913	-0.0900
	130.0	0.9839	40.0	1428	1240	-0.0903	-0.0889

TABLE 15
VALUES OF CORRECTION CONSTANTS

Correction is given by:

$$\eta = K e^{\alpha(t_n - t)}$$

Specimen No.	Concrete Type (Mix)	Age at Loading, Days	K	α
R1	1	14	0.137	-0.009
R2	1	28	0.108	-0.006
R3	2	28	0.0950	-0.0288
R4	2	60	0.1040	-0.0191
R5	2	90	0.0996	-0.0077

FIGURES

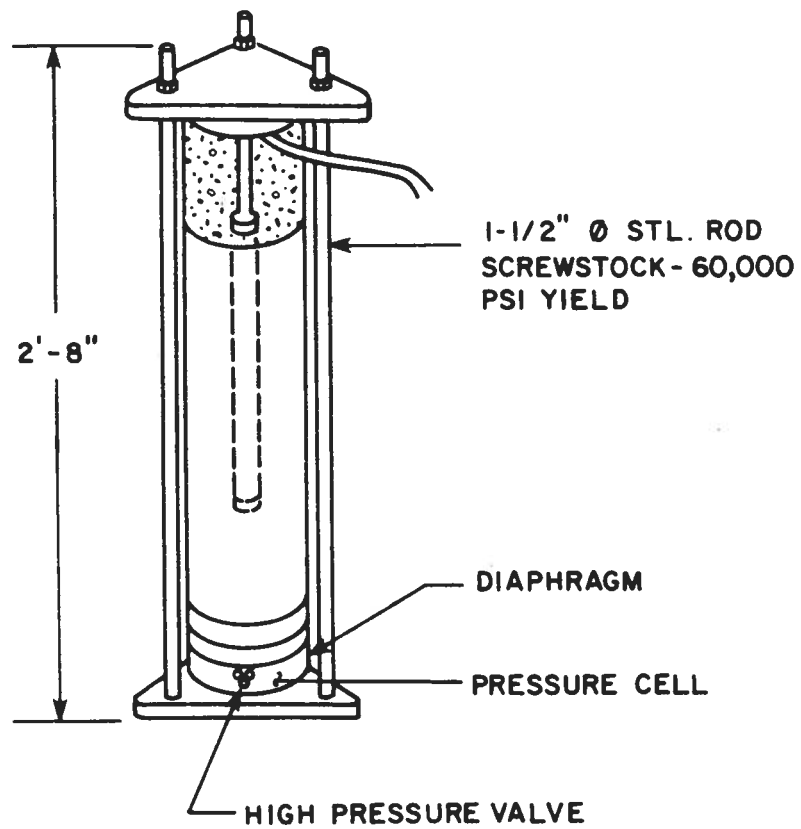


FIG. 1 SPECIMEN ASSEMBLY FOR STRESS RELAXATION TEST

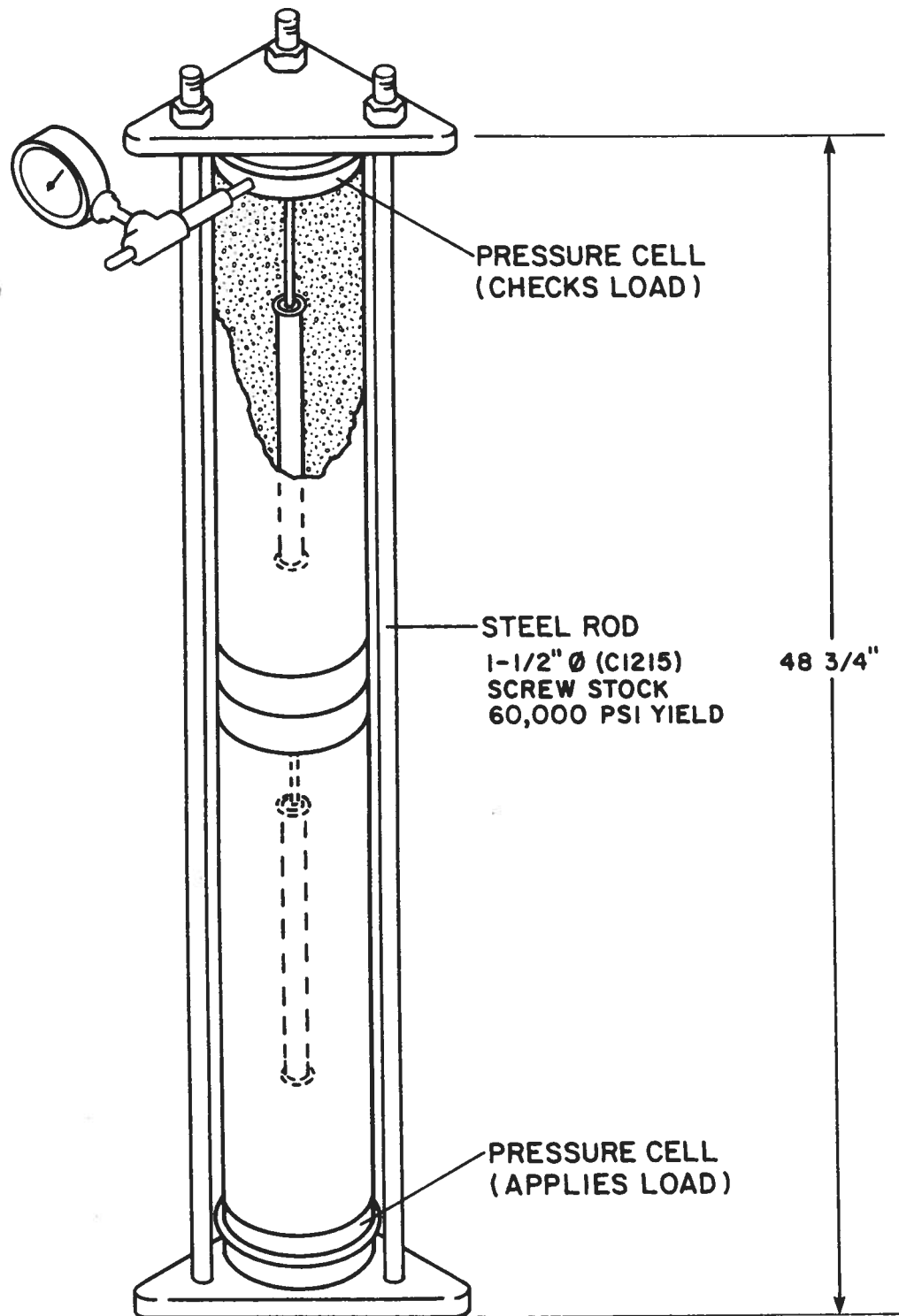


FIG. 2 SPECIMEN ASSEMBLY FOR CREEP TEST

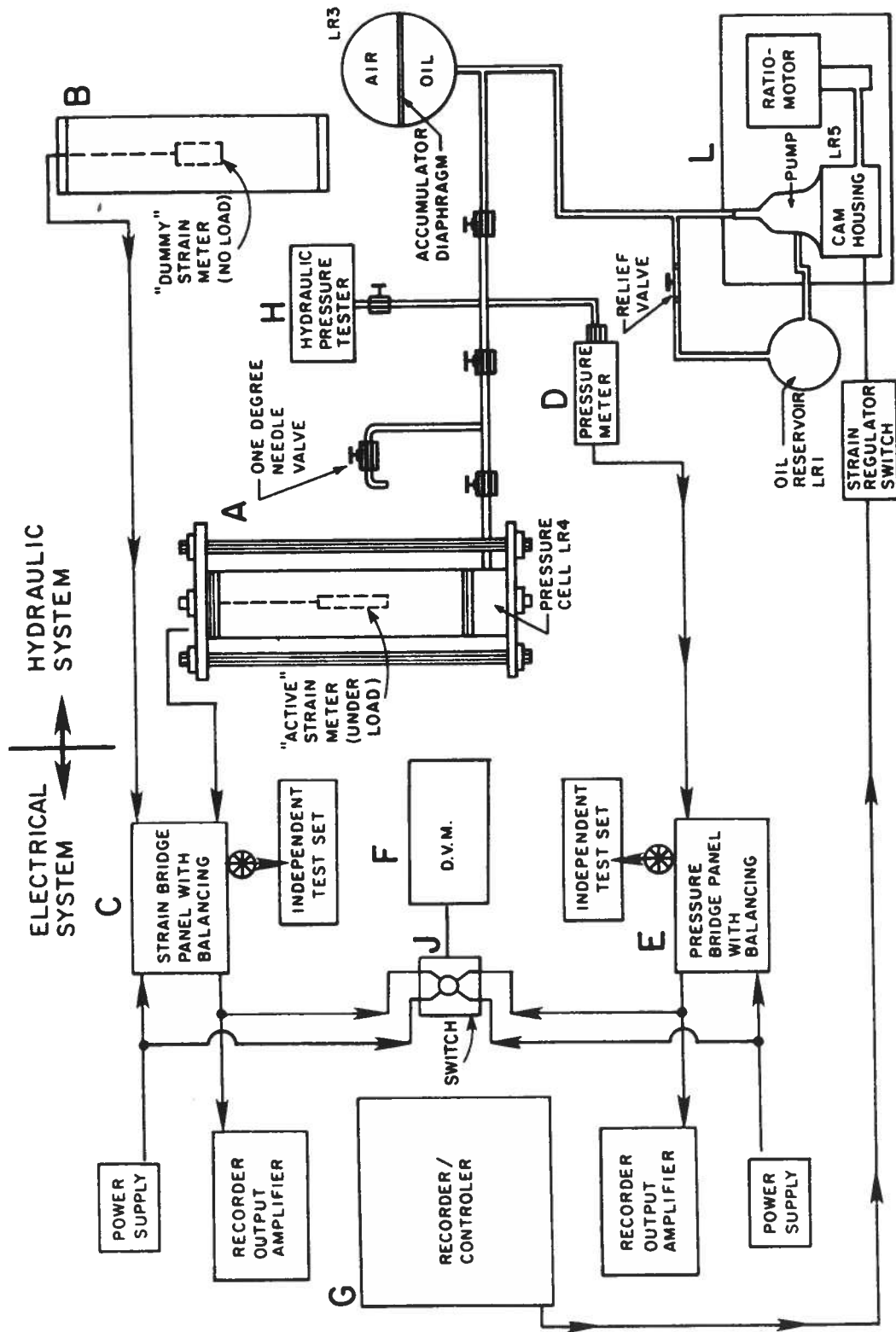


FIG. 3 SCHEMATIC LAYOUT OF UNITS FOR STRESS RELAXATION TEST

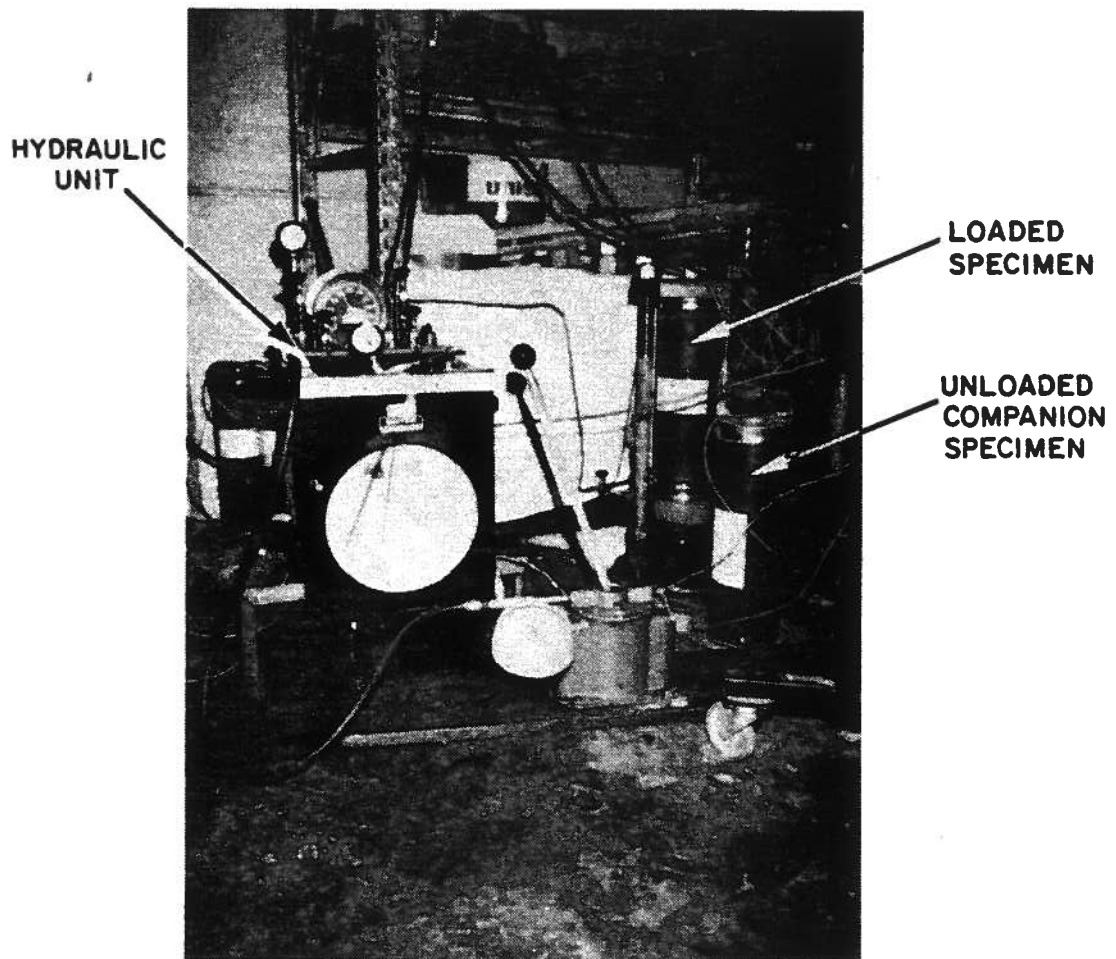


FIG. 5 PHOTOGRAPH SHOWING HYDRAULIC CONTROL UNIT

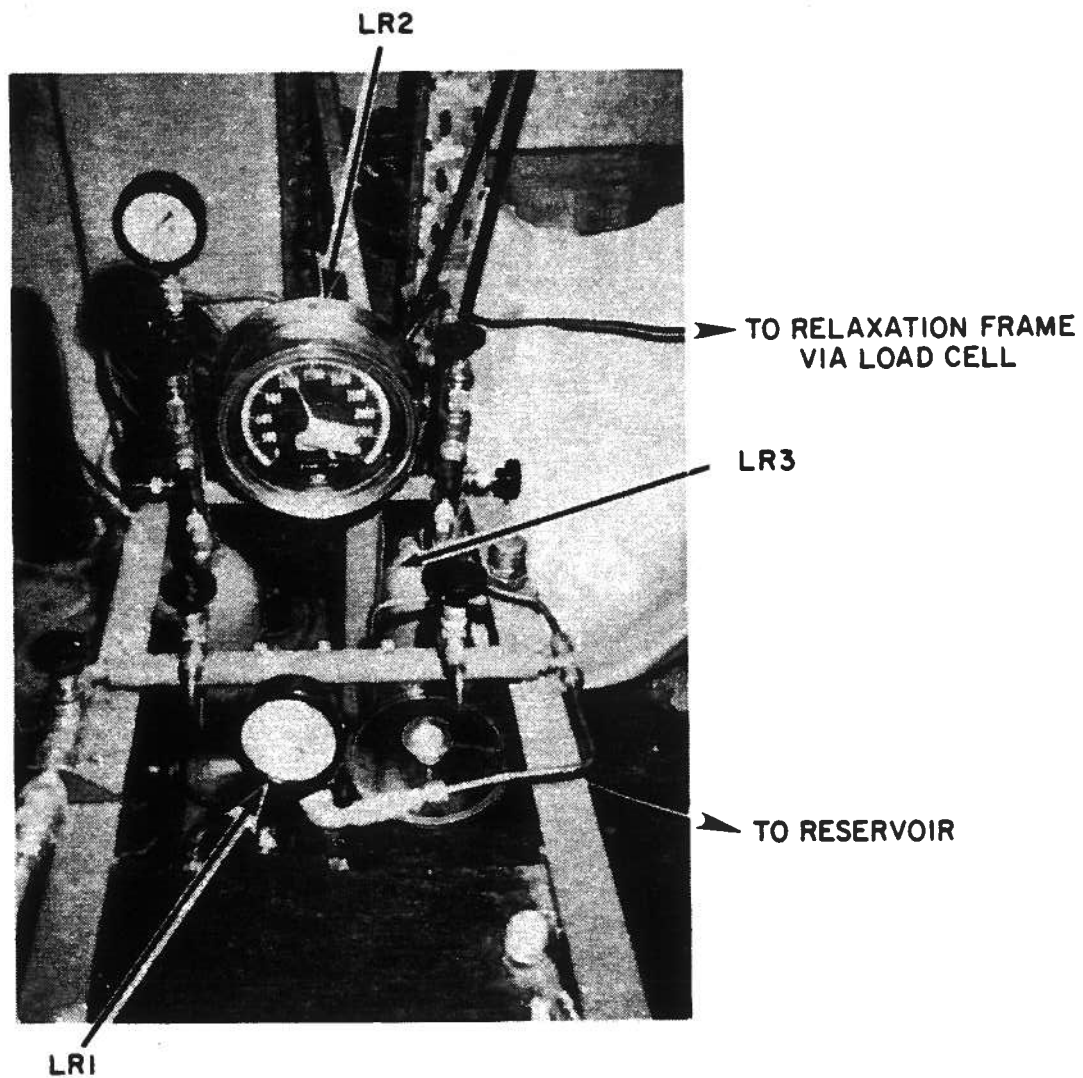
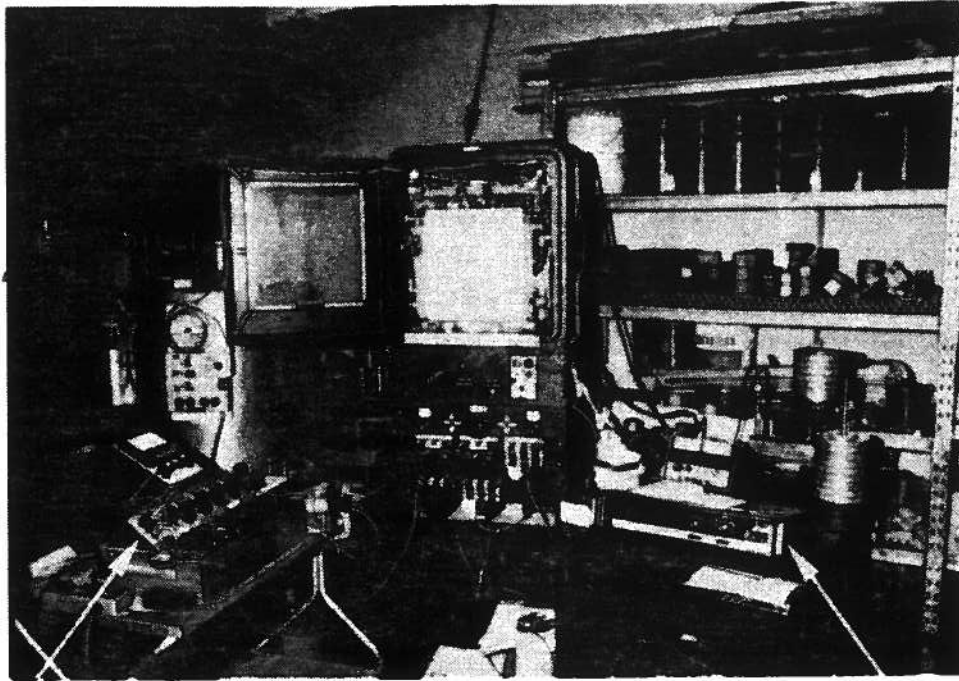


FIG. 6 PHOTOGRAPH SHOWING INSIDE VIEW OF HYDRAULIC CONTROL UNIT

SPEEDOMAX RECORDER



TEST SET

DVM

STRESS PEN

STRAIN PEN

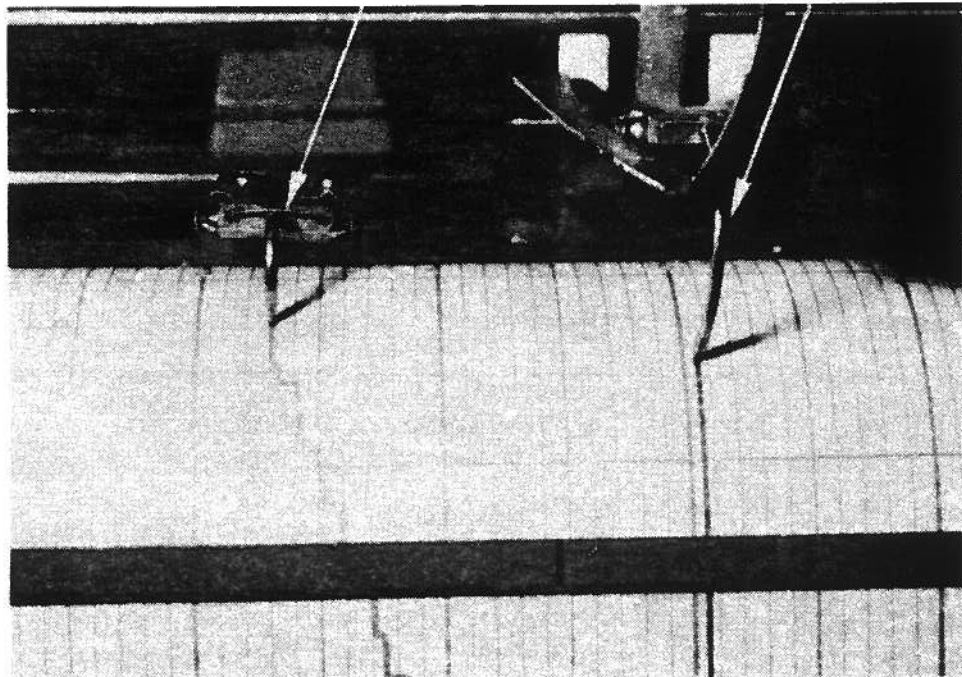


FIG. 7 PHOTOGRAPHS SHOWING SPEEDOMAX RECORDER, DIGITAL DISPLAY EQUIPMENT (DVM) AND INDEPENDENT TEST SET

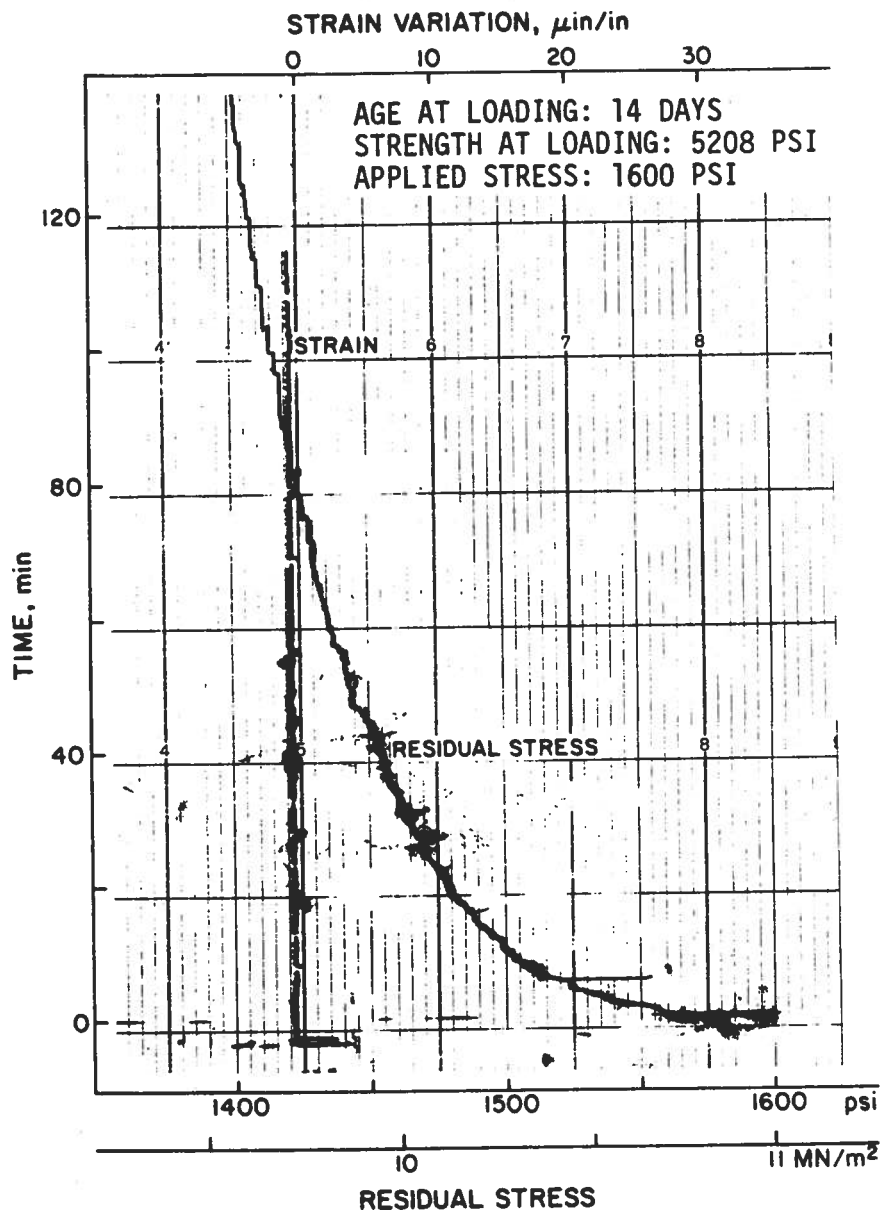


FIG. 8 AUTOMATIC PLOT OF EARLY STAGES OF STRESS RELAXATION OF CONCRETE--
SPECIMEN R1

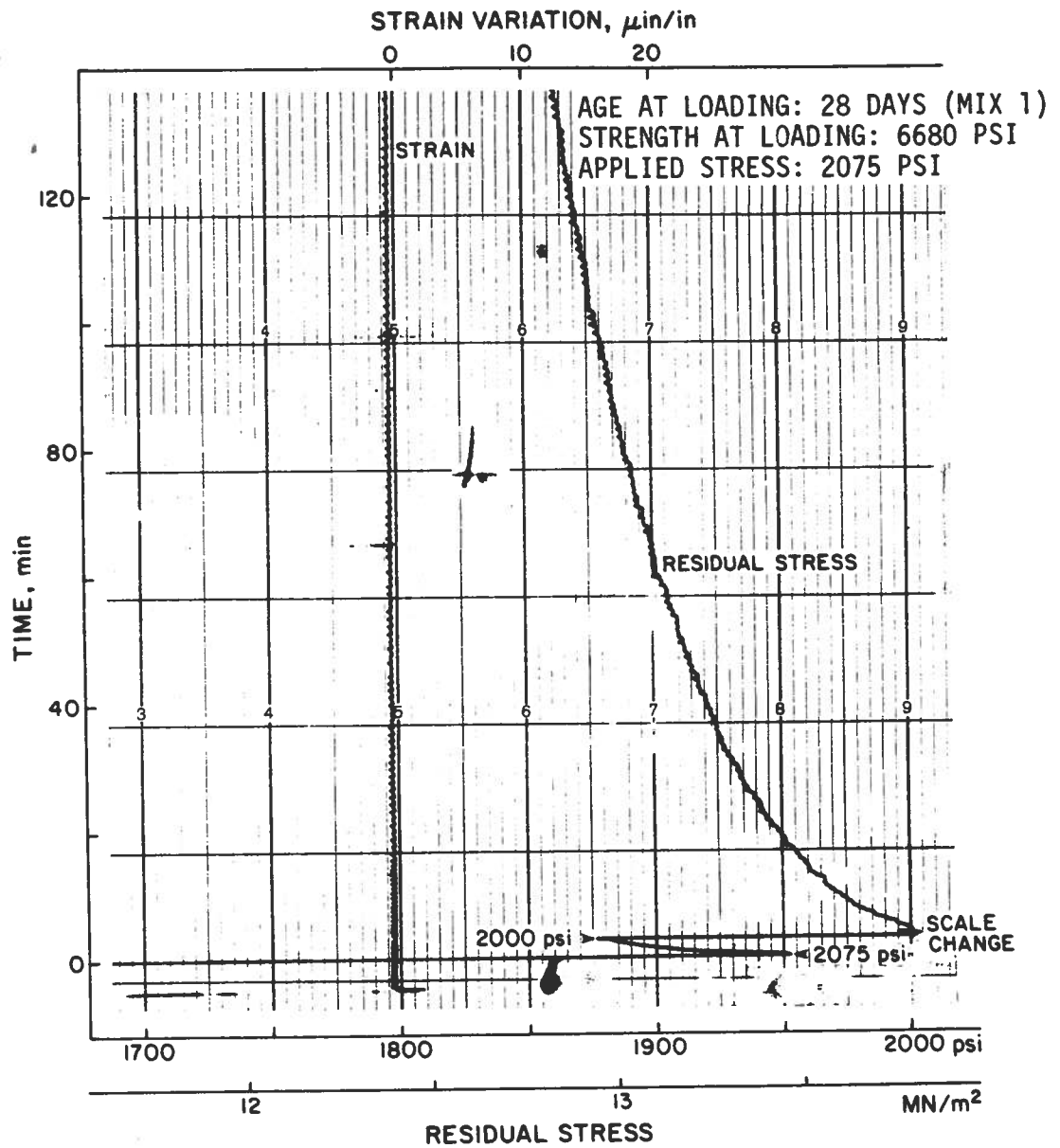


FIG. 9 AUTOMATIC PLOT OF EARLY STAGES OF STRESS RELAXATION OF CONCRETE--
SPECIMEN R2

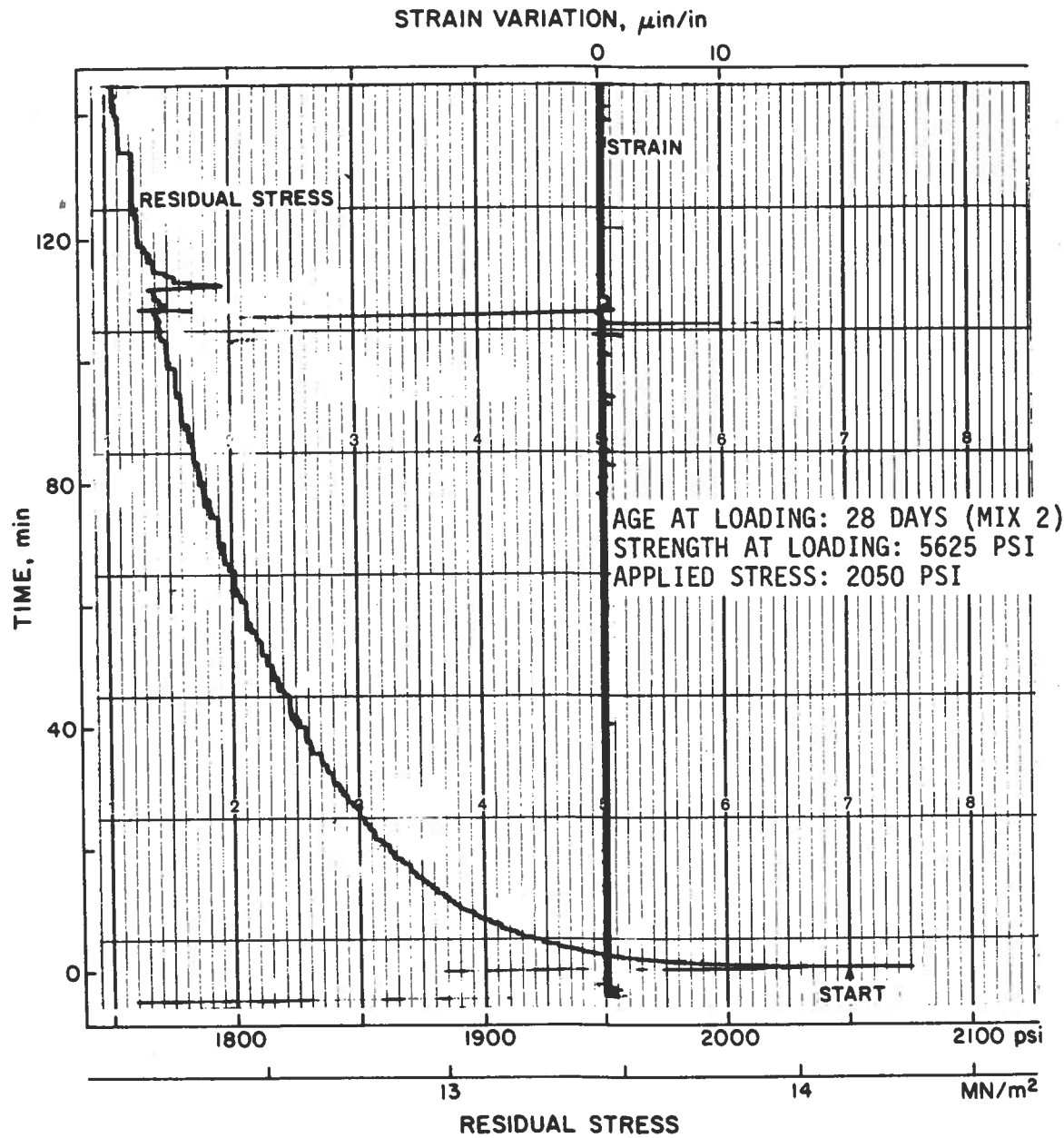


FIG. 10 AUTOMATIC PLOT OF EARLY STAGES OF STRESS RELAXATION OF CONCRETE
--SPECIMEN R3

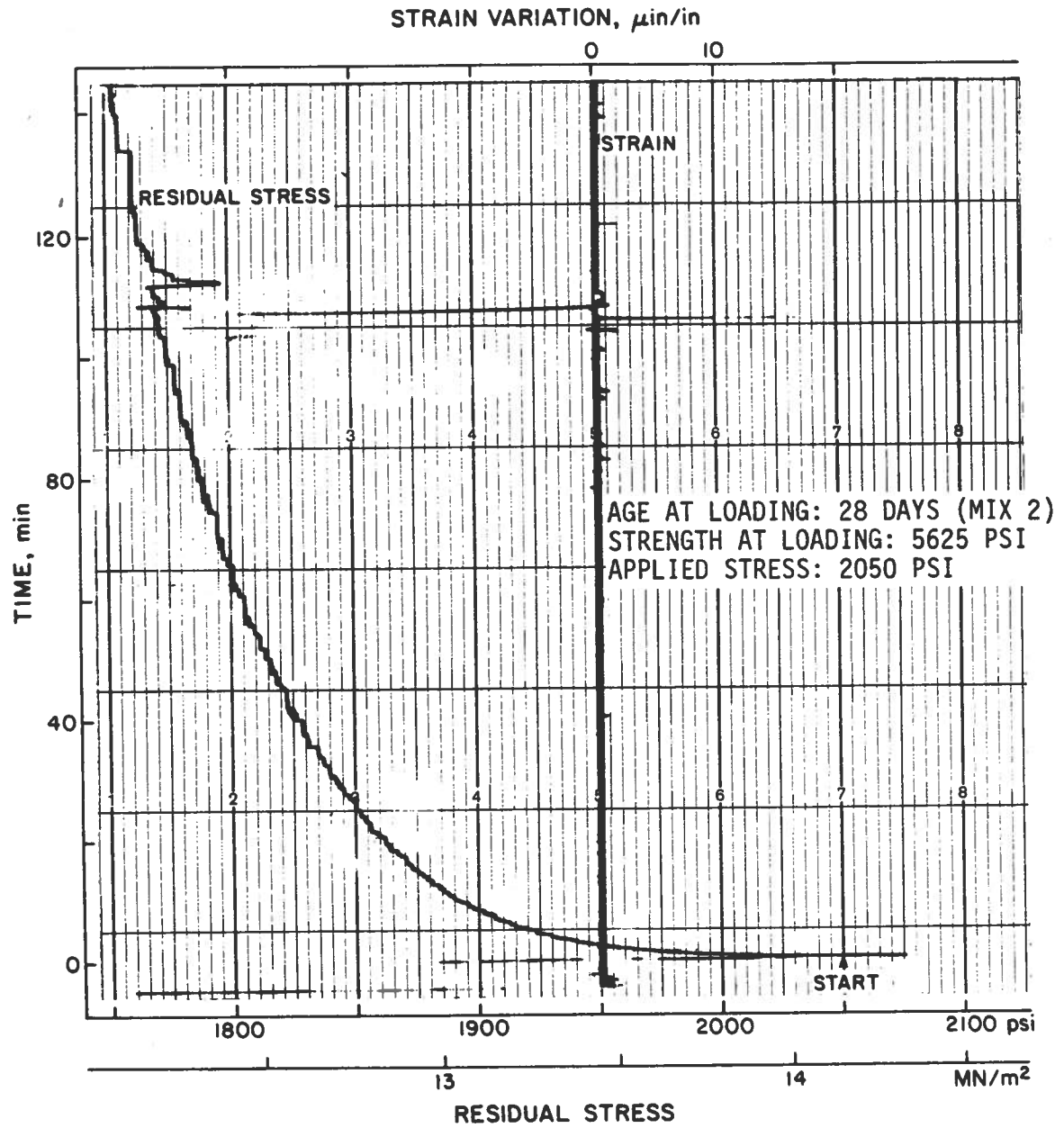


FIG. 10 AUTOMATIC PLOT OF EARLY STAGES OF STRESS RELAXATION OF CONCRETE
--SPECIMEN R3

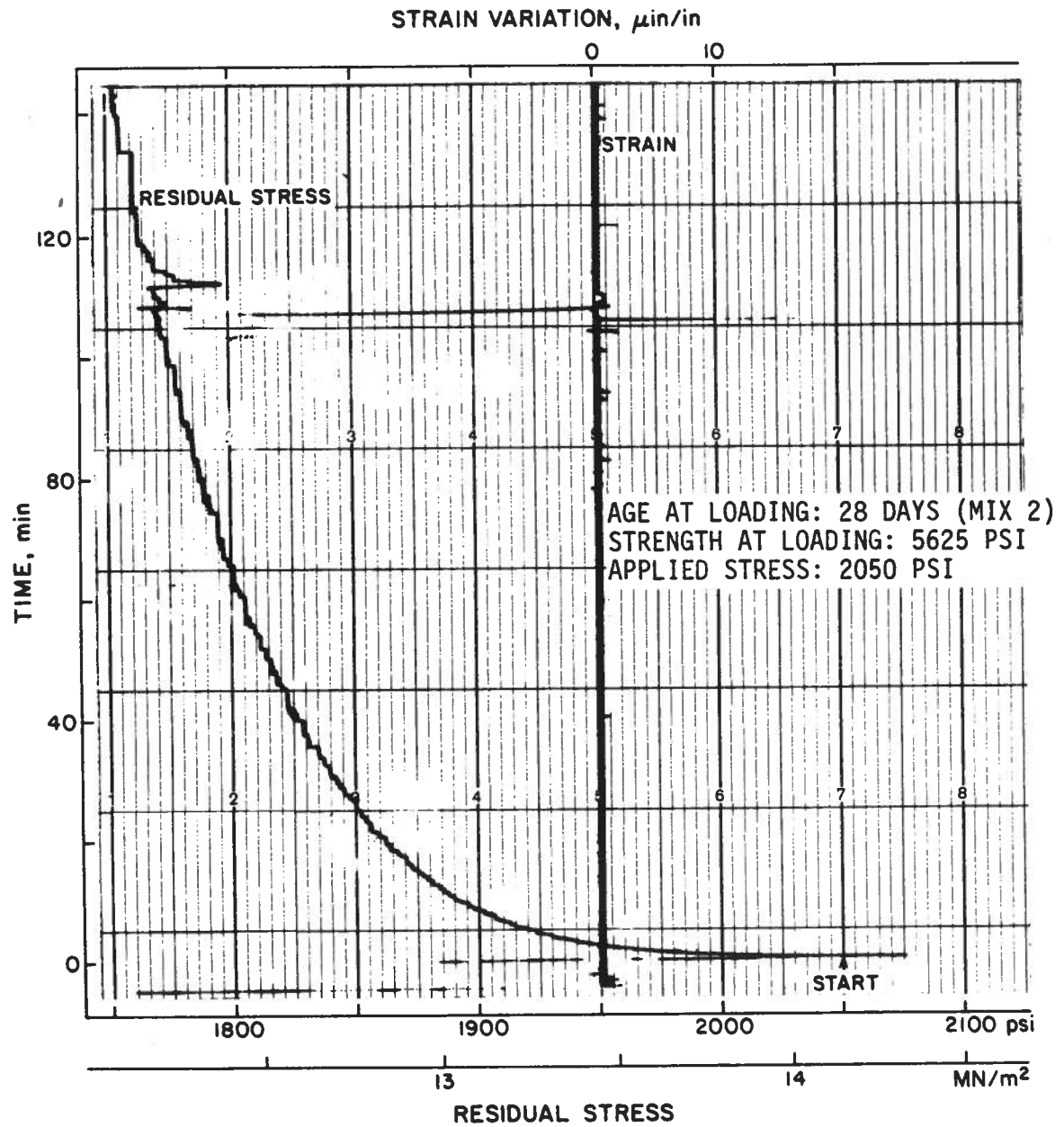


FIG. 10 AUTOMATIC PLOT OF EARLY STAGES OF STRESS RELAXATION OF CONCRETE
--SPECIMEN R3

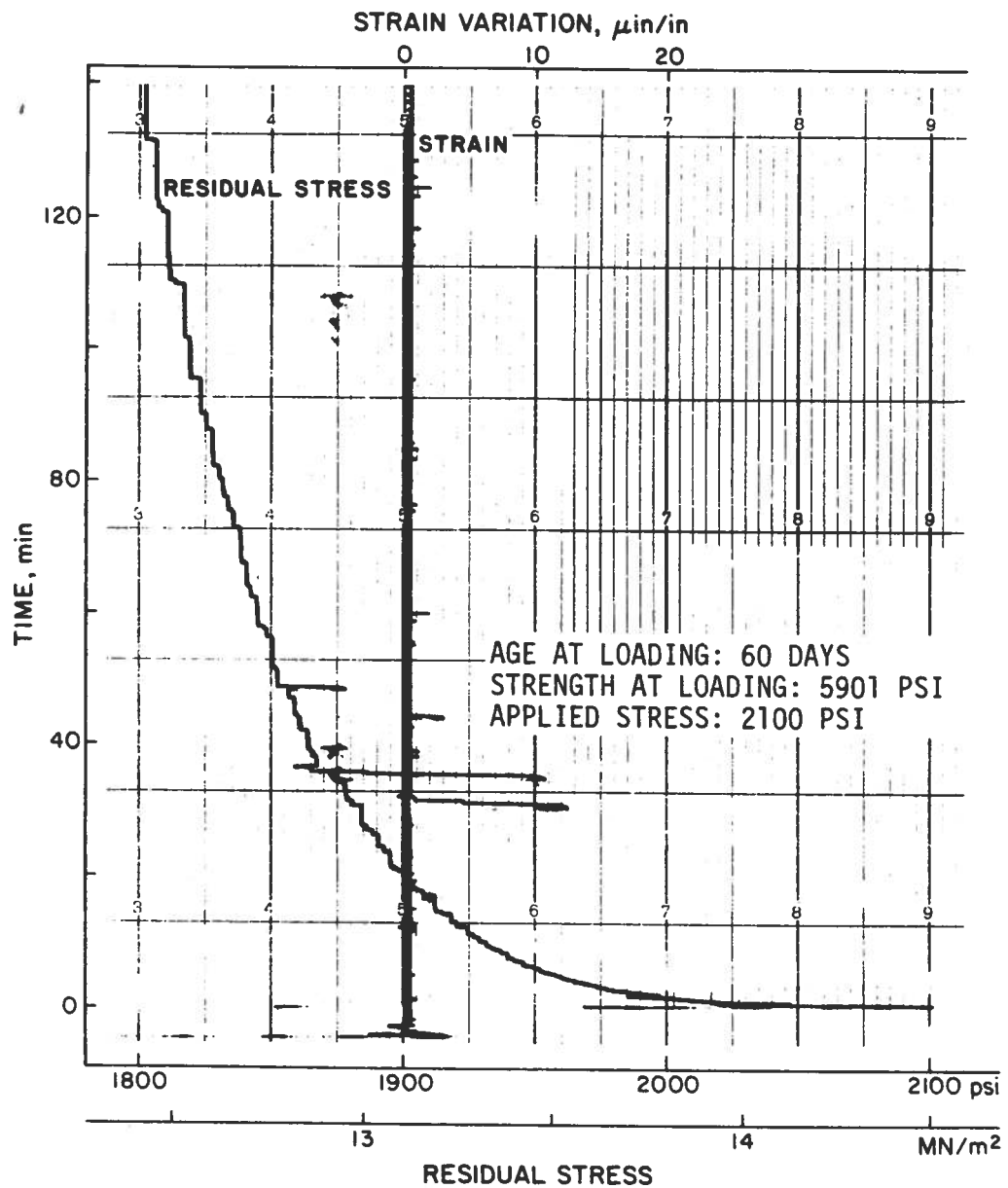


FIG. 11 AUTOMATIC PLOT OF EARLY STAGES OF STRESS RELAXATION OF CONCRETE
--SPECIMEN R4

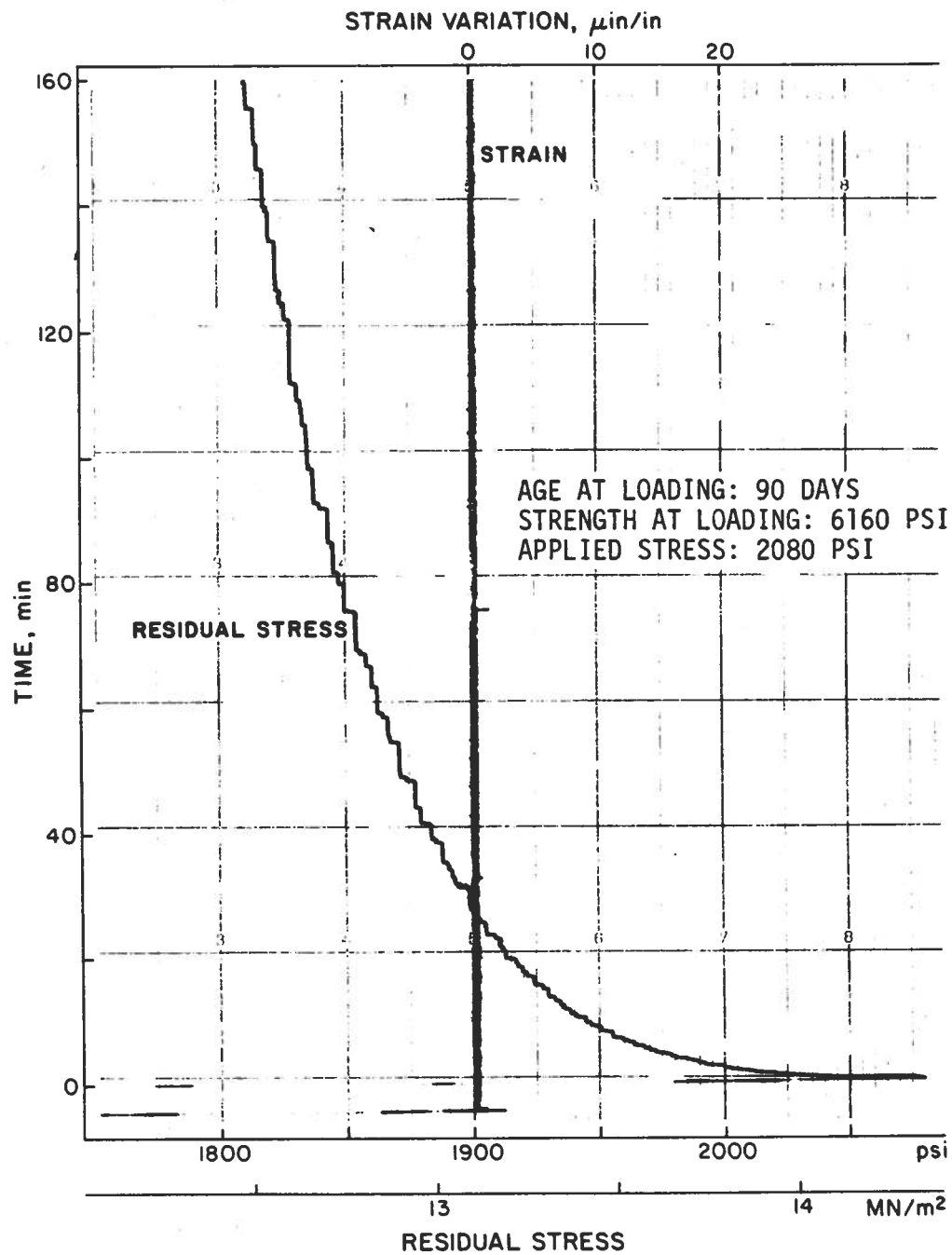


FIG. 12 AUTOMATIC PLOT OF EARLY STAGES OF STRESS RELAXATION OF CONCRETE
--SPECIMEN R5

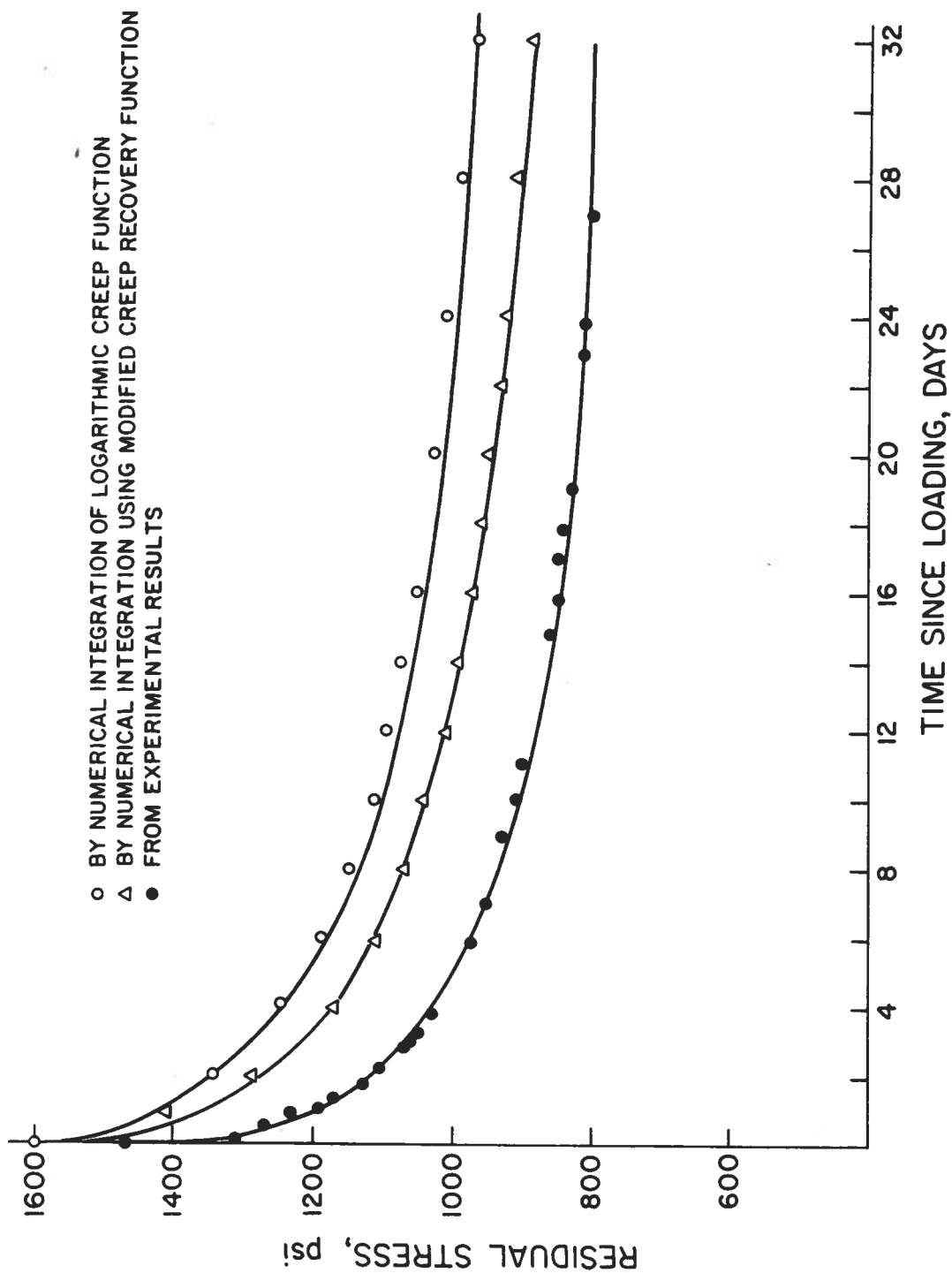


FIG. 13 RESIDUAL STRESS AGAINST TIME SINCE LOADING INCLUDING PREDICTIONS BY INTEGRATION OF CREEP FUNCTIONS--- SPECIMEN R1 (STRESS/STRENGTH RATIO: 0.31, AGE AT LOADING: 14 DAYS)

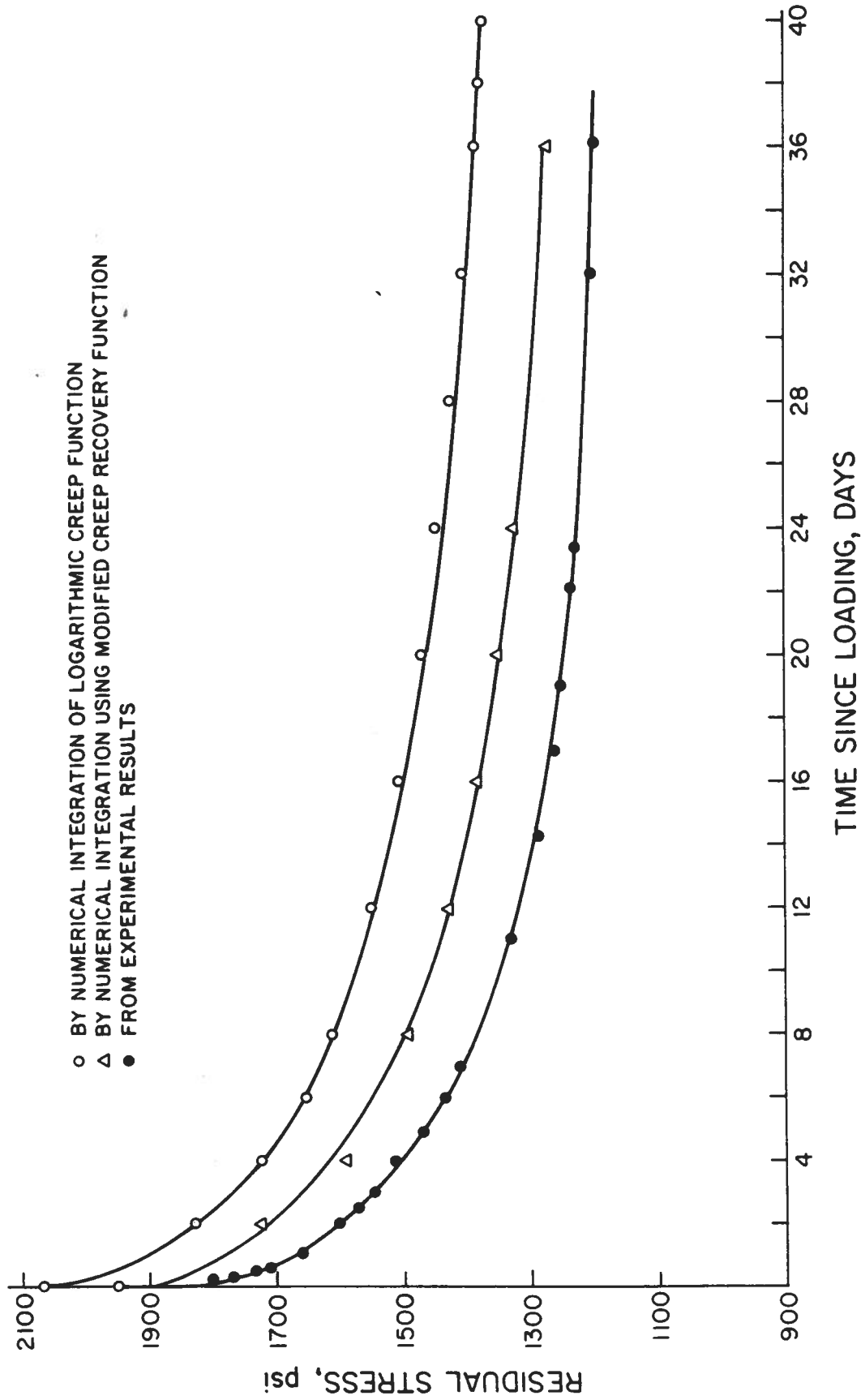


FIG. 14 RESIDUAL STRESS AGAINST TIME SINCE LOADING INCLUDING PREDICTIONS BY INTEGRATION OF CREEP FUNCTIONS-- SPECIMEN R2 (STRESS/STRENGTH RATIO: 0.31, AGE AT LOADING: 14 DAYS)

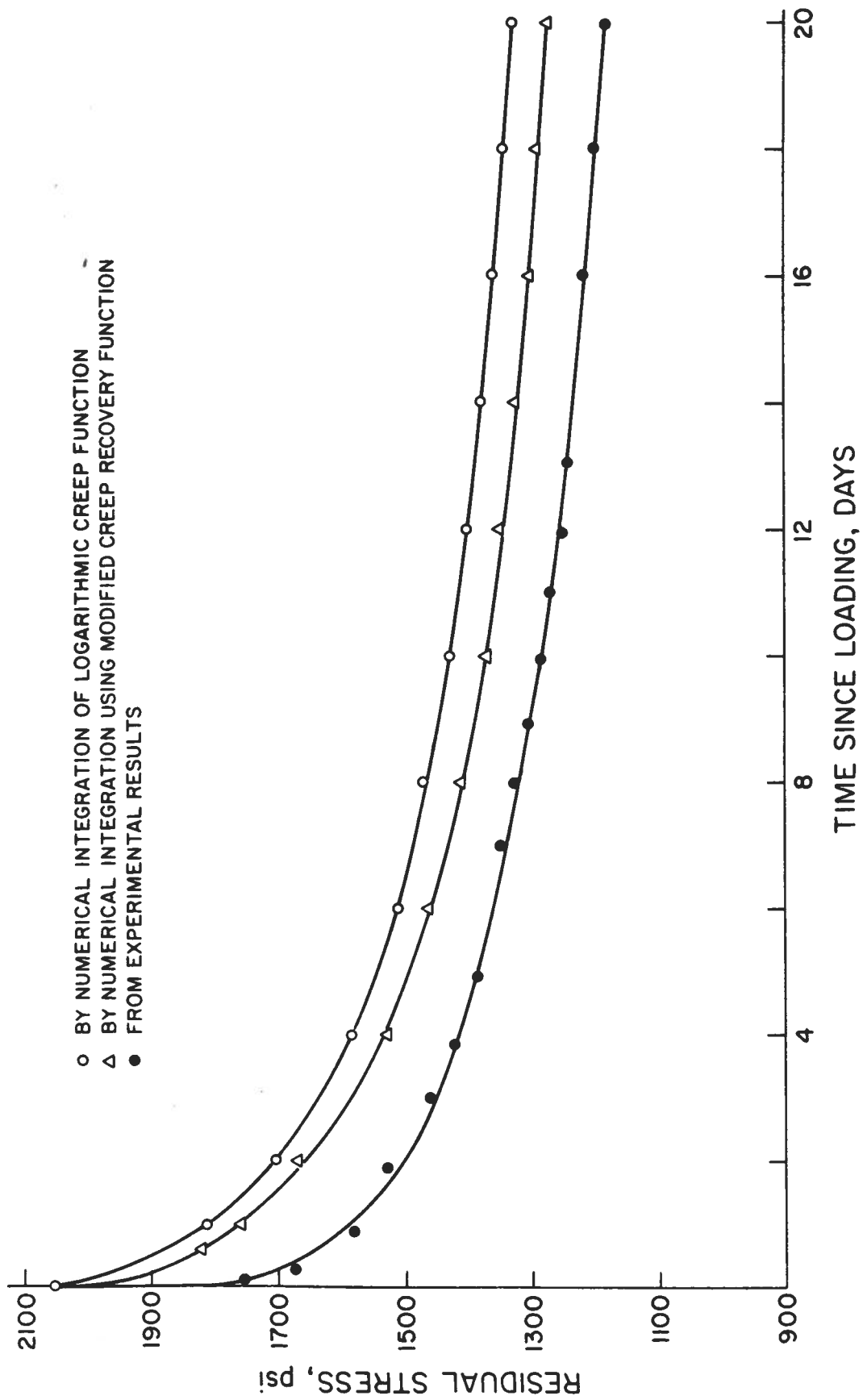


FIG. 15 RESIDUAL STRESS AGAINST TIME SINCE LOADING INCLUDING PREDICTIONS BY INTEGRATION OF CREEP FUNCTIONS--- SPECIMEN R3 (STRESS/STRENGTH RATIO: 0.31, AGE AT LOADING: 14 DAYS)

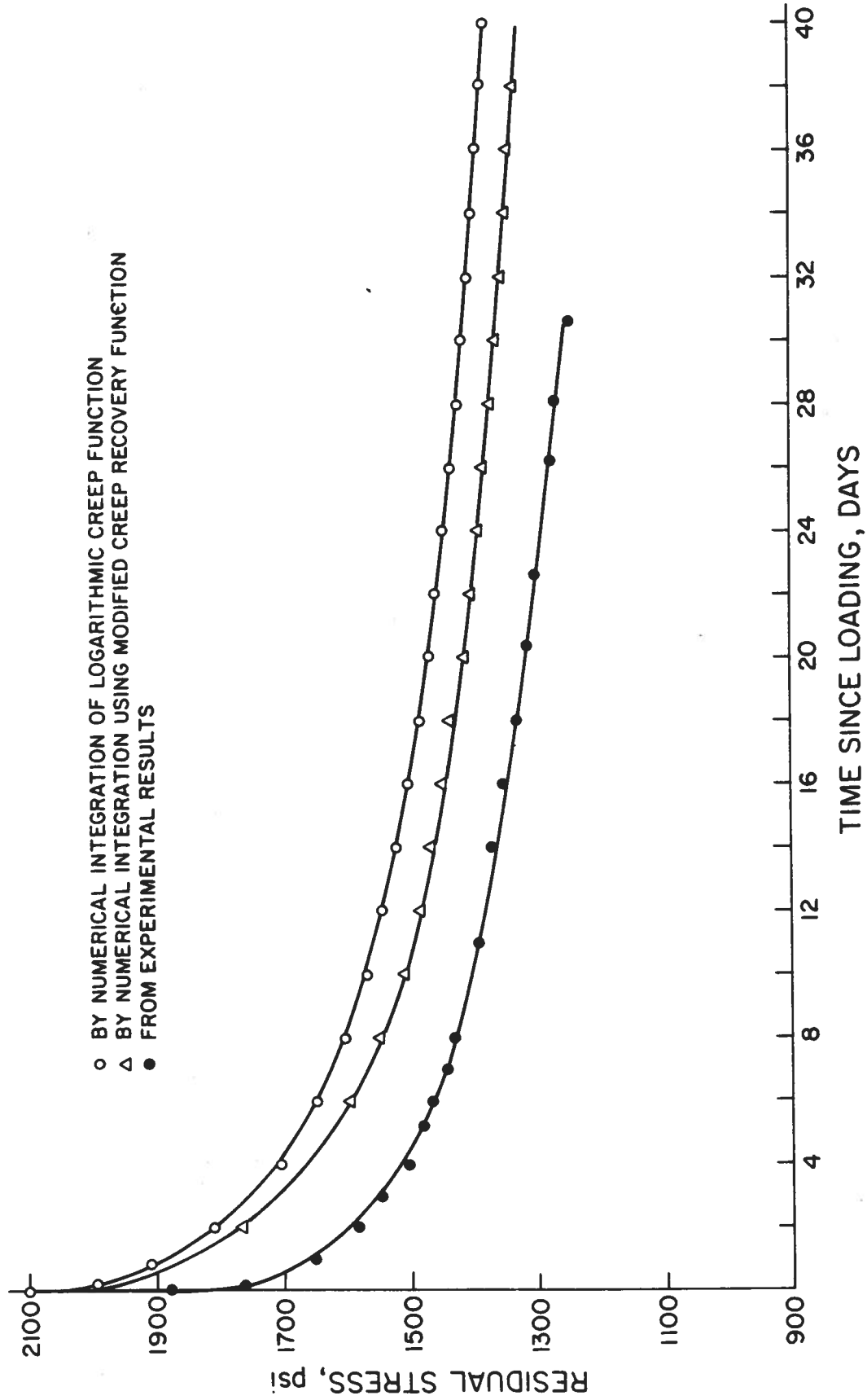


FIG. 16 RESIDUAL STRESS AGAINST TIME SINCE LOADING INCLUDING PREDICTIONS BY INTEGRATION OF CREEP FUNCTIONS--- SPECIMEN R4 (STRESS/STRENGTH RATIO: 0.31, AGE AT LOADING: 14 DAYS)

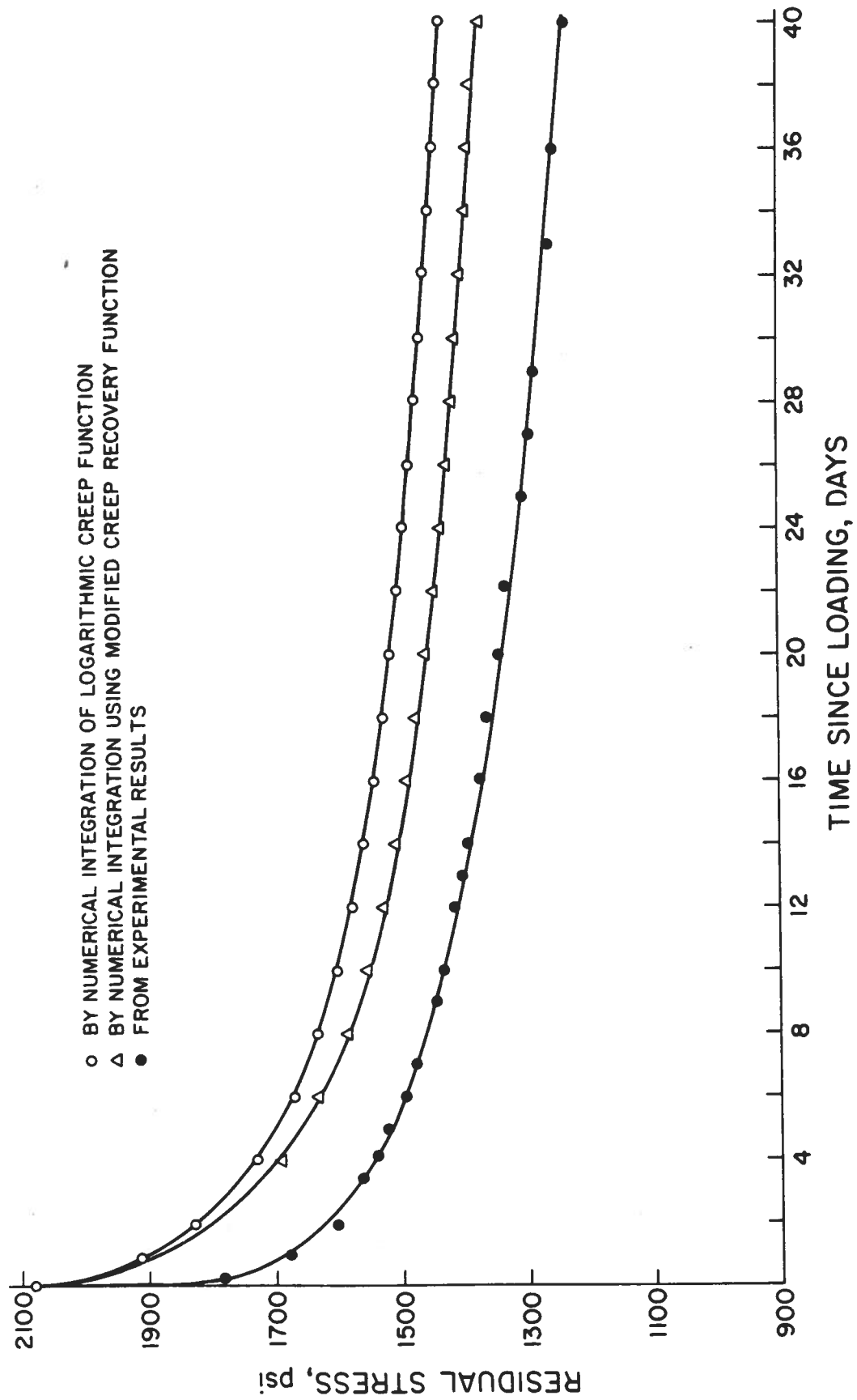


FIG. 17 RESIDUAL STRESS AGAINST TIME SINCE LOADING INCLUDING PREDICTIONS BY INTEGRATION OF CREEP FUNCTIONS--- SPECIMEN R5 (STRESS/STRENGTH RATIO: 0.31, AGE AT LOADING: 14 DAYS)

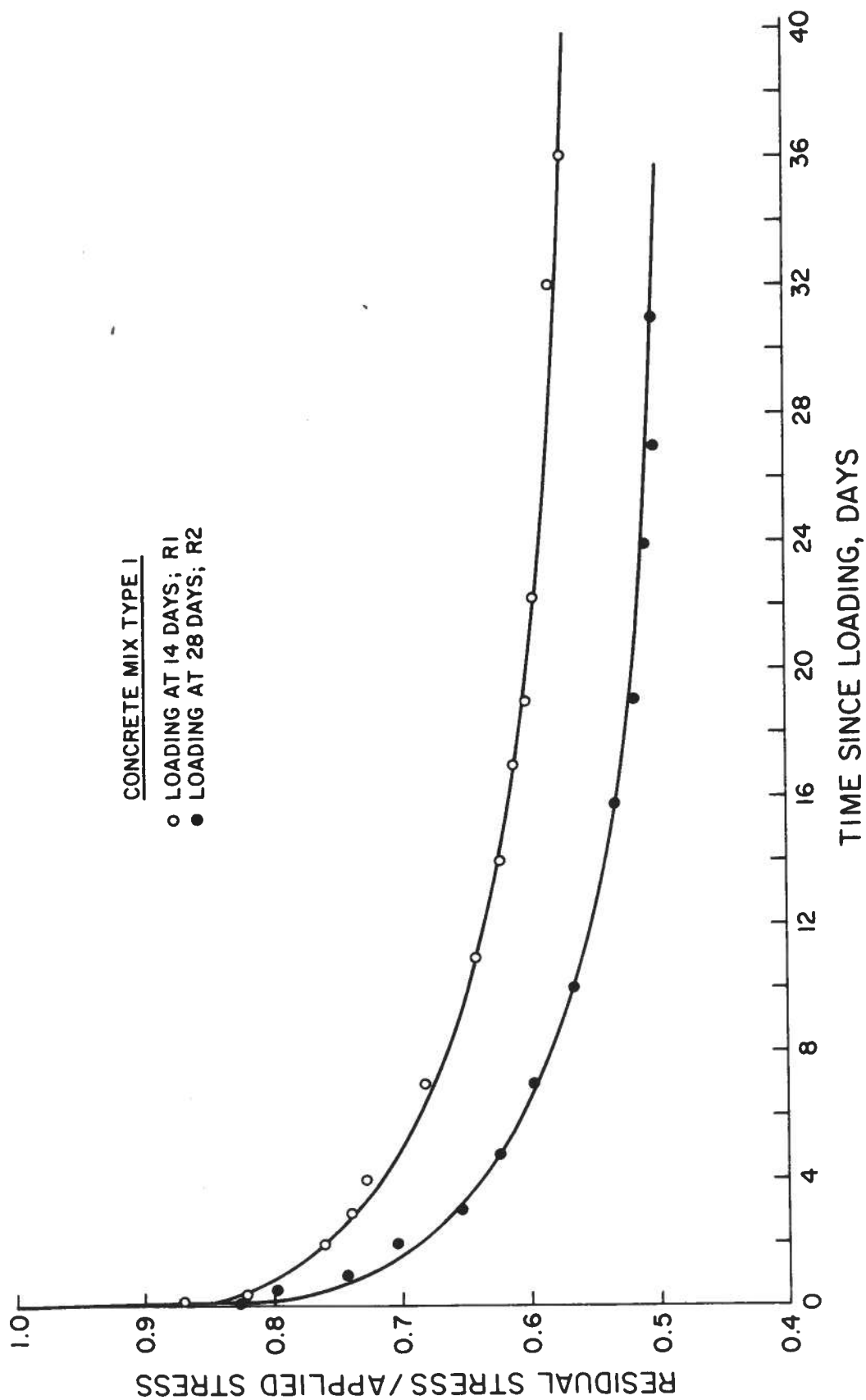


FIG. 18a COMPARATIVE CURVES OF RESIDUAL STRESS AGAINST TIME SINCE LOADING--MIX 1

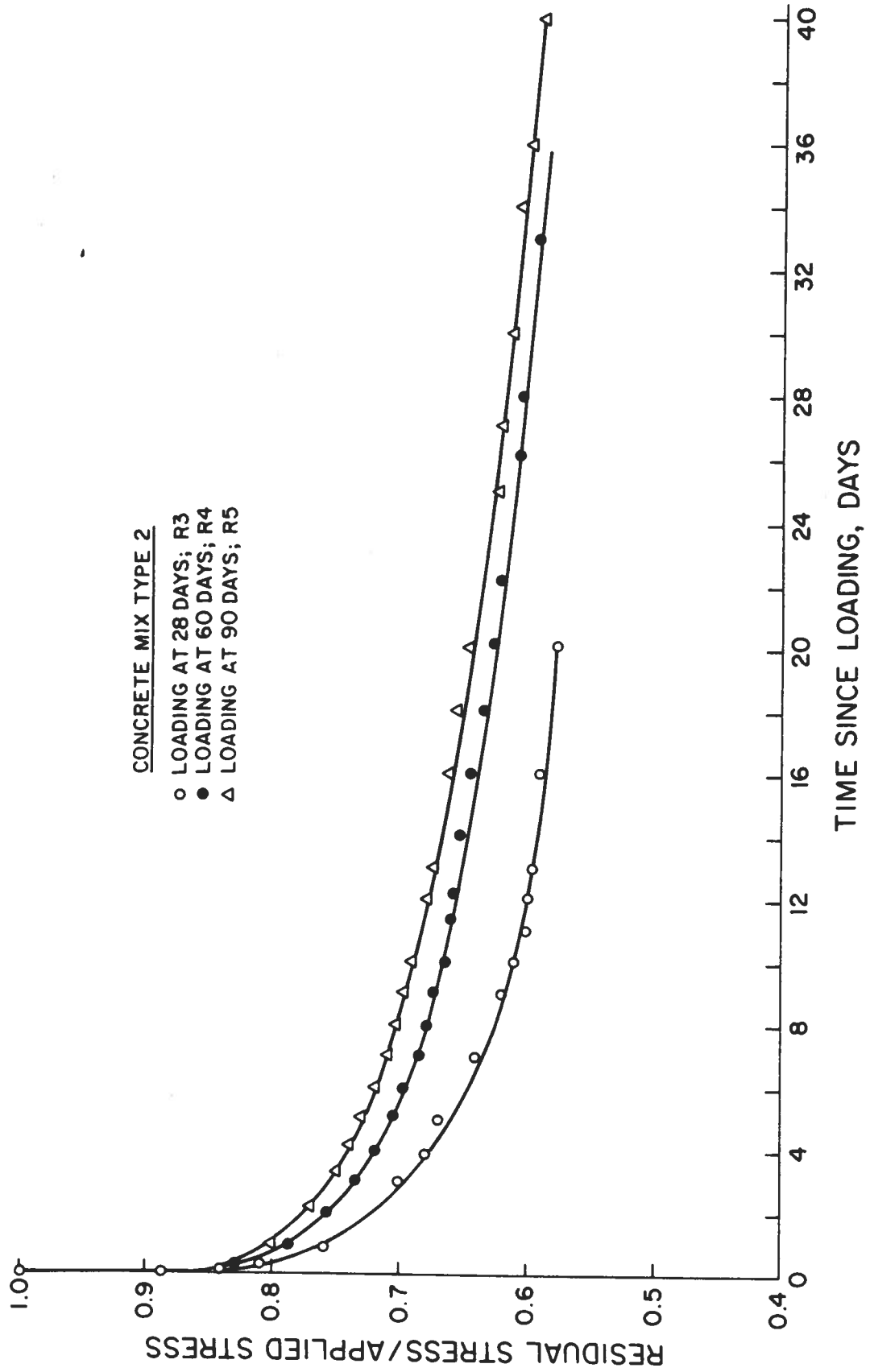


FIG. 18b COMPARATIVE CURVES OF RESIDUAL STRESS AGAINST TIME SINCE LOADING--MIX 2

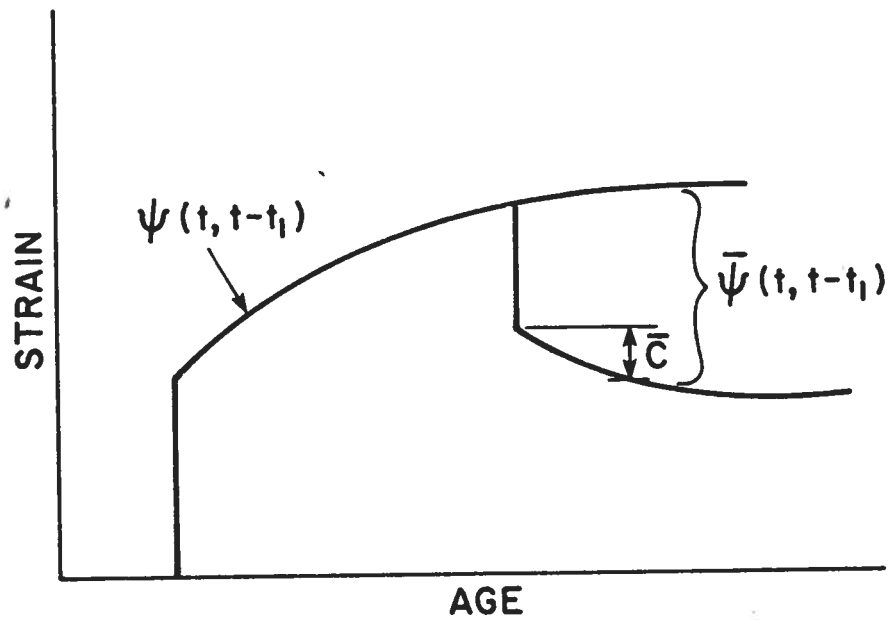


FIG. 19 IDEALIZED CREEP AND CREEP RECOVERY CURVE

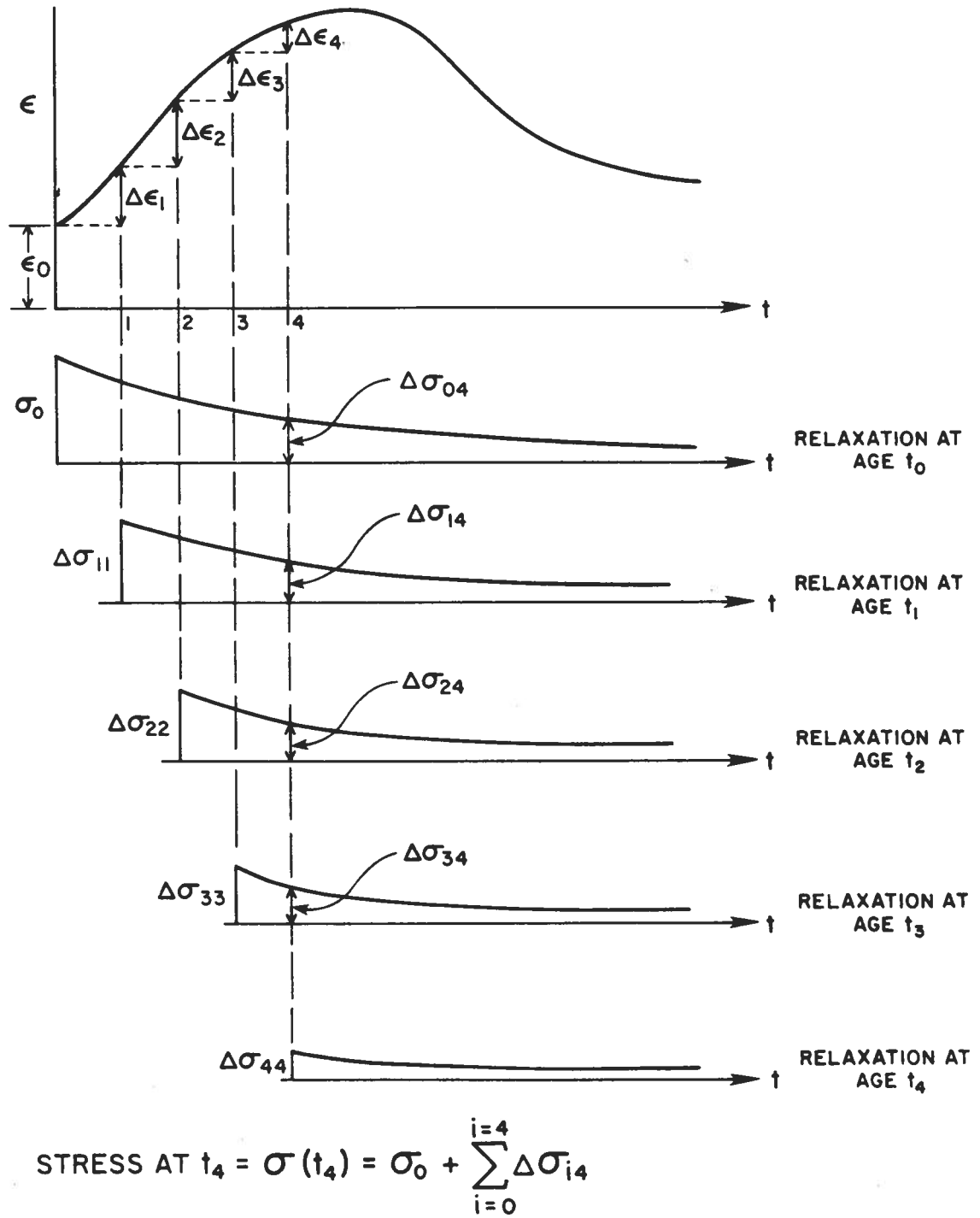


FIG. 20 PROCEDURE FOR THE USE OF STRESS RELAXATION CURVES FOR DETERMINING STRESS LEVELS IN MASS CONCRETE STRUCTURES

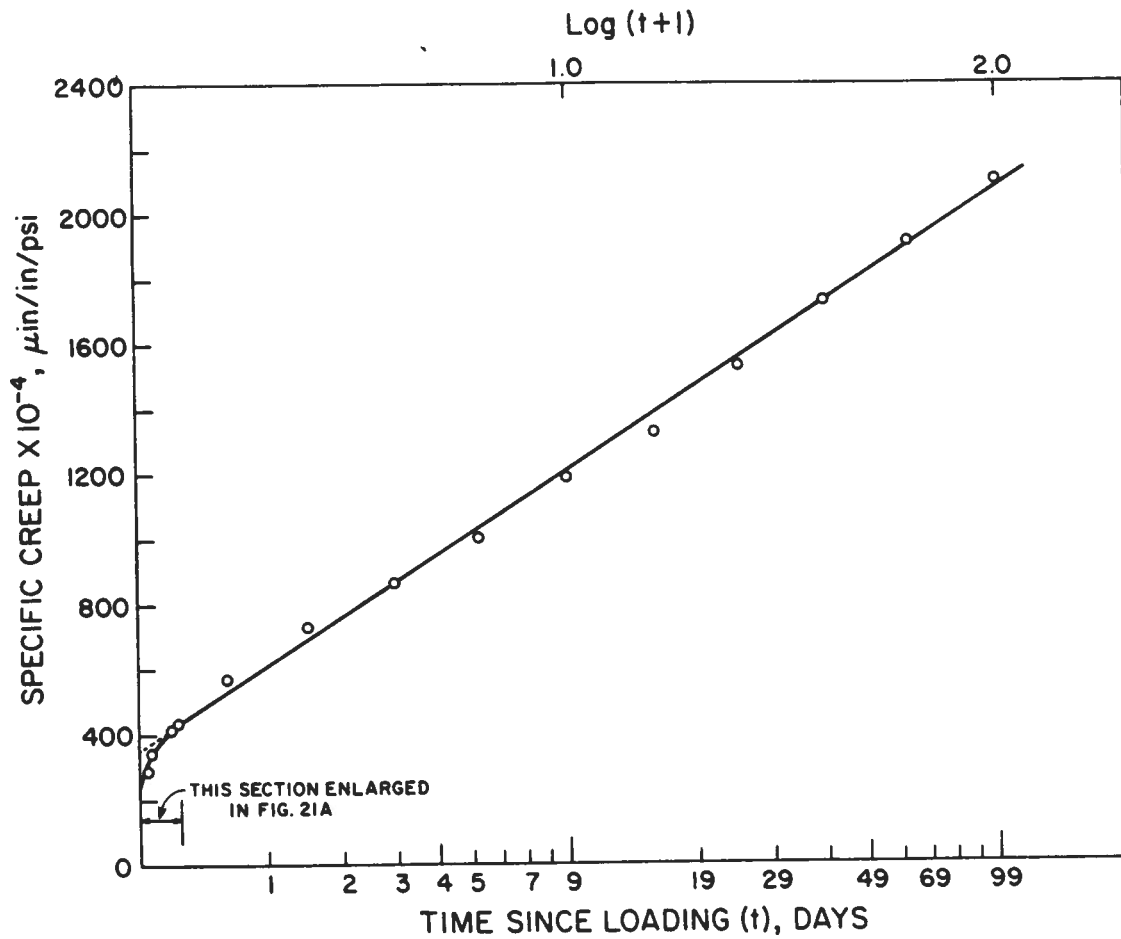


FIG. 21 CORRECTION FOR INACCURACY OF LOGARITHMIC CREEP FUNCTION AT EARLY AGES--28-DAY CREEP DATA FOR SPECIMEN C2 (MIX 1)

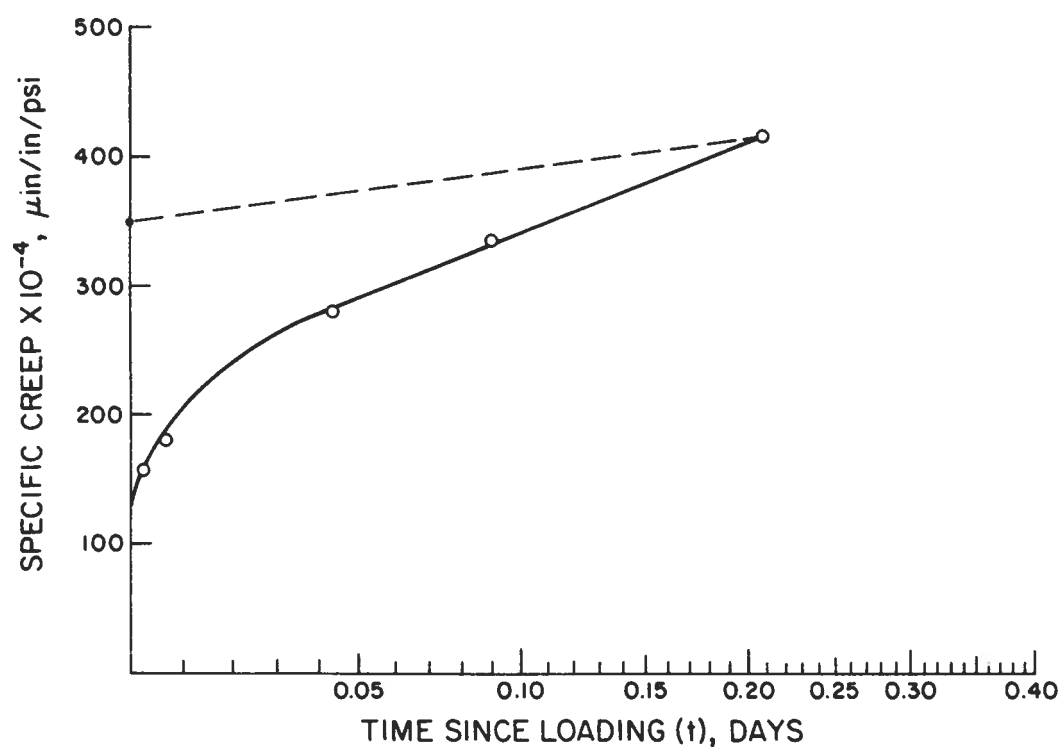


FIG. 21a SECTION OF 28-DAY CREEP DATA FOR SPECIMEN C2 ENLARGED

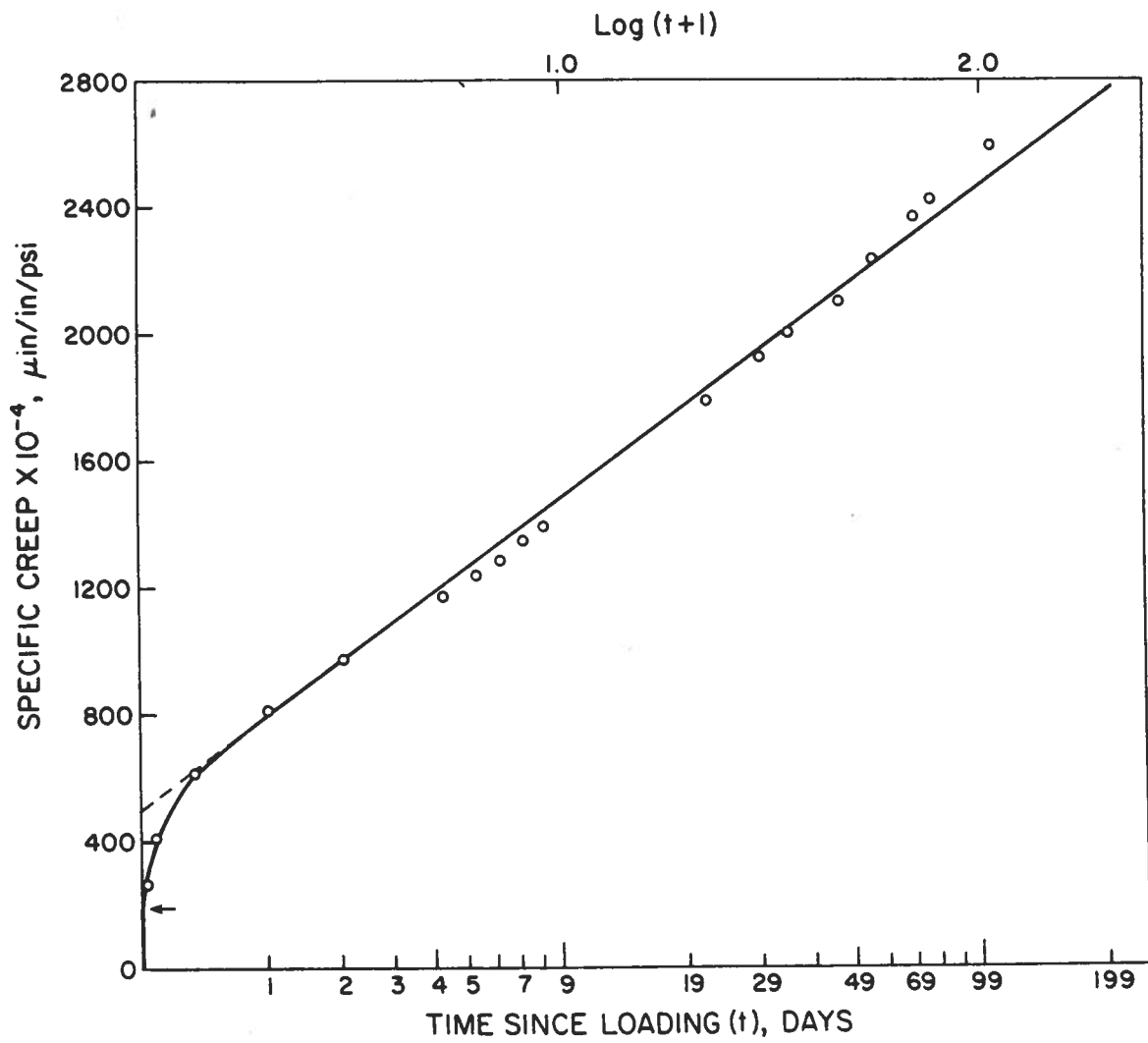


FIG. 22. CORRECTION FOR INACCURACY OF LOGARITHMIC CREEP FUNCTION AT EARLY AGES--28-DAY CREEP DATA FOR SPECIMEN C3 (MIX 2)

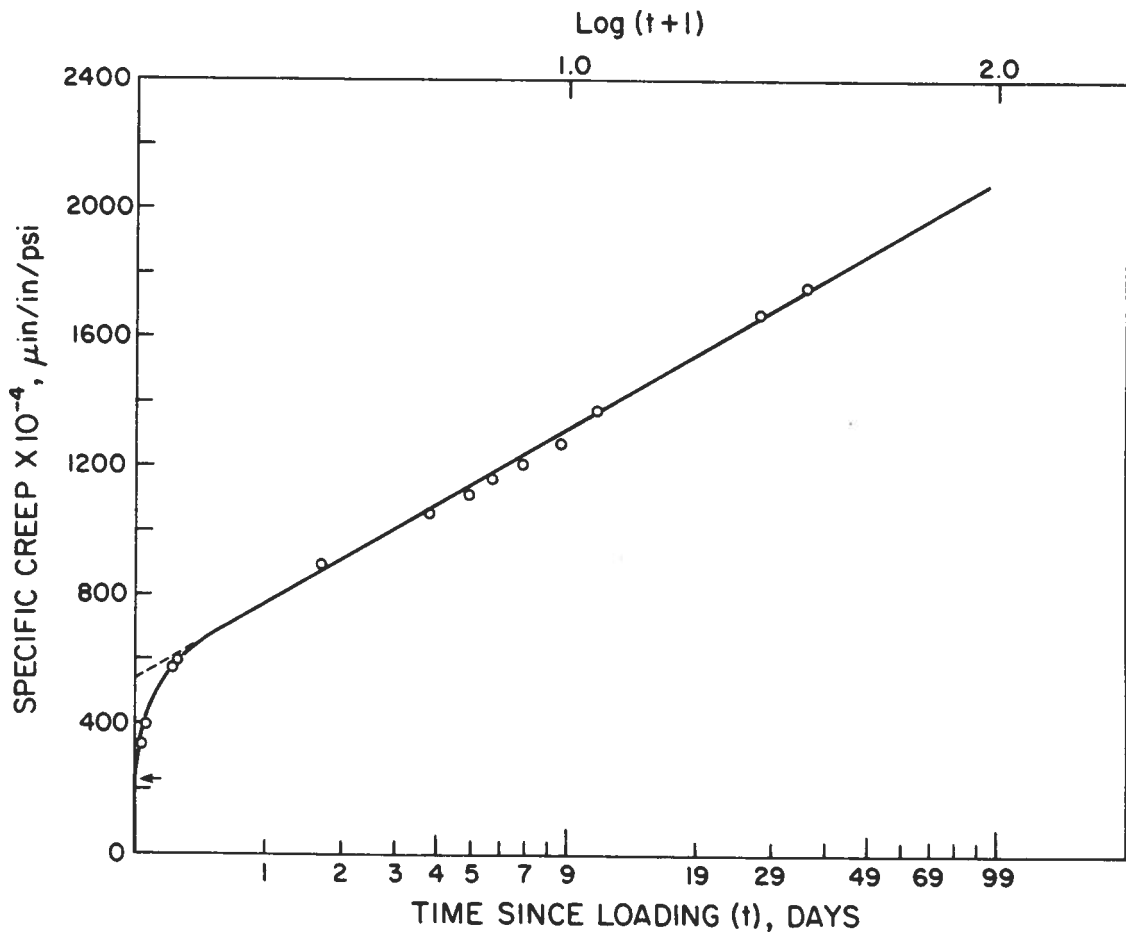


FIG. 23. CORRECTION FOR INACCURACY OF LOGARITHMIC CREEP FUNCTION AT EARLY AGES--90-DAY CREEP DATA FOR SPECIMEN C4

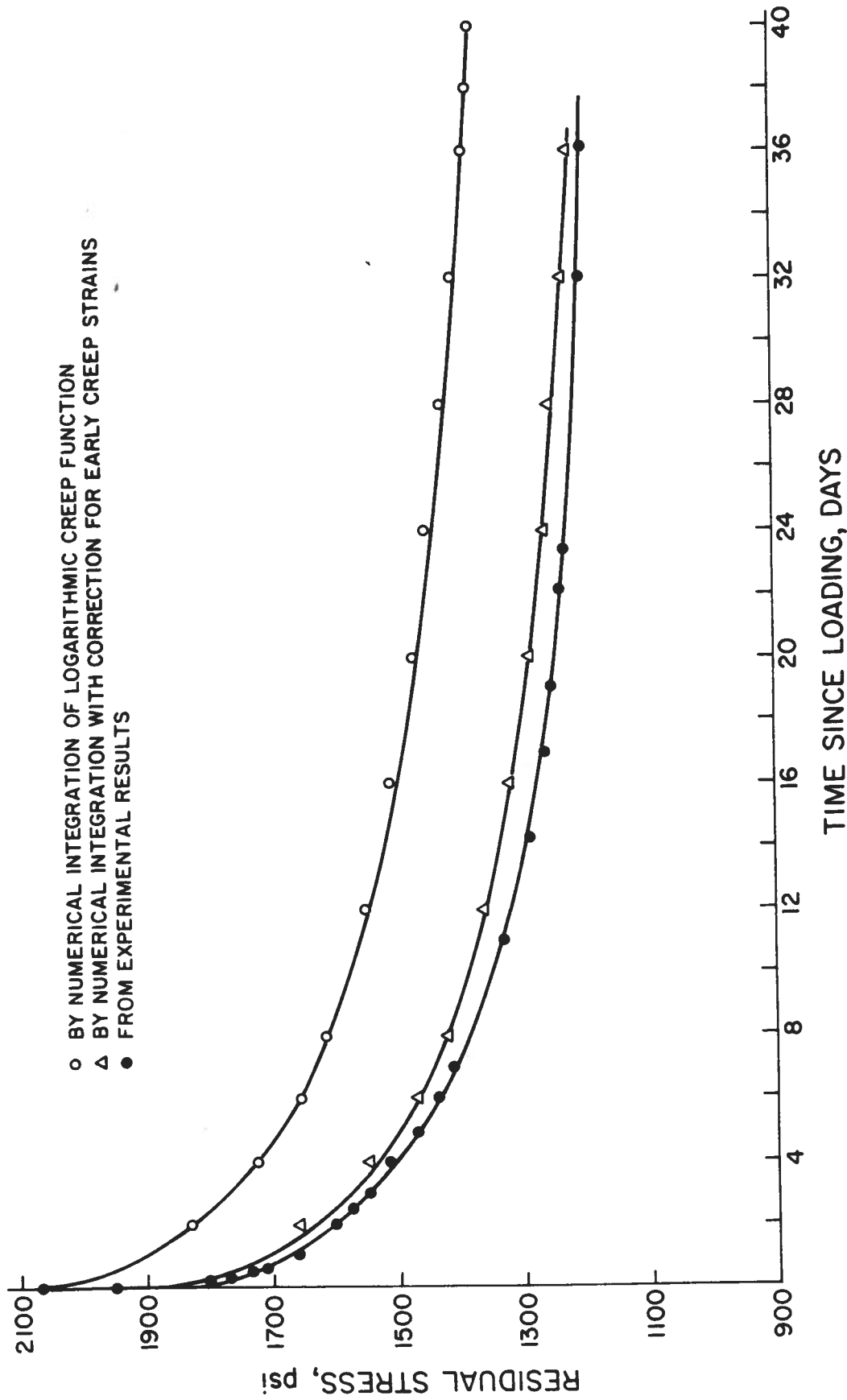


FIG. 24 RESIDUAL STRESS AGAINST TIME SINCE LOADING INCORPORATING CORRECTIONS FOR EARLY CREEP STRAINS--
SPECIMEN R2 (STRESS/STRENGTH RATIO: 0.31, AGE AT LOADING: 28 DAYS (MIX 1))

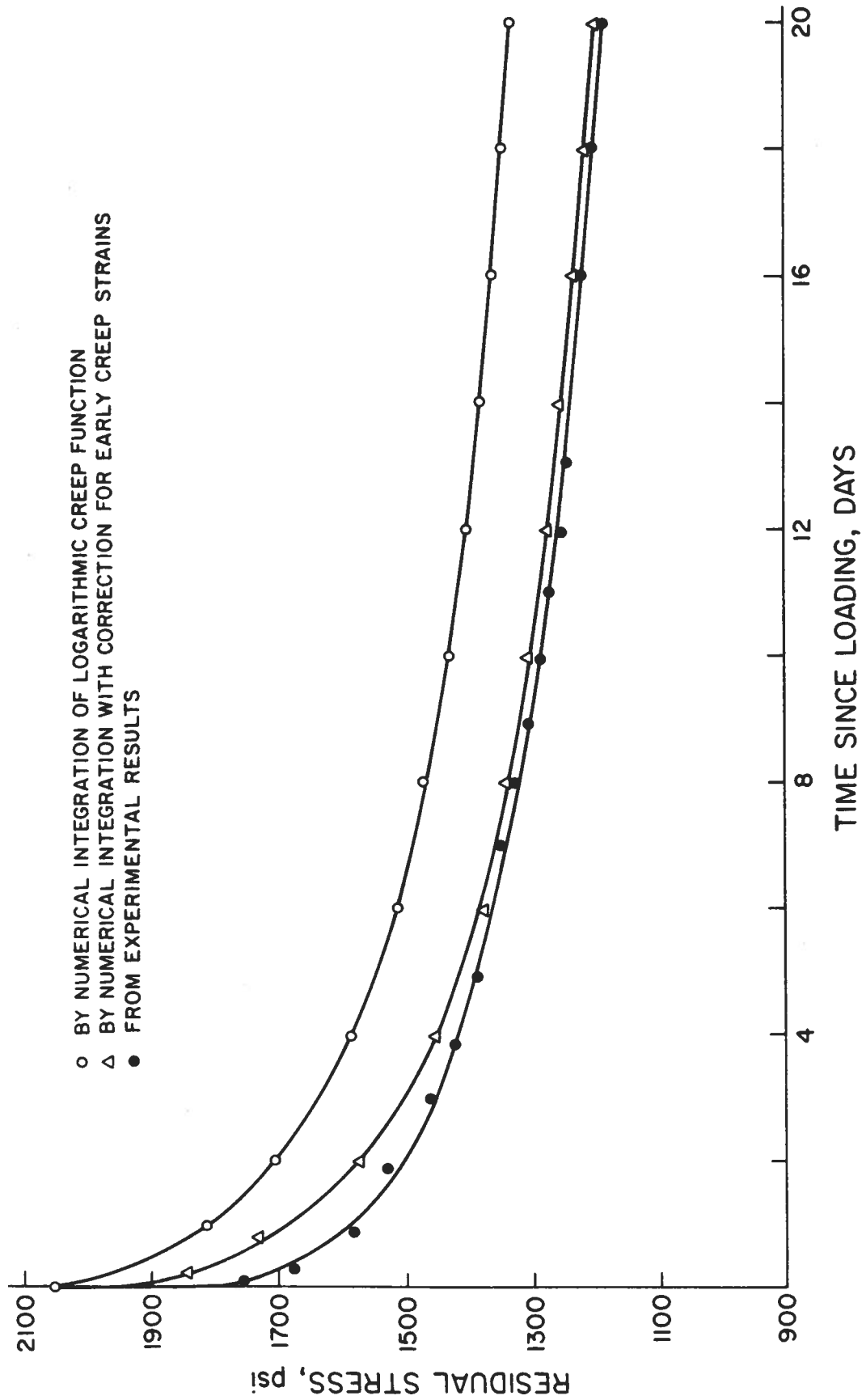


FIG. 25 RESIDUAL STRESS AGAINST TIME SINCE LOADING INCORPORATING CORRECTIONS FOR EARLY CREEP STRAINS-- SPECIMEN R3 (STRESS/STRENGTH RATIO: 0.36, AGE AT LOADING: 28 DAYS (Mix 2))

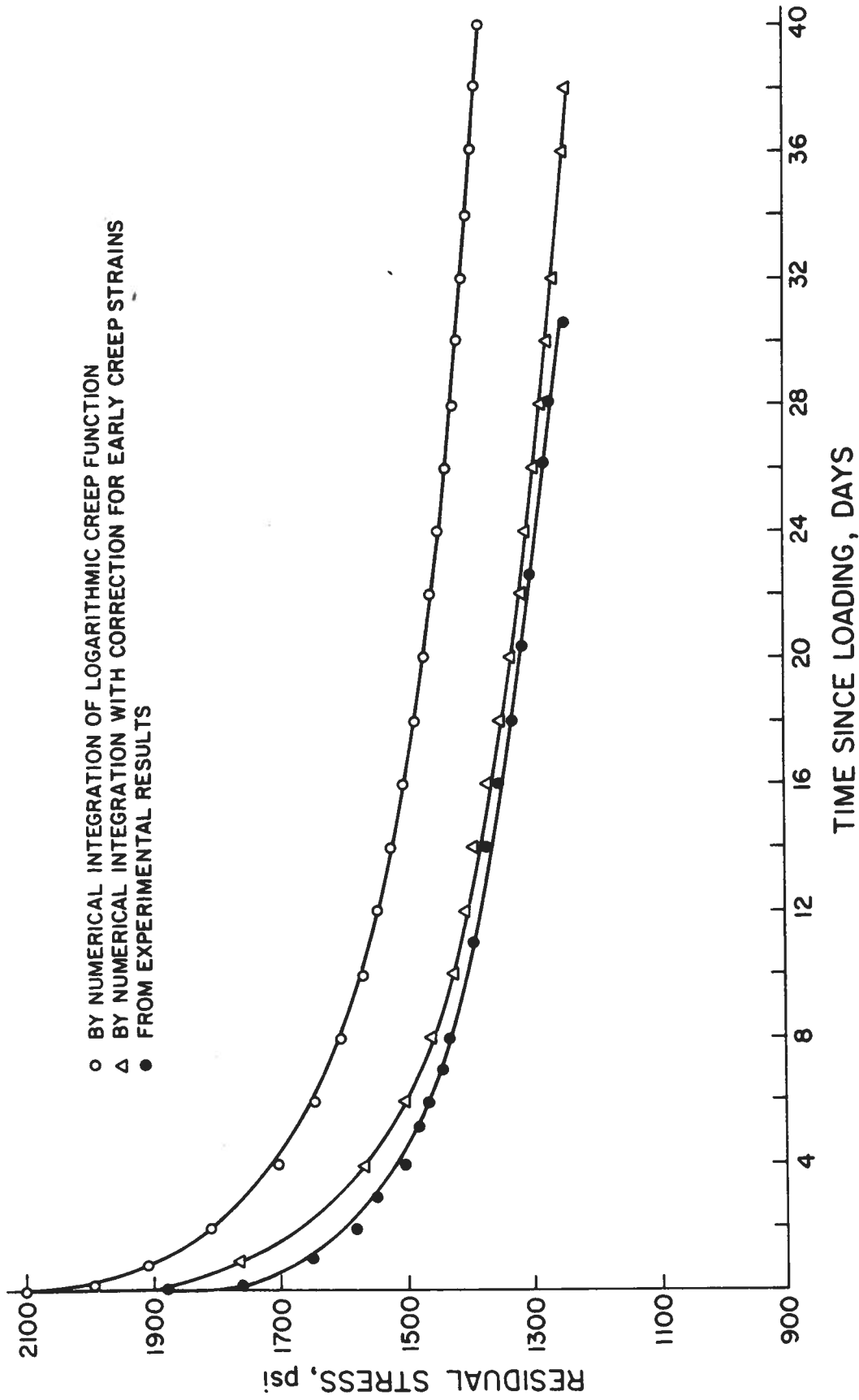


FIG. 26. RESIDUAL STRESS AGAINST TIME SINCE LOADING INCORPORATING CORRECTIONS FOR EARLY CREEP STRAINS---
SPECIMEN R4 (STRESS/STRENGTH RATIO: 0.36, AGE AT LOADING: 60 DAYS)

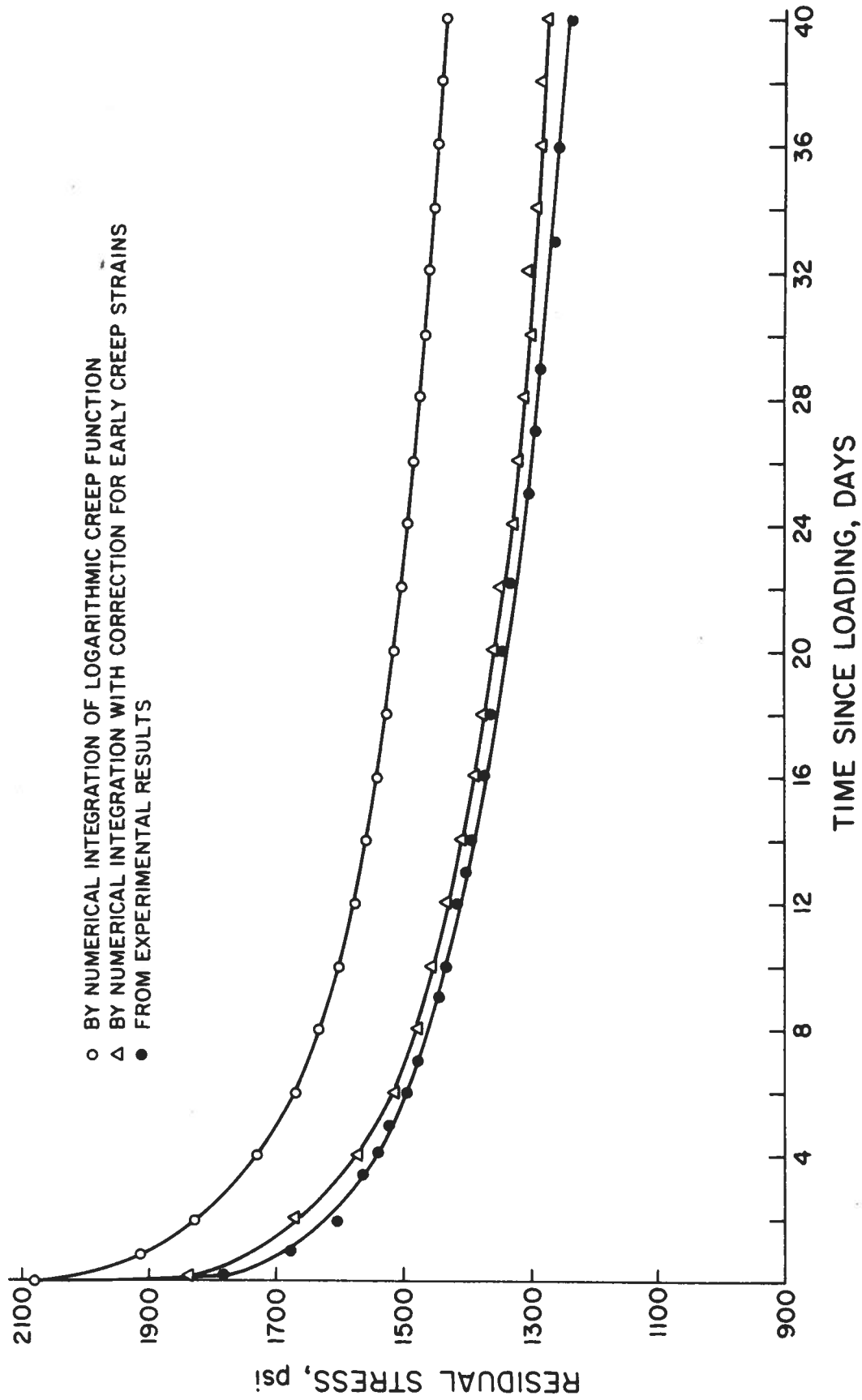


FIG. 27 RESIDUAL STRESS AGAINST TIME SINCE LOADING INCORPORATING CORRECTIONS FOR EARLY CREEP STRAINS-- SPECIMEN R5 (STRESS/STRENGTH RATIO: 0.34, AGE AT LOADING: 90 DAYS)

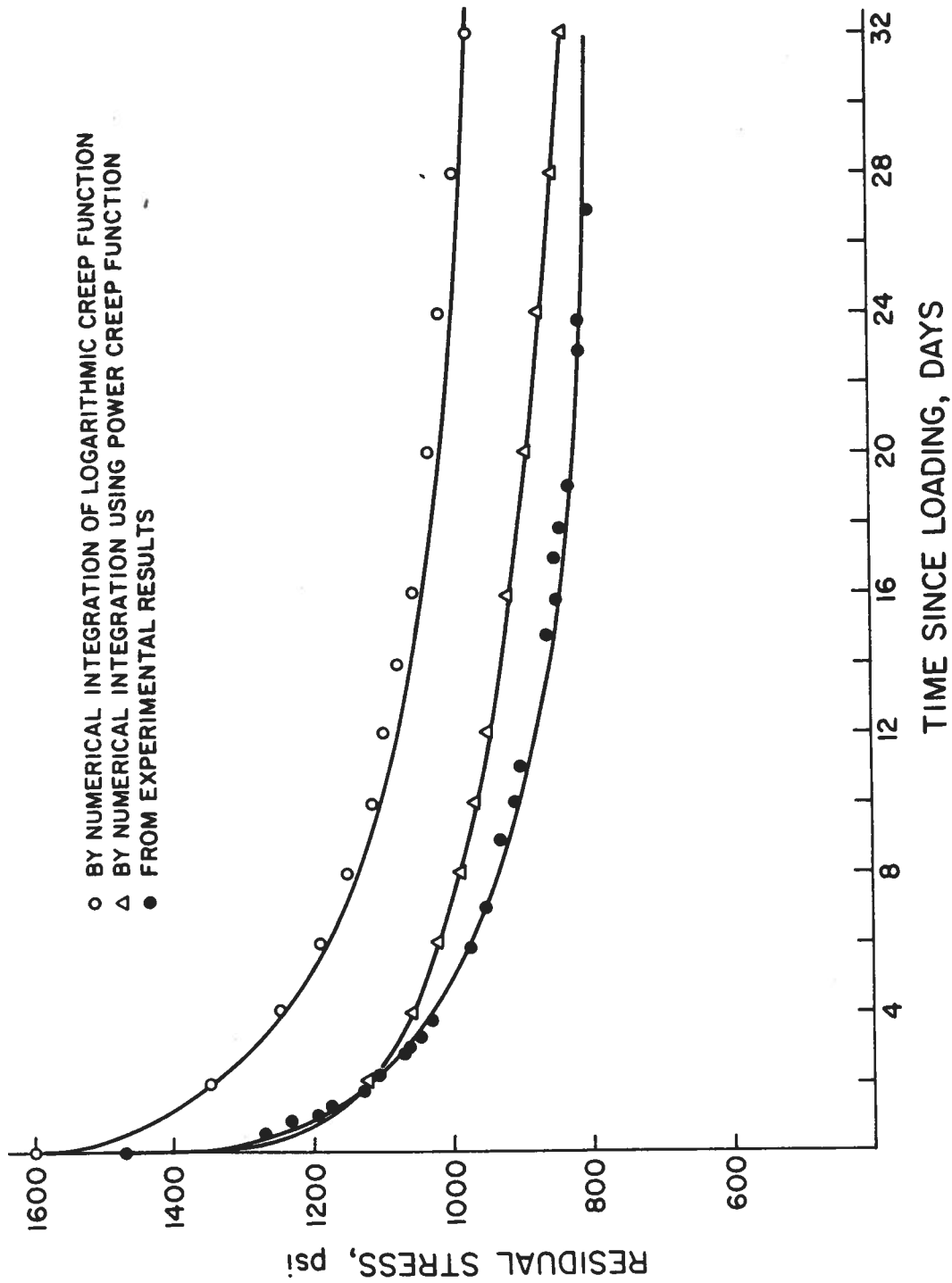


FIG. 28 RESIDUAL STRESS AGAINST TIME SINCE LOADING SHOWING RELATIVE EFFICIENCIES OF LOGARITHMIC AND POWER CREEP FUNCTIONS--SPECIMEN R1 (STRESS/STRENGTH RATIO: 0.31, AGE OF LOADING: 14 DAYS)

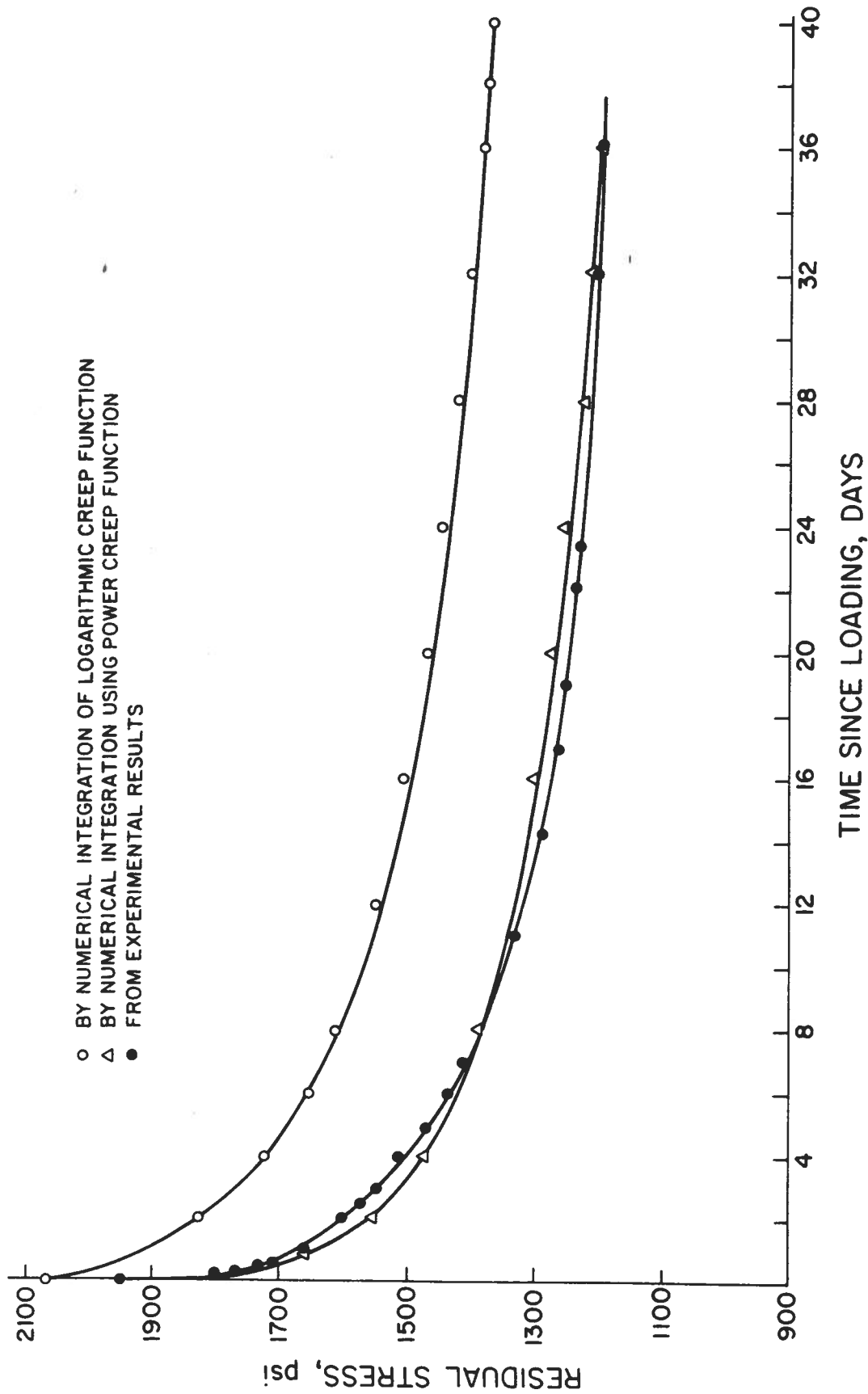


FIG. 29 RESIDUAL STRESS AGAINST TIME SINCE LOADING SHOWING RELATIVE EFFICIENCIES OF LOGARITHMIC AND POWER CREEP FUNCTIONS--SPECIMEN R2 (STRESS/STRENGTH RATIO: 0.31, AGE AT LOADING: 28 DAYS (MIX 1))

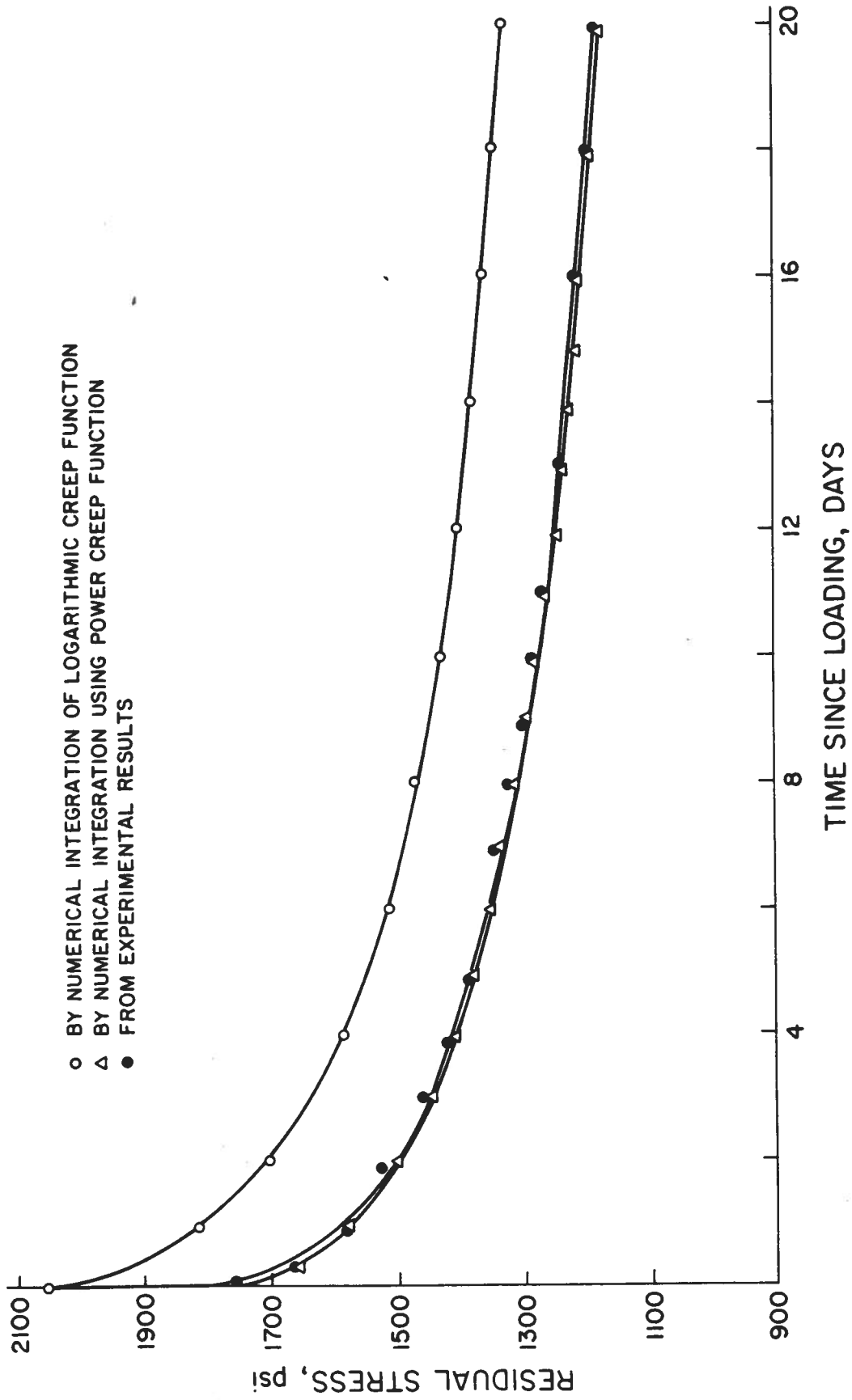


FIG. 30 RESIDUAL STRESS AGAINST TIME SINCE LOADING SHOWING RELATIVE EFFICIENCIES OF LOGARITHMIC AND POWER CREEP FUNCTIONS--SPECIMEN R3 (STRESS/STRENGTH RATIO: 0.36, AGE AT LOADING: 28 DAYS (MIX 2))

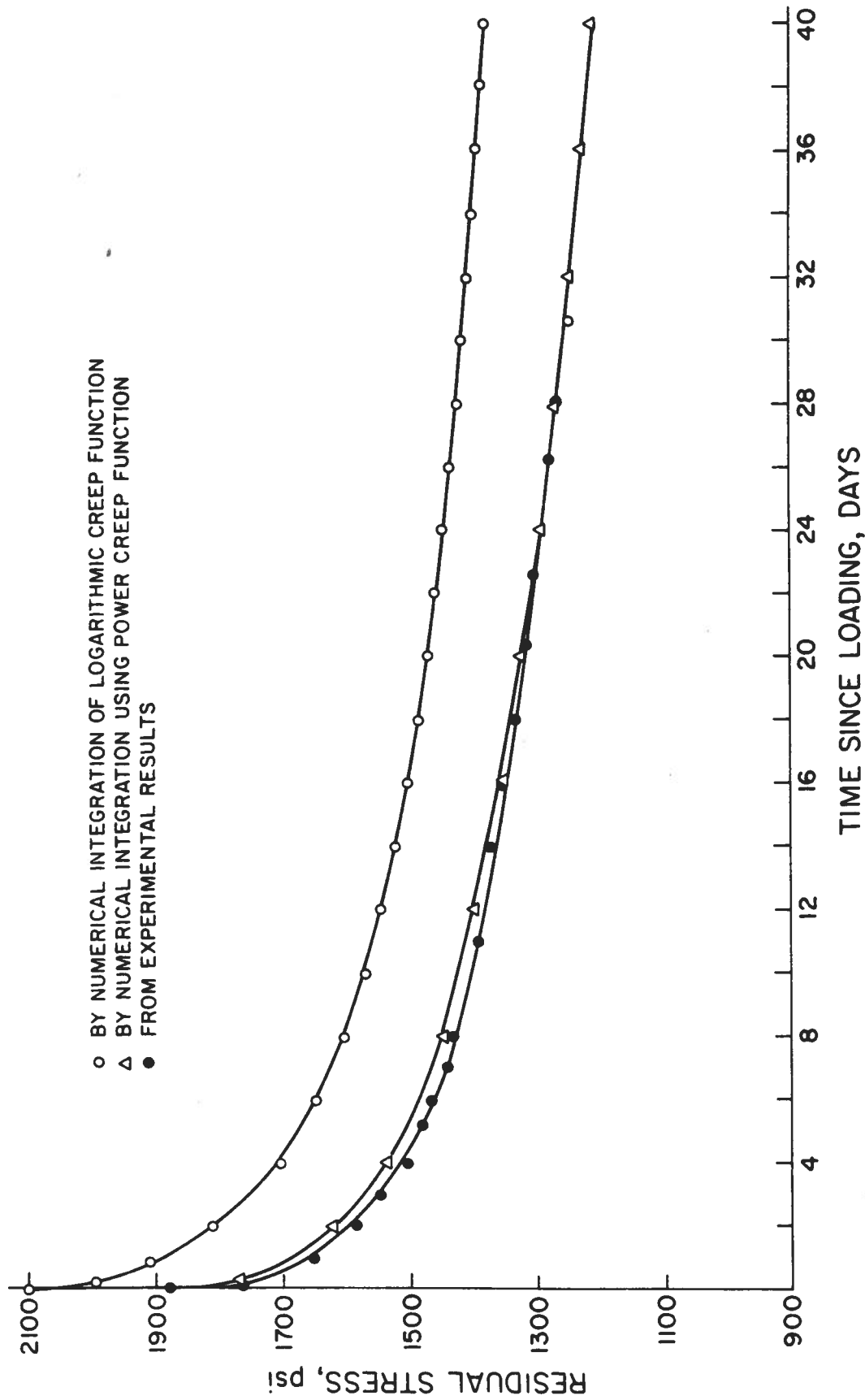


FIG. 31 RESIDUAL STRESS AGAINST TIME SINCE LOADING SHOWING RELATIVE EFFICIENCIES OF LOGARITHMIC AND POWER CREEP FUNCTIONS--SPECIMEN R4 (STRESS/STRENGTH RATIO: 0.36, AGE AT LOADING: 60 DAYS)

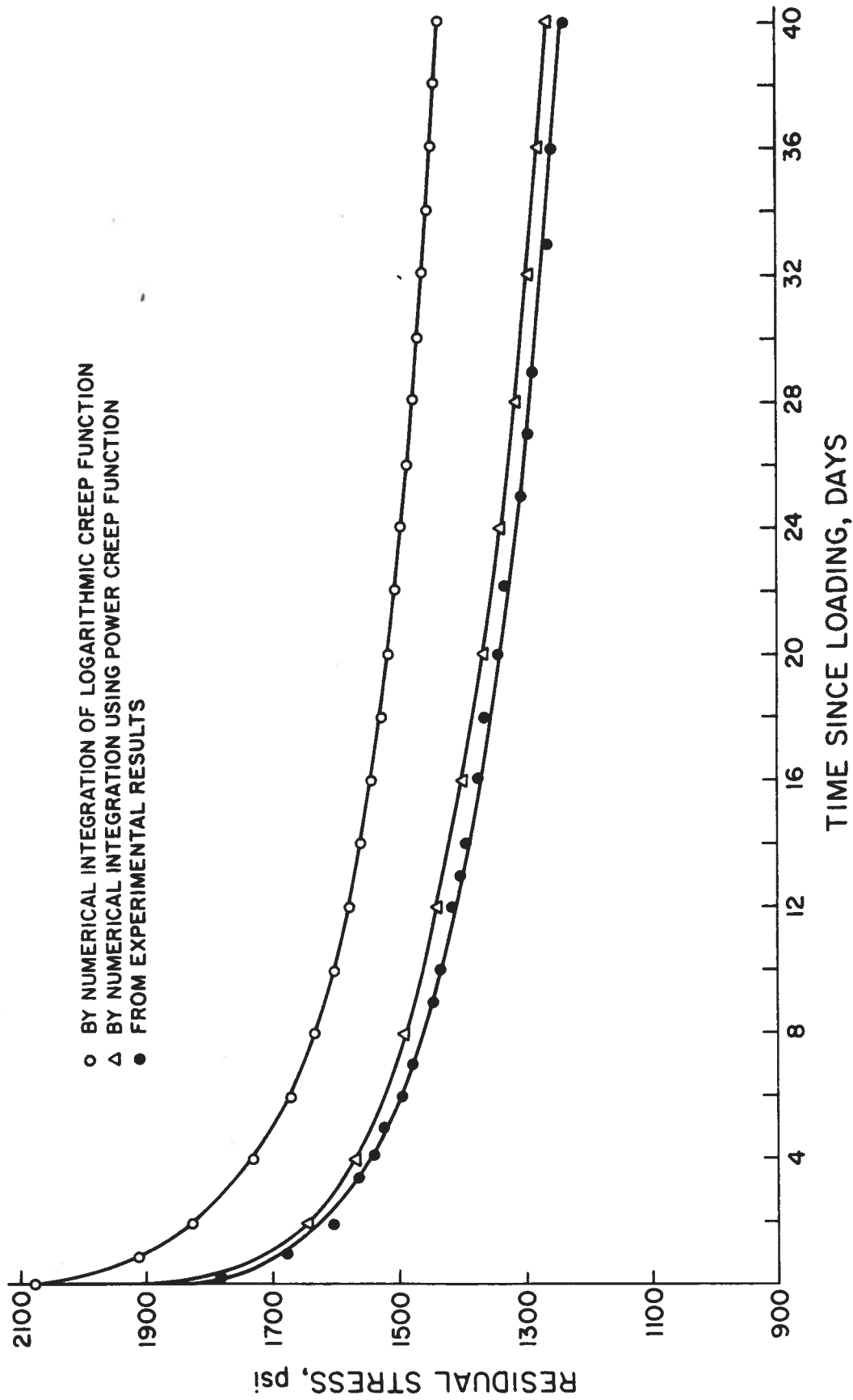


FIG. 32 RESIDUAL STRESS AGAINST TIME SINCE LOADING SHOWING RELATIVE EFFICIENCIES OF LOGARITHMIC AND POWER CREEP FUNCTIONS--SPECIMEN R5 (STRESS/STRENGTH RATIO: 0.34, AGE AT LOADING: 90 DAYS)

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